CTEQ/CFNS Summer School

June 5, 2023

Deep Inelastic Scattering & Collinear Factorization

See:

- **"Partons, "Factorization and Resummation", hep-ph-9606312**
- **"Handbook of Perturbative QCD", Rev. Mod. Phys. 67 (1995) 157.**
  (no arXive number, but PDF available on Inspire)

The Context of QCD: "Fundamental Interactions"

- Electromagnetic
- + Weak Interactions $\Rightarrow$ Electroweak
- + Strong Interactions (QCD) $\Rightarrow$ Standard Model
- + . . . = Gravity and the rest?
- QCD: A theory "off to a good start". Think of . . .

  – $\vec{F}_{12} = -GM_1M_2\hat{r}/R^2$ $\Rightarrow$ elliptical orbits
  . . . 3-body problem . . .

  – $L_{\rm QCD} = \bar{q}\ \not{D} q - (1/4)F^2$ $\Rightarrow$ asymptotic freedom
  . . . confinement . . .

I. Deep-inelastic Scattering and Collinear Factorization

IA. Nucleons to Quarks

IB. DIS: Structure Functions and Scaling

IC. Classic Parton Model Extensions: Fragmentation and Drell-Yan

Introduce concepts and results that predate QCD, led to QCD and were incorporated and explained by QCD.

IA. From Nucleons to Quarks

- **The pattern: nucleons, pions and isospin:**

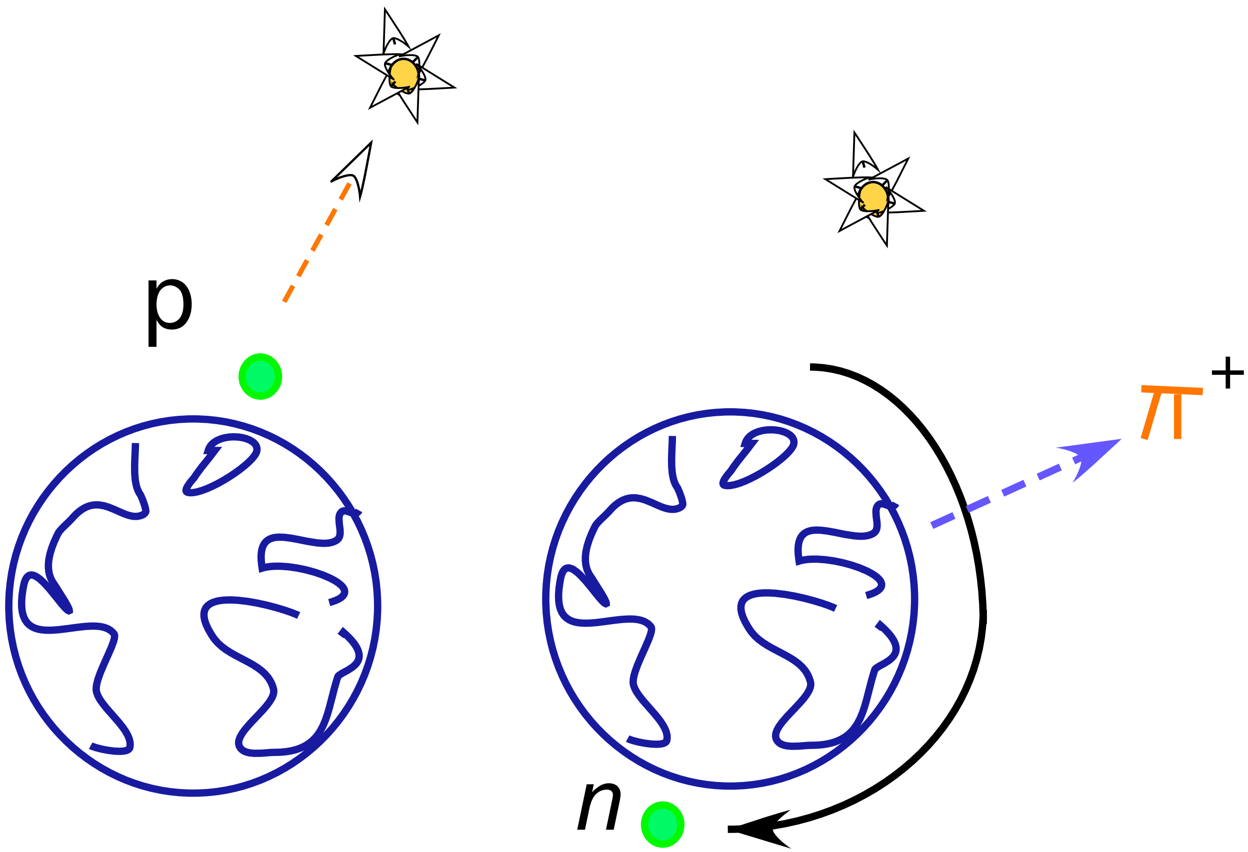
$$\begin{pmatrix} p \\ n \end{pmatrix}$$

  - **p: m=938.3 MeV,** $S = 1/2$, $I_3 = 1/2$
  - **n: m=939.6 MeV,** $S = 1/2$, $I_3 = -1/2$

$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

  - $\pi^{\pm}$**: m=139.6 MeV,** $S = 0$, $I_3 = \pm 1$
  - π^0**: m=135.0 MeV,** $S = 0$, $I_3 = 0$

- **Isospin space . . .**
- **Globe with a "north star" set by electroweak interactions:**



Analog: the rotation group (more specifically, $SU(2)$).

- **Explanation: π, N common substructure: quarks**
  (Gell Mann, Zweig 1964)

- **spin $S = 1/2$,**
  $I = 1/2$ (u, d) & $I = 0$ (s)
  with approximately equal masses (s heavier);

$$\begin{pmatrix} u \ (Q = 2e/3, I_3 = 1/2) \\ d \ (Q = -e/3, I_3 = -1/2) \\ s \ (Q = -e/3, I_3 = 0) \end{pmatrix}$$

$$\pi^+ = (u\bar{d}) \ , \quad \pi^- = (\bar{u}d) \ , \quad \pi^0 = \frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right) \ ,$$

$$p = (uud) \ , \quad n = (udd) \ , \quad K^+ = (u\bar{s}) \ldots$$

This is the quark model

- Quark model nucleon has symmetric spin/isospin wave function (return to this later)
- Early success: μ_p/μ_n = -3/2 (from $S = 1/2$, $I = 1/2$ uud, ddu wave functions; good to %)
- And now, six: 3 'light' (u, d, s), 3 'heavy': (c, b, t)
- Of these all but t form bound states of quark model type.

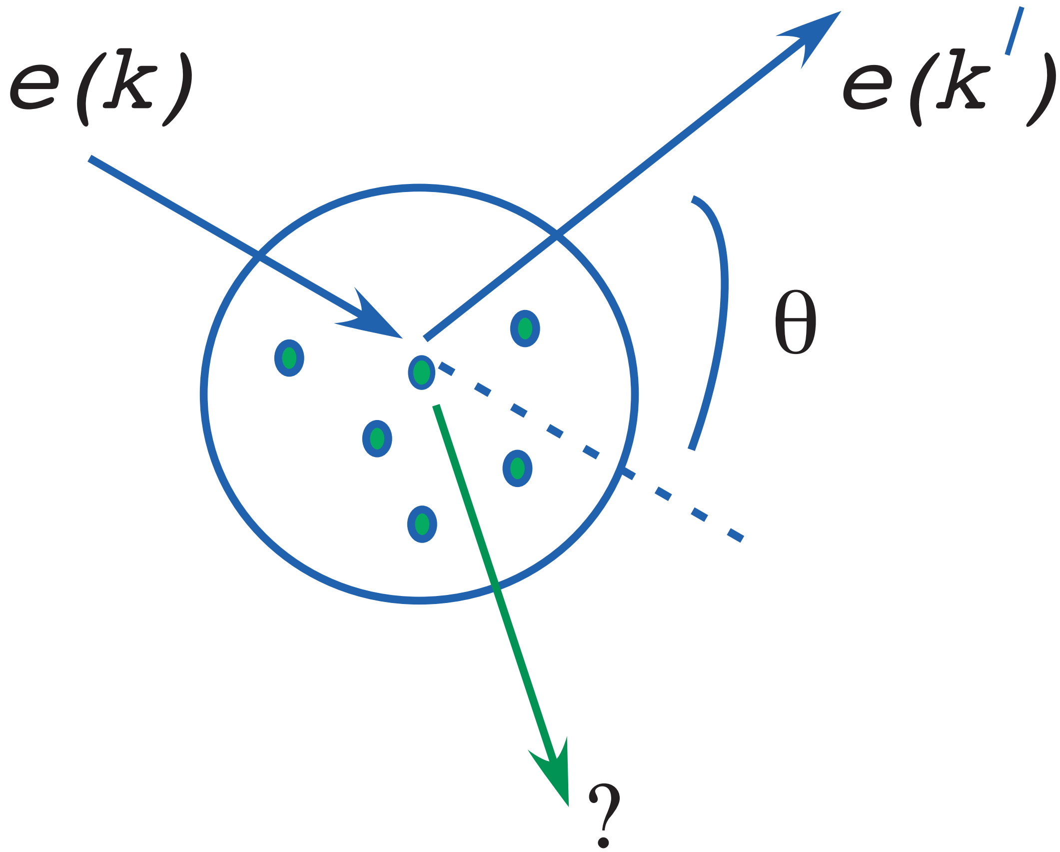
- **Quarks as Partons: “Seeing” Quarks.**

  No isolated fractional charges seen (“confinement.”)

  Could such a particle be detected?

  Look closer: do high energy electrons
  bounce off anything hard? (SLAC 1969 – ‘Rutherford-prime’)

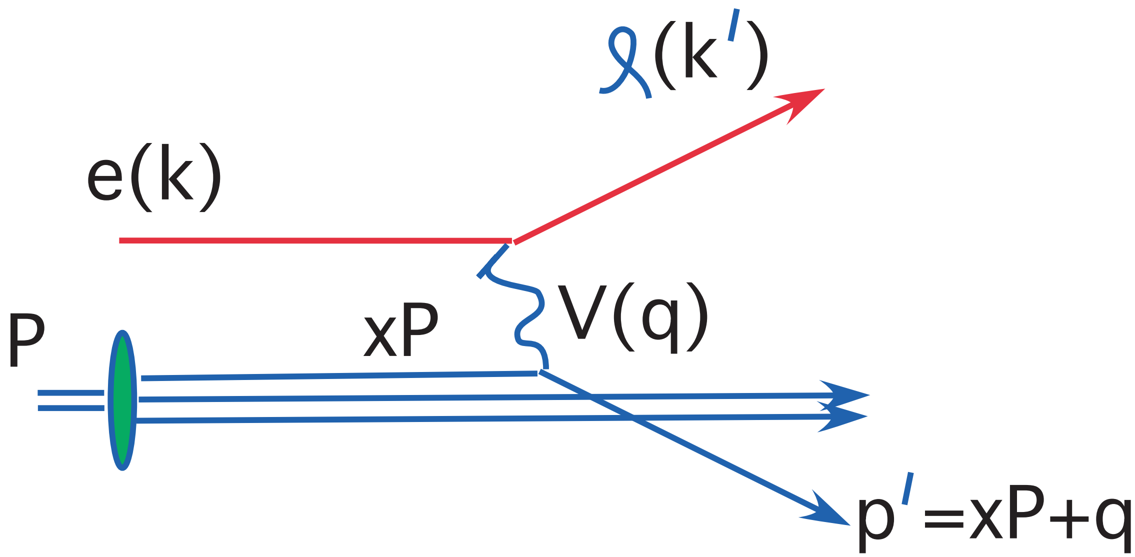
- **So look for:**



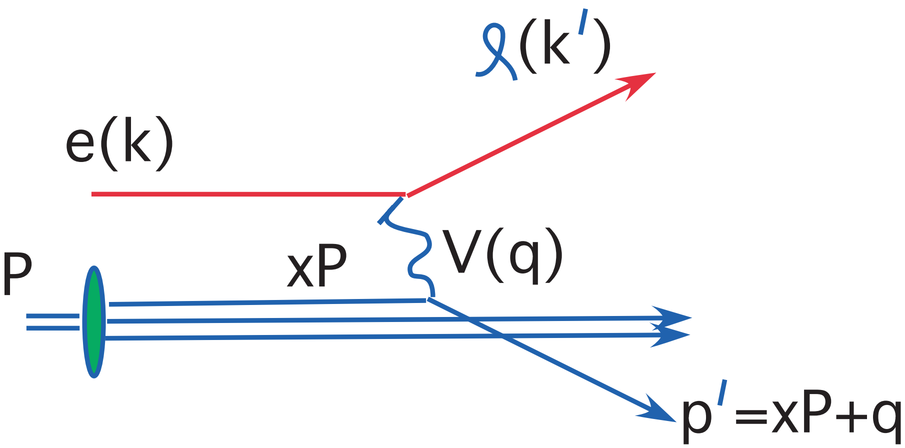
"Point-like' constituents.

The angular distribution gives information on the constituents.

Kinematics ($e + N(P) \to \ell + X$)



- $V = \gamma, Z_0 \Rightarrow \ell = e,\ \mu,$ **"neutral current" (NC).**
- $V = W^-(e^-, \nu_e)$, $V = W^+(e^+, \bar{\nu}_e)$, **also** (μ, ν_μ) **reactions. "charged current" (CC).**
- $W^2 \equiv (p+q)^2 \gg m^2_{\text{proton}}$: **Deep-inelastic scattering (DIS)**



$Q^2 = -q^2 = -(k - k')^2$ **momentum transfer.**

$x \equiv \frac{Q^2}{2p \cdot q}$ **momentum fraction (from** $p'^2 = (xp + q)^2 = 0$**).**

$y = \frac{p \cdot q}{p \cdot k}$ **fractional energy transfer in** p **rest frame.**

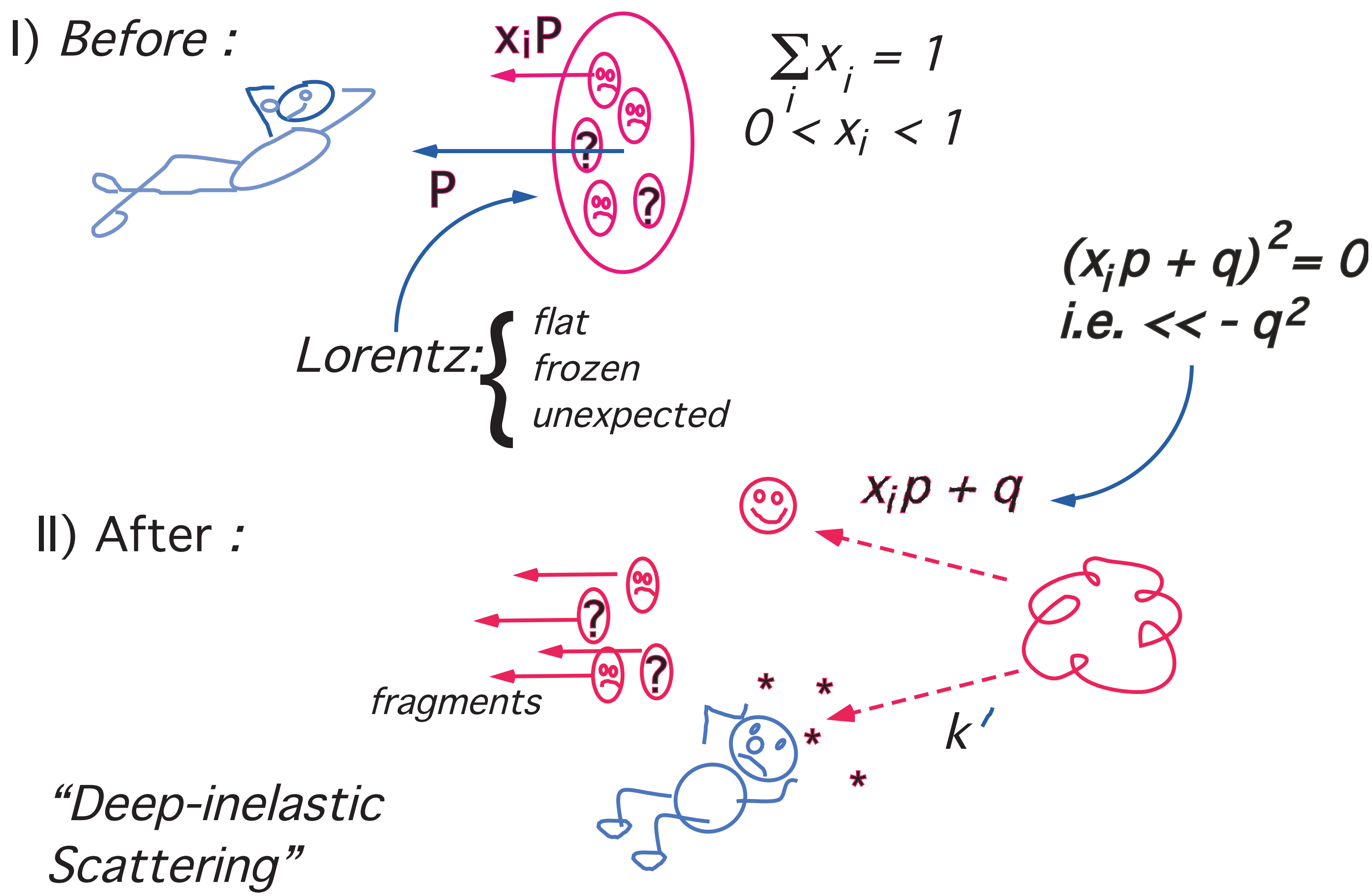
$W^2 = (p + q)^2 = \frac{Q^2}{x}(1 - x)$ **squared invariant mass of final-state hadrons.**

A useful identity:

$$xy = \frac{Q^2}{S}$$

From CTEQ Summer School 1992:

Parton Interpretation (Feynman 1969, 72)
Look in the electron's rest frame . . .



– Basic Parton Model Relation

$$\sigma_{\rm eh}(p,q) \;=\; \sum_{\text{partons } a} \int_0^1 d\xi\, \hat{\sigma}^{\rm el}_{ea}(\xi p, q)\, \phi_{a/h}(\xi)\,,$$

– **where**: $\sigma_{eh}(p,q)$ is the inclusive cross section $e(k) + h(p) \to e(k' = k - q) + X(p+q)$

– **and**: $\hat{\sigma}^{\rm el}_{ea}(\xi p, q)$ is the elastic cross section $e(k) + a(\xi p) \to e(k' - q) + a(\xi p + q)$ which sets $(\xi p + q)^2 = 0 \to \xi = -q^2/2p\cdot q \equiv x$.

– **and**: $\phi_{a/h}(\xi)$ is the distribution of parton a in hadron h, the “probability for a parton of type a to have momentum ξp”. It is independent of the details of the hard scattering – the hallmark of factorization.

- **in words: Hadronic INELASTIC cross section is the sum of convolutions of partonic ELASTIC cross sections with the hadron's parton distributions.**

- **The nontrivial assertion: quantum mechanical incoherence of large-q scattering and the partonic distributions. Multiply probabilities without adding amplitudes.**

- **Heuristic justification: the binding of the nucleon involves long-time processes that do not interfere with the short-distance scattering. Later we'll see how this works in QCD.**

The basic elastic scattering: electron with "quark" a, fractional charge e_a, from one-photon exchange:

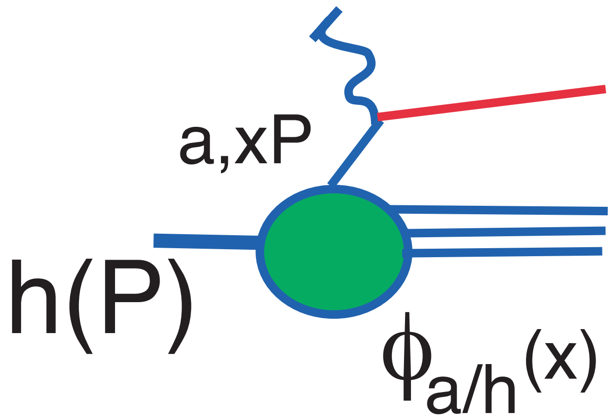
$$2\omega_{k'}(k,\xi p,q)\,\frac{d^3\sigma_{ea}^{(\mathrm{el})}}{d^3k'} \;=\; \frac{e_f^2\alpha_{EM}^2}{2s\;2p\cdot q}\left(\frac{s^2+u^2}{Q^4}\right)\delta(\xi-x)$$

where

$$s \;=\; (\xi p+k)^2\,,\quad u \;=\; (\xi p-k')^2 \quad t \;=\; -Q^2\,.$$

The "extra" delta function restricts the energy of the <u>incoming</u> quark, which isolates the parton distributions.
To analyze DIS in general terms, we will introduce a more general notation in terms of "structure functions" below.

– **The conventional picture for distributions:**



– **"QM incoherence" $\Leftrightarrow$ no interactions between the outgoing scattered quark and the rest.**

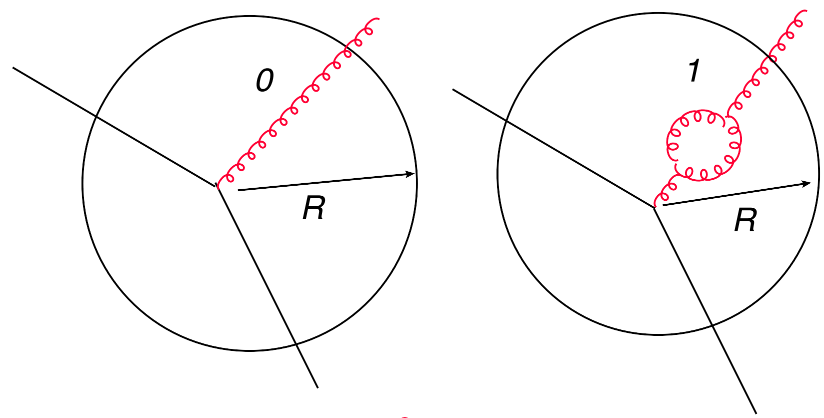
– **Note: cross section is like a area. One parton: $1/Q^2$, area covered by $a \sim \phi_a(x)/Q^2$. For this picture to work:**

$$\phi_a(x)/Q^2 \ll \pi R_p^2 \,.$$

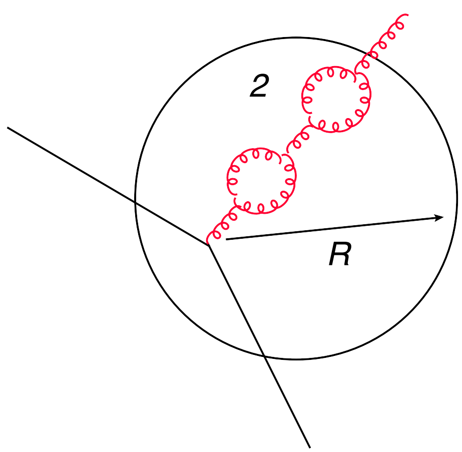
Otherwise, the partons cover the proton and we can assume only a single interaction. This is called "saturation".

What makes QCD different

QCD can confine and yet be nearly "free". Why? States with two extra gluons add up to infinity for R about 1 Fermi. There has to be a nearby source to absorb them. Quarks cannot appear alone; this is called "confinement".



This is not something proven, but demonstrated by "numerical lattice simulations" which provide beautiful agreement in hadronic mass differences



+ . . .

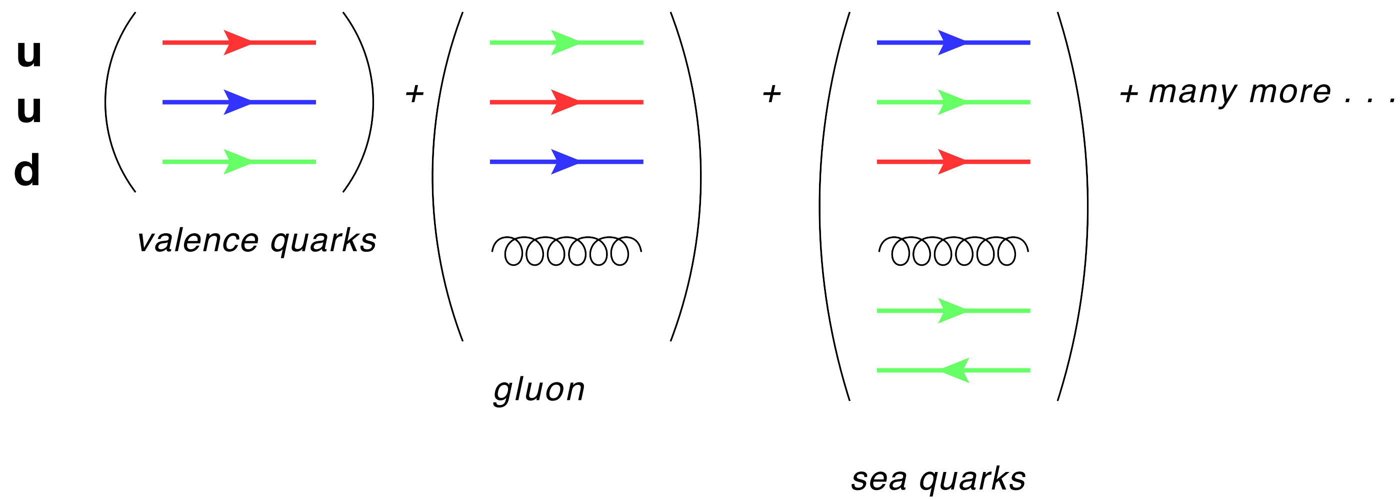
Each increases with R.

For R about 1 fm, they are all equal!

Asymptotic Freedom: Smaller (Larger) R gives weaker (stronger) forces

This means that the "states" of QCD are really different.
They are the protons, neutrons and other hadrons,
mostly made of three quarks (baryons). and quark-antiquark (mesons).

Our world, of course, is mostly protons, neutrons and the nuclei they can make. In our pictures, they are represented like:

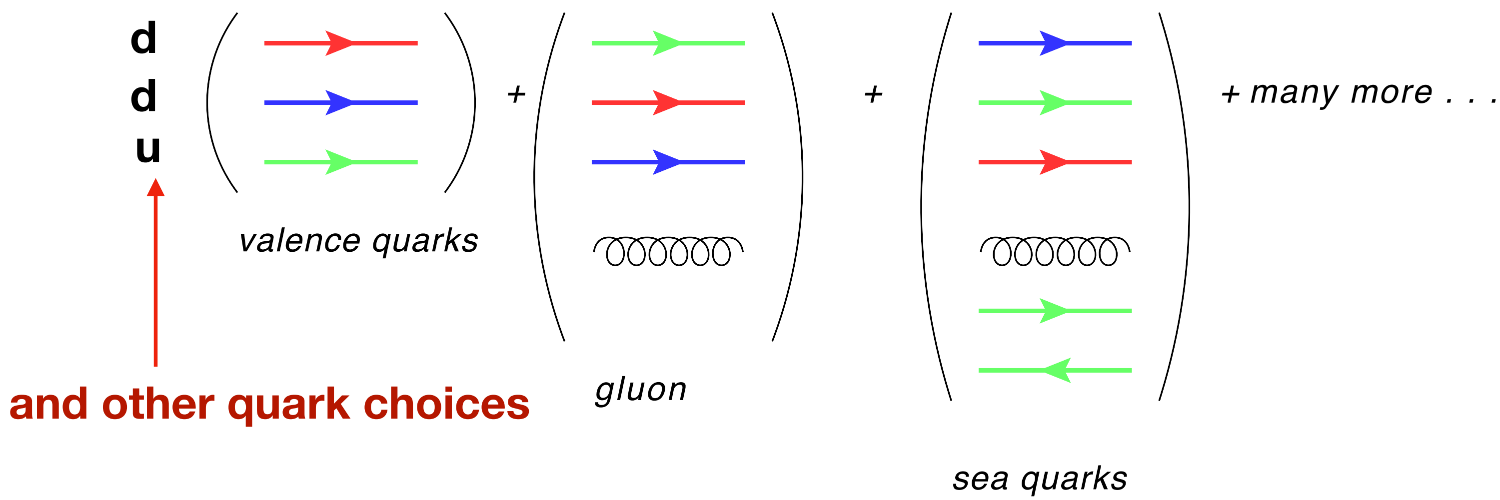


= | proton >

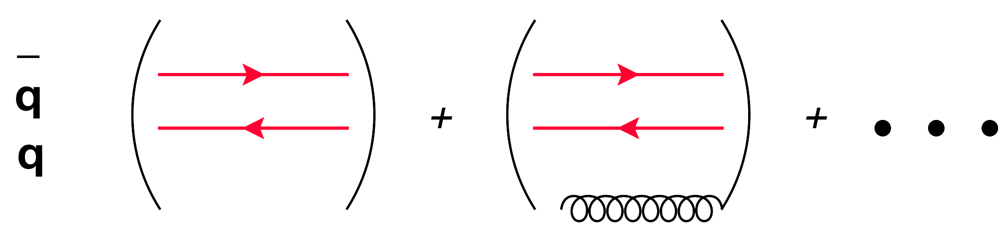
Taken all together, the proton has spin-1/2, the same as an electron or a single quark. It has a definite mass and charge +1. It is extraordinarily stable, and is the ultimate decay product for heavier solutions to the QCD Schrodinger equation.

The other “classic” states:

| neutron > =



| meson > =

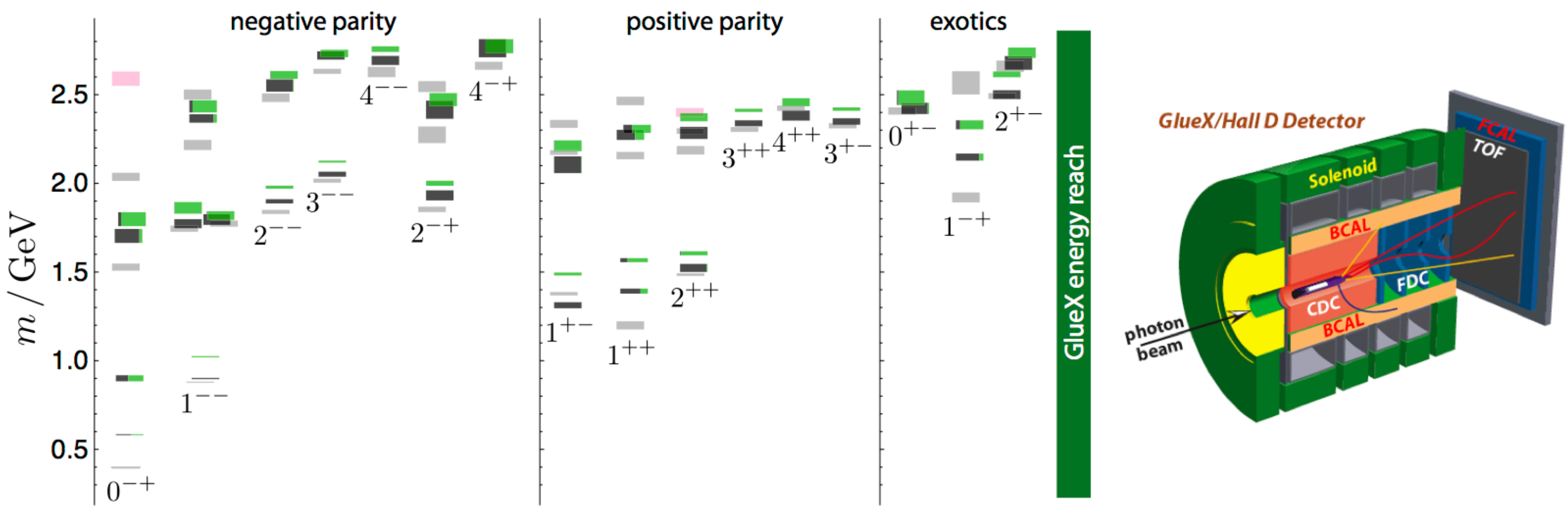


“On the lattice”: very roughly — the computer starts with list of just three quarks, or a quark and an antiquark fixed at some position. The state can be given “extra” properties, like spin and left-right symmetry (parity).

Fun part: “uncertainty principles” in QFT mean that states of all energies will emerge.

It then solves the Schoedinger equation (rules for how the list changes in time) and looks for the lowest energy state that is produced.

For example, from the USQCD Collaboration collaboration web site):



Lattice QCD calculations of the meson spectrum suggest the presence of many exotics.

In addition, our nucleons have spin angular momentum.

To describe for the scattering process, we start in the rest frame,

$p^\mu = (m , \underline{0})$ and take $S^\mu = m (0 , \underline{s})$. $p^2 = m^2 = - S^2$ $S \cdot p = 0$.

In a frame where p^μ has a large energy, S^μ also has large components.
At very high energy they are equal up to a sign.

This is positive or negative "Longitudinal polarization" or "helicity"

(projection of intrinsic angular momentum along the direction of motion)

even though S·p remains equal to 0.

- Often convenient to use “lightcone” coordinates and momenta

$$
\begin{aligned}
v^\mu &= \left(v^+, v^-, \mathrm{v}_T\right) \\
v^\pm &= \frac{1}{\sqrt{2}}\left(x^0 \pm x^3\right) \\
\mathrm{v}_T &= (v_2, v_1)
\end{aligned}
$$

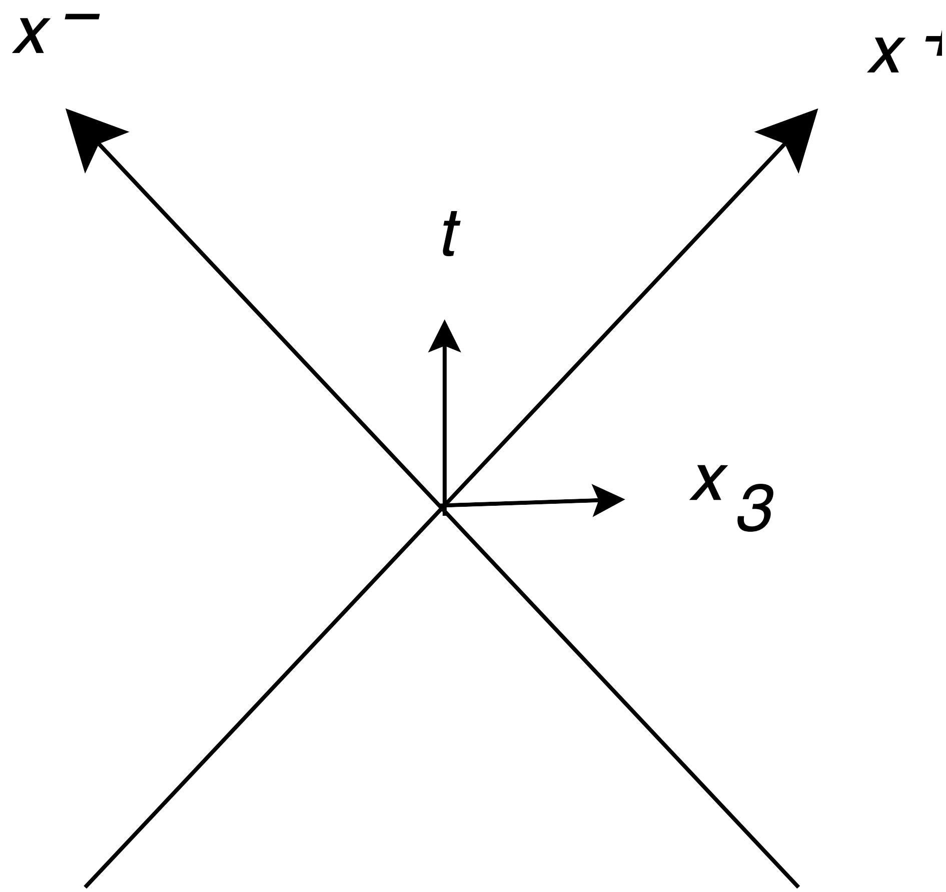
in terms of which

$$
\begin{aligned}
v^2 &= 2v^+v^- - v_T^2 \\
v \cdot w &= v^+w^- + v^-w^+ - \mathrm{v} \cdot \mathrm{w}
\end{aligned}
$$

- for DIS, common momentum assignments are

$$
\begin{aligned}
p^\mu &= \left(p^+, 0^-, 0\right) \\
q^\mu &= \left(-xp^+, \frac{Q^2}{2xp^+}, 0_T\right)
\end{aligned}
$$

The picture



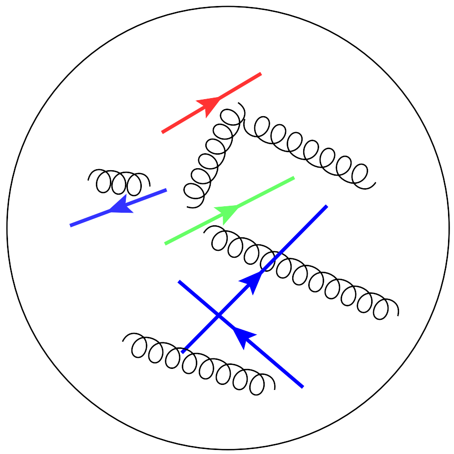
- **An interpretation we'll use: fermion field operators** $\Psi = e,\ u,\ d,\ \ldots$ **absorb particles** $e,\ u,\ d,\ \ldots$**. We can think of this is a "measurement".**

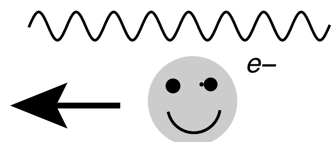
- **Similarly,** $(1 + \lambda\gamma_5)\Psi$ **absorbs fermions of helicity** $\lambda = \pm 1$**.**

- **Equivalently,** $\gamma_5 \Psi$ **absorbs particles and multiplies by** -1 **when the particle has negative helicity.**

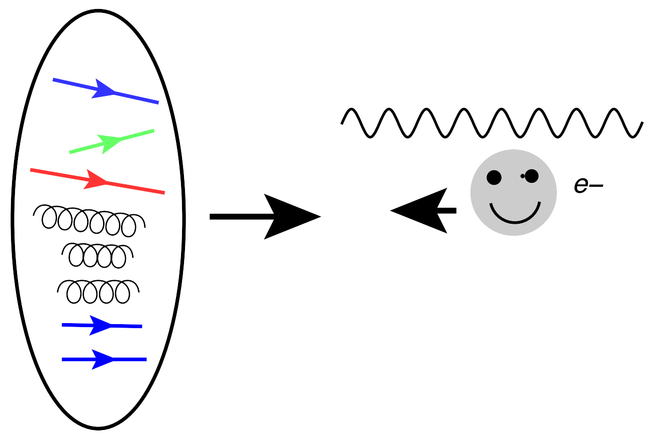
What a proton looks like, and why you need high energy to see inside:

At rest, a proton looks like this, with partonsgoing every which way.





But from the electron's point of view,they all line up (almost)



To a good approximation, an electron arrives in a virtual state with a single extra photon.   Only that photon interacts directly with quarks in the proton. How much can you get from that?

Quite a lot!   When that photon is absorbed by a quark

The proton may remain “whole”, but change direction: elastic scattering.

It may produce an “excited” heavier proton: quasi elastic scattering.

It may break up the proton: inelastic scattering, and produce other particles, anticipated or not in QCD.

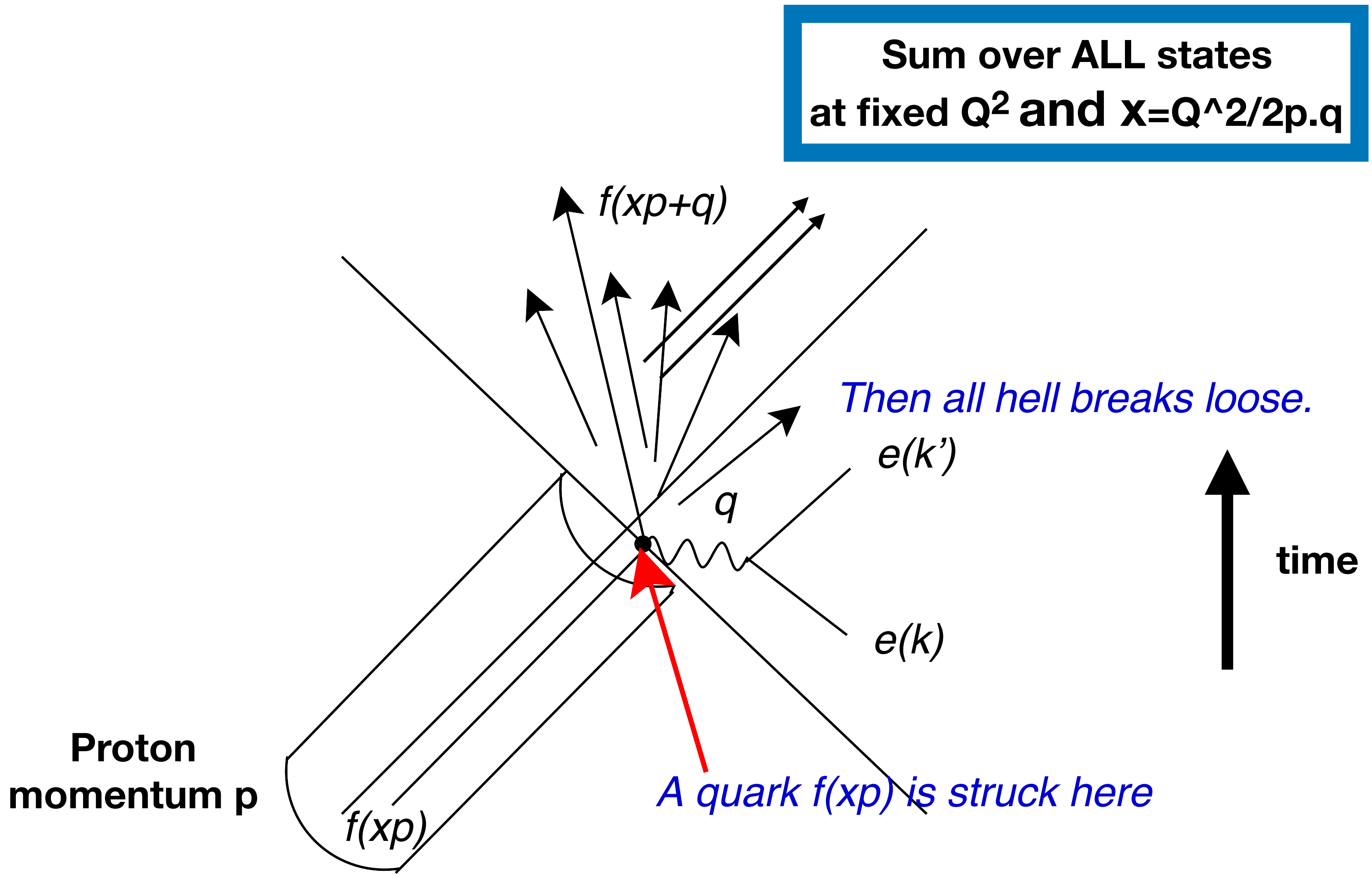
If it transfers a lot of energy: “deeply inelastic (DIS)”.

This may depend on the spin state of the nucleon.

Let’s see some of what we can learn from DIS.

DIS: fixed x and “large” Q^2
QM: Measuring xp spreads the quark’s position along the opposite light cone.

Picturing a typical “deep inelastic” ep event, in the usual variables



- Once the photon localizes the quark field in amplitude and complex conjugate, the cross sections for the sum over all final states add to unity, with corrections of order $\alpha_s(Q)$. This gives the generalization of the parton model relation:

$$\sigma_{\rm eh}(p,q) \;=\; \sum_{\text{partons } a} \int_0^1 d\xi\, \hat\sigma^{\rm el}_{ea}(\xi p,q)\; \phi_{a/h}(\xi)\,,$$

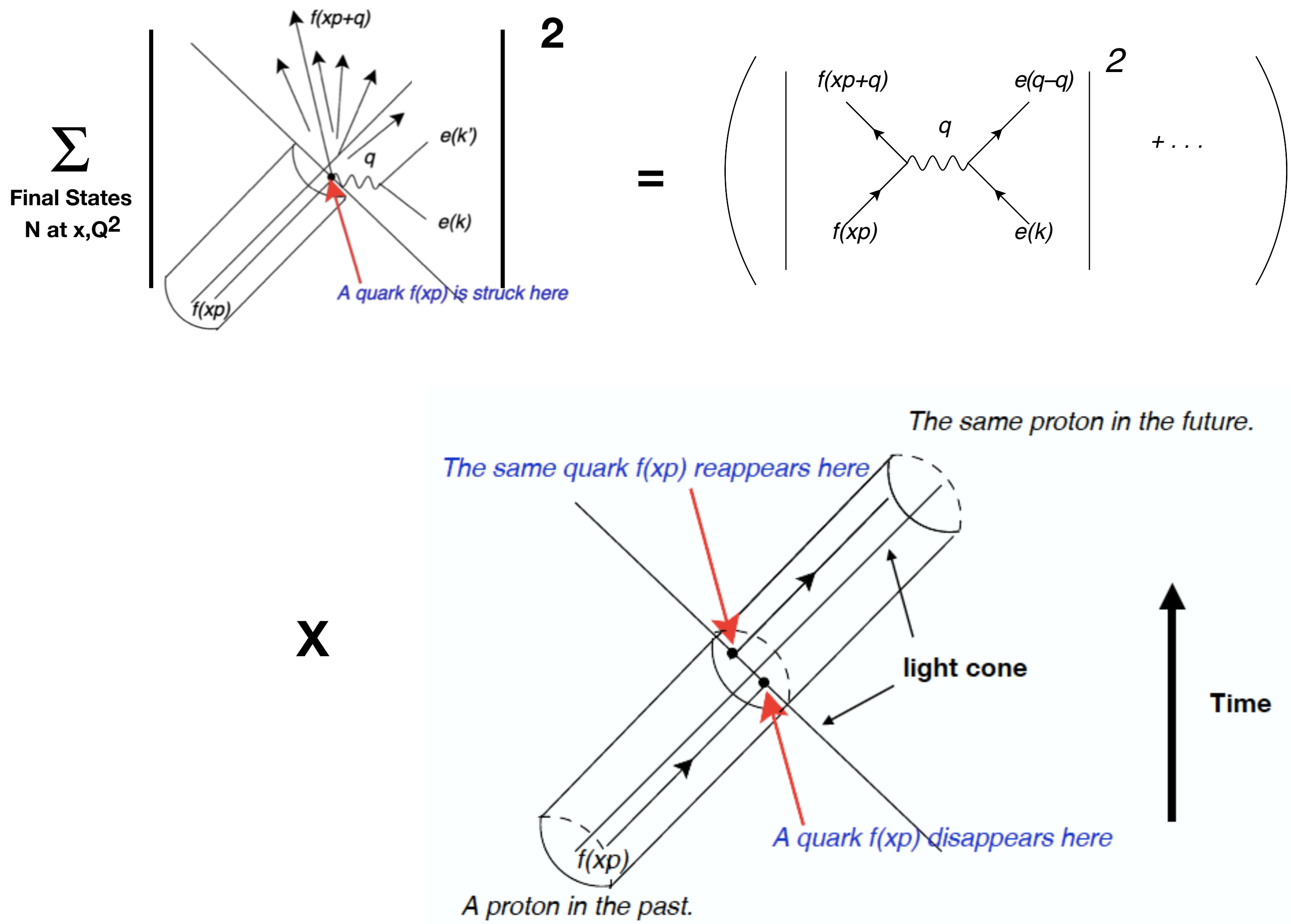
to the QCD relation:

$$\sigma_{\rm eh}(p,q) \;=\; \sum_{\text{partons } a} \int_0^1 d\xi\, \hat\sigma_{ea}(\xi p,q,\mu)\; \phi_{a/h}(\xi,\mu)\,,$$

where $\mu \sim Q$ is an otherwise arbitrary scale that defines the matrix element for $\phi_{a/n}$ and where (schematically)

$$\hat\sigma_{ea}(\xi p,q,\mu) \;=\; \hat\sigma^{\rm el}_{ea}(\xi p,q)\,\delta(\xi - x) + \mathcal{O}(\alpha_s(\mu))$$

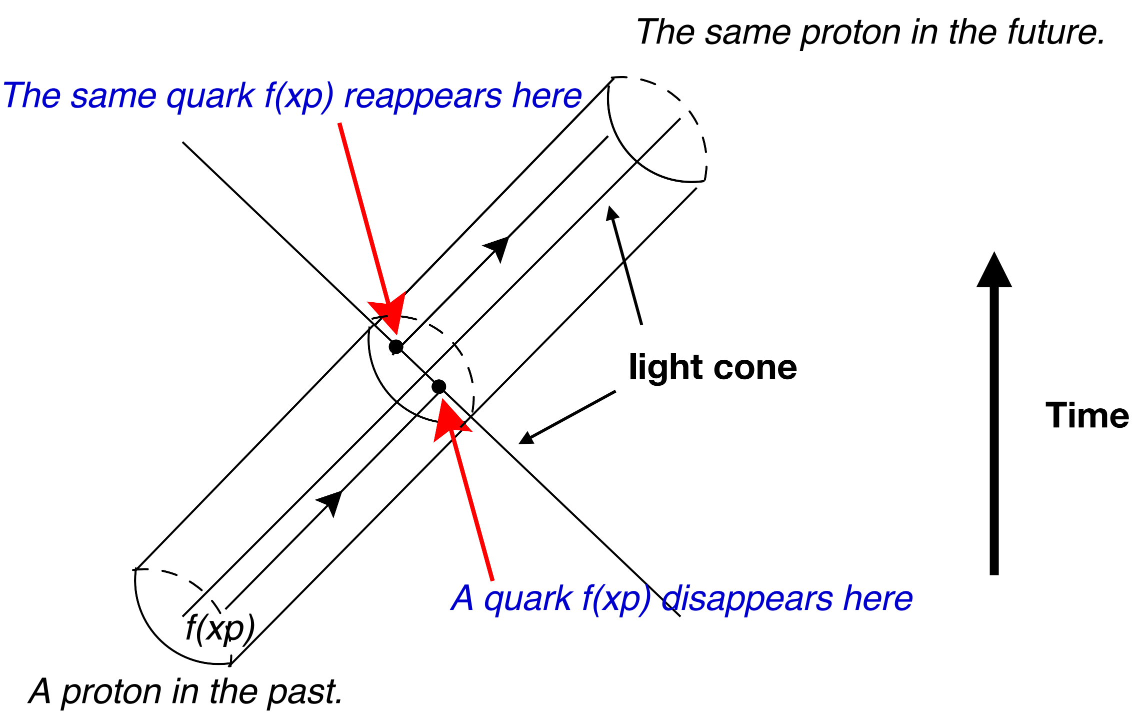
- In pictures . . .



- **Where the final factor $\phi(\xi, \mu)$ is the "collinear" parton distribution, with a matrix element defnition . . .**

Parton distribution as a matrix element for two measurements:

$$\phi(\xi,\mu) \quad = \quad \int d\lambda e^{-i\lambda\xi p^+} \; \left\langle p \left| \bar{q}_\beta(\lambda n^\mu)\gamma^+_{\beta\alpha} q_\alpha(0) \right| p \right\rangle$$



- **The operator $\bar{q}(\lambda n)$ is $\mathcal{O}(1/\mu)$ from the light cone $n^\mu = (0, 1^-, 0)$ (renormalization of the matrix element).**

- **A contemporary set of parton distributions "at different scales": see "evolution" (CTEQ 2015: 1506.07443):**

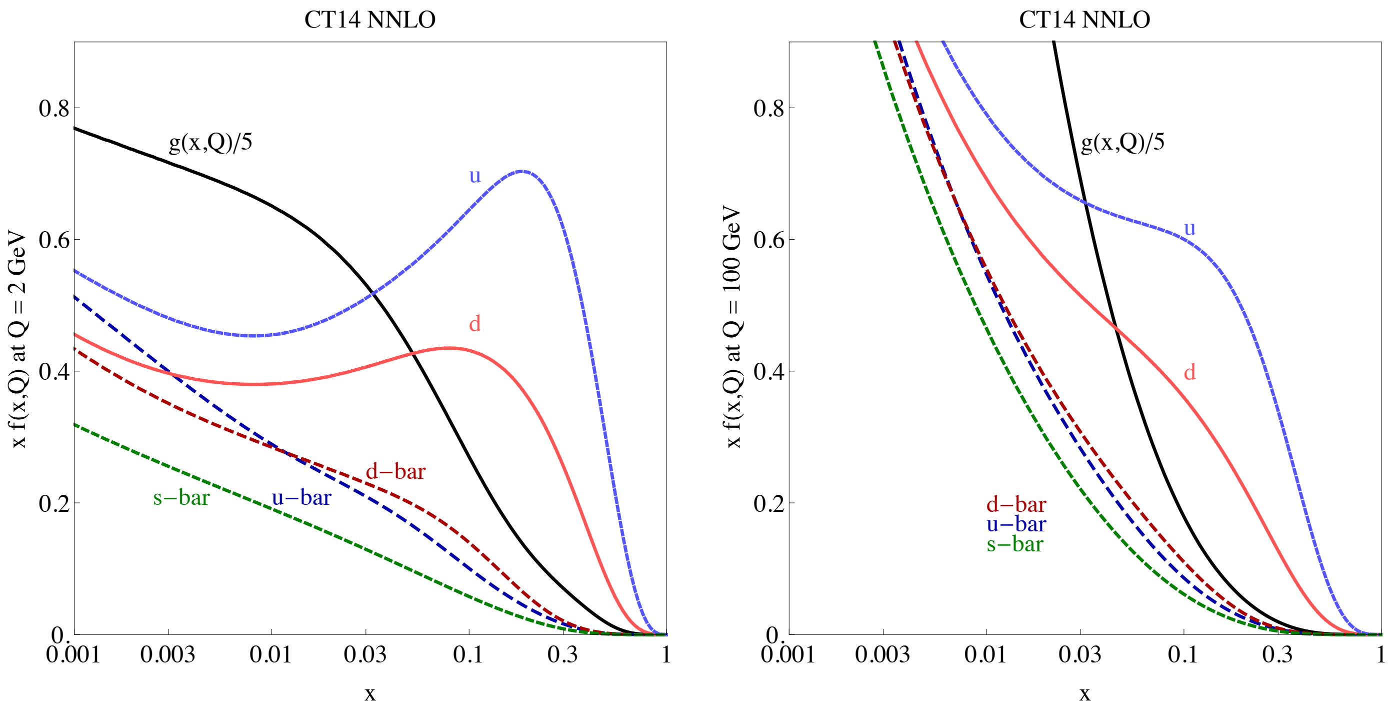
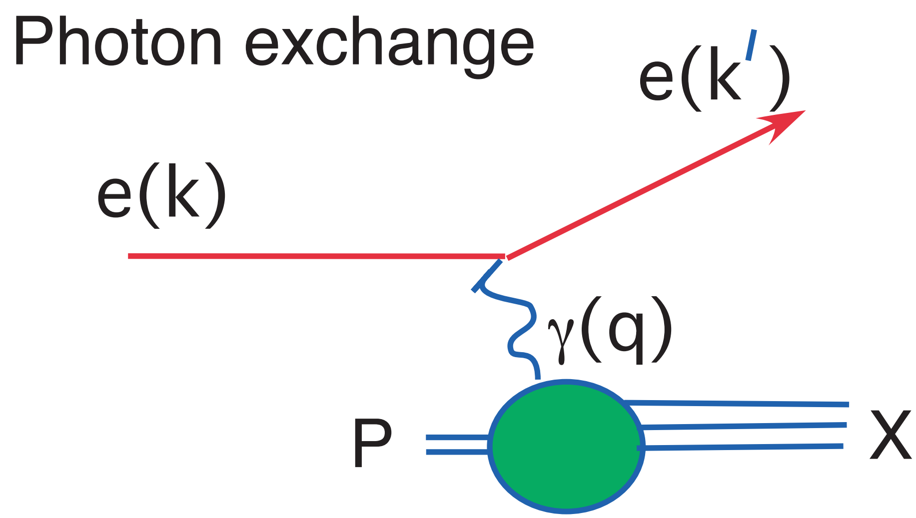


FIG. 4: The CT14 parton distribution functions at $Q = 2$ GeV and $Q = 100$ GeV for $u, \overline{u}, d, \overline{d}, s = \overline{s}$, and g.

- **The distributions change with Q: $\phi_a(x) \to \phi_a(x, Q)$ – we'll see where this comes from.**

B. DIS: Structure Functions and Scaling



$$A_{e+N\to e+X}(\lambda,\lambda',\sigma;q) = \bar{u}_{\lambda'}(k')(-ie\gamma_\mu)u_\lambda(k) \times \frac{-ig^{\mu\mu'}}{q^2} \times \langle X|\, eJ^{\mathrm{EM}}_{\mu'}(0)\, |p,\sigma\rangle$$

- **Historically an assuption that the photon couples to hadrons by point-like current operator. Now, built into the Standard Model.**

- **The cross section:**

$$d\sigma_{\rm DIS} = \frac{1}{2^2}\frac{1}{2s}\frac{d^3k'}{(2\pi)^3 2\omega_{k'}} \sum_X \sum_{\lambda,\lambda',\sigma} |A|^2$$
$$\times\ (2\pi)^4\,\delta^4(p_X + k' - p - k)$$

In $|A|^2$, separate the known leptonic part from the "unknown" hadronic part: $\sum |A|^2 \delta^4(\cdots) \equiv L^{\mu\nu} W_{\mu\nu}$.

- **The leptonic tensor:**

$$
\begin{aligned}
L^{\mu\nu} &= \frac{e^2}{8\pi^2}(\bar{u}_{\lambda'}(k')\gamma^\mu u_\lambda(k))^* \, (\bar{u}_{\lambda'}(k')\gamma^\nu u_\lambda(k)) \\
&= \frac{e^2}{8\pi^2} \, \mathrm{Tr}\left[\not{k}\gamma^\mu \frac{1}{2}(1+\lambda\gamma_5)\not{k}'\gamma^\nu\right] \\
&= \frac{e^2}{4\pi^2} \left(k^\mu k'^{\,\nu} + k'^{\,\mu} k^\nu - g^{\mu\nu} k\cdot k' \right) \\
&\qquad -i\lambda \frac{e^2}{4\pi^2} \epsilon^{\mu\nu\lambda\sigma} k_\lambda k'_\sigma
\end{aligned}
$$

- Leaves us with the "hadronic tensor":

$$W^{Vh}_{\mu\nu} = \frac{1}{8\pi} \sum_{\sigma,X} \langle X|J^{(V)}_{\mu}|p,S\rangle^* \langle X|J^{(V)}_{\nu}|p,S\rangle \\ \times\ (2\pi)^4\delta^4(p_X - p - q)$$

where $J^{(V)}_{\mu}$ is the electroweak current, coupled to vector: V = photon, Z_0 (or $\mathrm{W}^{\pm}$). It is dimensionless,

- And the cross section becomes:

$$2\omega_{k'}\frac{d\sigma}{d^3k'} = \frac{1}{s(q^2)^2}\ L^{\mu\nu}W_{\mu\nu}$$

- $W_{\mu\nu}$ has sixteen components,
but known properties of the strong interactions constrain $W_{\mu\nu}$ ...

- **An example: current conservation,**

$$\begin{aligned}
&\partial^\mu J_\mu^{\mathrm{EM}}(x) = 0 \\
&\quad \Rightarrow \langle X|\, \partial^\mu J_\mu^{\mathrm{EM}}(x)\, |p\rangle = 0 \\
&\quad \Rightarrow (p_X - p)^\mu \langle X|\, J_\mu^{\mathrm{EM}}(x)\, |p\rangle = 0 \\
&\quad \Rightarrow q^\mu W_{\mu\nu} = 0
\end{aligned}$$

- **With time-reversal, etc ...**

$$
\begin{aligned}
W_{\mu\nu} = & -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1^{(Vh)}(x, Q^2) \\
& + \left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) W_2^{(Vh)}(x, Q^2) \\
+\epsilon_{\mu\nu\lambda\sigma} q^\lambda & \left(\frac{s^\sigma}{p\cdot q} g_1(x, Q^2) + \frac{[p\cdot q s^\sigma - s\cdot q p^\sigma]}{(p\cdot q)^2} g_2(x, Q^2)\right)
\end{aligned}
$$

The final line is for polarized targets with spin s^σ.

- Often given in terms of the dimensionless structure functions,

$$
F_1 = W_1 \qquad F_2 = p\cdot q W_2 \qquad F_3 \qquad g_1 \qquad g_2 .
$$

- Note that if there is no other mass scale, the F's cannot depend on Q except indirectly through x.

The unpolarized cross section:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{xQ^4}\left[\left(1 - y + \frac{y^2}{2}\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right]$$

in terms of the "longitudinal" structure function:

$$F_L(x,Q^2) = F_2(x,Q^2) - 2xF_1(x,Q^2)$$

This comes from the symmetric parts of $W_{\mu\nu}$ and $L^{|mu\nu}$.

Only the spin-dependent parts of $W_{\mu\nu}$ and $L^{|mu\nu}$. are anti-symmetric, so the electron must also be polarized to measure hadronic spin-dependence:

$$\frac{1}{2}\left[\frac{d^2\sigma^{+-}}{dx dQ^2} - \frac{d^2\sigma^{++}}{dx dQ^2}\right] = \frac{4\pi\alpha^2}{Q^4} y(2-y) g_1(x, Q^2).$$

where the signs refer to electron and nucleon longitudinal spins. We neglect power-suppressed g_2 terms.

- **All of the structure functions inherit the same factorized form as the cross sections, in terms of polarized and unpolarized parton distributions:**

$$F_i(p,q) = \sum_{\text{partons } a} \int_0^1 d\xi \, C_i(\xi p, q, \mu) \, \phi_{a/h}(\xi, \mu)$$

and

$$g_1(x,Q) = \sum_{\text{partons } a} \int_0^1 d\xi \, C_g(\xi p, q, \mu) \, \Delta\phi_{a/h}(\xi, \mu)$$

- **The distributions as matrix elements for the quarks:**

$$\phi_{a/h}(\xi,\mu) = \frac{1}{2}\sum_S \int \frac{d\lambda}{2\pi} e^{-i\lambda\xi p\cdot n} \langle p,S|\bar{\Psi}_a(\lambda n) n\cdot\gamma\Psi_a(0)|p,S\rangle$$

$$\Delta\phi_{a/h}(\xi,\mu) = \frac{1}{2}\sum_S \int \frac{d\lambda}{2\pi} e^{-i\lambda\xi p\cdot n} \times \langle p,S|\bar{\Psi}_a(\lambda n)\gamma_5 n\cdot\gamma\Psi_a(0)|p,S\rangle$$

In both cases, the field $\Psi_a(0)$ absorbs particle a at the origin, and $\bar{\Psi}_a(\lambda n)$ recreates the particle at distance λn away along the n^μ lightcone, in the direction opposite to p^μ. The extra γ_5 in $\Delta\phi_a$ assigns an extra -1 for negative helicity quarks.

- **Structure functions in the Parton Model: The Callan-Gross Relation**

From the "basic parton model formula":

$$\frac{d\sigma_{eh}}{d^3k'} = \sum_{\text{quarks } f} \int d\xi \, \frac{d\sigma^{\text{el}}_{ef}(\xi p)}{d^3k'} \, \phi_{f/h}(\xi) \tag{1}$$

Can treat a quark of "flavor" f just like any hadron and get

$$\omega_{k'} \, \frac{d\sigma^{\text{el}}_{ef}(\xi p)}{d^3k'} = \frac{1}{2(\xi s)Q^4} \, L^{\mu\nu} \, W^{ef}_{\mu\nu}(k + \xi p \rightarrow k' + p')$$

Let the charge of f be e_f.

Exercise 1: Compute $W_{\mu\nu}^{\gamma f}$ to find:

$$W_{\mu\nu}^{\gamma f} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\,\delta\left(1 - \frac{x}{\xi}\right)\frac{e_f^2}{2}$$
$$+\left(\xi p_\mu - q_\mu\,\frac{\xi p\cdot q}{q^2}\right)\,\left(\xi p_\nu - q_\nu\,\frac{\xi p\cdot q}{q^2}\right)\,\delta\left(1 - \frac{x}{\xi}\right)\frac{e_f^2}{\xi p\cdot q}$$

Ex. 2: by substituting in (1), find the Callan-Gross relation,

$$F_2(x) = \sum_{\mathrm{quarks}\,f} e_f^2 x\,\phi_{f/p}(x) = 2xF_1(x)$$

Ex. 3: that this relation is quite different for scalar quarks.

- The Callan-Gross relation shows the compatibility of the quark and parton models.

- In addition: parton model structure functions are independent of Q^2, a property called “scaling”.

- With massless partons, there is no other massive scale. Then the F’s must be Q-independent; see above.

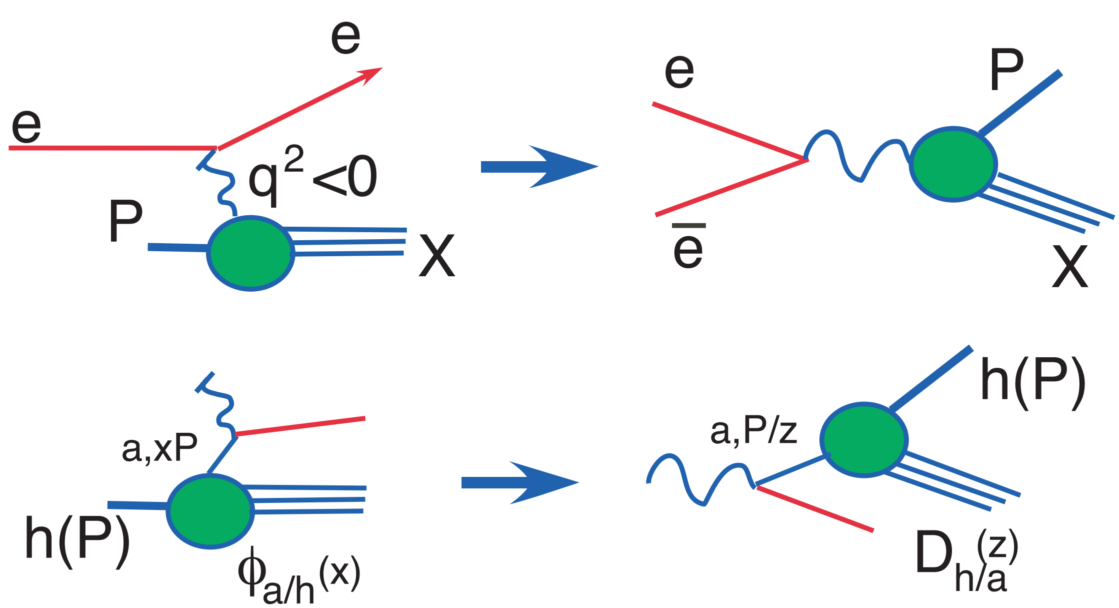
- Approximate properties of the kinematic region explored by SLAC in late 1960’s – early 1970’s.

- QCD “evolution” gives corrections to this picture.

D. Classic Parton Model Extensions: Fragmentation and Drell Yan

- **Fragmentation functions**
- **"Crossing" applied to DIS: "Single-particle inclusive" (1PI) From scattering to pair annihilation.**

Parton distributions become "fragmentation functions".



- **Parton model relation for 1PI: inclusive hadron from exclusive parton:**

$$\frac{d\sigma_h^{(\mathrm{incl})}(P,q)}{d^3P} = \sum_a \int_0^1 dz\, \frac{d\sigma^{(\mathrm{elas})}_{\mathrm{e^+e^-}\to a}(P/z,q)}{d^3P}\, D_{h/a}(z)$$

- **The direction of the hadron follows the direction of the parton!**

- $D_{h/a}$ **is "universal": could be in DIS, or hadron-hadron scattering.**

- **Heuristic justification from time dilation: Formation of hadron** $h(P)$ **from parton** $a(P/z)$ **takes a fixed time** τ_0 **in the rest frame of** a**, but much longer in the CM frame – this "fragmentation" thus decouples from** $\sigma_a^{(\mathrm{elastic})}$**, and is independent of** q **(scaling).**

- **For** $\mathrm{e}^{+}(\mathrm{k}_2)\mathrm{e}^{-}(\mathrm{k}_1) \rightarrow \mathrm{q}(\mathrm{p}_1)\bar{\mathrm{q}}(\mathrm{p}_2)$.
  Exercise 4: Start with the matrix element:
  $$\mathcal{M} \;=\; e_q \frac{e^2}{Q^2}\, \overline{u}(p_1,\sigma_1)\gamma_\mu v(p_2,\sigma_2)\; \overline{v}(k_2,s_2)\gamma^\mu u(k_1,s_1)$$

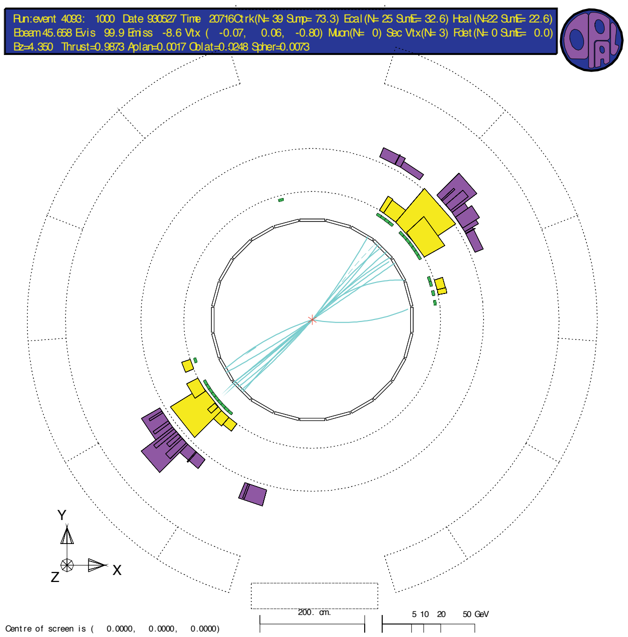
- **First square and sum/average spins in $\mathcal{M}$. Then evaluate phase space at fixed angle for the "quark" p_1 in the final state to get:**

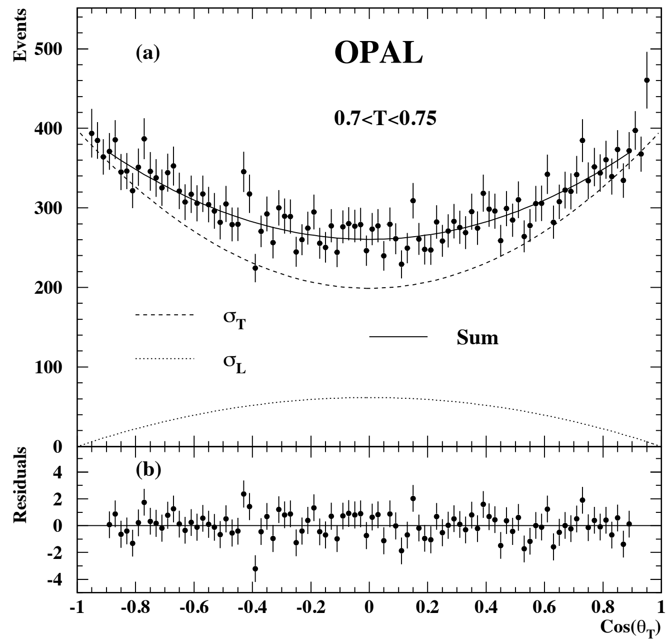
$$\frac{d\sigma^{(\mathrm{elastic})}_{\mathrm{q\bar{q}}\rightarrow\mu\bar{\mu}}(k_1,k_2)}{d\Omega} = \frac{1}{2Q^2}\frac{e_q^2 e^4}{32\pi^2}\, e_q^2 e^4 \left(1 + \cos^2\theta\right)$$

With $Q^2 = (k_1 + k_2)^2$, and θ the angle between the electron and the quark.

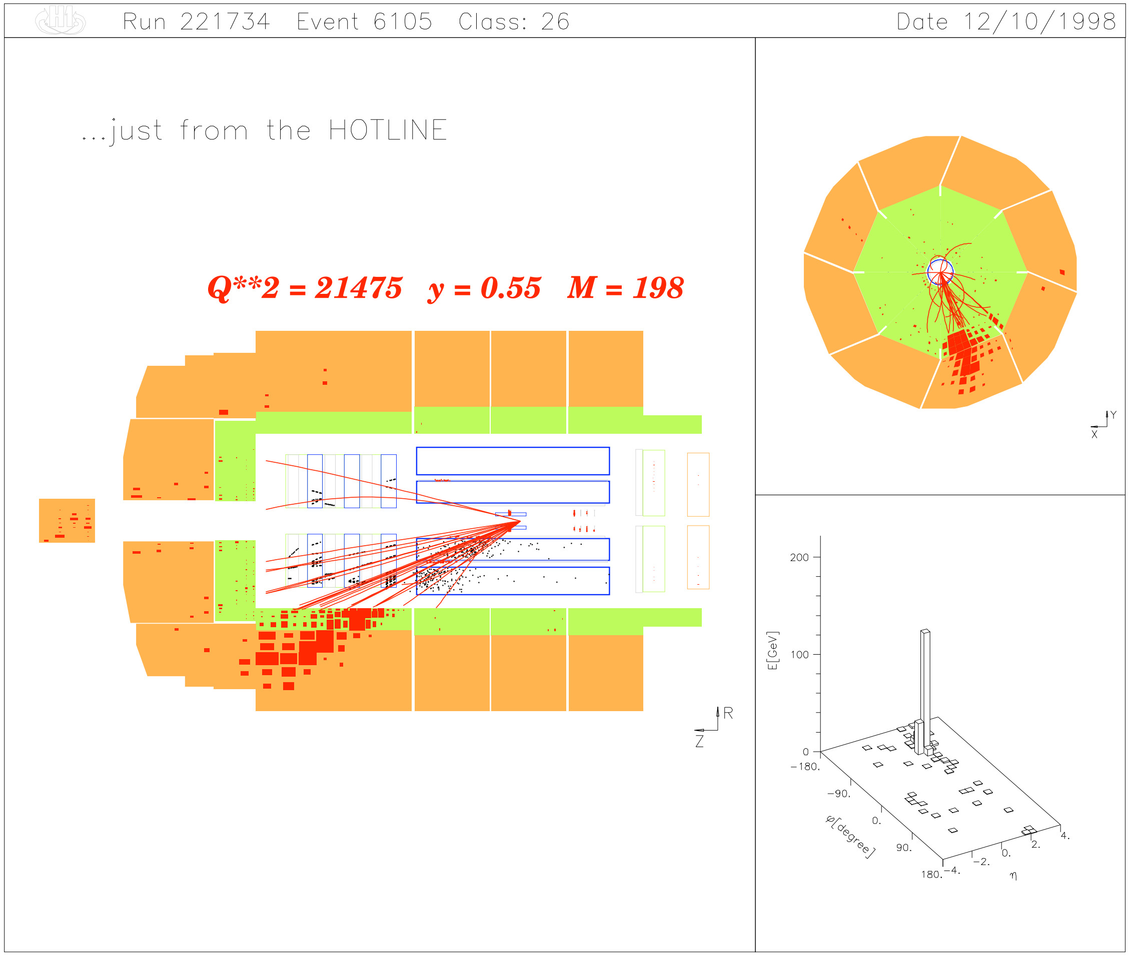
- The fragmentation picture suggests that almost all hadrons are aligned along parton directions $\Rightarrow$ most hadrons come out together as "jets", following the $1 + \cos^2\theta$ distribution relative to the incoming electron. And this is what happens.

- Hadrons should show up this way, and they do.



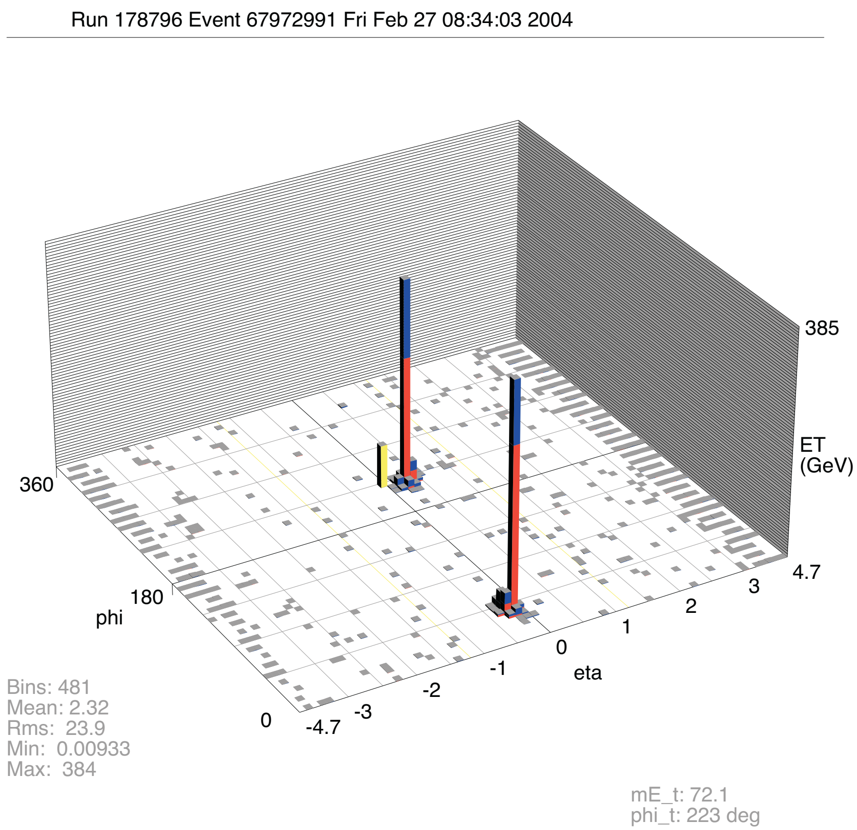


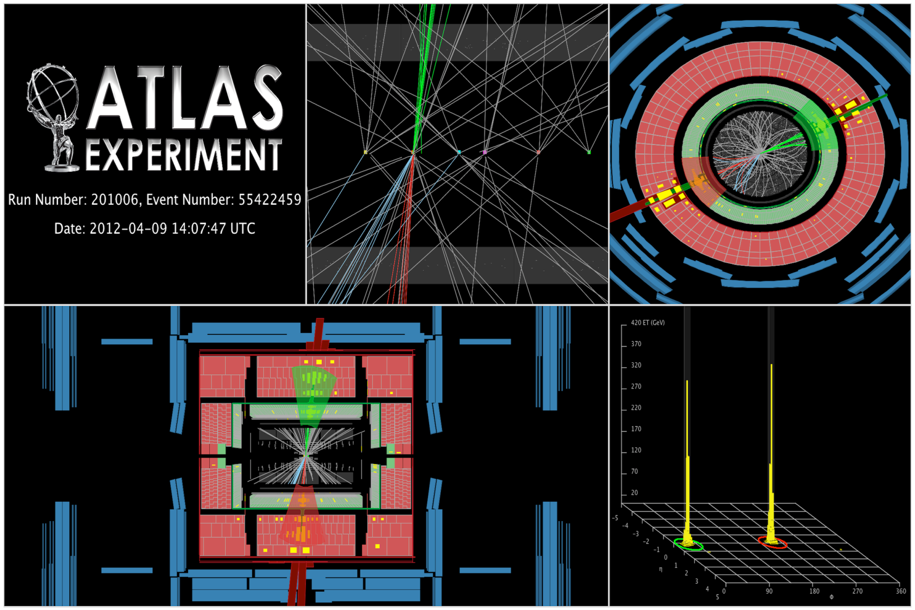
- **For DIS:**



Run 221734 Event 6105 Class: 26
Date 12/10/1998
...just from the HOTLINE
Q**2 = 21475 y = 0.55 M = 198
E[GeV]
φ[degree]
η

• And for nucleon-nucleon collisions:





- Finally: the Drell-Yan process

- **In the parton model (1970).**
  Drell and Yan: look for the annihilation of quark pairs into virtual photons of mass Q ... any electroweak boson in NN scattering.

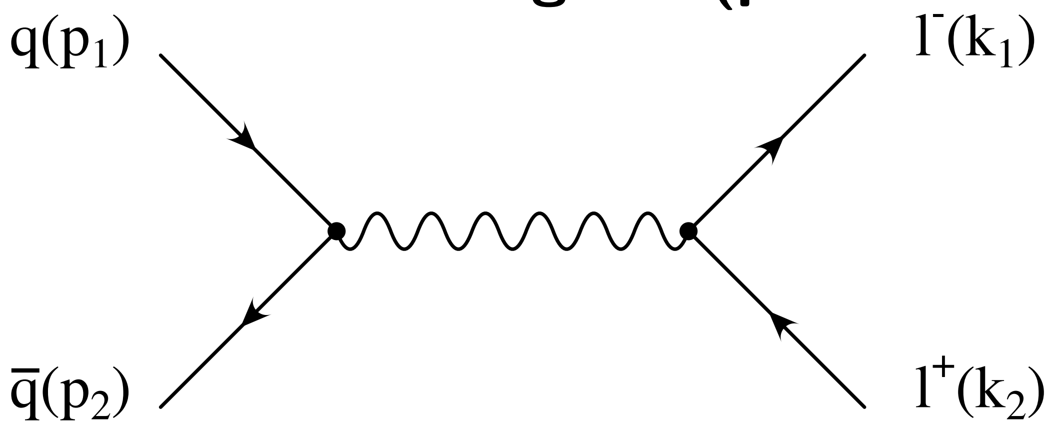
$$
\begin{aligned}
\frac{d\sigma_{NN\to\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2 d\ldots} \sim \\
&\int d\xi_1 d\xi_2 \sum_{a=\mathrm{q}\bar{\mathrm{q}}} \frac{d\sigma^{\mathrm{EW,\,Born}}_{\mathrm{a}\bar{\mathrm{a}}\to\mu\bar{\mu}}(Q,\xi_1 p_1,\xi_2 p_2)}{dQ^2 d\ldots} \\
&\times(\text{probability to find parton a}(\xi_1)\text{ in }N) \\
&\times(\text{probability to find parton }\bar{\mathrm{a}}(\xi_2)\text{ in }N)
\end{aligned}
$$

The probabilities are $\phi_{q/N}(\xi_i)$'s from DIS!

How it works (with colored quarks) . . .

- **The Born cross section**: $\mathrm{e}^+\mathrm{e}^-$ **backwards.**

$\boldsymbol{\sigma}^{\mathrm{EW,Born}}$ **is all from this diagram (parton x's set to unity):**



With this matrix element:

$$\boldsymbol{M \;=\; e_q \frac{e^2}{Q^2}\, \overline{u}(k_1,\sigma_1)\gamma_\mu v(k_2,\sigma_2)\; \overline{v}(p_2,s_2)\gamma^\mu u(p_1,s_1)}$$

- **First square and sum/average M. Then evaluate phase space.**

- **Total cross section at "pair mass"** $Q^2 = (x_1 p_1 + x_2 p_2)^2$

$$\sigma^{\mathrm{EW,\,elastic}}_{q\bar{q}\to\mu\bar{\mu}}(x_1 p_1, x_2 p_2) = \frac{1}{2\hat{s}} \int \frac{d\Omega}{32\pi^2} \frac{e_q^2 e^4}{3} (1 + \cos^2\theta)$$

$$= \frac{4\pi\alpha^2}{9Q^2} \sum_q e_q^2$$

With Q **the pair mass** and 3 for color average.

- **And measured rapidity:**

  Pair mass (Q) and rapidity

$$\eta \equiv (1/2)\ln\left(\frac{Q^+}{Q^-}\right) = (1/2)\ln\left(\frac{Q^0+Q^3}{Q^0-Q^3}\right)$$

- **ξ's are overdetermined $\rightarrow$ delta functions in the Born cross section**

$$\frac{d\sigma^{(PM)}_{NN\rightarrow\mu\bar{\mu}+X}(Q,p_1,p_2)}{dQ^2d\eta} =$$

$$\int_{\xi_1,\xi_2} \sum_{a=q\bar{q}} \sigma^{\text{EW, Born}}_{a\bar{a}\rightarrow\mu\bar{\mu}}(\xi_1 p_1, \xi_2 p_2)$$
$$\times\delta\left(Q^2 - \xi_1\xi_2 S\right)\ \delta\left(\eta - \frac{1}{2}\ln\left(\frac{\xi_1}{\xi_2}\right)\right)$$
$$\times\phi_{a/N}(\xi_1)\,\phi_{\bar{a}/N}(\xi_2)$$

- and integrating over rapidity, back to $d\sigma/dQ^2$,

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha^2_{\rm EM}}{9Q^4}\right) \int_0^1 d\xi_1\, d\xi_2\, \delta\left(\xi_1\xi_2 - \tau\right)$$
$$\times \sum_a \lambda_a^2\, \phi_{a/N}(\xi_1)\, \phi_{\bar{a}/N}(\xi_s)$$

Found by Drell and Yan in 1970 (aside from 1/3 for color).

Analog of DIS scaling in x is DY scaling in $\tau = Q^2/S$.

- Template for all hard hadron-hadron scattering

- Exercise 5: fill in the results for this parton model cross section, starting from the matrix element M.

- **Appendix I: Quarks in the Standard Model**
  Electroweak interactions of quarks: $SU(2)_L \times U(1)$. Their non-QCD interactions.

- **Quark and lepton fields: L(eft) and R(ight)**
  – $\psi = \psi^{(L)} + \psi^{(R)} = \frac{1}{2}(1-\gamma_5)\psi + \frac{1}{2}(1+\gamma_5)\psi$; $\psi = q, \ell$
  – **Helicity: spin along $\vec{p}$ (R=right handed) or opposite (L=left handed) in solutions to Dirac equation**

  – $\psi^{(L)}$: **expanded only in L particle solutions to Dirac eqn. R antiparticle solutions**

  – $\psi^{(R)}$: **only R particle solutions, L antiparticle**

  – **An essential feature: L and R have different interactions in general!**

– L quarks come in "weak $SU(2)$" = "weak isospin" pairs:

$$q_i^{(L)} = \begin{pmatrix} u_i \\ d_i' = V_{ij} dj \end{pmatrix} \qquad u_i^{(R)},\ d_i^{(R)}$$

$$(u, d') \qquad (c, s') \qquad (t, b')$$

$$\ell_i^{(L)} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \qquad e_i^{(R)},\ \nu_i^{(R)}$$

$$(\nu_e, e) \qquad (\nu_\mu, \mu) \qquad (\nu_\tau, \tau)$$

(We've neglected neutrino masses.)

– V_{ij} is the "CKM" matrix.

– The electroweak interactions distinguish L and R.

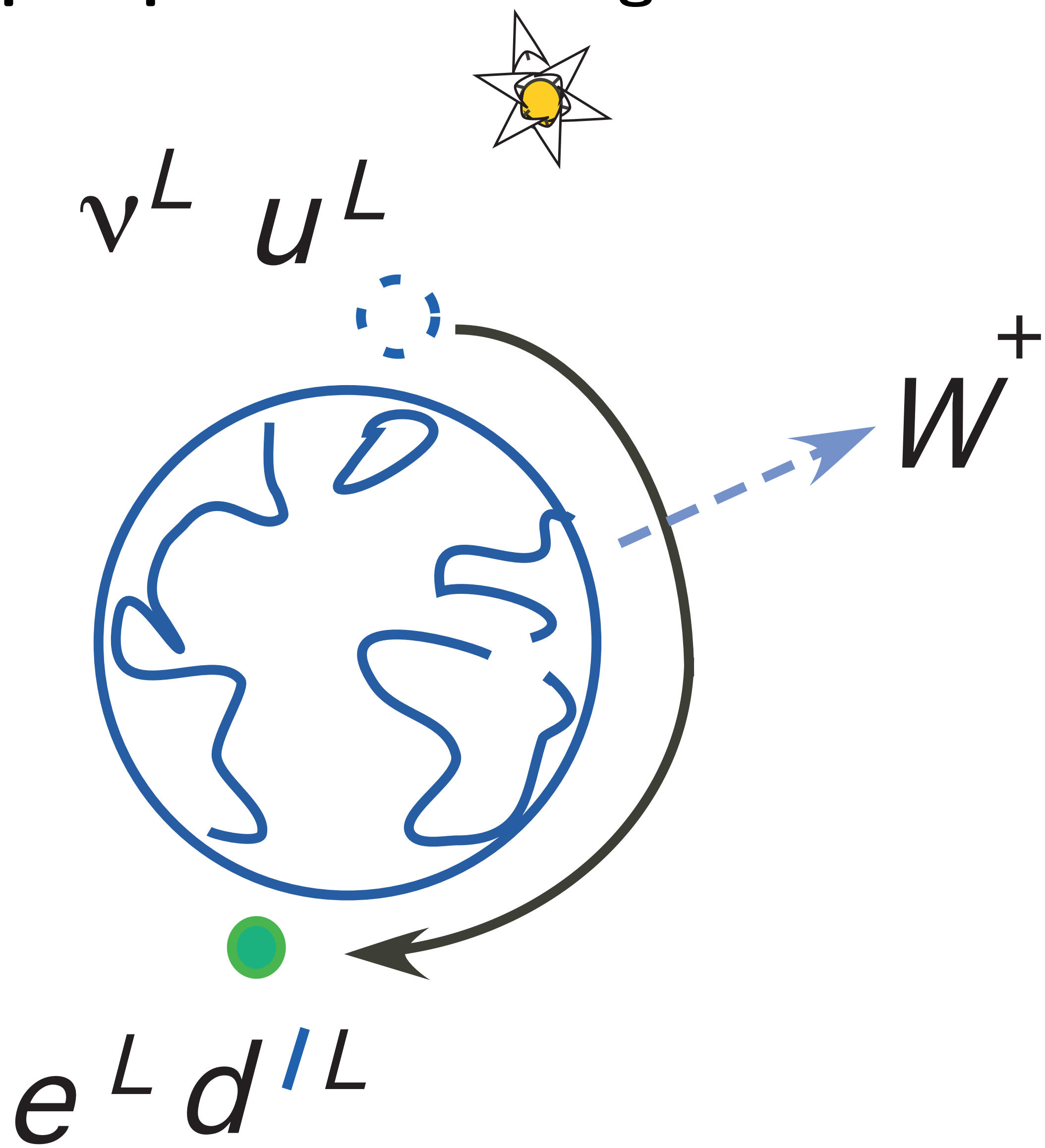
- **Weak vector bosons: electroweak gauge groups**
  - **SU(2): three vector bosons B_i with coupling g**
  - **U(1); one vector boson C with coupling g'**
  - **The physical bosons:**

$$W^{\pm} = B_1 \pm i B_2$$
$$Z = -C \sin\theta_W + B_3 \cos\theta_W$$
$$\gamma \equiv A = C \cos\theta_W + B_3 \sin\theta_W$$

$$\sin\theta_W = g'/\sqrt{g^2 + g'^2} \qquad M_W = M_Z/\cos\theta_W$$

$$e = gg'/\sqrt{g^2 + g'^2} \qquad M_W \sim g/\sqrt{G_F}$$

- **Weak isospin space: connecting u with d'**



- **Only left handed fields move around this globe.**

– The interactions of quarks and leptons with the photon, W, Z

$$\mathcal{L}_{\mathrm{EW}}^{(fermion)} = \sum_{\text{all } \psi} \bar{\psi} \left(i\not{\partial} - e\lambda_\psi \not{A} - (g m_\psi 2 M_W) h \right) \psi$$
$$-(g/\sqrt{2}) \sum_{q_i, e_i} \bar{\psi}^{(L)} \left(\sigma^+ \not{W}^+ + \sigma^- \not{W}^- \right) \psi^{(L)}$$
$$-(g/2\cos\theta_W) \sum_{\text{all } \psi} \bar{\psi} \left(v_f - a_f \gamma_5 \right) \not{Z} \, \psi$$

– Interactions with W are through ψ_L's only.

– Neutrino Z exchange depends on $\sin^2\theta_W$ even at low energy.

– This observation made it clear by early 1970's that $M_W \sim g/\sqrt{G_F}$ is large $\rightarrow$ a need for colliders.

– Coupling to the Higgs $h \propto$ mass (special status of t).

- **Symmetry violations in the standard model:**
  - W's interact through $\psi^{(L)}$ only, $\psi = q, \ell$.
  - These are left-handed quarks & leptons; right-handed antiquarks, antileptons.
  - Parity (P) exchanges L/R; Charge conjugation (C) exchanges particles, antiparticles.
  - CP combination OK ($L \rightarrow_P R \rightarrow_C L$) if all else equal, but it's not (quite) . . .

    Complex phases in CKM V result in CP violation.

• Appendix II: Structure Functions and Photon Polarizations

In the P rest frame can take

$$q^\mu = \left(\nu; 0, 0, \sqrt{Q^2 + \nu^2}\right) , \qquad \nu \equiv \frac{p \cdot q}{m_p}$$

In this frame, the possible photon polarizations ($\epsilon \cdot q = 0$):

$$\epsilon_R(q) = \frac{1}{\sqrt{2}} (0; 1, -i, 0)$$

$$\epsilon_L(q) = \frac{1}{\sqrt{2}} (0; 1, i, 0)$$

$$\epsilon_{\text{long}}(q) = \frac{1}{Q} \left(\sqrt{Q^2 + \nu^2}, 0, 0, \nu\right)$$

• **Alternative Expansion**

$$W^{\mu\nu} = \sum_{\lambda=L,R,long} \epsilon_\lambda^{\mu *}(q)\epsilon_\lambda^\nu(q)\, F_\lambda(x,Q^2)$$

• **For photon exchange (Exercise 4):**

$$F^{\gamma e}_{L,R} = F_1$$
$$F_{\mathrm{long}} = \frac{F_2}{2x} - F_1$$

• **So F_{long} vanishes in the parton model by the C-G relation.**

- **Generalizations: neutrinos and polarization**

- **Neutrinos: flavor of the "struck" quark is changed when a $W^{\pm}$ is exchanged. For W^{+}, a d is transformed into a linear combination of u, c, t, determined by CKM matrix (and momentum conservation).**

- **Z exchange leaves flavor unchanged but still violates parity.**

- **The Vh structure functions for $= W^+, W^-, Z$:**

$$
\begin{aligned}
W_{\mu\nu}^{(Vh)} - &\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W_1^{(Vh)}(x, Q^2) \\
&+ \left(p_\mu - q_\mu \frac{p \cdot q}{q^2}\right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2}\right) \frac{1}{m_h^2} W_2(x, Q^2) \\
&- i\epsilon_{\mu\nu\lambda\sigma} p^\lambda q^\sigma \; \frac{1}{m_h^2} W_3^{(Vh)}(x, Q^2)
\end{aligned}
$$

- **with dimensionless structure functions:**

$$
F_1 = W_1 \,, \qquad F_2 = \frac{p \cdot q}{m_h^2} W_2 \,, \qquad F_3 = \frac{p \cdot q}{m_h^2} W_3
$$

- $F_i^{(\nu h)}$ **gives $W^+ h$ scattering,** $F_i^{(\bar{\nu} h)}$ **gives $W^- h$**

- **And with spin (for the photon).**

$$W^{\mu\nu} = \frac{1}{4\pi} \int d^4z\, e^{iq\cdot z} \ \langle h(P,S) \,|\, J^\mu(z) J^\nu(0) \,|\, h(P,S)\rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \ F_1(x,Q^2)$$

$$+ \left(P^\mu - q^\mu \frac{P\cdot q}{q^2}\right)\left(P^\nu - q^\nu \frac{P\cdot q}{q^2}\right) F_2(x,Q^2)$$

$$+ \, i m_h \, \epsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{P\cdot q} g_1(x,Q^2) + \frac{S_\sigma (P\cdot q) - P_\sigma (S\cdot q)}{(P\cdot q)^2} g_2(x,Q^2)\right.$$

- **Parton model structure functions:**

$$F_2^{(eh)}(x) = \sum_f e_f^2 \, x \, \phi_{f/h}(x)$$
$$g_1^{(eh)}(x) = \frac{1}{2} \sum_f e_f^2 \left(\Delta\phi_{f/n}(x) + \Delta\bar{\phi}_{f/h}(x) \right)$$

- **Notation:** $\Delta\phi_{f/h} = \phi^{+}_{f/h} \; - \; \phi^{-}_{f/h}$ **with** $\phi^{\pm}_{f/h}(x)$ **probability for struck quark** f **to have momentum fraction** x **and helicity with (+) or against (−)** h**'s helicity.**