3D Hadronic Structure for EIC: QCD, SCET, resummation, ...

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Our Focus: Transverse Momentum Dependendent Parton Distributions ← TMD] TMDs

Refs:
- TMD Handbook (2304.03302), primarily Ch. 2
- EFT course (link posted) for more on SCET
- Can description of other 3D distributions important at EIC in Ch. 11 & Refs cited there. Eg GPD

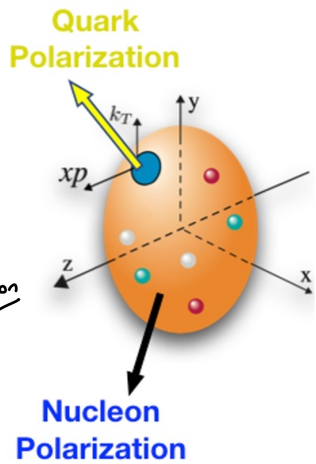
TMD Motivation

- explore mysterys of relativistic bound particles → proton

  momenta of partons → distributions $f(x, k_T)$

  spin - k_T quantum correlations, $k_T \cdot S_T \; g_{1T}^{\perp}$ Worm-gear

  $\epsilon_T^{\alpha\beta} k_{T\alpha} S_{T\beta} \; f_{1T}^{\perp}$ Sivers



- Precision physics, Higgs q_T, Drell-Yan q_T

  $pp \to H + X$ $\qquad$ $pp \to (\mu^+\mu^-) + X$

  motion

- Improve understanding of Confinement $f(x, k_T)$

  & Hadronization $D(z_h, p_T)$ → EIC!

- Goals:

  Connect Measurements to TMDs → Factorization

  TMD Universality → Wilson lines & loops

  Perturbative & Non-Pert. QCD → Expansions

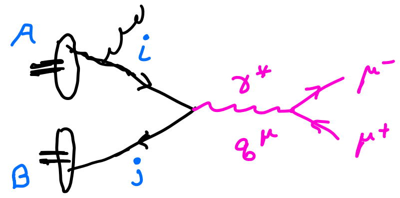
  Obtain Accurate Predictions → Large Logs & Resummation

  Interesting QFT rigor!

Drell-Yan

$pp \to (\mu^+\mu^-) + X$ CM frame P_A^+, P_B^- big

$s = 2P_A^+ P_b^-$



$$P_T^A = P_T^B = 0$$

$$q^\mu = (q^+, q^-, \vec{q}_T), \qquad q^\pm = \frac{q^0 \pm q^z}{\sqrt{2}}$$

$$Q^2 = q^2 = 2q^+q^- - q_T^2 \qquad q_T = |\vec{q}_T|$$

vars: $\{Q^2, Y, \vec{q}_T\}$

rapidity $Y = \frac{1}{2}\ln\left(\frac{q^+}{q^-}\right)$

$\boxed{\text{TMD}}$ when valid?

$$d\sigma \sim \int d^2k_T\, d^2k_T' \; \underbrace{f_{i/p}(x_a, \vec{k}_T, \ldots)}_{\text{acts like probability to find quark } i \text{ with } x_a, \vec{k}_T} \; f_{j/p}(x_b, \vec{k}_T', \ldots)\; \delta^2(\vec{q}_T - \vec{k}_T - \vec{k}_T')$$

$x_a = \dfrac{Q e^Y}{\sqrt{s}} \simeq \dfrac{q^+}{P_A^+}$ quark momentum fraction @ hard collision

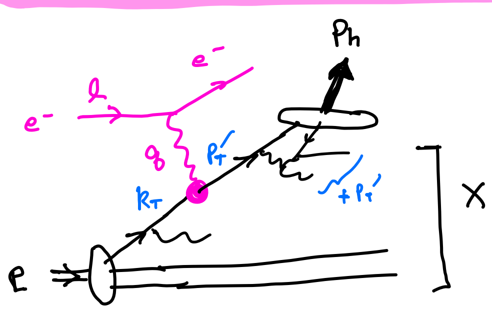
(similar $x_b = Qe^{-Y}/\sqrt{s}$)

$q_T \neq 0 \rightarrow$ intrinsic $k_T \sim \Lambda_{QCD}$, or from radiation

small q_T most sensitive to intrinsic

Semi-Inclusive DIS (SIDIS)

$e^- + p \to e^- + \underbrace{h(P_h)}_{\pi} + X$



$$q^2 = -Q^2 < 0$$

$$x = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot \ell}, \quad z_h = \frac{P\cdot P_h}{P\cdot q}$$

Hadron-Hadron Frame: $P_T = 0$, $P_{hT} = 0$: $\quad q_T = P_T' - k_T$

Photon-Hadron Frame: $P_T = 0$, $q_T = 0$: $\quad \left(-\dfrac{P_{hT}}{z_h} = -\dfrac{P_T}{z_h} - k_T\right)$

$P_{hT} = P_T + z_h k_T$

hadron Trans. Mom.

$$d\sigma \sim \int d^2k_T\, d^2P_T \; f_{i/p}(x, k_T)\; \underbrace{D_{i/h}(z_h, P_T)}_{} \; \delta^2(P_{hT} - z_h k_T - P_T)$$

fragmentation of quark i to hadron h

with momentum fraction z_h & trans. momentum P_T relative to quark

[Handbook pg. 57-58]

"Peel the Onion"

Use DY for simplicity

Handbook
1. § 2.1, 2.2
2. § 2.2
3. § 2.3
4. § 2.4

⋮

Layer

① Factorization

Observables ⟺ TMDs

ⓐ Collinear Factorization $\int d^2 q_T$ $\quad Q^2 \gg \Lambda_{QCD}^2$

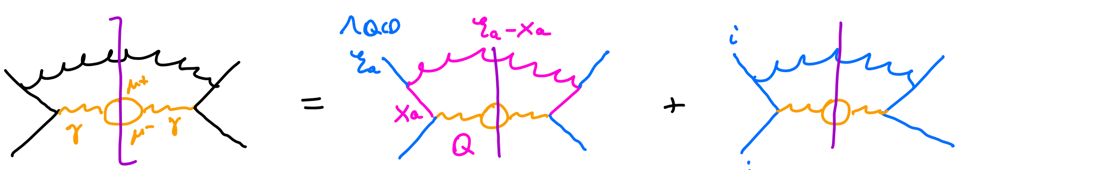
$$\frac{d\sigma}{dQ^2 dY} = \int_{x_a}^{1} d\xi_a \int_{x_b}^{1} d\xi_b \, f_{i/p}(\xi_a, \mu)\, f_{j/p}(\xi_b, \mu)\, \frac{d\hat{\sigma}_{ij}(\xi_{a,b}, \mu)}{dQ^2 dY} \left[1 + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{Q^2}\right)\right]$$

Fact. Scale = Renormalization Scale for PDF operator.

$\xi_a = x_a$ at tree level

$\xi_a > x_a$ due to gluon radiation, two scales Q, Λ_{QCD}

"hard" "collinear"



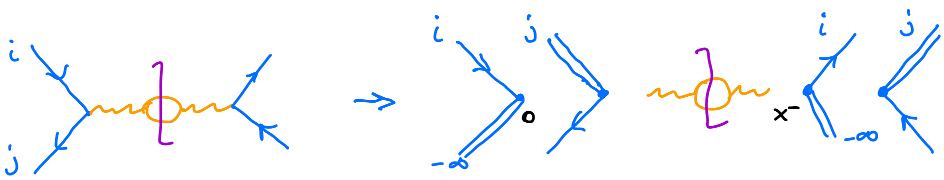
final state cut

$p^2 \gtrsim \mu^2$ $\quad$ $p^2 \lesssim \mu^2$

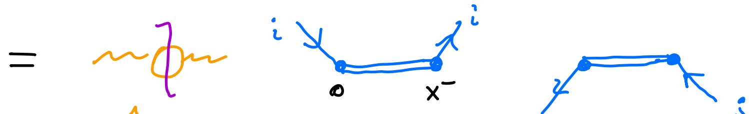
$\hat{\sigma}_{ij}^{(1)}\ f_i^{(0)}\ f_j^{(0)}$ $\quad$ $\hat{\sigma}_{ij}^{(0)}\ f_i^{(1)}\ f_j^{(0)}$

Why do PDF gluons decouple / factorize? Heuristically ...

From point of view of i's radiation, fast moving j looks like line of color charge // (along light-cone)



[SCET building block fields]

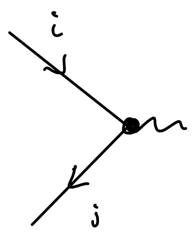


$\hat{\sigma}_{ij}$ $\quad$ $f_i(\xi_a)$ $\quad$ $f_j(\xi_b)$

Exercise ①: Wilson-Line from One-Gluon

See TMD handbook page 34 for Light-Cone basis

Consider how collinear gluon with kinematics of parton j looks from point of view of parton i.



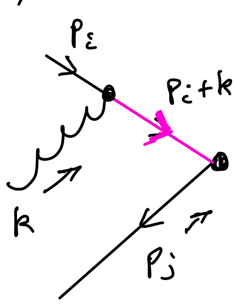
$$p_i^\mu \sim (\overset{+}{Q}, \overset{-}{\Lambda^2/Q}, \overset{\perp}{\Lambda}) \qquad \bar{n}\text{-collinear} \qquad \bar{n}^\mu = (1,0,0)$$

$$p_j^\mu \sim \underbrace{\left(\frac{\Lambda^2}{Q}, Q, \Lambda\right)}_{} \qquad n\text{-collinear} \qquad n^\mu = (0,1,0)$$

momentum compatible with energetic protons Q with $p_\perp \sim \Lambda$ fluctuations

Let gluon have $k^\mu \sim (\overset{+}{\frac{\Lambda^2}{Q}}, \overset{-}{Q}, \Lambda)$ like p_j^μ.

Try to attach it to p_i^μ:



$$= \underbrace{\bar{v}(p_j)\gamma^\mu}_{\substack{\text{plays no}\\ \text{role, keep}\\ \text{fixed!}}} \frac{i(\not{p}_i + \not{k})}{(p_i+k)^2 + i0} \left(-i g T^A \not{\epsilon}^A(k)\right) u(p_i)$$

Since gluons appear from covariant derivatives their size tracks their momentum $\quad iD^\mu = \underbrace{i\partial^\mu}_{k^\mu} + gA^\mu$

So $\not{\epsilon}^A(k) = \not{\alpha}\, \underbrace{\bar{n}\cdot\epsilon^A(k)}_{\epsilon^-\text{ largest part}} + \dots \qquad \therefore\ k^- \ \&\ A^-$ big!

Expand and keep leading term:

$$\frac{(+g)(\not{p}_i + \not{k})}{(p_i+k)^2 + i0} \not{\alpha}\, \bar{n}\cdot\epsilon^A T^A = \underbrace{\overset{\text{Expand}}{\dots}}_{\text{Exercise ⓐ}} = \underbrace{\frac{g\, \bar{n}\cdot\epsilon^A T^A}{\bar{n}\cdot k + i0}}_{*} \quad \frac{\not{\bar{n}}\not{\alpha}}{2}$$

Good component projector. Relevant to later Exercise.

Exercise ⓑ Check that Momentum Space one gluon Feynman Rule for Wilson line reproduces $*$.

Even with arbitrary gluon & quark radiation!

$$/\!\!/_\gamma = W[\gamma] = \text{Wilson Line} = P\exp\left[-ig\int_\gamma dx^\mu A_\mu^c(x)\, t^c\right]$$

γ = path.

↑ L1
↓ L2

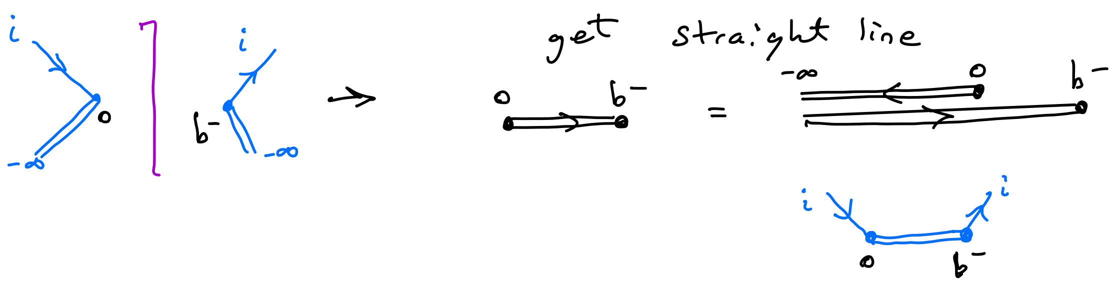
Lecture 2

Iain Stewart

Recap

ⓐ Collinear Factorization $\int d^2 q_T$ $\quad Q^2 \gg \Lambda^2_{QCD}$

$$\frac{d\sigma}{dQ^2 dY} = \int_{x_a}^1 d\xi_a \int_{x_b}^1 d\xi_b\, f_{i/p}(\xi_a,\mu)\, f_{j/p}(\xi_b,\mu)\, \frac{d\hat\sigma_{ij}(\xi_{a,b},\mu)}{dQ^2\, dY}\left[1 + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)\right]$$



$b^- = x^-$ in Lecture 1 (sorry)

$$f_i^{bare}(\xi) \equiv \int \frac{db^-}{2\pi}\, e^{-i\xi P^+ b^-} \langle p|\, \underbrace{\bar\psi_i(b^-)\frac{\gamma^+}{2} W[\gamma]\,\psi_i(0)}_{\text{gauge invariant}}\, |p\rangle$$

prob. of finding parton i with momentum fraction ξ inside proton

ⓑ $q_T^2 \sim Q^2$

$$\frac{d\sigma}{dQ^2\, dY\, dq_T^2} = \int d\xi_a\, d\xi_b\, f_{i/p}(\xi_a,\mu)\, f_{j/p}(\xi_b,\mu)\, \frac{d\hat\sigma_{ij}(\xi_{a,b},\mu)}{dQ^2\, dY\, dq_T^2}\left\{1 + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{q_T^2 \text{ or } Q^2}\right)\right]$$

ⓒ $q_T^2 \ll Q^2$ $\boxed{\text{2 scales}}$ TMDs

$q_T^2 \gg \Lambda_{QCD}^2$ <u>pert</u>
or
$q_T^2 \sim \Lambda_{QCD}^2$ <u>non pert.</u>

$$\frac{d\sigma}{dQ^2\,dY\,d^2q_T} = H_{i\bar{\iota}}(Q^2,\mu) \int d^2k_T \; f_{i/p}(x_a, \vec{k}_T, \mu, \ldots)\; f_{\bar{\iota}/p}(x_b, \vec{q}_T - \vec{k}_T, \mu, \ldots)$$

$$* \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}, \frac{\Lambda_{QCD}^2}{Q^2} \right) \right]$$

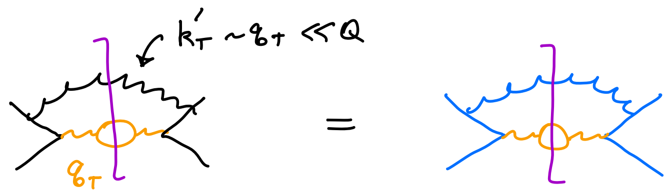
remember onion

$$= H_{i\bar{\iota}}(Q^2,\mu) \int d^2b_T \; e^{i\vec{q}_T\cdot\vec{b}_T}\; \tilde{f}_{i/p}(x_a, \vec{b}_T, \mu, \ldots)\; \tilde{f}_{\bar{\iota}/p}(x_b, \vec{b}_T, \mu, \ldots)$$

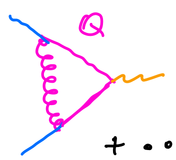
- no ξ_a, ξ_b ?

  $\xi_a = x_a$

  $\xi_b = x_b$



hard scale corrections
purely virtual



$+ \ldots$ $\xrightarrow{\text{squared}}$ $H_{i\bar{\iota}}(Q^2,\mu)$

- What if we expand ⓑ with $\Lambda_{QCD}^2 \ll q_T^2 \ll Q^2$ & compare with ⓒ ?

TMD PDF $\Leftrightarrow$ collinear PDF

$$f_i(x, k_T, \mu, \ldots) = \sum_j \int_x^1 \frac{d\xi}{\xi} \, \underbrace{C_{ij}\left(\frac{x}{\xi}, k_T, \mu, \ldots\right)}_{\text{perturbative } k_T} \, f_j(\xi, \mu) * \left[1 + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{k_T^2} \right) \right]$$

$$H_{i\bar{\iota}} \int_T C_{ij} \otimes C_{\bar{\iota}\bar{J}} = \frac{d\hat{\sigma}_{j\bar{J}}(\xi_{a,b}, \mu)}{dQ^2\,dY\,dq_T^2}$$ when expanded $q_T^2 \ll Q^2$

Layer

(2) TMD Factorization Thm II : rapidity scales

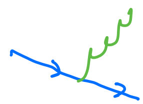
- What type of radiation is allowed? infrared

$2p^+p^- = \vec{p}_T^{\,2}$

<u>collinear</u> $\sim \left(Q, \frac{q_T^2}{Q}, q_T \right)$ with components $+$, $-$, $\perp$

$\theta \ll 1$ collimated

<u>soft</u> $\sim \left(q_T, q_T, q_T \right)$



soft, wide angle

distinguish? <u>by rapidity</u> ν = "rapidity cutoff"

$$Y_{collinear} \simeq \frac{1}{2} \ln \left(\frac{Q^2}{q_T^2} \right) \gg Y_{soft} \simeq \frac{1}{2} \ln (1)$$

think: $q_T e^{2Y} \geq \nu$ $\qquad \nu \gtrsim q_T e^{2Y}$

- Full factorization must decouple "Glauber" region, not covered here
- Traditional QCD Fact, approx for graphs in different mom. regions
  SCET = fields for diff. mom. regions with expanded $\mathcal{L}$.

$\boxed{\text{Draw Figure first}}$

$$\frac{d\sigma}{dQ\, dY\, d^2q_T} = H_{i\bar{i}}(Q^2,\mu) \int d^2b_T\, e^{i \vec{q}_T \cdot \vec{b}_T}\, \tilde{B}_{i/p}\left(x_a, \vec{b}_T, \mu, \frac{\zeta_a}{\nu^2}\right) B_{\bar{i}/p}\left(x_b, \vec{b}_T, \mu, \frac{\zeta_b}{\nu^2}\right) * \tilde{S}(b_T, \mu, \nu)$$

$$= H_{i\bar{i}}(Q^2,\mu) \int d^2b_T\, e^{i \vec{q}_T \cdot \vec{b}_T}\, \tilde{f}_{i/p}(x_a, \vec{b}_T, \mu, \zeta_a)\, f_{\bar{i}/p}(x_b, \vec{b}_T, \mu, \zeta_b)$$

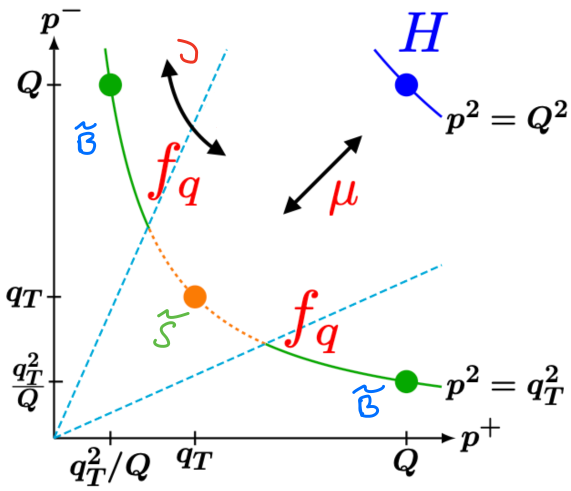
- $\tilde{f} = \tilde{B} \sqrt{\tilde{S}}$ indep. of ν
- Collins-Soper Scales

$$\zeta_a \simeq 2\,(x_a P_A^+)^2 \quad e^{-2y_n}$$

$$\zeta_b \simeq 2\,(x_b P_B^-)^2 \quad e^{+2y_n}$$

residual freedom in how we split soft fn. (ignore)

$$\zeta_a \zeta_b = Q^4$$



• Mention RGE in μ & ν

Layer ③ Operators (finally) use bare $\tilde B$, $\tilde S$

• UV regulator dim. reg. $d = 4 - 2\epsilon$, scale μ

• Rapidity regulator τ, scale ν

It's an onion...

$$\tilde f_{i/p}(x, \vec b_T, \mu, \nu) = \lim_{\substack{\epsilon \to 0 \\ \tau \to 0}} \underbrace{Z^i_{uv}(\mu, \nu, \epsilon)}_{\text{UV counterterm}} \; \underbrace{\tilde B_{i/p}(x, b_T, \epsilon, \tau, x p^+)}_{= f^{(u)}_{i/p} / \tilde S^{subt}} \; \sqrt{\tilde S(b_T, \epsilon, \tau)}$$

Unsub. PDF

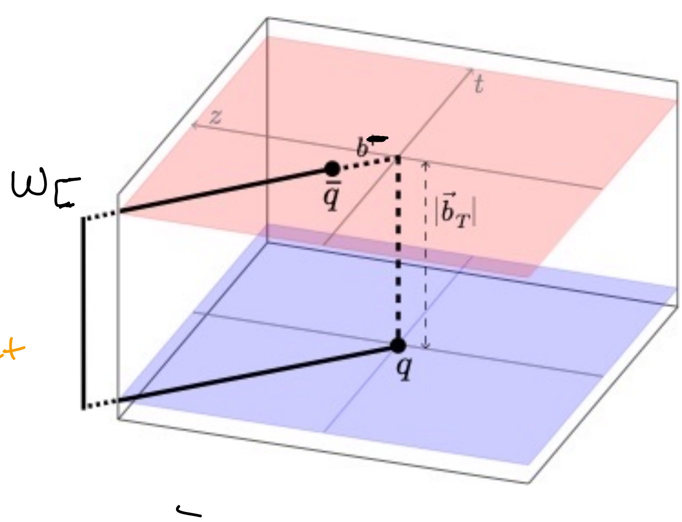
$$f^{(u)}_{i/p} = \int \frac{db^-}{2\pi} e^{-ib^- x p^+} \langle p | \left[\bar\psi_i(b^\mu) \frac{\gamma^+}{2} W_\Gamma \, \psi_i(0) \right]_\tau | p \rangle$$

• $b^\mu = (0, b^-, \vec b_T)$

separated also by $\vec b_T$

• Staple shaped Wilson line

• generated by expansions that replace fields from other proton (like before)



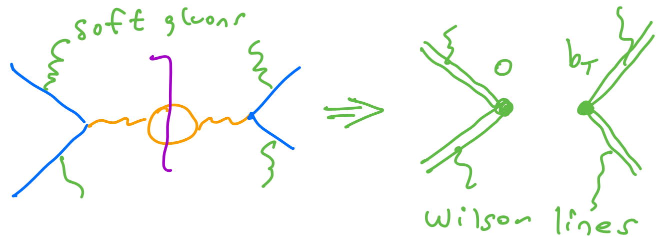
- encode initial state interactions   gauge invariant operator

- reduces to bare collinear PDF for $b_T \to 0$ (not true for renormalized)

Soft Fn

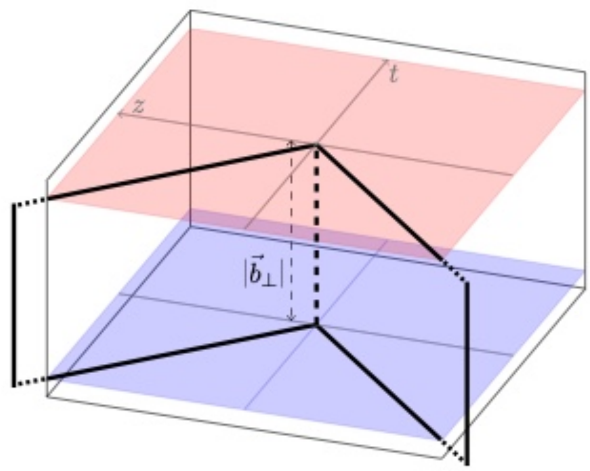
$$\tilde{S}(b_T, \epsilon, \tau) = \frac{1}{N_c} \langle 0 | \left[\mathrm{Tr}\ W_{\gg}(b_T) \right]_\tau | 0 \rangle$$

- 2 staples pasted together $\Leftrightarrow$ soft approx to fields for both protons



- closed loop, 6 sides (gauge inv.)

- $\tilde{S}(b_T \to 0, \epsilon, \tau) = 1$



skip

Subtraction

$\tilde{S}^{subt} = \ ?$ depends on choice for τ

- remove infrared double counting btwn $\tilde{f}^{(u)}$ and $\tilde{S}$
- many choices possible for τ (Sec. 2.4 of Handbook)

Common : $\tilde{S}^{subt} = \tilde{S}$

but also : $\tilde{S}^{subt} = 1$ possible

Layer 4 Rapidity Regulators

Why do we need τ?

$$\int_{q_T}^{Q} \frac{dk^+}{k^+} = \lim_{\tau \to 0} \Big[\underbrace{\int_{0}^{Q} \frac{dk^+}{k^+} R_c(k,\tau,\nu)}_{\text{collinear approx}} + \underbrace{\int_{q_T}^{\infty} \frac{dk^+}{k^+} R_s(k,\tau,\nu)}_{\text{soft approx}} \Big]$$

regulators

skip

must expand to derive factorization

Examples

- Collins, Space-Like Wilson Lines $\quad \dfrac{\sqrt{\tilde{S}}}{\tilde{S}^{\text{subt}}} \to \dfrac{1}{\sqrt{\tilde{S}}}$

light-cone $(0, 1, 0_T) \to (-e^{2y_B}, 1, 0_T)$ with $y_B \to -\infty$

$$\tilde{f}_{i/p}(x, b_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{uv}(\mu, \zeta, \epsilon) \lim_{y_B \to -\infty} \frac{\tilde{f}_{i/p}^{(u)}(x, b_T, \epsilon, y_B, xP^+)}{\sqrt{\tilde{S}(b_T, \epsilon, 2y_n - 2y_B)}}$$

here $\zeta = 2(xP^+)^2 e^{-2y_n}$

- η regulator Chiu, Jain, Neill, Rothstein $\eta \to 0$

introduce $|\sqrt{2}k^+/\nu|^{-\eta}$ in $\underset{\text{Wilson Lines}}{W_\sqsubset} \to \int \dfrac{dk^+}{k^+} \left|\dfrac{\sqrt{2}k^+}{\nu}\right|^{-\eta}$

$|k^z/\nu|^{-\eta/2}$ in $W_{\gtrdot} \to \int \dfrac{dk^+ dk^-}{k^+ k^-} \left|\dfrac{k^z}{\nu}\right|^{-\eta}$

$$\tilde{f}_{i/p}(x, b_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{uv}(\mu, \zeta, \epsilon) \lim_{\eta \to 0} \tilde{f}^{(u)}(x, b_T, \epsilon, \eta, xP^+) \sqrt{\tilde{S}(b_T, \epsilon, \eta)}$$

$\tilde{S}^{\text{subt}} = 1$ here

- different $\tilde{f}^{(u)}$, $\tilde{S}$ but same $\tilde{B}_{i/p}$ & Z_{uv}
- many constructions (§2.4.1) yield same $\tilde{f}_{i/p}$, but not all (§2.5)

$\boxed{\text{One-Loop}}$ Illustration of Concepts

proton → quark state $\qquad p^\mu = (p^+, 0, 0)$, $p^2 = 0$

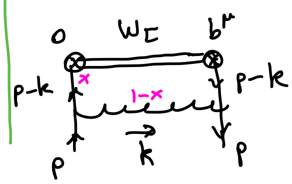
$d = 4 - 2\epsilon$ for UV & IR , Feynman Gauge

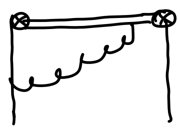
η-regulator: $\tilde{f}^{(u)}$, $\tilde{S}$, $\tilde{S}^{subt} = 1$

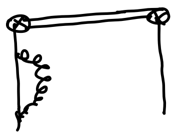
Exercise #2 Go through this, see §2.4.2 of Handbook

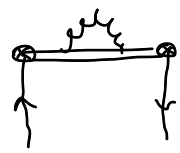
bare

$$\tilde{f}^{(u)}_{q/q'}(x, b_T, \epsilon, \eta, xp^+) = \int \frac{db^-}{2\pi}\, e^{-ib^- x p^+} \langle q'(p) | \left[\bar{\psi}(b^\mu)\, W_\square\, \frac{\gamma^+}{2}\, \psi(0) \right]_\eta | q(p) \rangle$$

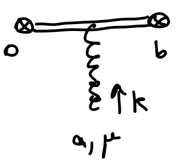






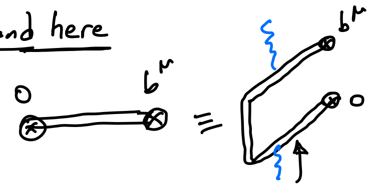


(a) $\qquad$ + mirror (b) $\qquad$ $0,\ \frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}}$ (careful) $\qquad$ 0 as light-like lines $n^\mu n_\mu = 0$, transv @ ∞ = 0



$$= -g_0 \frac{n_b^\mu t^a\, e^{-ik\cdot b}}{n_b \cdot k + i0} + \frac{g_0\, n_b^\mu t^a}{n_b \cdot k - i0}$$

shorthand here



$n_b = \frac{1}{\sqrt{2}}(1, 0, 0, -1)$

$$M_a = -i g_0^2 C_F \int d\!\!^-{}^d k \int d\!\!^- b^-\, e^{-ib^- x p^+} e^{i(p-k)\cdot b} \frac{\bar{u}\, \gamma^\mu (\not{p} - \not{k}) \gamma^+ (\not{p} - \not{k}) \gamma_\mu u}{2\left[(p-k)^2 + i0\right]^2 (k^2 + i0)} \quad R_c$$

$$M_b = -2 i g_0^2 C_F \int d\!\!^-{}^d k \underbrace{\int d\!\!^- b^-\, e^{-ib^- x p^+} e^{i(p-k)\cdot b}}_{\delta[(1-x)p^+ - k^+]\, e^{i\vec{b}_T \cdot \vec{k}_T}} \frac{\bar{u}\, \gamma^+ (\not{p} - \not{k}) \gamma^+ u}{2(k^+ + i0)\left[(p-k)^2 + i0\right](k^2 + i0)} \quad R_c$$

+ scaleless

$\overline{MS}$ $\quad g_0 = Z_g\, \mu^\epsilon\, g(\mu) \left(\frac{e^{\gamma_E}}{4\pi}\right)^{\epsilon/2}$,

$$M_a + M_b = \frac{\alpha_s(\mu) C_F}{2\pi} \Big[\underbrace{\frac{1+x^2}{1-x}}_{P_{qq}(x)} - \epsilon(1-x) \Big] \Gamma(-\epsilon) \left(\frac{b_T^2 \mu^2}{4 e^{-\gamma_E}} \right)^\epsilon \quad R_c$$

singular as $x \to 1$, ie $k^+ \to 0$

"η-reg" $R_c = \omega^2 \left|\frac{\sqrt{2}k^+}{\nu}\right|^{-\tau} = \omega^2\left(\frac{(1-x)p^+}{\nu/\sqrt{2}}\right)^{-\tau}$

Expand

$\tau \to 0$

$\epsilon \to 0$

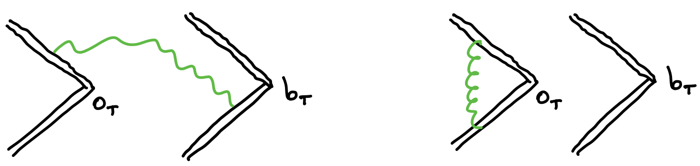
use: $(1-x)^{-1-\tau} = -\frac{1}{\tau}\delta(1-x) + \left(\frac{1}{1-x}\right)_+ + O(\tau)$

$$(1+x^2)(1-x)^{-1-\tau} = -\left(\frac{2}{\tau}+\frac{3}{2}\right)\delta(1-x) + \left(\frac{1+x^2}{1-x}\right)_+ + O(\tau)$$

where $[f(x)]_+ = f(x), \quad x\neq 1$

$$\int_0^1 dx\, [f(x)]_+ g(x) = \int_0^1 dx\, f(x)\,[g(x)-g(1)] \,, \quad \text{any } g$$

$$\tilde S(b_T,\epsilon,\tau) = \frac{1}{N_c}\langle 0| \left[\mathrm{Tr}\, W_{\gtrless}(b_T)\right]_\tau |0\rangle$$



$R_s^{tot} = \omega^2 \left|\frac{k^- - k^+}{\nu/\sqrt{2}}\right|^{-\tau}$

+ mirror   + mirror

$\Downarrow$ $0 = \frac{1}{\epsilon} - \frac{1}{\epsilon}$

$$M_s = 2g_0^2 C_F \int d^dk\, e^{ib_T\cdot k_T} \frac{-i}{(2k^+k^- - k_T^2 + i0)} \frac{1}{(k^+ - i0)(-k^- + i0)} \quad R_s$$

$$k^+ = \frac{k_\perp^2}{2k^-} - i0 \quad \text{for } k^- > 0$$

$$= \frac{g_0^2 C_F}{\pi}\int d^{d-2}k_T\, \frac{e^{ib_T\cdot k_T}}{k_T^2}\int_0^\infty \frac{dk^+}{k^+}\, \omega^2\left|\frac{k_T^2}{2k^+} - k^+\right|^{-\tau}\left(\frac{\nu}{\sqrt{2}}\right)^{\tau}$$

bare

$$\tilde f^{(u)}_{q/q} = \delta(1-x) + \frac{\alpha_s(\mu)C_F}{2\pi}\Big\{ -\left(\frac{1}{\epsilon_{IR}} + L_b\right)[P_{qq}(x)]_+ + (1-x)$$
$$+ \delta(1-x)\left(\frac{1}{\epsilon_{UV}}+L_b\right)\left(\frac{2}{\tau}+\frac{3}{2}+\ln\frac{\nu^2}{y}\right) + O(\tau) + O(\epsilon)\Big\}$$

$$L_b \equiv \ln\frac{b_T^2\mu^2}{b_0^2}\,, \quad b_0 = 2e^{-\gamma_E}\,, \quad y = 2(xp^+)^2\,, \quad C_F = 4/3$$

$$\tilde S_q = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}\left[\frac{2}{\epsilon_{UV}^2} + 2\left(\frac{1}{\epsilon_{UV}}+L_b\right)\left(\frac{-2}{\tau} + \ln\frac{\mu^2}{\nu^2}\right) - L_b^2 - \frac{\pi^2}{6}\right]$$
$$+ O(\tau) + O(\epsilon)$$

$$Z_{uv}^{q} = 1 - \frac{\alpha_s(\mu) C_F}{2\pi} \left[\frac{1}{\epsilon_{uv}^2} + \frac{1}{\epsilon_{uv}} \left(\frac{3}{2} + \ln \frac{\mu^2}{\mathcal{J}} \right) \right]$$

$$\tilde{f}^{(\text{to 1-loop})}(x, b_T, \mu, \mathcal{J}) = Z_{uv}^{q} \, \tilde{f}^{(u)}_{q/q} \sqrt{\tilde{S}}$$

$$= \delta(1-x) + \frac{\alpha_s(\mu) C_F}{2\pi} \left[- \left(\frac{1}{\epsilon_{IR}} + L_b \right) \left[P_{qq}(x) \right]_+ + (1-x) + \delta(1-x) \left\{ - \frac{L_b^2}{2} + L_b \left(\frac{3}{2} + \ln \frac{\mu^2}{\mathcal{J}} \right) - \frac{\pi^2}{12} \right\} \right]$$

- expected IR div.
- $\frac{1}{\tau}$'s cancel, $\ln \mathcal{J}^2$ cancel

Lecture 3

Recap L2 $\quad L_b \equiv \ln \frac{b_T^2 \mu^2}{b_0^2}$, $\quad b_0 = 2e^{-\gamma_E}$, $\quad \zeta = 2(xP^+)^2$, $\quad C_F = 4/3$

bare

$$\tilde{f}^{(u)}_{q/q} = \delta(1-x) + \frac{\alpha_s(\mu) C_F}{2\pi} \Big\{ -\Big(\frac{1}{\epsilon_{IR}} + L_b\Big)[P_{qq}(x)]_+ + (1-x) + \delta(1-x)\Big(\frac{1}{\epsilon_{uv}} + L_b\Big)\Big(\frac{2}{\tau} + \frac{3}{2} + \ln\frac{\nu^2}{\zeta}\Big) \Big\}$$

$$\tilde{S}_q = 1 + \frac{\alpha_s(\mu) C_F}{2\pi}\Big[\frac{2}{\epsilon_{uv}^2} + 2\Big(\frac{1}{\epsilon_{uv}} + L_b\Big)\Big(\frac{-2}{\tau} + \ln\frac{\mu^2}{\nu^2}\Big) - L_b^2 - \frac{\pi^2}{6}\Big]$$

$$Z^q_{uv} = 1 - \frac{\alpha_s(\mu) C_F}{2\pi}\Big[\frac{1}{\epsilon_{uv}^2} + \frac{1}{\epsilon_{uv}}\Big(\frac{3}{2} + \ln\frac{\mu^2}{\zeta}\Big)\Big]$$

$$\tilde{f}^{(\text{to 1-loop})}(x, b_T, \mu, \zeta) = Z^q_{uv}\, \tilde{f}^{(u)\,\text{bare}}_{q/q} \sqrt{\tilde{S}^{\text{bare}}}$$

$$= \delta(1-x) + \frac{\alpha_s(\mu) C_F}{2\pi}\Big[-\Big(\frac{1}{\epsilon_{IR}} + L_b\Big)[P_{qq}(x)]_+ + (1-x) + \delta(1-x)\Big\{-\frac{L_b^2}{2} + L_b\Big(\frac{3}{2} + \ln\frac{\mu^2}{\zeta}\Big) - \frac{\pi^2}{12}\Big\}\Big]$$

Renormalization Group Evolution

consider anomalous dimensions in α_s expansion

- $\mu \frac{d}{d\mu} \tilde{f}_{i/p}(x, b_T, \mu, \zeta) = \Big(\mu \frac{d}{d\mu} Z^q_{uv}\Big)\, \tilde{f}^{\text{bare}} \sqrt{\tilde{S}^{\text{bare}}}$

$$\mu \frac{d}{d\mu} \ln \tilde{f}_{i/p} = \gamma^q_\mu(\mu, \zeta) = +\,(Z^q_{uv})^{-1} \mu \frac{d}{d\mu} Z^q_{uv}$$

$$\overset{\text{1-loop}}{=} \cdots = \frac{\alpha_s(\mu) C_F}{\pi}\Big(\frac{3}{2} + \ln \mu^2/\zeta\Big)$$

$$\overset{\text{all orders form}}{=} \Gamma_{\text{cusp}}[\alpha_s] \ln \mu^2/\zeta + \gamma^q_\mu[\alpha_s]$$

Note: $\gamma^q_\mu(\mu, \zeta)$ always perturbative for $\mu \gg \Lambda_{QCD}$.

evolve from $\mu \simeq 1\,\text{GeV} \to \mu \simeq Q$

- rapidity RGE = Collins-Soper Eqtn

$$y \frac{d}{dy} \ln \tilde{f}_{i/p}(x, b_T, \mu, y) = \frac{1}{2} \gamma_y^q(\mu, b_T)$$ Known to 4-loops

$$\overset{1\text{-loop}}{=} -\frac{\alpha_s(\mu) C_F}{2\pi} L_b = -\frac{\alpha_s(\mu) C_F}{2\pi} \ln \frac{\mu^2 b_T^2}{b_0^2}$$

evolve from $\sqrt{y} \simeq 1\,\text{GeV} \to \sqrt{y} \simeq Q$

all orders form $$\gamma_y^q(\mu, b_T) = -2 \int_{1/b_T}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{cusp}[\alpha_s(\mu')] + \gamma_y^q[\alpha_s(1/b_T)]$$

Note : $\gamma_y^q(\mu, b_T)$ is non-perturbative for $b_T^{-1} \sim \Lambda_{QCD}$

- Both equations ~~needed~~ needed to sum large logs

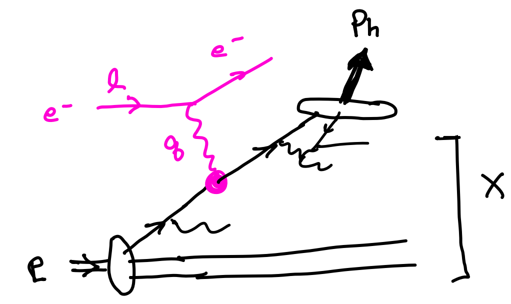
Resummation $\alpha_s \ln\left(\frac{Q}{q_T}\right) \sim \alpha_s \ln(Q b_T) \sim 1$

$$\widetilde{\sigma}^W(\boldsymbol{b}_T) = f_q(x_1) f_{\bar{q}}(x_2) C[\alpha_s] \exp\Big\{ \frac{\alpha_s}{4\pi} \Big(d_{12} L_b^2 + d_{11} L_b\Big) + \Big(\frac{\alpha_s}{4\pi}\Big)^2 \Big(d_{23} L_b^3 + d_{22} L_b^2 + d_{21} L_b\Big) + \Big(\frac{\alpha_s}{4\pi}\Big)^3 \Big(d_{34} L_b^4 + d_{33} L_b^3 + d_{32} L_b^2 + d_{31} L_b\Big) \Big\} + \ldots,$$

LL NLL NNLL N^{3}LL

Back to SIDIS

$e^- + p \to e^- + h(P_h) + X$



$$q^2 = -Q^2 < 0$$

$$x = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot \ell}, \quad z_h = \frac{P\cdot P_h}{P\cdot q}$$

$$\frac{d\sigma}{dx\,dy\,dz\,dP_{hT}^2} = \sigma_0^{SIDIS}(x,y,Q)\, H_{ii}(Q^2,\mu) \int_0^{2\pi} d\phi_h \int d^2b_T\, e^{i\vec{b}_T\cdot\vec{P}_{hT}/z_h}$$

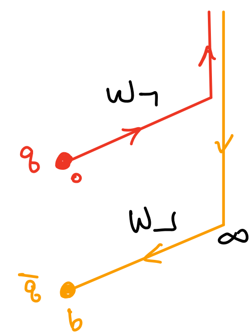
$$* \tilde{f}_{i/p}(x,b_T,\mu,\zeta_p)\, \underbrace{\tilde{D}_{h/i}(z_h,b_T,\mu,\zeta_h)}_{\text{TMD fragmentation fn.}}$$

$$\tilde{D}_{h/i}^{ren} = \lim_{\epsilon\to 0}\lim_{\tau\to 0} \underbrace{Z_{uv}^{i}(\mu,\zeta,\epsilon)}_{\text{same as PDF}}\, \tilde{D}_{h/i}^{bare}\, \sqrt{\tilde{S}^{bare}}$$

$$\tilde{D}_{h/i}^{(u)} = \frac{1}{4N_c z}\,\mathrm{tr}\int \frac{db^-}{2\pi}\, e^{ib^-(P_h^+/z)}\, \gamma^+_{\alpha\alpha'}$$

$$* \sum_X \langle 0|\, [(W_\lrcorner\, \psi_i^\alpha)(b)]_\tau\, |h(P_h)\,X\rangle$$

$$* \langle X\, h(P_h)|\, [(\bar\psi_i^{\alpha'} W_\urcorner)(0)]_\tau\, |0\rangle$$



- Analogous except for final state hadron $\quad b = (0, b^-, \vec{b}_T)$
- Use outgoing Wilson lines to $+\infty$
- In fact $\tilde{f}_{i/p}$ in SIDIS also has outgoing Wilson lines $+\infty$ $W_\urcorner$ (rather than prev. $-\infty$ $W_\sqsubset$)

(Does it matter? We'll see it does sometimes!)

Spin Polarized TMDs

- Consider polarized protons & quarks
- Have 8 TMD PDFs at leading order in $q_T \ll Q$

Consider

proton spin

$$\Phi_{\alpha\alpha'} = \int db^- \, d^2b_T \, e^{-ib^- x P^+} e^{i b_T \cdot k_T} \langle P(P,S) | [\bar{\psi}^i_\alpha(b) \, W_C \, \psi^i_{\alpha'}(0)]_T | P(P,S) \rangle$$

spinor indices

$$n_{a,b} = \frac{(1,0,0,\pm 1)}{\sqrt{2}}$$

Spin vector S^μ

$$S^\mu = S_L \frac{(P^- n_a^\mu - P^+ n_b^\mu)}{M} + S_T^\mu$$

longitudinal spin — transverse spin

$$-S^2 = S_L^2 + S_T^2 = \begin{cases} 1 & \text{pure state} \\ <1 & \text{mixed state} \end{cases}$$

$$\bar{u} \gamma^\mu u = 2 P^\mu$$
$$\bar{u} \gamma^\mu \gamma_5 u = 2M S^\mu$$

$$u(P,S)\, \bar{u}(P,S) = \frac{\not{P} + M}{2} (1 + \gamma_5 \not{S})$$

unpolarized — hadron spin polariz.

Constraints on $\Phi_{\alpha\alpha'}$

- no S^μ or linear in S^μ
- hermiticity $\quad \Phi^\dagger = \gamma_0 \Phi \gamma_0$
- Parity $\quad \Phi^P = \gamma_0 \Phi \gamma_0$

* no Time reversal constraint used here *

See Exercise #1 (see also SCET in EFT course)

- Good Quark Fields for Leading Order: $\frac{\gamma^- \gamma^+}{2} \psi^i = \psi^i$

$$\Phi = \frac{1}{2} \Big\{ f_1 \gamma^- - f_{1T}^\perp \epsilon_T^{\rho\sigma} \frac{k_{T\rho}}{M} S_{T\sigma} \gamma^- + \Big(S_L g_1 - \frac{k_T \cdot S_T}{M} g_{1T}^\perp \Big) \gamma_5 \gamma^-$$
$$+ h_1 \not{S}_T \gamma^- \gamma_5 + \Big(S_L h_{1L}^\perp - \frac{k_T \cdot S_T}{M} h_{1T}^\perp \Big) \frac{\not{k}_T \gamma^- \gamma_5}{M} + i h_1^\perp \frac{\not{k}_T \gamma^-}{M} \Big\}$$

8 TMDs

Homework #3: Check this!

Leading Quark TMDPDFs

Nucleon Spin   Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity; $h_{1T}^\perp$ = − Pretzelosity

Contracting $f_{i/p_S}^{[\Gamma]} = \Gamma_{\alpha\alpha'}\,\Phi_{\alpha\alpha'}$ we can write

$$
\begin{aligned}
f_{i/p_S}^{[\gamma^+]}(x,\mathbf{k}_T,\mu,\zeta) &= f_1(x,k_T) - \frac{\epsilon_T^{\rho\sigma}k_{T\rho}S_{T\sigma}}{M}\,\kappa\, f_{1T}^\perp(x,k_T)\,, \qquad \leftarrow |\vec{k}_T| \\
f_{i/p_S}^{[\gamma^+\gamma_5]}(x,\mathbf{k}_T,\mu,\zeta) &= S_L\, g_1(x,k_T) - \frac{\mathbf{k}_T\cdot\mathbf{S}_T}{M}\, g_{1T}^\perp(x,k_T)\,, \\
f_{i/p_S}^{[i\sigma^{\alpha+}\gamma_5]}(x,\mathbf{k}_T,\mu,\zeta) &= S_T^\alpha h_1(x,k_T) + \frac{S_L k_T^\alpha}{M} h_{1L}^\perp(x,k_T) \\
&\quad - \frac{\mathbf{k}_T^2}{M^2}\left(\frac{1}{2}g_T^{\alpha\rho} + \frac{k_T^\alpha k_T^\rho}{\mathbf{k}_T^2}\right)S_{T\rho}\, h_{1T}^\perp(x,k_T) - \frac{\epsilon_T^{\alpha\rho}k_{T\rho}}{M}\,\kappa\, h_1^\perp(x,k_T)
\end{aligned}
\qquad (2.124)
$$

Under T = T-reversal $S^\mu \to -S^\mu$. Consider $\boxed{TP}$ where P = Parity :

- naively $f_{1T}^\perp$, $h_1^\perp$ <u>odd</u> , rest <u>even</u>
- but also switches Wilson lines $W_{\sqsupset_{+\infty}} \leftrightarrow W_{\sqsubset_{-\infty}}$

$\therefore$ $\left(f_{1T}^\perp\right)^{SIDIS} = -\left(f_{1T}^\perp\right)^{DY}$ Famous SIDIS sign-flip

$\left(h_1^\perp\right)^{SIDIS} = -\left(h_1^\perp\right)^{DY}$

<u>Encoded above by</u>: $\kappa = \begin{matrix} +1 & DY \\ -1 & SIDIS \end{matrix}$

others TMDs + sign $\therefore$ equal

For later use, the Fourier transform

$|\vec{b}_T|$

$$
\begin{aligned}
\tilde f_{i/p_S}^{[\gamma^+]}(x,\mathbf{b}_T,\mu,\zeta) &= \tilde f_1(x,b_T) + i\epsilon_{\rho\sigma} b_T^\rho S_T^\sigma M \tilde f_{1T}^\perp(x,b_T)\,, \\
\tilde f_{i/p_S}^{[\gamma^+\gamma_5]}(x,\mathbf{b}_T,\mu,\zeta) &= S_L \tilde g_1(x,b_T) + i b_T\cdot S_T M \tilde g_{1T}^\perp(x,b_T)\,, \\
\tilde f_{i/p_S}^{[i\sigma^{\alpha+}\gamma_5]}(x,\mathbf{b}_T,\mu,\zeta) &= S_T^\alpha \tilde h_1(x,b_T) - i S_L b_T^\alpha M \tilde h_{1L}^\perp(x,b_T) + i\epsilon^{\alpha\rho} b_{\perp\rho} M \tilde h_1^\perp(x,b_T) \\
&\quad + \frac{1}{2}\mathbf{b}_T^2 M^2 \left(\frac{1}{2} g_T^{\alpha\rho} + \frac{b_T^\alpha b_T^\rho}{\mathbf{b}_T^2}\right) S_{\perp\rho} \tilde h_{1T}^\perp(x,b_T)\,.
\end{aligned}
\qquad (2.127)
$$

The k_T prefactors complicate the Fourier transform:

$$
\begin{aligned}
&\tilde f_1(x,b_T) \equiv \tilde f_1^{(0)}(x,b_T)\,, && \tilde f_{1T}^\perp(x,b_T) \equiv \tilde f_{1T}^{\perp(1)}(x,b_T)\,, && \tilde h_{1T}^\perp(x,b_T) \equiv \tilde h_{1T}^{\perp(2)}(x,b_T) \\
&\tilde g_{1L}(x,b_T) \equiv \tilde g_{1L}^{(0)}(x,b_T)\,, && \tilde h_1^\perp(x,b_T) \equiv \tilde h_1^{\perp(1)}(x,b_T)\,, && \\
&\tilde h_1(x,b_T) \equiv \tilde h_1^{(0)}(x,b_T) && \tilde g_{1T}(x,b_T) \equiv \tilde g_{1T}^{(1)}(x,b_T)\,, && \\
& && \tilde h_{1L}^\perp(x,b_T) \equiv \tilde h_{1L}^{\perp(1)}(x,b_T)\,. &&
\end{aligned}
\qquad (2.128)
$$

$$
\begin{aligned}
\tilde f^{(n)}(x,b_T,\mu,\zeta) &\equiv n!\left(\frac{-1}{M^2 b_T}\partial_{b_T}\right)^n \tilde f(x,b_T,\mu,\zeta) \\
&= \frac{2\pi\, n!}{(b_T M)^n}\int_0^\infty \mathrm{d}k_T\, k_T \left(\frac{k_T}{M}\right)^n J_n(b_T k_T)\, f(x,k_T,\mu,\zeta)
\end{aligned}
\qquad (2.129)
$$

Bessel Fn. order n from $\int_0^{2\pi} d\phi$

A similar spin decomposition can be done for

- Quark TMD FFs
- Gluon TMD PDFs & FFs

see § 2.7 of Handbook

We'll need:

Leading Quark TMDFFs — Hadron Spin, Quark Spin

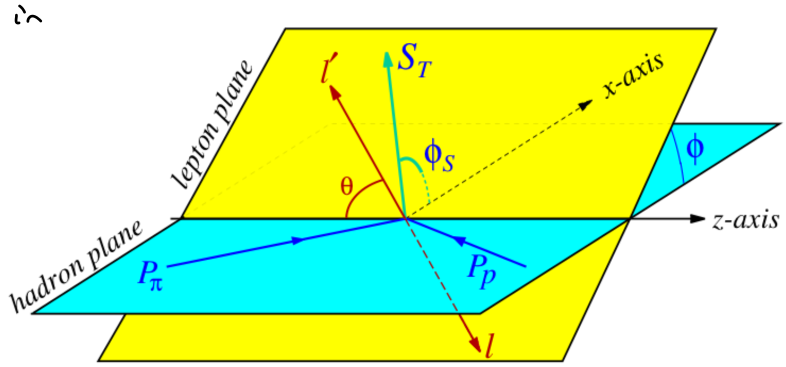
	Quark Polarization		
	Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons	D_1 = Unpolarized		$H_1^\perp$ = − Collins

Polarized Drell-Yan $\pi(P_\pi) + p(P_p, S) \to \gamma^*/Z \xrightarrow{q^\mu} \ell^+\ell^- X$

Angles θ, ϕ, ϕ_S defined in Collins-Soper frame

- $(\ell^+\ell^-)$ at rest
- $P_{\pi T} = P_{pT} = \frac{q_T}{2}$

Leading for $q_T \ll Q$



Structure Functions $F = F(x_\pi, x_p, q_T, Q^2)$

$$\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{\mathcal{F}\,Q^2}\Big\{\Big[(1+\cos^2\theta)F_{UU}^1 + \sin^2\theta\cos(2\phi)F_{UU}^{\cos 2\phi}\Big]$$
$$+ S_L \sin^2\theta\sin(2\phi)F_{UL}^{\sin 2\phi}$$
$$+ S_T(1-\cos^2\theta)\sin\phi_S F_{UT}^{\sin\phi_S}$$
$$+ S_T\sin^2\theta\Big[\sin(2\phi+\phi_S)F_{UT}^{\sin(2\phi+\phi_S)} + \sin(2\phi-\phi_S)F_{UT}^{\sin(2\phi-\phi_S)}\Big]\Big\}$$

π unpol. polarization of proton

Factorization

$$F_{UU}^1 = \mathcal{B}\big[\tilde{f}_{1,\pi}^{(0)}\,\tilde{f}_{1,p}^{(0)}\big],$$ ← Unpol.²

$$F_{UU}^{\cos 2\phi} = M_\pi M_p\,\mathcal{B}\big[\tilde{h}_{1,\pi}^{\perp(1)}\,\tilde{h}_{1,p}^{\perp(1)}\big],$$ ← Boer-Mulders²

$$F_{UL}^{\sin 2\phi} = -M_\pi M_p\,\mathcal{B}\big[\tilde{h}_{1,\pi}^{\perp(1)}\,\tilde{h}_{1L,p}^{\perp(1)}\big],$$ ← " " & Worm-Gear

$$F_{UT}^{\sin\phi_S} = M_p\,\mathcal{B}\big[\tilde{f}_{1,\pi}^{(0)}\,\tilde{f}_{1T,p}^{\perp(1)}\big],$$ ← Unpol. & Sivers

$$F_{UT}^{\sin(2\phi-\phi_S)} = -M_\pi\,\mathcal{B}\big[\tilde{h}_{1,\pi}^{\perp(1)}\,\tilde{h}_{1,p}^{(0)}\big],$$ ← Boer-Mulders & Transversity

$$F_{UT}^{\sin(2\phi+\phi_S)} = -\frac{M_\pi M_p^2}{4}\,\mathcal{B}\big[\tilde{h}_{1,\pi}^{\perp(1)}\,\tilde{h}_{1T,p}^{\perp(2)}\big].$$ ← Boer Mulders & Pretzelosity

$$\mathcal{B}[\tilde{f}_\pi^{(m)}\,\tilde{f}_p^{(n)}] \equiv \sum_i H_{i\bar{i}}(Q,\mu)\int_0^\infty \frac{db_T}{2\pi}\,b_T\,b_T^{m+n}\,J_{m+n}(q_T b_T)\,\tilde{f}_{i/p}^{(m)}(x_a,b_T,\mu,\zeta_a)\,\tilde{f}_{\bar{i}/\pi}^{(n)}(x_b,b_T,\mu,\zeta_b)$$

Polarized SIDIS

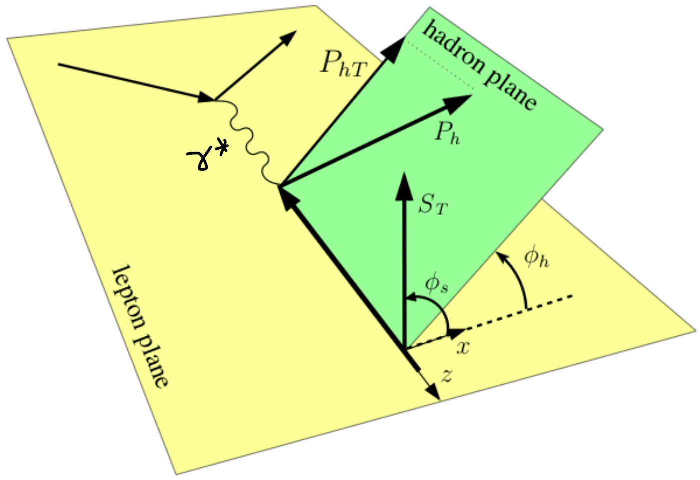
$$\ell(l, \lambda) + p(P, S) \to \ell(l') + h(P_h) + X$$

massless   helicity   unpol.

Frame for angles ϕ_h, ϕ_S

= Trento convention

- $\vec{q} \parallel \hat{z}$
- Leptons : x-z plane



Consider $Q \ll M_{W,Z}$, just γ^*

Leading Order in $P_{hT} \ll Q$

Structure Functions

$$F = F(x, z_h, P_{hT}^2, Q^2)$$

$$\frac{d^6\sigma}{dx\,dy\,dz_h\,d\phi_S\,d\phi_h\,dP_{hT}^2} = \frac{\alpha_{em}^2}{x\,y\,Q^2}\left(1-y+\frac{1}{2}y^2\right)\Big[F_{UU,T} + \cos(2\phi_h)\,p_1\,F_{UU}^{\cos(2\phi_h)}$$
$$+ S_L \sin(2\phi_h)\,p_1\,F_{UL}^{\sin(2\phi_h)} + S_L\,\lambda\,p_2\,F_{LL}$$
$$+ S_T \sin(\phi_h - \phi_S)\,F_{UT,T}^{\sin(\phi_h-\phi_S)}$$
$$+ S_T \sin(\phi_h + \phi_S)\,p_1\,F_{UT}^{\sin(\phi_h+\phi_S)} + \lambda\,S_T \cos(\phi_h - \phi_S)\,p_2\,F_{LT}^{\cos(\phi_h-\phi_S)}$$
$$+ S_T \sin(3\phi_h - \phi_S)\,p_1\,F_{UT}^{\sin(3\phi_h-\phi_S)}\Big]$$

beam pol. $\ell(\lambda)$ target pol. $p(S)$

$p_i = p_i(y)$
see Handbook Eq.(2.186)

Factorization

$$F_{UU}(x,z_h,P_{hT},Q^2) = \mathcal{B}\left[\tilde{f}_1^{(0)}\,\tilde{D}_1^{(0)}\right], \quad \leftarrow \text{unpol.}^2$$

$$F_{UU}^{\cos 2\phi_h}(x,z_h,P_{hT},Q^2) = M_N M_h\,\mathcal{B}\left[\tilde{h}_1^{\perp(1)}\,\tilde{H}_1^{\perp(1)}\right], \quad \leftarrow \text{Boer-Mulders} * \text{Collins}$$

$$F_{UL}^{\sin 2\phi_h}(x,z_h,P_{hT},Q^2) = M_N M_h\,\mathcal{B}\left[\tilde{h}_{1L}^{\perp(1)}\,\tilde{H}_1^{\perp(1)}\right], \quad \leftarrow \text{Worm-Gear} * \text{Collins}$$

$$F_{LL}(x,z_h,P_{hT},Q^2) = \mathcal{B}\left[\tilde{g}_1^{(0)}\,\tilde{D}_1^{(0)}\right], \quad \leftarrow \text{Helicity} * \text{Unpol.}$$

$$F_{LT}^{\cos(\phi_h-\phi_S)}(x,z_h,P_{hT},Q^2) = M_N\,\mathcal{B}\left[\tilde{g}_{1T}^{\perp(1)}\,\tilde{D}_1^{(0)}\right], \quad \leftarrow \text{Worm-Gear} * \text{Unpol}$$

$$F_{UT}^{\sin(\phi_h+\phi_S)}(x,z_h,P_{hT},Q^2) = M_h\,\mathcal{B}\left[\tilde{h}_1^{(0)}\,\tilde{H}_1^{\perp(1)}\right], \quad \leftarrow \text{Transversity} * \text{Collins}$$

$$F_{UT}^{\sin(\phi_h-\phi_S)}(x,z_h,P_{hT},Q^2) = -M_N\,\mathcal{B}\left[\tilde{f}_{1T}^{\perp(1)}\,\tilde{D}_1^{(0)}\right], \quad \leftarrow \text{Sivers} * \text{Unpol}$$

$$F_{UT}^{\sin(3\phi_h-\phi_S)}(x,z_h,P_{hT},Q^2) = \frac{M_N^2 M_h}{4}\,\mathcal{B}\left[\tilde{h}_{1T}^{\perp(2)}\,\tilde{H}_1^{\perp(1)}\right], \quad \leftarrow \text{Pretzelosity} * \text{Collins}$$

$$\mathcal{B}[\tilde{f}^{(m)}\,\tilde{D}^{(n)}] \equiv x \sum_i e_i^2 \mathcal{H}_{ii}(Q^2,\mu) \int_0^\infty \frac{db_T}{2\pi}\, b_T\, b_T^{m+n}\, J_{m+n}(q_T b_T)\; \tilde{f}^{(m)}_{i/N}(x,b_T,\mu,\zeta_1)\, \tilde{D}^{(n)}_{h/i}(z_h,b_T,\mu,\zeta_2).$$

Other Processes : $e^+e^- \to h_1 + h_2 + X$ see § 2.11

Implementation

ⓐ Pert. Orders LO, NLO (α_s), NNLO (α_s^2), ...

in H_{ij}, C_{ij}

ⓑ Resummation $\alpha_s \ln\left(\frac{Q}{q_T}\right) \sim \alpha_s \ln(Q b_T) \sim 1$

$$\widetilde{\sigma}^W(\boldsymbol{b}_T) = f_q(x_1) f_{\bar{q}}(x_2) C[\alpha_s] \exp\Big\{ \frac{\alpha_s}{4\pi}\left(d_{12}L_b^2 + d_{11}L_b\right) + \left(\frac{\alpha_s}{4\pi}\right)^2\left(d_{23}L_b^3 + d_{22}L_b^2 + d_{21}L_b\right) + \left(\frac{\alpha_s}{4\pi}\right)^3\left(d_{34}L_b^4 + d_{33}L_b^3 + d_{32}L_b^2 + d_{31}L_b\right)\Big\} + \dots,$$

LL NLL NNLL N^{3}LL

Perturbative $\gamma_\mu^q(\mu,\zeta)$

Pert. + Non-Pert. $\gamma_\zeta^q(\mu,b_T)$

ⓒ Combine Pert. & Non-Pert

($k_T \sim b_T^{-1} \gg \Lambda_{QCD}$) ($k_T \sim b_T^{-1} \sim \Lambda_{QCD}$)

fit to TMD data

$$f_{i/p}(x,b_T,\mu,\zeta) = \underbrace{f^{\text{pert}}_{i/p}(x,b^*(b_T),\mu,\zeta)}\; f_i^{NP*}(x,b_T)$$

$$\sum_j \int \frac{d\xi}{\xi}\, C_{ij}\left(\frac{x}{\xi}, b_T,\mu,\zeta\right) f_j(\xi,\mu)$$

pert. Global fits

- $b^*(b_T)$ shields pert. from Landau Pole $\alpha_s(\Lambda_{QCD}) = \infty$
- f_i^{NP*}'s meaning depends on choice of b^*

(d) Y term (eg. DY)

$$\frac{d\sigma}{dQ\,dY\,dq_T} = \frac{d\sigma^W}{dQ\,dY\,dq_T} + \frac{d\sigma^Y}{dQ\,dY\,dq_T}$$

Factorized, dominant $q_T \ll Q$

$\mathcal{O}(q_T^2/Q^2) + \cdots$ important when $q_T \sim Q$

Global Fits

SV19 = Scimemi, Vladimirov (1912.06532)

Pavia19 = Bachetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici (1912.07550)

Common features:

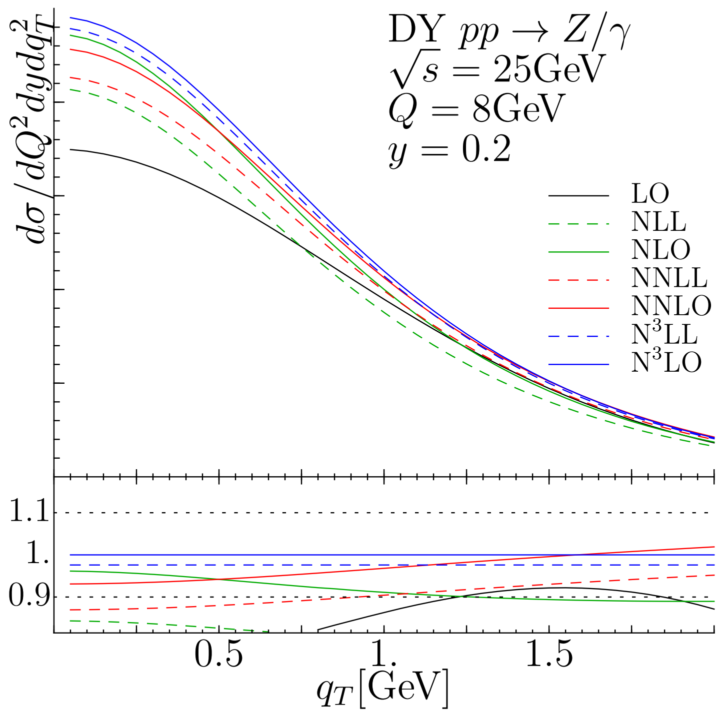
- Unpolarized Data with constraint: $q_T/Q < 0.2 - 0.25$ (4-6% power corrections)
- Perturbative accuracy: N3LL resummation + NNLO matching to PDF
- Neglect small contributions from Boer-Mulders terms (higher twist for pert. b_T)
- Common b_T dependence for all flavors

Global Fits

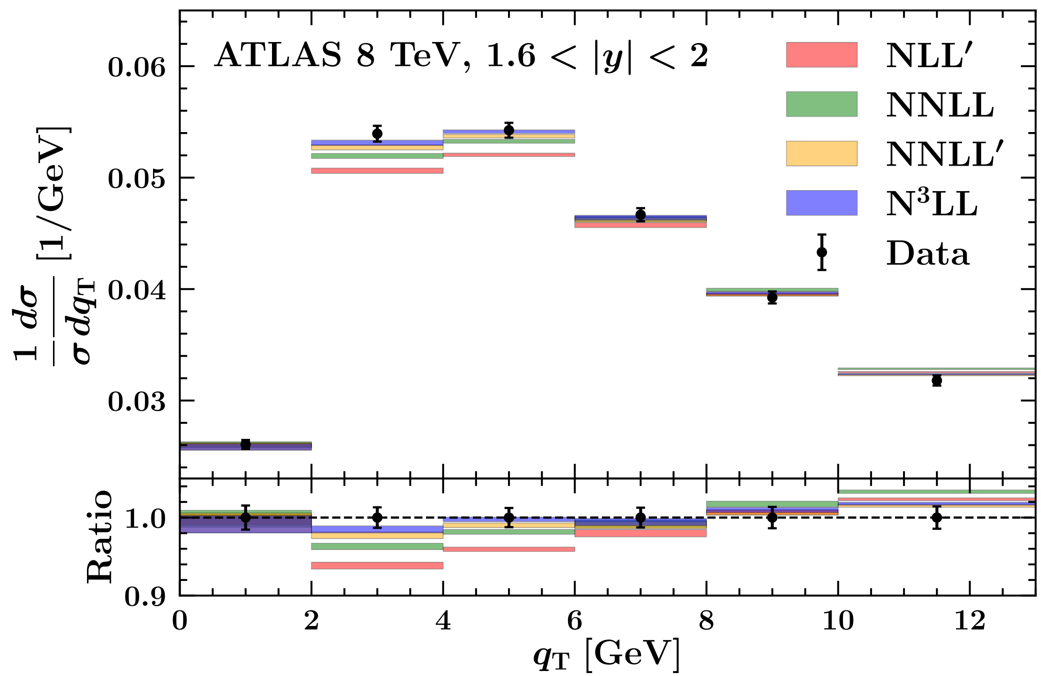
Common features:

Good Perturbative convergence:

SV19



Pavia19



Global Fits

Differences:

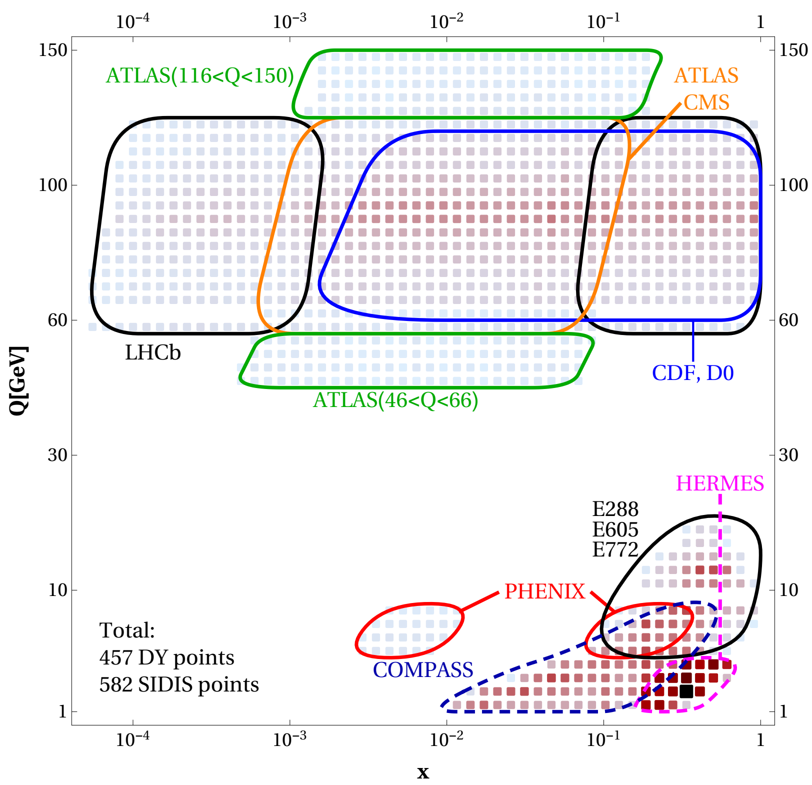
Some differences in solution of evolution equations

Datasets used

SV19

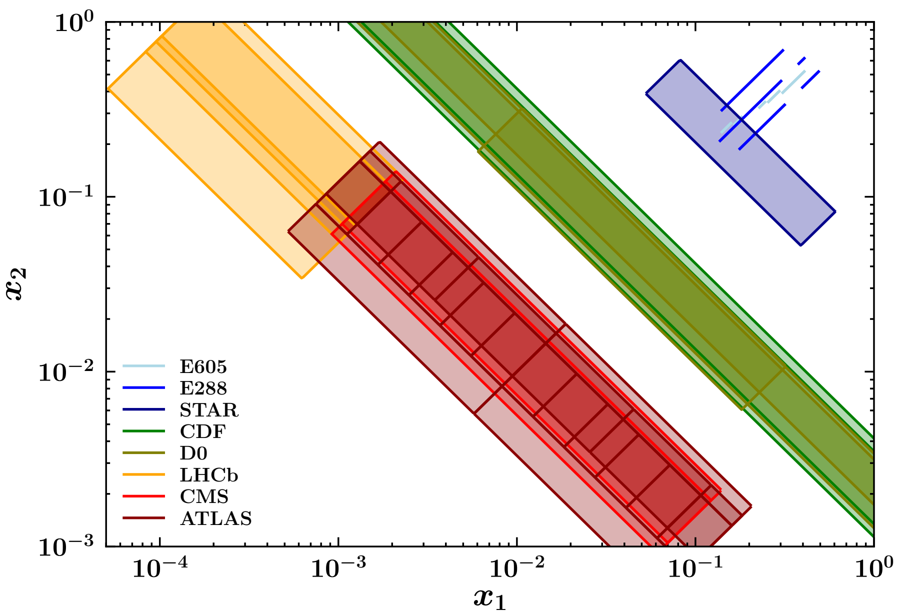
Drell-Yan (457 bins)

SIDIS (582 bins)



Pavia19

Drell-Yan (353 bins)



$$x_1 = Qe^{y}/\sqrt{s}, \quad x_2 = Qe^{-y}/\sqrt{s}$$

Global Fits

Differences:

Non-perturbative Models

SV19

TMDPDF: 5
TMDFF: 4
CS kernel: 2

$$f_{NP}(x,b) = \exp\left(-\frac{\lambda_1(1-x)+\lambda_2 x + x(1-x)\lambda_5}{\sqrt{1+\lambda_3 x^{\lambda_4}\boldsymbol{b}^2}}\boldsymbol{b}^2\right)$$

$$D_{NP}(x,b) = \exp\left(-\frac{\eta_1 z + \eta_2(1-z)}{\sqrt{1+\eta_3(\boldsymbol{b}/z)^2}}\frac{\boldsymbol{b}^2}{z^2}\right)\left(1+\eta_4\frac{\boldsymbol{b}^2}{z^2}\right)$$

$$\gamma_\zeta^q(\mu,b) = \gamma_\zeta^{q\,\mathrm{pert}}(\mu,b^*) - \frac{1}{2}c_0 b b^*$$

$$b^*(b) = \frac{b}{\sqrt{1+\boldsymbol{b}^2/B_{\mathrm{NP}}^2}}$$

Pavia19

TMDPDF: 7
CS kernel: 2

$$f_{\mathrm{NP}}(x,b_T) = \left[\frac{1-\lambda}{1+g_1(x)\frac{b_T^2}{4}} + \lambda\exp\left(-g_{1B}(x)\frac{b_T^2}{4}\right)\right]$$

$$g_1(x) = \frac{N_1}{x\sigma}\exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{x}{\alpha}\right)\right]$$

$$g_{1B}(x) = \frac{N_{1B}}{x\sigma_B}\exp\left[-\frac{1}{2\sigma_B^2}\ln^2\left(\frac{x}{\alpha_B}\right)\right]$$

$$\gamma_\zeta^q(\mu,b) = \gamma_\zeta^{q\,\mathrm{pert}}(\mu,b_*) - \frac{1}{2}\left(g_2 b_T^2 + g_{2B} b_T^4\right)$$

$$b_*(b_T) = b_{\mathrm{max}}\left(\frac{1-\exp\left(-\frac{b_T^4}{b_{\mathrm{max}}^4}\right)}{1-\exp\left(-\frac{b_T^4}{b_{\mathrm{min}}^4}\right)}\right)^{\frac{1}{4}}$$

Note: model form for b* used to split perturbative & non-perturbative parts

Global Fits

Fit Results:

SV19

$$\chi^2/N_{pt} = 1.06$$

NP-parameters			
RAD	$B_{\rm NP} = 1.93 \pm 0.22$	$c_0 = (4.27 \pm 1.05) \times 10^{-2}$	
TMDPDF	$\lambda_1 = 0.224 \pm 0.029$	$\lambda_2 = 9.24 \pm 0.46$	$\lambda_3 = 375. \pm 89.$
	$\lambda_4 = 2.15 \pm 0.19$	$\lambda_5 = -4.97 \pm 1.37$	
TMDFF	$\eta_1 = 0.233 \pm 0.018$	$\eta_2 = 0.479 \pm 0.025$	
	$\eta_3 = 0.472 \pm 0.041$	$\eta_4 = 0.511 \pm 0.040$	

Pavia19

$$\chi^2/N_{pt} = 1.02$$

Parameter	Value
g_2	0.036 ± 0.009
N_1	0.625 ± 0.282
α	0.205 ± 0.010
σ	0.370 ± 0.063
λ	0.580 ± 0.092
N_{1B}	0.044 ± 0.012
α_B	0.069 ± 0.009
σ_B	0.356 ± 0.075
g_{2B}	0.012 ± 0.003

Low and High energy data are well described

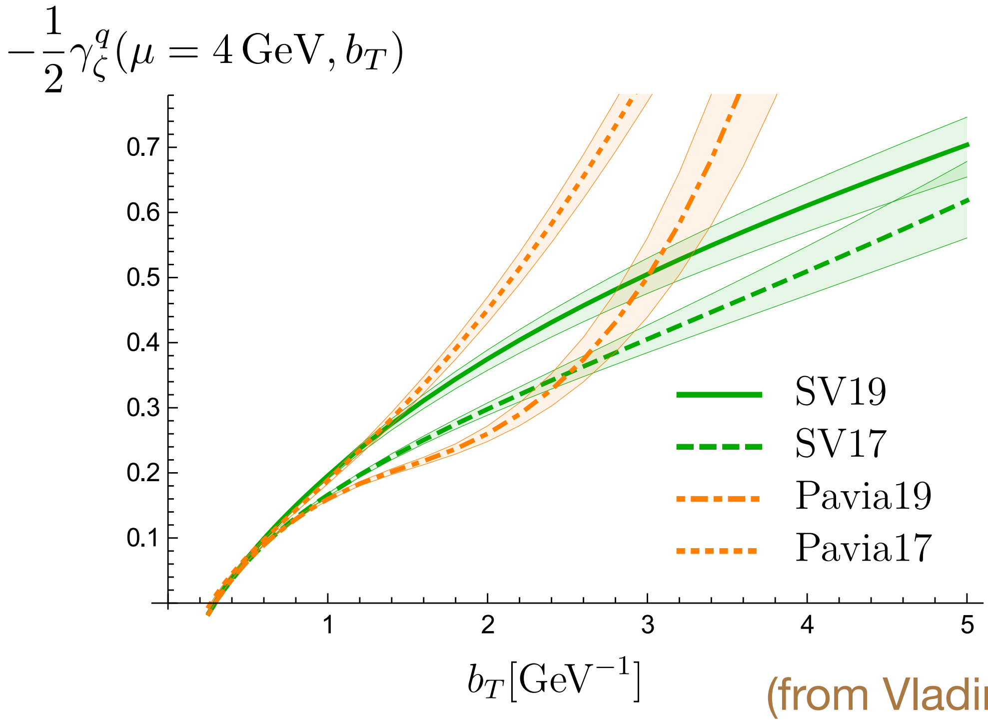
RAD parameters are less sensitive to input PDF set

Universality of RAD satisfied by DY vs. SIDIS data

Global Fits

Fit Results:

Comparison of results for CS Kernel in non-perturbative regime:



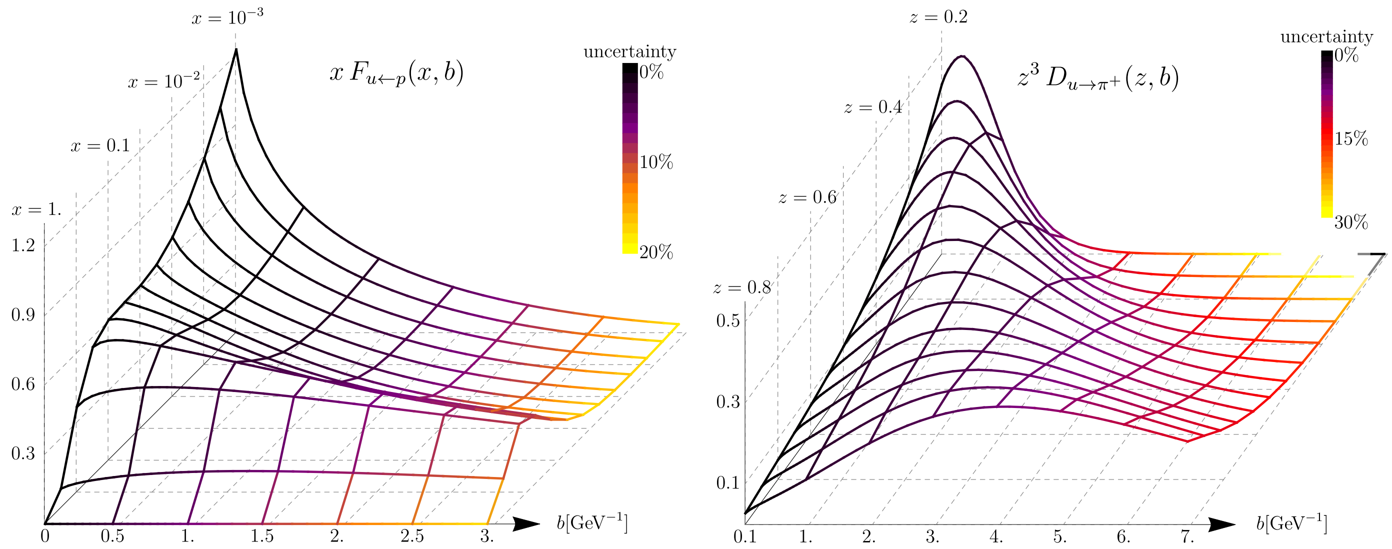
(from Vladimirov, 2003.02288)

Global Fits

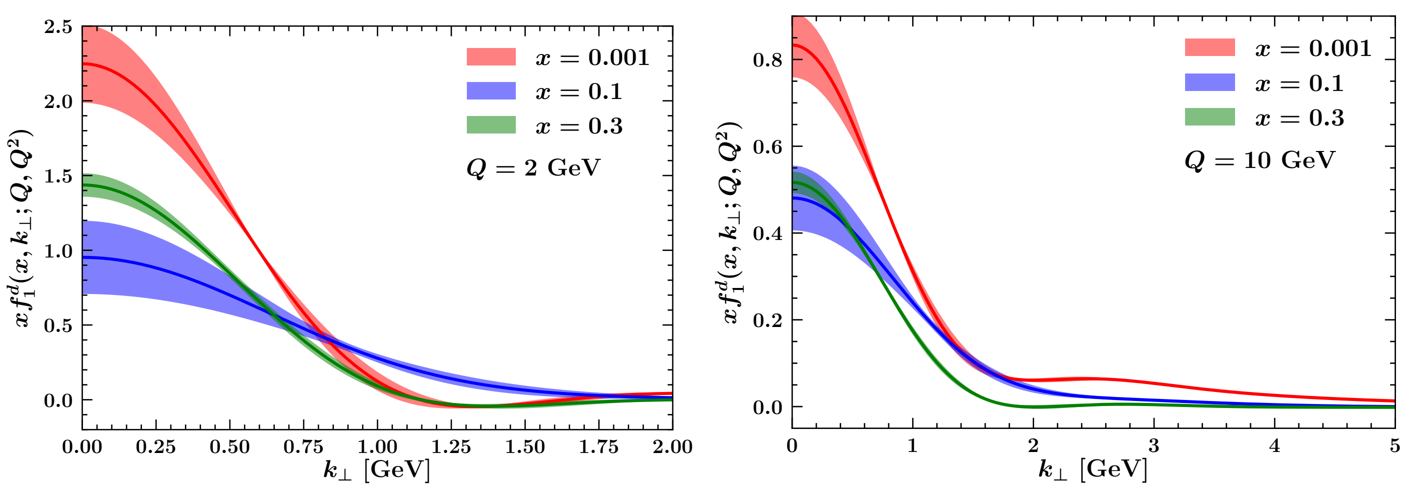
Fit Results:

Results for intrinsic TMDPDF (& TMDFF)

SV19



Pavia19



Quite precise determinations if we assume a given fit form.

Global Fits

Bury, Prokudin, Vladimirov (2012.05135)

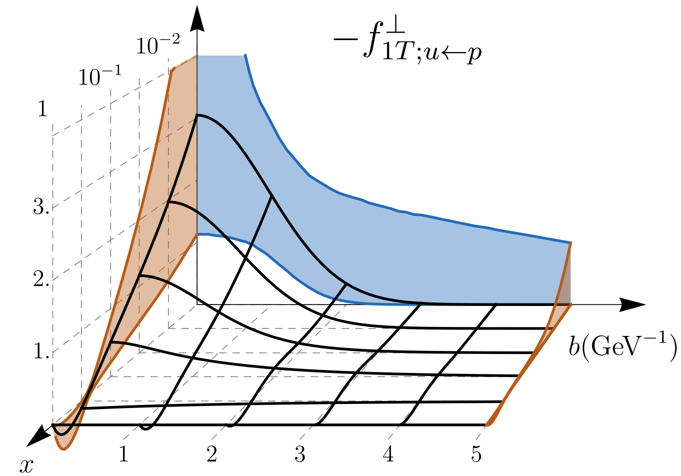
Extraction of Sivers function from global fit to SIDIS, DY, and W/Z data [76 bins: HERMES, COMPASS, Jlab (SIDIS); STAR(W/Z); COMPASS (DY)]

$$f_{1T}^{\perp\,\text{SIDIS}} = -f_{1T}^{\perp\,\text{DY}}$$

N3LL analysis following SV19

Flavor dependent parametrization (no matching)

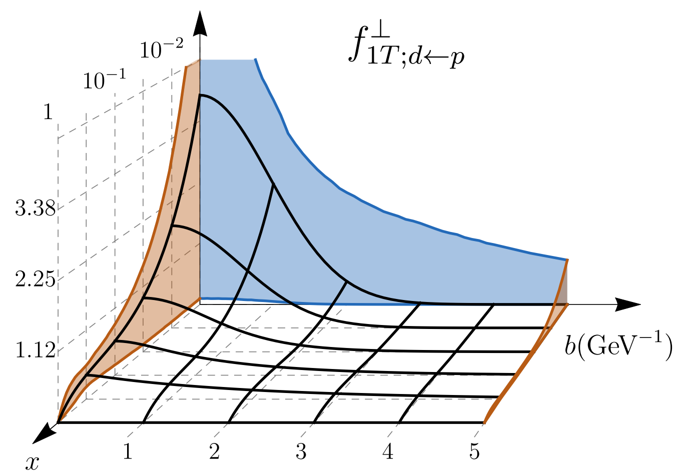
$$f_{1T;q\leftarrow h}^{\perp}(x,b) \quad = N_q \frac{(1-x)x^{\beta_q}(1+\epsilon_q x)}{n(\beta_q,\epsilon_q)}\ \exp\left(-\frac{r_0+x r_1}{\sqrt{1+r_2 x^2 b^2}} b^2\right)$$



Results:

Good global fit: $\chi^2/N_{pt} = 0.88$

Opposite signs for up and down Sivers functions



Data not precise enough to confirm sign flip

$f_{1T}^{\perp\,\text{SIDIS}} = +f_{1T}^{\perp\,\text{DY}}$ **gives** $\chi^2/N_{pt} = 1.0$

The End