The Chase for Large Dynamic Aperture vFFA Lattices...

(or)

Exploring Nonlinear Optics using Active Deep Learning for vFFA Lattice Design

Adrian Oeftiger, Andrea Santamaria Garcia,
Jean-Baptiste Lagrange, Simon Hirlaender

Links

This talk links to and builds on

- Feb '21 S. Machida et al., PRAB **24**, 021601: *"Optics design of vertical excursion fixed-field alternating gradient accelerators"* ↗
- 30.09.21 S. Machida, FFA'21: *"Optics Design of vFFA"*
  ⟶ vFFA triplet lattice, half-integer resonance, simulation analysis
- 12.09.23 M. Topp-Mugglestone, FFA'23: *"Analytic model of vertical FFAs"* ↗
  ⟶ vFFA triplet lattice, analytical analysis
- 15.09.23 M. Topp-Mugglestone, FFA'23: *"The FFA Code FixField"* ↗
  ⟶ tracking code used to compute closed orbit + dynamic aperture

Structure

- Introduction
- Results
  - Domain of Stable Closed Orbits
  - Dynamic Aperture
  - Tune Space
- Approach

Motivation

Starting point:

→ vFFA reference lattice for ISIS2-FETS parameters:

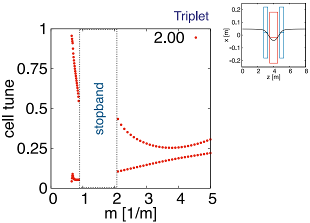
- has very good dynamic aperture (DA), but
- tunes are close to half-integer resonance stopband?

→ first simplistic space charge studies:

- beam intensity seems strongly limited by half-integer stopband

S. Machida, FFA'21

Straight lattice without total bending



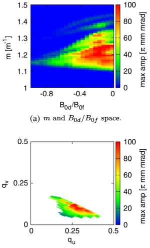
Motivating question for this study!

How can we "tune" the betatron tunes of the vFFA (at design stage) while maximising DA?

Context

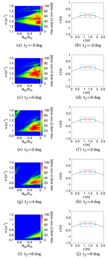
Starting from S. Machida et al., PRAB **24**, 021601:

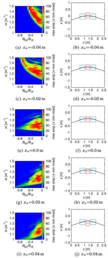
- 2D sections through multi-dimensional (5D) lattice parameter space investigated for **max. DA**
- simulation parameters scanned on regular 2D grids

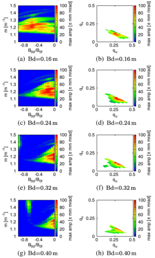


(a) m and B_{0d}/B_{0f} space.

(b) q_u and q_v space.







⟶ first significant insights for best DA (but grid scanning in full 5D space requires considerable computational effort)

⇝ can one improve coverage of parameter space by educated "guessing"?

Overview

Goal of present study: explore parameter space guided by data-driven approach to investigate relation between tunes and DA

$\Longrightarrow$ accommodate for space charge tune spread away from stopbands

$\Longrightarrow$ get lattice parameters for chosen tunes where DA is max!

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$\Longrightarrow$ get lattice parameters for chosen tunes where DA is max!

Approach: active learning (iterative supervised learning)

- build **uncertainty-aware** surrogate model to predict DA for lattice
- query model for largest uncertainties
- run DA simulations for these parameters

$\Longrightarrow$ improve surrogate model accuracy iteratively!

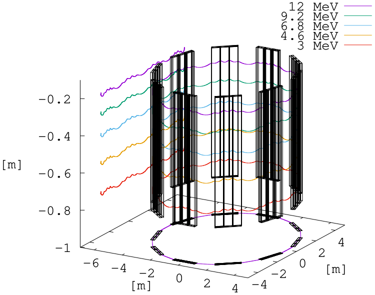
Status:

$\Longrightarrow$ simulated 200 000 lattices, collected 51 356 stable closed orbits & DA

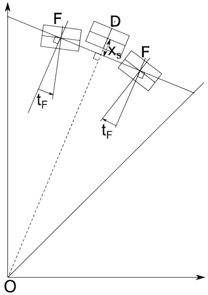
Machine Simulation Model

- 10-cell FDF triplet lattice, magnet length 50 cm, 3 MeV proton
- arctan fringe field model with $\lambda = 15\,\mathrm{cm}$, simulated in `FixField`
- 5D lattice parameter space (similar to PRAB'21):

$$(B_{0f}, B_{0d}, m, x_s, t_f) \in \mathscr{L}$$



(a) closed orbits for $(-1\,\mathrm{T}, 1.15\,\mathrm{T}, 1.31\,\mathrm{m}^{-1}, 0, 0)$ (reference lattice for ISIS2 FETS-vFFA)



(b) 5 parameters

Results

all_data

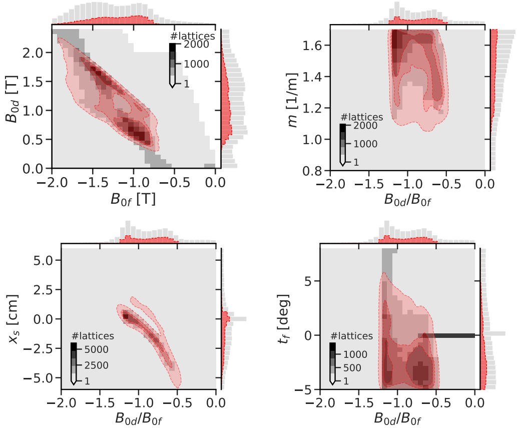
	b0f	b0d	m	xs	tilt	ID_clo	has_CO	x	xp	y	yp	stable_CO	qu	qv	ID_da	u	v	beta_u	beta_v	DA
0	-1.71821	0.0659491	1.05303	-0.016848	6.42728	0_0000	False	NaN	NaN	NaN	NaN	False	NaN	NaN		0	0	NaN	NaN	NaN
1	-1.46663	1.09767	1.28828	-0.0176404	2.70019	0_0001	False	NaN	NaN	NaN	NaN	False	NaN	NaN		0	0	NaN	NaN	NaN
2	-0.0500765	0.0486	1.09101	-0.041357	6.90459	0_0002	False	NaN	NaN	NaN	NaN	False	NaN	NaN		0	0	NaN	NaN	NaN
3	-0.822423	0.715469	1.16307	0.0538947	1.24005	0_0003	False	NaN	NaN	NaN	NaN	False	NaN	NaN		0	0	NaN	NaN	NaN
4	-1.05525	1.20707	0.859408	0.0410143	6.1225	0_0004	False	NaN	NaN	NaN	NaN	False	NaN	NaN		0	0	NaN	NaN	NaN
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
199856	-0.892178	0.71482	1.10493	-0.0147489	-4.2395		True	4.33672	8.80784e-06	-0.874347	7.4742e-06	True	0.79981	0.100356	6_9996	0.019886	0.047535	1.201174	5.355880	0.000751109
199857	-0.985175	0.579237	1.58108	-0.0415676	-3.81063		True	4.27842	2.20019e-06	-0.615888	3.91223e-07	True	0.864784	0.179517	6_9997	0.016077	0.007984	2.190085	2.894207	0.000140043
199858	-1.31767	1.02342	1.49853	-0.0189886	-1.45044		True	4.3177	-7.44249e-11	-0.792047	9.14914e-10	True	0.862953	0.164953	6_9998	0.019981	0.016847	1.954631	3.200540	0.000292933
199859	-1.34421	0.86187	1.66586	-0.0325432	-4.93544		True	4.29268	-9.85006e-06	-0.740032	-1.16912e-06	True	0.885052	0.208064	6_9999	0.012257	0.004212	2.687962	2.630656	6.26354e-05
199860	-1	1.15	1.31	0	0		True	4.35951	2.90234e-10	-0.728627	-3.27974e-10	True	0.243428	0.120039	7_0	0.026527	0.05986	0.951543	4.381166	0.00155739

199861 rows × 20 columns

$\Longrightarrow$ collected data for 199861 simulated lattices (available on github!):

- closed orbit coordinates
- betatron tunes: q_u, q_v
- maximum stable amplitudes in decoupled space: A_u, A_v
- decoupled beta functions: β_u, β_v
- transverse dynamic aperture: $D = \frac{A_u^2}{\beta_u} + \frac{A_v^2}{\beta_v}$

Domain of Stable CO



Distribution of identified **stable** closed orbits in 5D lattice parameter space. The number of lattices simulated per bin is shown in grey, where non-white bins contain at least one simulation. The red contours show the kernel density estimation for the distribution of valid closed orbits.

Decoupled β-functions

From `FixField` get 1-cell transfer matrix:

- matrix in real (coupled) space, $\mathbb{T}$
  - $\longrightarrow$ eigenvalues $\Rightarrow$ betatron tunes q_u, q_v (chosen such that $q_u > q_v$) if stable
- decoupled diagonal block matrix (with Parzen procedure [PAC'97]), $\widehat{\mathbb{T}} = \mathbb{R}\mathbb{T}\mathbb{R}^{-1}$
  - $\longrightarrow$ decoupled β-functions from

$$\widehat{\mathbb{T}}_{12} = \beta_u \sin(2\pi q_u)$$

and

$$\widehat{\mathbb{T}}_{34} = \beta_v \sin(2\pi q_v)$$

(or vice versa)

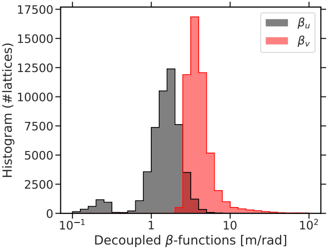
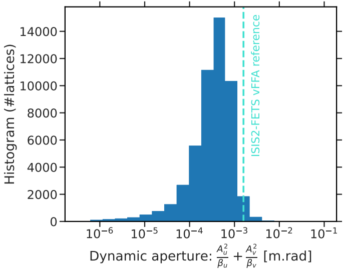


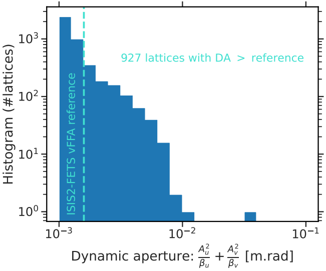
Figure: decoupled β-functions histogram

Dynamic Aperture

Dynamic aperture from interval bisection in u and v, separately:



(a) DA histogram

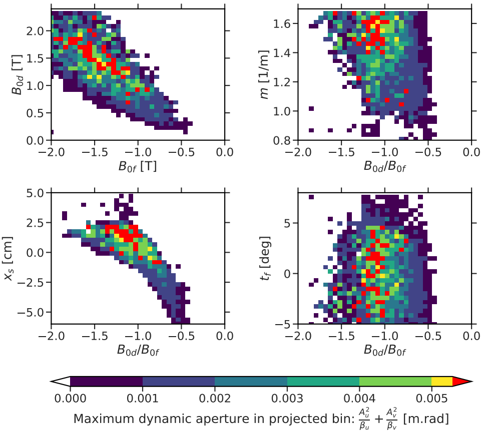


(b) zoom on upper DA histogram end

- reference vFFA ISIS2-FETS lattice: $\mathscr{L} = (-1\,\mathrm{T}, 1.15\,\mathrm{T}, 1.31\,\mathrm{m}^{-1}, 0, 0)$ features a dynamic aperture of $D = 1.6 \times 10^{-3}$

$\Longrightarrow$ found nearly 1000 lattices with larger DA

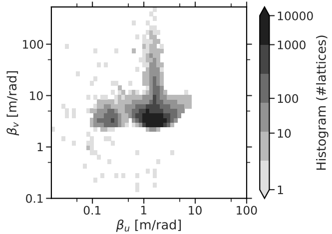
Dynamic Aperture in $\mathscr{L}$



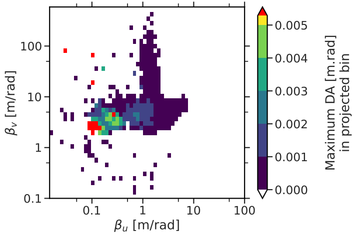
Simulation results for dynamic aperture (DA) in 5D lattice parameter space. Each panel plots the maximum DA per projected bin for the nearly 50 000 valid closed orbits. Red bins contain the **top 50 lattices** (overall maximum DA).

Decoupled β-function Space

Stable lattices in decoupled β-function space:



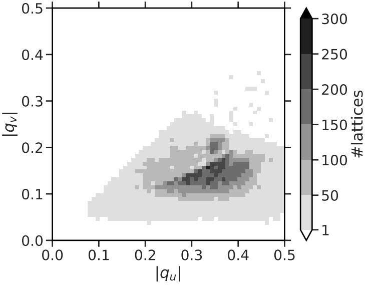
(a) histogram of stable lattices



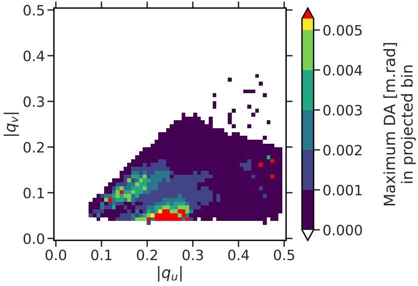
(b) max. DA vs. decoupled β-functions

$\Longrightarrow$ most large DA lattices located around $\beta_u = 0.1\,\text{m}$ and $\beta_v = 3\,\text{m}$

Stable lattices in betatron tune space:



(a) histogram of stable lattices



(b) maximum DA vs. tunes

$\Longrightarrow$ most large DA lattices are located close to half-integer resonance

$\Longrightarrow$ few "outliers" feature very large $\beta_{u,v}$ functions, to be understood...

Approach

Approach: Details

Approach in 3 steps (each iteration = 10k simulations):

1. uniform random sampling: **20k samples**

   $\Longrightarrow$ 429 lattices with CO

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   - classifier: 5D input $\mathscr{L} \to$ output = probability of CO existence
   - reject lattices with too low predicted probability!

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3. rejection sampling + prediction with DA surrogate model: **30k more samples**
   - ensemble of 5 deep neural networks predict DA
   - ensemble mean = prediction, ensemble std = uncertainty

   $\Longrightarrow$ $\approx$ 99.5% lattices with CO

$\Longrightarrow$ 200k total lattices with $\approx$ 25% stable CO

Simulation Duration

Simulations with `FixField` on the GSI Green Cube (virgo cluster):

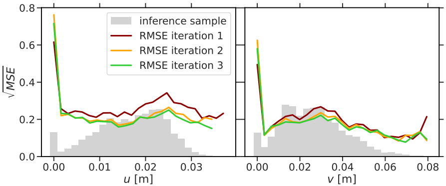
- CO simulation only: $\approx 1\,\text{min}$ CPU wall time
- CO + 2D DA simulation: $\approx 5\,\text{h}$ CPU wall time

Step	Lattice samples	Total CPU wall time
1. uniform random sampling	20 000	$\approx$ 2 weeks
2. rejection sampling with classifier	150 000	$\approx$ 3.5 months
3. rejection sampling + DA surrogate	30 000	$\approx$ 17 years

+ evaluating DA for $\approx$ 20 000 lattice samples from steps 1.+2. = 11 years CPU wall time

Iterations with NN

Prediction on unseen test data (last 10 000 data points):



Observation:

- → uncertainty vs. DA amplitude goes down with iteration
- ⇝ can we accelerate the improvement with better neural network structure?

Prediction Quality

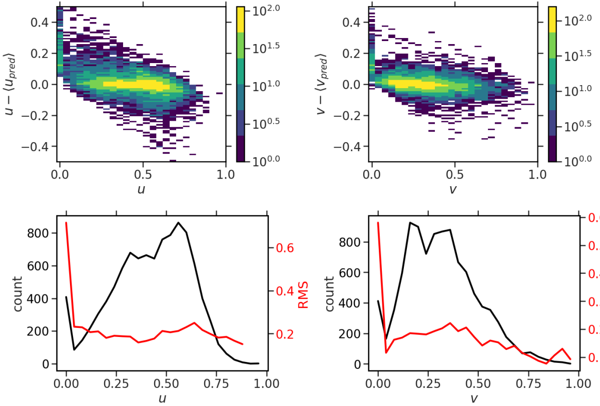


Figure: Prediction Accuracy vs. (normalised) DA amplitude per plane (model trained on ≈ 40000 data points)

⇝ remove unstable closed orbits (DA= 0) from NN prediction model?

Summary:

- data-driven approach successively builds a prediction model: $\mathscr{L} \mapsto \mathrm{DA}$
- predict most interesting area in lattice space $\mathscr{L}$ for next simulations
- collected $\approx 50\,000$ lattices with stable closed orbit (CO)
- trained model is $> 99.5\%$ successful in predicting CO
- established $\approx 50\,000$ links between tunes & DA

Conclusion

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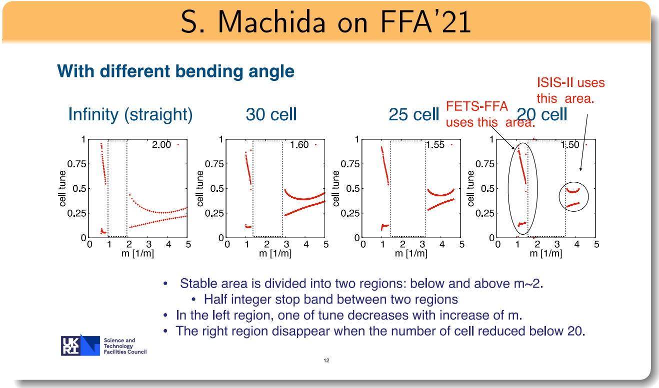
Next Steps:

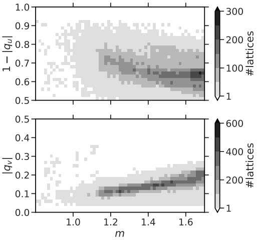
1. include stability criterion in CO classification
   (so NNs only need to predict finite DA values and not zeros)
2. improve NN prediction model with quantitative analysis
   (metric: uncertainty prediction between nearest neighbour data points in lattice space)
3. develop inverse prediction model: tunes $\mapsto \mathscr{L}|_{\text{max. DA}}$

Thank you for your attention!

Acknowledgements: Shinji Machida

Tunes vs. *m*





histogram of stable lattices