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Start-to-end polarized bunch acceleration/deceleration simulation in eRHIC ERL

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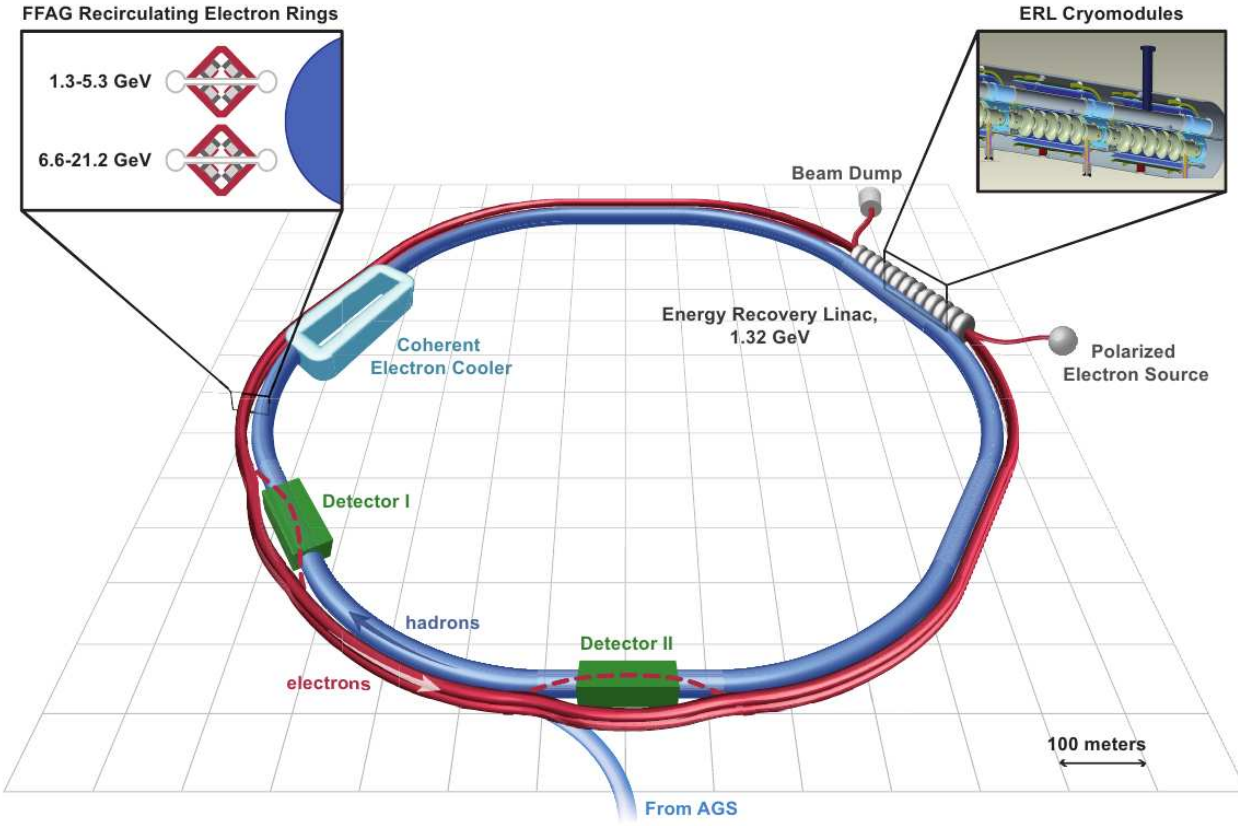
Contents

1 Introduciton

eRHIC ERL with its two recirculation loops alongside RHIC. The top left box shows a cross-section of the FFAG1 (low energy) and FFAG2 (high energy) re-circulating loops. The 1.322 GeV linac is located in RHIC IR2, it is connected to the FFAG loops by a merger section (resp. spreader) at its upstream (resp. downstream) end.

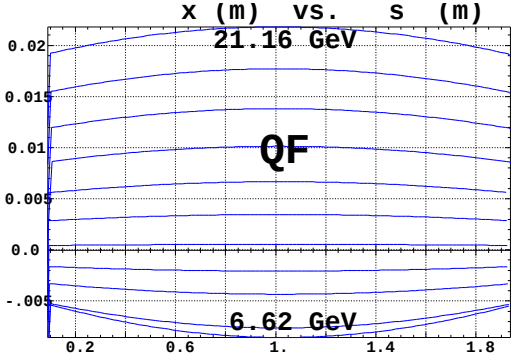
$$5.3 \xrightarrow{up} 21.164 \xrightarrow{down} 5.3\,\mathrm{GeV}$$



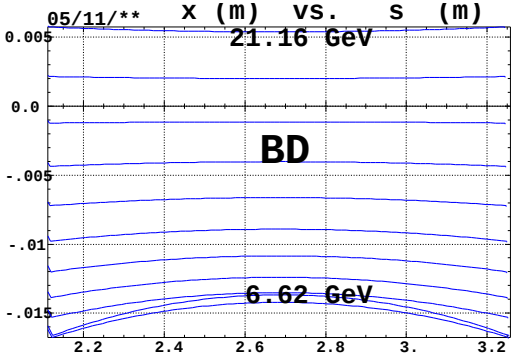
2 FFAG2 loop

• Quadrupole doublet optics

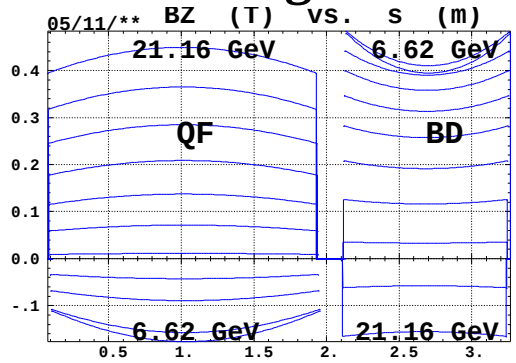
Orbits in QF

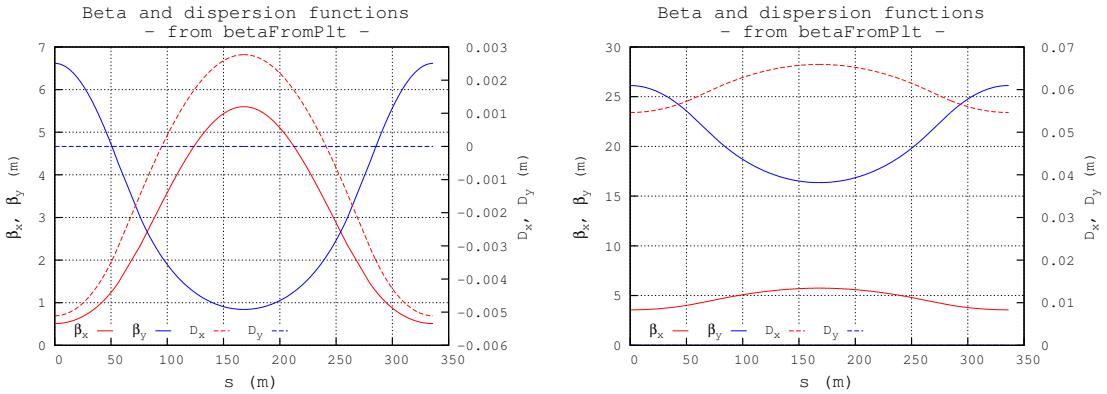


Orbits in BD

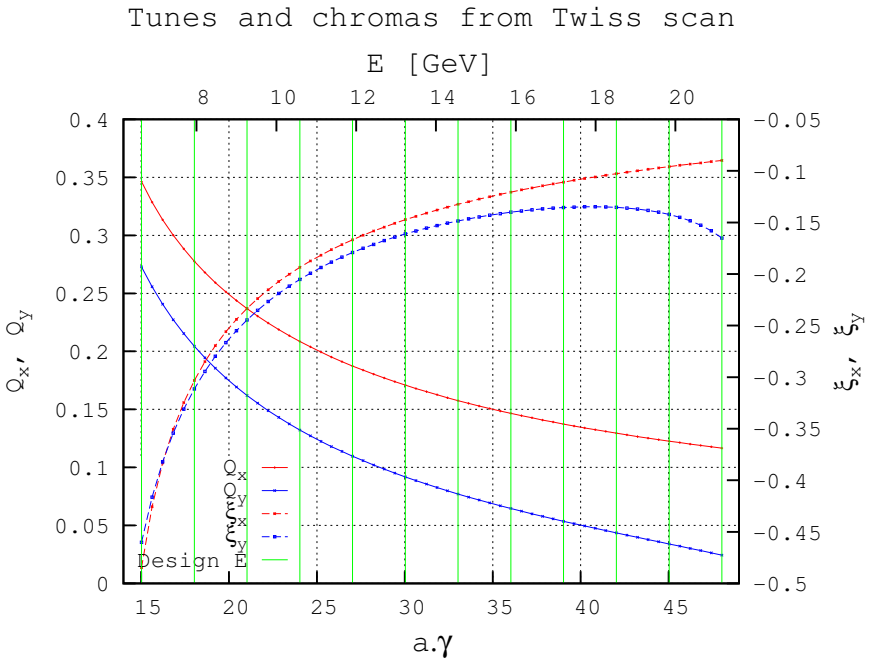


Field along orbits



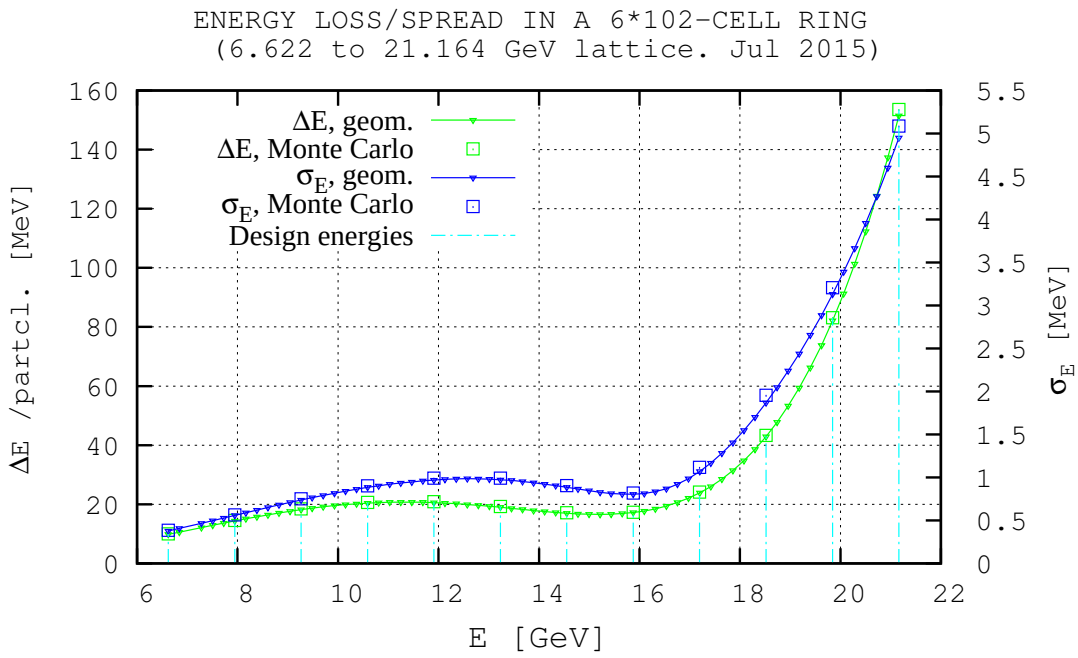


Optical functions at 6.622 GeV (left) and 21.164 GeV (right)



Tunes and chromaticities as a function of energy.

• SR induced ΔE and σ_E, from cell geometry. Comparison with Monte Carlo tracking



The solid lines are from theoretical model, Eq. 5 and Eq. 2, they cover 67 different energies evenly spread in [6.622,21.164]GeV.

$$\overline{\Delta E}[MeV] = \overline{\Delta E_{QF}} + \overline{\Delta E_{BD}} \approx 0.96 \times 10^{-15}\gamma^4\left(\frac{\theta_{\rm QF}}{|\overline{\rho_{\rm QF}}|} + \frac{\theta_{\rm BD}}{|\overline{\rho_{\rm BD}}|}\right) \quad \textbf{over cell} \overset{\times 6\text{arcs}\times 120\text{cells}}{\longrightarrow} \textbf{over ring} \tag{1}$$

$$\sigma_E \approx \sqrt{\sigma^2_{E,\rm QF} + \sigma^2_{E,\rm BD}} \approx 1.94 \times 10^{-14}\gamma^{7/2}\sqrt{\frac{\theta_{\rm QF}}{\overline{\rho_{\rm QF}}^2} + \frac{\theta_{\rm BD}}{\overline{\rho_{\rm BD}}^2}} \quad \textbf{over cell} \overset{\sqrt{\times 6\text{arcs}\times 120\text{cells}}}{\longrightarrow} \textbf{over ring} \tag{2}$$

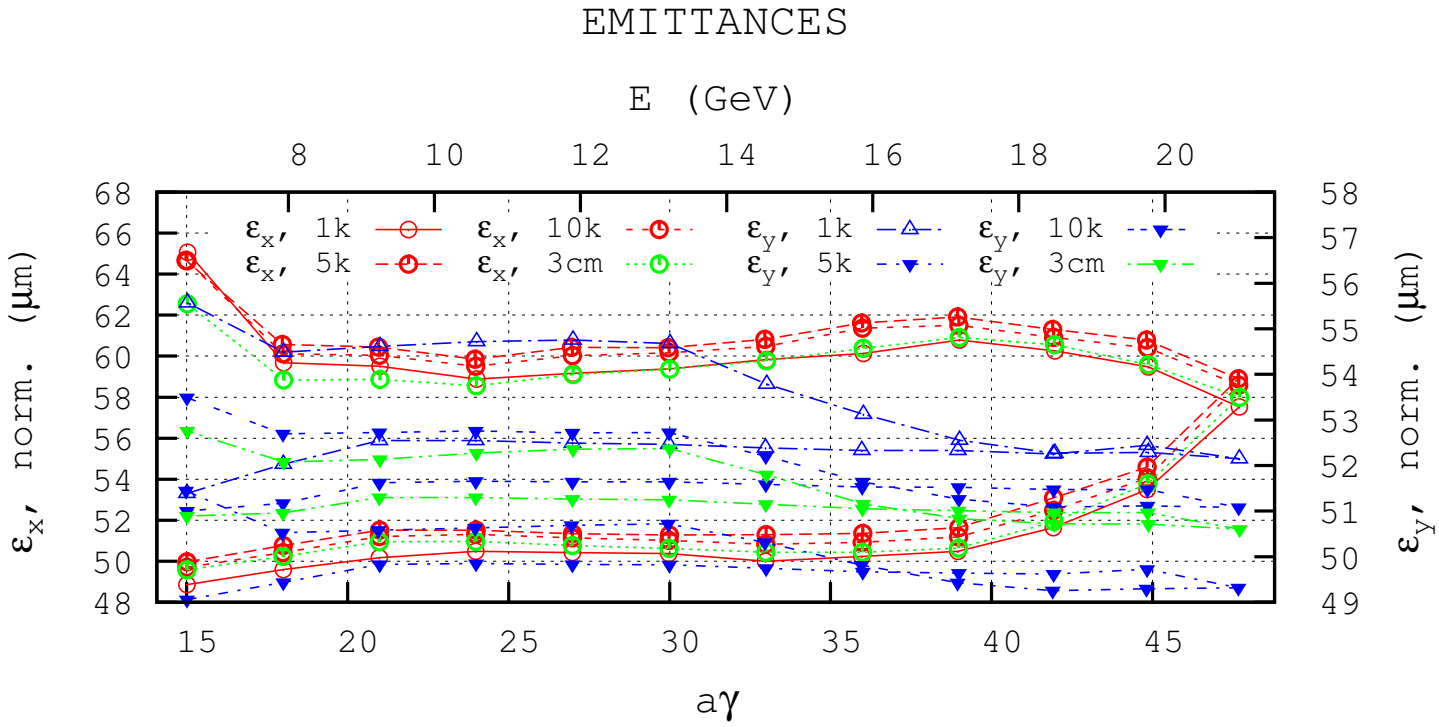
"Monte Carlo" data at design energies (square markers) are shown for comparison, taken from 10^4 particle tracking in a single cell, and then $\times 6\text{arcs}\times 120\text{cells}$ for energy loss, or $\times\sqrt{6\text{arcs}\times 120\text{cells}}$ for energy spread.

• End-to-end ($6.62 \overset{11\times1.322}{\rightarrow} 21.16 \overset{11\times(-1.322)}{\rightarrow} 6.62$ GeV) in a 6-arc ring

The game here : a 1000- or 5000-particle bunch is launched for a 23 FFAG-loop up-down cycle.

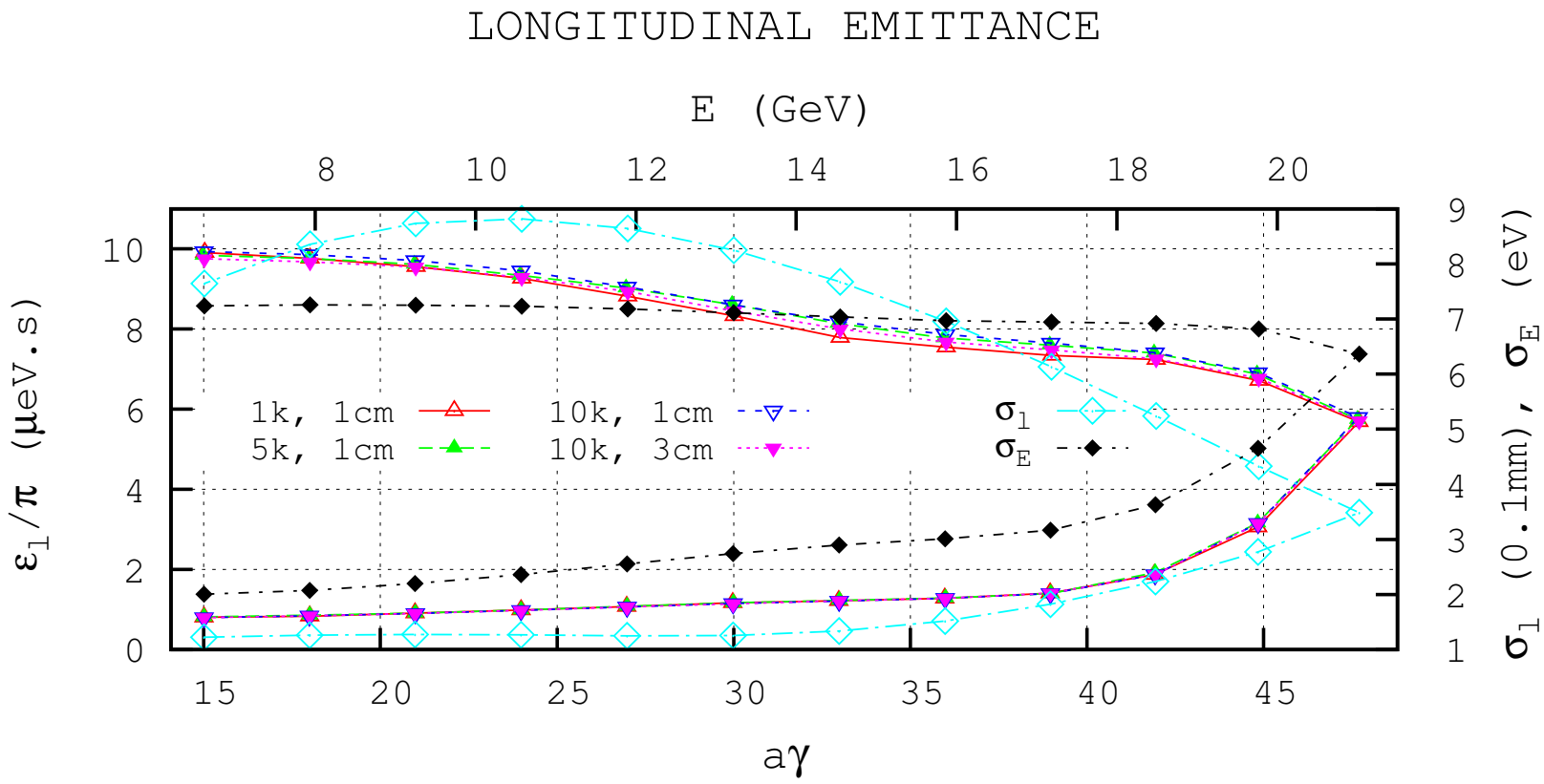
- **Initial emittances : $\approx 50\mu$m ; initial dp/p : random uniform in $[-3\times10^{-4}, +3\times10^{-4}]$.**
- **SR is compensated at end of each turn, based on Eqs. in slide #4**
- **The bunch is re-centered on the theoretical FFAG orbit after each turn.**
- **In the figure, red curves, left V axis : horizontal emittance ; blue curves, right V axis : vertical emittance**

Reason for the various similar curves in the plot below :
Comparing convergence for different numbers of particles (1k, 5k and 10k), and different step sizes (1cm and 3cm). Conclusion to this comparison : no big difference, the main effect is rather slight translation of the curves.



• Evolution of longitudinal emittance, bunch is tracked over a 23-loop end-to-end up-down cycle in a simplified 6-arc ring (6.622 $\overset{up}{\rightarrow}$ 21.164 $\overset{down}{\rightarrow}$ 6.622 GeV).

Initial conditions are : $\epsilon_x = \epsilon_y = 50\,\pi\mu$**m normalized,** $\delta E/E$ **uniform random in** $\pm 3\,10^{-4}$ **and** $\sigma_l = 0$**.**



- Longitudinal bunch emittance : left axis, left four markers,

- σ_l **and** σ_E **: right axis, right two markers,**

• **Some values (orders of magnitude) regarding the cumulated effect of SR on transverse and longitudinal motions, at top energy and back to 6.62 GeV, in a 6×120 cell ring. The "6.622 (start)" rows give the initial conditions in the two tracking simulations.**

bunch energy (GeV)	ϵ_x/π**, norm. (**μ**m)**	ϵ_y/π**, norm (**μ**m)**	ϵ_l/π **(**μ**eV.s)**	$\frac{\sigma_E}{E}$ **(**10^{-4}**)**	σ_l **(mm)**
Initial point object					
6.622 (start)	**0**	**0**	**0**	**0**	**0**
21.16	**2.7**	**0**	**4.2**	**3**	**0.3**
6.62 (down end)	**4.4**	**0**	**7.5**	**11**	**0.8**
Initial extended object					
6.622 (start)	**50**	**50**	**0**	± 3**, unif.**	**0**
21.16	**59**	**50**	**6**	**3**	**0.4**
6.62 (down end)	**65**	**52**	**10**	**11**	**0.9**

• Spin diffusion on the way up

Solution of the diffusion equations [*cf.* V. Ptitsyn, EIC'14]:

$$V = MV_0 + W,\ V = \begin{pmatrix} \overline{\Delta E^2} \\ \overline{\Delta E \Delta \phi} \\ \overline{\Delta \phi^2} \end{pmatrix} \overset{\text{Cst field}}{\Longrightarrow} \begin{pmatrix} \overline{\Delta E^2} \\ \overline{\Delta E \Delta \phi} \\ \overline{\Delta \phi^2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \alpha s & 1 & 0 \\ \alpha^2 s^2 & 2\alpha s & 1 \end{pmatrix} \begin{pmatrix} \overline{\Delta E^2} \\ \overline{\Delta E \Delta \phi} \\ \overline{\Delta \phi^2} \end{pmatrix}_{s=0} + \omega \times \begin{pmatrix} s \\ \alpha s^2/2 \\ \alpha^2 s^3/3 \end{pmatrix}$$

where $\omega = \dfrac{C}{\rho^3} \lambda\!\!\!^{-}_c r_e \gamma^5 E^2 \approx 1.44 \times 10^{-27} \dfrac{\gamma^5}{\rho^3} E^2$

(with $\lambda\!\!\!^{-}_c = \hbar / m_e c$ **the electron Compton wavelength,** $C = 110\sqrt{3}/144$**),**

$\alpha = \dfrac{a}{\rho E_0} \approx \dfrac{1}{0.4406\rho}$ **(with** $a = 1.16 \times 10^{-3}$ **anomalous gyromagnetic factor, electron mass** $E_0 = 0.511 \times 10^{-3}$ **GeV).**

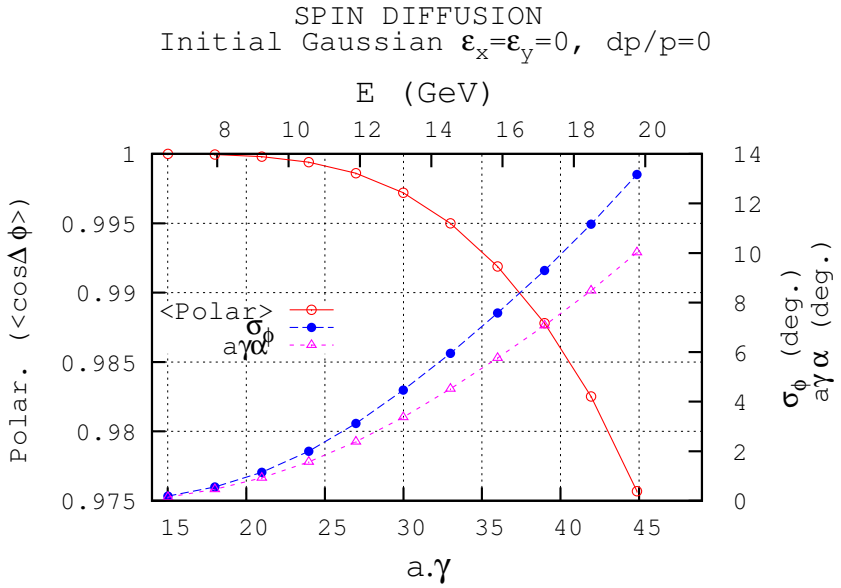
Assuming $\begin{pmatrix} \overline{\Delta E^2} \\ \overline{\Delta E \Delta \phi} \\ \overline{\Delta \phi^2} \end{pmatrix}_{s=0} = 0$ **yields for instance :**

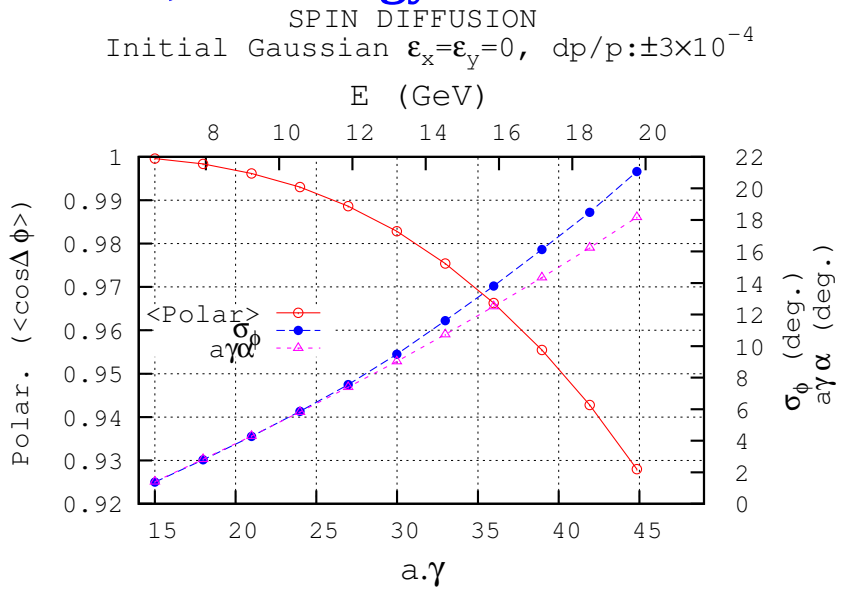
$$\sigma_E = \overline{\Delta E^2}^{1/2} = (\omega s)^{1/2}, \qquad \sigma_\phi = \overline{\Delta \phi^2}^{1/2} = \sqrt{\frac{\omega \alpha^2 s^3}{3}}^{1/2} = \frac{\alpha s}{\sqrt{3}} \sigma_E = \frac{\alpha \sigma_E^3}{\sqrt{3}\omega}$$

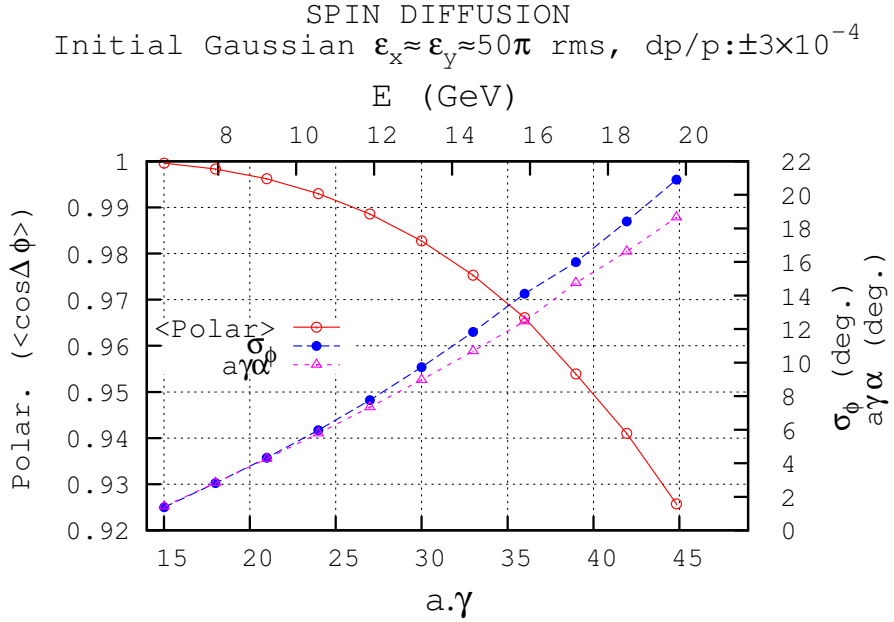
• **Tracking polarization in a 6-arc loop**

Starting spins are all aligned, horizontal, they precess around vertical $\vec{B}$ at a rate $G\gamma\alpha$

Cumulated effect over turns, vs. energy :







3 Linac

- **Chambers matrices are used (same as simulation in CEBAF linacs, for instance)**

  to transport the bunch in the 42 cavities of the 120 m long linac

- **For both planes, 1 cavity :**

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} \cos(u) - \sqrt{2}\sin(u)\cos(\phi) & vW_i \sin(u)\cos(\phi) \\ -\frac{\sin(u)}{vW_o}(2\cos(\phi) + \frac{1}{\cos(\phi)}) & \frac{1}{W_o W_i}(\cos(u) + \sqrt{2}\sin(u)\cos(\phi)) \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in} \quad (3)$$

with $u = \log(W_o/W_i)/(\sqrt{8}\cos(\phi))$**,** $v = \sqrt{8}L_{cav}/(W_o - W_i)$**,** W_i**,** W_o **respectively the incoming and outgoing kinetic energies,** L_{cav} **the cavity length,** ϕ **the particle phase at the cavity.**

- **If** $\Delta W/W \ll 1$ **this can be simplified in**

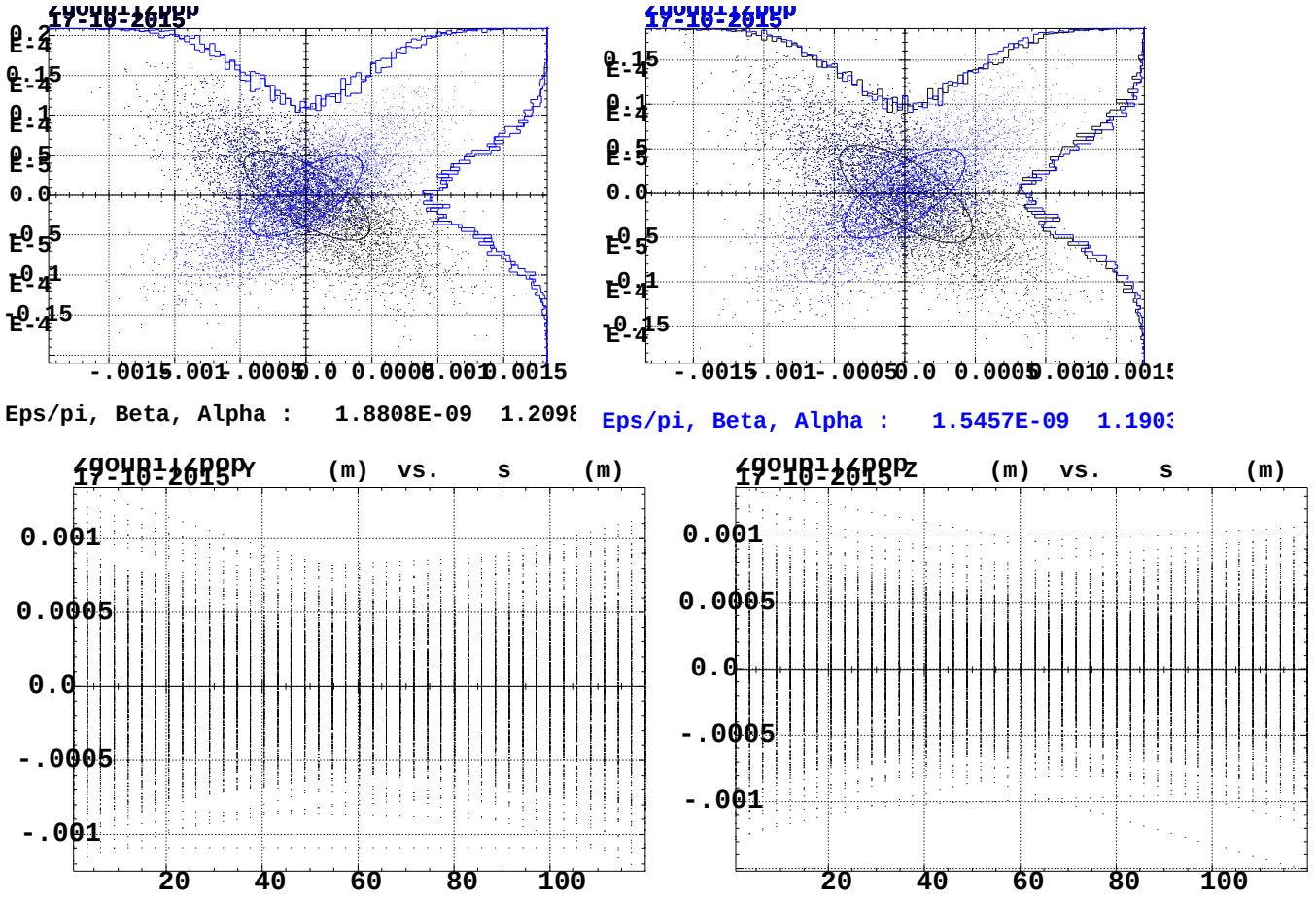
$$\begin{pmatrix} x \\ x' \end{pmatrix}_{out} = \begin{pmatrix} \sqrt{(W_i/W_o)} & L_{cav} \times \sqrt{(W_i/W_o)} \\ 0. & \sqrt{(W_i/W_o)} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_{in} \quad (4)$$

• **Now check tracking over two different passes in linac**

- Normalized emittances considered are $25\pi\mu$m. Initial bunch length 4 mm, $\Delta E/E \approx \pm 6 \times 10^{-4}$.

Low energy linac pass, 5.3→6.62 GeV

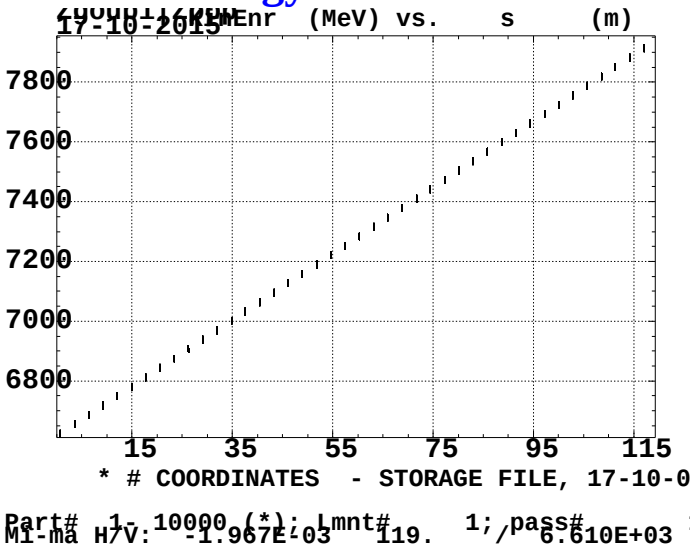
Phase spaces in and out of linac, beam bundle along linac :



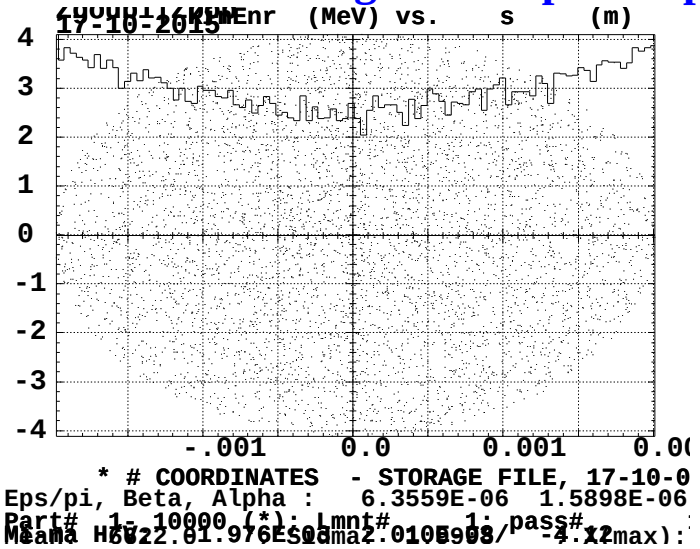
x and y normalized rms emittances are preserved.

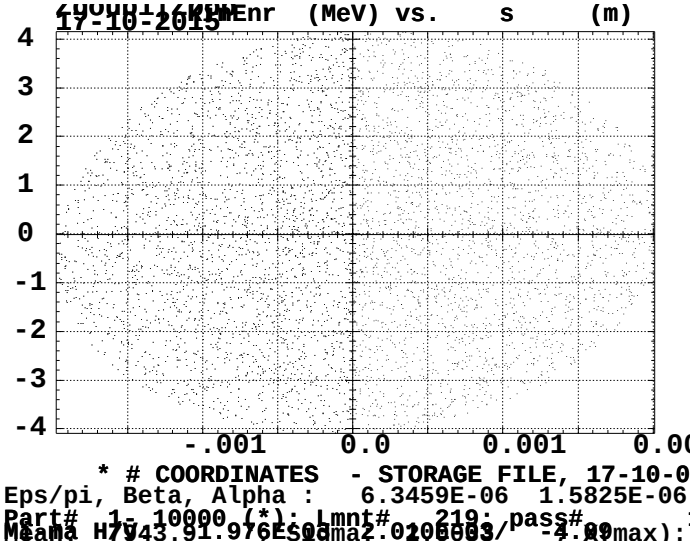
• **Initial** $\Delta E/E = \pm 4/6622$**.**

• **Energy vs. distance**



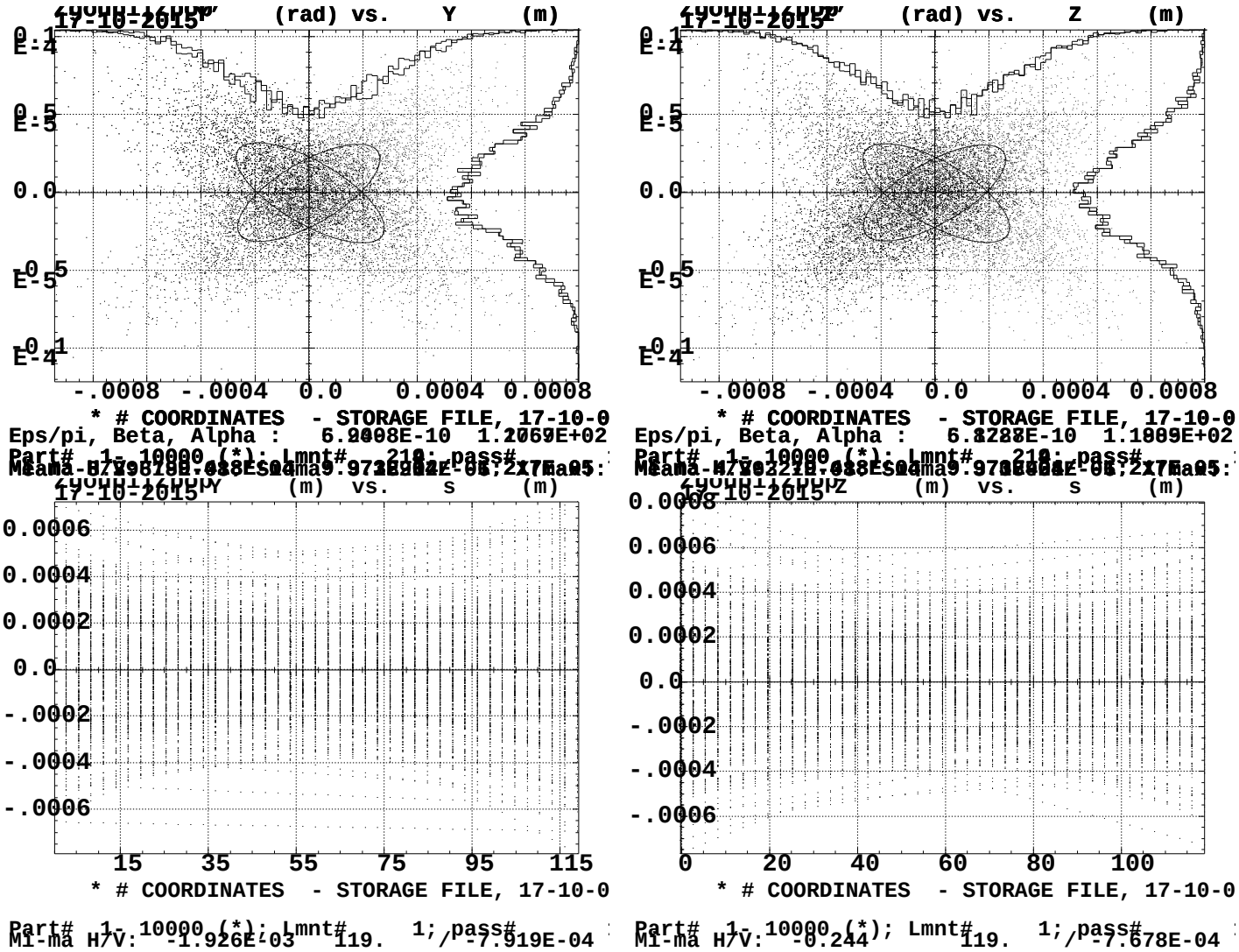
• **Longitudinal phase space in and out of linac.**





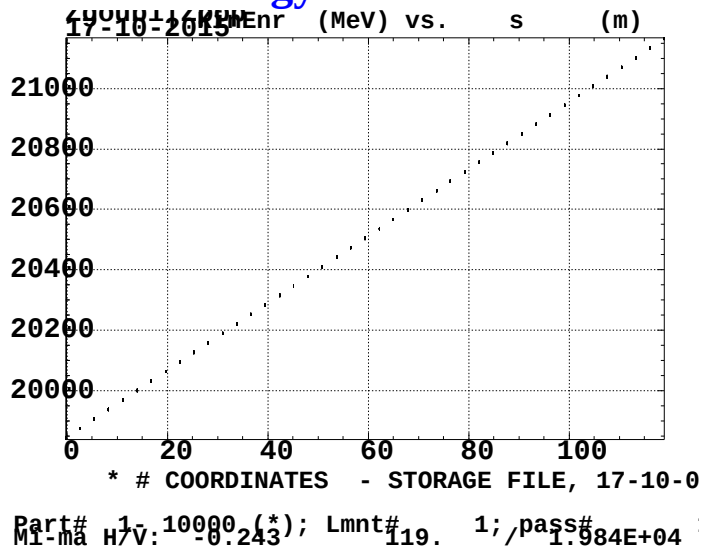
High energy linac pass, 19.8→21.16 GeV

- **Phase spaces in and out of linac, and beam bundle along linac :**

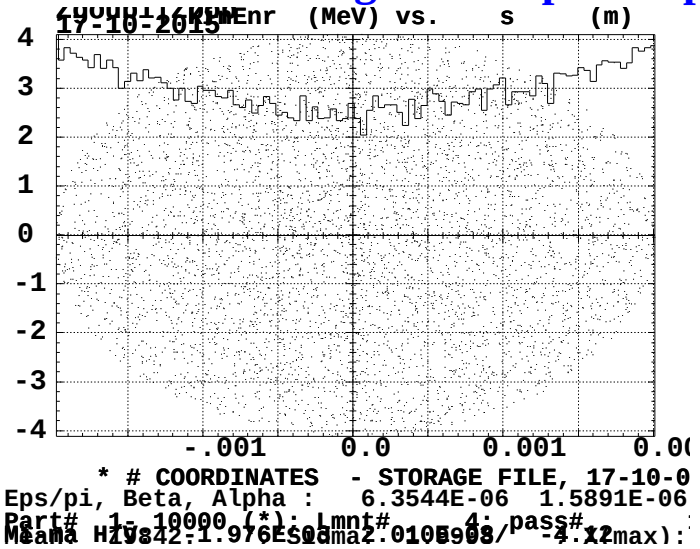


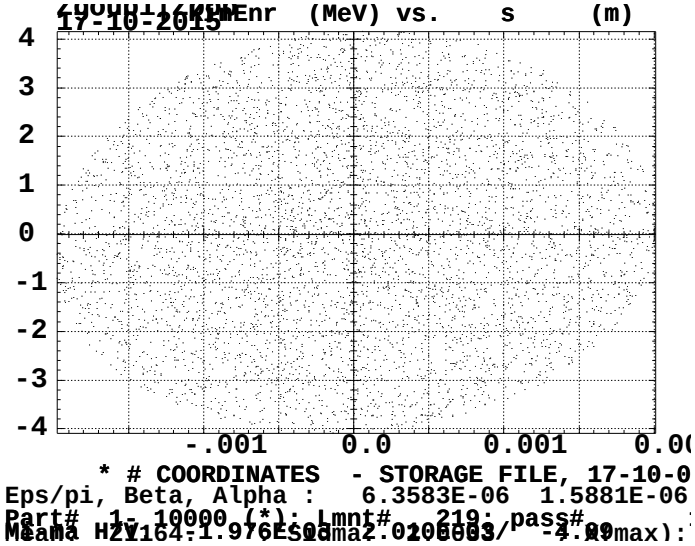
• **Initial** $\Delta E/E = \pm 4/6622$**.**

• Energy vs. distance

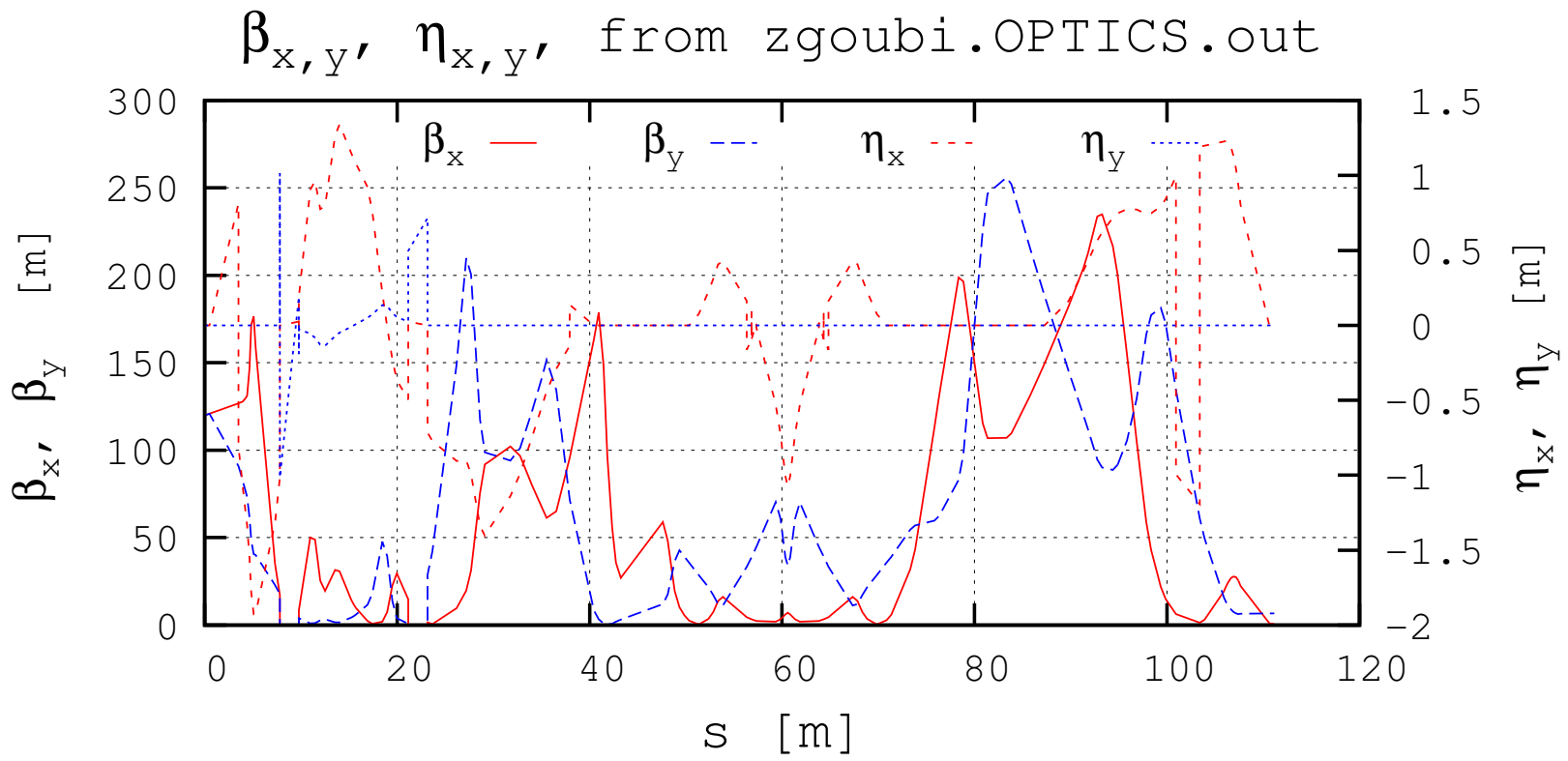


• Longitudinal phase space in and out of linac.





4 Spreader/merger lines



Optical functions in the 21.164 GeV spreader line (connected to the linac to the left, to the FFAG2 loop to the right). $\beta_x = \beta_y = 120$ **m and** $\alpha_x = \alpha_y = 1$ **to the left, and (FFAG loop side)** $\alpha_x = \alpha_y = 0$**,** $\beta_x = 3.5659$ **m,** $\beta_y = 26.1247$ **m,** $D_x = 5.46 \times 10^{-2}$ **m,** $D'_x = 0$ **to the right.**

- **The symmetric of this line is taken for the 21.164 GeV merger.**
- **That very line is used for all energies, with magnet settings scaled accordingly.**
- **Note : some local discontinuities are observed in the optical functions, artifacts.**

5 Multiturn optics

• The lattice in the up-down ERL tracking simulations has the following form

$$\text{ERL} = \underbrace{\text{merger} + \downarrow \text{linac} + \text{spreader}}_{\text{RHIC IR2 region}} + \text{FFAG2} \quad \text{(Observation point)}$$

with

LINAC : **42 cavities**

spreader / merger : all lines identical, scaled in rigidity

$$\text{FFAG2} = \text{ARC}-\text{DS}-\tfrac{1}{2}\text{LSS} + \underbrace{\left[\tfrac{1}{2}\text{LSS}-\text{DS}-\text{ARC}-\text{DS}-\tfrac{1}{2}\text{LSS}\right]}_{\textbf{4 times}} + \tfrac{1}{2}\text{LSS}-\text{DS}-\text{ARC}$$

and

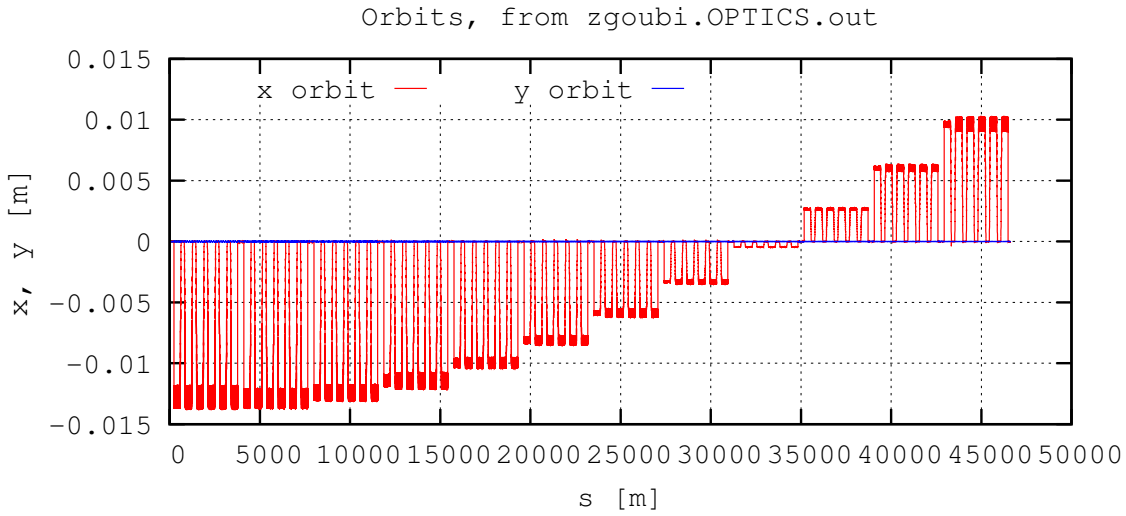
$$\text{ARC} = \textbf{102} \times [\tfrac{1}{2}\text{BD} - \textbf{drift} - \text{QF} - \textbf{drift} - \tfrac{1}{2}\text{BD}]$$

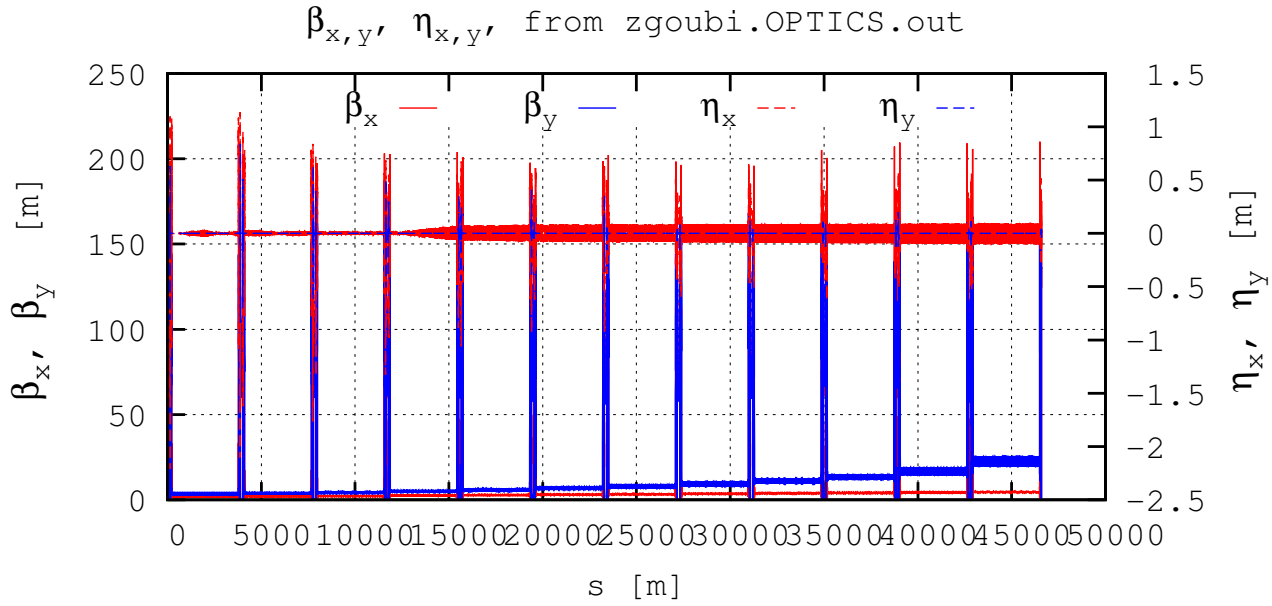
LSS : 52 quadrupole-doublet cells

DS : 18 transition FFAG cells

• Orbits and optical functions : complete transport of initial values

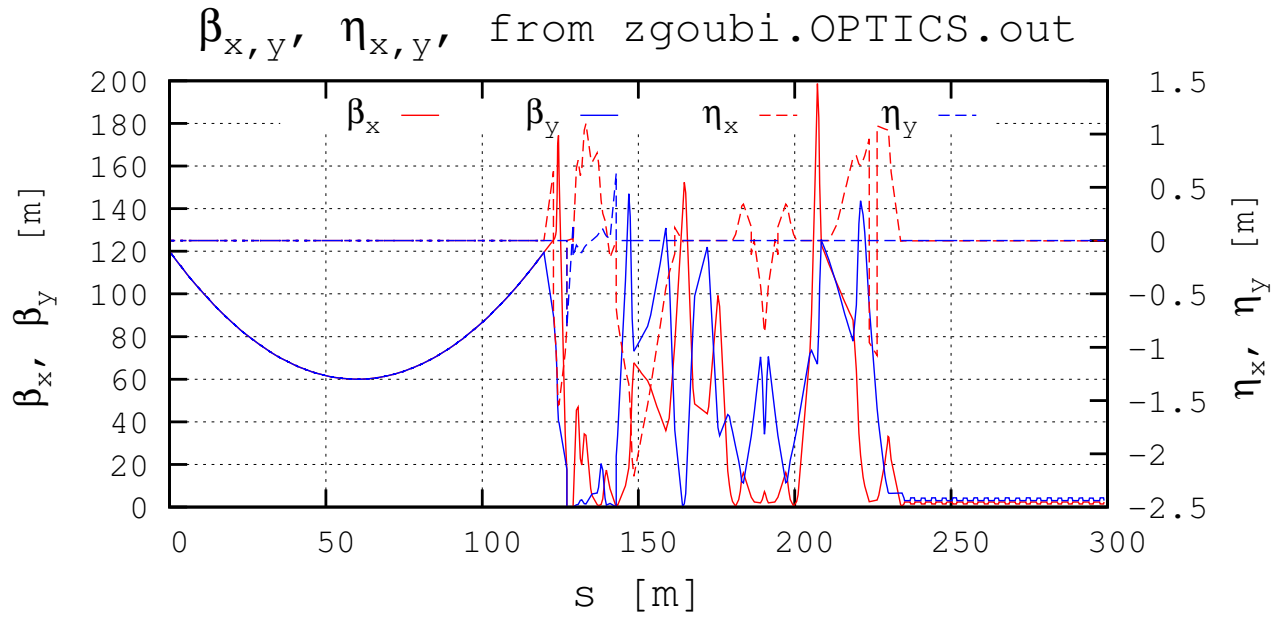
5.3 to 21.1 GeV - 12 loops



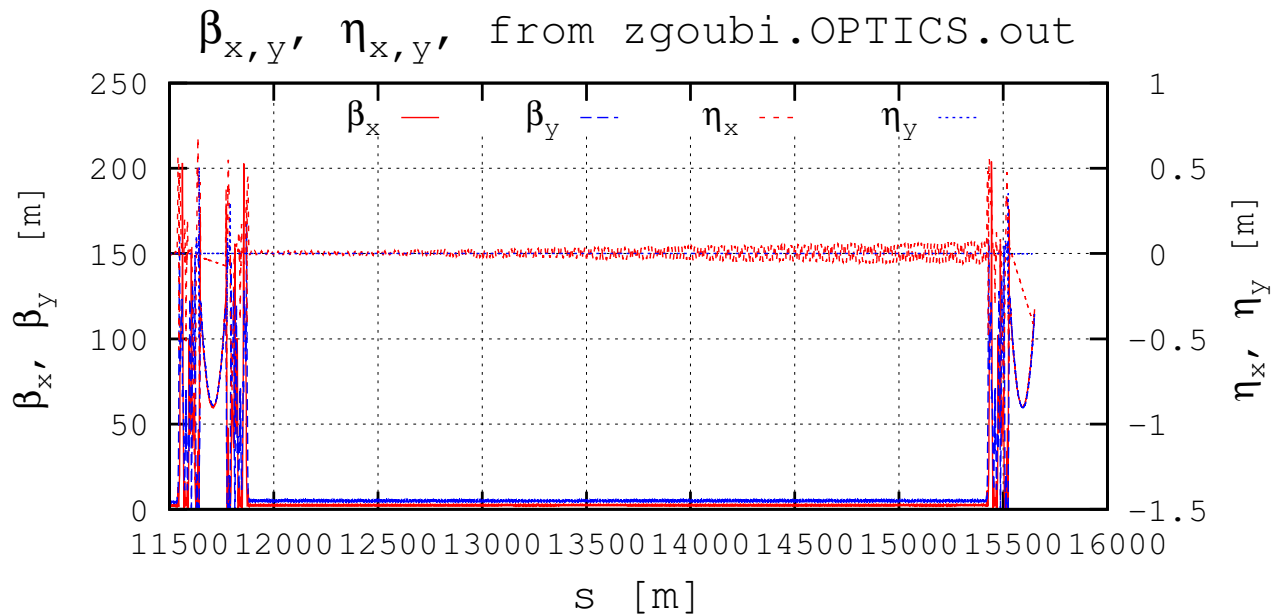


• DETAILS - make sure cumulating the many optical segments ends up in consistent start-to-end lattice

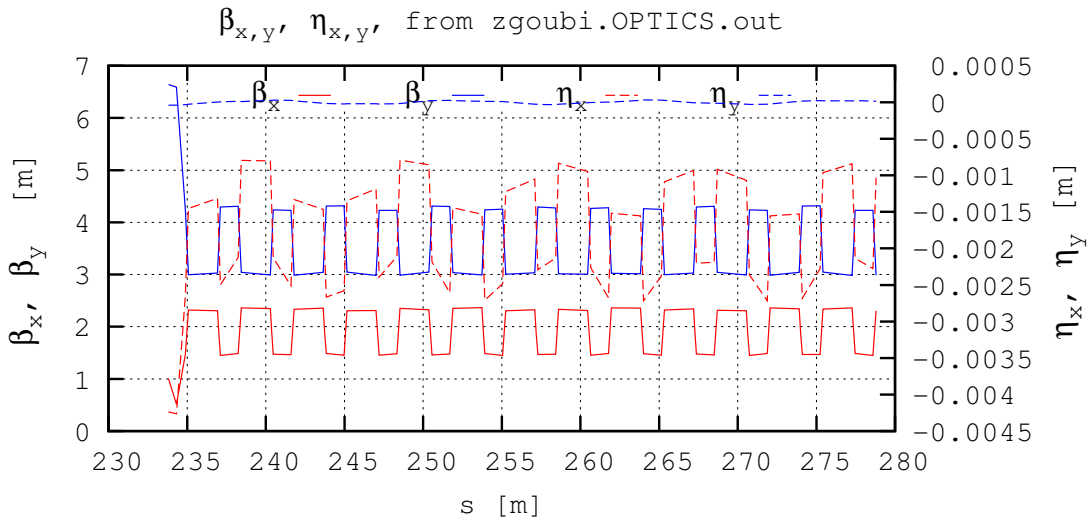
• 5.3 to 6.622 GeV linac pass



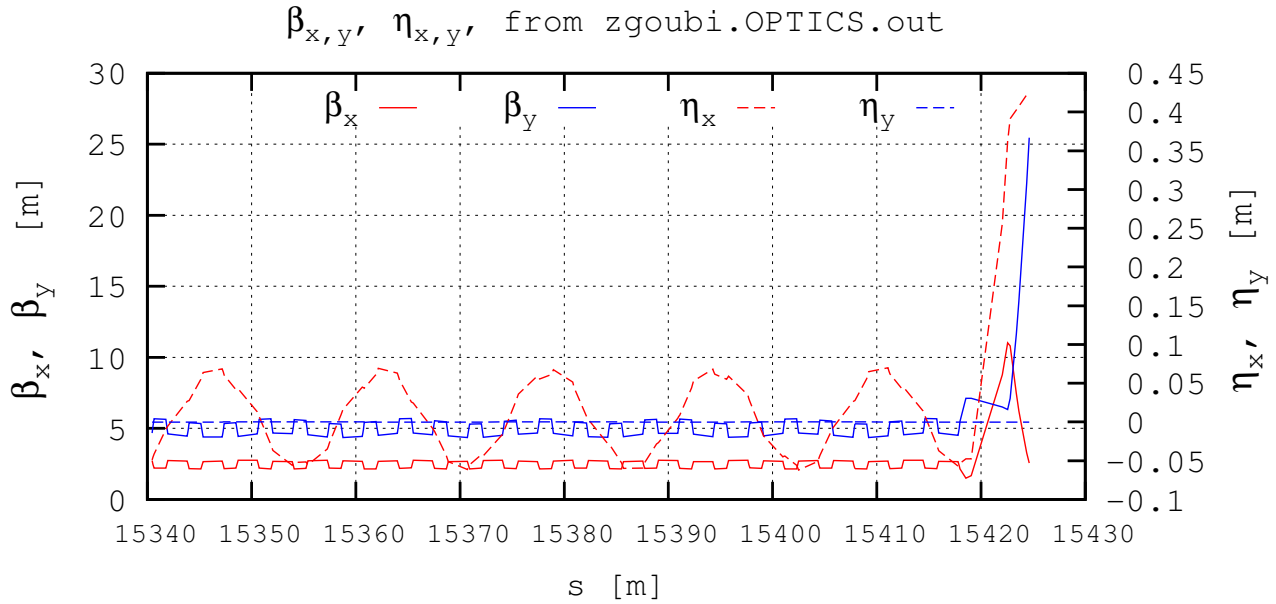
• 9.2 to 11.9 GeV linac passes



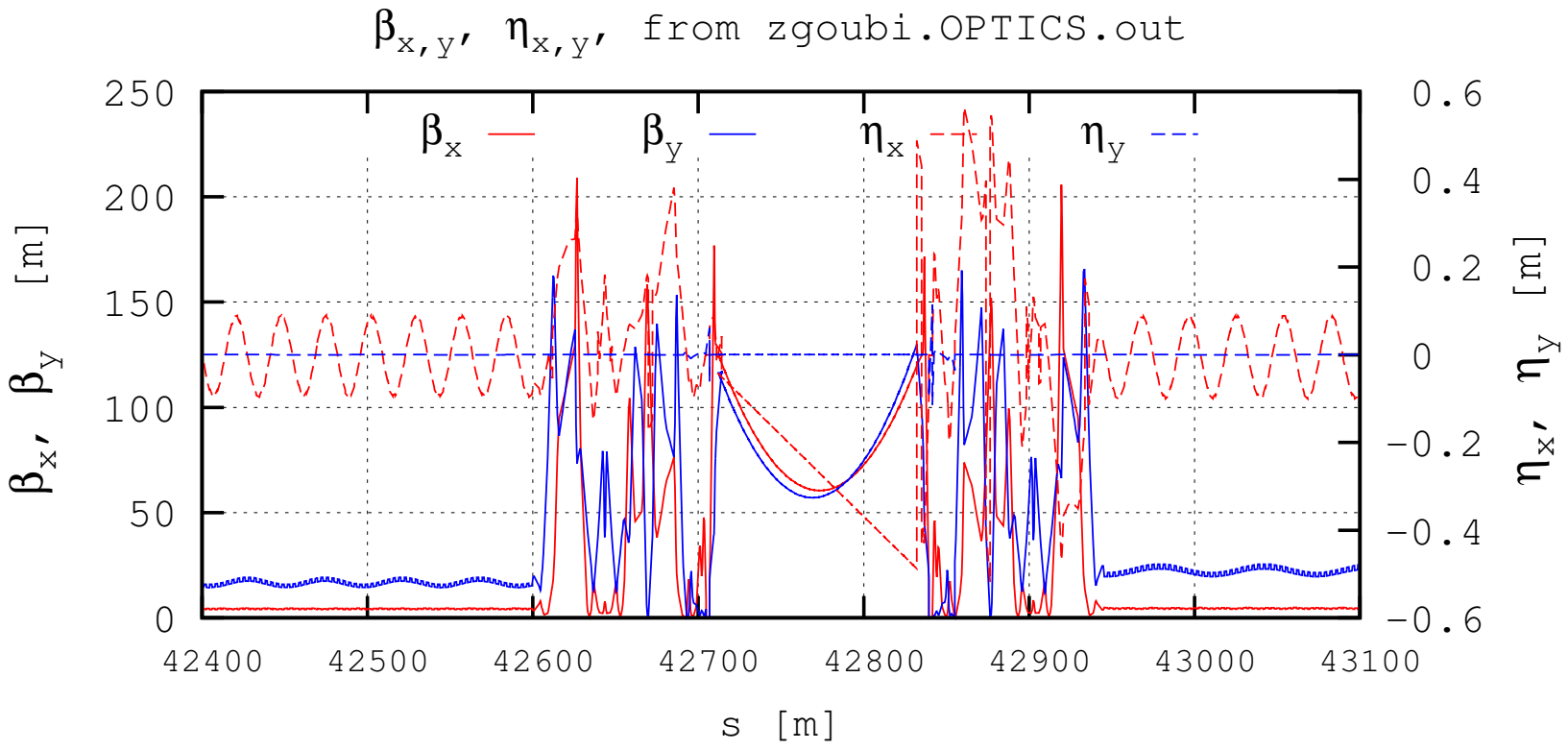
• Exit of 6.622 spreader line



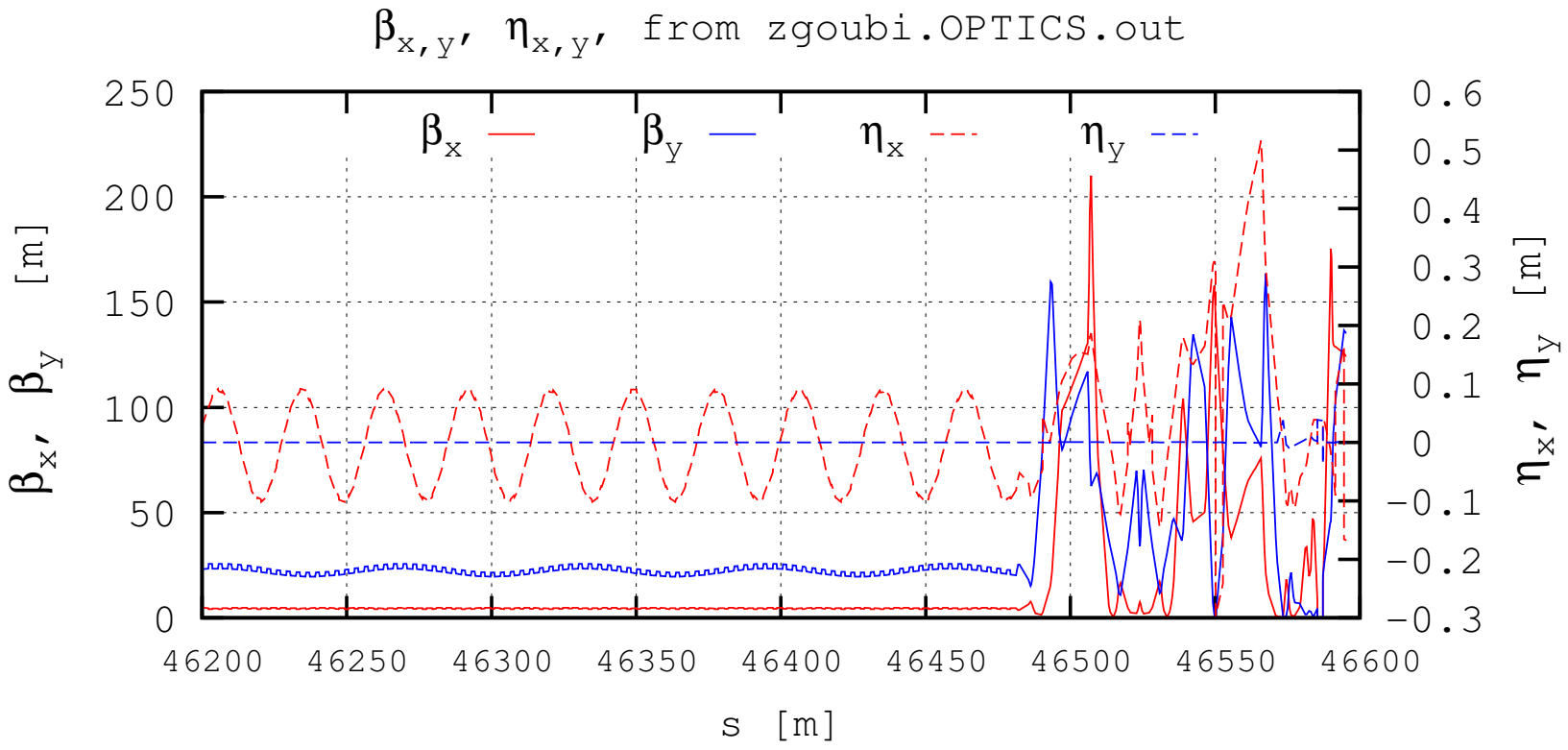
• Entrance of 10.5 GeV merger line



• 19.842 to 21.164 GeV linac pass



• **Merger region at 21.164 GeV, start of deceleration cycle**



• **Conclusion of this optics "tour" : transport of the initial orbit and optical funcitons over the 12 passes behaves correctly.**

6 Bunch tracking : Twelve linac passes and loops, up

Working hypotheses

In addition to the optical settings reviewed, namely, $\begin{cases} \text{FFAG2 loop} \\ \text{Linac} \\ \text{spreader/merger} \end{cases}$ **, we consider the following :**

• Artificial beam centroid centering is applied (Zgoubi's "AUTOREF") as follows :

- at entrance of linac (i.e., exit of any merger line) : bunch centering on linac axis, phase set on RF crest

- at exit of any splitter : horizontal centering on appropriate FFAG orbit and on design momentum

- at entry into any merger : horizontal (x,x') and vertical (y,y') centering on zero, momentum centering (that compensates SR loss)

- at downstream end of all dispersion suppressors (i.e., from arc to straight, at straight entrance) : horizontal (x,x') and vertical (y,y') centering on LSS axis.

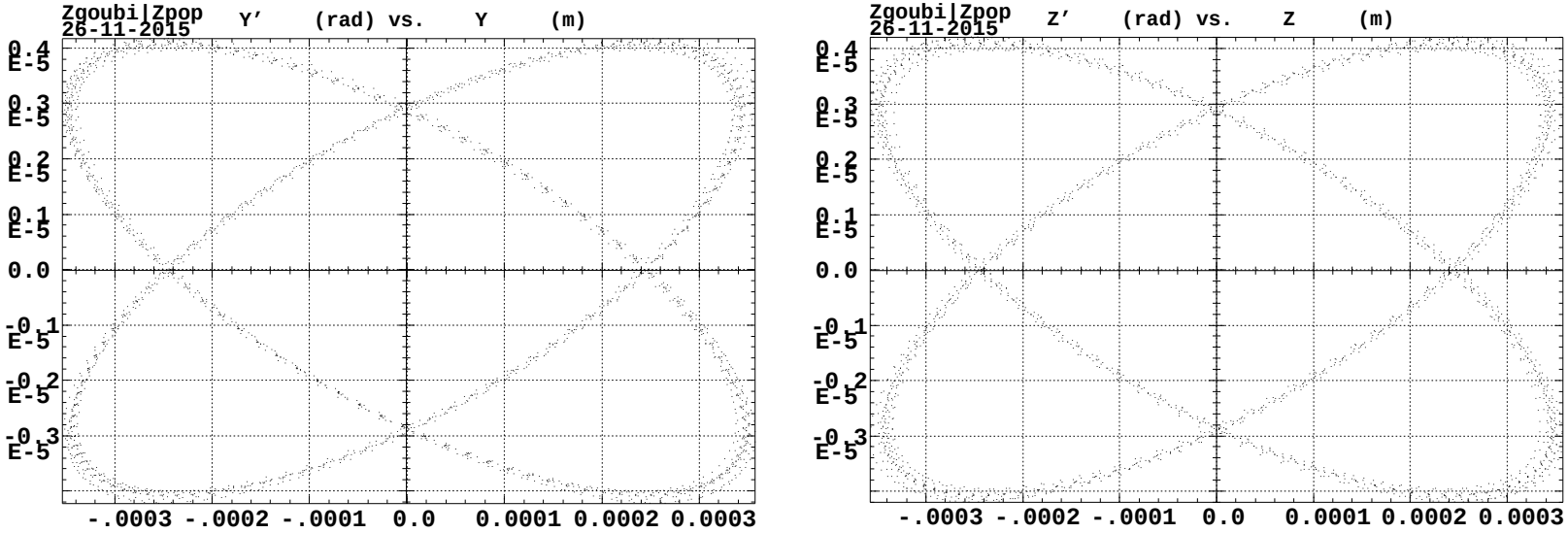
- SR is maintained off, always, in spreader and merger lines

• Beam ellipses at linac ends

• One hundred particles evenly distributed on paraxial invariant (either H or V) with $\beta = 120$ m, $\alpha = 1$ are launched at linac entrance with E=5.3 GeV, for 12 linac-pass tracking up to 21.164 GeV.

Result : Optical function values do remain $\beta = 120$ m, $\alpha = 1$ at % level, both planes, all the way, figure below

A superimposition of H ellipses (converging) and V ellipses (diverging) observed at linac ends
at the 12 linac passes (linac damping has been inhibited).
Obtained from the tracking of 100 particles launched evenly spread on a common invariant at 5.300 GeV.



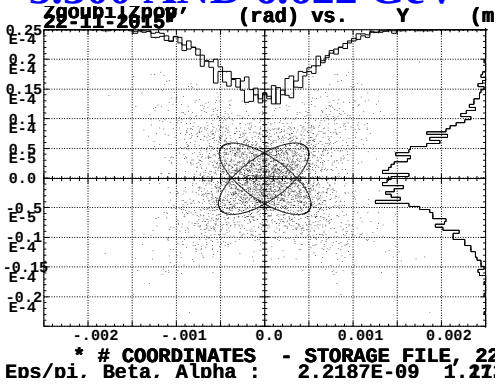
- **Initial 23μm transverse, zero longitudinal**

- **<u>No SR</u>**

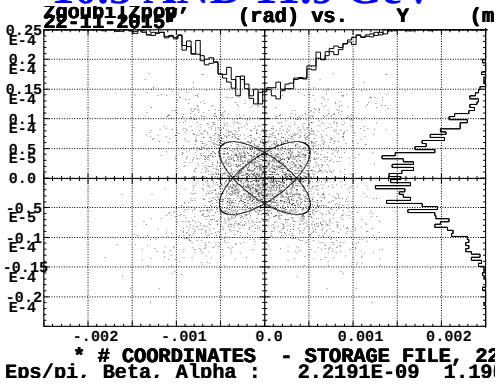
LINAC DAMPING OFF

(x,x'), AT LINAC ENDS

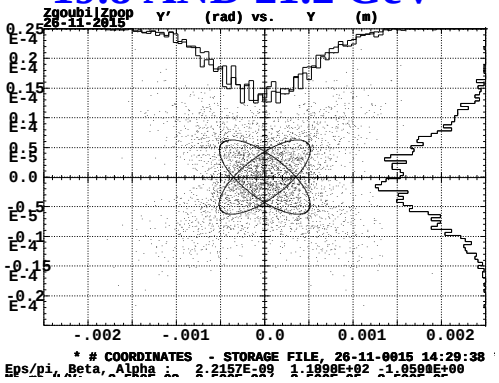
5.300 AND 6.622 GeV



10.5 AND 11.9 GeV

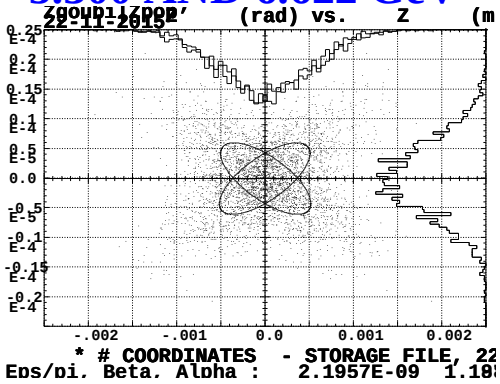


19.8 AND 21.2 GeV

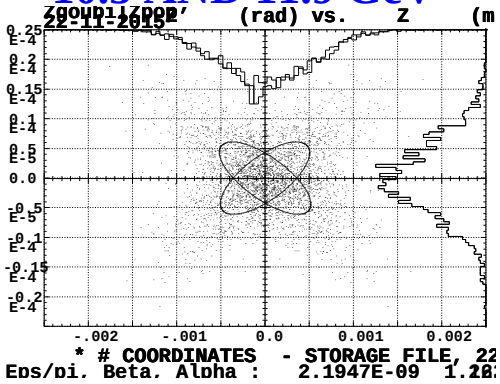


(y,y'), AT LINAC ENDS

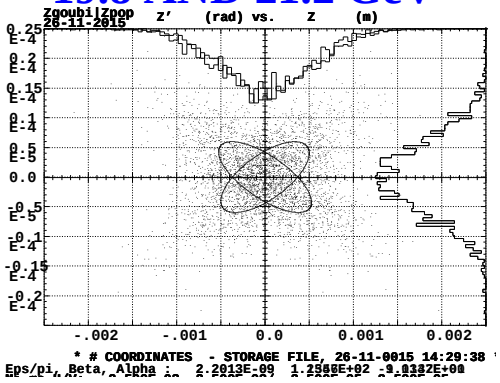
5.300 AND 6.622 GeV



10.5 AND 11.9 GeV

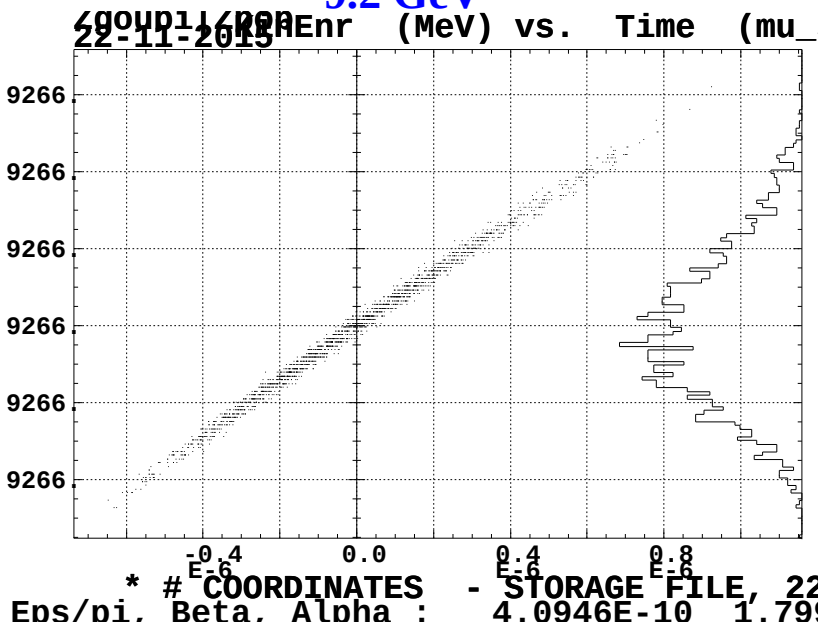


19.8 AND 21.2 GeV

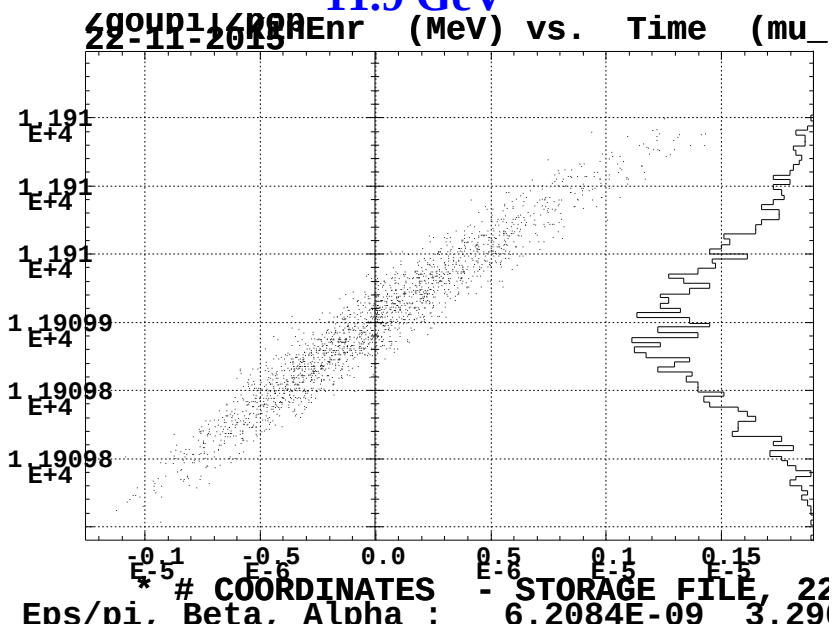


(time,kin-E), AT LINAC EXIT

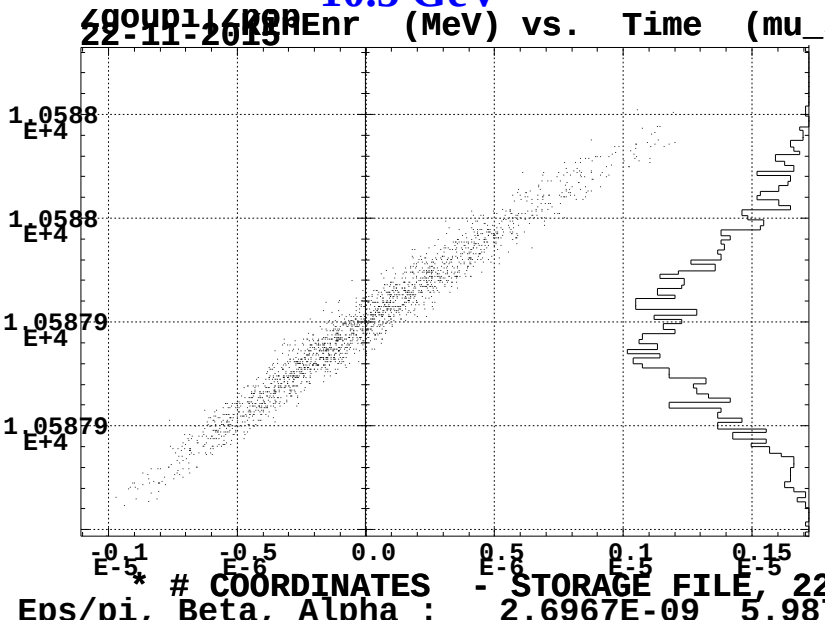
9.2 GeV



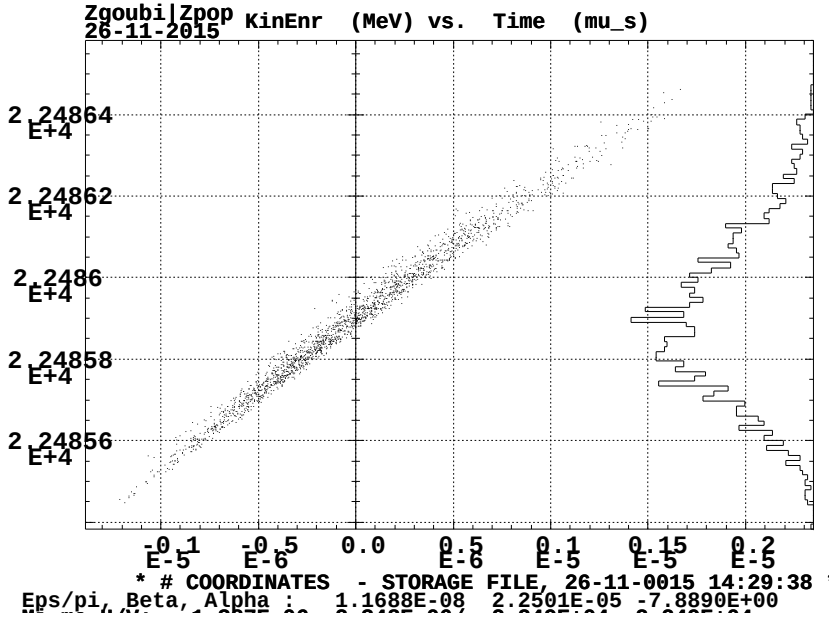
11.9 GeV



10.5 GeV



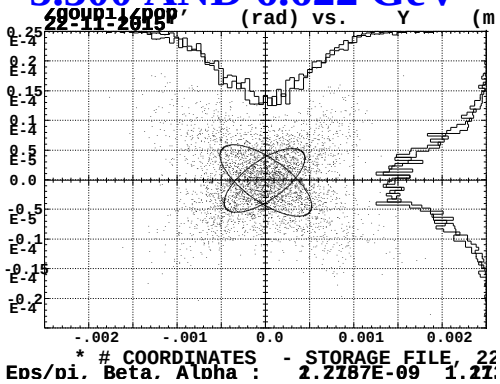
21.164 GeV



- **Including SR, linac damping set**

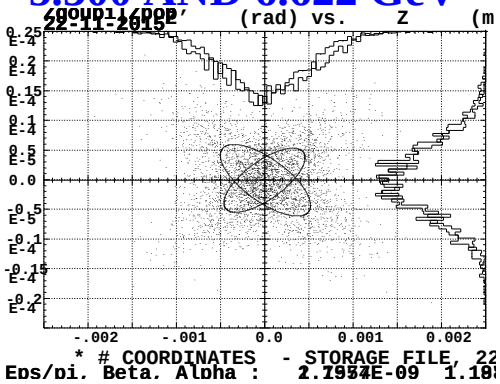
(x,x'), AT LINAC ENDS

5.300 AND 6.622 GeV

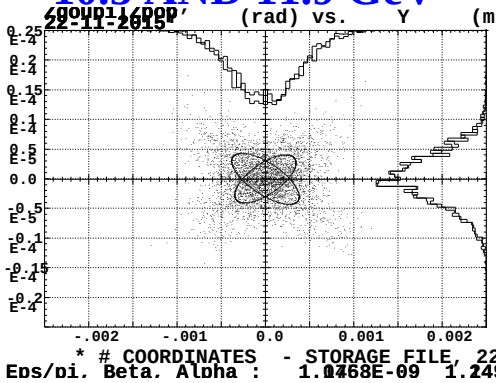


(y,y'), AT LINAC ENDS

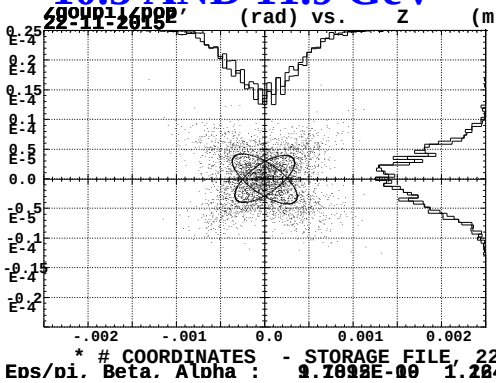
5.300 AND 6.622 GeV



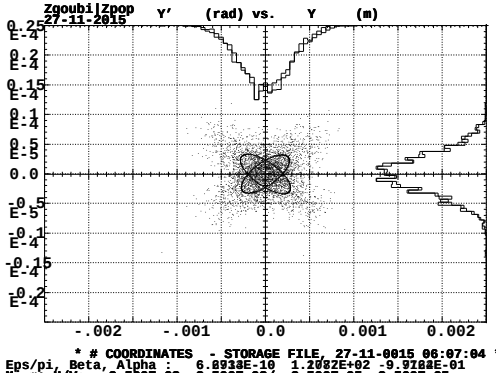
10.5 AND 11.9 GeV



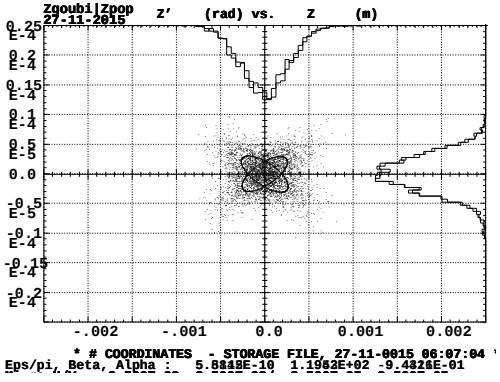
10.5 AND 11.9 GeV



19.8 AND 21.164 GeV

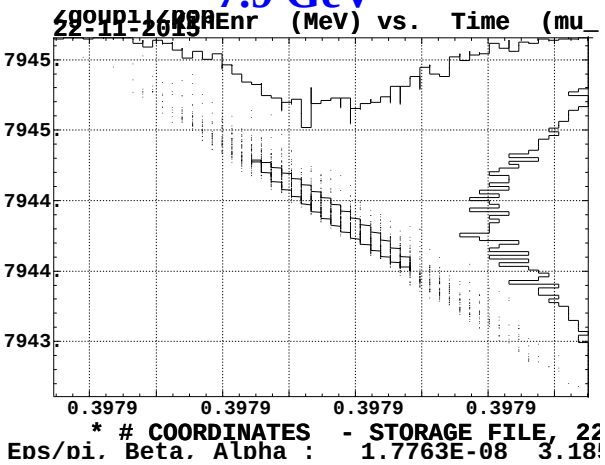


19.8 AND 21.164 GeV

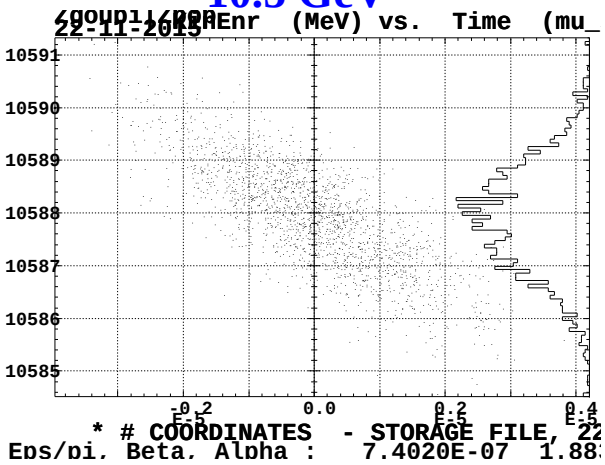


(time,kin-E), AT LINAC EXIT

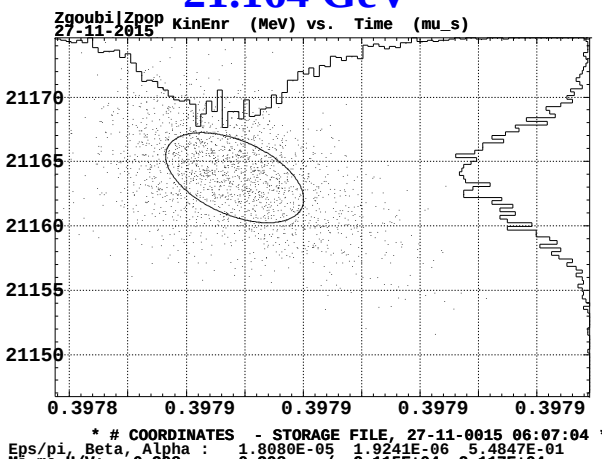
7.9 GeV



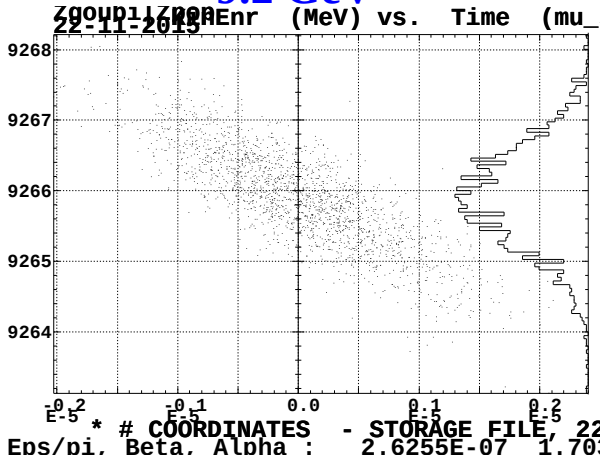
10.5 GeV



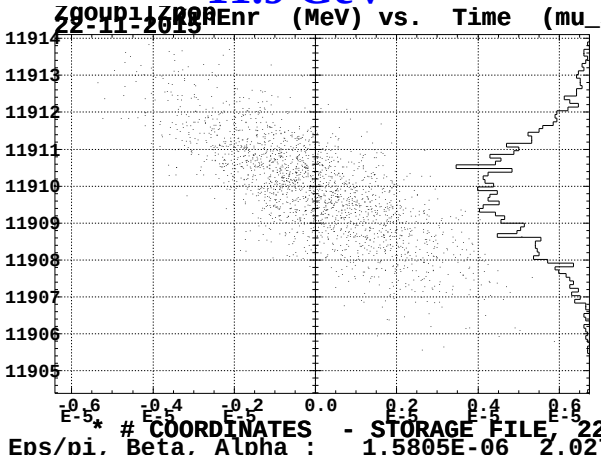
21.164 GeV



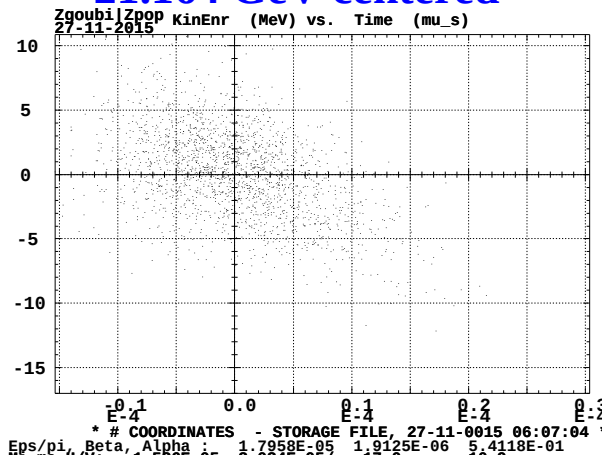
9.2 GeV



11.9 GeV

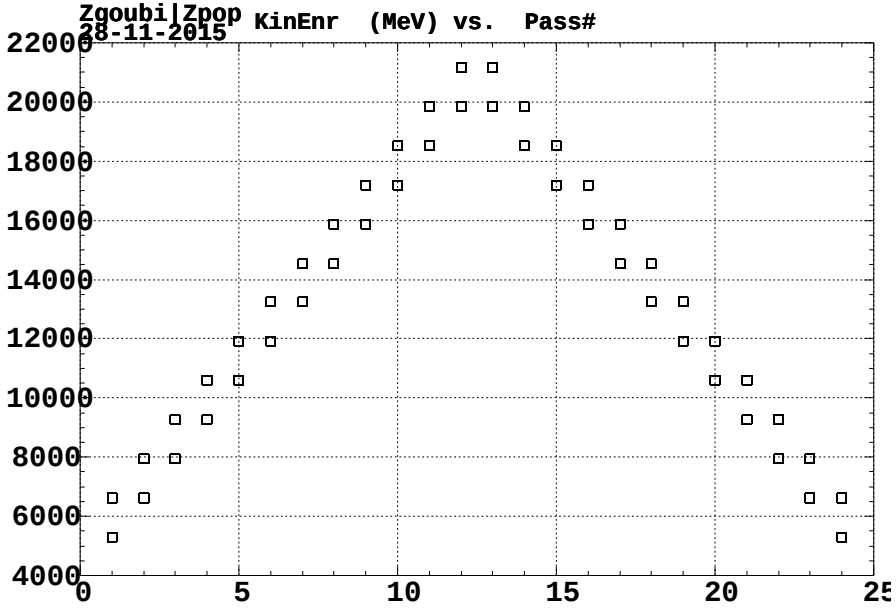


21.164 GeV centered



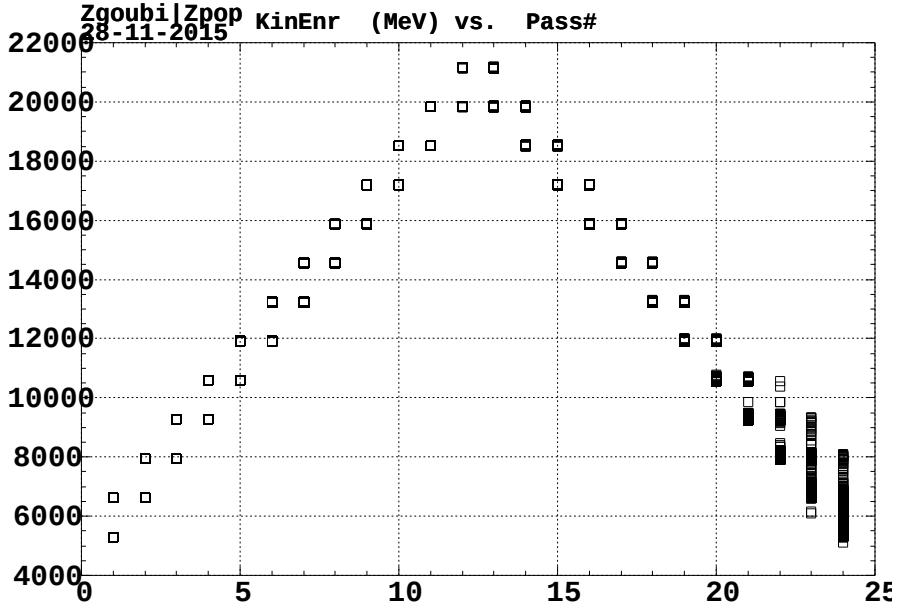
7 Start-to-end, up-down

SR off



Each square is actually a superimposition of 2000 particles, thus the beam remains well confined all the way from the first linac pass to extraction at 5.3 GeV.

SR on

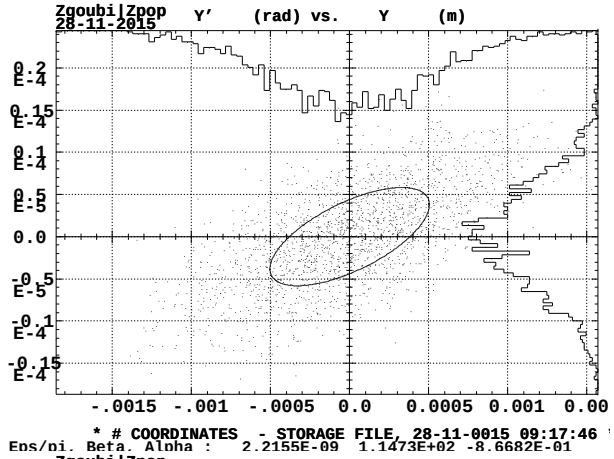


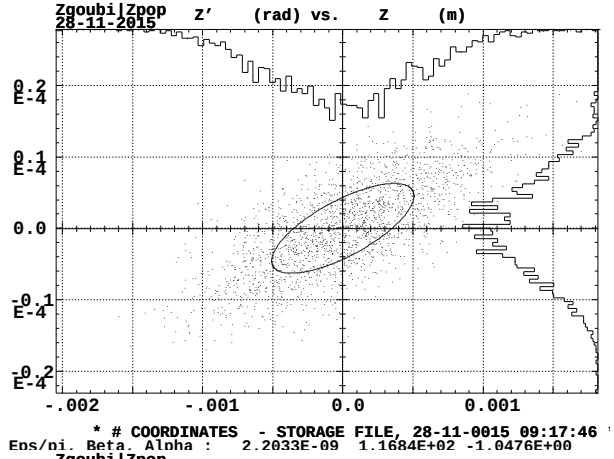
Serious beam spreading starts around linac pass number 19∼20.

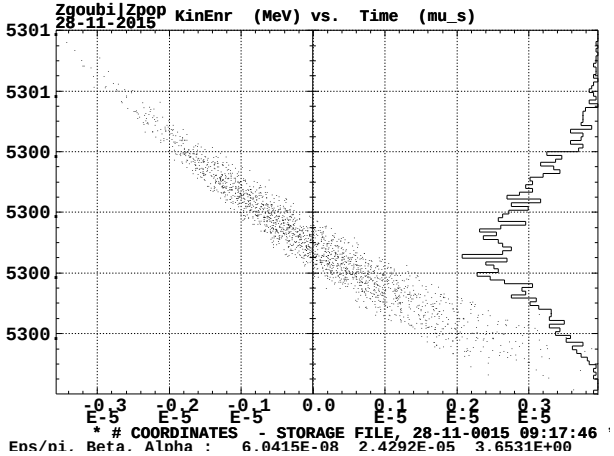
Note : there is no attempt in these simulations, of any longitudinal dynamics management using RF phasing. This improves longitudinal motion [Ref. Y. Hao, BNL].

• Final phase spaces after ER, 5.3 GeV

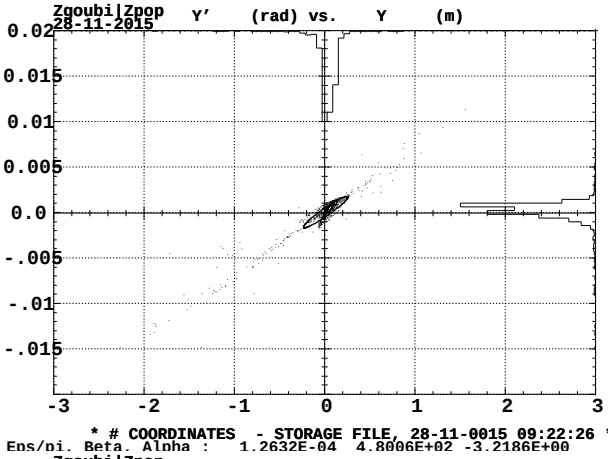
SR off

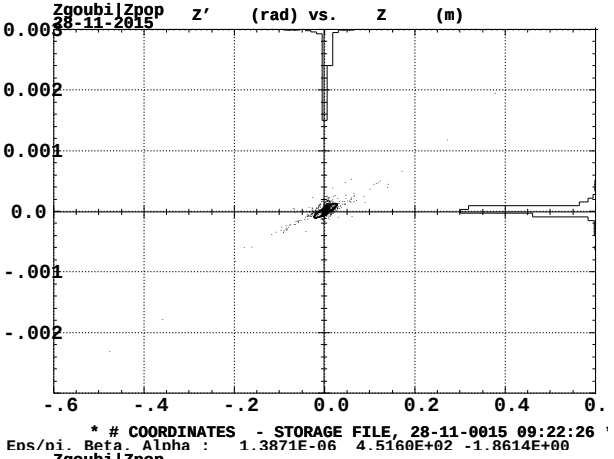


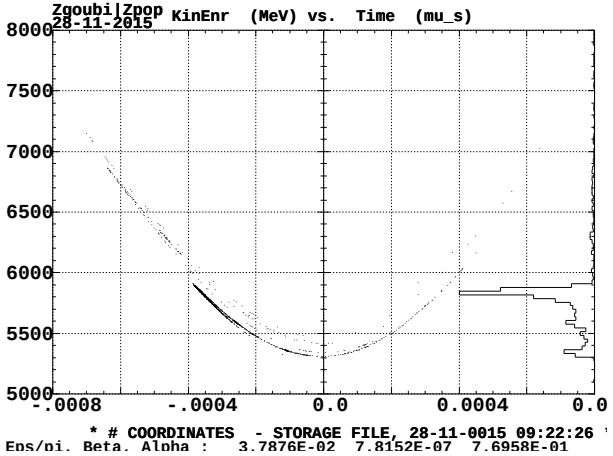


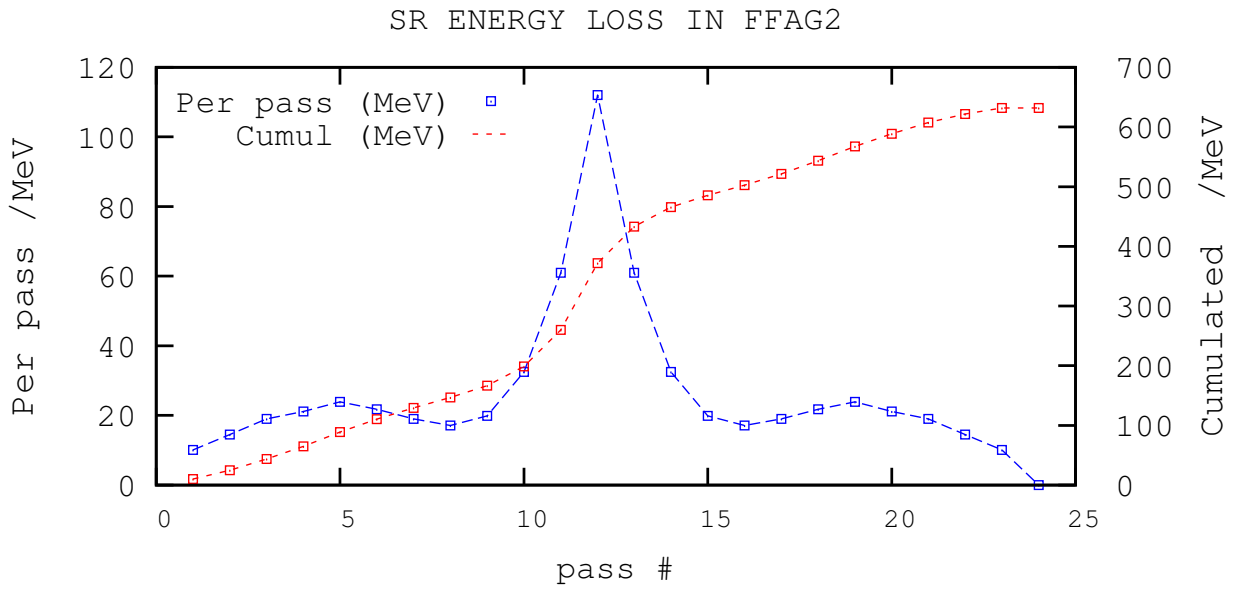


SR on

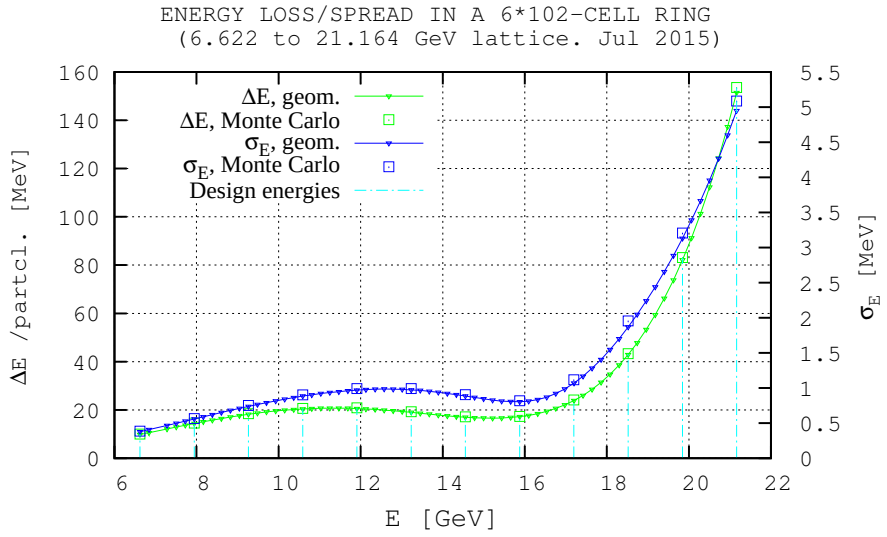








For the record, see slide# 4 (note, 120 cells/arc vs 102 cell/arc in FFAG2) :



$$\overline{\Delta E}[MeV] = \overline{\Delta E_{QF}} + \overline{\Delta E_{BD}} \approx 0.96 \times 10^{-15}\gamma^4\left(\frac{\theta_{\rm QF}}{|\overline{\rho_{\rm QF}}|} + \frac{\theta_{\rm BD}}{|\overline{\rho_{\rm BD}}|}\right) \quad \textbf{over cell} \overset{\times 6\text{arcs}\times 120\text{cells}}{\longrightarrow} \textbf{over ring} \tag{5}$$