mitp Mainz Institute for Theoretical Physics

MPA MAINZ PHYSICS ACADEMY

Cluster of Excellence PRISMA+ Precision Physics, Fundamental Interactions and Structure of Matter

Flavour Probes of Axion-like Particles

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HET Seminar, BNL, Feb. 10, 2023

Based on work with M.Bauer, M.Neubert,

S.Renner, A.Thamm

arXiv: 1908.00008, 2012.12272,

2102.13112, **2110.20698**

JG|U

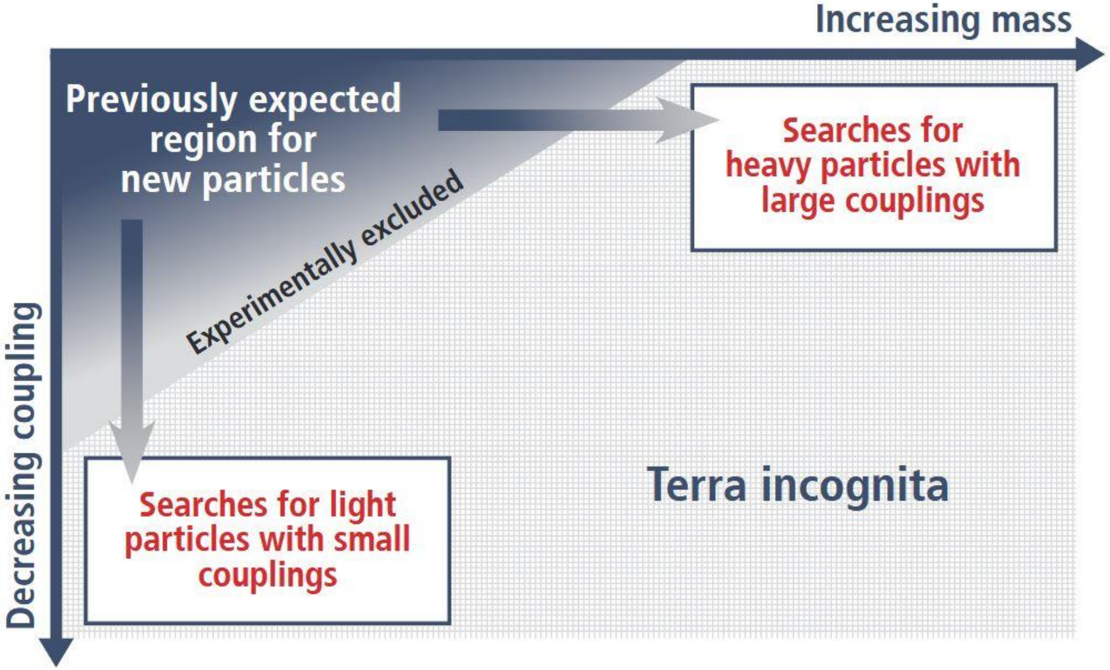
OUTLINE

- Motivation and Introduction
- From the EFT to Observables
- Quark Flavour Probes of ALPs
- ALPs and low-energy anomalies
- If time allows: Lepton Flavour Probes

MOTIVATION

- Plenty of (in)direct hints for new physics, e.g. ν-oscillations, $(g\text{-}2)_\mu$, dark matter…

- Light and weakly coupled particles provide an interesting alternative to heavy new physics



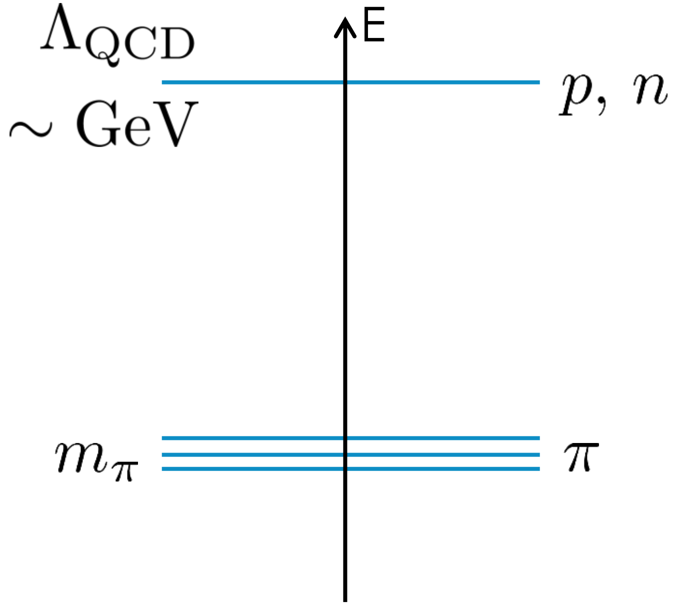
- Many explicit models for ALPs: QCD axion, flavon, familon, comp. Higgs

[Peccei, Quinn (1977)] [Alanne, Blasi, Goertz(2019)] [Gherghetta, Nguyen (2020)]

WHY AXION-LIKE PARTICLES (ALPS)?

Any dynamics with a spontaneously broken (approximate) global symmetry will produce light spinless particles (Goldstone theorem)

Analogy: Pions in QCD



Pions are pNGBs of approximate chiral Symmetry

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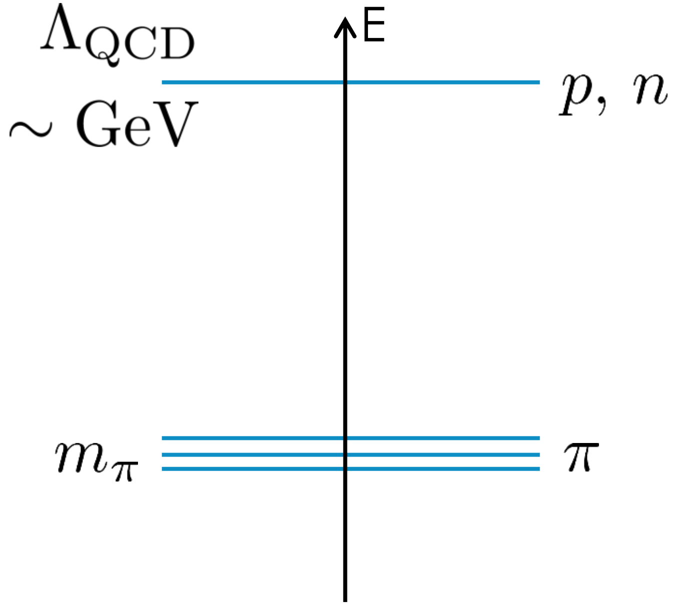
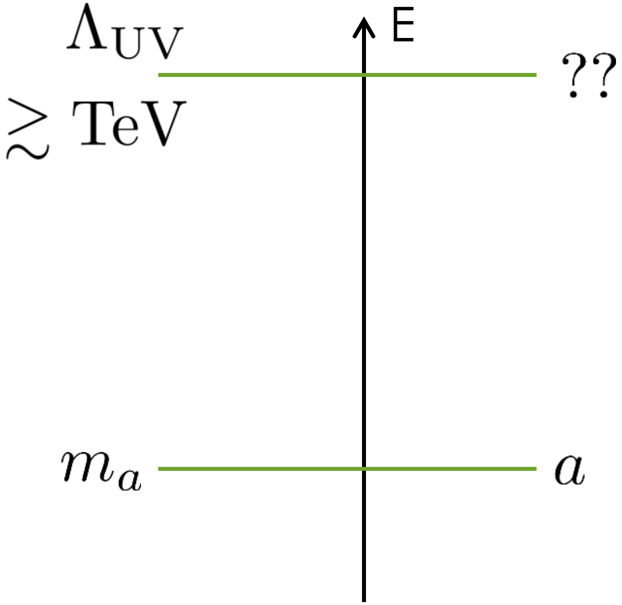


Pions are pNGBs of approximate chiral Symmetry

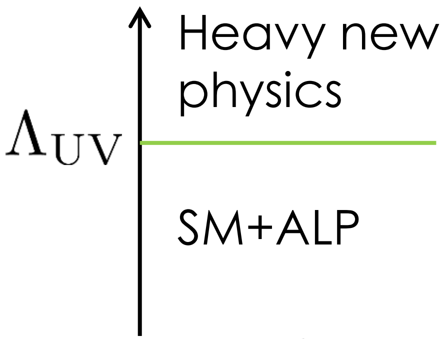
BSM physics



ALP is a pNGB coming from the spontaneous breaking of a global U(1) symmetry

THE ALP LAGRANGIAN

- Don't need to know the details of the UV physics to study ALPs



- ALP is pseudoscalar
- Invariant under shift symmetry $a \to a + c$
- Softly broken by mass term

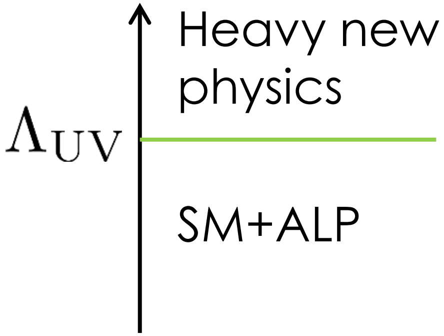
➢ $$\mathcal{L}_{\rm eff}^{D\leq 5} = \frac{1}{2}\left(\partial_\mu a\right)\left(\partial^\mu a\right) - \frac{1}{2} m_a^2 a^2 + \frac{\partial_\mu a}{f}\sum_F \bar{\psi}_F \mathbf{c}_F \gamma^\mu \psi_F$$
$$+ c_{GG}\frac{\alpha_s}{4\pi}\frac{a}{f} G^a_{\mu\nu}\tilde{G}^{\mu\nu,a} + c_{WW}\frac{\alpha_2}{4\pi}\frac{a}{f} W^A_{\mu\nu}\tilde{W}^{\mu\nu,A} + c_{BB}\frac{\alpha_1}{4\pi}\frac{a}{f} B_{\mu\nu}\tilde{B}^{\mu\nu}$$

[Georgi, Kaplan, Randall (1986)]

- Parameter space depends on $m_a, f, \mathbf{c}_F, c_{VV}$

THE ALP LAGRANGIAN

- Don‘t need to know the details of the UV physics to study ALPs



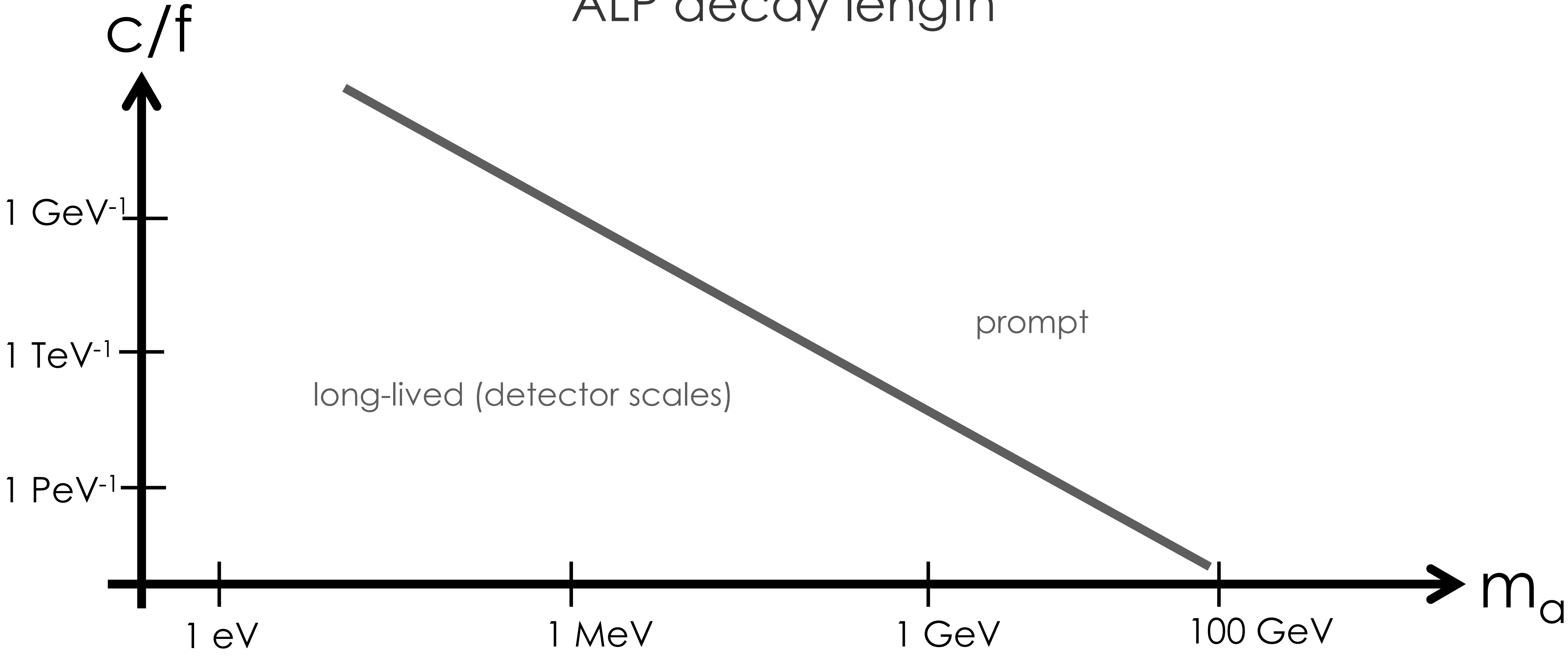
- ALP is pseudoscalar
- Invariant under shift symmetry $a \to a + c$
- Softly broken by mass term

➢ In low-energy effective theory, use alternate form:

$$\mathcal{L}_{\text{eff}}^{D\leq 5}(\mu \leq \mu_W) \supset -\frac{ia}{2f}\sum_f \Big[(m_{f_i} - m_{f_j})[k_f - k_F]_{ij}\,\bar{f}_i f_j \\ + (m_{f_i} + m_{f_j})[k_f + k_F]_{ij}\,\bar{f}_i \gamma_5 f_j\Big]$$

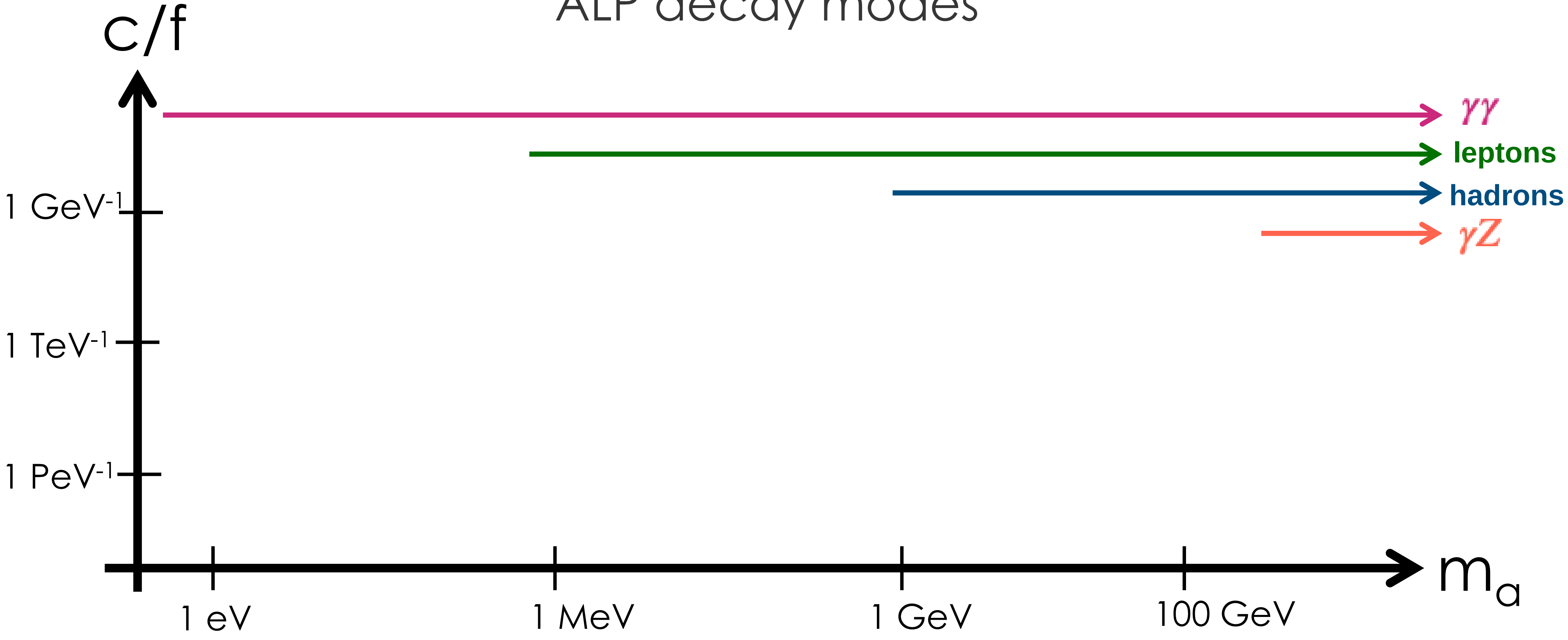
ALP PHENO IN A NUTSHELL

ALP decay length



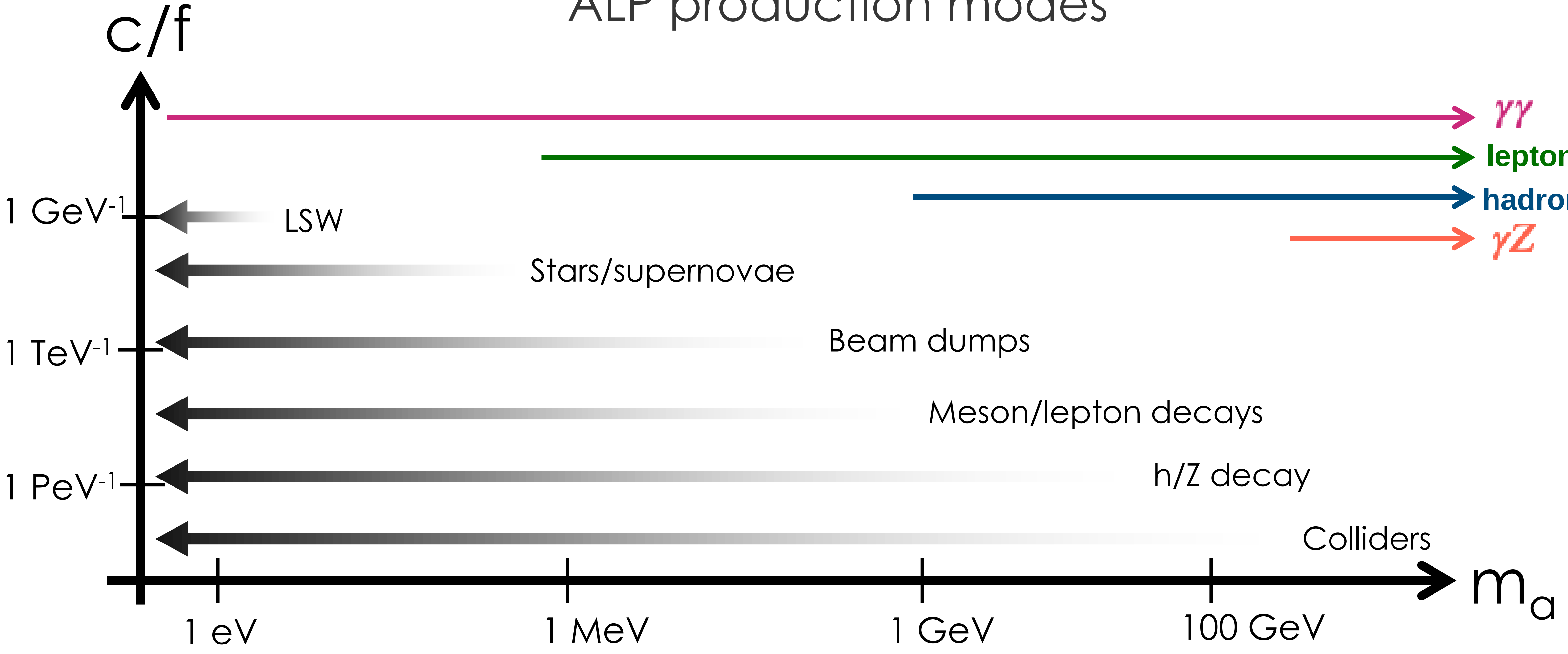
ALP PHENO IN A NUTSHELL

ALP decay modes



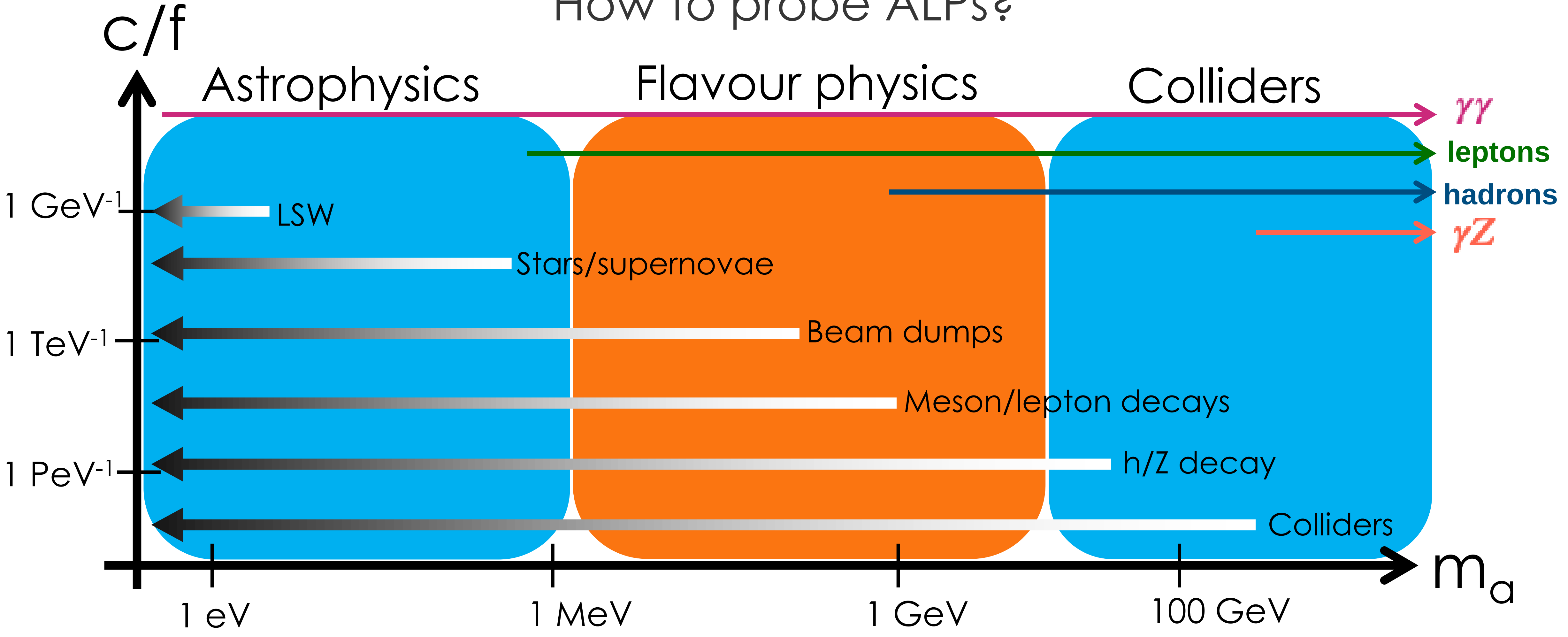
ALP PHENO IN A NUTSHELL

ALP production modes



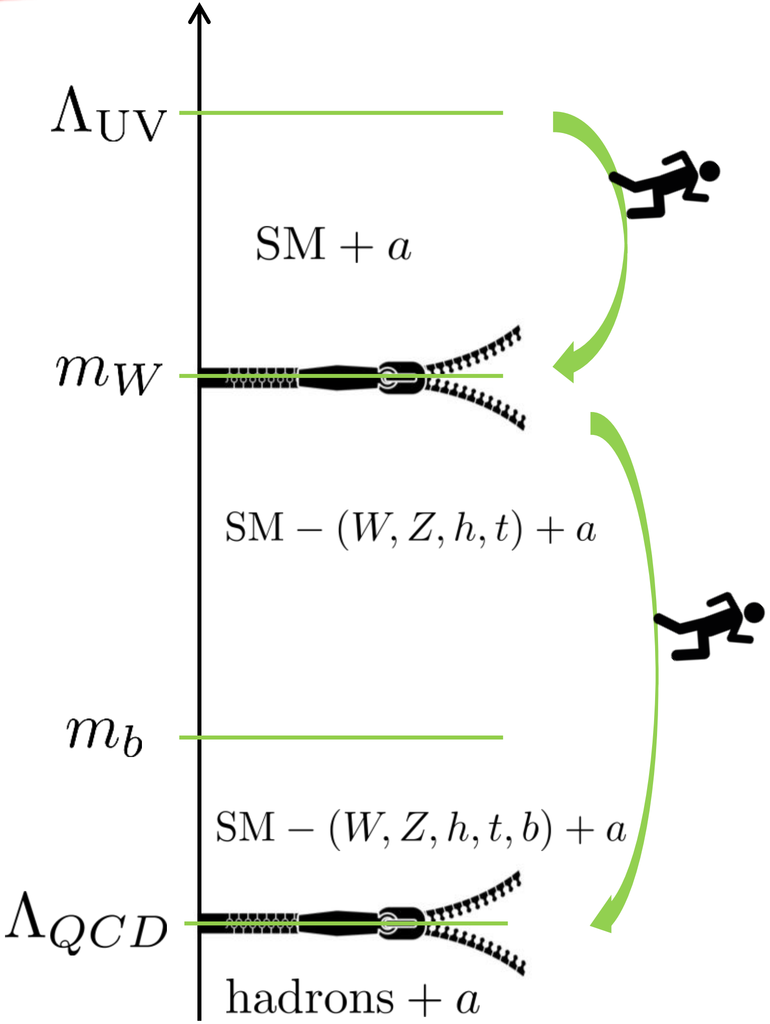
ALP PHENO IN A NUTSHELL

How to probe ALPs?



FROM EFT TO OBSERVABLES

RUNNING FROM UV



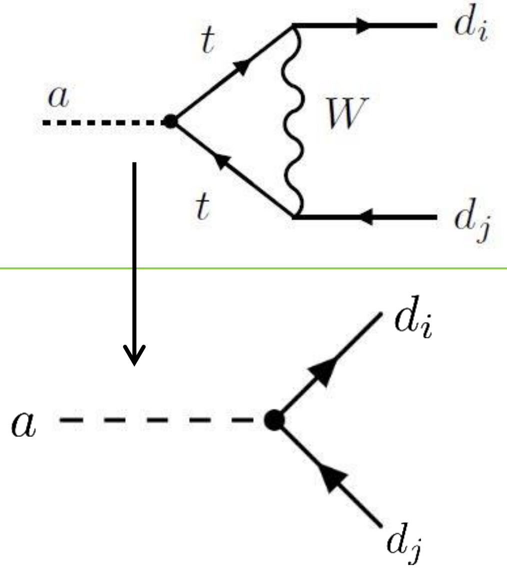
- ALP couplings determined by UV physics
- Connection to observables by running and matching to lower scales

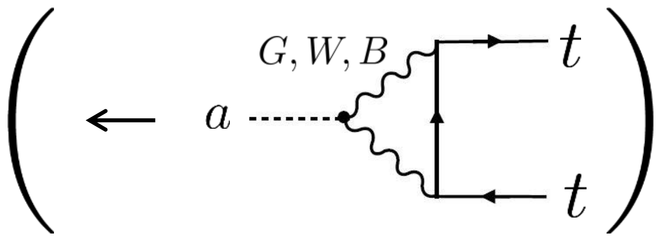
See also [Choi, Im, Park, Yun (2017)
Chala, Guedes, Ramos, Santiago (2020)]

- Flavour pheno: Focus on MeV-GeV mass range (below strong constraints from astro/cosmology)

GENERATING FLAVOUR CHANGE

- Inevitably generate flavour-changing effects:





$\mathbf{c}_F = c_F \mathbb{1}$

||

- Starting with flavour universal coupling at $\Lambda = 4\pi f = 4\pi\,\mathrm{TeV}$

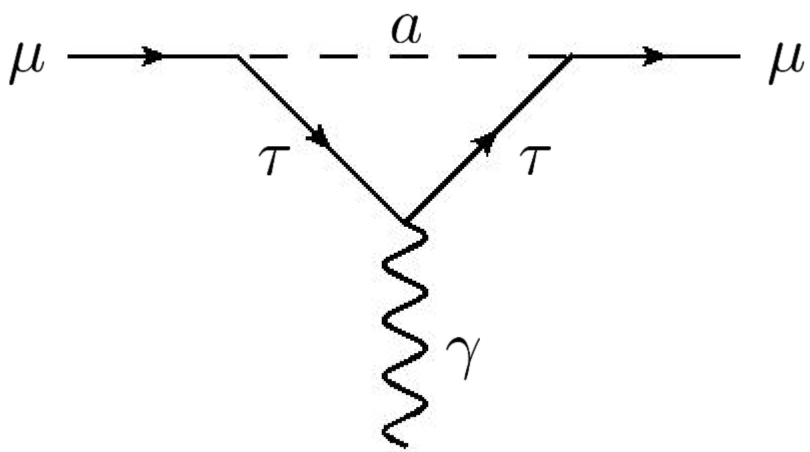
Vector couplings don‘t run

$$[k_D(m_t)]^{\mathrm{univ}}_{ij} \simeq 10^{-5} V^*_{ti} V_{tj} \big[-6.1 c_{GG} - 2.8 c_{WW} - 0.02 c_{BB} + \underline{1.9 \times 10^3} c_u(\Lambda) - 9.2 c_d(\Lambda) - \underline{1.9 \times 10^3} c_Q(\Lambda) - 0.05 c_e(\Lambda) + 4.2 c_L(\Lambda) \big]$$

Largest contribution from tops

WHAT ABOUT LEPTONS?

- SM is Lepton flavour conserving → cannot generate LFV through RGE
- There can still be connections between LFV couplings and flavour-conserving observables: $a - \tau - \mu$ coupling can mediate $(g-2)_\mu$



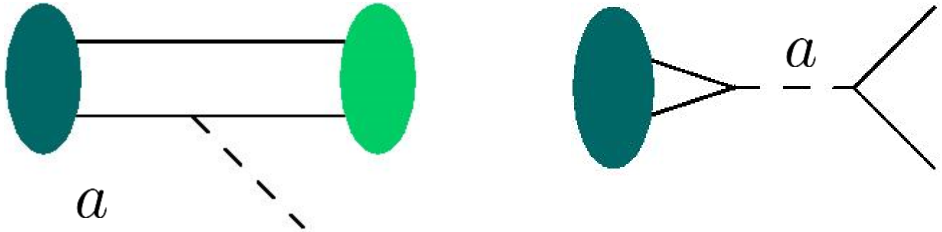
ALPS IN QUARK FLAVOUR PROCESSES

- RGs and matching calculations allow us to
  - Calculate all observables in terms of UV Lagrangian parameters
  - Compare to other constraints from non-flavour experiments
- ALPs can be found in:

Meson decays

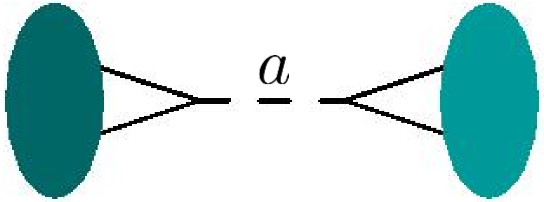
$K \to \pi a,\ B \to K a$ (on-shell)

$B_{(s,d)} \to \mu^+\mu^-$ (off-shell)



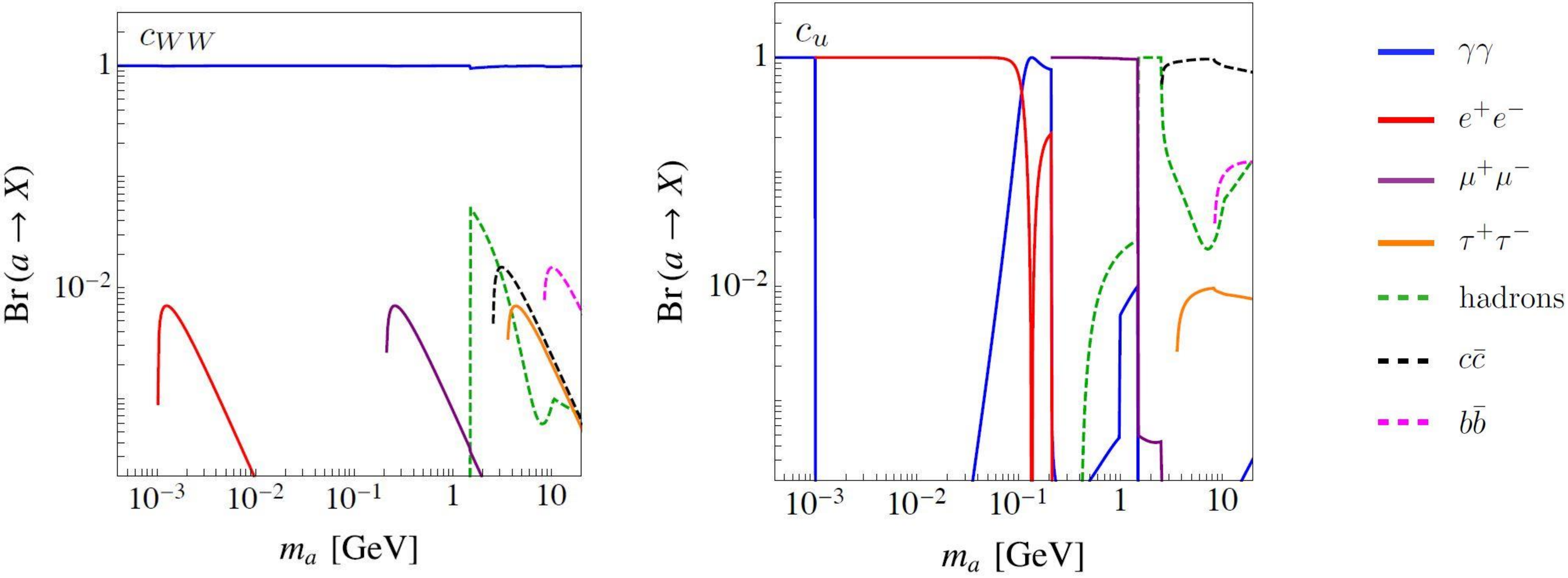
Meson mixing

$B^0_{(s,d)} \leftrightarrow \bar{B}^0_{(s,d)}$



ALP PHENO - EXAMPLE

- Branching ratios of ALPs for two different UV couplings

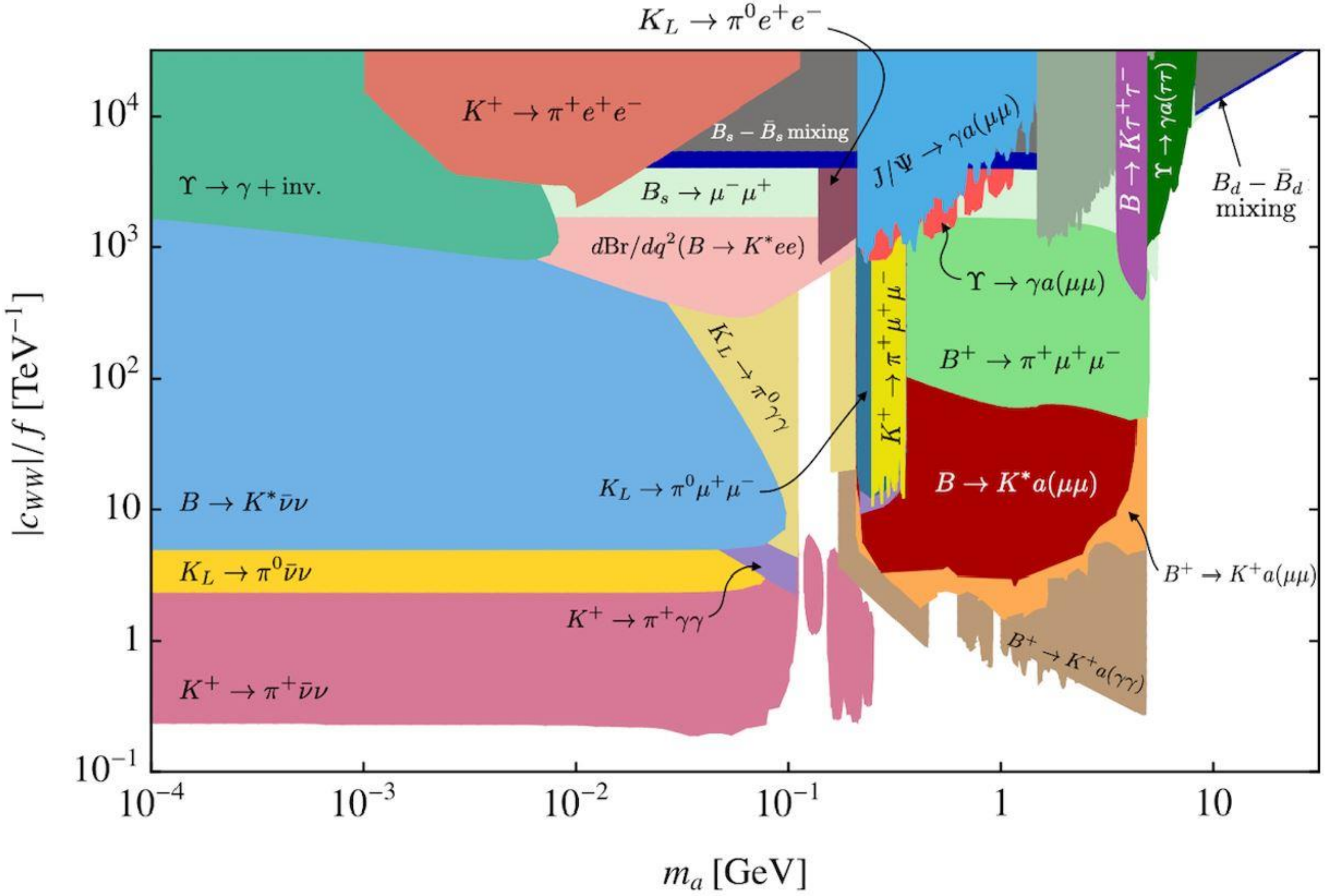


QUARK FLAVOUR PROBES OF ALPS

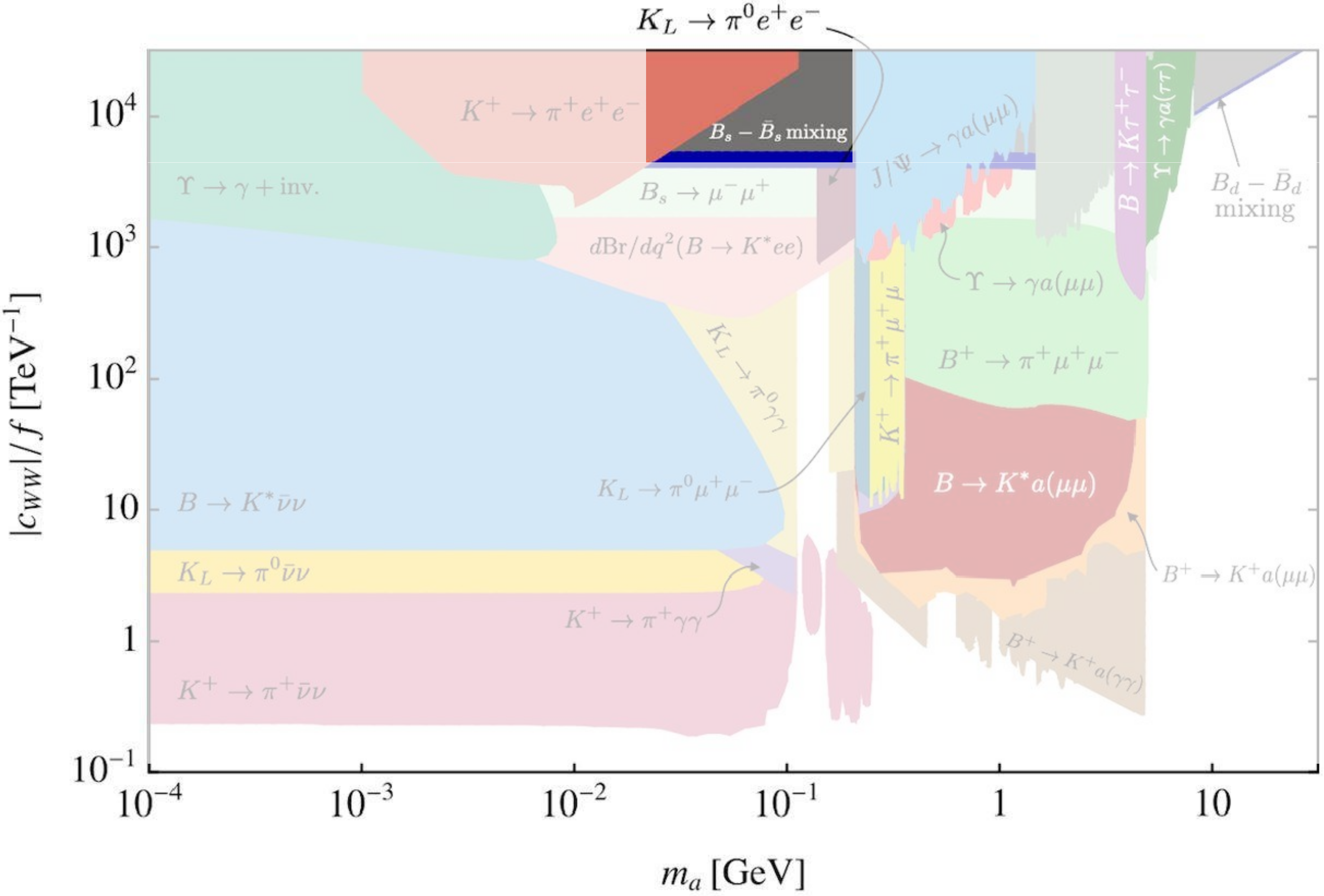
GENERAL REMARKS

- Assume one dominant flavourless or flavour-universal coupling in the UV with $\Lambda = 4\pi f,\ f = 1\mathrm{TeV}$ (scale dependence of induced couplings is small)
- Generate other couplings through RG running
- Effective branching ratios often depend on experimental cuts (e.g. time of flight, energy…) or the ALP decay length (in subsequent decays)

CONSTRAINTS FROM SU(2)$_L$ COUPLING



CONSTRAINTS FROM $SU(2)_L$ COUPLING



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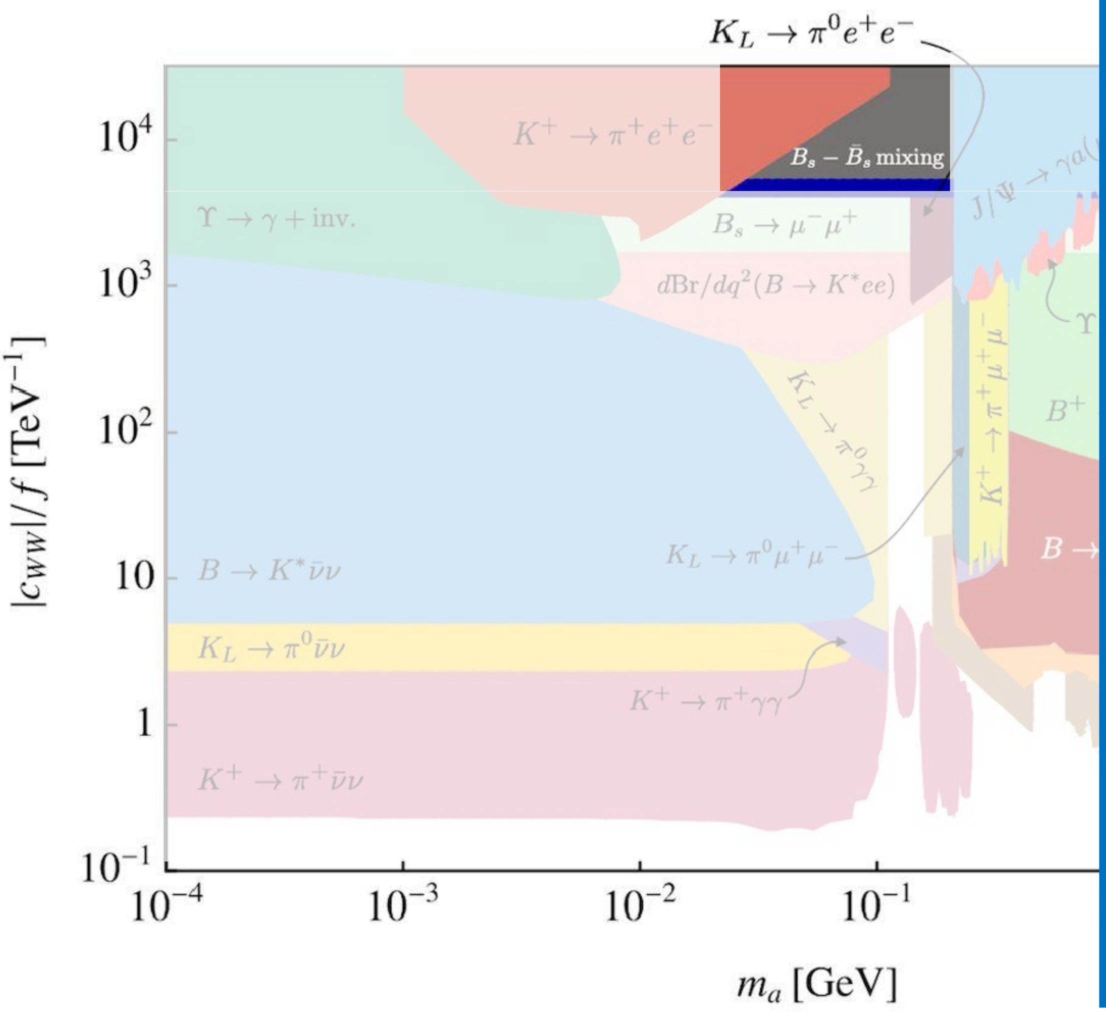
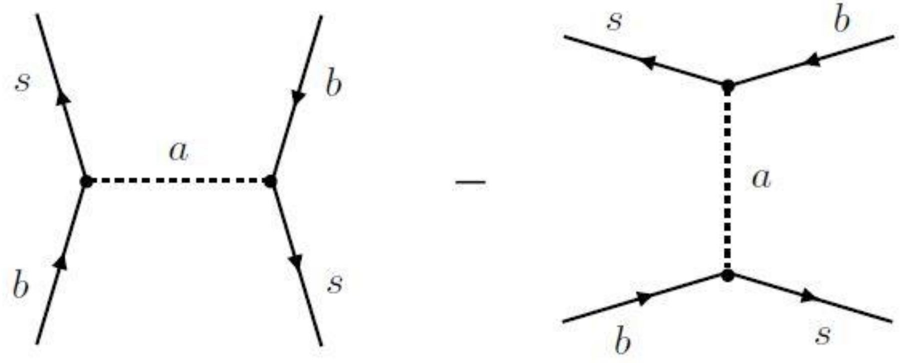


$$B_s - \bar{B}_s - \text{mixing}$$

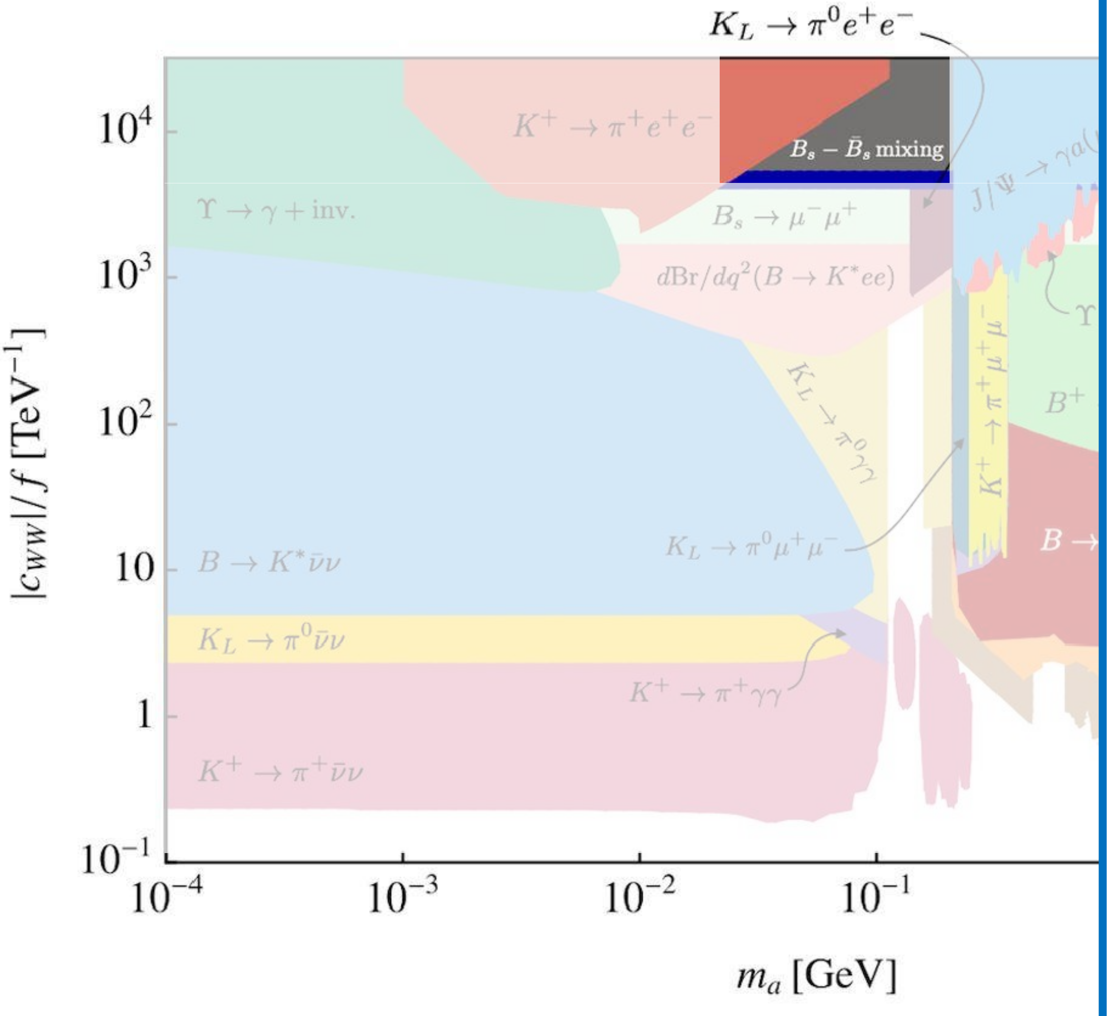


- Measurement is in small tension with SM prediction from weighted average (HQET, sum rules) and lattice (FLAG) of order ~1σ [HFLAV (2016)]

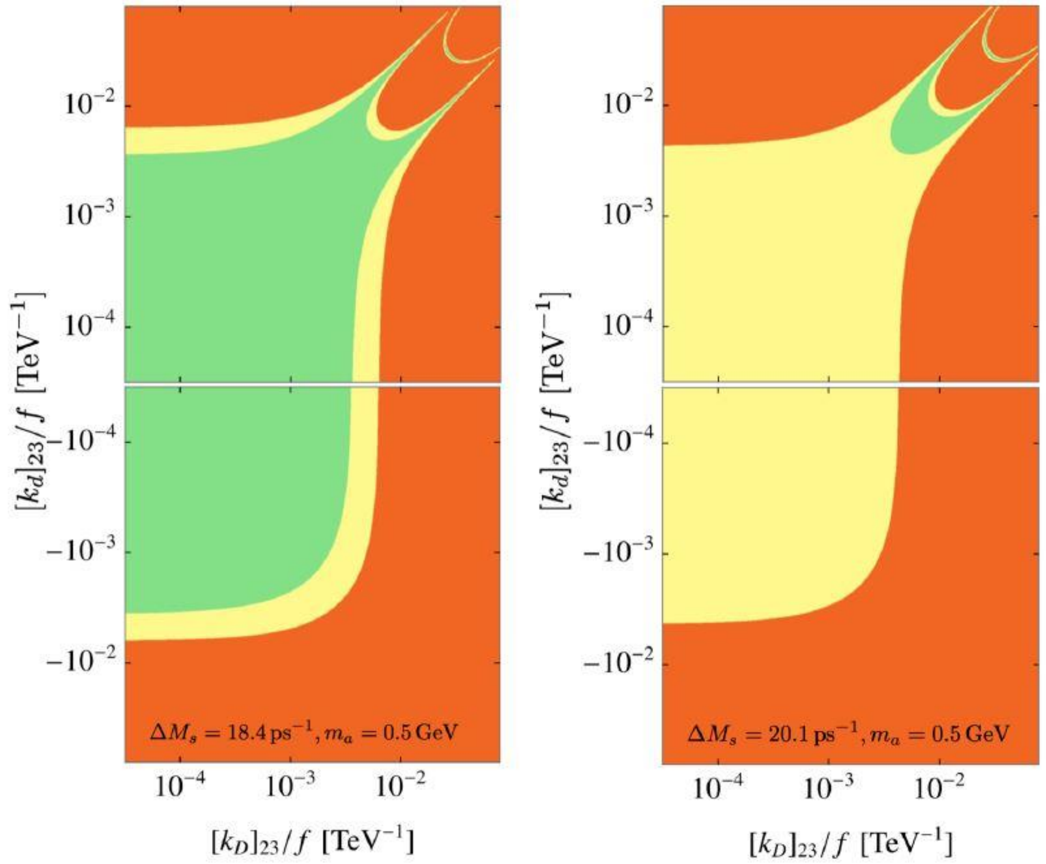
➢ ALP can explain tension but must not overshoot

➢ Can get constraints on ALP

CONSTRAINTS FROM SU(2)$_L$ COUPLING



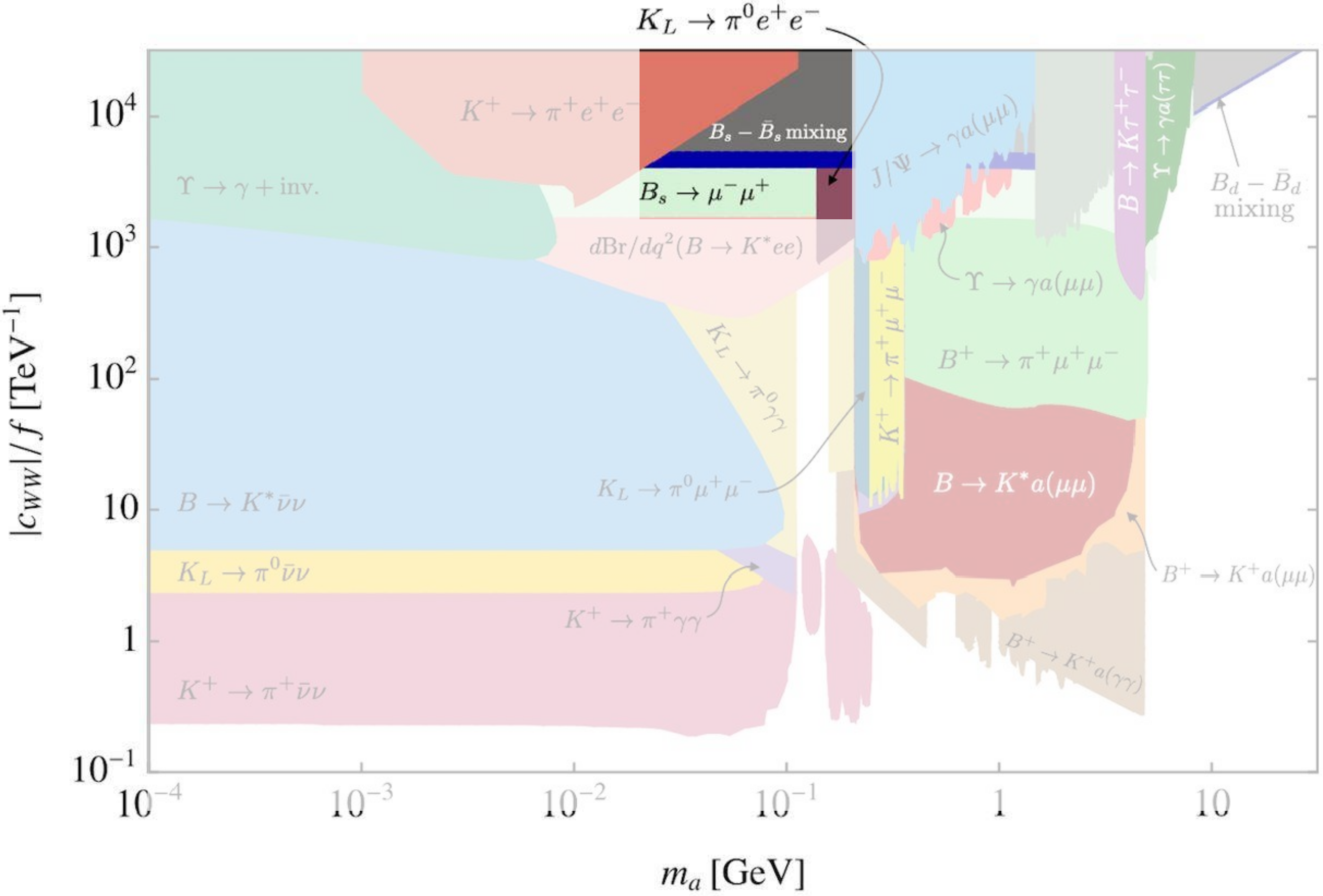
$B_s - \bar{B}_s - \text{mixing}$



weighted average

FLAG 2019

CONSTRAINTS FROM SU(2)$_L$ COUPLING



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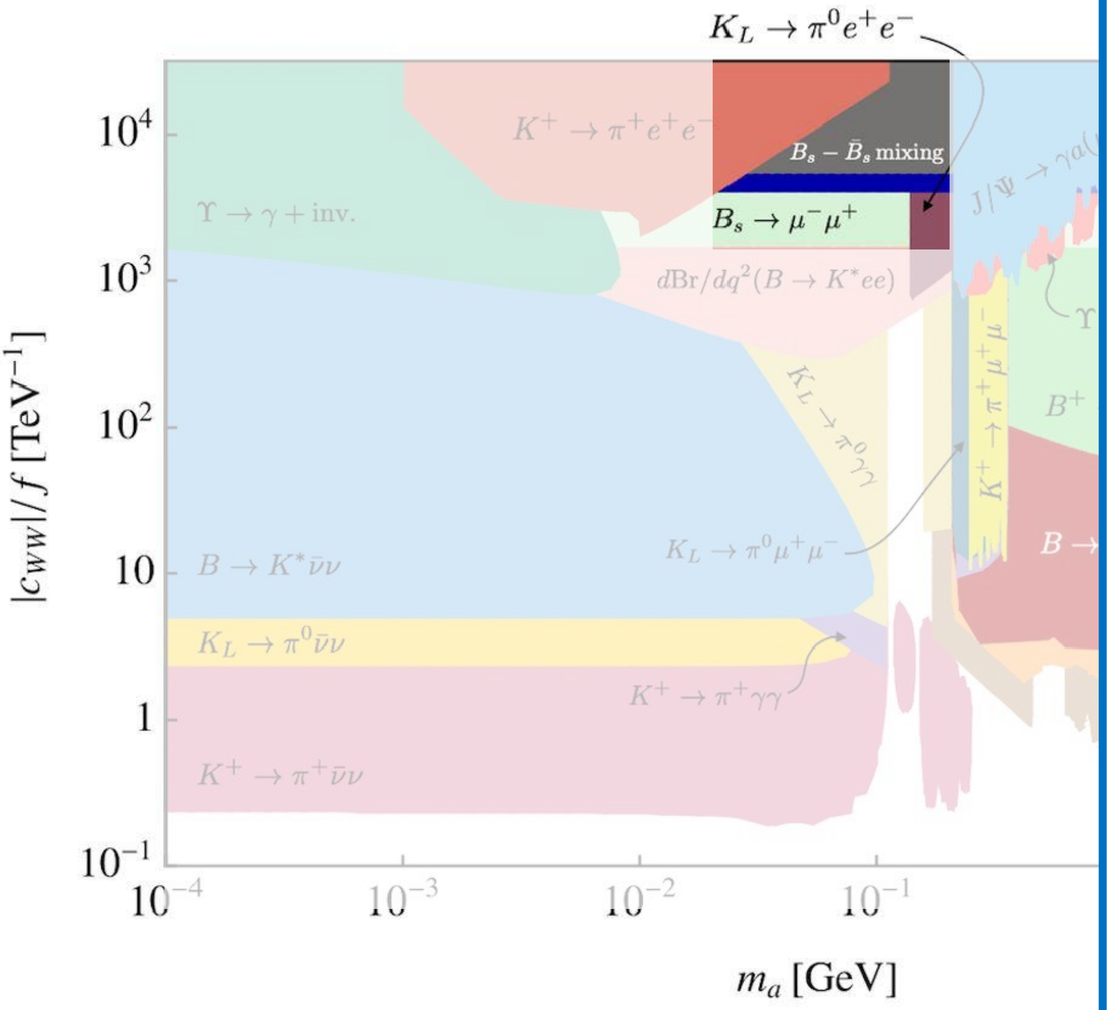
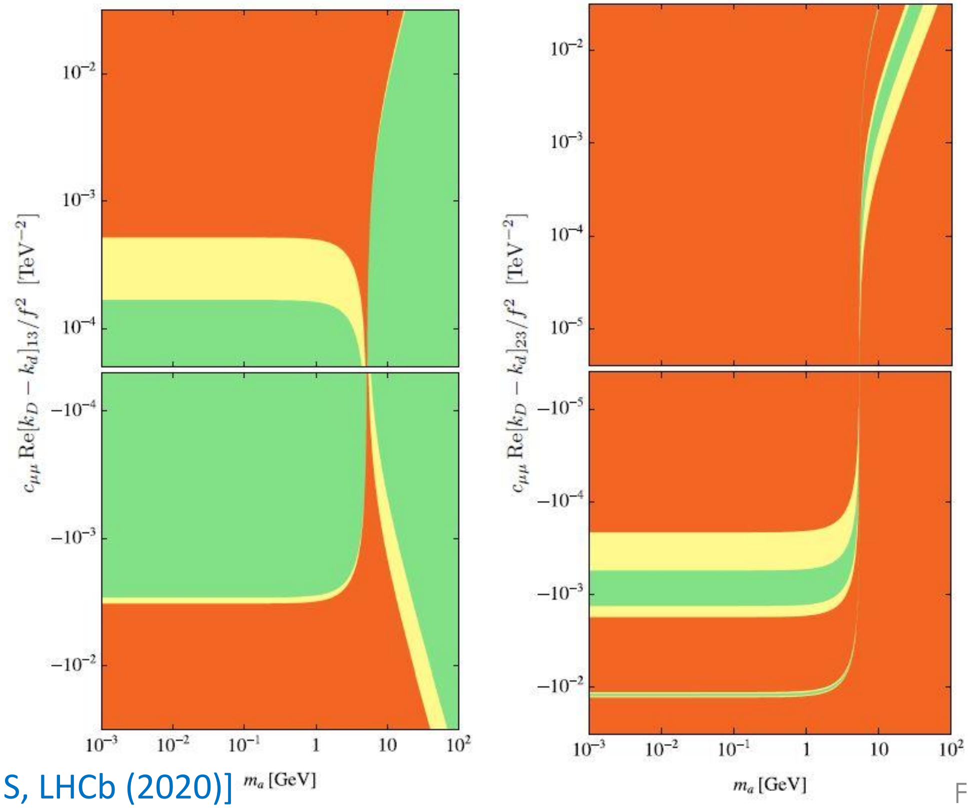


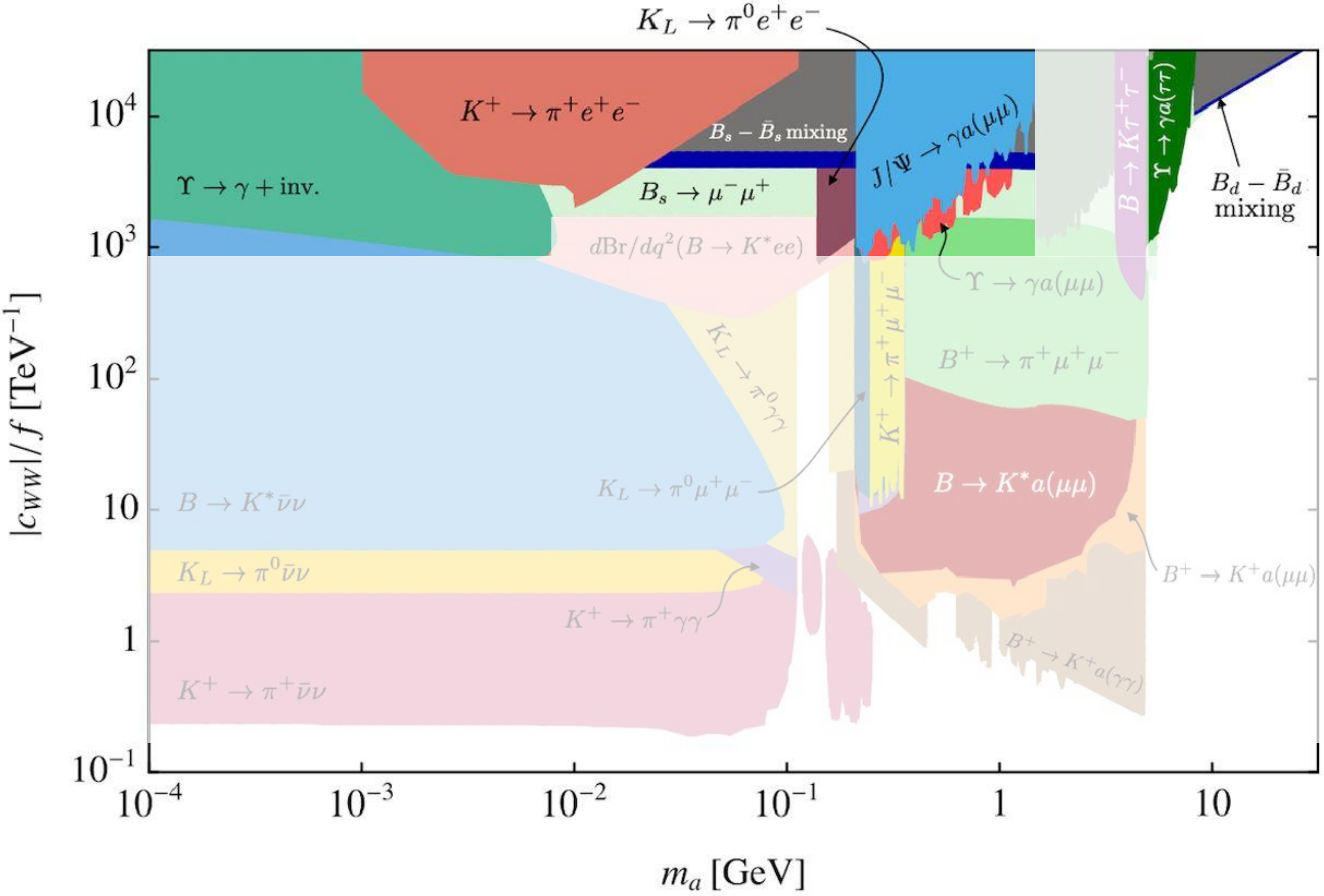
$$B_s \to \mu^+ \mu^-$$

- Measurements differ from SM prediction by 0.64σ (B_d) and 2.4σ (B_s)



[ATLAS, CMS, LHCb (2020)]

CONSTRAINTS FROM SU(2)$_L$ COUPLING



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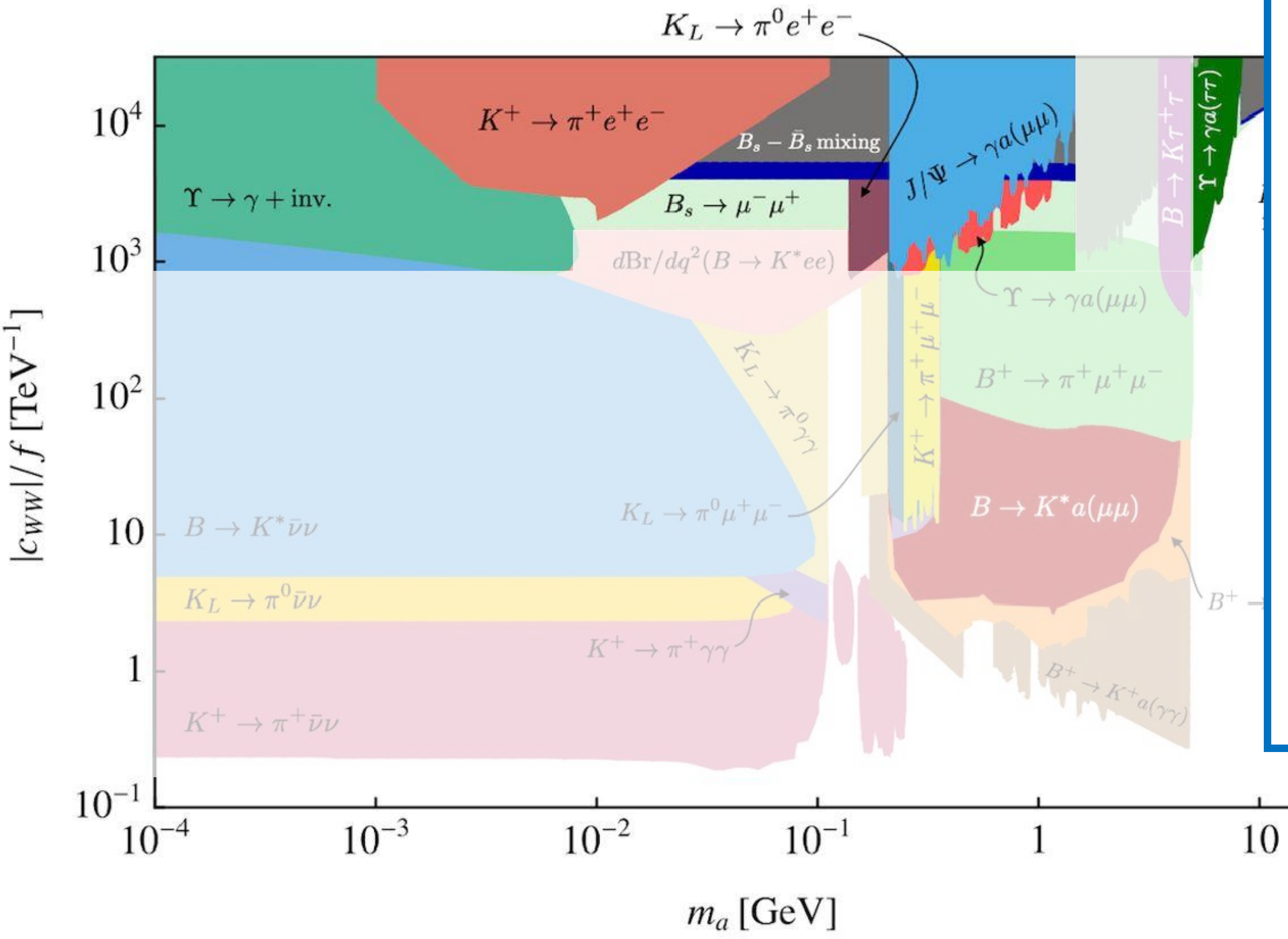
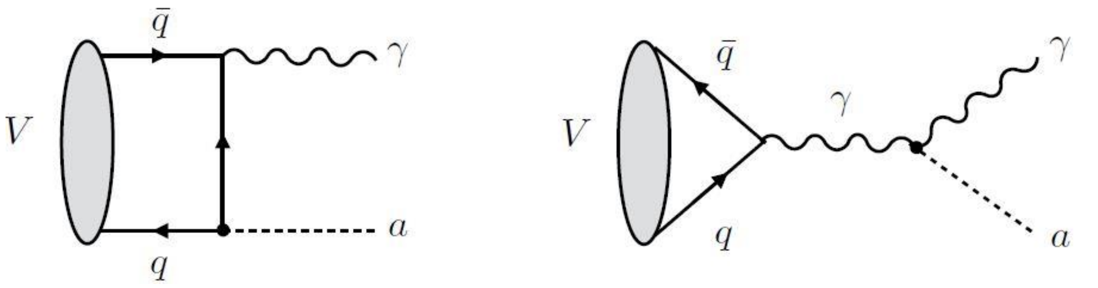


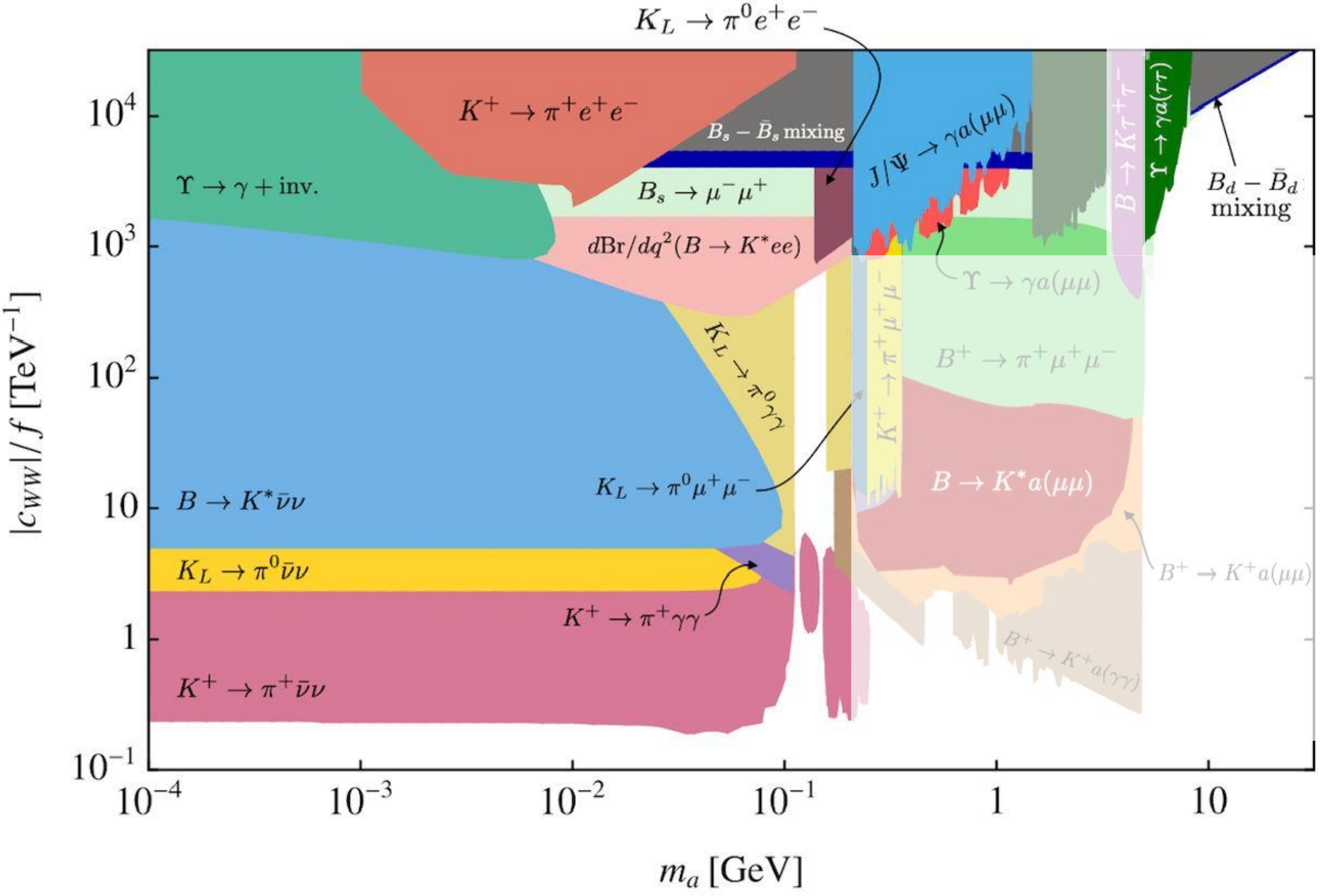
Radiative J/ψ and Υ decays



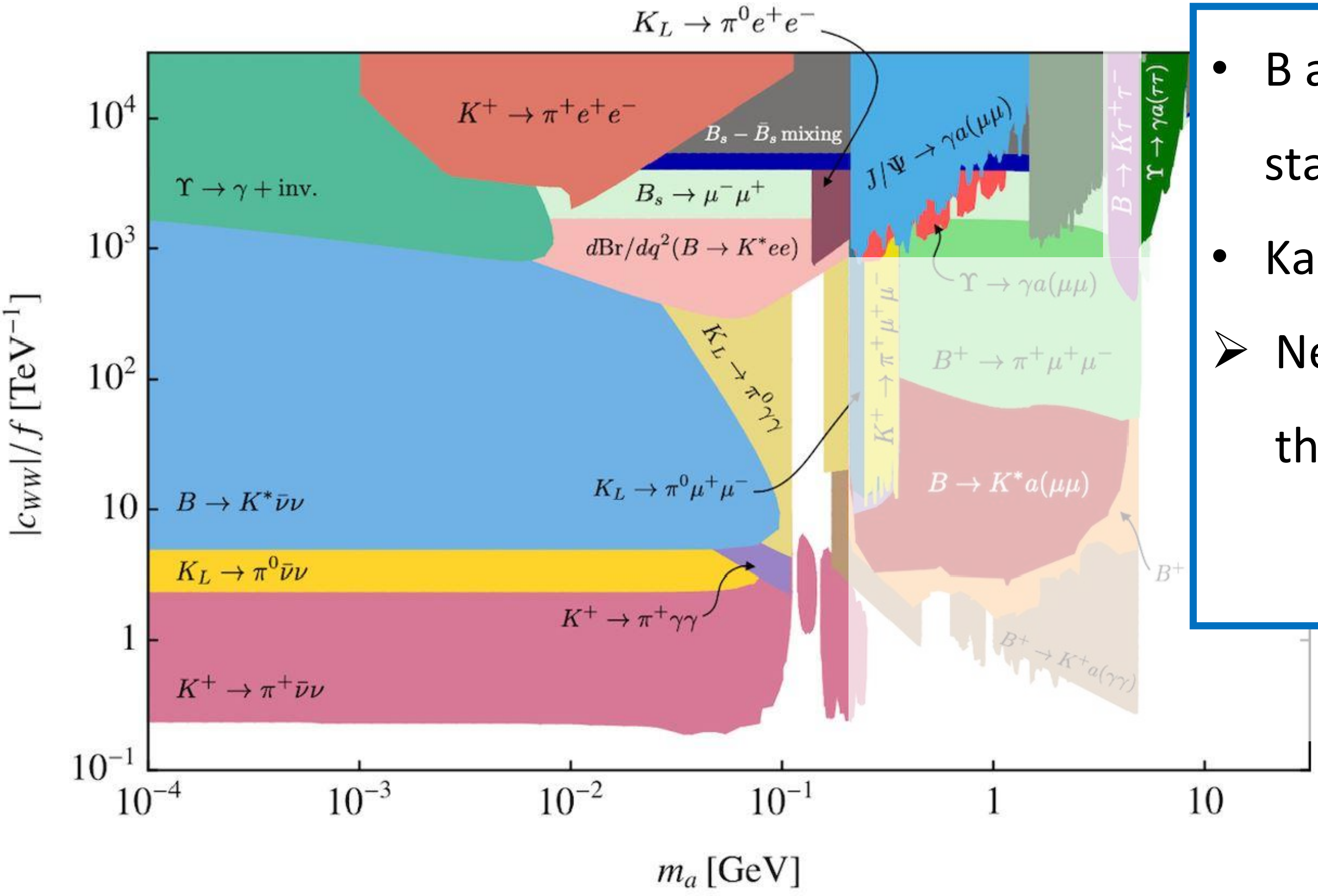
- Searches done in di-muon channel for J/ψ and di-muon, di-tau and hadronic for Υ

[BaBar(2011, 2012), BESIII (2015)]

CONSTRAINTS FROM SU(2)$_L$ COUPLING



CONSTRAINTS FROM $SU(2)_L$ COUPLING



- B and K decays with invisible or photon final states give strongest constraints for $m_a \lesssim m_K$
- Kaon decays studied within ALP + χPT

➢ Need to consistently implement the ALP in the weak chiral Lagrangian

THE WEAK DECAY K→ΠA

- Strongest particle-physics constraint on ALP couplings for masses below

$$m_a < m_K - m_\pi \approx 354\mathrm{MeV}$$

- Despite a 35 year history, we find that most recent works are based on inconsistent equations [Georgi, Kaplan, Randall (1986)]

- Chiral implementation of leading SU(3) octet weak interaction operator

$$\mathcal{L}_{\mathrm{weak}} = \frac{-4G_F}{\sqrt{2}} V_{ud}^* V_{us} g_8 \left[L_\mu L^\mu\right]^{32}$$

Experimental constant

s-d-transition

with L_μ^{ij} the chiral representation of left-handed current $\bar{q}_L^i \gamma_\mu q_L^j$

[Bernhard, Draper, Soni, Politzer, Weis (1985); Crewther (1986); Kambor, Missimer, Wyler (1990)]

THE WEAK DECAY K→ΠA

- Georgi, Kaplan, Randall used $L_\mu^{ij} = -\frac{if_\pi^2}{4} e^{-i(\kappa_{q_j}-\kappa_{q_i})c_{GG}a/f}\left[\boldsymbol{\Sigma}\partial_\mu\boldsymbol{\Sigma}^\dagger\right]^{ij}$

$$\boldsymbol{\Sigma}(x) = \exp\left[\frac{i\sqrt{2}}{f_\pi}\lambda^a\pi^a(x)\right]$$

Phase used in chiral rotation to eliminate ALP-gluon coupling, involves auxiliary parameters κ_q

- The Noether theorem gives instead:

$$L_\mu^{ji} = -\frac{if_\pi^2}{4} e^{i\left(\kappa_{q_j}-\kappa_{q_i}\right)c_{GG}\frac{a}{f}}\left[\boldsymbol{\Sigma}\,\left(\boldsymbol{D}_\mu\boldsymbol{\Sigma}\right)^\dagger\right]_{ji}$$

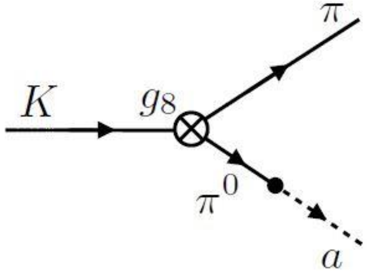
$$\ni -\frac{if_\pi^2}{4}\left[1+i\left(\kappa_{q_j}-\kappa_{q_i}\right)c_{GG}\frac{a}{f}\right]\left[\boldsymbol{\Sigma}\,\partial_\mu\boldsymbol{\Sigma}^\dagger\right]_{ji} + \frac{f_\pi^2}{4}\frac{\partial^\mu a}{f}\left[\hat{\boldsymbol{k}}_Q - \boldsymbol{\Sigma}\,\hat{\boldsymbol{k}}_q\boldsymbol{\Sigma}^\dagger\right]_{ji}$$

Contains also derivative ALP interactions

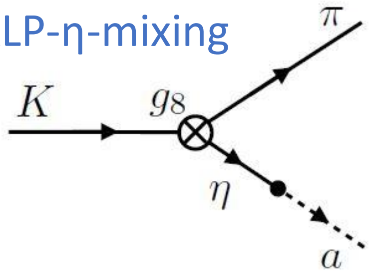
Extra terms from Noether-procedure!

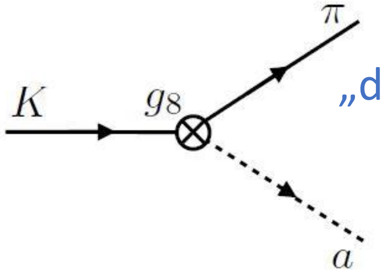
THE WEAK DECAY K→ΠA

ALP-π-mixing



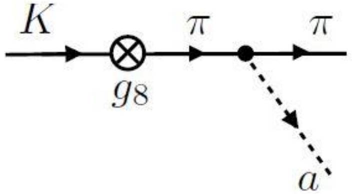
ALP-η-mixing

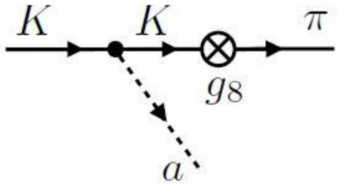




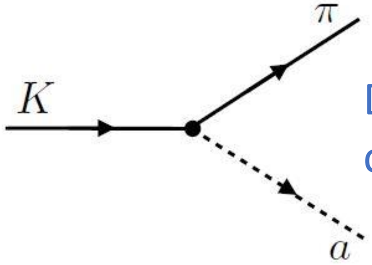
„direct“ contribution

Final state radiation





Initial state radiation



Direct flavour changing contribution

- Only in the sum of all diagrams the auxiliary parameters cancel
- Including only the first diagrams (kinetic mixing) gives in general an uncontrolled approximation (except for special cases)

THE WEAK DECAY K→ΠA

- Decay amplitude

$$
\begin{aligned}
i\mathcal{A}_{K^-\to\pi^- a} = \frac{N_8}{4f}\Bigg[& 16c_{GG}\frac{(m_K^2-m_\pi^2)(m_K^2-m_a^2)}{4m_K^2-m_\pi^2-3m_a^2} \\
& + 6(c_{uu}+c_{dd}-2c_{ss})m_a^2\frac{m_K^2-m_a^2}{4m_K^2-m_\pi^2-3m_a^2} \\
& + (2c_{uu}+c_{dd}+c_{ss})(m_K^2-m_\pi^2-m_a^2)+4c_{ss}m_a^2 \\
& + (k_d+k_D-k_s-k_S)(m_K^2+m_\pi^2-m_a^2)\Bigg] \\
& - \frac{m_K^2-m_\pi^2}{2f}[k_q+k_Q]^{23}.
\end{aligned}
$$

UV flavour changing coupling

Previously used:

$$\mathcal{A}_{K^-\to\pi^- a} \approx \frac{iN_8 m_K^2}{4f_a}\frac{m_u}{m_u+m_d}.$$

underestimates amplitude by

factor $\frac{m_u}{m_u+m_d} \approx 0.16$

leading to factor ≈40 in BR

THE WEAK DECAY K→ΠA

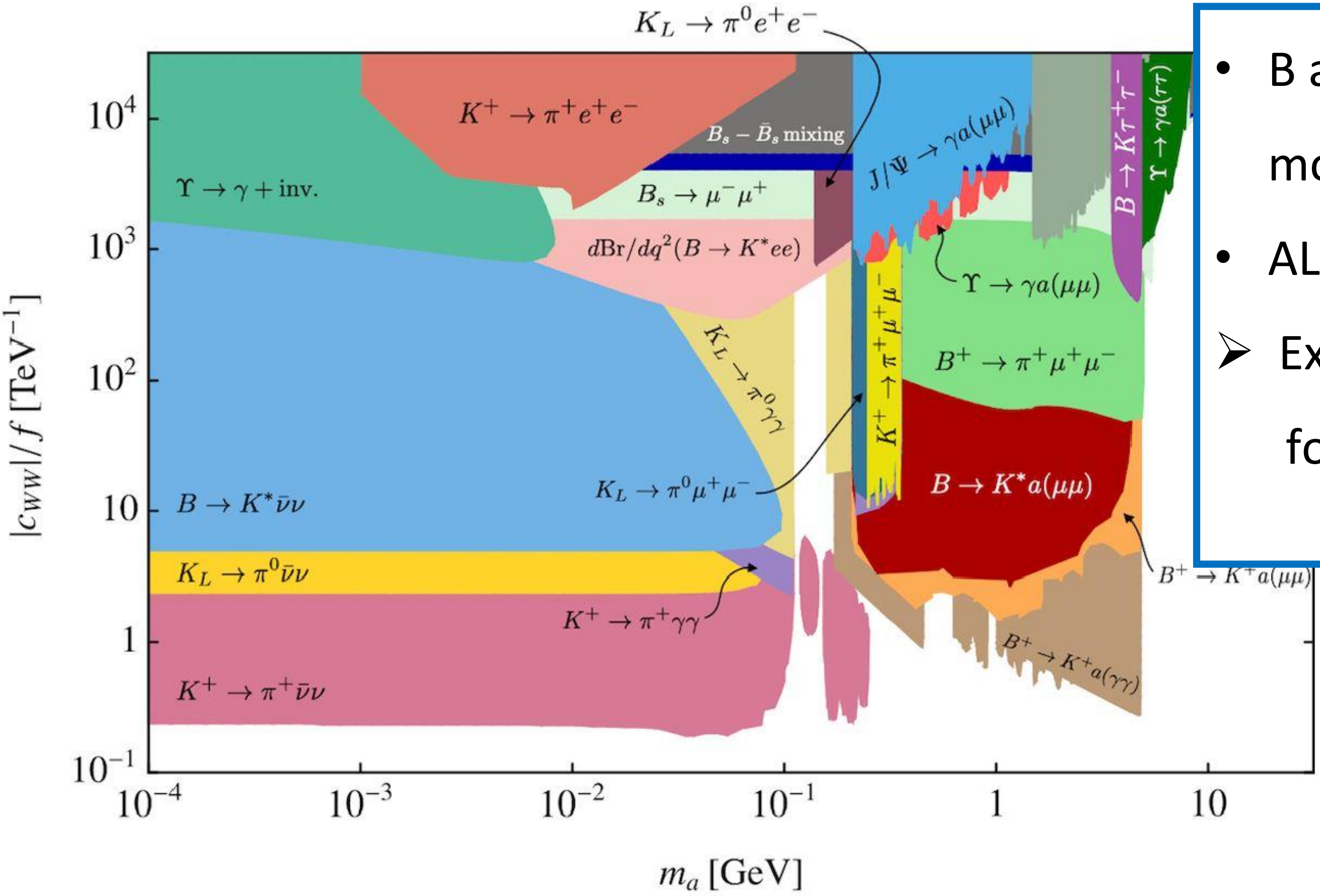
- Using m$_a$=0 approximation we can derive bounds on $|c_i|/f < \left(\Lambda_i^{\text{eff}}\right)^{-1}$

  from NA62 upper limit $\text{Br}(K^- \to \pi^- X) < 2.0 \times 10^{-10}$

Assuming MFV

c_i	c_{GG}	c_{WW}	c_{uu}	c_{dd}	k_D^{12}	$k_D^{12}/\lvert V_{td}V_{ts}\rvert$
Λ_i^{eff} [TeV]	61.3	6.5	1126	31.0	1.9 x 10^8	60 000

CONSTRAINTS FROM $SU(2)_L$ COUPLING

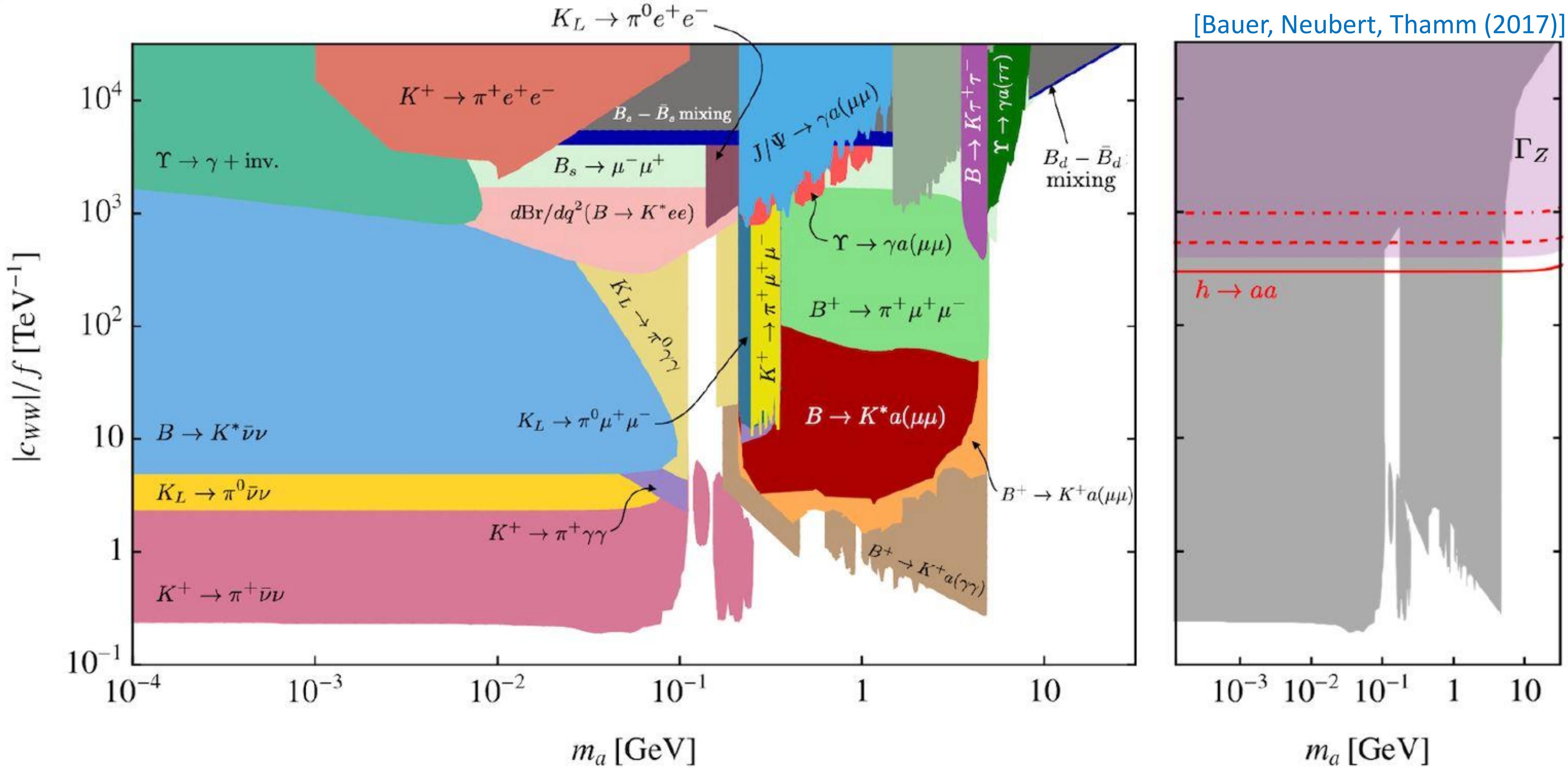


- B and K decays with charged final states are most constraining for $m_a \gtrsim m_K$
- ALP-lepton coupling induced at 1-loop

➢ Exclude couplings $|c_{WW}/f| \gtrsim 0.25 \text{TeV}^{-1}$ for $m_a < m_B$

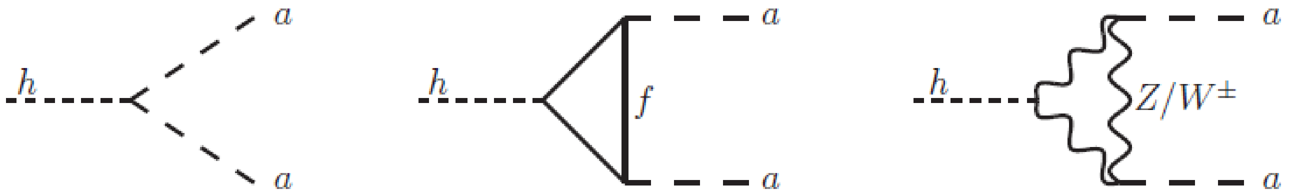
COMPARISON WITH COLLIDER EXPERIMENTS

[Bauer, Neubert, Thamm (2017)]

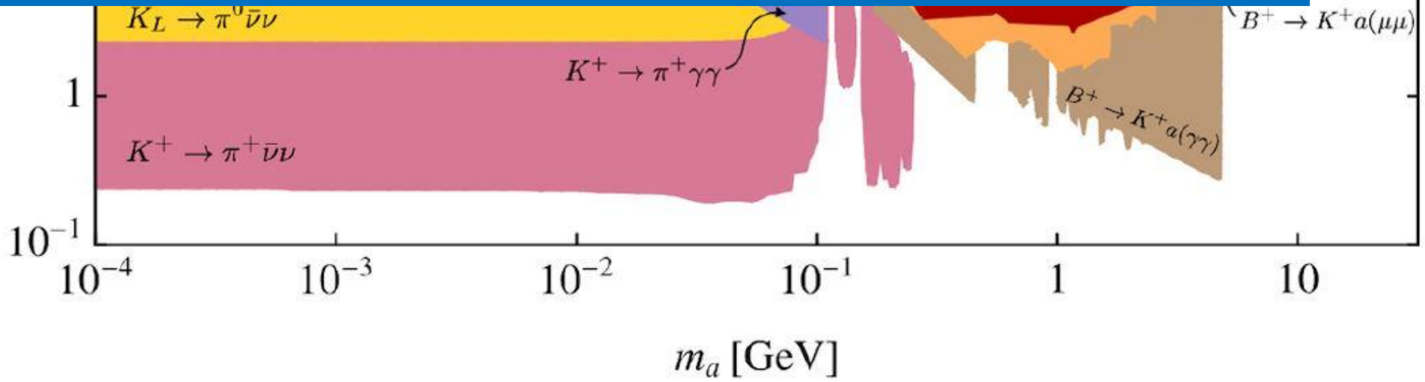


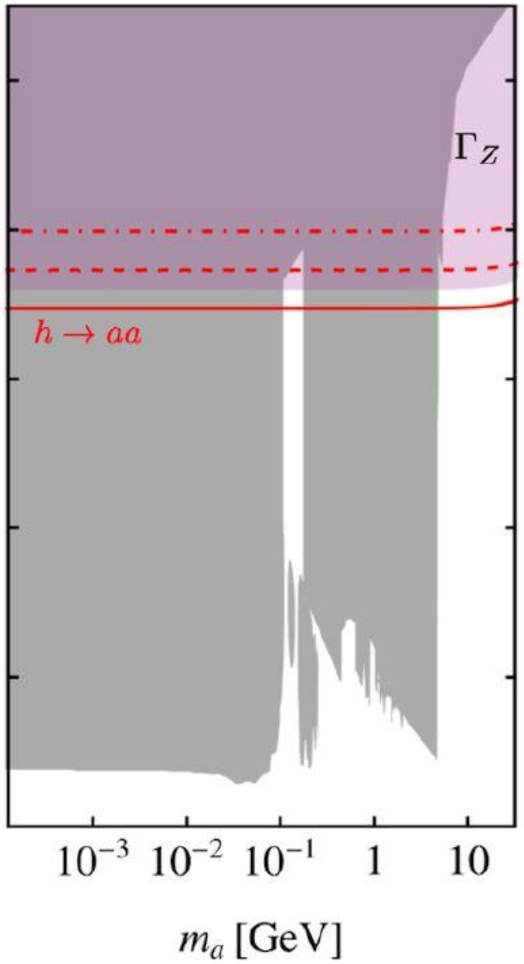
COMPARISON WITH COLLIDER EXPERIMENTS

- Comparison with Z and h decays

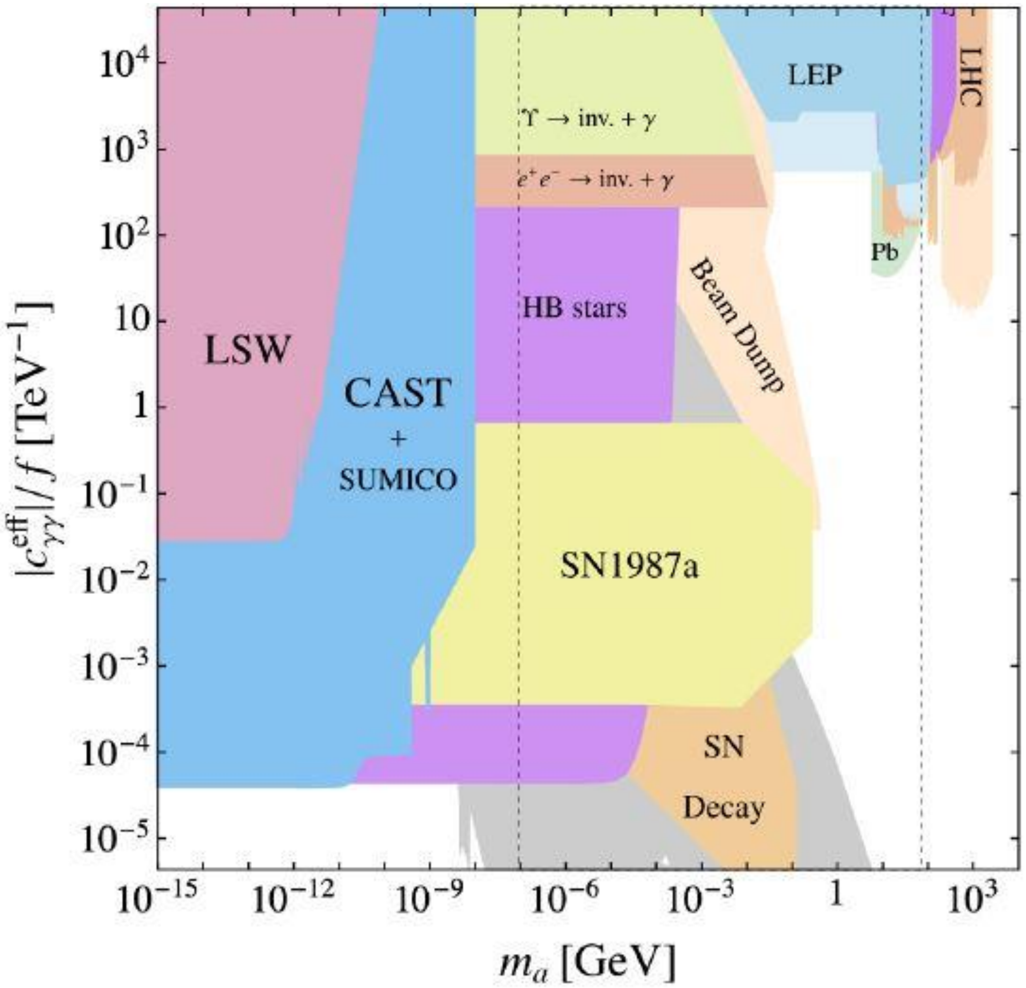


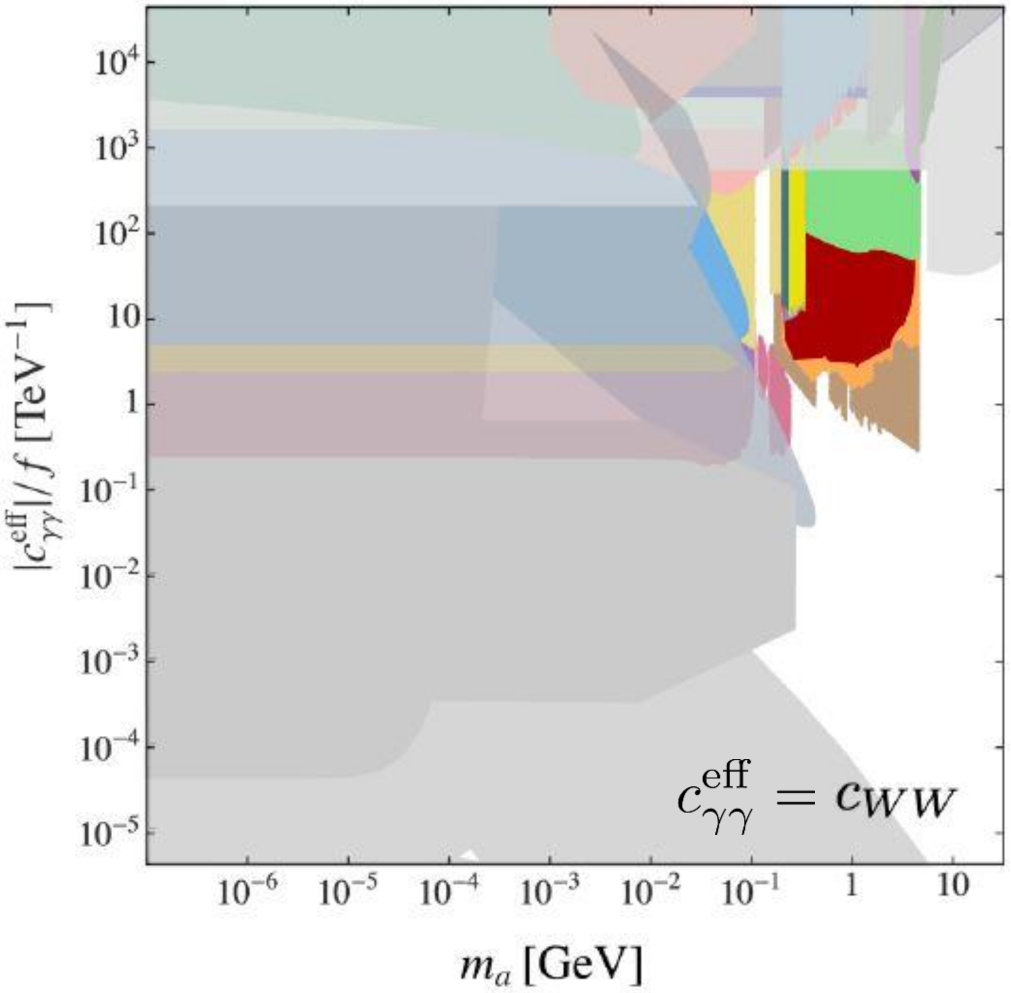
- Constraints from Z from EW precision tests



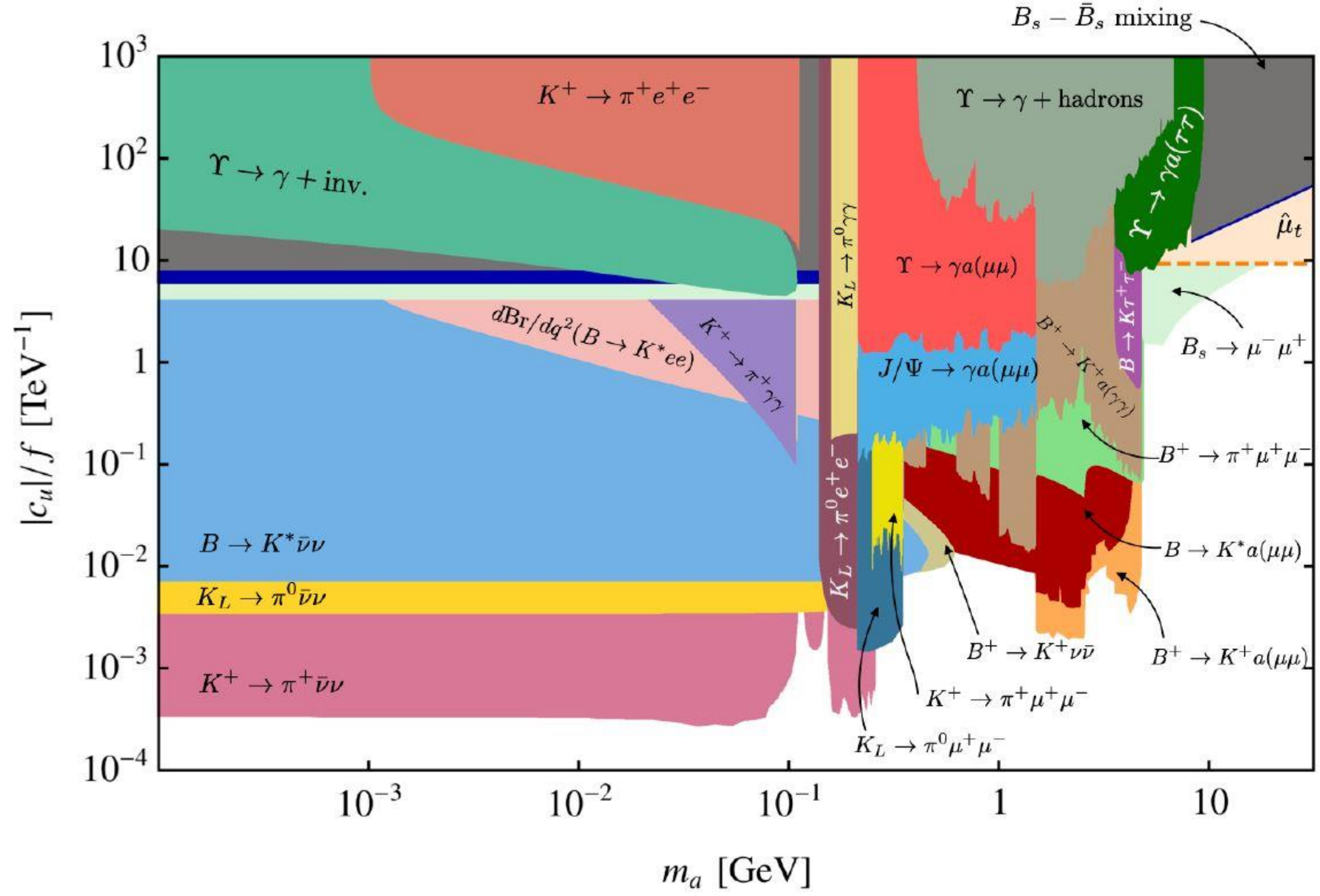


COMPARISON WITH OTHER EXPERIMENTS



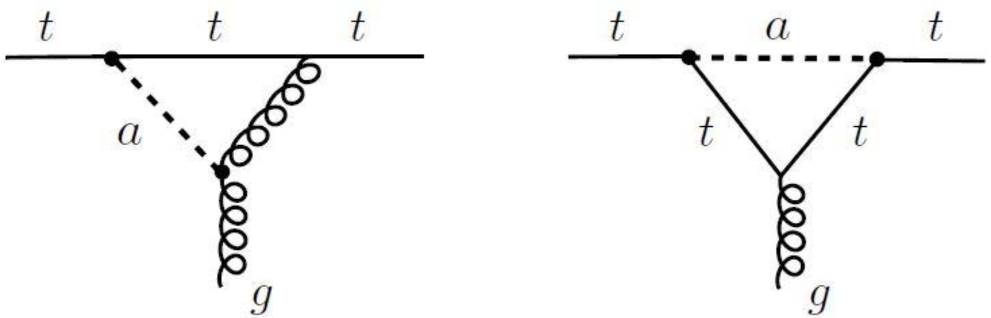


CONSTRAINTS FROM RH UP-TYPE QUARKS



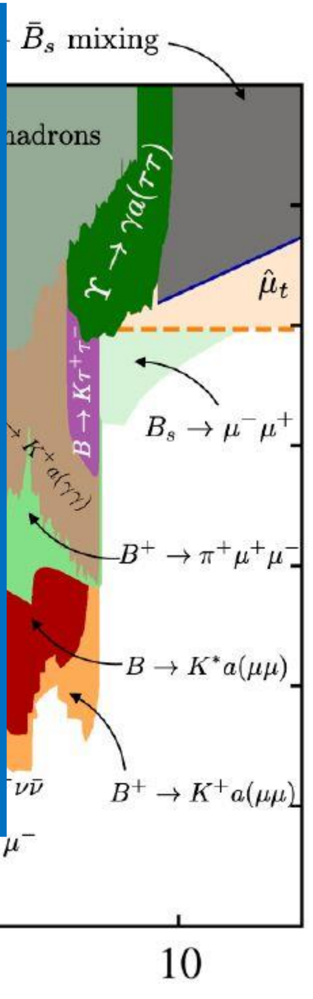
CONSTRAINTS FROM RH UP-TYPE QUARKS

Chromomagnetic moment of the top quark

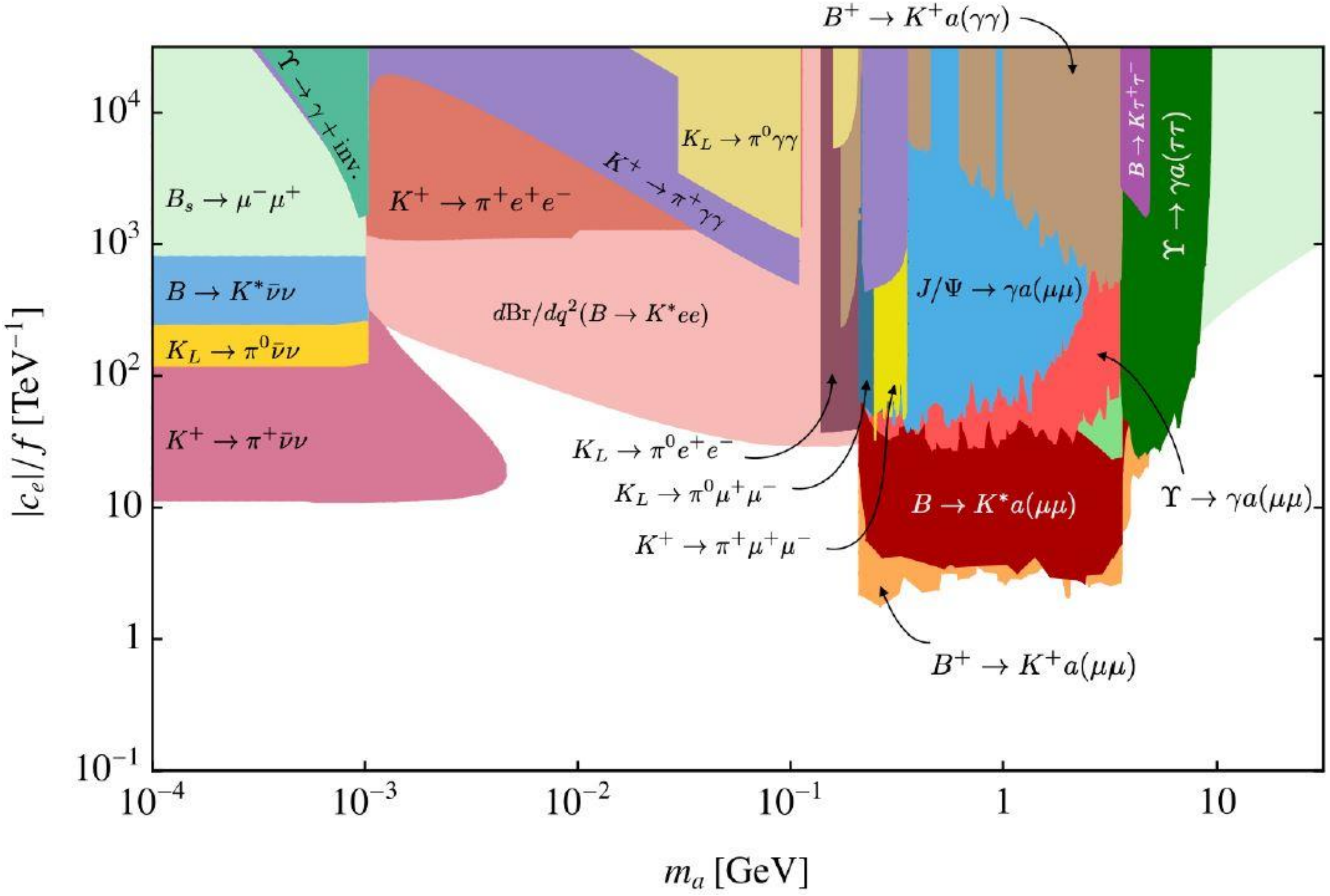


- Defined via coefficient of $\mathcal{L} \supset -\hat{\mu}_t \frac{g_s}{2m_t}\, \bar{t}\sigma^{\mu\nu}T^a t\, G^a_{\mu\nu}$
- Experiment assumes this operator is the only contribution

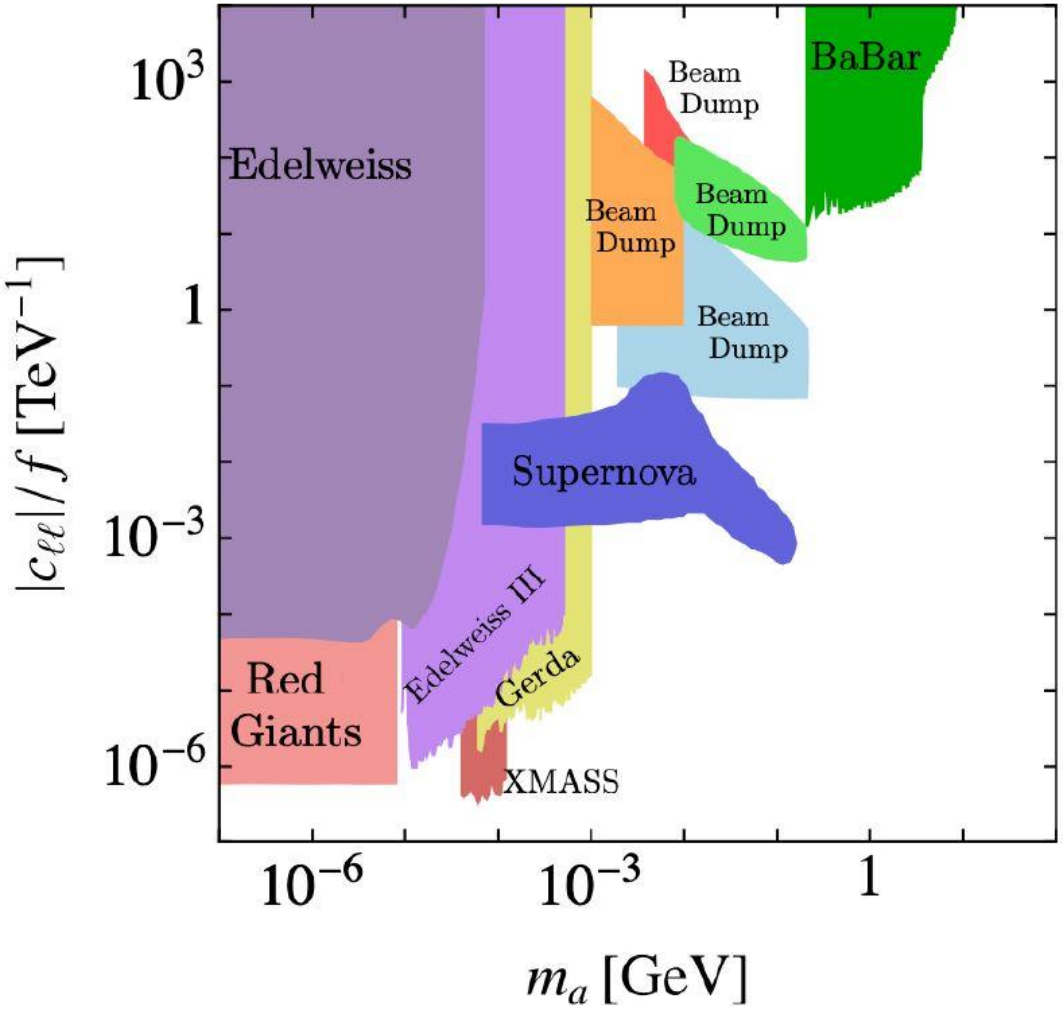
[CMS (2019)]

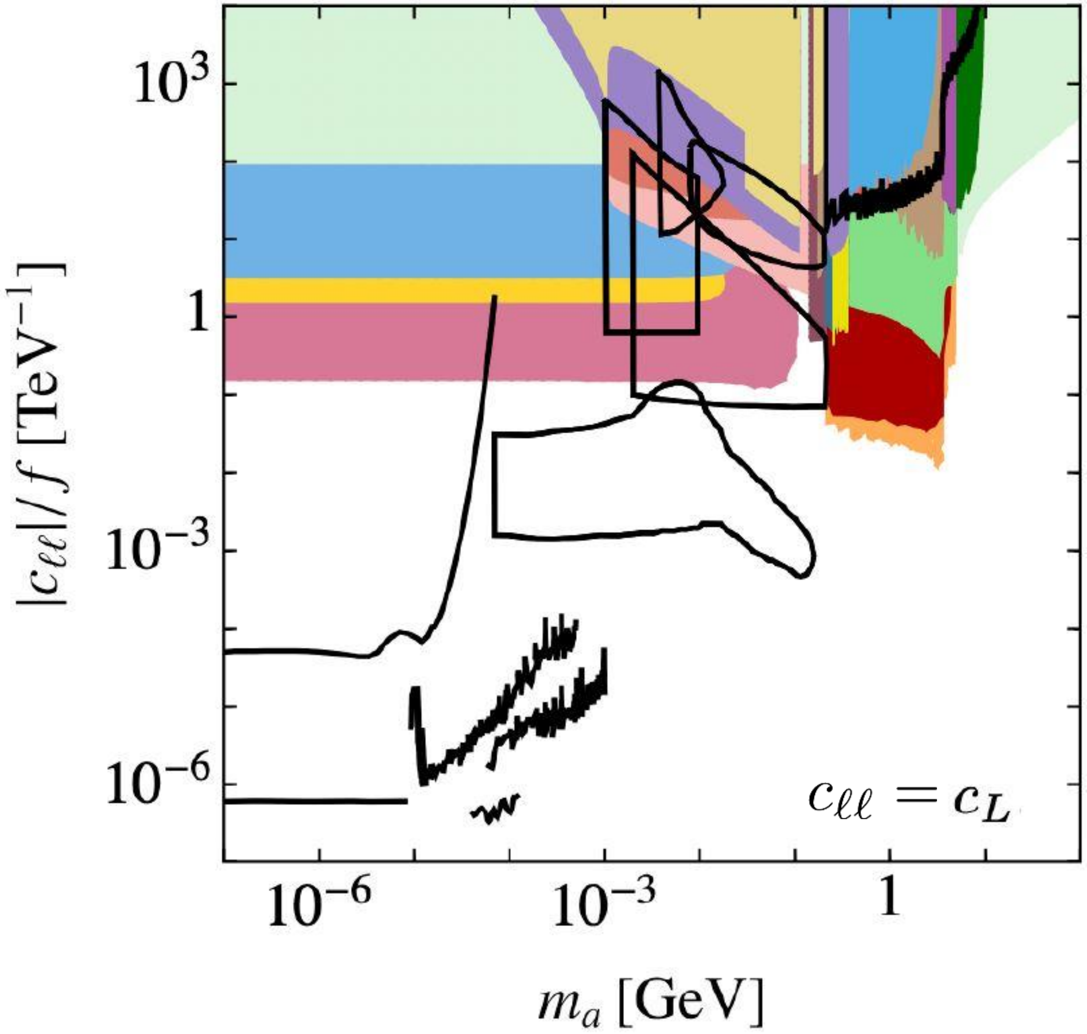


CONSTRAINTS FROM LEPTONS



CONSTRAINTS FROM LEPTONS





ALPS AND LOW-ENERGY ANOMALIES

FORMER ANOMALIES IN RARE B-DECAYS

- Anomalies seeming to indicate lepton flavour universality violation

$$R_K = \frac{\mathrm{Br}\left(B^+ \to K^+ \mu^+ \mu^-\right)}{\mathrm{Br}\left(B^+ \to K^+ e^+ e^-\right)} = 0.846\,^{+0.042}_{-0.039}\,^{+0.013}_{-0.012} \qquad \text{for } 1.1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2, \quad [$$

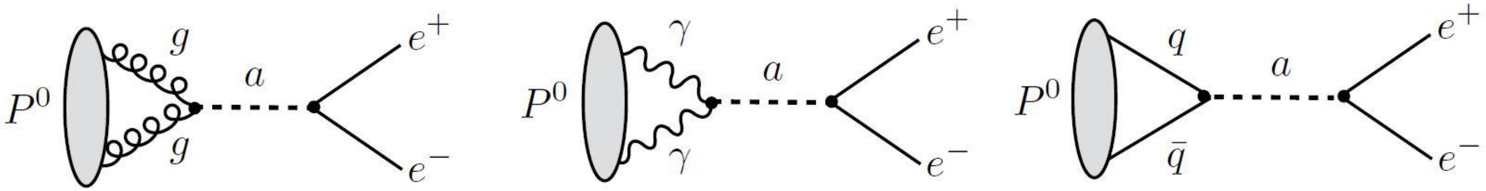
[LHCb (2017 and ongoing)]

$$R_{K^*} = \frac{\mathrm{Br}\left(B^0 \to K^{*0} \mu^+ \mu^-\right)}{\mathrm{Br}\left(B^0 \to K^{*0} e^+ e^-\right)} = \begin{cases} 0.66\,^{+0.11}_{-0.07} \pm 0.03 & \text{for } 0.045\,\mathrm{GeV}^2 < q^2 < 1.1\,\mathrm{GeV}^2 \\ 0.69\,^{+0.11}_{-0.07} \pm 0.05 & \text{for } 1.1\,\mathrm{GeV}^2 < q^2 < 6\,\mathrm{GeV}^2 \end{cases}$$

- In principle, ALPs could address these issues as they naturally couple non-universally to different lepton flavours
- Explanation by heavy ALP is ruled out by a combination of $\mathrm{Br}(B_s \to e^+ e^-)$, $B_s - \bar{B}_s$ mixing, and $(g-2)_e$

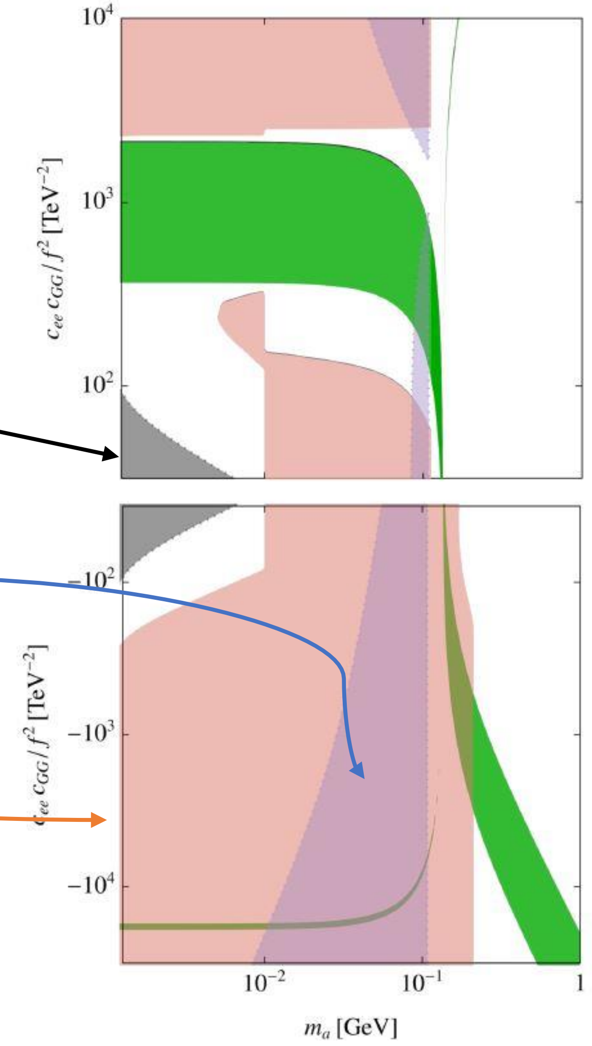
THE KTEV ANOMALY

- Anomaly between measurement and prediction of $\mathrm{Br}(\pi^0 \to e^+e^-)$ [KTeV (2007)]
- ALP can interfere with SM amplitude via



THE KTEV ANOMALY

Coupling to gluon & electron

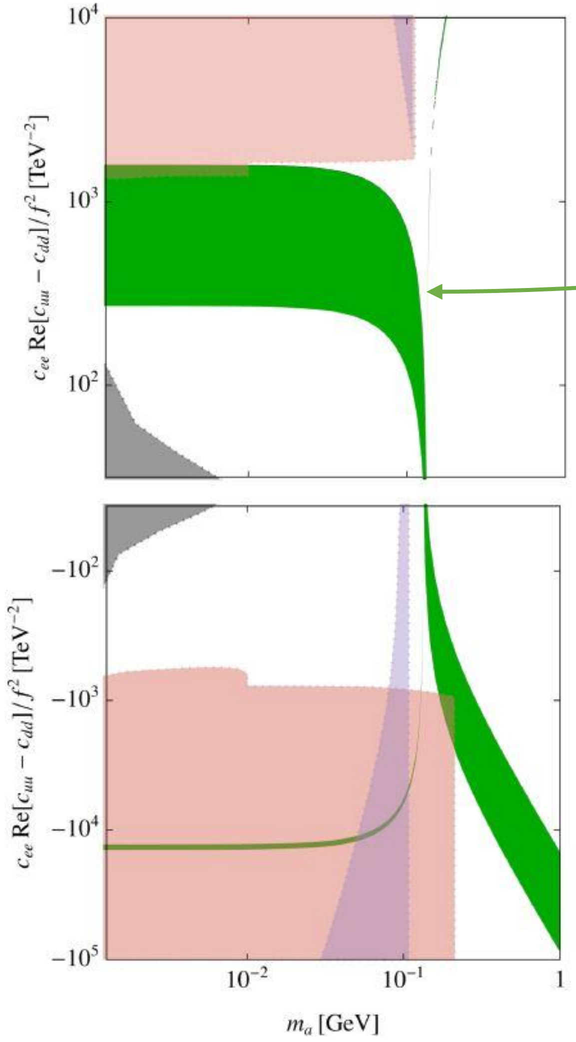


$K^+ \longrightarrow \pi^+ + \text{inv.}$

$K^+ \longrightarrow \pi^+ \gamma\gamma$

$K^+ \longrightarrow \pi^+ e^+ e^-$

Coupling to quarks & electron



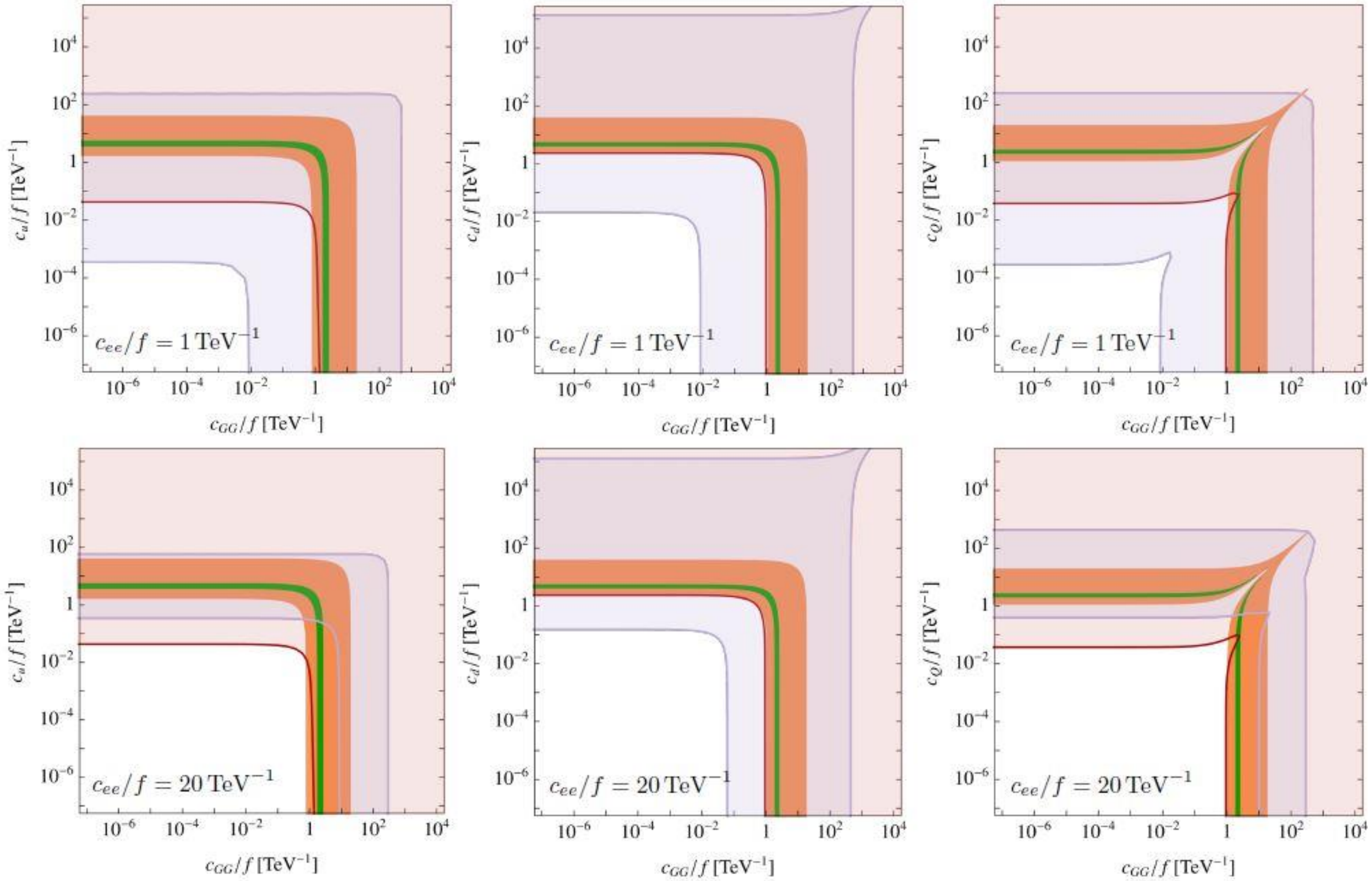
Anomaly explained at 1σ

THE ATOMKI ANOMALIES

- Transition amplitudes of excited Berillium and Helium to respective ground states deviate from SM prediction by ~7σ [ATOMKI (2016)]
- In principle, a $\sim 17\,\mathrm{MeV}$ ALP has all properties needed for simultaneous explanation
- ALP explanation (mostly) ruled out by Kaon measurements

THE ATOMKI ANOMALIES

Green: Berillium explained
Orange: Helium explained
Blue: ruled our by $K_L \rightarrow \pi\nu\nu$
Red: ruled out by $K^- \rightarrow \pi^- a(e^+e^-)$

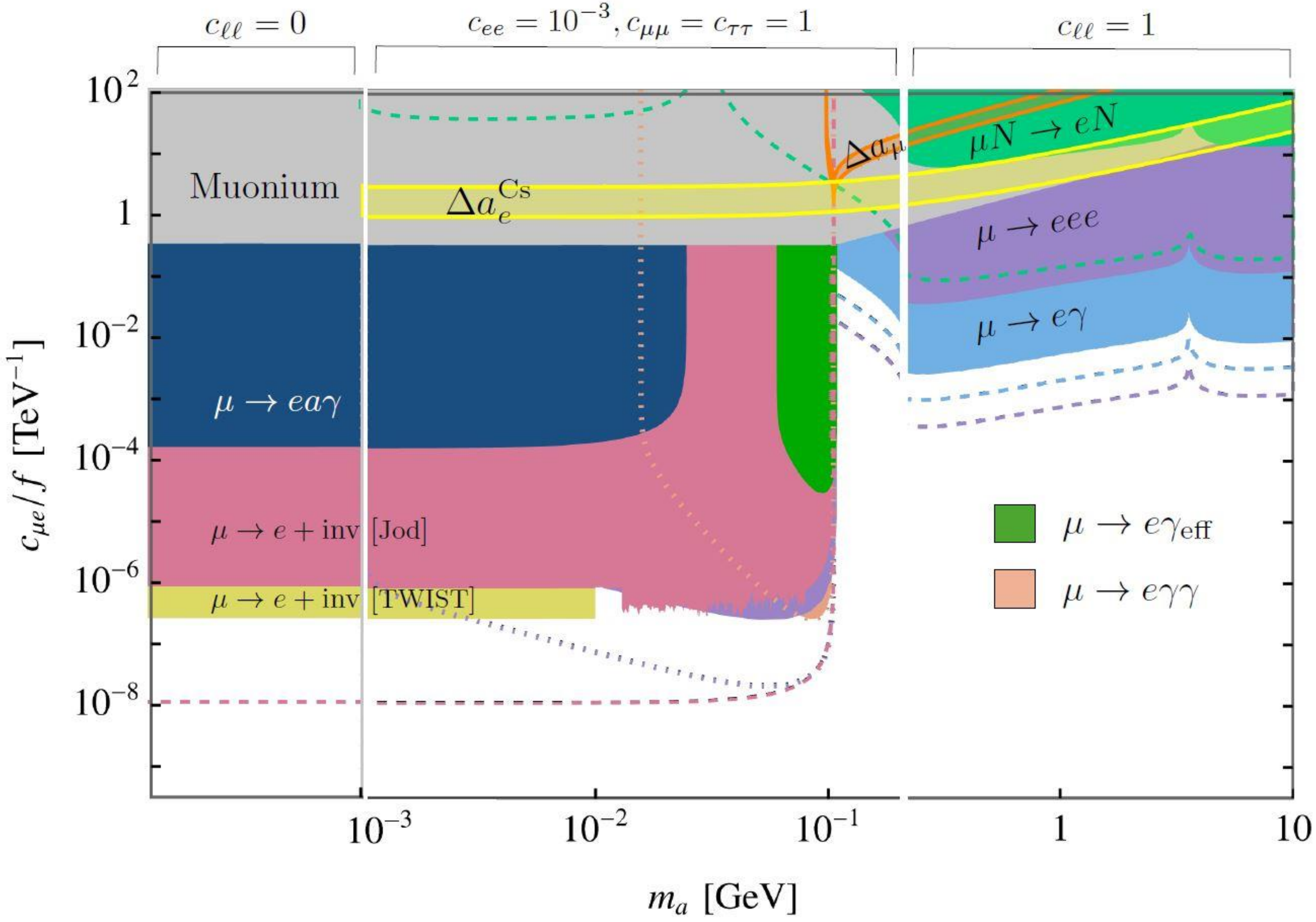


LEPTON FLAVOUR PROBES OF ALPS

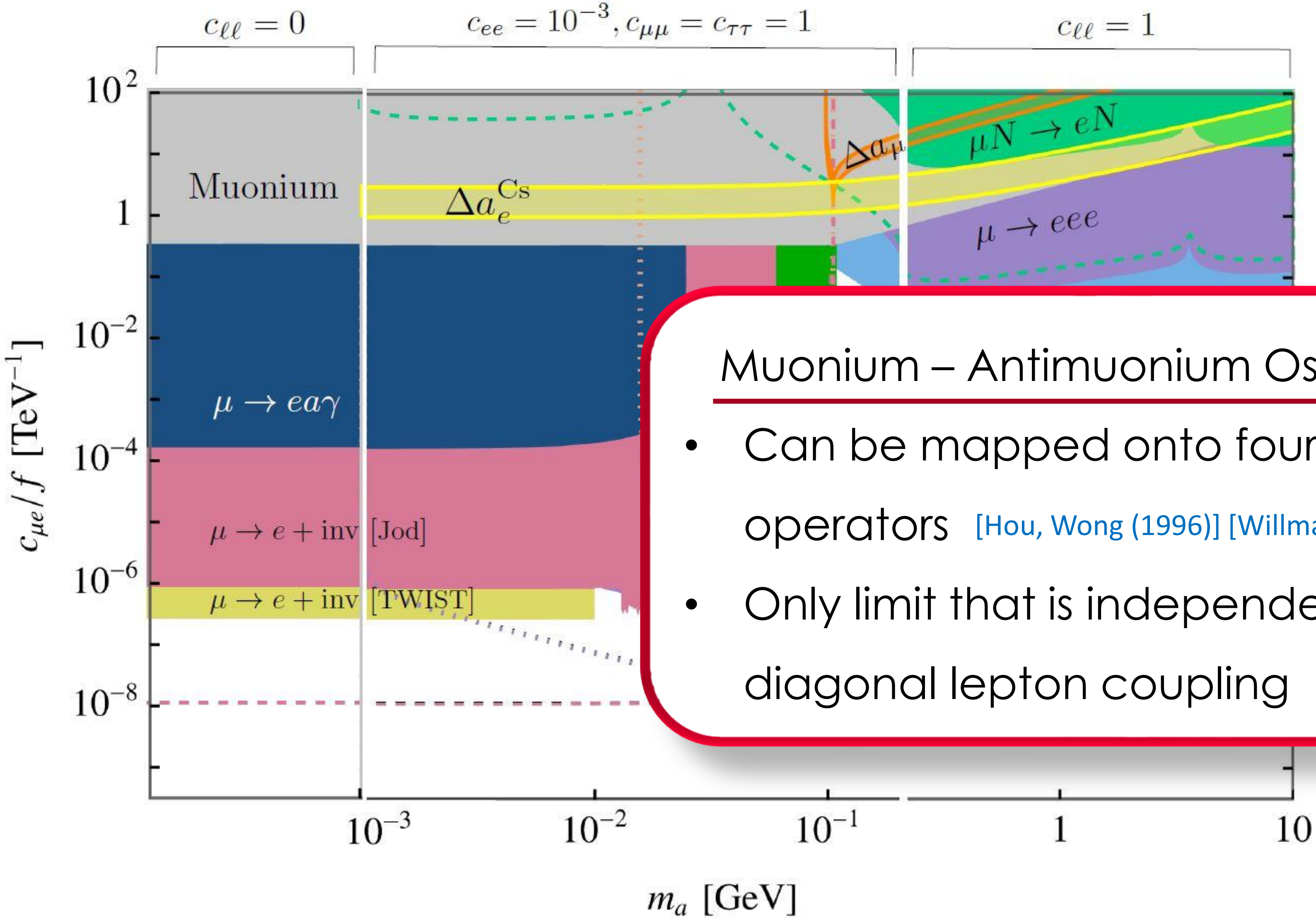
GENERAL REMARKS

- Flavour change can‘t be generated by loops → no mixing with SM
- Some observables only generated at loop level
- Heavy leptons in internal loops can magnify ALP effects
- Assume one LFV coupling to be dominant, include diagonal couplings, no tree-level coupling to gauge bosons
- Focus on muons, results can easily be translated to tau-sector

CONSTRAINTS FROM LFV



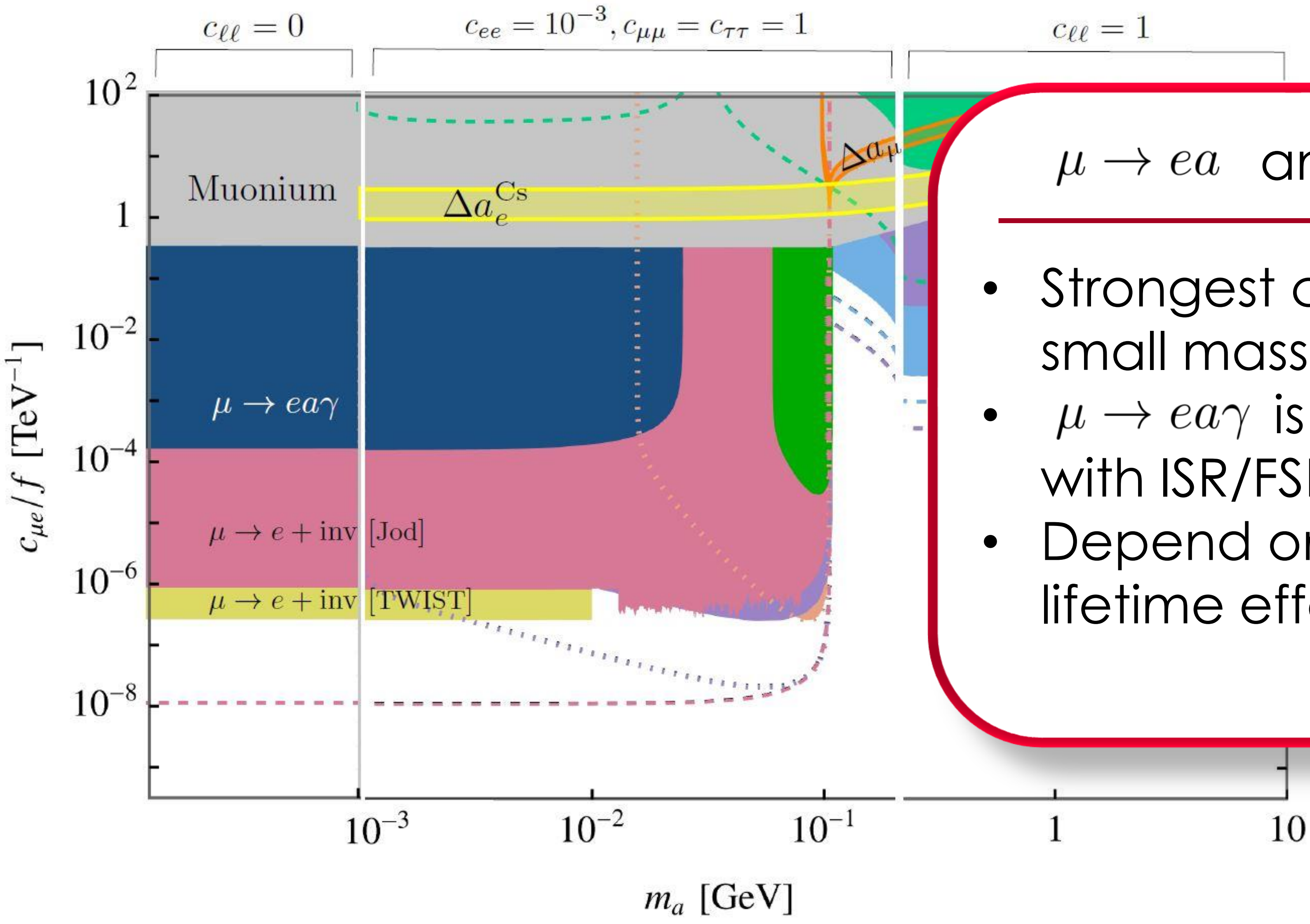
CONSTRAINTS FROM LFV



Muonium – Antimuonium Oscillation

- Can be mapped onto four-fermion operators [Hou, Wong (1996)] [Willmann et al. (1999)]
- Only limit that is independent of $c_{\ell\ell}$ diagonal lepton coupling

CONSTRAINTS FROM LFV



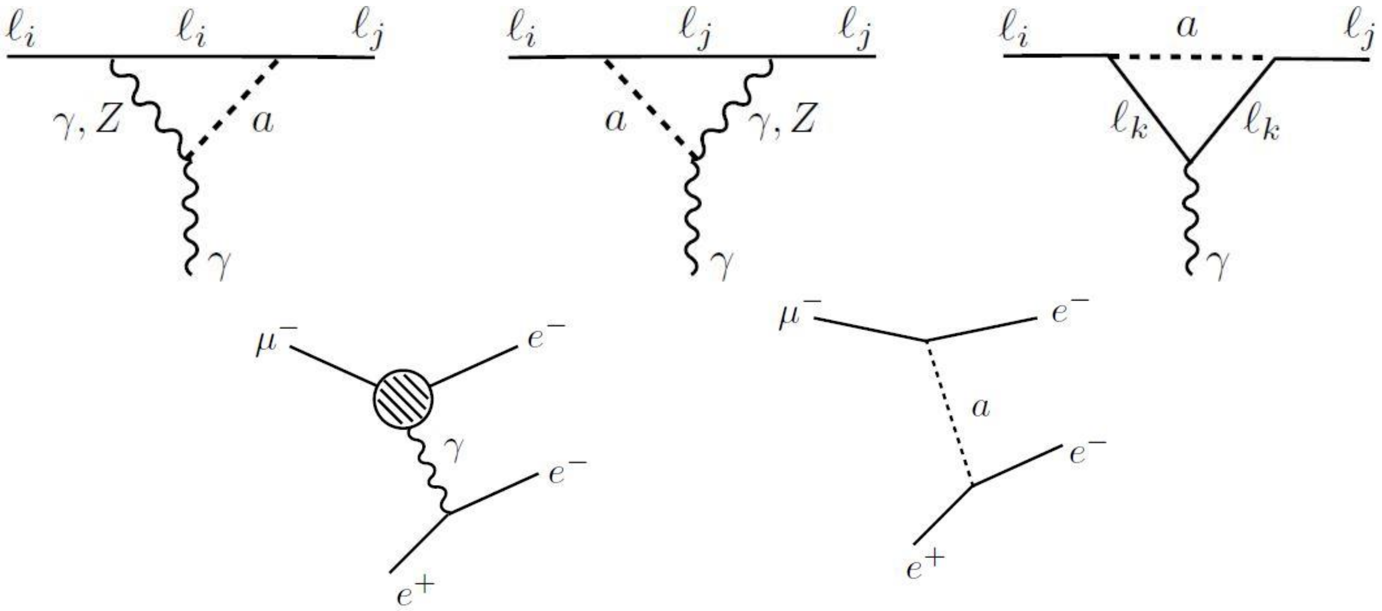
$\mu \to ea$ and $\mu \to ea\gamma$

- Strongest constraints in small mass region
- $\mu \to ea\gamma$ is $\mu \to ea$ with ISR/FSR
- Depend on $c_{\ell\ell}$ via lifetime effect

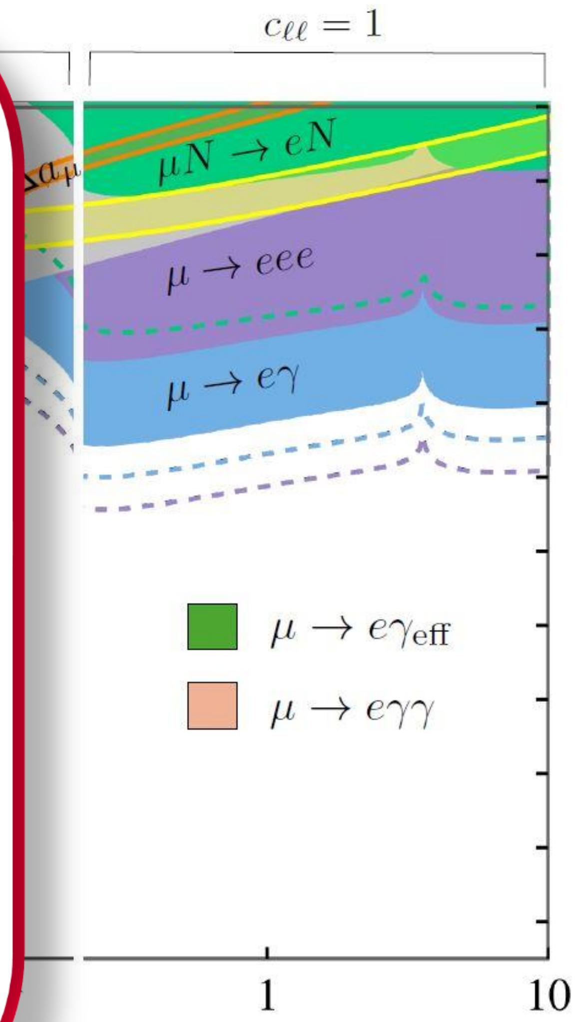
CONSTRAINTS FROM LFV

Muon decays

- Strongest bounds for heavy ALPs

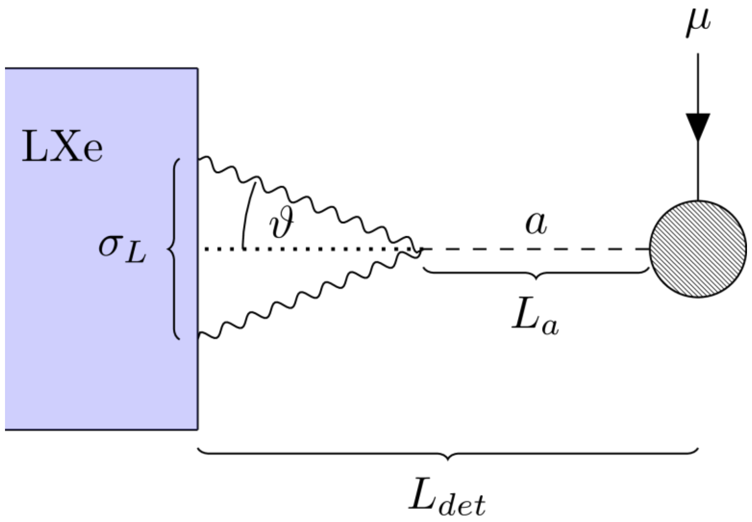


- Strongest limit achieved with on-shell ALP

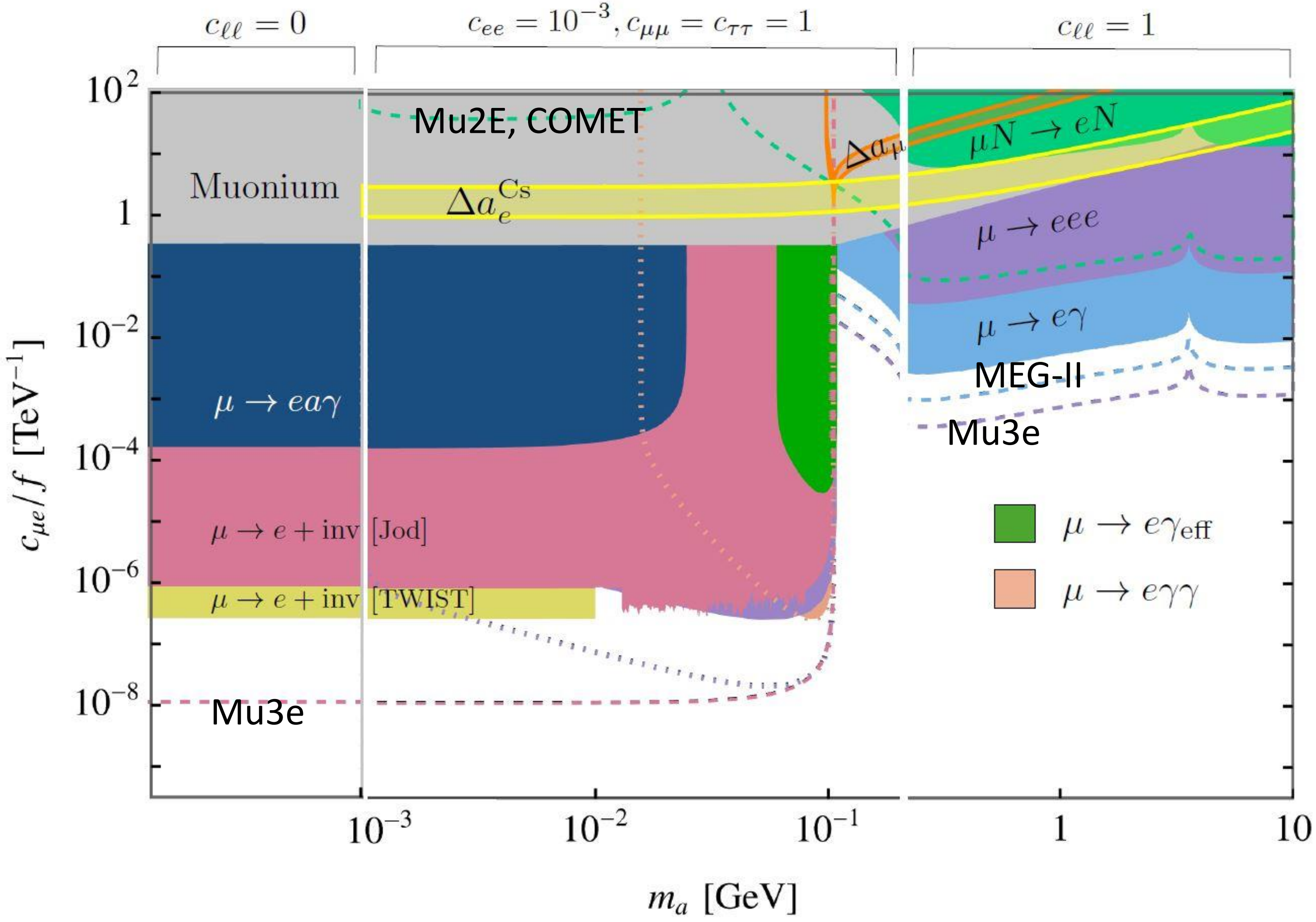


DETAILS ON MUON DECAYS

- For $2m_e<m_a<m_\mu$ can have subsequent $\mu \to ea,\ a \to ee$ decay
- $\mathrm{Br}(\mu \to 3e) \approx \mathrm{Br}(\mu \to ea) \times \mathrm{Br}(a \to ee)$
- Many orders of magnitude more sensitive to LFV couplings than e.g. $\mu \to e\gamma$
- Overcomes phase-space suppression of 3-body decay
- If ALP is boosted or decays close to detector, $\mu \to e\gamma\gamma$ can mimic $\mu \to e\gamma$



CONSTRAINTS FROM LFV



ANOMALOUS MAGNETIC MOMENTS

- Currently, there is a tension between experiment and theoretical prediction for the anomalous magnetic moment of the muon $(g\text{-}2)_\mu$ of $4.2\sigma^{WP}/1.5\sigma^{BMW}$, and the electron $(g\text{-}2)_e$ of $2.4\sigma^{Cs}$ / $1.6\sigma^{Rb}$

  [Bennet *et al* (2006), Kesharvarzi *et al* (2018), Davier *et al* (2020), BMW (2020)]
  [Hanneke, Fogwell, Gabrielse (2008) and (2011)]

- a_μ and a_e receive contribution from both flavour-conserving and -violating couplings

  [Bauer, Neubert, Thamm (2017), Chang *et al* (2001), Marciano *et al* (2016)]

- Explanation of both anomalies with $c_{e\mu}$ coupling or $c_{e(\mu)\tau}$ couplings is ruled out by Muonium oscillations or constraints from $\mu \to e\gamma$ [Endo, Iguro, Kitahara (2020)]

- Can explain both with
  - Non-universal ALP-lepton coupling $c_{ee} \sim -(10-30) \times c_{\mu\mu}$
  - Quite small $c_{\ell\ell}$, explain a_μ with $c_{\mu\tau}$ and a_e with $c_{e\mu}$

CONCLUSION

- MeV-GeV ALPs are well motivated
- Flavour physics can probe ALP parameter space that is complementary to collider and/or astro-physics
- RG running can have major effects and inevitably generates quark flavour changes

CONCLUSION

Thank you for your attention!

- MeV-GeV ALPs a ted
- Flavour physics can probe A r space that is complementary to collider and/or as
- RG running can have major effects and inevitab quark flavour changes

BACKUP

ALP COUPLINGS TO THE CHIRAL LAGRANGIAN

- At μ≈2GeV match to a chiral effective theory
- Remove ALP-gluon coupling by $q(x) \to \exp\left[-i\,\boldsymbol{\kappa}_q \gamma_5\, c_{GG}\, \frac{a(x)}{f}\right] q(x)$

  subject to $\mathrm{Tr}\,\boldsymbol{\kappa}_q = 1$

➢ Obtain

$$\mathcal{L}_{\text{eff}}^{\chi} = \frac{f_\pi^2}{8}\,\mathrm{Tr}\left[\boldsymbol{D}^\mu \boldsymbol{\Sigma}\,(\boldsymbol{D}_\mu \boldsymbol{\Sigma})^\dagger\right] + \frac{f_\pi^2}{4} B_0\,\mathrm{Tr}\left[\hat{\boldsymbol{m}}_q(a)\,\boldsymbol{\Sigma}^\dagger + \text{h.c.}\right]$$
$$+ \frac{1}{2}\,\partial^\mu a\,\partial_\mu a - \frac{m_{a,0}^2}{2}\,a^2 + \hat{c}_{\gamma\gamma}\,\frac{\alpha}{4\pi}\,\frac{a}{f}\,F_{\mu\nu}\,\tilde{F}^{\mu\nu}\,,$$

$$i\boldsymbol{D}_\mu \boldsymbol{\Sigma} = i\partial_\mu \boldsymbol{\Sigma} + e\,A_\mu\,[\boldsymbol{Q}, \boldsymbol{\Sigma}] + \frac{\partial_\mu a}{f}\left(\hat{\boldsymbol{k}}_Q\,\boldsymbol{\Sigma} - \boldsymbol{\Sigma}\,\hat{\boldsymbol{k}}_q\right)$$

$$\boldsymbol{\Sigma}(x) = \exp\left[\tfrac{i\sqrt{2}}{f_\pi}\,\lambda^a \pi^a(x)\right]$$

ALP COUPLINGS TO THE CHIRAL LAGRANGIAN

- Left-handed quark currents are given by

$$
\begin{aligned}
L_\mu^{ji} &= -\frac{i f_\pi^2}{4}\, e^{i\left(\kappa_{q_j}-\kappa_{q_i}\right) c_{GG} \frac{a}{f}} \left[\boldsymbol{\Sigma}\, \left(\boldsymbol{D}_\mu \boldsymbol{\Sigma}\right)^\dagger\right]_{ji} \\
&\ni -\frac{i f_\pi^2}{4} \left[1 + i\left(\kappa_{q_j}-\kappa_{q_i}\right) c_{GG}\frac{a}{f}\right] \left[\boldsymbol{\Sigma}\, \partial_\mu \boldsymbol{\Sigma}^\dagger\right]_{ji} + \frac{f_\pi^2}{4}\frac{\partial^\mu a}{f} \left[\hat{\boldsymbol{k}}_Q - \boldsymbol{\Sigma}\, \hat{\boldsymbol{k}}_q\, \boldsymbol{\Sigma}^\dagger\right]_{ji}
\end{aligned}
$$

- Leading order operators for Kaon decays are

$$
\mathcal{L}_{s\to d} = -\frac{4G_F}{\sqrt{2}}\, V_{ud}^* V_{us} \left(g_8\, \mathcal{O}_8 + g_{27}^{1/2}\, \mathcal{O}_{27}^{1/2} + g_{27}^{3/2}\, \mathcal{O}_{27}^{3/2}\right)
$$

ALP COUPLINGS TO THE CHIRAL LAGRANGIAN

- With

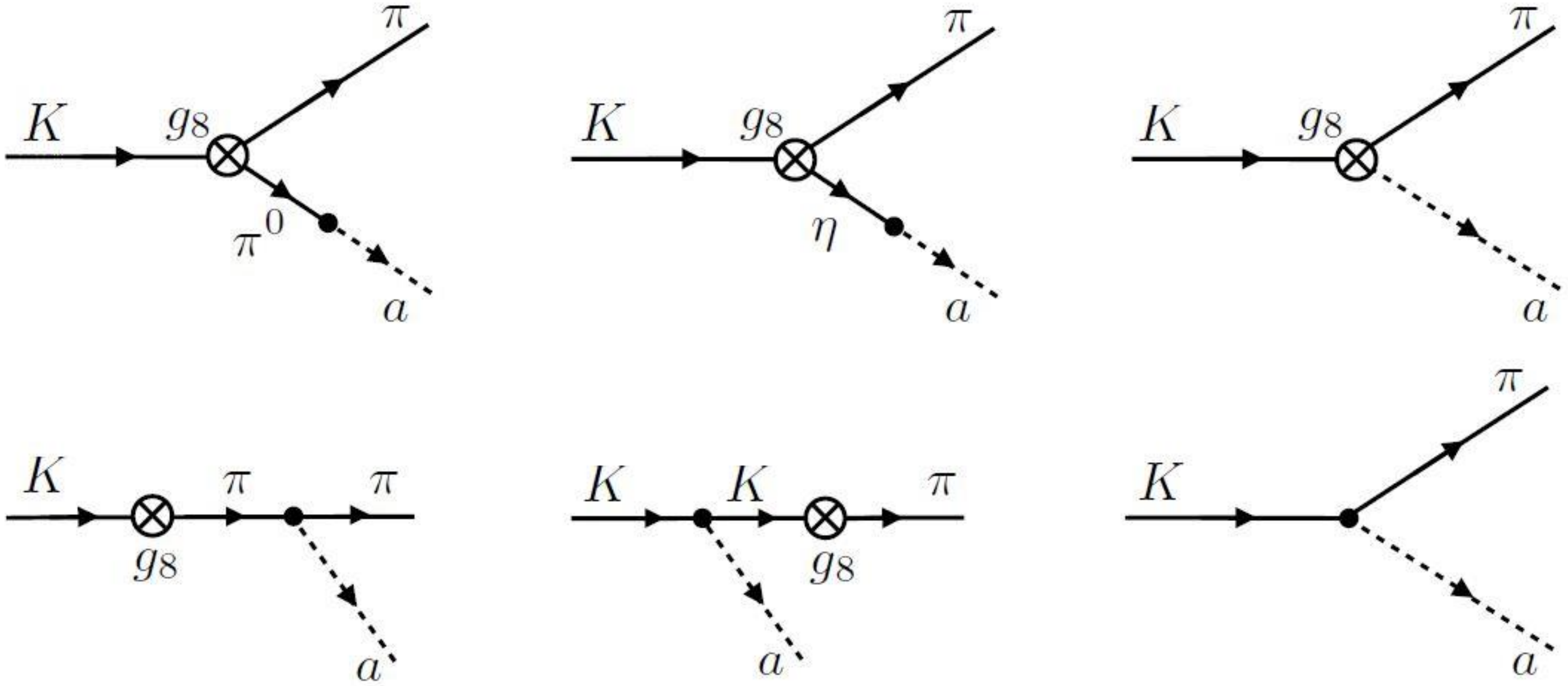
$$\mathcal{O}_8 = \sum_i L_{3i} L_{i2} \,,$$

$$\mathcal{O}_{27}^{1/2} = L_{32} L_{11} + L_{31} L_{12} + 2L_{32} L_{22} - 3L_{32} L_{33} \,,$$

$$\mathcal{O}_{27}^{3/2} = L_{32} L_{11} + L_{31} L_{12} - L_{32} L_{22} \,,$$

- $\mathcal{O}_{27}^{3/2}$ is weaker by a factor of 30 (ΔI=½ selection rule)
- Coefficient of $\mathcal{O}_{27}^{1/2}$ is further suppressed

ALP COUPLINGS TO THE CHIRAL LAGRANGIAN



ELECTRIC DIPOLE MOMENTS

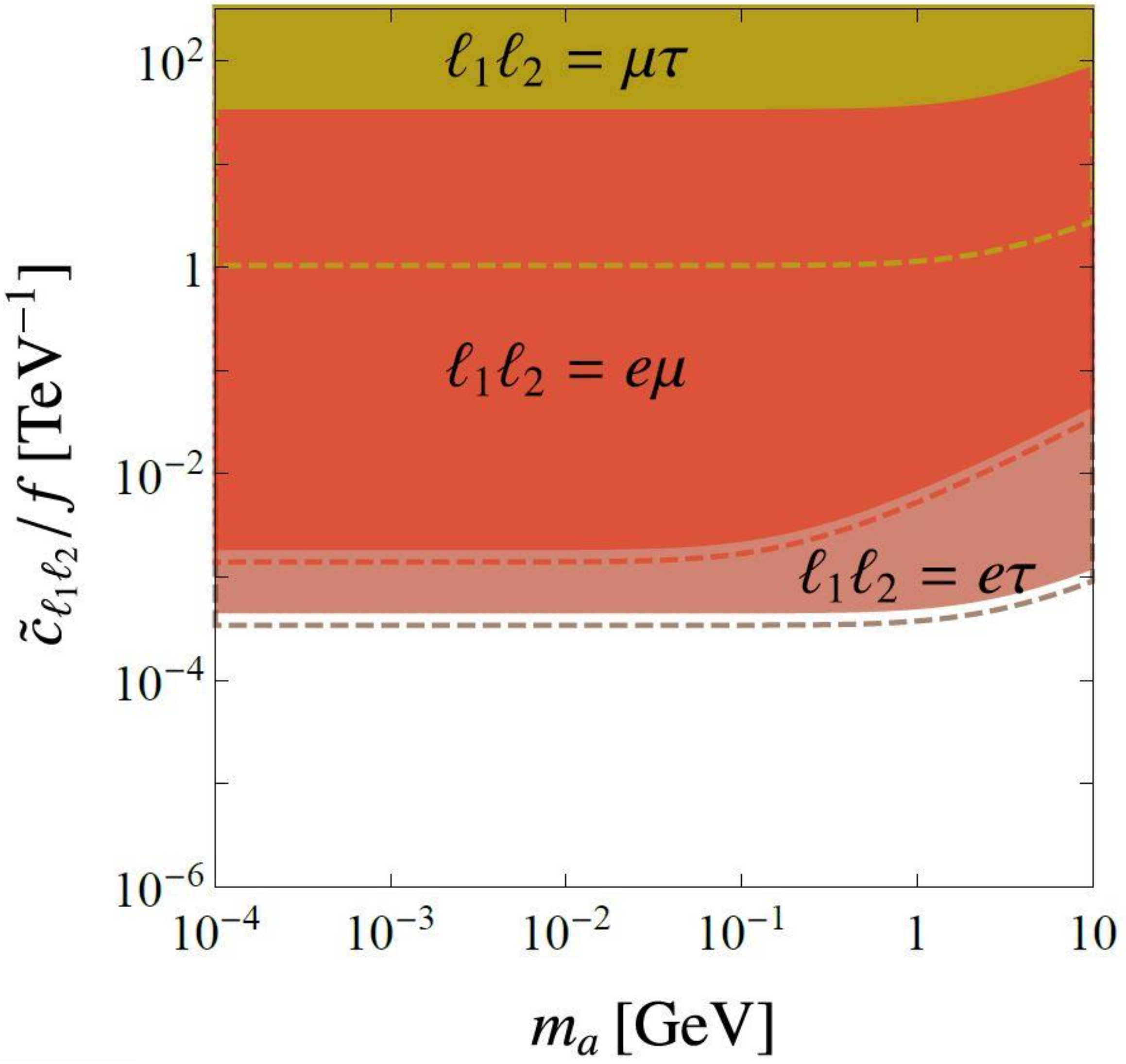
- Current measurements limit EDMs to

$$|d_e| < 1.1 \times 10^{-29}\mathrm{ecm}$$
$$|d_\mu| < 1.9 \times 10^{-20}\mathrm{ecm}$$

- SM predictions are ~10 orders of magnitude weaker than these limits.
  - ➢ parameter space for ALP-interactions, occur already at 1-loop

[Bernreuther, Suzuki (1991), Booth (1993), ACME Collaboration (2018)]

$$\tilde{c}_{\ell_1\ell_2} = \sqrt{\left|\mathrm{Im}\left([k_E]^*_{21}[k_e]_{21}\right)\right|}$$



OVERVIEW OVER LFV EXPERIMENTS

LFV Channel	Current limit	Projection
$\mu \to e\gamma$	4.2×10^{-13} [MEG Coll. (2016)]	6×10^{-14} [MEGII Coll. (2018)]
$\mu \to 3e$	1.0×10^{-12} [SINDRUM Coll. (1988)]	1×10^{-16} [Perrevoort, Mu3e (2018)]
$\mu \to ea, m_a < 13\,\mathrm{MeV}$	5.8×10^{-5} [Bayes *et al* (2014)]	1×10^{-8} [Perrevoort, Mu3e (2018)]
$\mu \to ea, m_a > 13\,\mathrm{MeV}$	9.0×10^{-6}	
$\mu \to ea\gamma$	1.1×10^{-9} [Bolton *et al* (1988)]	
$\mu \to e\gamma\gamma$	7.2×10^{-11} [LAMPF Coll (1986)]	
$\mu N \to eN$	7.0×10^{-13} [SINDRUM-II (2006)]	1×10^{-17} [Mu2e (2014)] [COMET (2020)]

ANOMALOUS MAGNETIC MOMENTS