Transverse coasting instabilities in scaling FFAs

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with thanks to N. Biancacci (CERN)

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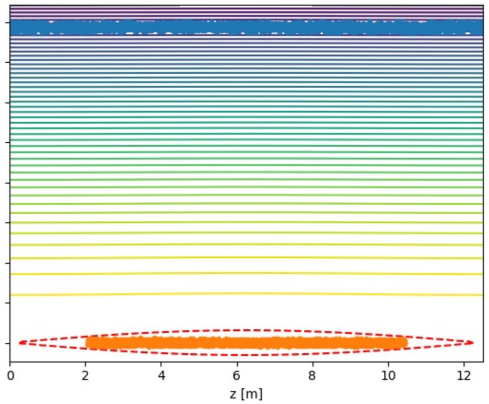
Stacking in FFAs

- FFAs naturally accommodate stacking because of their large momentum acceptance, DC magnets and flexible RF.
- This allows the output repetition rate to be varied and the space charge limit at injection to be circumvented (e.g. stack 4 beams in ISIS-II to extract at 25 Hz instead of 100 Hz).
- The stacked beam coasts while newly injected beam is accelerated. How do coasting beam instabilities affect the stacked beam?
- Here we consider transverse instabilities only.



Extraction energy

coasting stacked beam

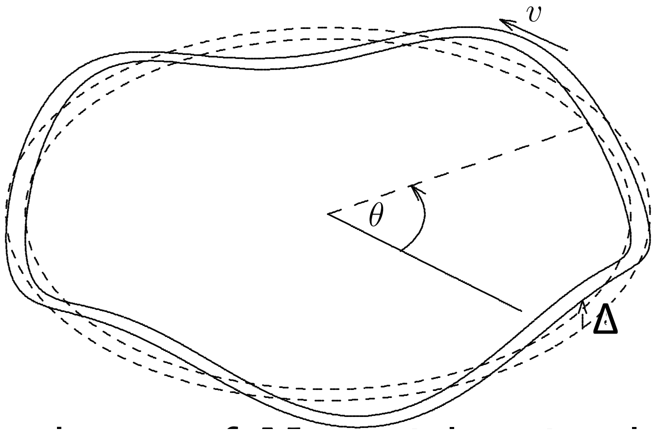
Injection energy

bunched beam

Relevant features of scaling FFAs

Zero chromaticity	Removes one source of Landau damping.
Nonlinear fields (B ~ r^k)	Tune shift with amplitude could provide Landau damping
Wide aperture magnet	Parallel plate aperture can often be assumed, $Z_V = 2*Z_H$

Transverse coasting beam instability



- Consider a proton beam of charge qN in a ring of circumference C =2πR. Assume uniform line density ρ(s) = qN/C. Assume a transverse perturbation d(s,t) of form

$$d(s,t) = \Delta e^{j(\Omega t - ns/R)}$$

n: mode number, Ω: angular frequency of perturbation

- The corresponding dipole moment is $D(s,t) = d(s,t)\rho(s) = \frac{qN\Delta}{C} e^{j(\Omega t - ns/R)}$

- The equation of motion in transverse plane, including wakefield produced by the perturbation, is

$$\ddot{y} + \omega_\beta^2 y(s,t) = \frac{\langle F_y \rangle (s,t)}{\gamma m_0}$$

where the force term is given by the driving and detuning wake function

source particle displacement

test particle displacement

$$F_y = -\frac{q^2}{C} \left(\frac{\partial W}{\partial \hat{y}} \bigg|_{y=0} \hat{y} + \frac{\partial W}{\partial y} \bigg|_{\hat{y}=0} y \right)$$

driving/dipolar wake

detuning/quadrupolar wake

N. Biancacci et al, PRAB 23, 124402 (2020)

Driving/dipolar impedance

- Impedance is given by Fourier transform of wake function

$$Z_y^{driv/det}(\omega) = -j\int_0^{\infty} W_y^{driv/det}(z)e^{j\omega z/v}dz/v$$

- Rewrite equation of motion with force replaced by $\left\langle F^{driv}\right\rangle(s,t) = -\frac{qv}{C}D(s,t)jZ_y^{driv}(\Omega)$

$$\ddot{y} + \omega_\beta^2 y(s,t) = -j\frac{qv}{\gamma m_0 C}D(s,t)Z_y^{driv}(\Omega)$$

- Defining the mode frequency $\Omega = n\omega_0 - \omega_\beta + \Delta\Omega_n$ and assuming small frequency shift

$$\boxed{\Delta\Omega_n^{driv} = -\frac{q^2vN}{2\gamma m_0 C^2\omega_\beta}jZ_y^{driv}[(n-Q_y)\omega_0]}$$

- Complex frequency shift can result in exponential growth (neg. imaginary part, i.e. positive real Z) and frequency shift (real part) via $e^{j\Delta\Omega_n^{driv}}$

Growth rate (turns) : $\tau_n = -\frac{1}{T_0\mathrm{Im}(\Delta\Omega^{driv})}$ **Norm. frequency shift:** $\frac{Re(\Delta\Omega^{driv})}{\omega_0}$

Resistive wall impedance

(circular aperture, δ_{skin} << wall thickness)

- The longitudinal impedance follows from resistance per unit length = 1/σA . Note the skin depth $\delta_{skin} = \sqrt{\frac{2}{|\omega|\mu\sigma_c}}$

$$\frac{Z_{\parallel}^{circ}(\omega)}{L} = \beta\frac{sgn(\omega)+j}{2\pi b\delta_{skin}\sigma}$$

- The corresponding transverse impedance is given by

$$\frac{Z_{\perp}^{circ}(\omega)}{L} = Z_0\beta\delta_{skin}\frac{sgn(\omega)+j}{2\pi b^3}$$

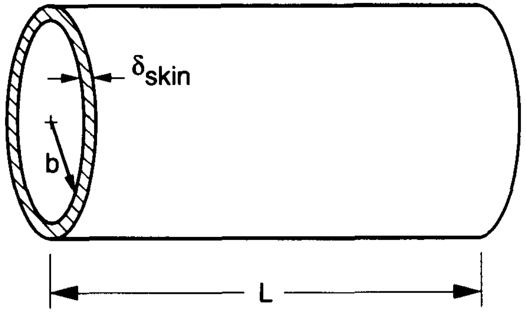
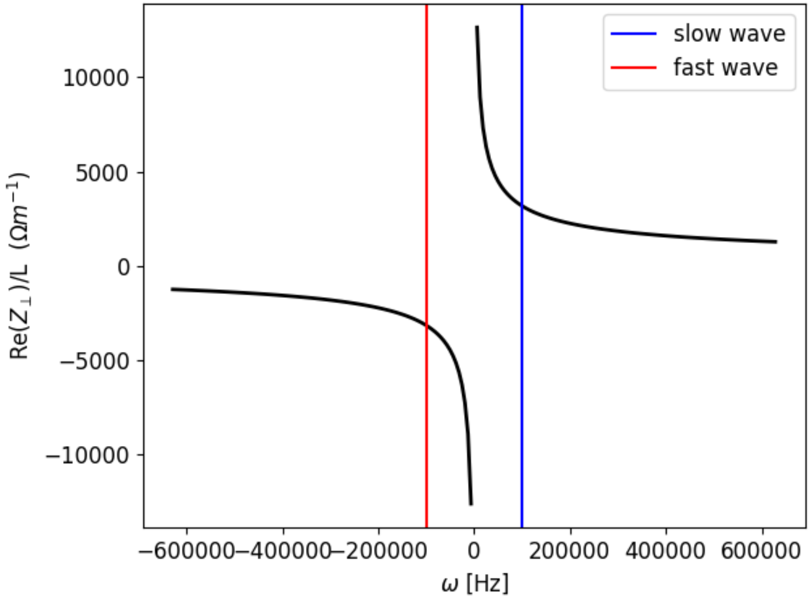


Figure: A. Chao



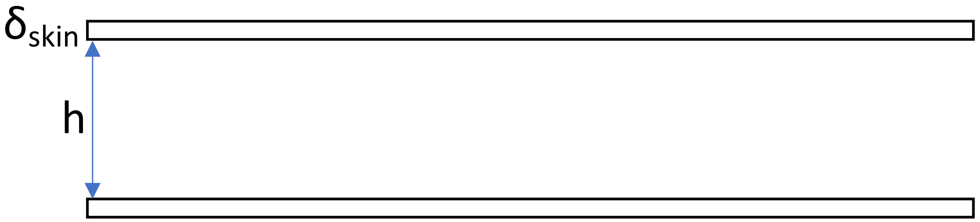
Resistive wall impedance

(parallel plate aperture, δ_{skin} << wall thickness)

- In the parallel plate case, the vertical impedance is double the horizontal

$$\frac{Z^{circ}_{V,H}(\omega)}{L} = Z_0 \beta \delta_{skin} F_{V,H} \frac{sgn(\omega)+j}{2\pi(h/2)^3} \qquad \text{where } F_{V,H} = \frac{\pi^2}{12}, \frac{\pi^2}{24}$$

- Assuming a parallel plate geometry is a reasonable starting point for a rectangular aperture.



- Implies vertical instabilities will have a faster growth rate – in fact this is not necessarily true.

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	Fast wave	Slow Wave
	$n - Q_y < 0$	$n - Q_y > 0$
$\mathrm{Re}(Z^{RW})$	Negative	Positive
$\mathrm{Im}(\Delta\Omega)$	Positive	Negative
Stability	Stable	Unstable

PS study – driving/dipolar impedance only

- Coasting beam study in PS where a parallel aperture is assumed. Assume zero momentum spread and zero chromaticity.
- Instability at the lowest slow mode, n=7 (just above the working points $Q_H=Q_V=6.4$).
- Growth rate/frequency shift calculated by modified version of PyHEADTAIL is in agreement with theory.

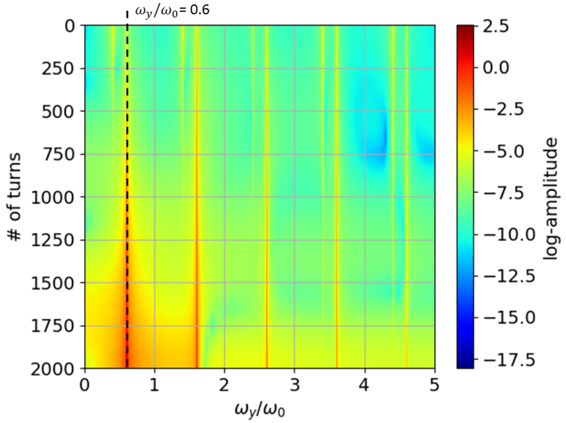

FIG. 2. Amplitude spectrogram of the instability simulated with PyHEADTAIL accounting for the driving impedance only at the intensity of 1×10^{13} charges. The most unstable line corresponds to the slow wave $n = 7$, for which $(n - Q_y) = 0.6$.

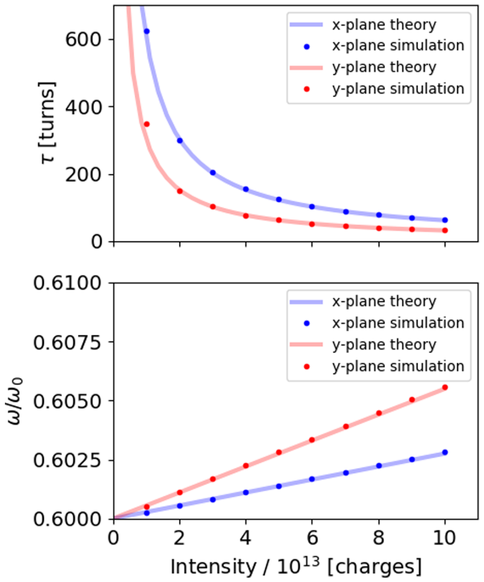

FIG. 3. PyHEADTAIL simulations (with dots) compared to theory [16] for driving impedance only (with full lines) for the horizontal (blue) and the vertical (red) planes. The rise time of the most unstable mode is shown at the top while the normalized frequency shift is shown at the bottom.

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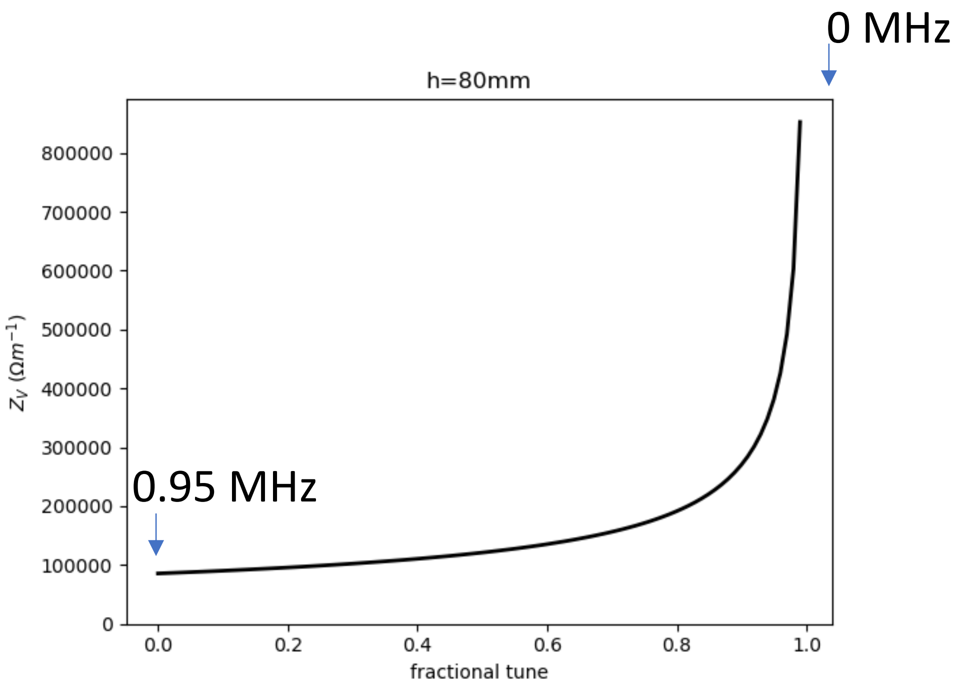
“ISIS-II” vs PS

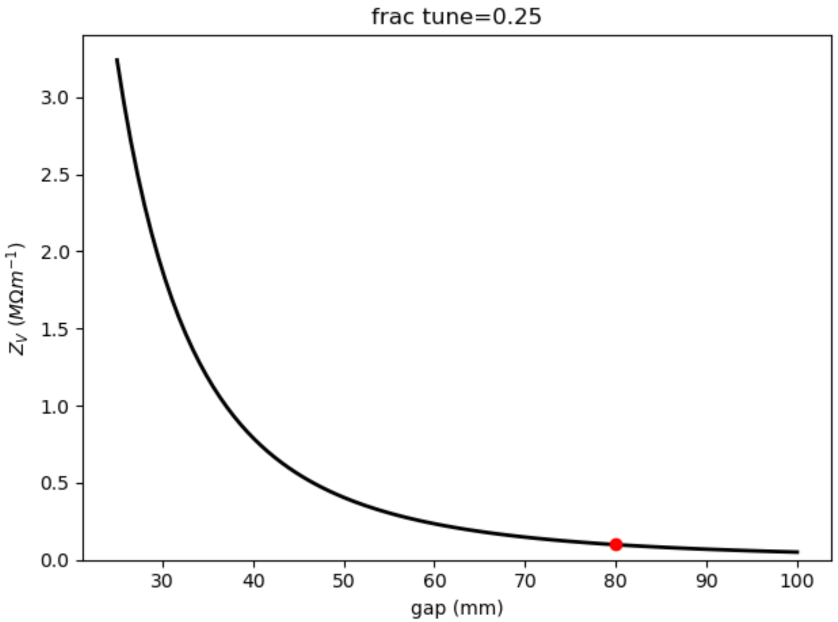
	PS	ISIS-II
Kinetic energy	1.4 GeV	1.2 GeV
Radius	100m	45m
Revolution frequency	0.44 MHz	0.95 MHz
Intensity	1e13 – 1e14	1.3e14
Magnet half gap	27.5 mm	40 mm (FETS-FFA)
Tunes (H,V)	6.4, 6.4	10.25, 10.25 (?)

- No lattice established for ISIS-II FFA as yet.

Effect of tune and aperture on impedance



- Choice of fractional tune determines the slow wave frequency = $(n - Q_v)\omega_0$.
- The closer the fractional part is to 1, the higher the impedance.

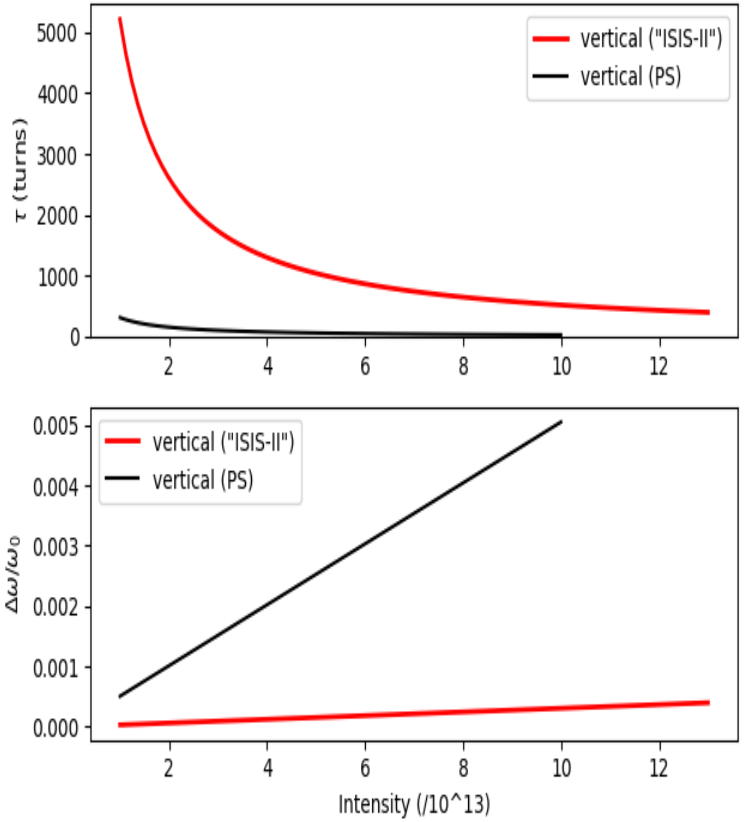


- Impedance varies with 1/h^3.

“ISIS-II” growth rate/frequency shift

	PS	“ISIS-II”
N	1.3e13	1.3e14
f_{slow}[MHz]	0.26	0.71
$Z_{V,H}$ [MΩ/m]	1.1	0.1
τ (turns)	242	400

- Order of magnitude increase in intensity in ISIS-II offset by small impedance (higher frequency and aperture).



Detuning/quadrupolar impedance

- The distortion of symmetric modes in a non-circular aperture introduces a detuning impedance.
- In addition, the classical impedance model no longer applies as ω->0 and skin depth > wall thickness. Correct impedance calculated by *ImpedanceWake2D*.
- The detuning impedance is sampled at DC for coasting beam.
- The resulting tune shift can cause mode coupling between fast and slow waves. In the PS case, this mode coupling results in an instability in the horizontal plane.

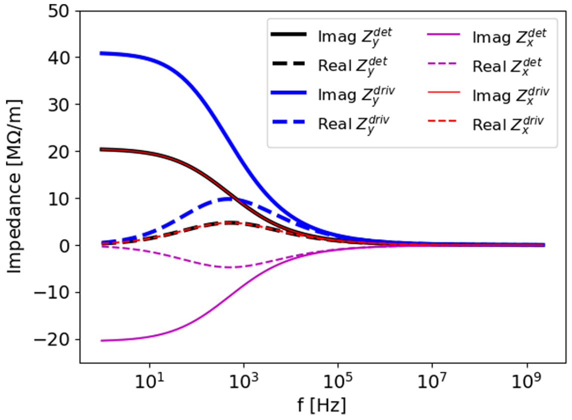


FIG. 1. Vertical driving and detuning impedances (respectively in blue and black), and horizontal driving and detuning impedances (in red and magenta) for a flat pipe made of two parallel plates of stainless steel (electrical resistivity of 7.2×10^{-7} Ωm, 27.5 mm half gap, infinite thickness, 628.32 m long).

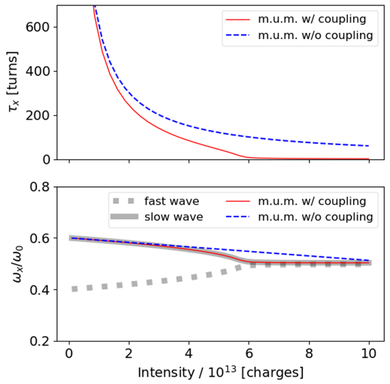


FIG. 4. At the top, the rise time of the most unstable mode (m.u.m.) in the horizontal plane is shown respectively with (red line) and without (dashed blue line) coupling of fast and slow waves. At the bottom, the frequency normalized to the revolution frequency of the m.u.m. with and without coupling is shown together with the normalized frequency shift of the fast (dashed gray line) and slow (full gray line) waves corresponding respectively to $n = 6$ and $n = 7$.

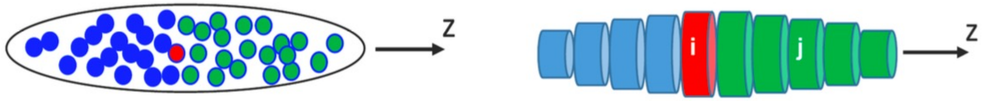
N. Biancacci et al, PRAB 23, 124402 (2020)

PyHEADTAIL: simulate coasting beam with resistive wall wake

- 6D tracking code used to simulate collective effects including instabilties.
- "ParallelHorizontalPlatesResistiveWall" wakefield class in PyHEADTAIL assumes skin depth << wall thickness and a bunched beam.
- Update wake calculation in code to simulate coasting beam.

PyHEADTAIL wakefield (bunched beam)

Figure: M. Schenk



(a) Bunch before slicing. (b) Bunch after slicing.

Figure 2.15: Principle of beam slicing for in `PyHEADTAIL`. On the left the bunch is represented as a collection of macroparticles. The effect of the wakefield generated by all the macroparticles on the red one must be evaluated. To decrease computation time, the bunch is instead sliced as pictured on the right. Each slice contains several thousands of macroparticles, and it is the effect of all slices on the red one which is computed. Pictures courtesy of M.Schenk [131].

- Dipolar kick experience by macroparticle i due to all other macroparticles

$$\Delta x'_i = -\frac{e^2}{\beta^2 E_0}\sum_{j=1}^{N} W_x^{dip}\left(z_i - z_j\right)\Delta x_j$$

- Reduce computational time by slicing bunch. Assume wake is constant within slice.

$$\Delta x'_i = -\frac{e^2}{\beta^2 E_0}\sum_{j=1}^{N_{slices}} N_j W_x^{dip}\left(i - j\right)\left\langle \Delta x_j \right\rangle$$

convolution of wake with dipole moment

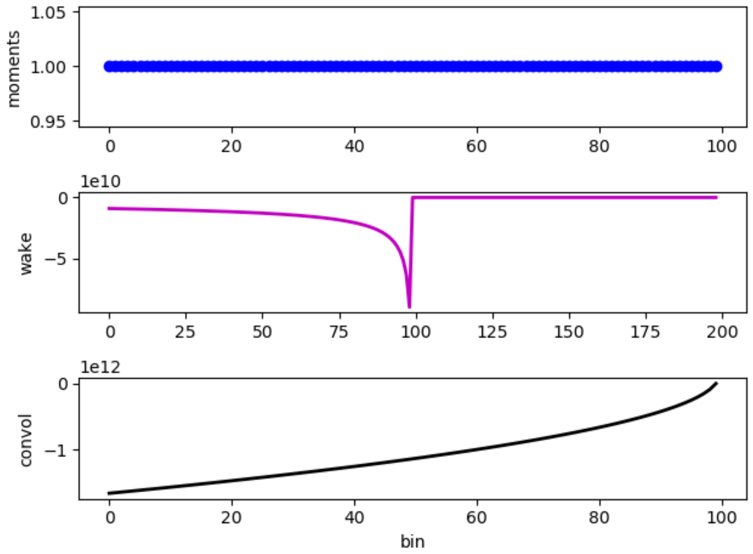
PyHEADTAIL wake kick calculation (bunched)

$$N_{bin} \langle \Delta x_{bin} \rangle$$

Assume uniform dipole moment

$$W_{\perp}(z_{bin} - z_0)$$

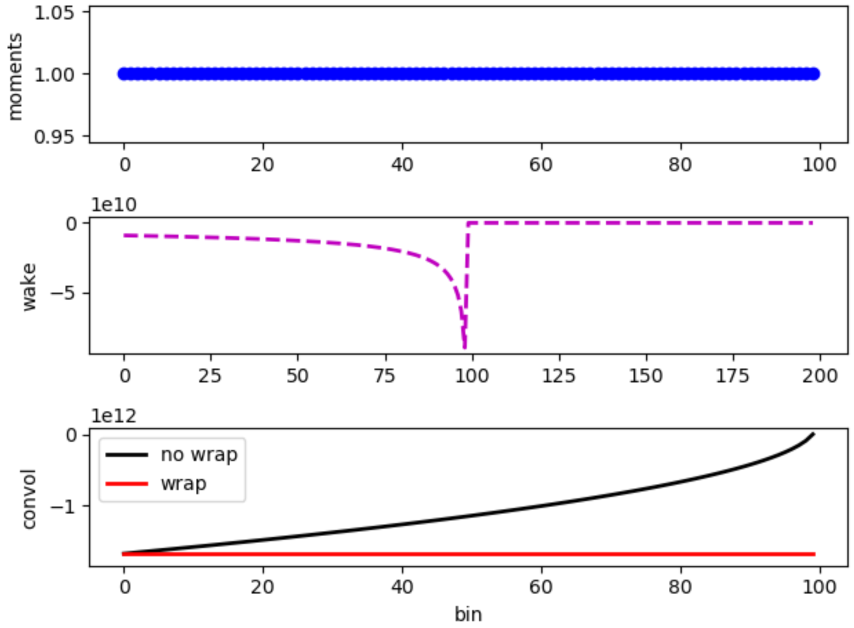
Convolution



convol = numpy.convol(source_moments, wake, ‘valid’)

PyHEADTAIL wakefield (coasting)

- Wrap convolution so that there is no head or tail.



Assume uniform dipole moment

convol_wrap = numpy.convol(np.tile(source_moments,2), wake, ‘valid’)[n:2*n]

- After making this change in the code, check if growth rate matches prediction

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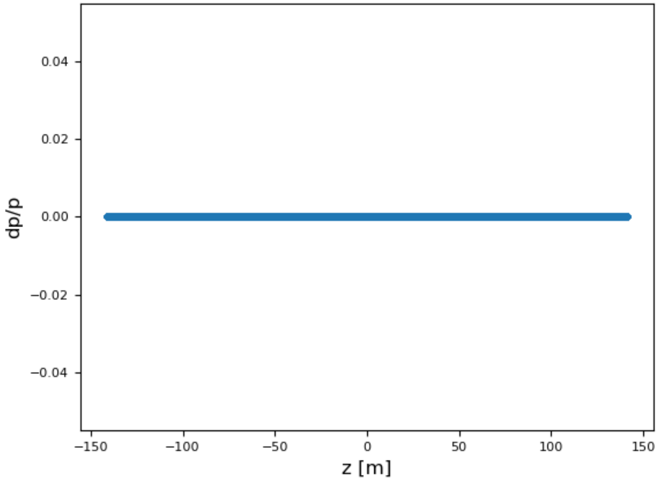
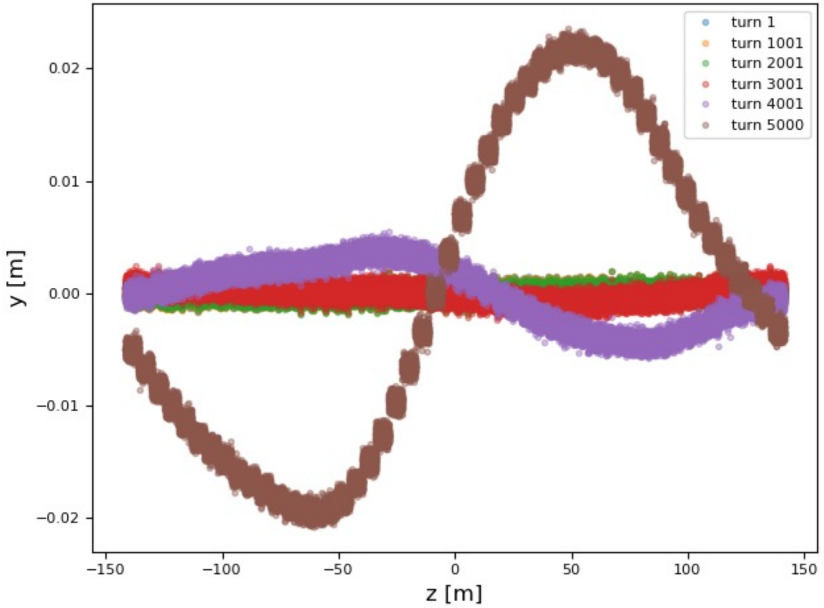
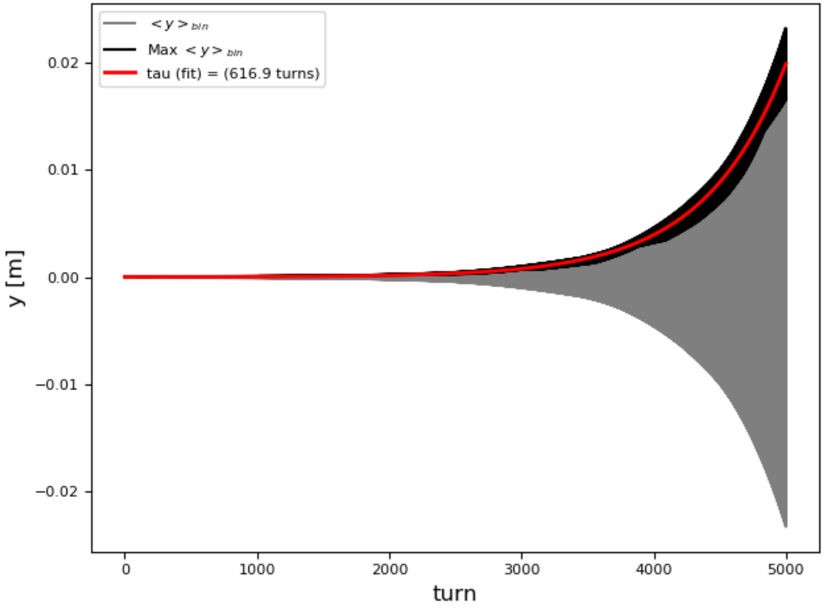
PyHEADTAIL growth rate







- Start with uniform distribution in z, zero momentum spread and a small transverse emittance (1e-7 mm mrad).
- Track 5000 turns.
- The growth rate is of same order as prediction (predicted growth rate τ=400)

Landau damping

- Slide 5 showed the coherent frequency shift ΔΩ from impedance in the absence of tune spread. Define a coherent tune shift $\Delta Q_{coh} = -\Delta\Omega/\omega_0$ in this case (minus sign to change convention).
- In the absence of tune spread, the instability grows for any $\mathrm{Im}(\Delta Q_{coh}) > 0$.
- Introducing tune spread changes the coherent tune shift. There can be some region of $\mathrm{Im}(\Delta Q_{coh}) > 0$ where the instability is damped.
- This region is found by solving the dispersion relation. In the case of amplitude dependent tune spread

$$1 = -\Delta Q_{coh} \int_0^\infty dJ_x \int_0^\infty dJ_y \frac{J_x \frac{\partial \rho(J_x, J_y)}{\partial J_x}}{Q - Q_x(J_x, J_y)}$$

where Q is the coherent tune, $\rho(J_x, J_y)$ is the density distribution and Qx(Jx, Jy) is the amplitude dependent tune. Including just linear tune shift with amplitude,

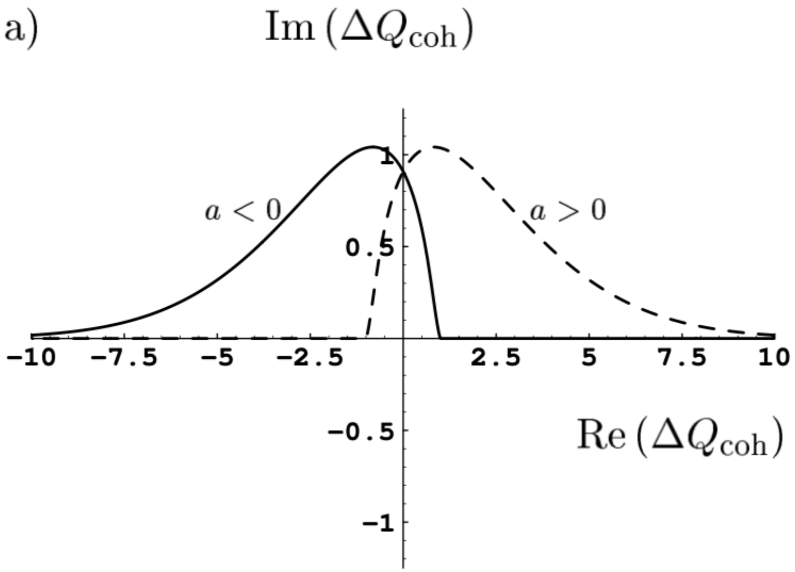
$$Q_x(J_x, J_y) = Q_0 + aJ_x + bJ_y$$

direct term cross term

J.S Berg, F. Ruggiero, CERN report SL-AP-96-71, 1996

Stability region with octupoles

- J.S. Berg and F. Ruggiero derived analytic expressions for the stability limit for the case of 2D amplitude dependent tunes.
- Example below shows a case where there is a direct term only (b=0). Sign of octupole determines



J.S Berg, F. Ruggiero, CERN report SL-AP-96-71, 1996

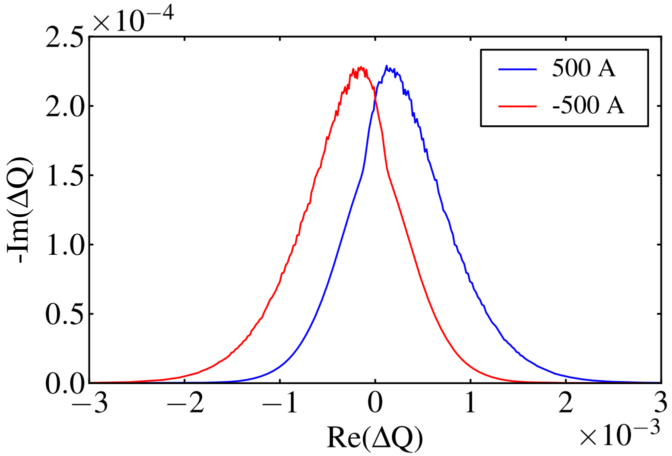


FIG. 4. Stability diagrams from octupoles powered with opposite polarities.

X. Buffat et al, PRAB 17, 111002 (2014)

- PyHEADTAIL already includes amplitude tune shift as an option.

Conclusions

- Coasting beams in scaling FFA may be subject to transverse instabilities.
- Since the scaling FFA is normally rectangular, a parallel geometry may be assumed. In this case the vertical dipolar RW impedance is twice the horizontal.
- The detuning impedance can be significant for the case of a coasting beam in a non-circular aperture. It may result in fast-slow mode coupling (N. Biancacci paper).
- Work to update the wake calculation to deal with a coasting beam in PyHEADTAIL in underway.
- Check if tune shift with amplitude from nonlinear field in scaling FFA will result in Landau damping for the instability.