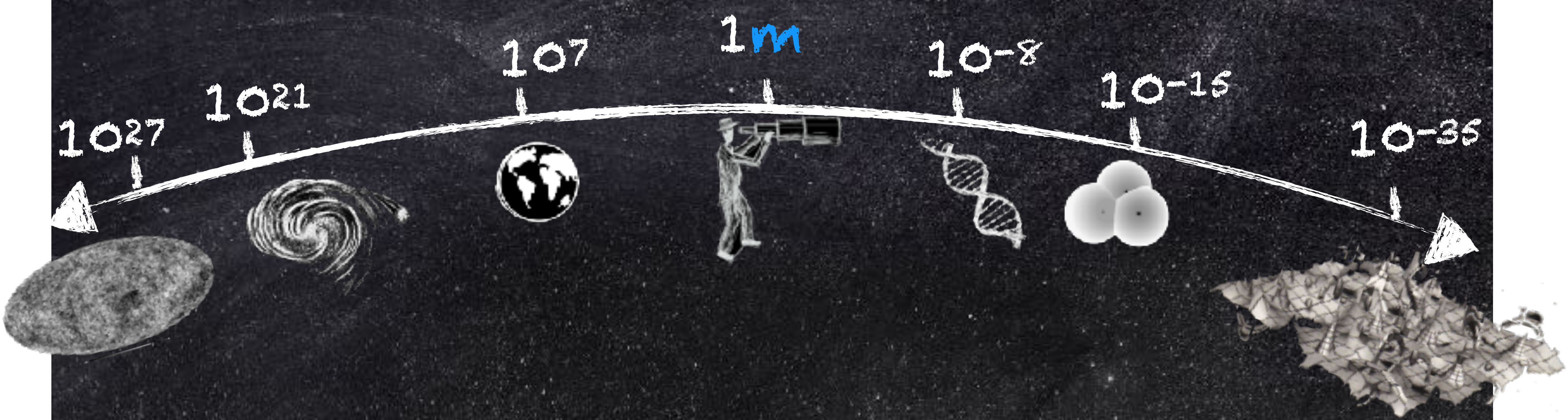
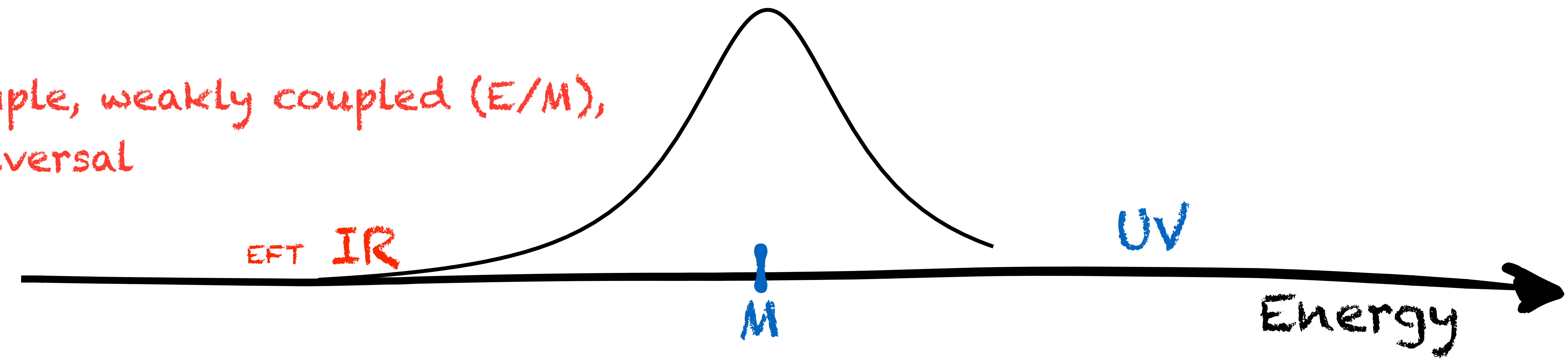
Positivity of Effective Field Theories and Large-N QCD



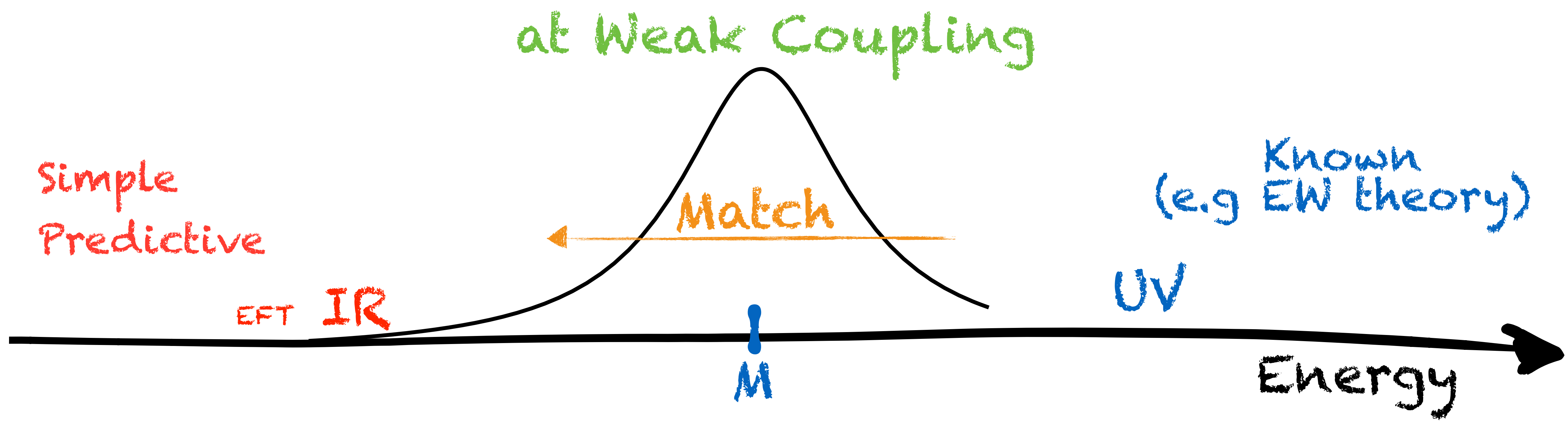
Francesco Riva
(UNIGE)

Effective Field Theories

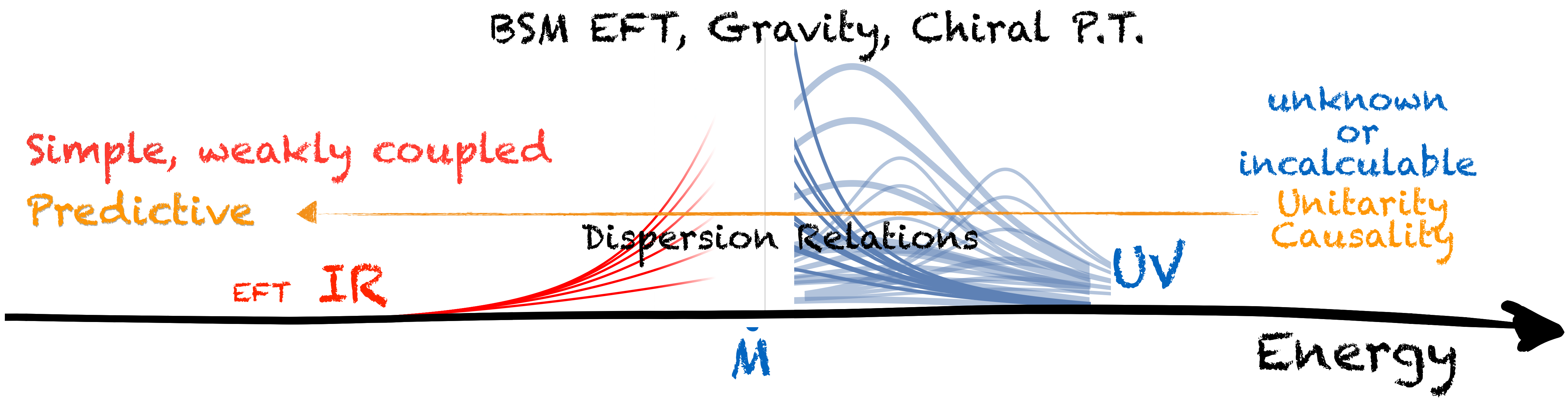
Simple, weakly coupled (E/M),
universal



Effective Field Theories



Effective Field Theories



U(1) Goldstone

Galileon (massive/modified gravity)

$$A^{Tree}_{2\to 2} = c_2 \frac{(s^2+t^2+u^2)}{2} - c_3 stu + c_4 \frac{(s^2+t^2+u^2)^2}{4} + \cdots$$

Lore: all E-behaviours IR consistent!

But not all are UV completable!

Outline

1. Dispersion relations for EFTs

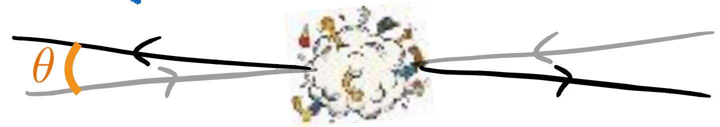
2. Positivity for BSM phenomenology

3. Large-N QCD

UV-IR Connection

Froissart,Martin',...60s
Adams,Arkani-Hamed,Dubovsky,
Nicolis,Rattazzi'06,
...

Forward Scattering $t \sim \theta^2 = 0$ ⟶



Total energy $s = E^2$

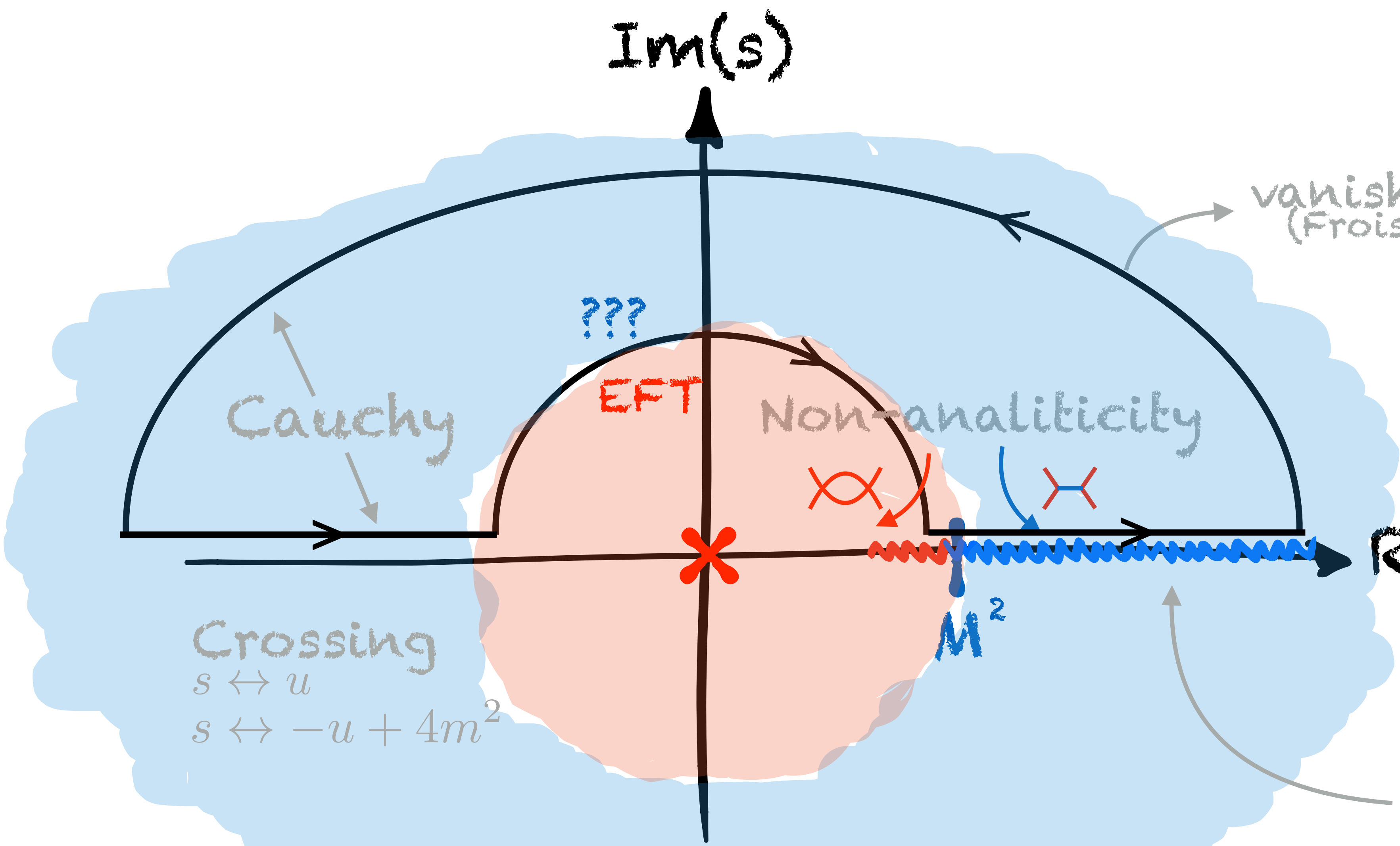
Physical Properties

Mathematical Properties of 2→2 forward amplitude $A(s)/s^n$

Causality ▶ Analytic in $s \in \mathbb{C}/phys$

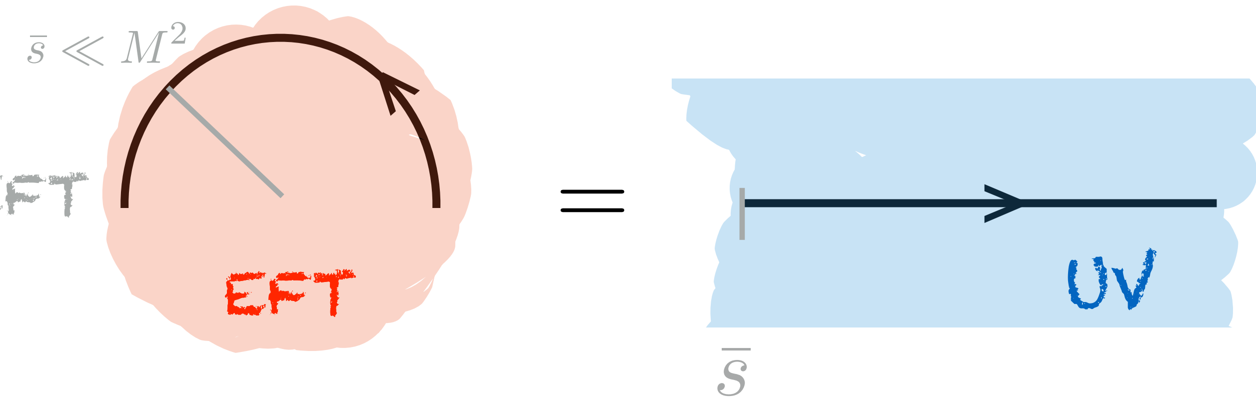
Unitarity ▶ Positive across $s \in \mathbb{R}$

(optical theorem)



$UV_{BSM} \Longleftrightarrow IR_{EFT}$ (Dispersion) Relations

$\bar{s}$ = any energy within EFT



positive

Arcs: $$\mathcal{A}_n(\bar{s}) \equiv \int_{\cap_{\bar{s}}} \frac{ds}{\pi i}\frac{A(s)}{s^{n+1}} = \frac{2}{\pi}\int_{\bar{s}}^{\infty} ds \frac{\mathrm{Im}A(s)}{s^{n+1}}$$

Calculable in EFT

(coefficients of s^n in tree amplitude)

$\mathcal{A}_n > 0 \quad (n \geq 2)$

$\mathcal{A}_n > 0 \quad (n \geq 2)$

Adams,Arkani-Hamed,Dubovsky,
Nicolis,Rattazzi'06,

- Consistency condition for EFTs

More than positivity

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

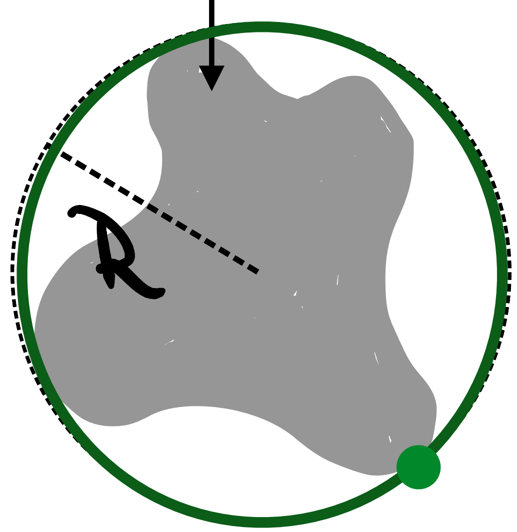
Moments

$$\int_0^1 d\mu(x)x^n \quad \xleftarrow{\text{variables change}} \quad \frac{2}{\pi}\int_{\bar{s}}^{\infty} ds \frac{\text{Im}A(s)}{s^{n+1}}$$

positive ($\text{Im}A(s)$)

Moments appear everywhere in physics...

e.g. stones $d\mu(x)$ = mass distributions



n=0: total mass M (sets units)

n=1: centre of mass <RM

n=2: moment of inertia <R²M

Bounded

What bounds do moments satisfy?

Bounds ↔ Positive Polynomials

Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20

Bounds on EFT Coeff. ↔ Bounds on Moments ↔ Positive polynomials in [0,1]

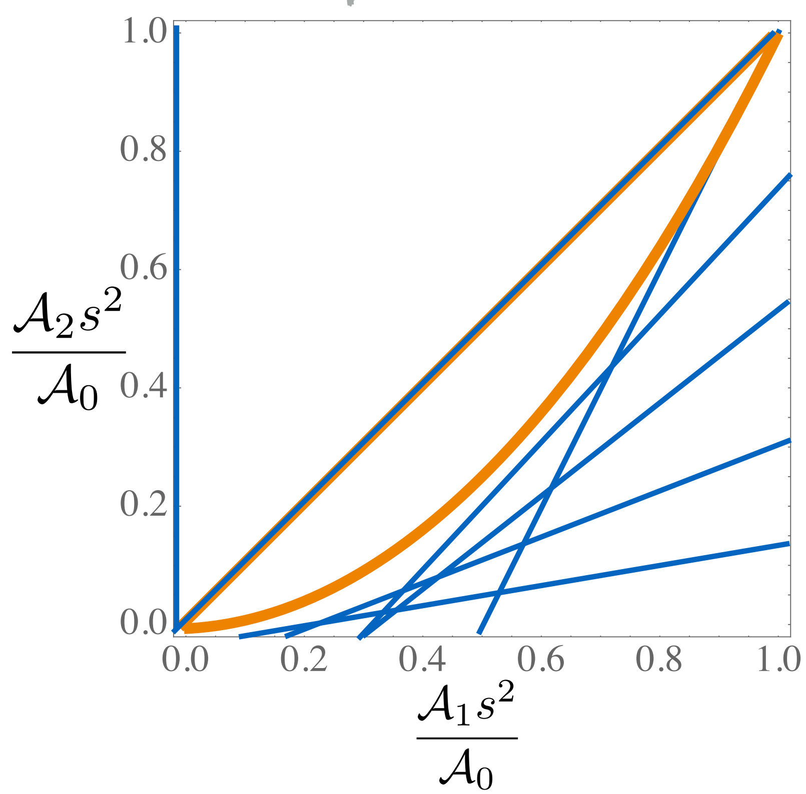
e.g. $\int d\mu(x) \quad 1 - x > 0 \qquad \Rightarrow \qquad \mathcal{A}_0 - s^2\mathcal{A}_1 > 0$

$$p = x^n(1-x)^m$$

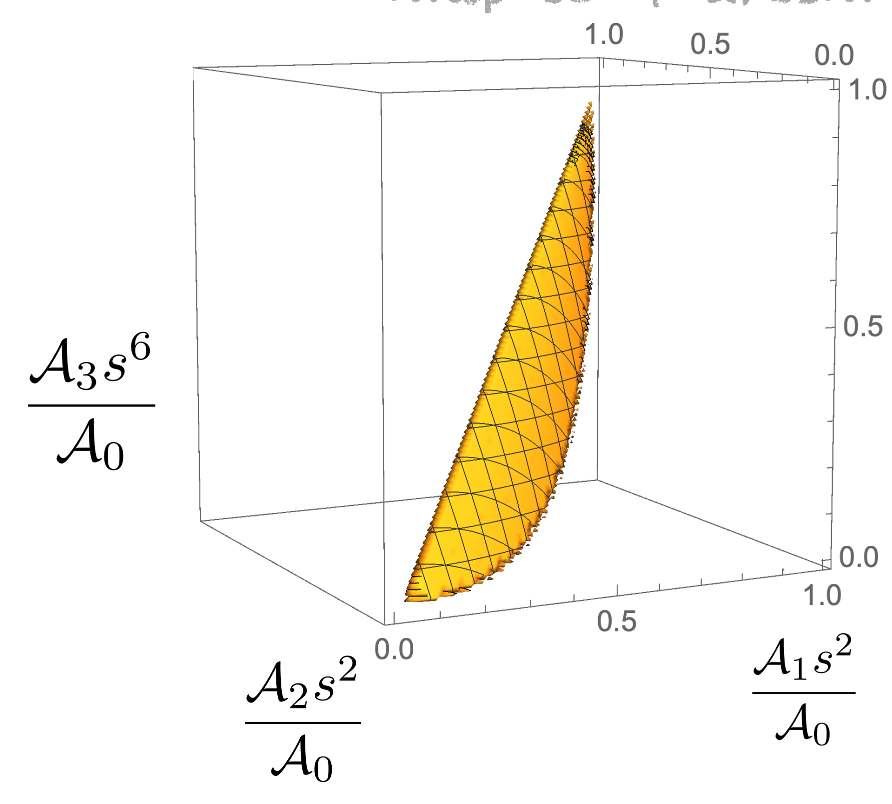
(Bernstein Polynomials)

$p = x(1-x) \to \mathcal{A}_1 \geq s^2 \mathcal{A}_2 \geq 0$

...up to 3 arcs...



...up to 4 arcs...

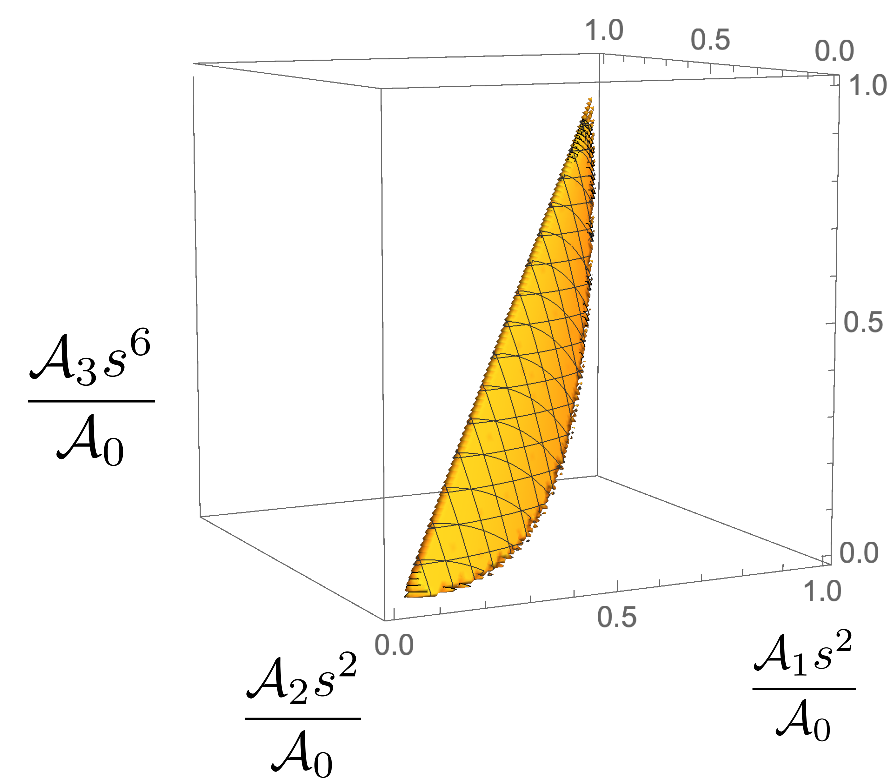


1. Bounds on EFTs

Bounds on $\mathcal{A}_n$ and
on EFT Wilson coefficients

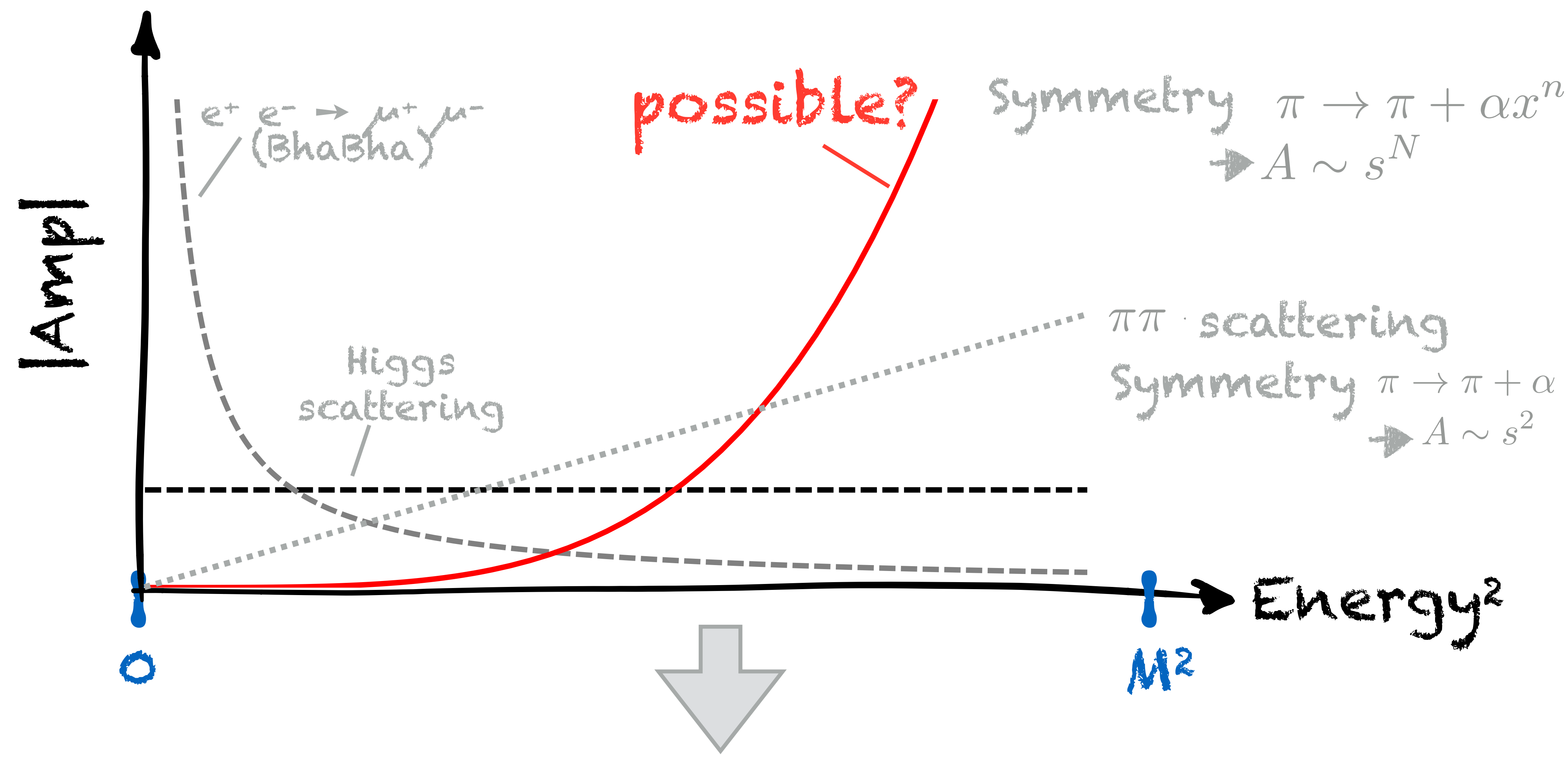
⇐

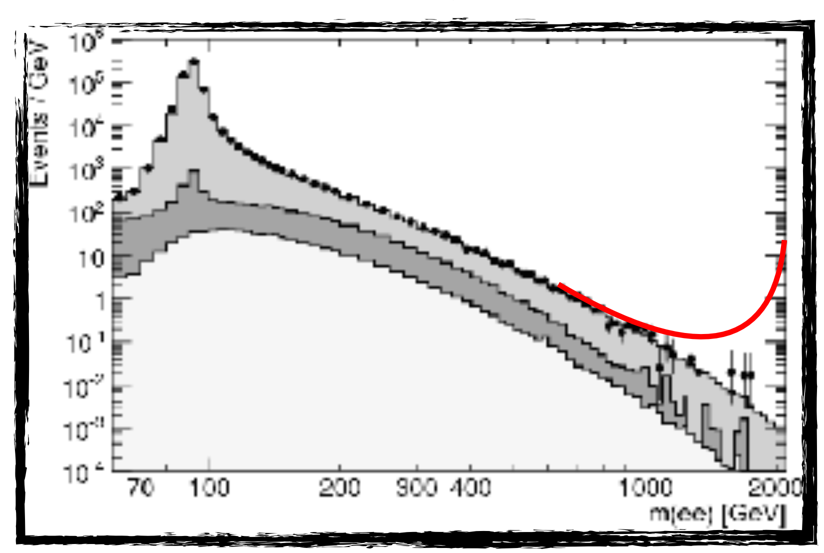
Bounds on $\mathcal{A}_n$



1. How "soft" can EFTs be?

Tree-level: $$A(s) = c_0 + c_2 s^2 + c_4 s^4 + c_6 s^6 + c_8 s^8 + \cdots$$

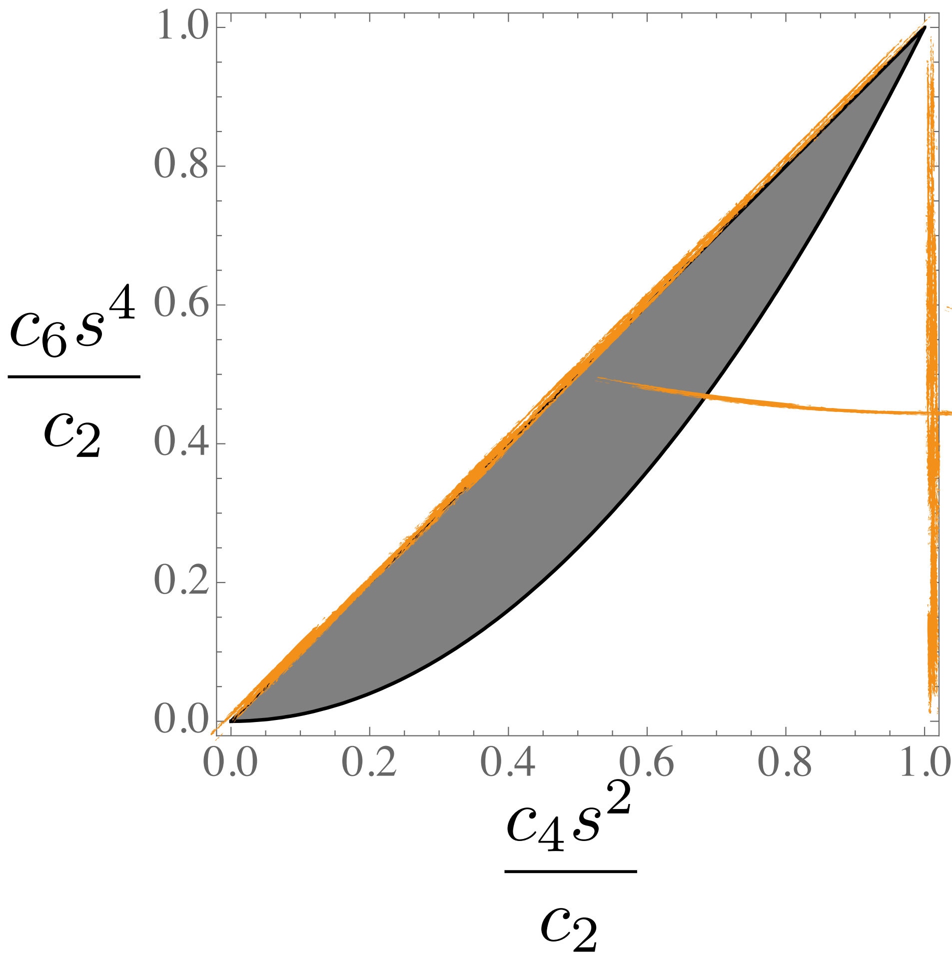




1. How "soft" can EFTs be?

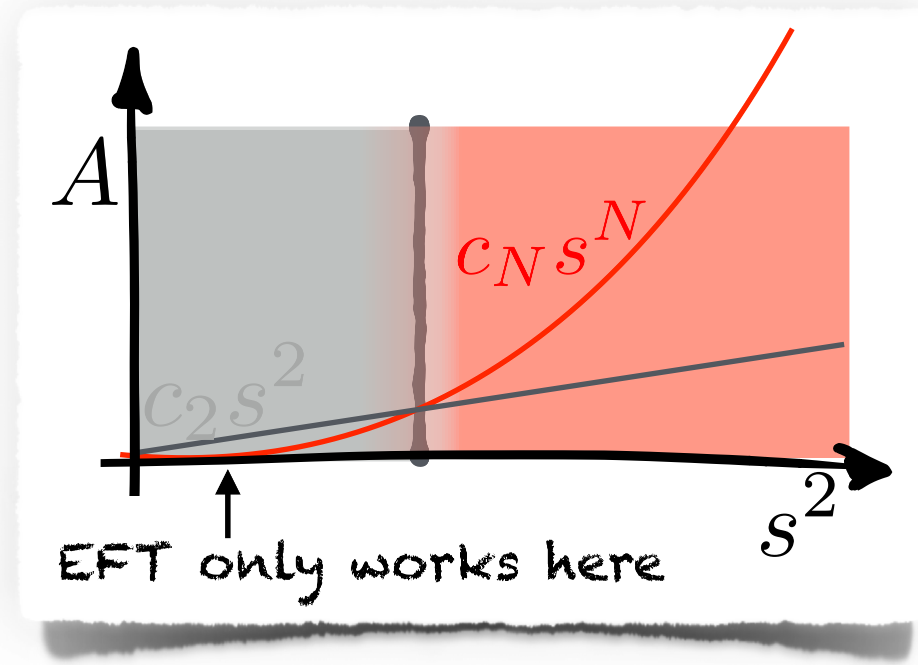
Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20

$$A(s) = \cancel{c_0} + c_2 s^2 + c_4 s^4 + c_6 s^6 + \cdots$$



$$c_2 s^2 > c_4 s^4 > c_6 s^6 > \cdots$$

Froissart



Very soft theories have low cutoff...

... so low that they are unobservable!

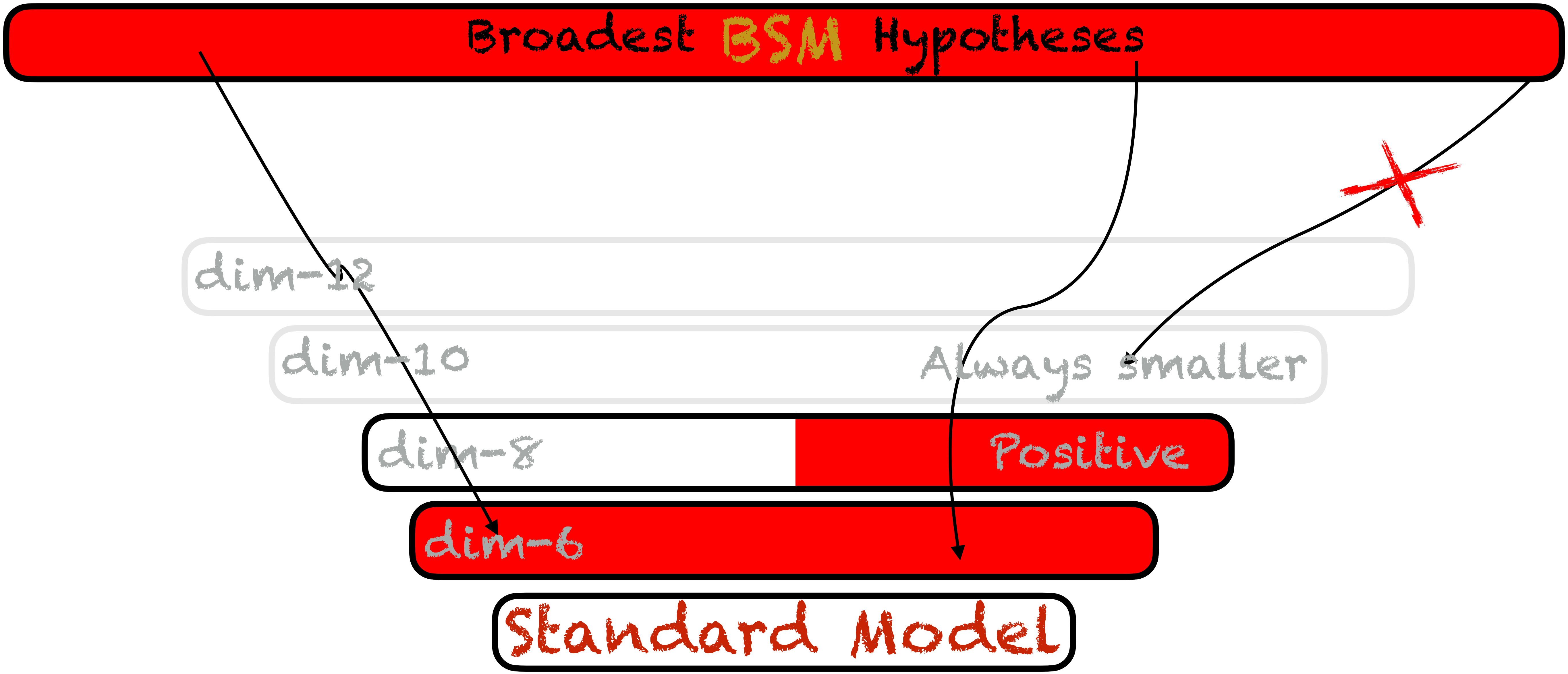
(dimension>8 operators cannot dominate)

Obvious in specific weakly coupled models (E/M<<1), Less obvious in general:

Goldstone bosons appeared to have naturally soft amplitudes

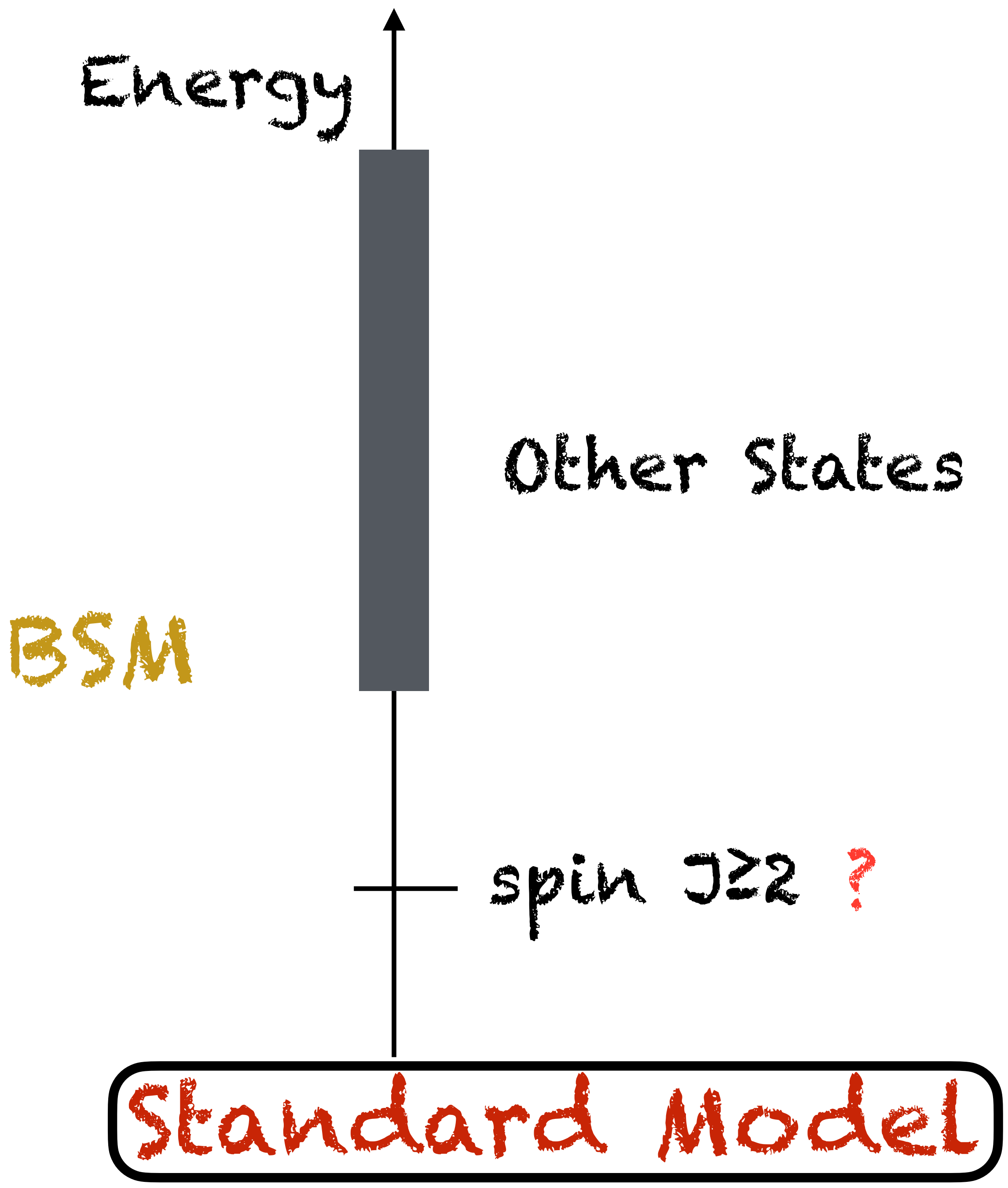
1. How "soft" can EFTs be?

Bellazzini, Elias-Miro, Rattazzi, Riembau, FR'20



2. Massive Higher Spin

Why are we not searching for higher-spin particles at the LHC?



2. Massive Higher Spin

Bellazzini,Serra,Sgarlata,FR'19
Bertucci,McPeak,Ricossa,FR,Vichi'to appear

$$\Phi^{\mu_1\cdots\mu_J}$$

Higher Spin resonances exist in QCD, Nuclei/atoms, Strings,...
(J>2)

$$m_{HS} \gtrsim \frac{1}{L_{HS}}$$

Can there be lighter HS states?

Interactions grow with $E \lesssim \frac{1}{L_{HS}}$

$$A(\Phi\Phi \to \Phi\Phi) \propto \frac{s^2}{m_{HS}^4} + \cdots + \frac{s^{2J}}{m_{HS}^{4J}}$$

$$m_{HS} \gtrsim \frac{1}{L_{HS}} \leftarrow \propto \; c_2 > \bar{s}^{2J-2} \; c_{2J}$$

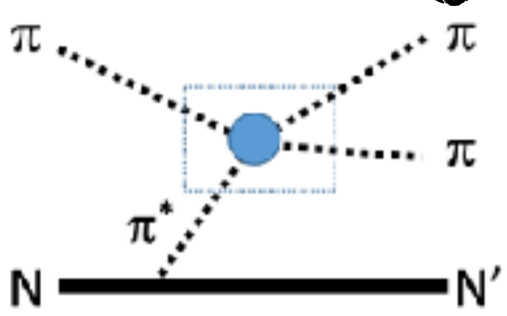
Resonances of Spin>2 always heavier than their size^{-1}

No light higher spin particles ▶ No dedicated LHC searches

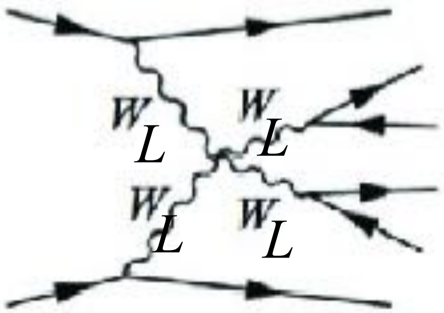
▶ No Massive Gravity

3. Chiral Perturbation Theory

Understanding pions?



Predictions for composite Higgs models?



3. Dispersion relations in ChPT

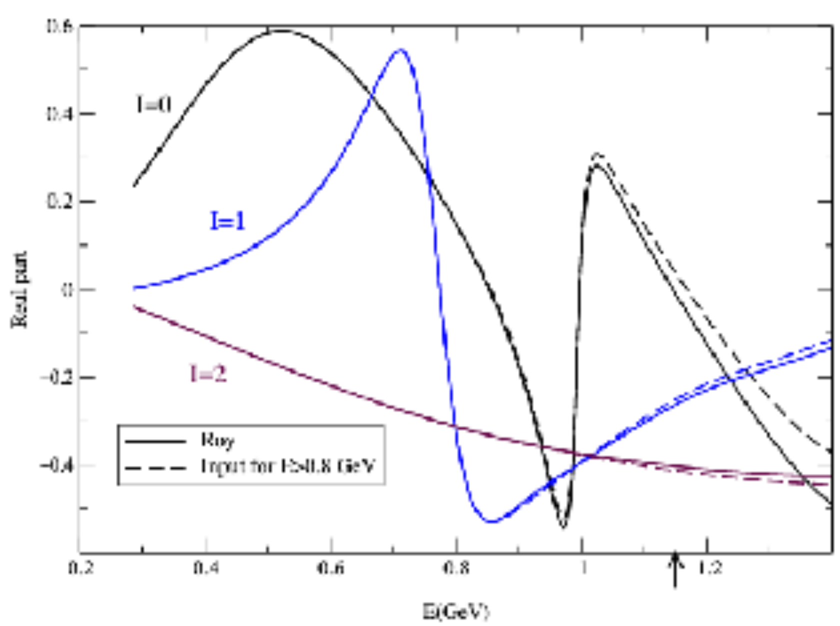
Roy Equations to improve fit to experimental data:

$$2\, a_\ell^{(I)}(s) \frac{\sqrt{s}}{k} = \frac{1+(-)^{\ell+I}}{2}\Big[\int_0^1 dx\, P_\ell(x) \sum_{I'=0,2} g_1^{II'}\big(s, \tfrac{4-s}{2}(1-x)\big)\, a_0^{(I')} + 4\int_0^1 dx\, P_\ell(x) \int_4^\infty ds' \sqrt{\frac{s'}{s'-4}} \sum_{\ell'=0}^{\infty}(2\ell'+1) \sum_{I'=0,1,2} \Big\{ g_2^{II'}\big(s, \tfrac{4-s}{2}(1-x), s'\big)\, \mathrm{Im}\, a_{\ell'}^{(I')}(s') + g_3^{II'}\big(s, \tfrac{4-s}{2}(1-x), s'\big)\, \mathrm{Im}\, a_{\ell'}^{(I')}(s')\, P_{\ell'}\Big(1 + \frac{(4-s)(1-x)}{(s'-4)}\Big) \Big\} \Big], \qquad (13)$$

complicated kernel without manifest generic properties

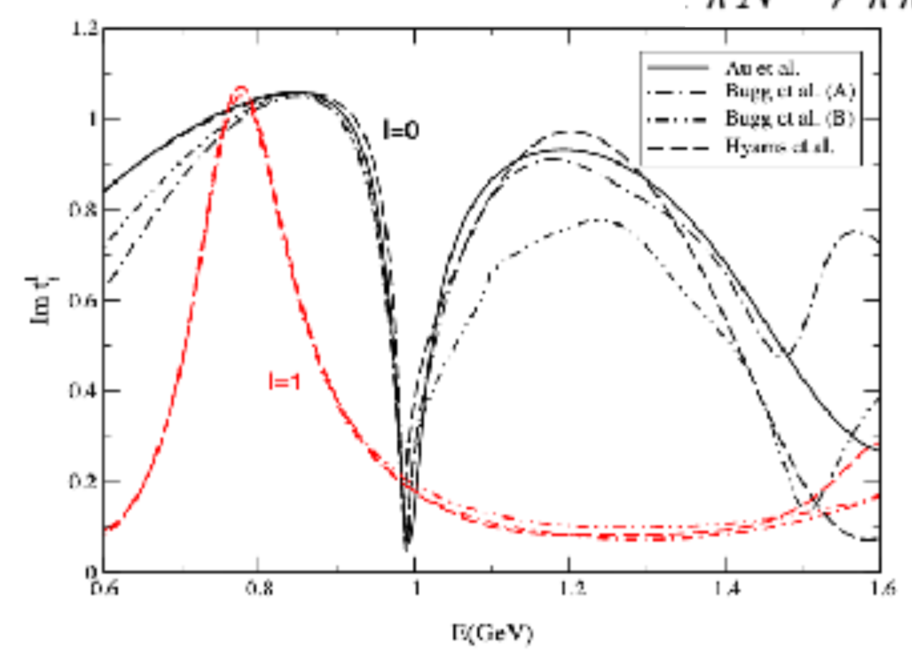
Roy'71

obtain a_l in IR



measure a_l at all energies

$\pi N \to \pi\pi N$.



Ananthanarayan, Colangelo, Gasser, Leutwyler '01

Can we learn something about the structure of QCD?

- separate theory predictions from experimental input

3. Dispersion relations in ChPT

Theory "Positivity" bounds on pion amplitude coefficients:

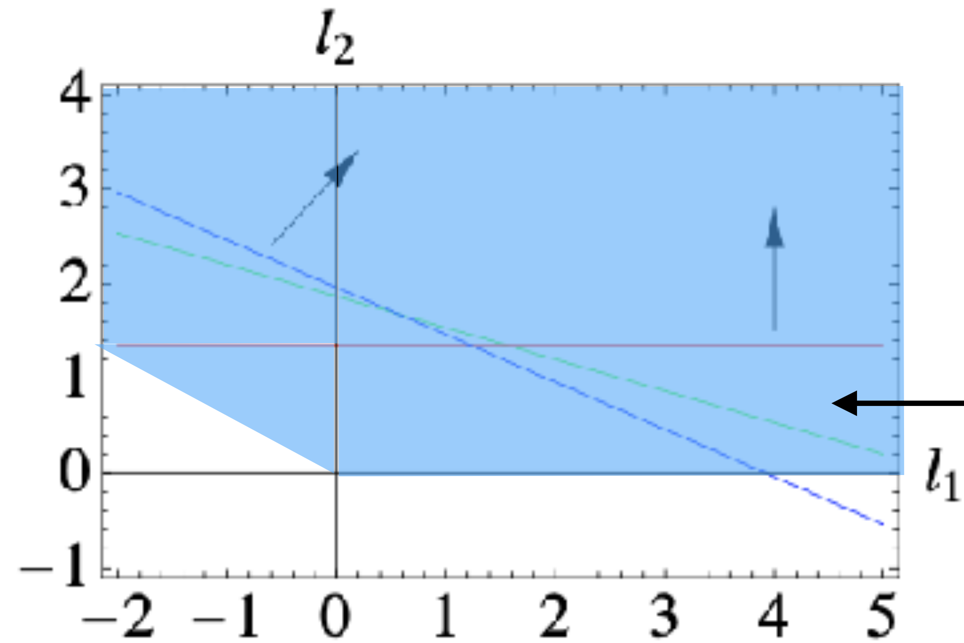
Mateu, Manohar '08
Distler, Grinstein, Porto, Rothstein '06

$$\mathcal{L}_2 = \ell_1 (\nabla^\mu U^T \nabla_\mu U)^2 + \ell_2 (\nabla^\mu U^T \nabla^\nu U)(\nabla_\mu U^T \nabla_\nu U)$$

$$A(s,t) = \frac{s+t}{f^2} + (l_1 + \frac{3}{2} l_2)\frac{s^2+t^2}{f^4} + l_2 \frac{st}{f^4} + \cdots$$

Th: $l_1 + 2l_2 > 0$ Exp: 8.2 ± 0.6

Th: $l_2 > 0$ Exp: 4.3 ± 0.1



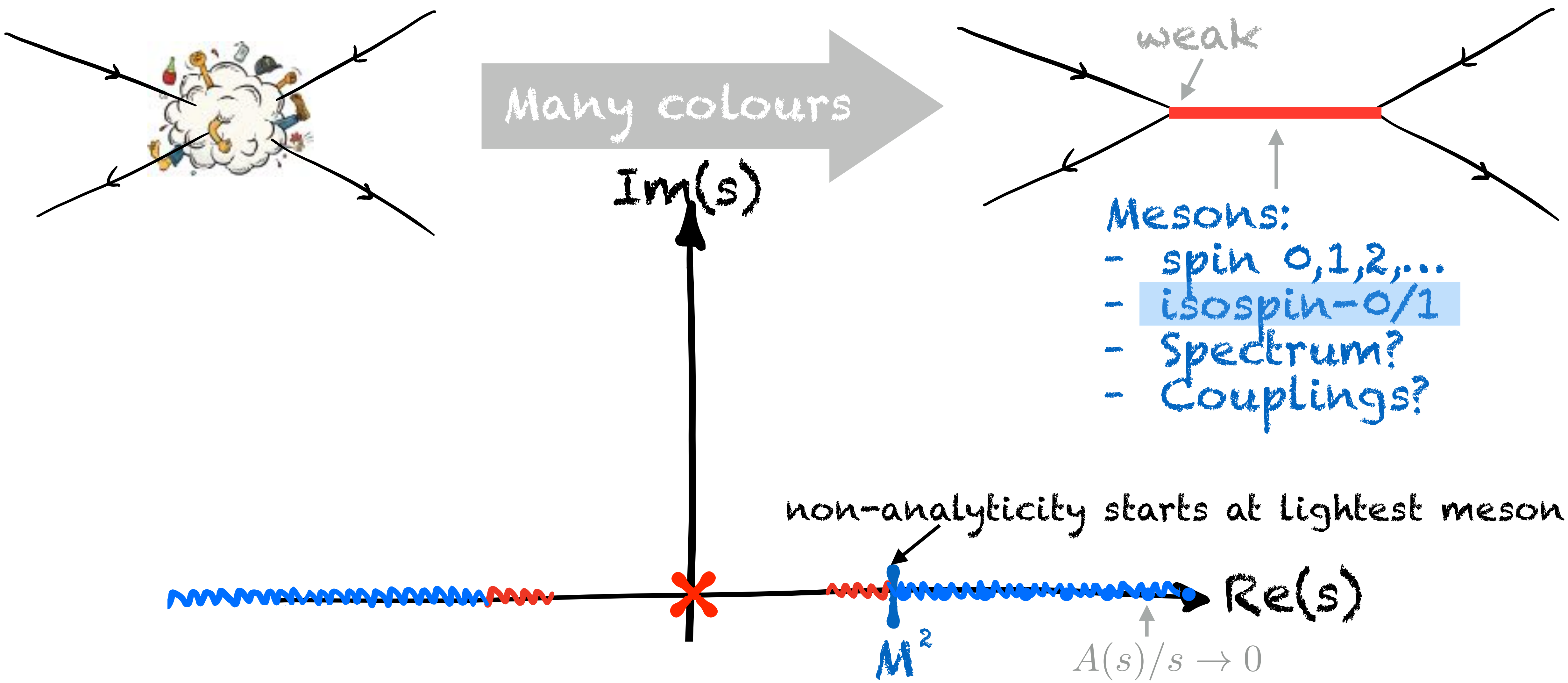
ChPt loops contribute a small calculable piece (more later)

...now, more constraints from non-forward amplitude (efficient if IR weakly coupled)

e.g
deRham,Melville,Tolley,Zhou'17
Bellazzini,Elias-Miro,Rattazzi,Riembau,FR'20
Arkani-Hamed,Huang,Huang'20
Tolley,Wang,Zhou'20
Caron-Huot,vanDuong'20
Caron-Huot,Mazac,Rastelli,Simons-Duffin'21
Chiang,Huang,Li,Rodina,Weng'21
Bellazzini,Riembau,FR'21

3. Large-N QCD

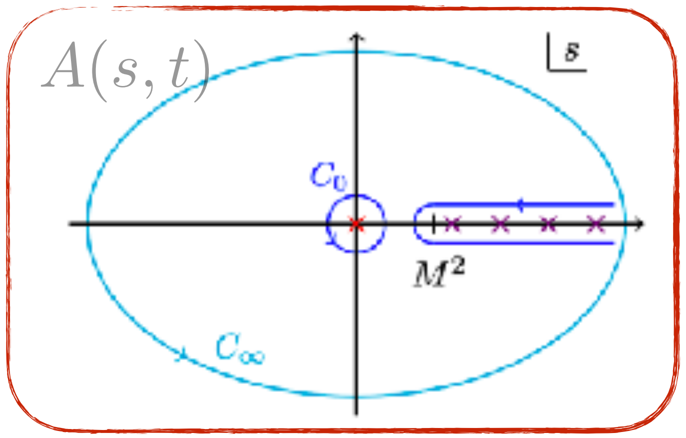
Dispersive approach complemented with Large-N_c info:



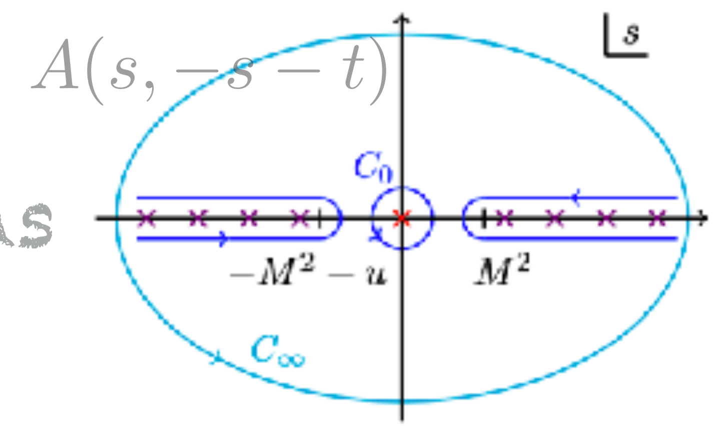
3. Positivity for Pions

Albert,Rastelli'22
Fernandez, Pomarol, FR, Sciotti '22

$$A(s,t) = \frac{(s+t)}{f^2} + g_{2,0}\left(s^2+t^2\right) + g_{2,1}st + \cdots$$



2 dispersion relations



$$\int_{\cap_{\bar{s}_t}} \frac{ds}{\pi s i}\frac{A(s,t)}{s^{1+n}} = \frac{2}{\pi s}\int_{\bar{s}}^{\infty} ds \sum_{\ell} \mathrm{Im} f_\ell(s) \frac{P_\ell(1+\frac{2t}{s})}{s^{1+n}}$$

$t \approx 0$ UV→IR dispersion relations

$t \approx 0$ UV→UV null constraints

$n=0$ $\quad g_{1,0} + g_{2,1}t + g_{3,1}t^2 + \ldots = \left\langle \frac{P_J(1)}{s} + 2\frac{P'_J(1)}{s^2}t + 2\frac{P''_J(1)}{s^3}t^2 + \ldots \right\rangle$

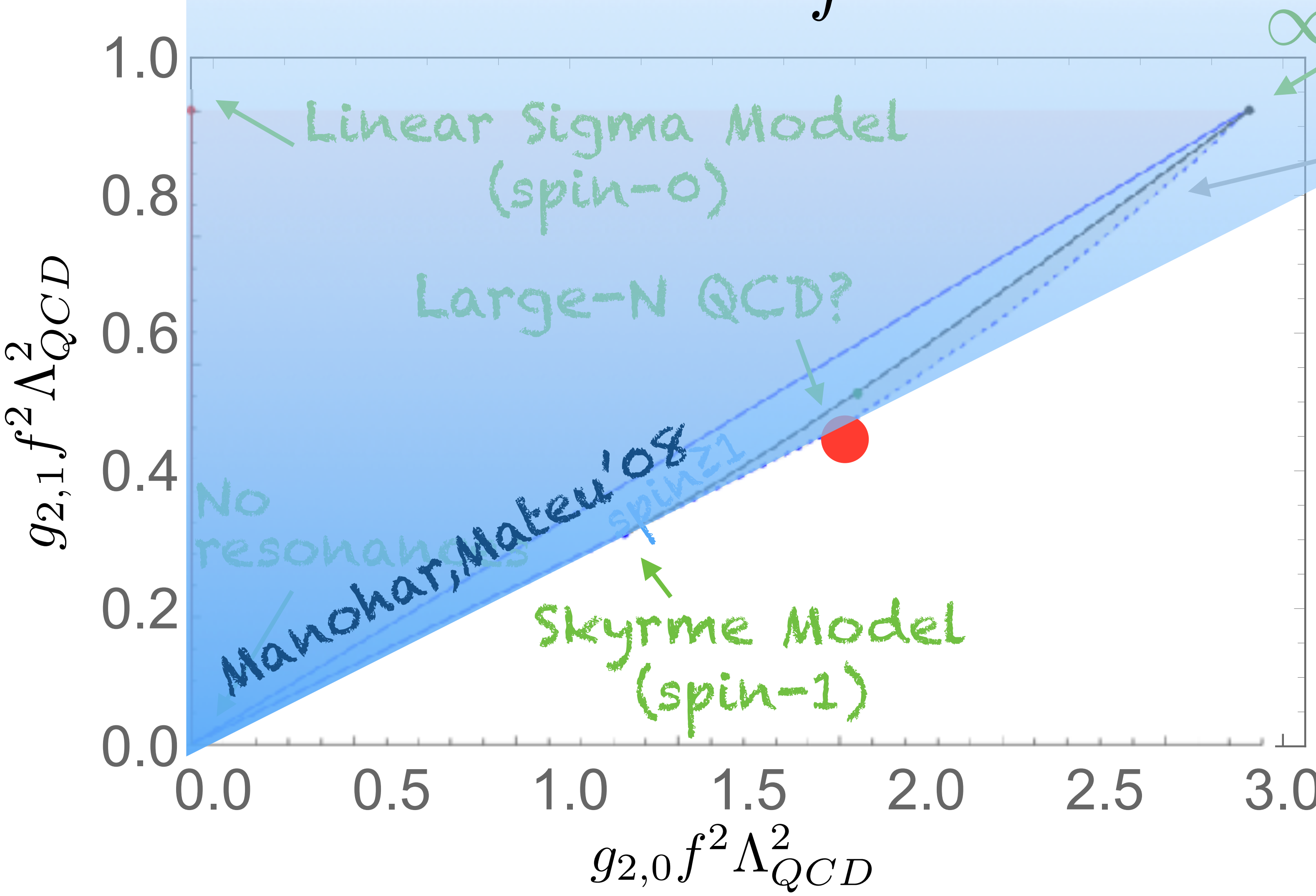
$n=1$ $\quad g_{2,0} + g_{3,1}t + g_{4,2}t^2 + \ldots = \left\langle \frac{P_J(1)}{s^2} + 2\frac{P'_J(1)}{s^3}t + 2\frac{P''_J(1)}{s^4}t^2 + \ldots \right\rangle$

$n=2$ $\quad g_{3,0} + g_{4,1}t + g_{5,2}t^2 + \ldots = \left\langle \frac{P_J(1)}{s^3} + 2\frac{P'_J(1)}{s^4}t + 2\frac{P''_J(1)}{s^5}t^2 + \ldots \right\rangle$

3. Positivity for Pions

Fernandez, Pomarol, FR, Sciotti '22

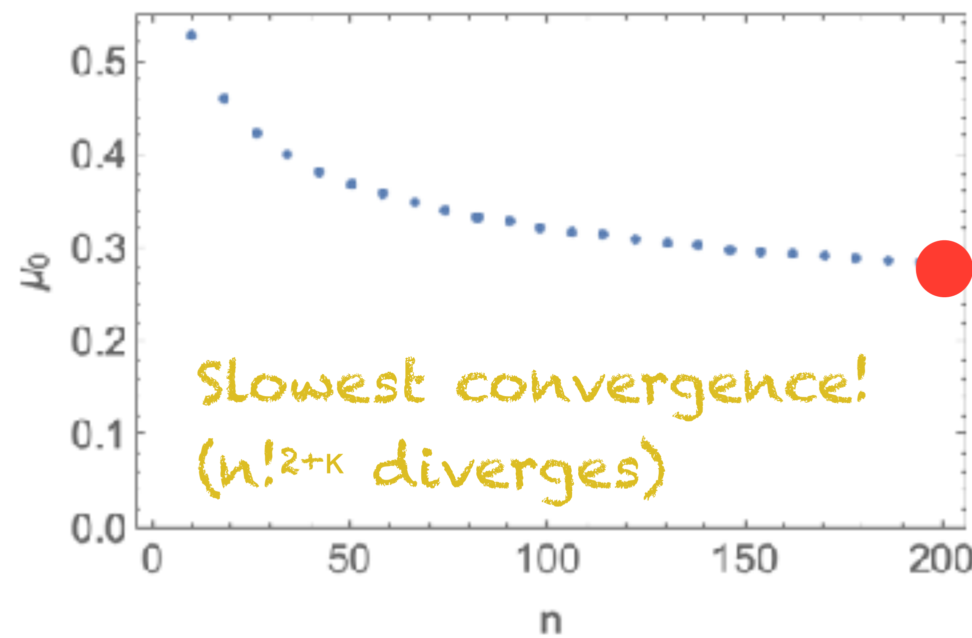
$$A(s,t) = \frac{(s+t)}{f^2} + g_{2,0}\,(s^2+t^2) + g_{2,1}st + \cdots$$



∞ resonance model

Numerical Results

Albert,Rastelli'22



$$\mu_0 = \frac{\begin{vmatrix} 0!^2 & 1!^2 & \cdots & n!^2 \\ 1!^2 & 2!^2 & \cdots & (n+1)!^2 \\ \cdots & \cdots & \ddots & \vdots \\ n!^2 & (n+1)!^2 & \cdots & (2n)!^2 \end{vmatrix}}{\begin{vmatrix} 2!^2 & 3!^2 & \cdots \\ 3!^2 & 4!^2 & \cdots \\ \cdots & \cdots & \ddots \end{vmatrix}} \to 0$$

Theories at the corner are special

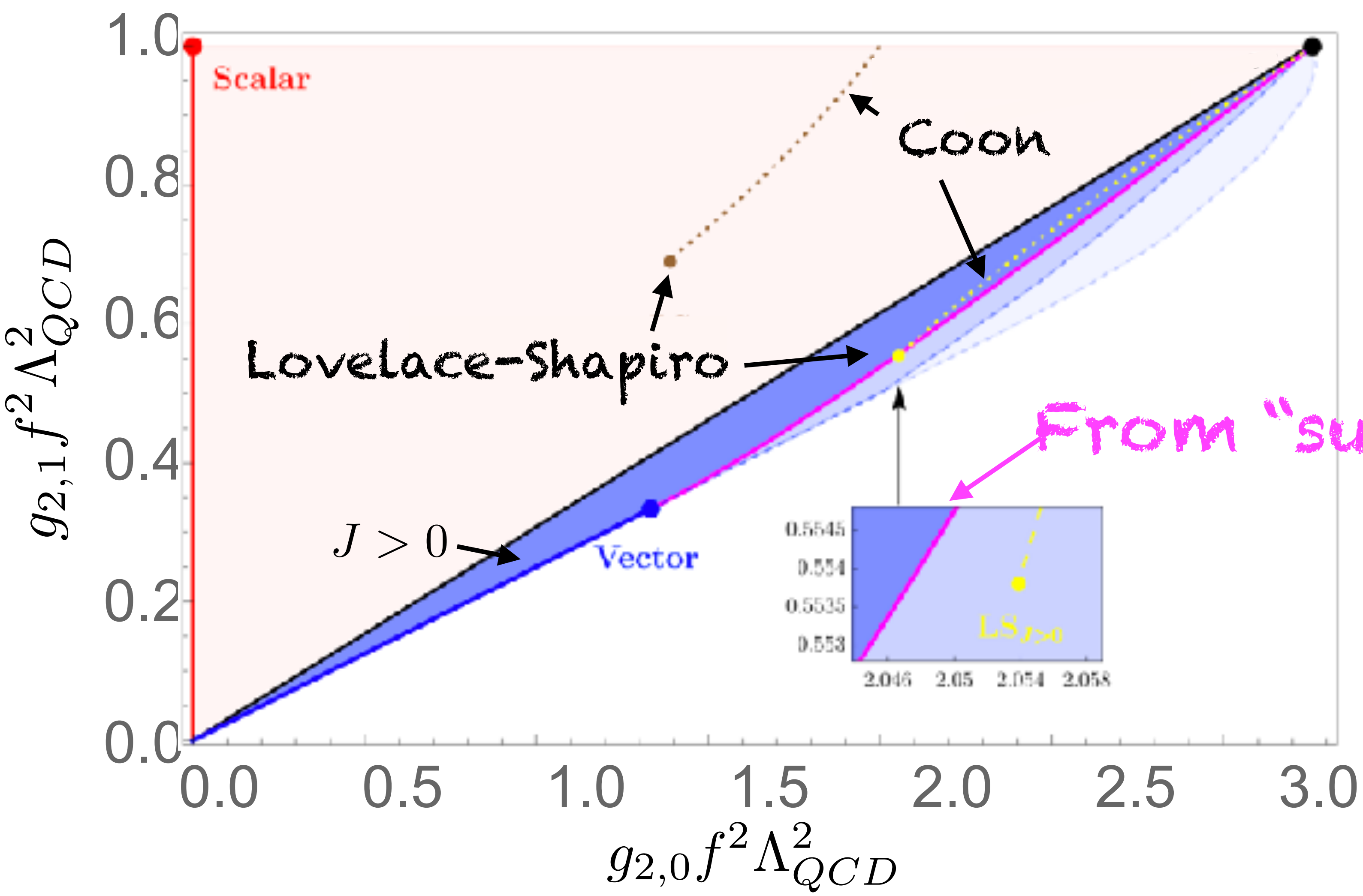
Analytic limit unreachable numerically

→ No new kink, only vector model

3. Positivity for Pions

Fernandez, Pomarol, FR, Sciotti '22

$$A(s,t) = \frac{(s+t)}{f^2} + g_{2,0}\,(s^2+t^2) + g_{2,1}st + \cdots$$



J>0 "su-model"

Albert,Rastelli'22
Caron-Huot,vanDuong'20

$$A(s,t) \sim \frac{1}{s-M^2}\frac{1}{t-M^2} - \text{spin-0}$$

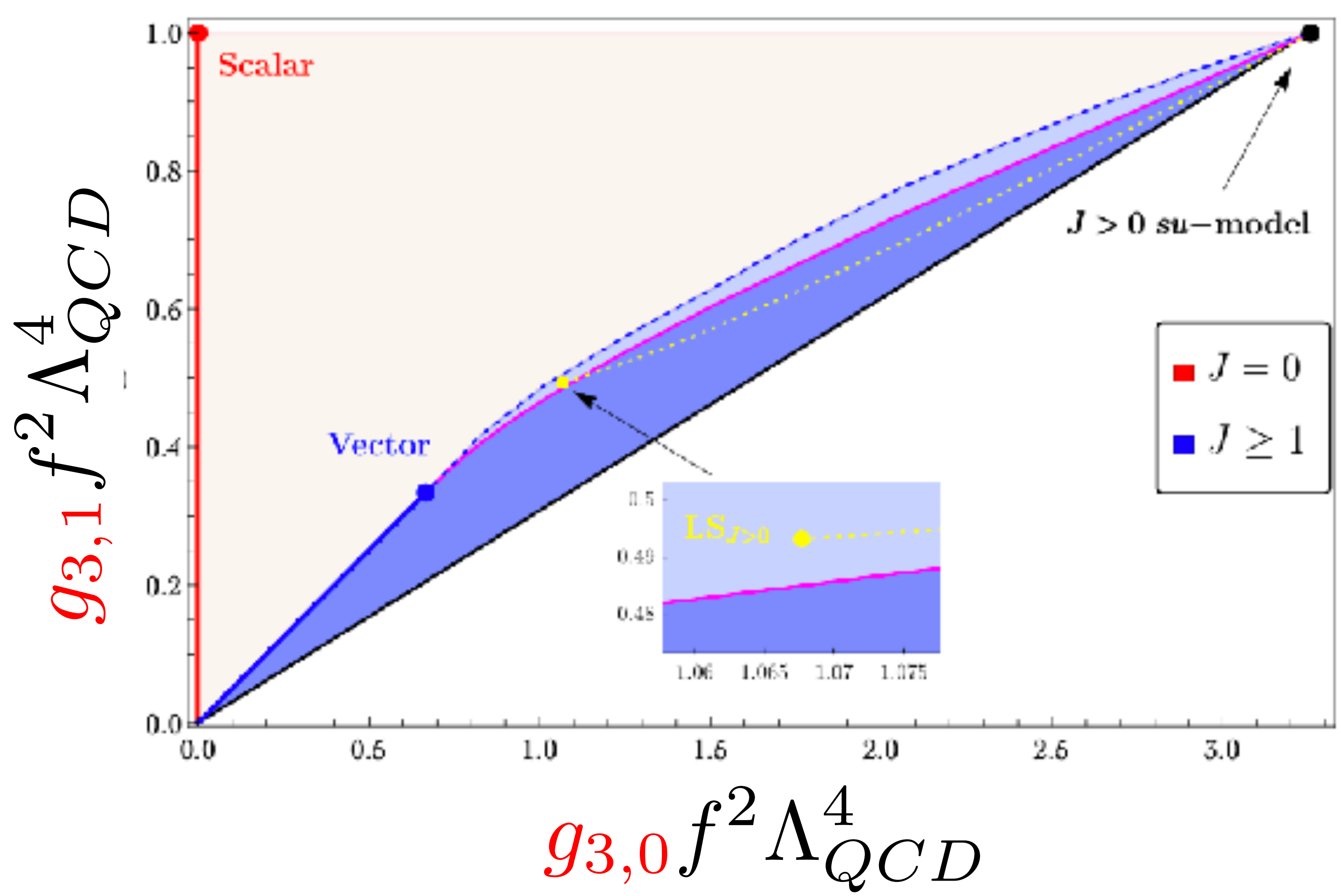
Extremal theories (edges/corners) provide a reference

- multi-spin extremal theories still unknown!

3. Positivity for Pions

Fernandez, Pomarol, FR, Sciotti '22

$$A(s,t) = \frac{(s+t)}{f^2} + g_{2,0}\,(s^2+t^2) + g_{2,1}st + g_{3,0}(s^3+t^3) + g_{3,1}(s^2t+t^2s) + \cdots$$



Extremal theories (edges/corners) provide a reference

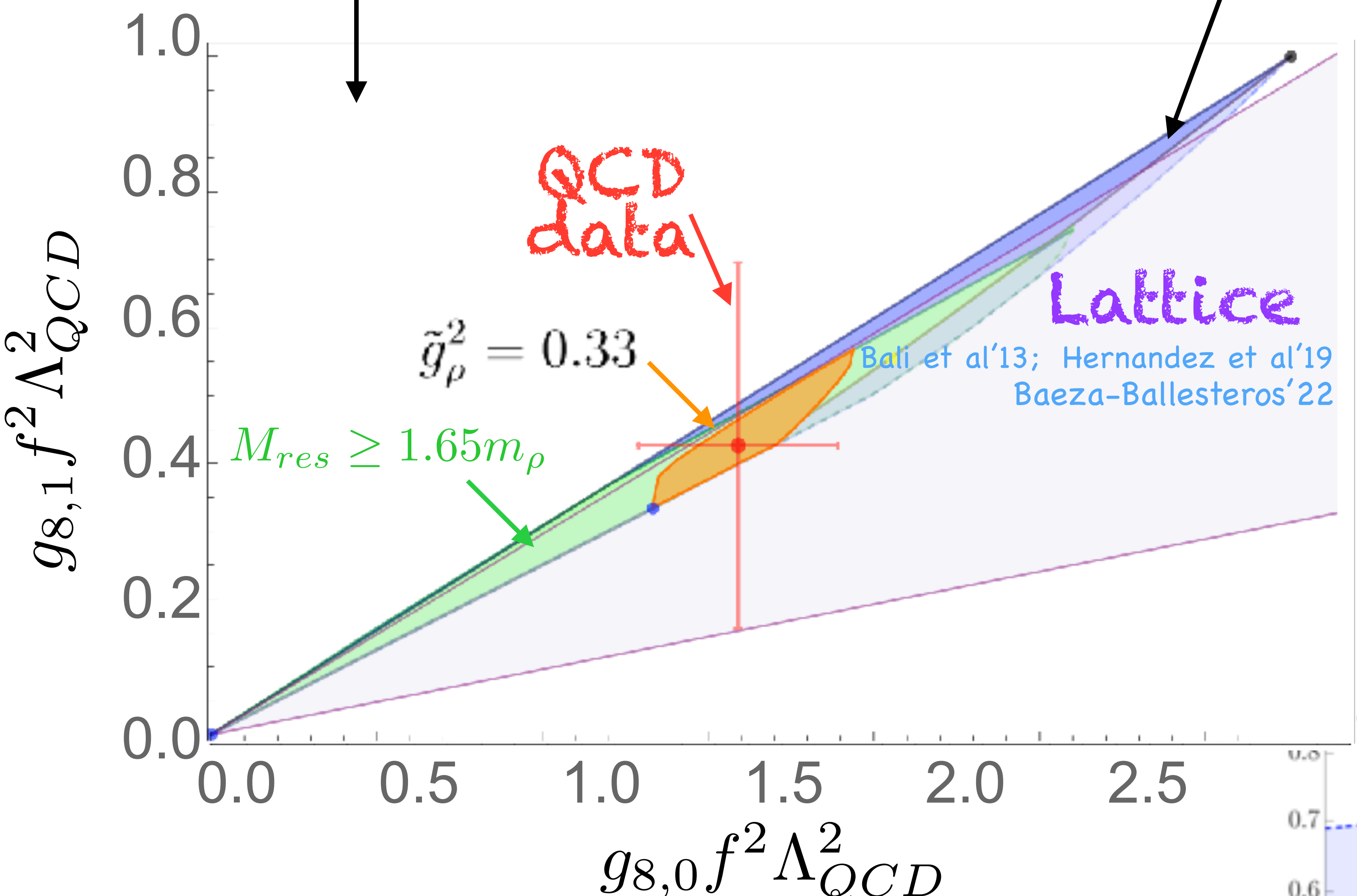
→ multi-spin extremal theories still unknown!

3. Vector Meson Dominance

Fernandez, Pomarol, FR, Sciotti '22

Spin-0 easy to model

Spin>0 info most important



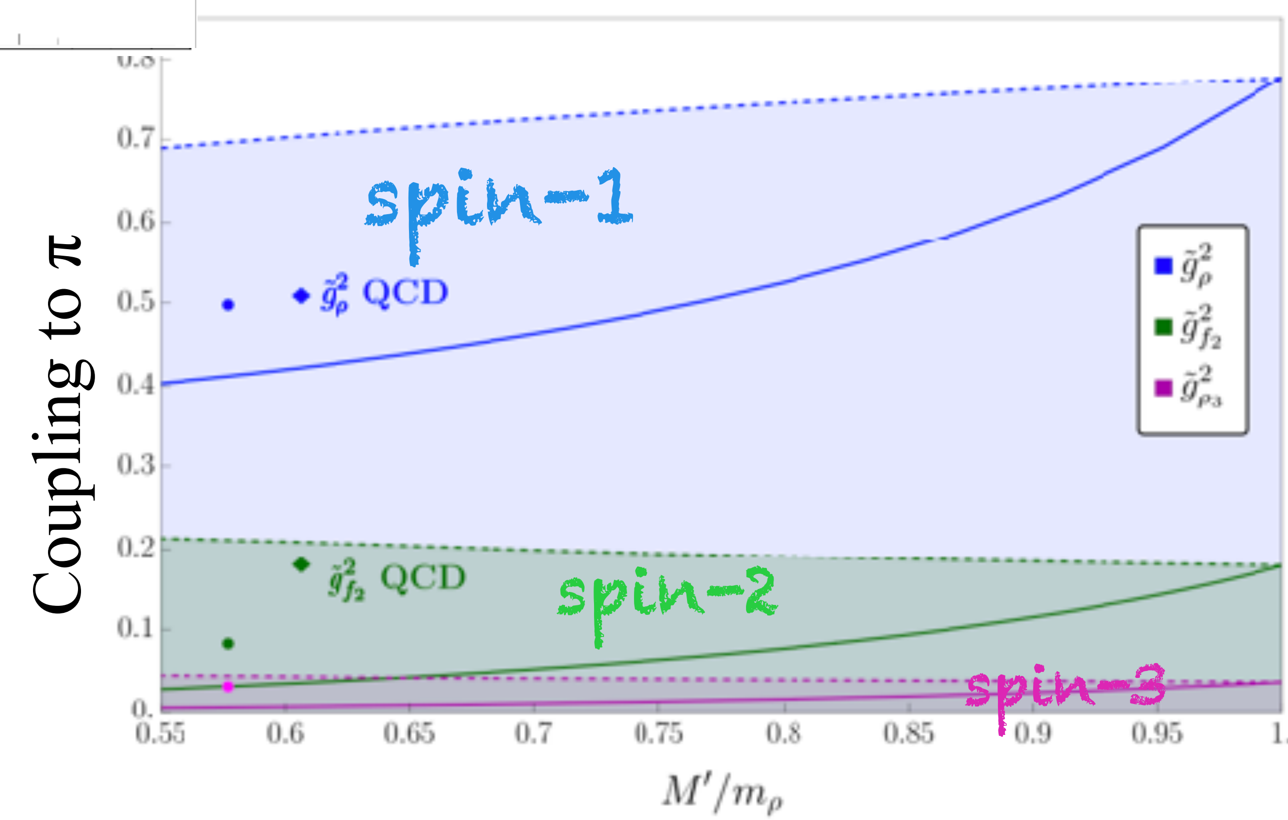
Null-Constraints:

UV Spin-1 UV Spin>1

$$\int_0^\infty ds \frac{a_1(s)}{s^n} = \sum_{J=1}^\infty \int_0^\infty ds \frac{a_J(s)}{s^n} \frac{J^2}{s^2}$$

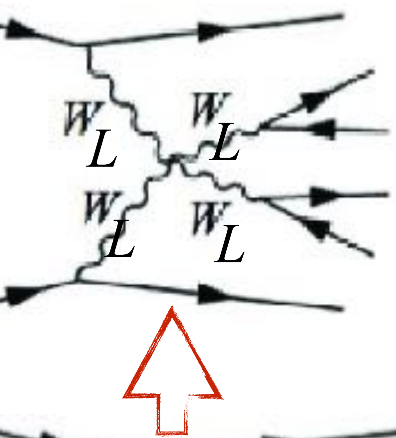
spin>1 always together... ... and smaller!

- IR coefficients dominated by UV spin-1 (VMD)
- Holographic models good
  (light states have s=0,1)

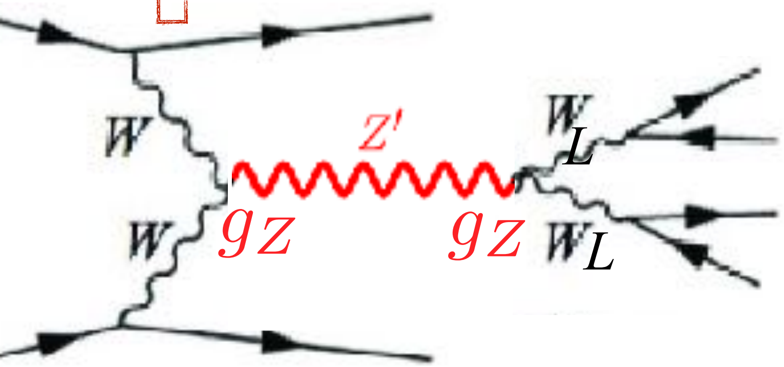


3. From pions to $W_L W_L$ scattering

Fernandez, Pomarol, FR, Sciotti '22



$$A(s,t) = c_H(s+t) + g_{8,0}\,(s^2+t^2) + g_{8,1}st + \cdots$$

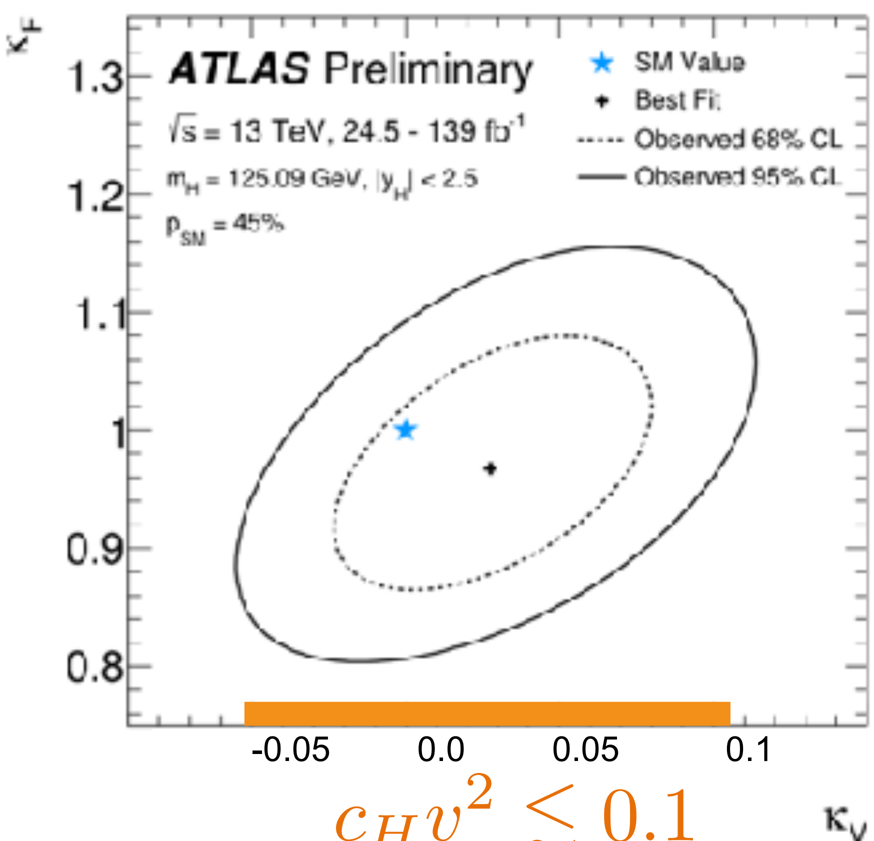


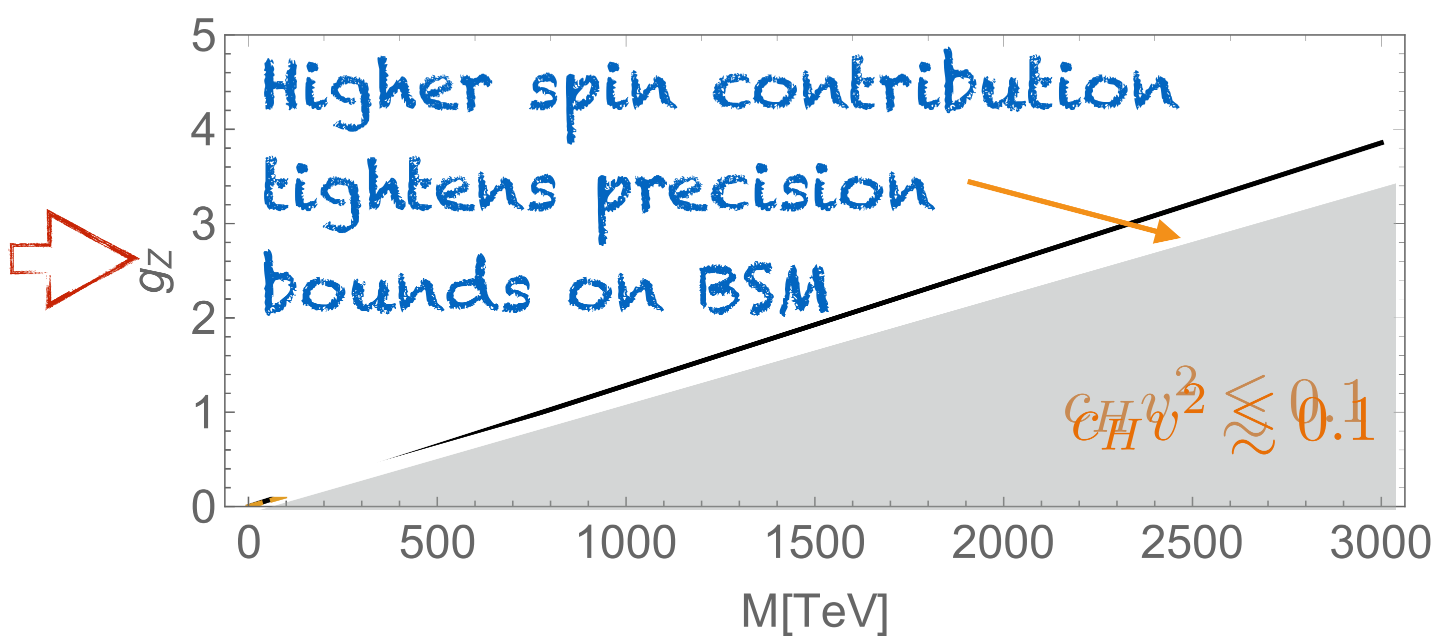
UV Spin-1 | UV Spin>1

$$\int_0^\infty ds \frac{a_1(s)}{s^n} = \sum_{J=1}^\infty \int_0^\infty ds\, \frac{a_J(s)}{s^n}\frac{J^2}{s^2}$$

spin>1 always together... | ... and smaller!

$$c_H = \frac{g_Z^2}{M^2} \qquad c_H > 1.3\frac{g_Z^2}{M^2}$$



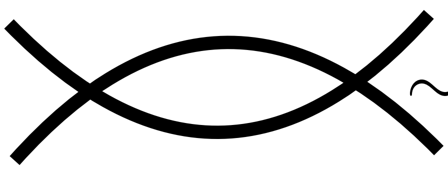


3. Beyond Large-N

Falkowski, Henning, Lombardo, FR, Rodriguez-Sanchez'to appear
Bellazzini, Riembau, FR' 21

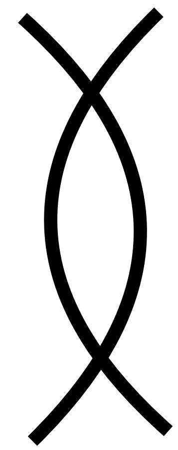
Loops change relation Wilson coeff. ⟷ arcs (on which bounds apply)

Running



$$\sim \frac{1}{(4\pi f^2)^2} s^2 \log\left(\frac{-s}{\mu^2}\right)$$

Forward Divergences



$$\sim \frac{1}{(4\pi f^2)^2} st \log\left(\frac{-s}{4m_\pi^2 + t}\right)$$

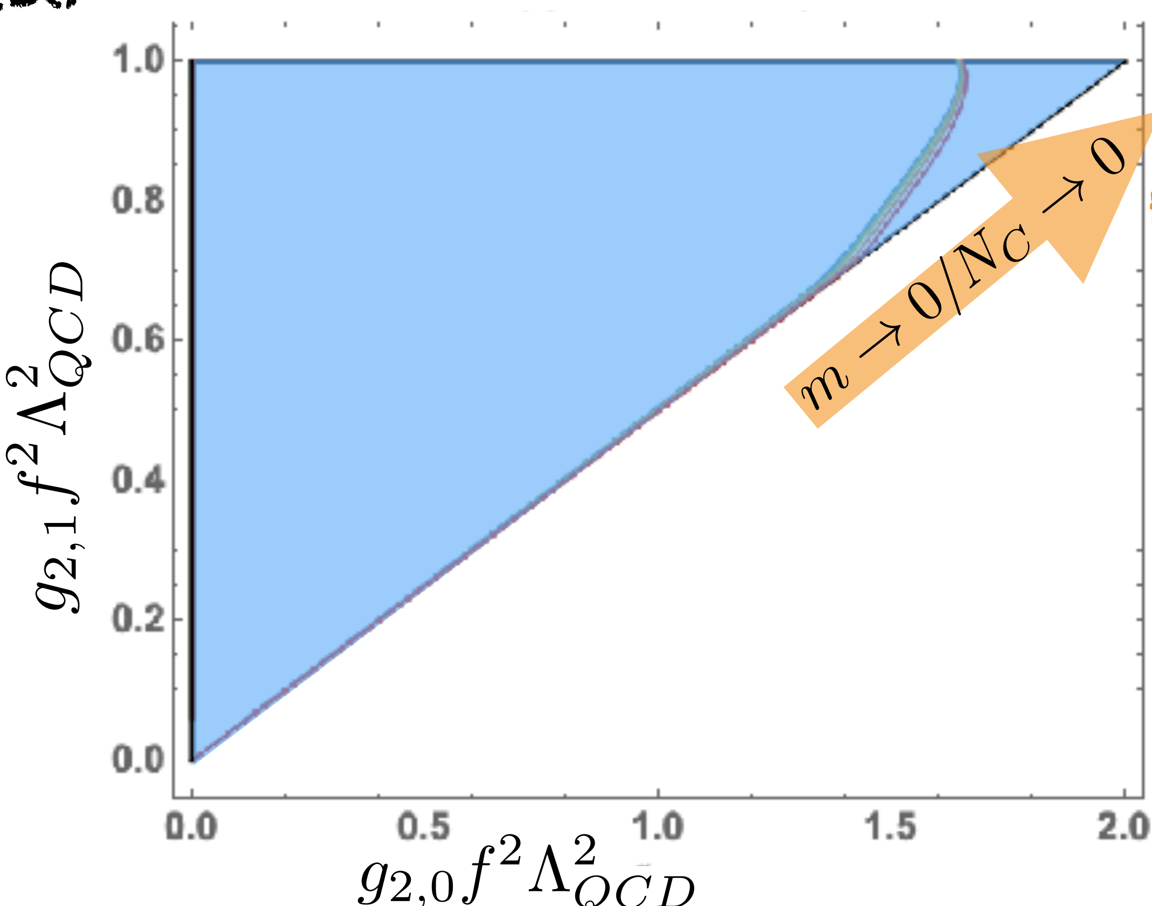
Larger in higher arcs

▶ $g_{n>3}$ loop-dominated!

▶ Running Logs in g_2

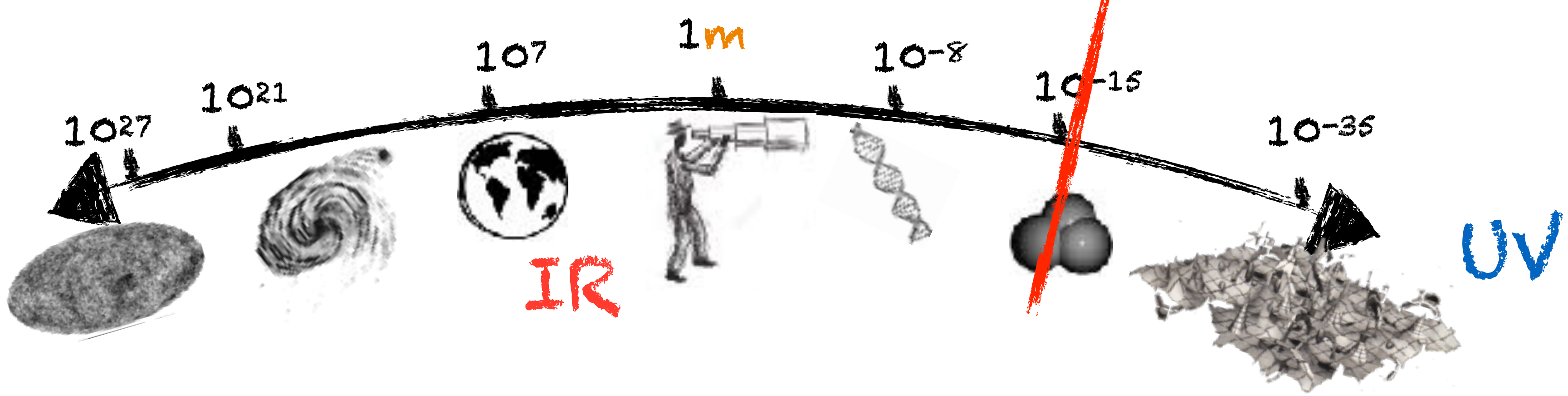
For m=0 singular in t≈0

▶ No null constraints



For m≠0 bounds weaker for smaller N_c

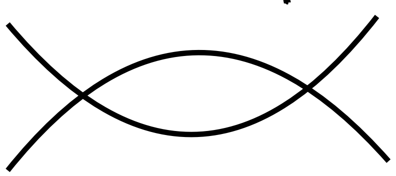
Summary and outlook

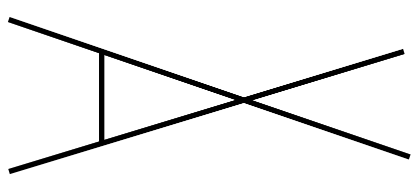


- Dispersion relations to explore BSM
  - Operators with dim>8 dont matter
  - Particles with spin$\geq$2 Cant be lighter that others

- Combine Large-N and dispersion relations
  - Corner QCD
  - Explain Vector Meson Dominance

- Development of a method that survives IR (t=0) loop non-analyticities





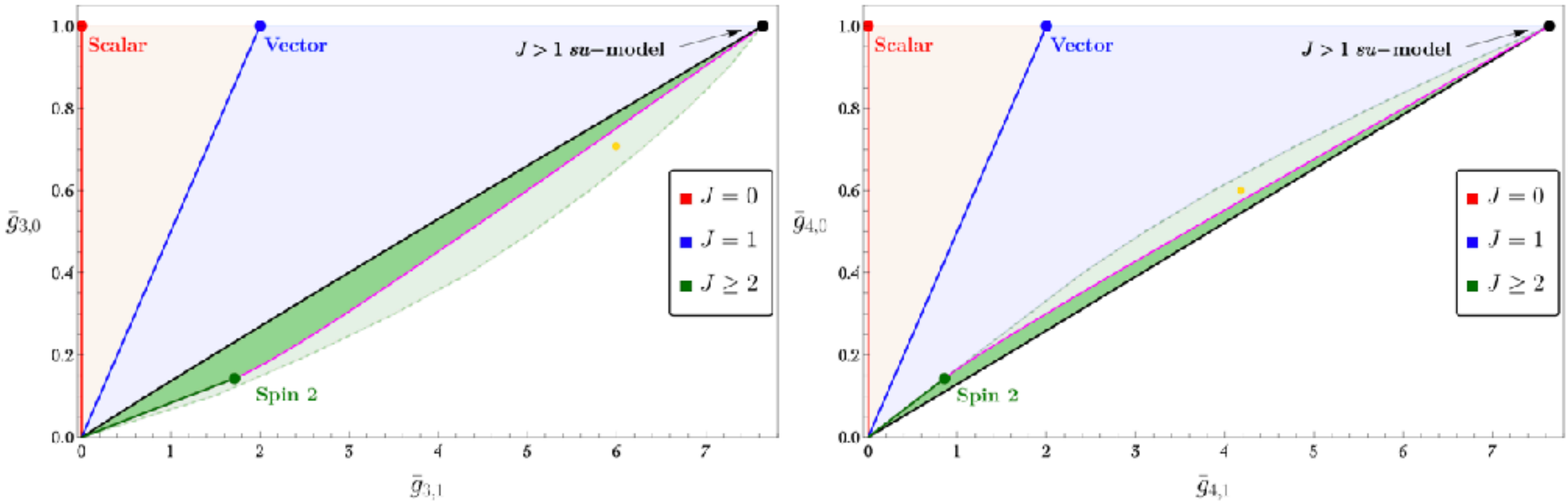
$$\sim s^m t^n \log s \log t$$

non-analytic

Bellazzini,FR,Riembau'21
Beadle,Isabella,Perrone,
Ricossa,FR,Serra'to appear

- Understanding Pi-Photon amplitudes

Large-N QCD with $A(s)/s^2 \to 0$



Couplings to pions:

$$\bar g_s^2, \bar g_\rho^2 \leq 1 . \quad \bar g_{f_2}^2 \lesssim 0.80 . \qquad \bar g_{\rho_3}^2 \lesssim 0.14 , \quad \bar g_{f_4}^2 \lesssim 0.04 .$$

LECs:

$$\frac{\bar g_{2,1}}{\bar g_{2,1}|_\rho} \lesssim 1 + 0.17\,\frac{1-\bar g_\rho^2}{\bar g_\rho^2} .$$

(e.g for grho=0.5 the rho contributes 80% to LECs)

SU(2) amplitude

Albert,Rastelli'22

$$\mathcal{T}_{ab}^{cd} = A(s|t,u)\left(\frac{2}{N_f}\delta_{ab}\delta^{cd} + d_{abe}d^{cde}\right) + A(t|s,u)\left(\frac{2}{N_f}\delta_a^d\delta_b^c + d_a{}^d{}_e d_b{}^{ce}\right) + A(u|s,t)\left(\frac{2}{N_f}\delta_a^c\delta_b^d + d_a{}^c{}_e d^d{}_b{}^e\right).$$

$$\mathcal{M}^0(s|t,u) = 3A(s|t,u) + A(t|s,u) + A(u|s,t) = 3M(s,t) + 3M(s,u) - M(t,u),$$

$$\mathcal{M}^1(s|t,u) = A(u|s,t) - A(t|s,u) = 2\Big(M(s,u) - M(s,t)\Big),$$

$$\mathcal{M}^2(s|t,u) = A(t|s,u) + A(u|s,t) = 2M(t,u).$$

$$A(s|t,u) = M(s,t) + M(s,u) - M(t,u), \quad 2M(s,u) = A(s|t,u) + A(u|s,t)$$