The FFA code
FIXFIELD

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FixField code

- Single particle tracking code
- C language
- Runge Kutta 4 integration
- Standard libraries + CLapack
- Portable to different OS
- Crosschecked results with experiments for horizontal scaling FFA (circular and straight)
- Crosschecked results with OPAL/S-CODE/ZGOUBI for vertical scaling FFA

Integrated equations of motion

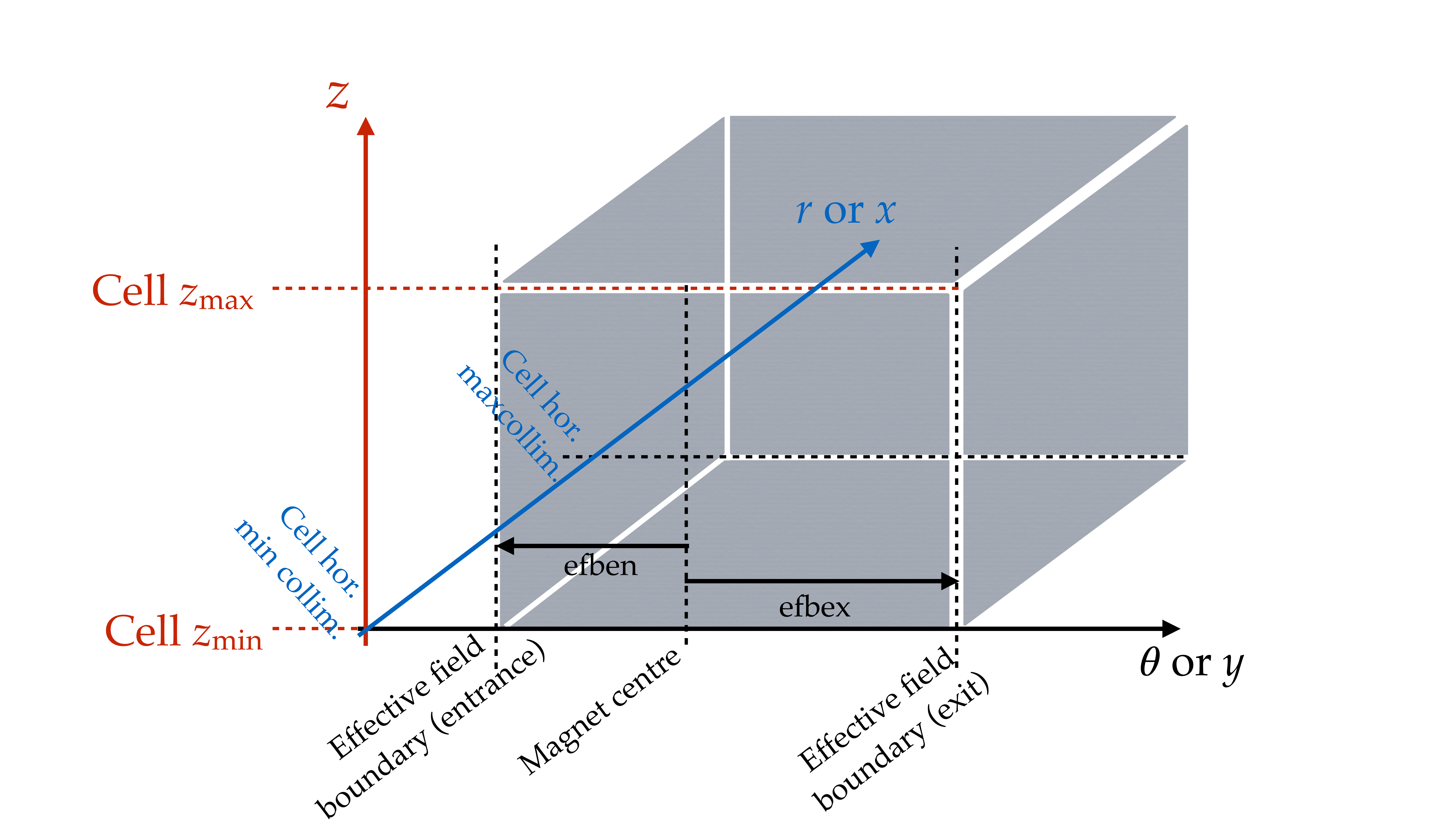
$$\begin{cases} \dfrac{\mathrm{d}u_x}{\mathrm{d}s} = \dfrac{(u_y B_z - u_z B_y)}{B\rho} \\ \dfrac{\mathrm{d}u_y}{\mathrm{d}s} = \dfrac{(u_z B_x - u_x B_z)}{B\rho} \\ \dfrac{\mathrm{d}u_z}{\mathrm{d}s} = \dfrac{(u_x B_y - u_y B_x)}{B\rho} \\ \dfrac{\mathrm{d}x}{\mathrm{d}s} = u_x \\ \dfrac{\mathrm{d}y}{\mathrm{d}s} = u_y \\ \dfrac{\mathrm{d}z}{\mathrm{d}s} = u_z \end{cases}$$

- x: horizontal, y: longitudinal, z: vertical
- Integration in *s*, longitudinal abscissa measured along the orbit of the particle (Frenet-Serret framework)
- Unit vector $\overrightarrow{u} = \dfrac{\overrightarrow{p}}{p}$
- Lattice composed of independent cells with superposition of magnetic field contribution from each cell component
- Thin RF gaps used for acceleration

Available magnetic components

- Horizontal circular scaling FFA (radial and spiral)
- Horizontal straight scaling FFA
- Vertical scaling FFA
- Separated function magnet up to octopole
- Cylindrical and Cartesian field maps with linear interpolation

Magnet geometry



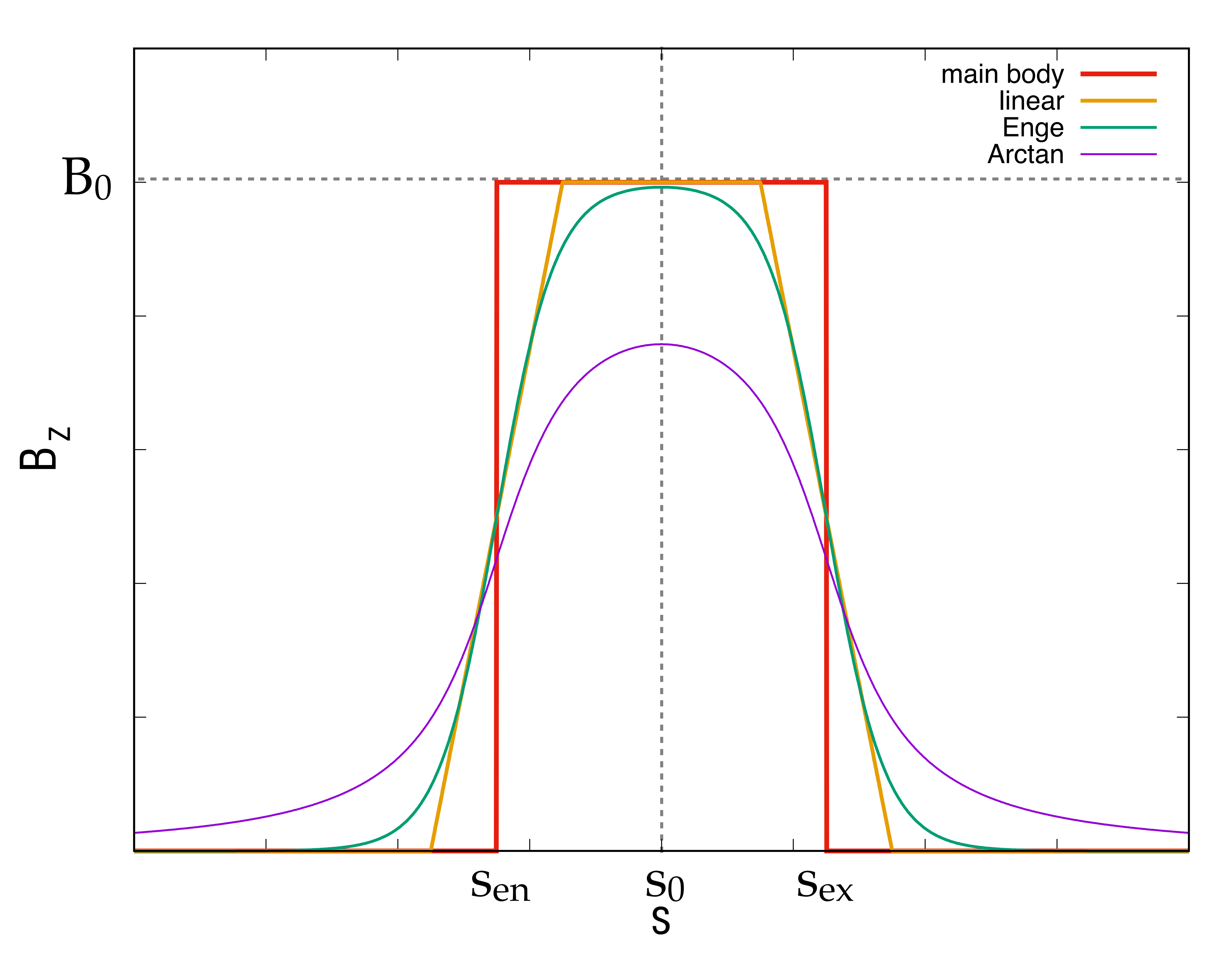
Fringe field models

- Main body only (no fringe field)
- Linear fringe field
- Enge fringe field

$$\mathcal{F}(s) = \frac{1}{1+e^{P(s)}}, P(s) = C_0 + C_1 \frac{s}{\lambda} + C_2 \left(\frac{s}{\lambda}\right)^2 + C_3 \left(\frac{s}{\lambda}\right)^3$$

  - Only $C_1 \neq 0$, equivalent to tanh
  - Full Enge fringe field (C_0, C_1, C_2, C_3)
- Arctan fringe field (vFFA only)

$$\mathcal{F}(s) = \frac{1}{\pi} \arctan\left(\frac{s}{\lambda}\right) + \frac{1}{2}$$



Field options

- Extrapolation off the median plane to 16th order (vFFA Arctan, hFFA spiral Enge)
- Generation of field maps from field model
- Maxwellian test of a given field (div B, Curl B)
- Longitudinal cell boundary:
    - Mirror symmetry
    - Zero-field boundary
- Generation of alignment errors (translation and/or rotation)

Particle distribution

- Can generate different beam distributions:
  - Waterbag
  - Gaussian
  - 2D ellipse (phase space)

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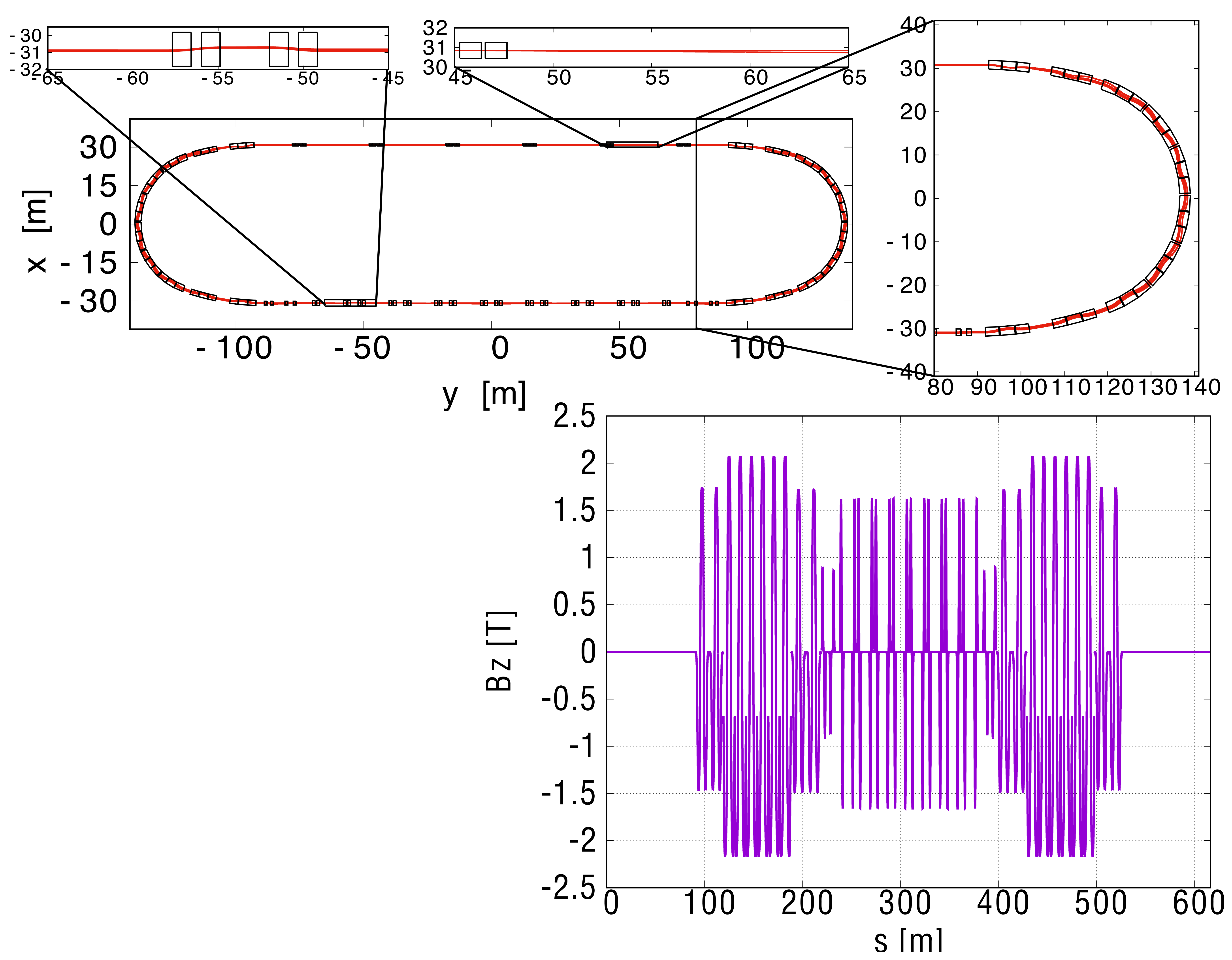
Tracking and plots

- Input with text files (lattice file, beam file)
- Generate output as text files and/or terminal
- Particle position and direction
- magnetic field seen by the particle
- Position of the magnets
- Available internal functions to generate plots with Gnuplot



nuSTORM storage ring, https://inspirehep.net/literature/2052496

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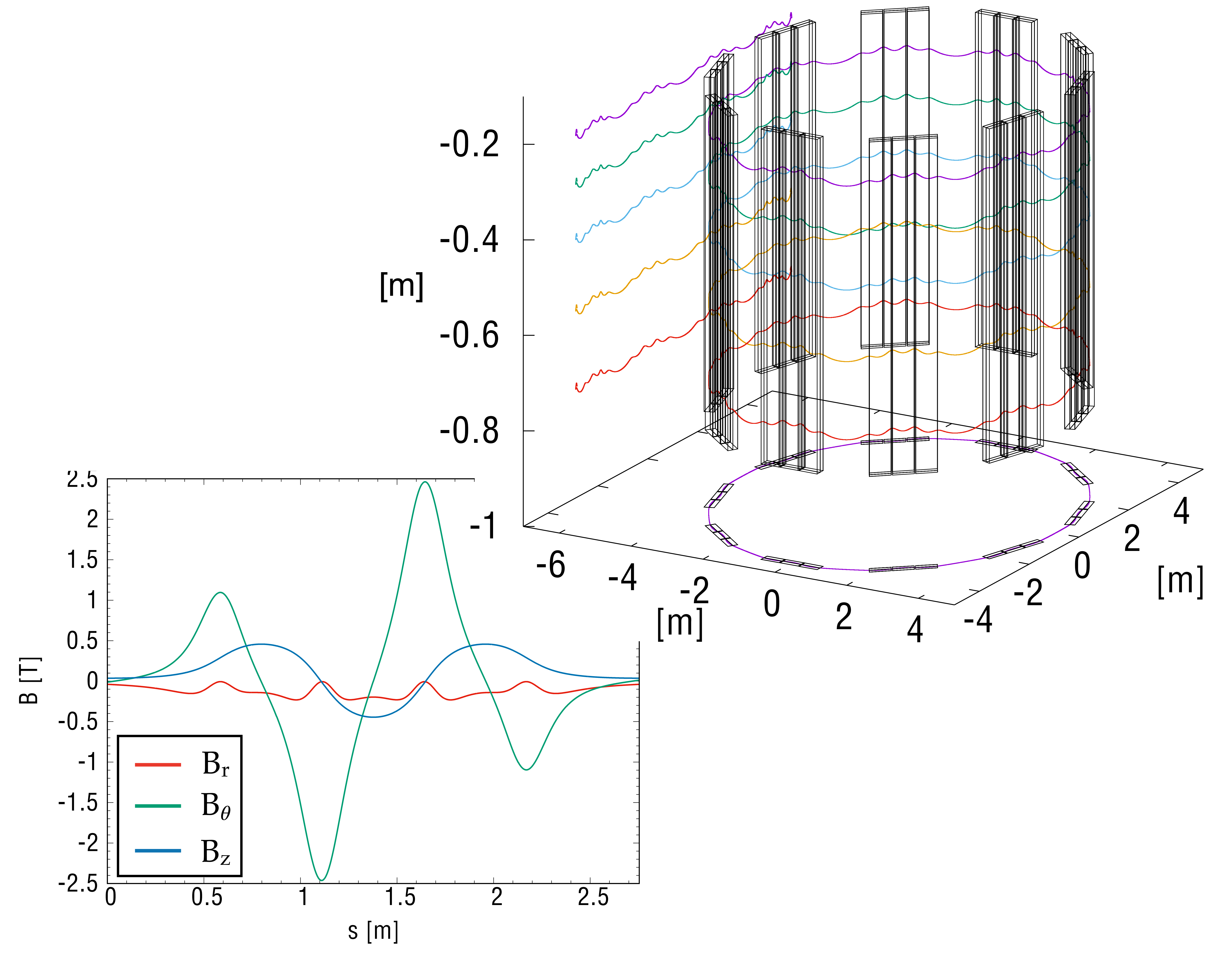

FETS-VFFA, RAL internal report

Periodic orbit search

- Line search method (iterative method with averaged operator)
- Nelder-Mead method
- Both methods can be confined to specific initial particle parameters (median-plane, zero-angle)

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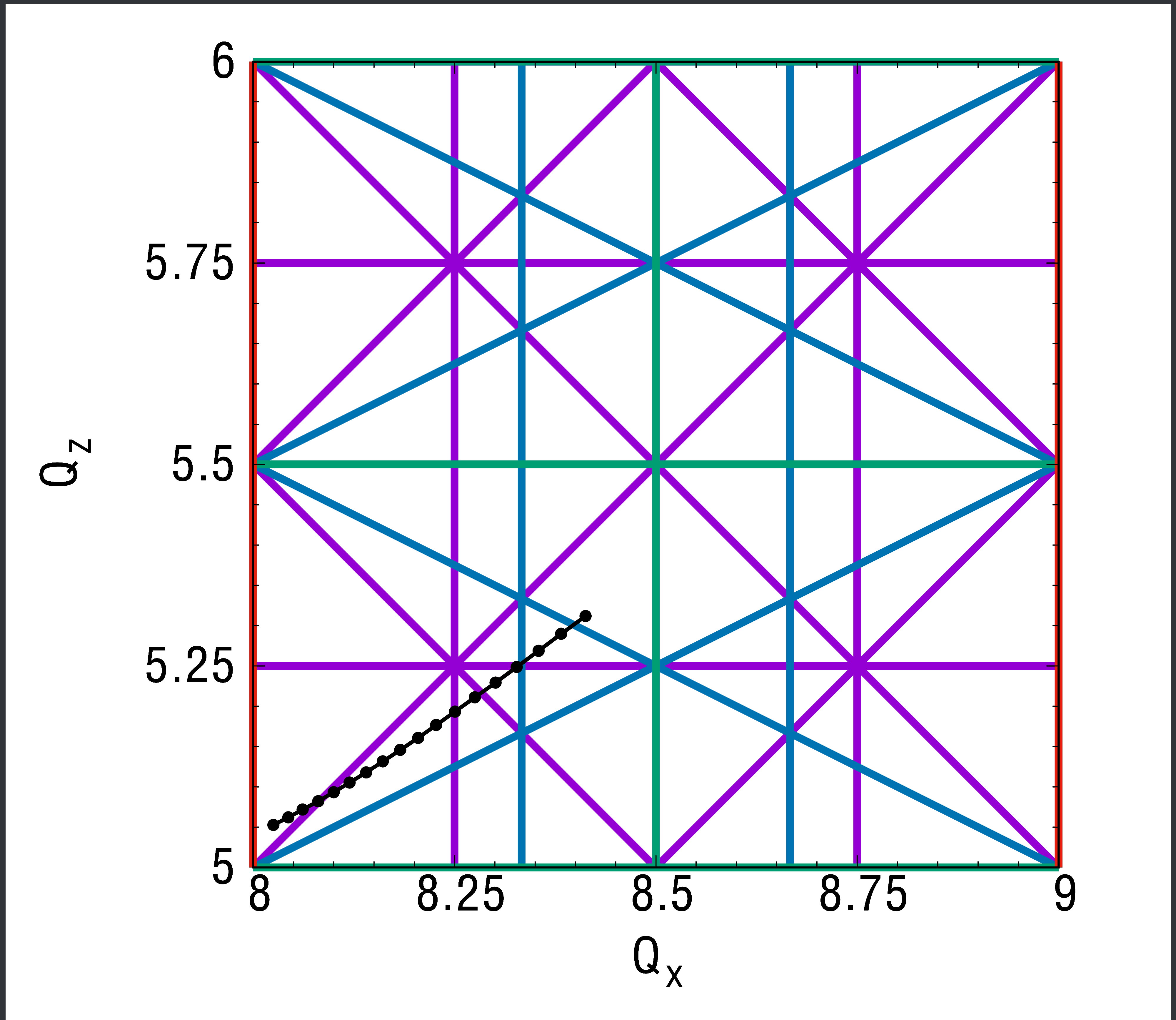
Transfer matrix computation

- Computed from tracking
- Particles tracked with offset from reference trajectory to determine the coefficients of the matrix
- Contribution of the second order can be cancelled by tracking particles with ±offsets (e.g. $m_{11} = \frac{x_{+\delta x} - x_{-\delta x}}{2\delta x} + \mathcal{O}(\delta x^3)$)
- Parzen decoupling procedure (vFFA)
- Symplectification function available

Betatron tunes and phase advances

- 2 available methods:
  - Computed by FFT (number of turns must be 2^n)
  - Computed from transfer matrix
- Tune diagramme can be plotted with resonances lines (systematic and/or non-systematic)



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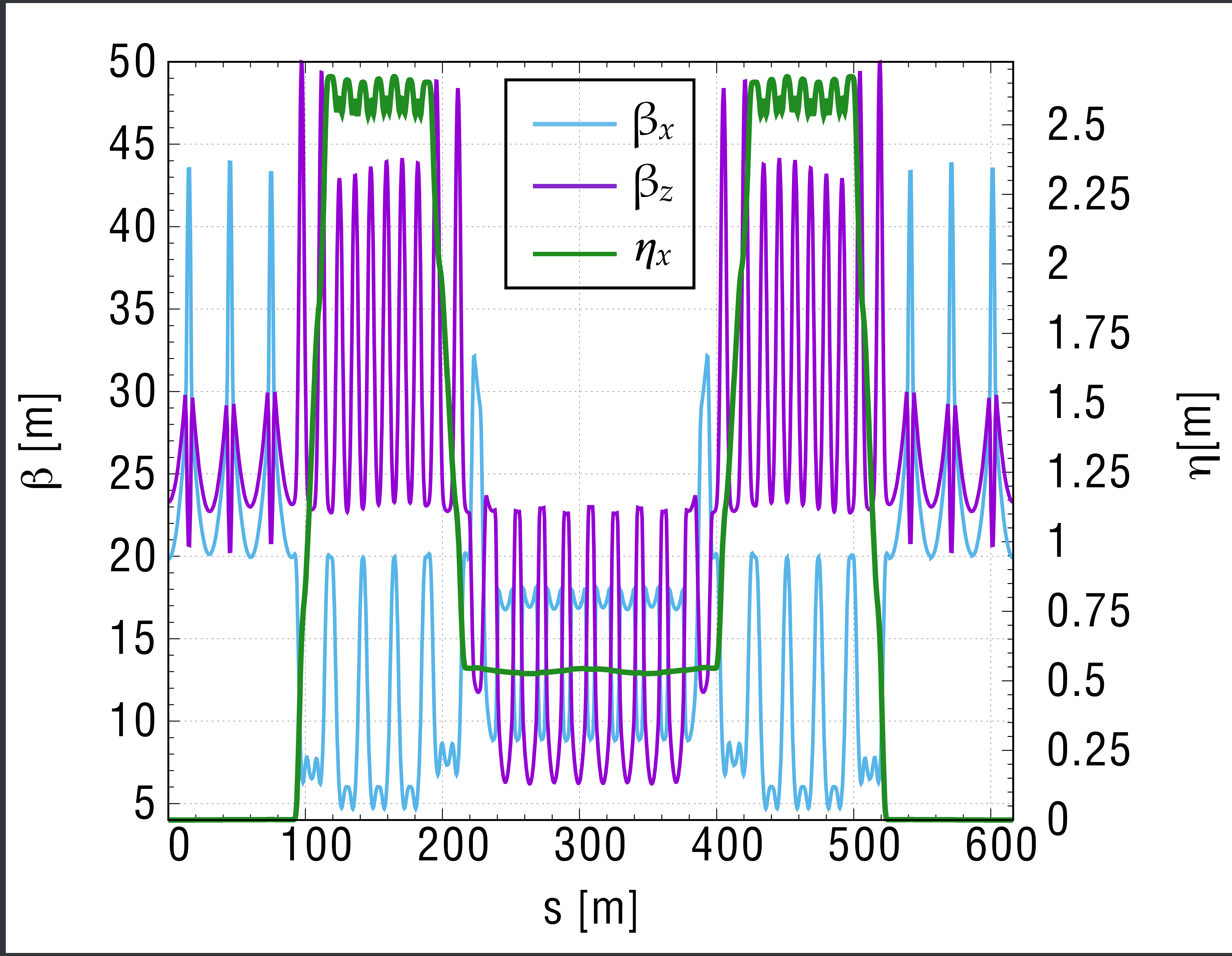
Beta-functions and dispersion functions

- Computed from transfer matrix along the reference orbit
- Possibility to compute effective field boundary of magnets as a function of s



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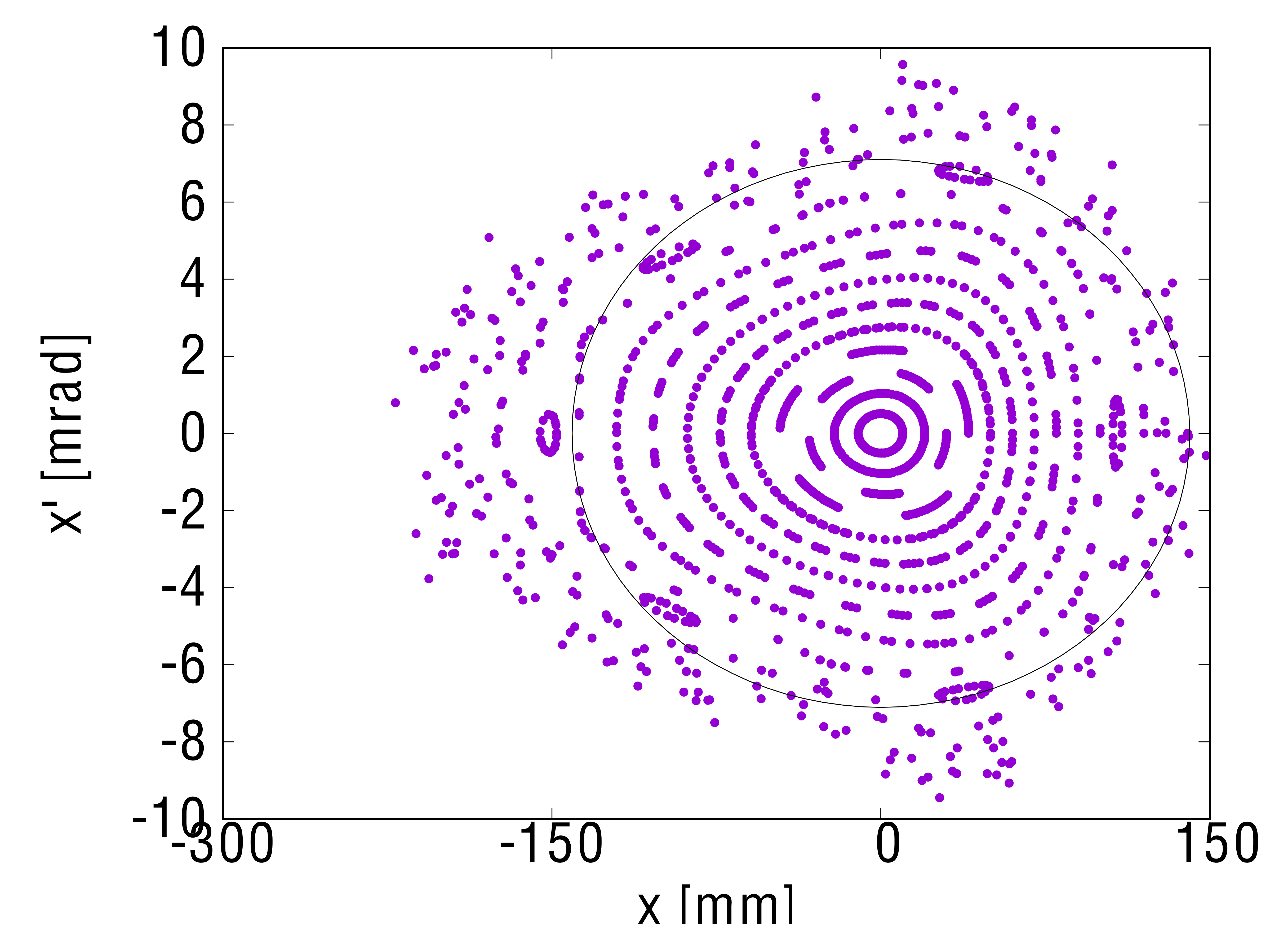
Dynamic aperture (DA) search

- Computation of the largest initial amplitude away from the reference orbit with stable motion over a set number of turns

- Possibility to compute and use decoupled space amplitude (vFFA)



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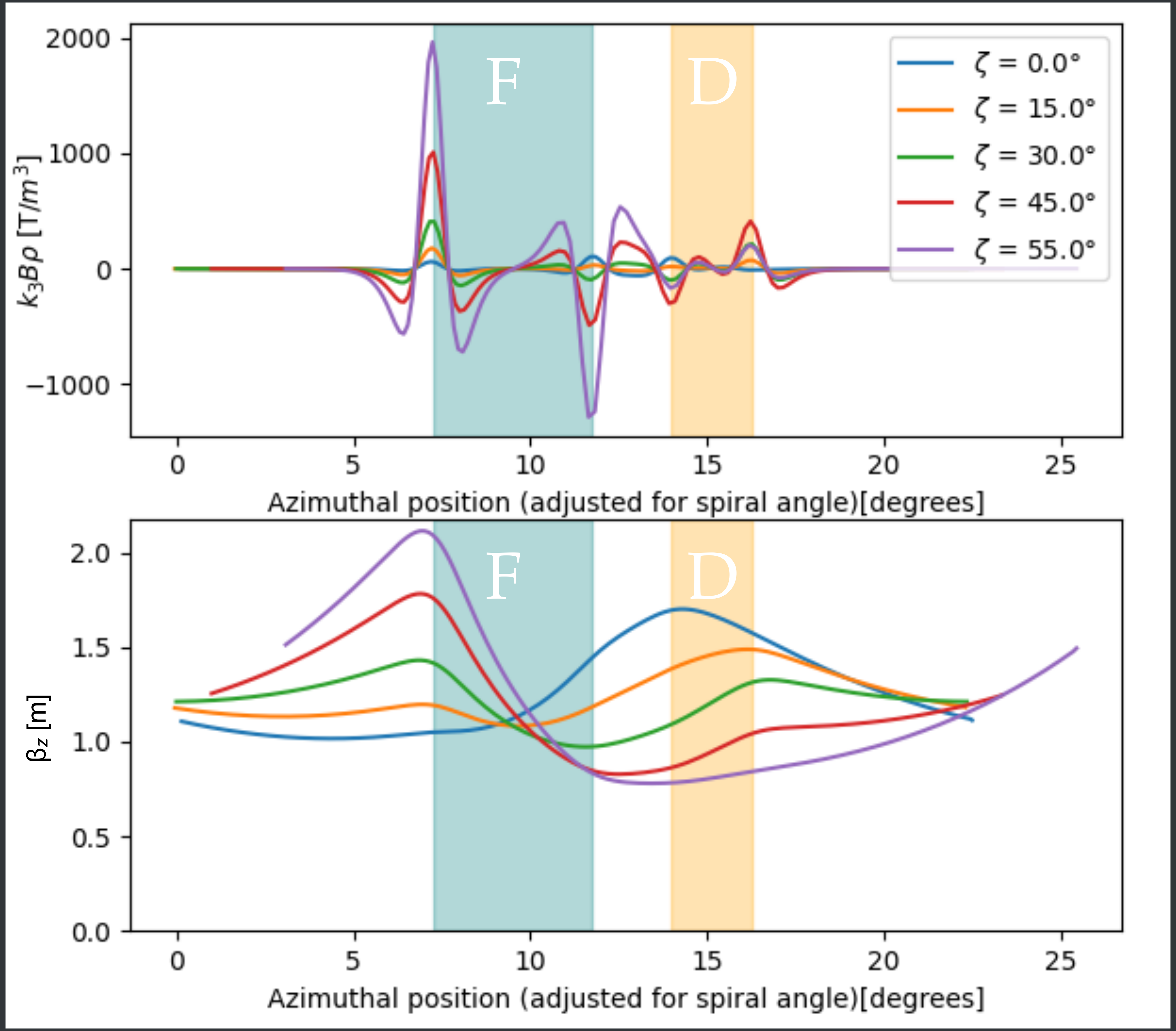
Harmonic analysis

- Decomposition of field components along the orbit to compute harmonic analysis
- Understanding of focusing components
- Can predict DA by computing octopolar components



FETS-hFFA octopolar harmonic analysis for different spiral angles (internal report)

Python scripts

- Call Fixfield from a python script for lattice optimisation (tunes and DA optimisation) with python libraries (e.g. scipy)

Summary

- Tracking/design code for FFAs
- Available on line (Github): https://github.com/lagrangejb/fixfield
- Suggestions/requests: jean-baptiste.lagrange@stfc.ac.uk