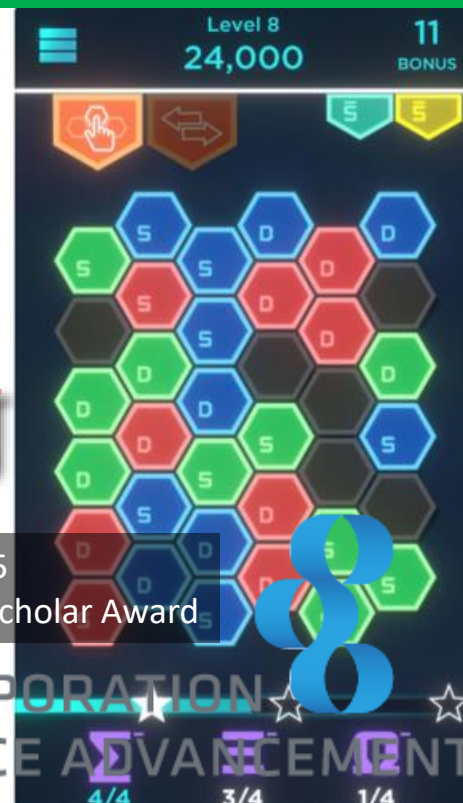
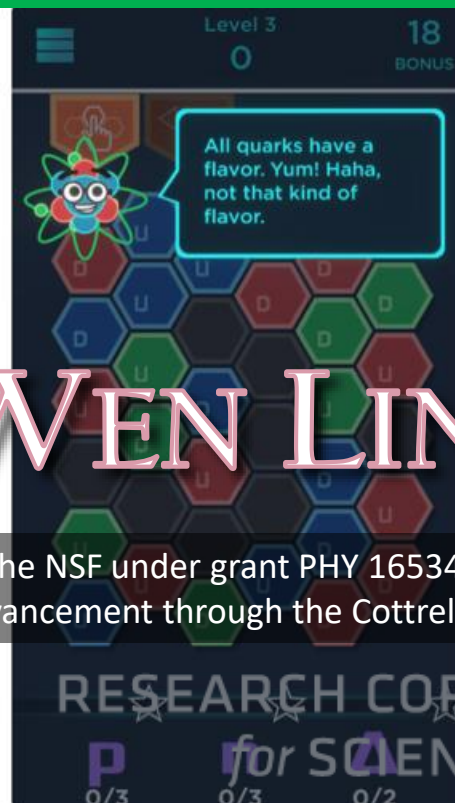
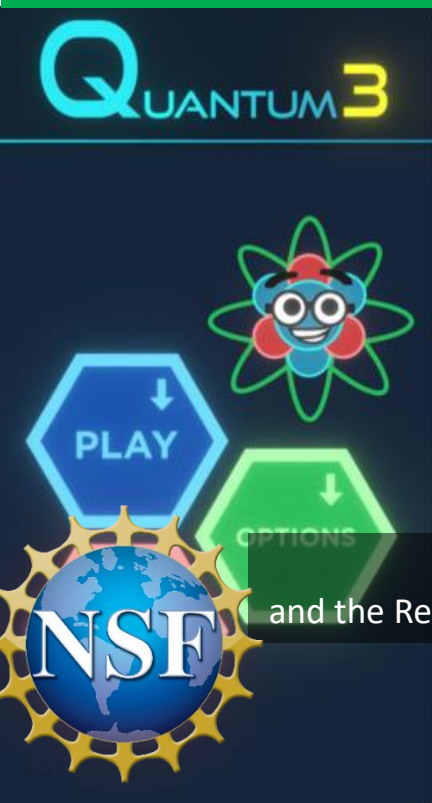
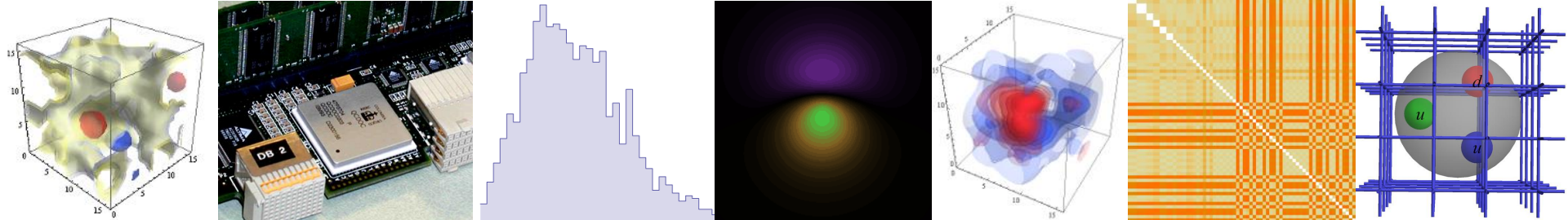





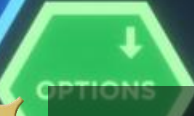

# Lattice QCD for EIC Physics

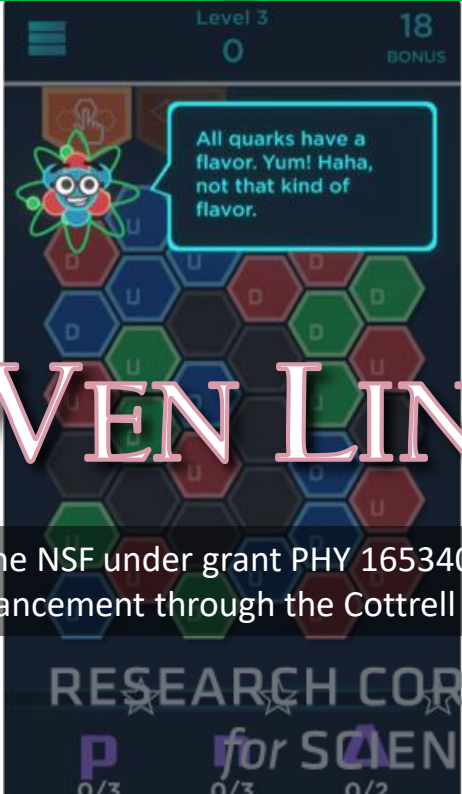
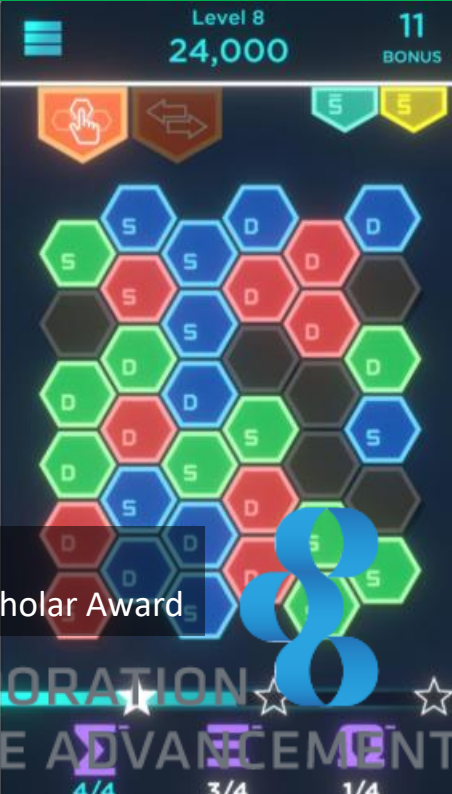


This work of HL is supported by the NSF under grant PHY 1653405 and the Research Corporation for Science Advancement through the Cottrell Scholar Award




# Probing the Heart of Matter with Supercomputers



**HUEY-WEN LIN**  
 This work of HL is supported by the NSF under grant PHY 1653405  
 and the Research Corporation for Science Advancement through the Cottrell Scholar Award



PROGRESS BAR: ☆ ☆ ☆

PROGRESS BAR: ☆ ☆ ☆

PROGRESS BAR: ☆ ☆ ☆

PROGRESS BAR: ☆ ☆ ☆

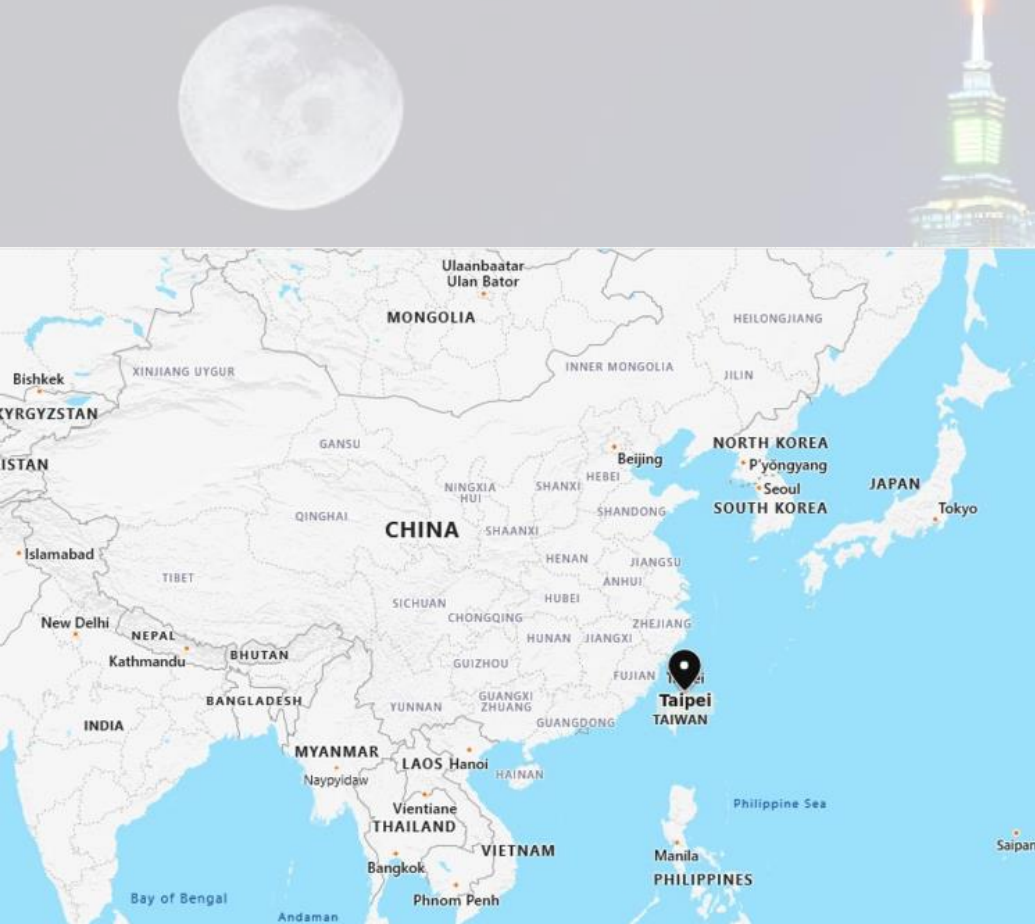
PROGRESS BAR: ☆ ☆ ☆

PROGRESS BAR: ☆ ☆ ☆



# *Few Personal Facts*

§ I am from a small town in Taiwan; PhD at Columbia U



# Few Personal Facts

- § I am from a small town in Taiwan; PhD at Columbia U
- § Preferred pronoun: she/her
- § Since NYC, I've lived in Virginia, Seattle, and the Bay Area
  - ∞ My biggest worry when I moved to the Midwest: snowstorms
- § Like many women in physics, I married a physics PhD, and often find myself the only female in the room
  - ∞ I started a number of diversity-related activities (social events, surveys, codes of conduct, etc.)





# Few Personal Facts

§ I am from a small town in Taiwan; PhD at Columbia U

§ Preferred pronoun: she/her

§ Since NYC, I've lived

∞ My biggest worry when

§ Like many women in physics,  
often find myself the

∞ I started a number of  
(social events, surveys)



§ My research is in **quantum chromodynamics (QCD)**

∞ I use high-performance supercomputers to study  
the properties of the quarks and gluons in the hadrons  
(nucleon, pion, ...)

# My Journey as a Physicist

## **Season 3:** Nuclear-Science Advisory Committee Long Range Plan (LRP)

This season's interviews were conducted by Bill Good and Kinza Hasan, and edited by Kiran Sakorikar and Esther Cohen-Lin.

**Episode 1** ([flyer](#)) ([transcript](#))

**Prof. Gail Dodge** (she/her)

Old Dominion University



### Prof. Gail Dodge (she/her): Season 3 Episode 1

My Journey as a Physicist • By [Bryan Stanley](#) & [Huey-Wen Lin](#) • Oct 20, 2022



00:00



Share

28:01



# *Probing the Heart of Matter*

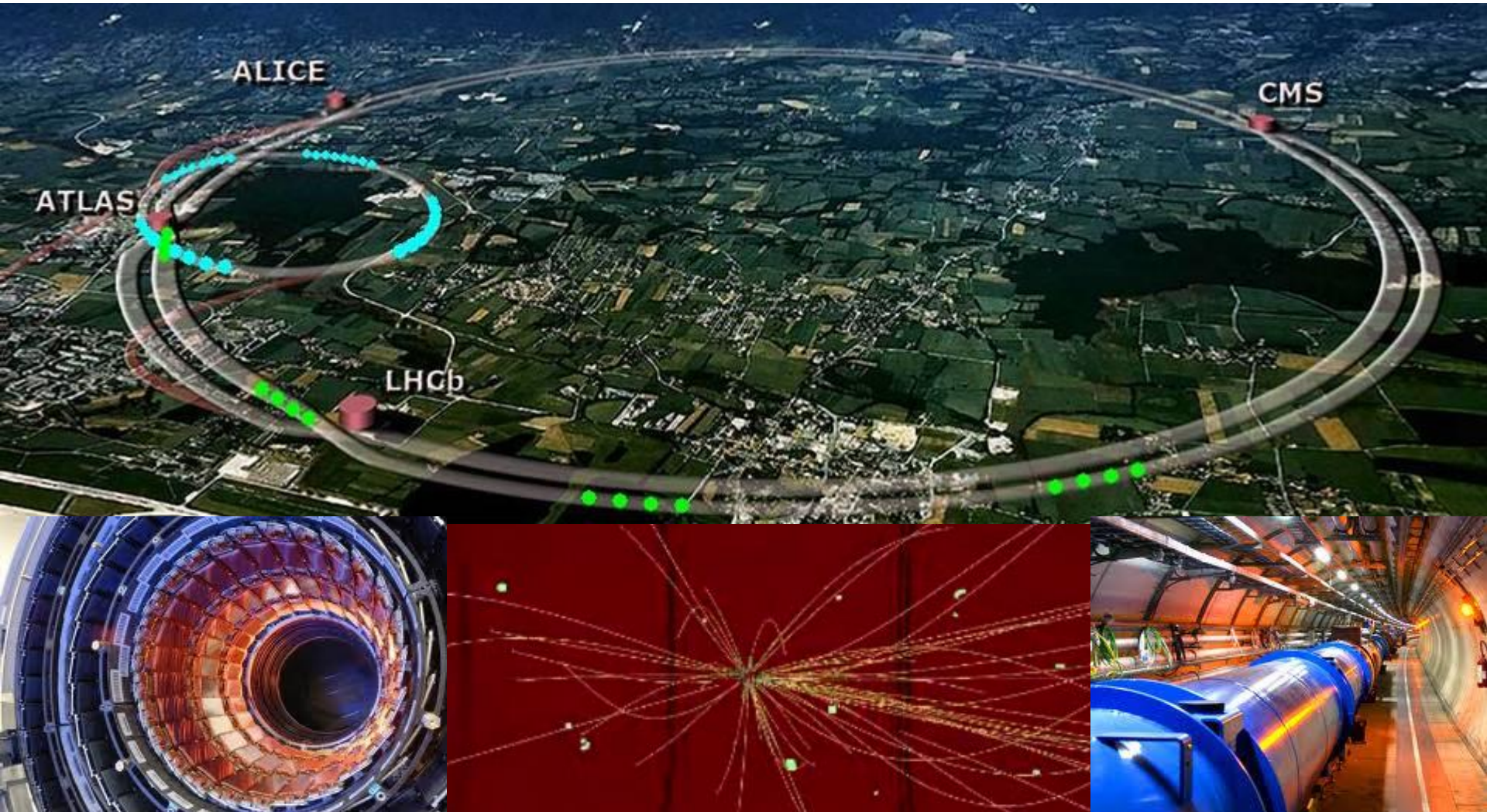
§ Our natural instinct: Smash!





# *Probing the Heart of Matter*

§ LHC strikes out onto the high-energy frontier (13 TeV)



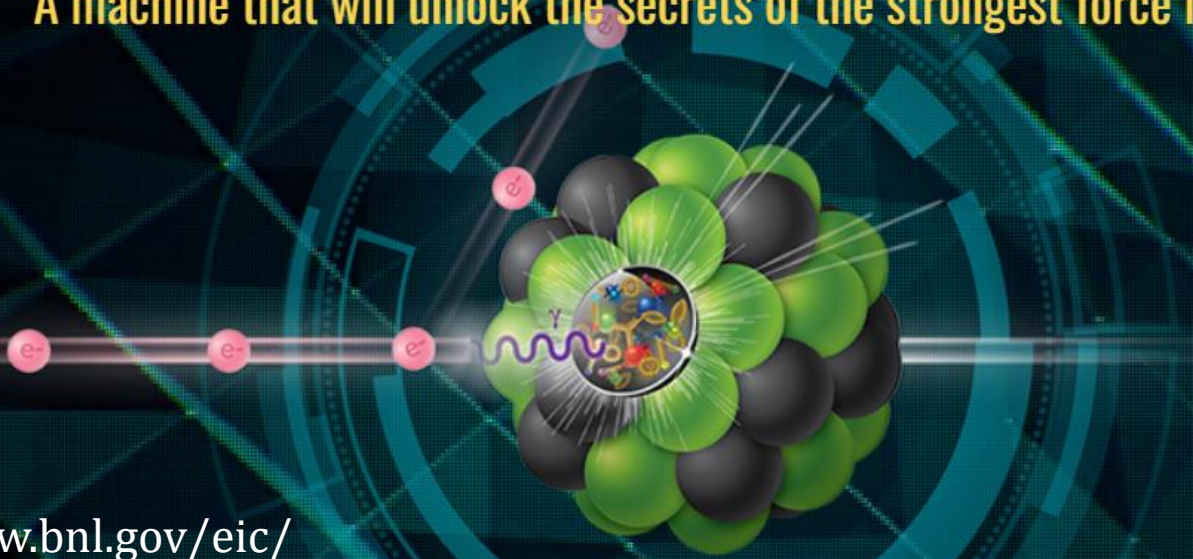


# *Probing the Heart of Matter*

## § The Electron-Ion Collider (EIC): The Ultimate QCD Microscope

### The Electron-Ion Collider

A machine that will unlock the secrets of the strongest force in Nature



<https://www.bnl.gov/eic/>

EIC White Paper, 1212.1701; EIC Yellow Report (2103.05419);  
[The Present and Future of QCD](#) (2303.02579)

# Probing the Heart of Matter

§ Calculations done using world's largest supercomputers

↻ Many millions of CPU/GPU hours





# Outline

## § Lecture 1

- Why lattice QCD?
- Anatomy of a Lattice Calculation
- Examples of Common Lattice Calculations
  - Spectroscopy
  - Moments and form factors
  - Proton spin/mass decomposition

## § Lecture 2

- x-dependent Parton Distributions
  - Recent Lattice PDFs Progress
  - Applications to Generalized Parton Distributions
  - Future Prospects and Challenges



# *From Thy Standard Model to Lattice QCD*





# The Standard Model and QCD

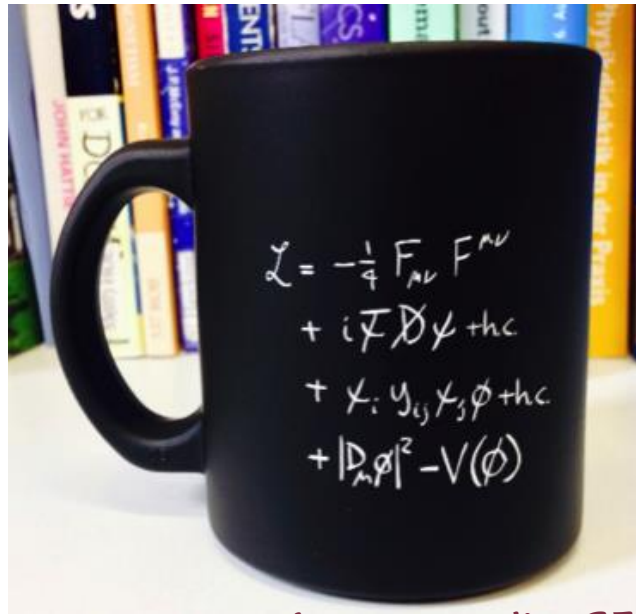


Image credit: CERN

& Quantum chromodynamics (QCD)

↻ The strong interactions of quarks and gluons (SU(3) gauge)



Image source: <https://www.particlezoo.net>

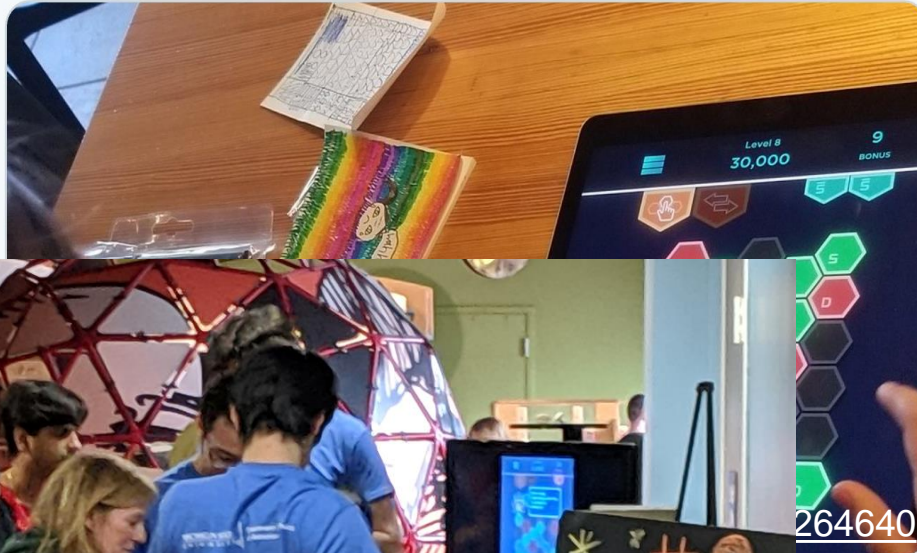
# Learn QCD on Your Phone!



**Chris Oakley** @DrPhysOaks · Mar 21

Replying to @NSF\_MPS and @michiganstateu

...and my seven year old is explaining to me how to create Xi - ...



<https://msutoday.msu.edu/news/2019/new-game-app-gets-young-girls-involved-in-stem/>



# Learn QCD on Your Phone!

§ Learn QCD on your phone

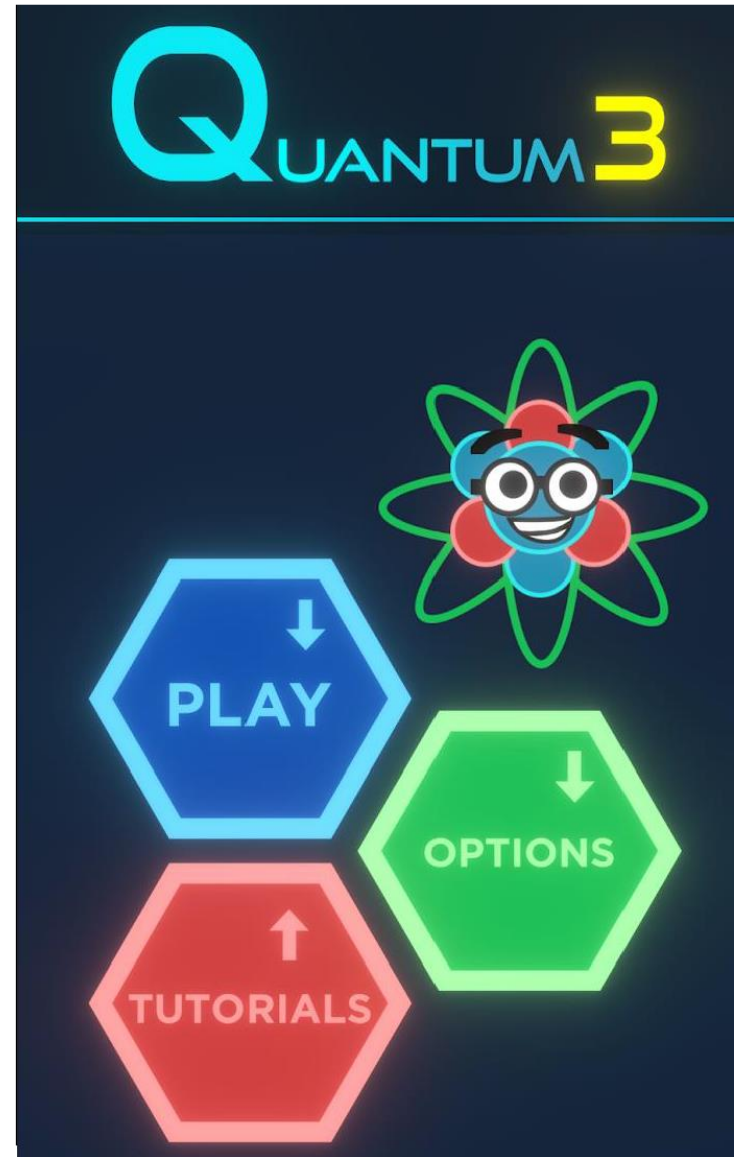
[Google Play Store](#)



[Apple Appstore](#)



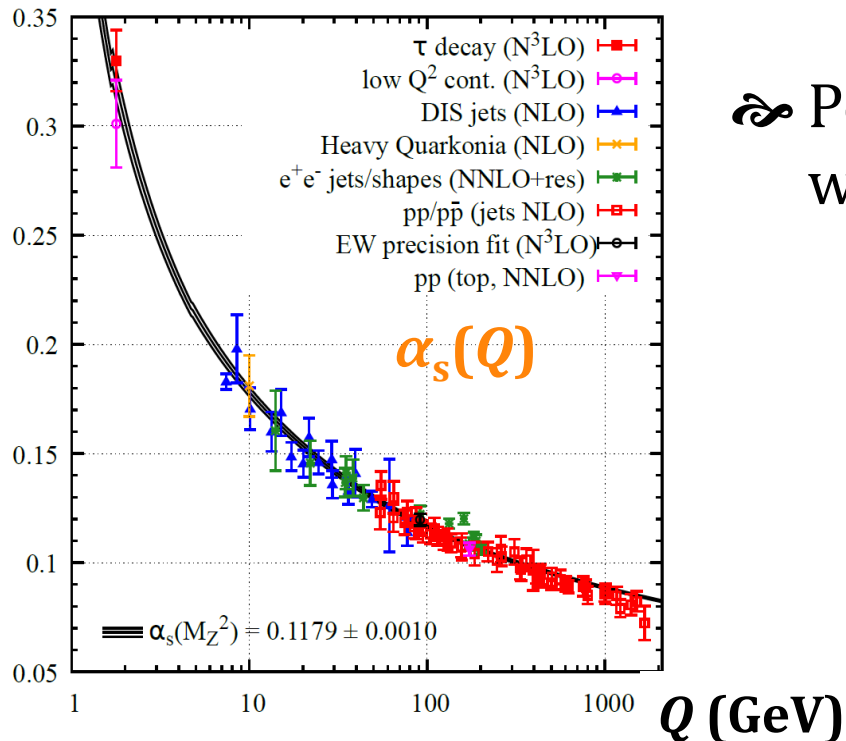
Supported by the NSF under grant PHY 1653405



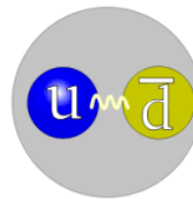
# The Color Force

§ Quantum chromodynamics (QCD) describes strong interactions of *quarks* and *gluons* (SU(3) gauge)

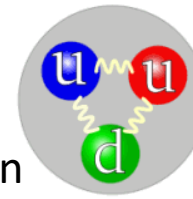
∞ “Confinement”  
no free quarks allowed



**Hadrons**



Pion



Proton



∞ Perturbation theory (like QED)  
works well at high energies

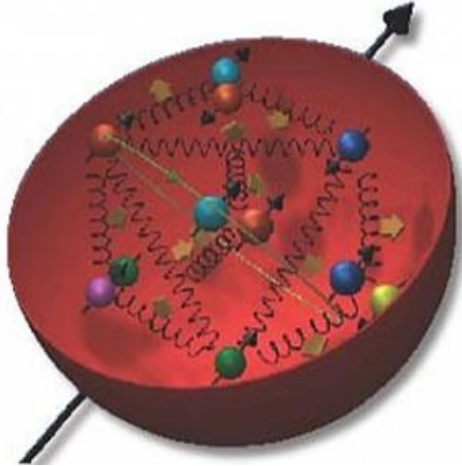


**The Nobel Prize in Physics  
2004**



# Wide-Scale Applications

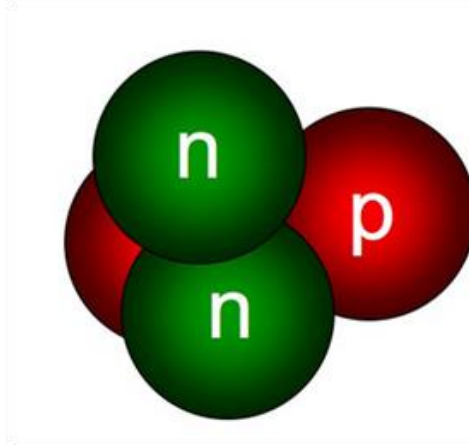
## § What can we learn from it?



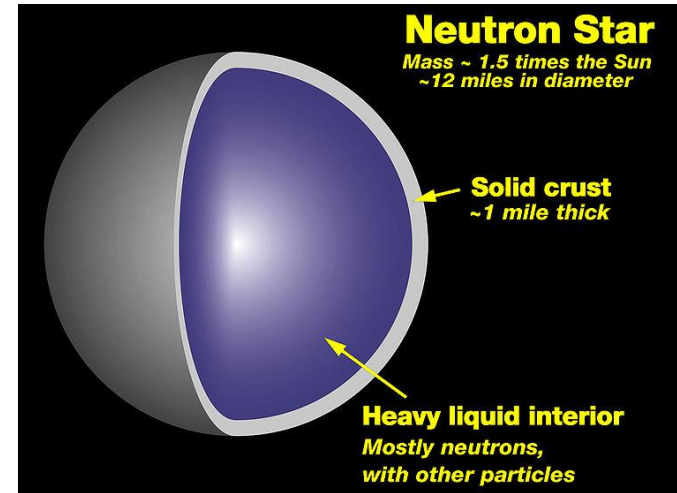
$10^{-15}$  m



Origin of proton mass and spin, imaging of proton, Parton distribution functions For new-physics searches, ...



Nuclei and why we exist



$10^4$  m



Neutron matter How they evolve

## § HOWEVER...

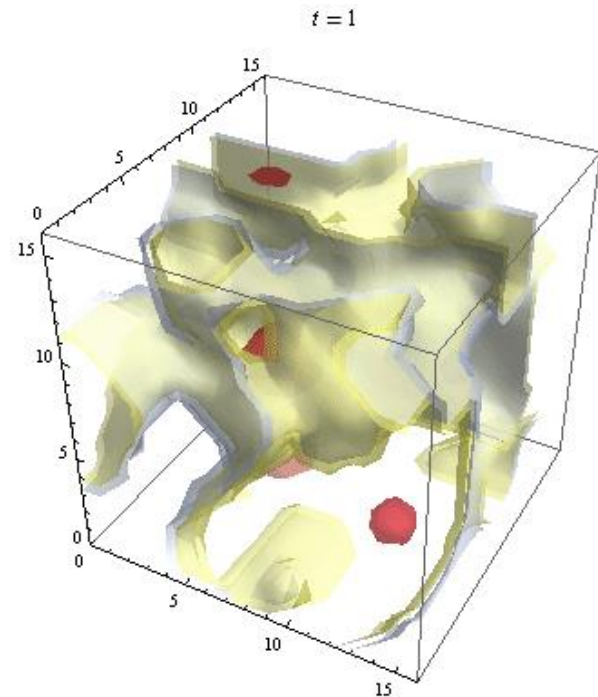
# *Difficulties at Low Energy*

§ Even just the vacuum of QCD is complicated

Classical



QCD

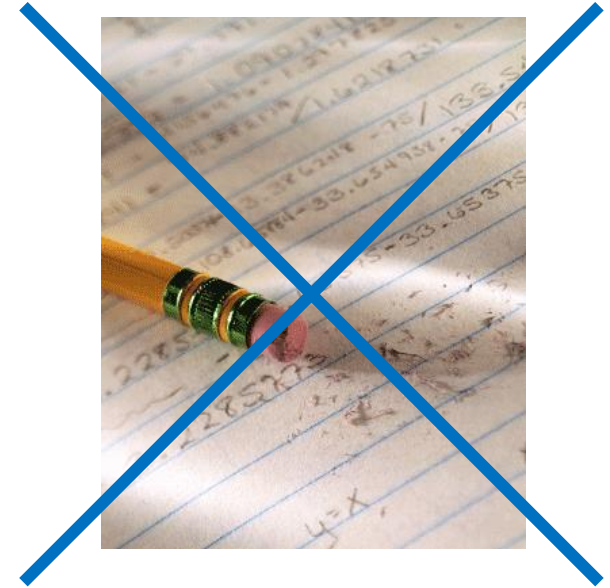




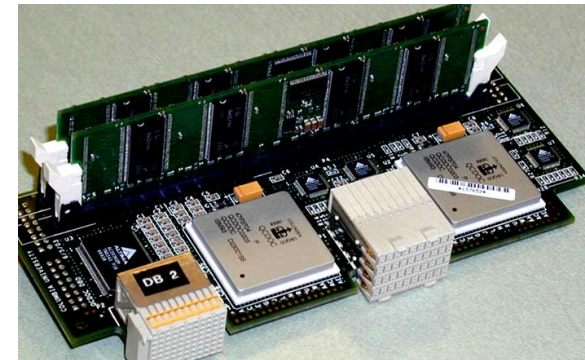
# Difficulties at Low Energy

§ Strong interactions make analytic calculation impossible

§ Direct QCD calculation is desired  
→ Lattice QCD

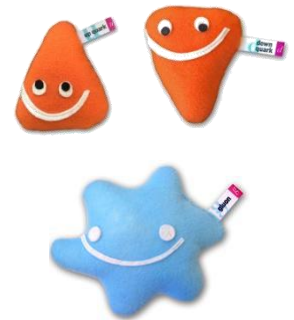
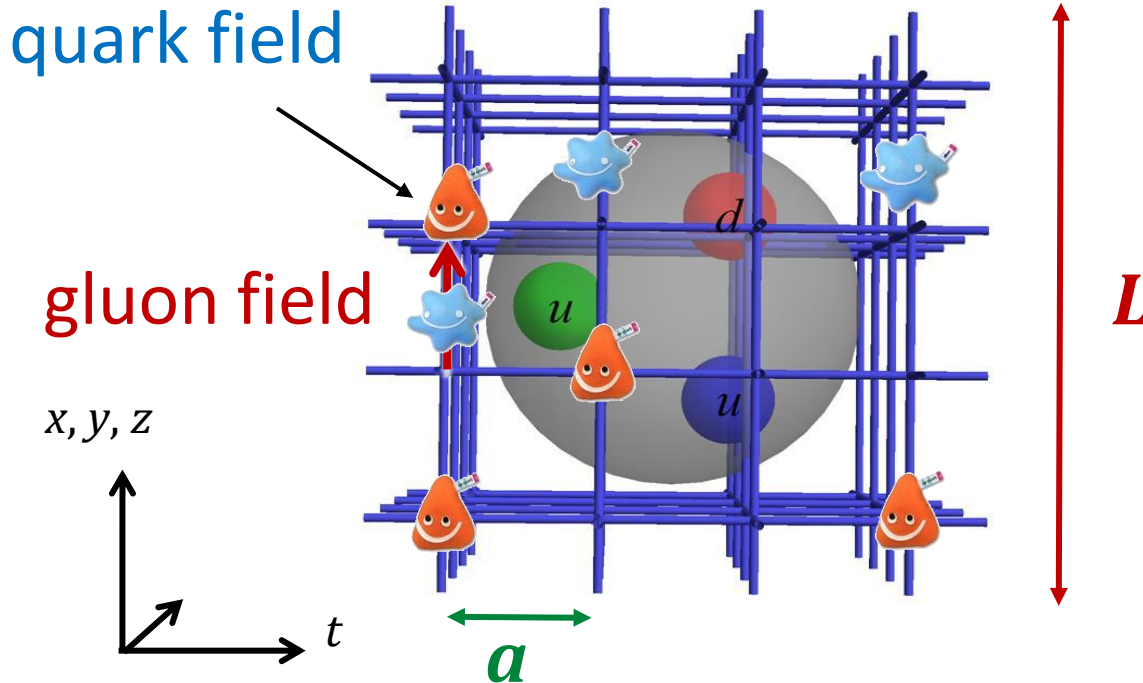


→ 0100101010  
10111010... →



# Wilson Comes to Rescue!

§ Lattice gauge theory was proposed in the 1970s by Wilson



§ Recover physical limit

$$m_q \rightarrow m_q^{\text{phys}}, \quad a \rightarrow 0, \quad L \rightarrow \infty$$



# Introducing the Lattice

- § Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories
- § Physical observables are calculated from the path integral

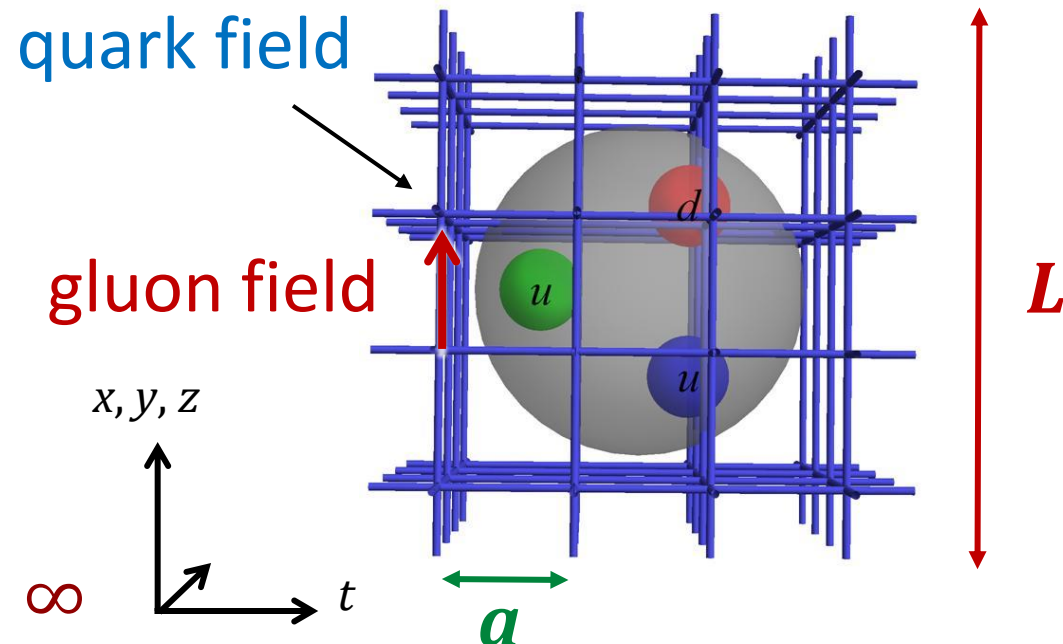
$$\langle 0 | O(\bar{\psi}, \psi, A) | 0 \rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS(\bar{\psi}, \psi, A)} O(\bar{\psi}, \psi, A)$$

in **Euclidian** space

- ∞ Quark mass parameter (described by  $m_\pi$ )
- ∞ Impose a UV cutoff  
discretize spacetime
- ∞ Impose an infrared cutoff  
finite volume

§ Recover physical limit

$$m_\pi \rightarrow m_\pi^{\text{phys}}, a \rightarrow 0, L \rightarrow \infty$$



# Are We There Yet?

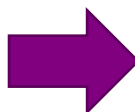
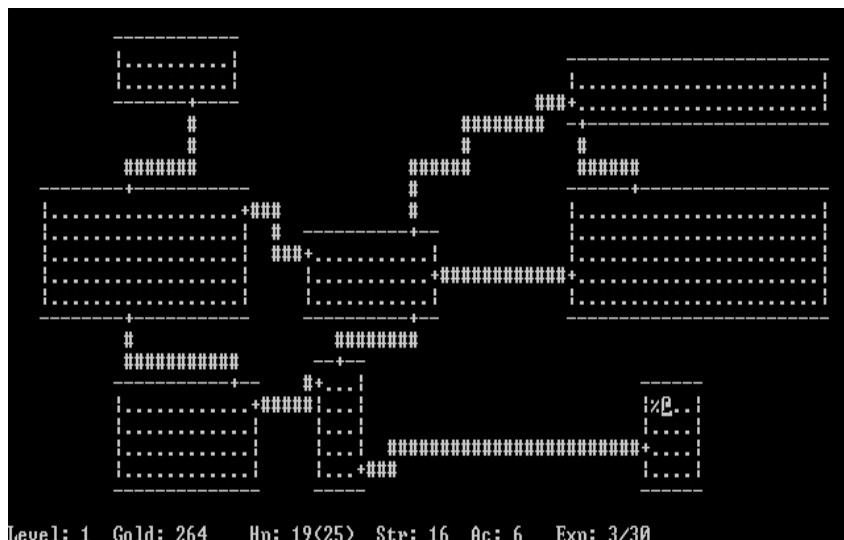
§ Lattice gauge theory was proposed in the 1970s by Wilson

☿ Why haven't we solved QCD yet?

## § Progress is limited by computational resources

## 1980s

# Today



## § Greatly assisted by advances in algorithms

☞ Physical pion-mass ensembles are not uncommon!



# Anatomy of a Lattice Calculation

## 1. Hardware (computational resources) and Software (Code)

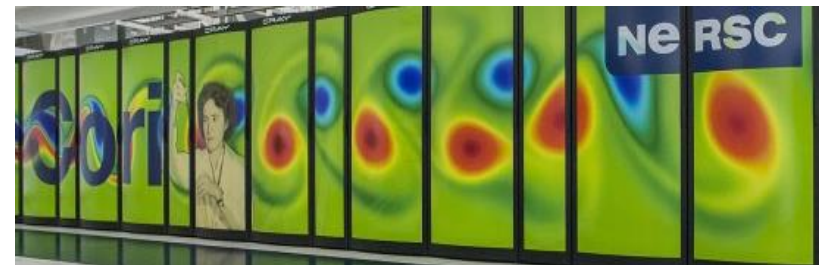
If you are young and energetic...



**FNAL pi0 Cluster  
(USQCD SciDAC)**



**Stampede2@TACC (NSF ACCESS)**



**Cori@NERSC (DOE ERCAP, ALCC)**

# Anatomy of a Lattice Calculation

## 1. Hardware (computational resources) and Software (Code)

### US Lattice Quantum Chromodynamics

USQCD has developed a suite of software enabling lattice QCD computations to be performed with high performance across a variety of architectures, including both custom facilities and commodity clusters. This software is made up of software library modules that can be re-used by higher level applications. The approximate organisation of the packages into layers is depicted below (we omit here 3rd party packages or packages auxiliary to another package). Please click on the plaquettes below to find project web pages of the individual software modules, as well as to complete lattice QCD packages which use them.

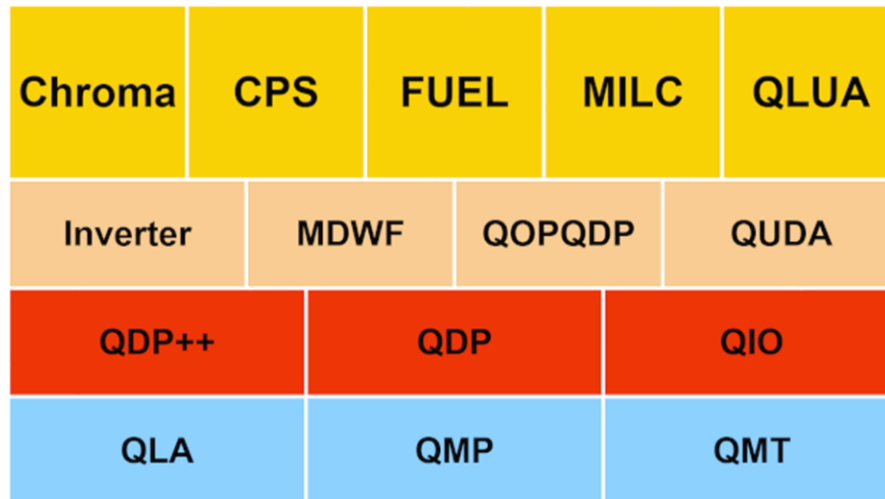


Figure 1: The SciDAC Layers and the software module architecture.

§ The lattice community shares much software

↪ for example, many in the US lattice community use code from

<http://usqcd-software.github.io>



# Anatomy of a Lattice Calculation

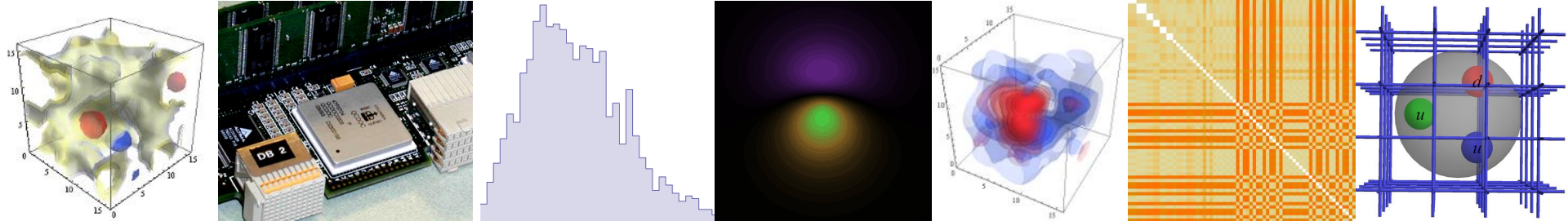
## 1. Hardware (computational resources) and Software (Code)

Online tutorials available:

<http://www.int.washington.edu/PROGRAMS/12-2c/>

### Week 3 (Aug. 20–24, 2012)

- ▶ "Hadron Structure", James Zanotti (University of Adelaide)
- ▶ "Lattice QCD+QED", Taku Izubuchi (BNL)
- ▼ "Computational Lattice QCD", Balint Joo (Thomas Jefferson Lab)
  - Exercises: [seattle\\_tut.tar.gz](#) (2012 Aug 24)
  - Code: [package-int.tar.gz](#) (110 MB)
  - Lecture 1: [Slides](#) [Video](#)
  - Lecture 2: [Slides](#) [Video](#)
  - Lecture 3: [Slides](#) [Video](#)
  - Lecture 4: [Slides](#) [Video](#)
  - Lecture 5: [Slides](#) [Video](#)
- ▶ "Extreme Computing Trilogy: Nuclear Physics", [Martin Savage](#) (University of Washington)
- ▶ "Cold Atoms and Unitary Fermi Gas", [Michael M. Forbes](#) (INT)
- ▶ "Introduction to GPU Computing", Mike Clark (NVIDIA)
- ▶ "Introduction to QUDA — GPU Computing for LQCD", Mike Clark (NVIDIA)
- ▶ "Extreme Computing Trilogy: Infrastructure", Kenneth Roche (PNNL)



# Tutorials @ INT Summer School on Problem Solving in Lattice QCD

Lecture playlist:

<https://www.youtube.com/watch?v=FsN34Sm6Ldo&list=PLDi14w7i5C3Bm3U1IQ4n596UZQh0pr1Cx>



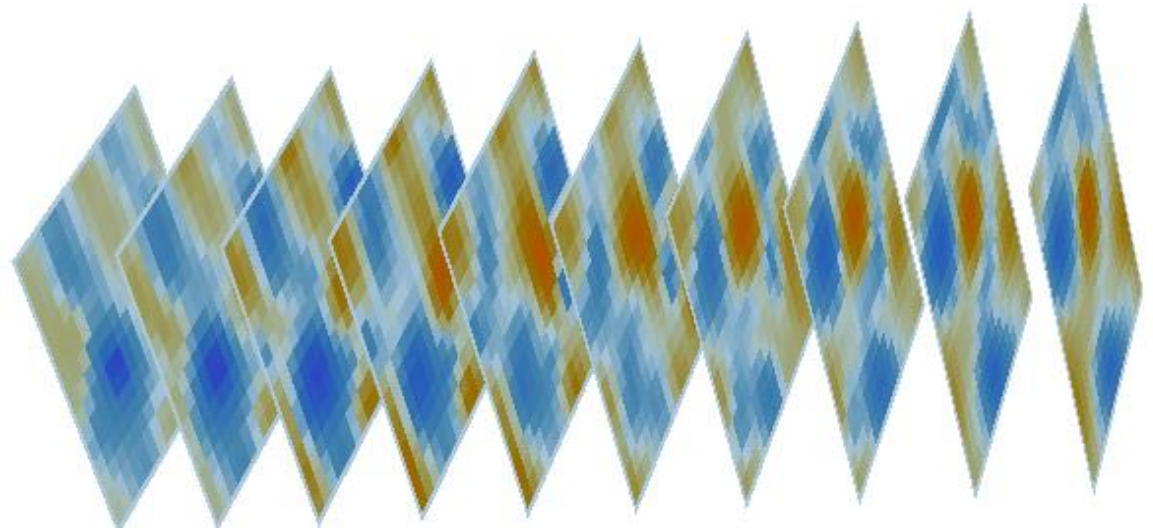
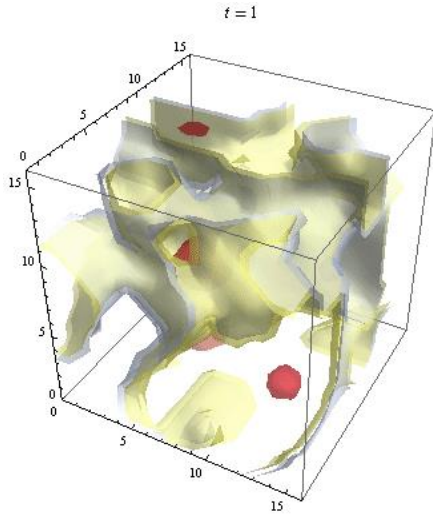
 **Anchor**  
by Spotify





# Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)
2. Some QCD Vacuum (gauge configurations)



# Anatomy of a Lattice Calculation

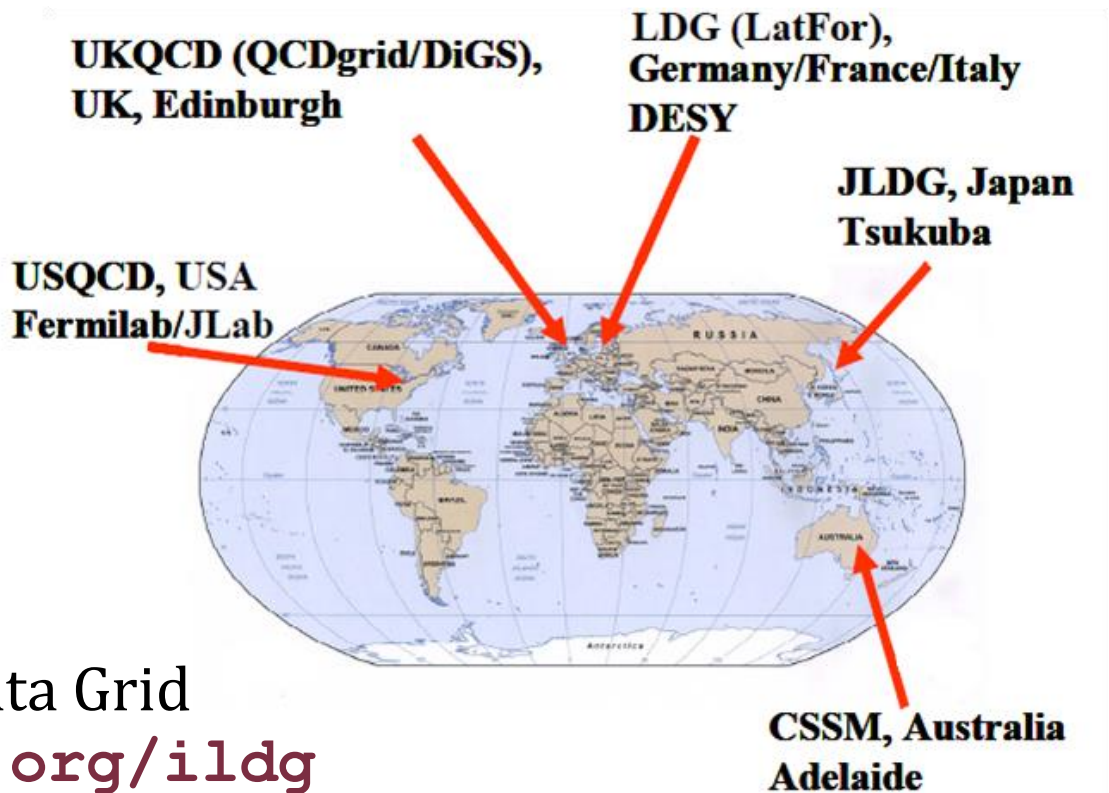
1. Hardware (computational resources) and Software (Code)
2. Some QCD Vacuum (gauge configurations)

↻ WWW (ILDG)



International Lattice Data Grid

<http://www.usqcd.org/ildg>

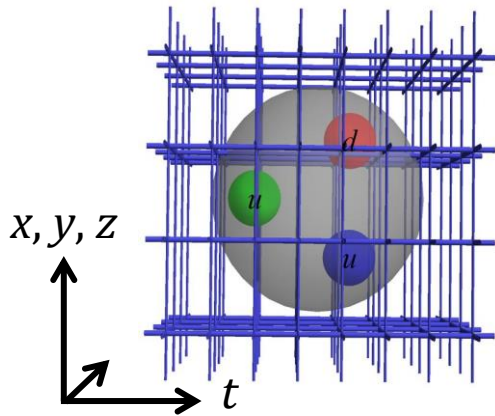


# Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)

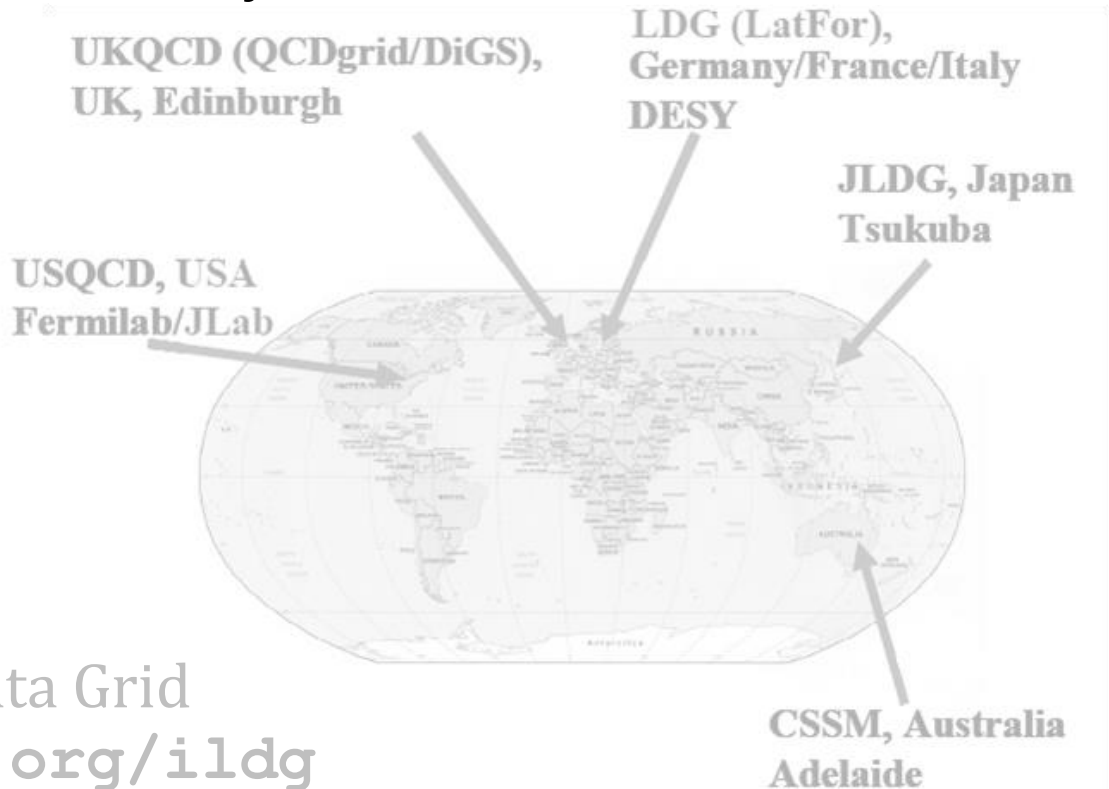
2. Some QCD Vacuum (gauge configurations)

↻ WWW (ILDG) or generate it yourself 0810.3588 350+



International Lattice Data Grid

<http://www.usqcd.org/ildg>





# Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)
2. Some QCD Vacuum (gauge configurations)

## 3. Correlators (hadronic observables)

- ↻ Invert Dirac operator matrix (rank  $10^{12}$ )
- ↻ Combine using color, spin and momentum into hadrons

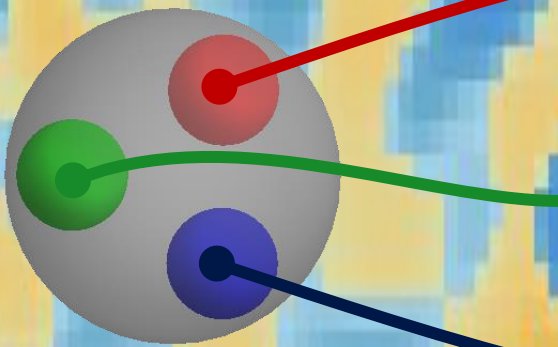
### § Simple hadron operators (not unique)

$$\text{↻ } \pi^+(x) = \bar{u}^a(x) \gamma_5 d^a(x)$$

$$\text{↻ } \rho^+(x) = \bar{u}^a(x) \gamma_\mu d^a(x)$$

$$\text{↻ } p_\delta(x) = \left( \bar{u}^a(x) C \gamma_5 d^b(x) \right) u_\delta^c \epsilon_{abc}$$

$$\text{↻ } \Delta_{\mu,\delta}^{++}(x) = \left( \bar{u}^a(x) C \gamma_\mu u^b(x) \right) u_\delta^c \epsilon_{abc}$$



Time

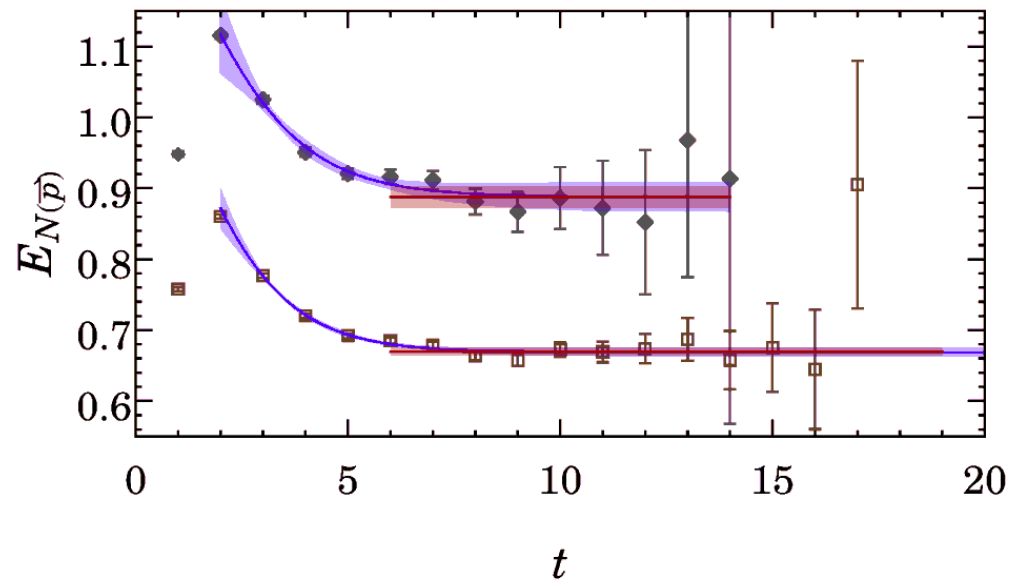
# Anatomy of a Lattice Calculation

Exercise: 1) Derive the time-dependent correlator formula. How does it differ between the pion and nucleon?  
2) Are there other ways to define  $M_{\text{eff}}$ ? Can you find an  $M_{\text{eff}}$  that gives the mass of the first radially excited state?

## 4. Analysis (extraction of masses or couplings)

$$\langle J_N J_N \rangle \propto \sum_n \langle J_N | n \rangle \langle n | J_N \rangle e^{-E_n t}$$

$$M_{\text{eff}} = -\log \left( \frac{\langle J_N J_N \rangle(t+1)}{\langle J_N J_N \rangle(t)} \right)$$

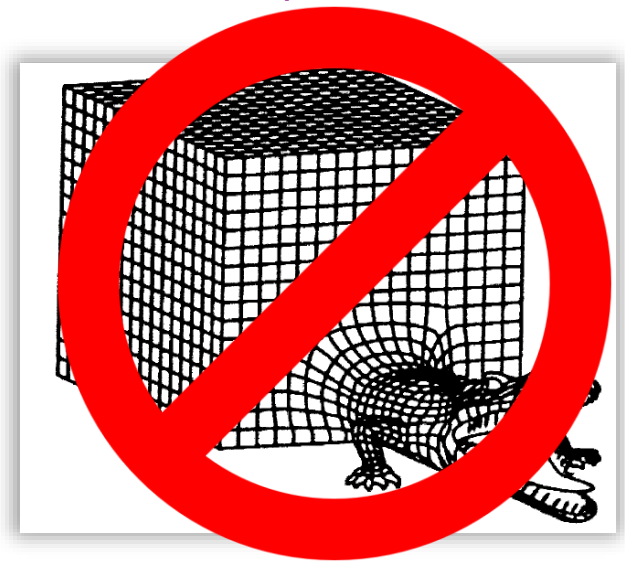


# Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)
2. Some QCD Vacuum (gauge configurations)
3. Correlators (hadronic observables)
4. Analysis (extraction of masses or couplings)
5. Systematic Uncertainty (nonzero  $a$ , finite  $L$ , etc.)

↻ Extrapolation to the continuum limit

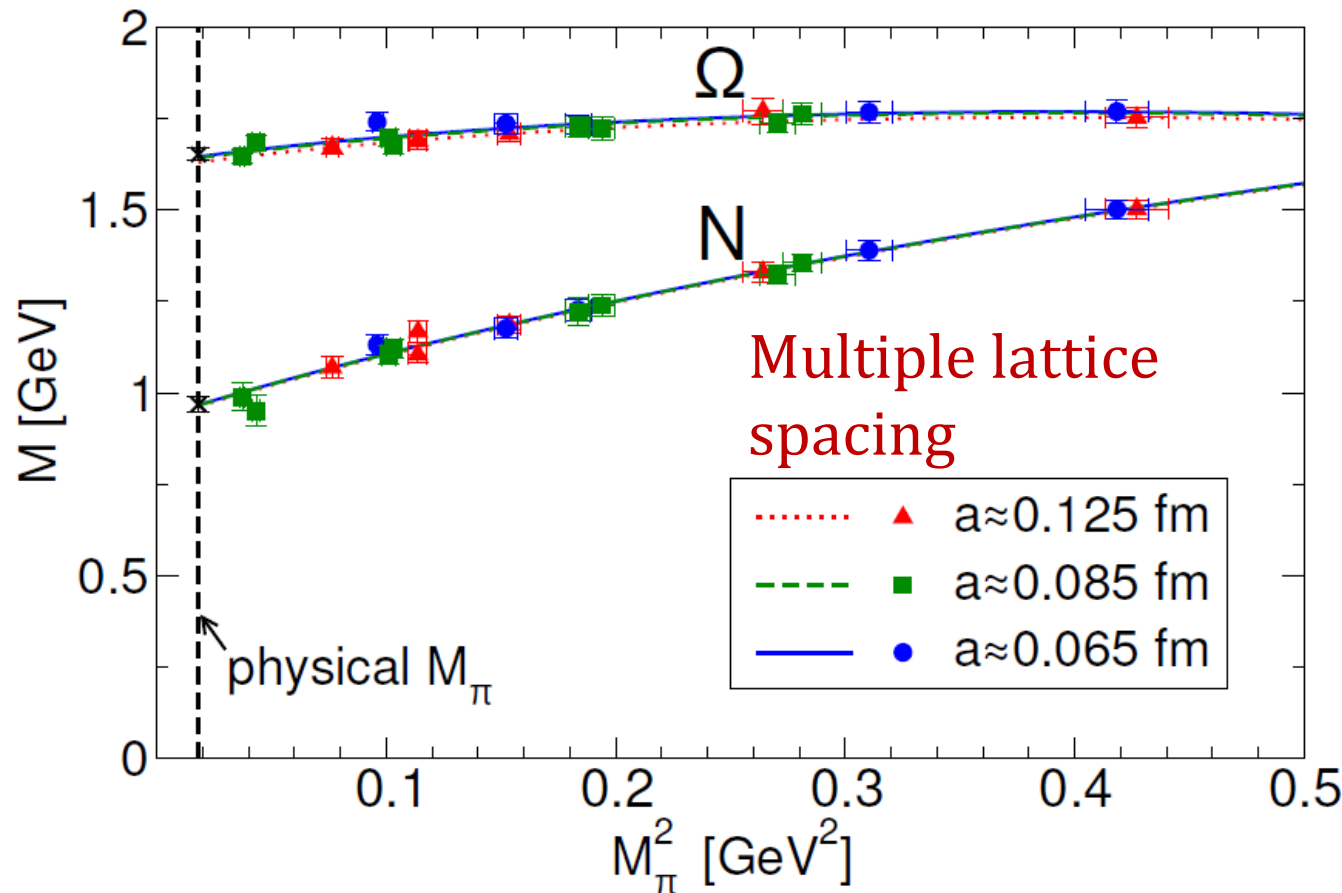
$$(m_\pi \rightarrow m_\pi^{\text{phys}}, L \rightarrow \infty, a \rightarrow 0)$$





# Anatomy of a Lattice Calculation

## 1. Hardware (computational resources) and Software (Code)



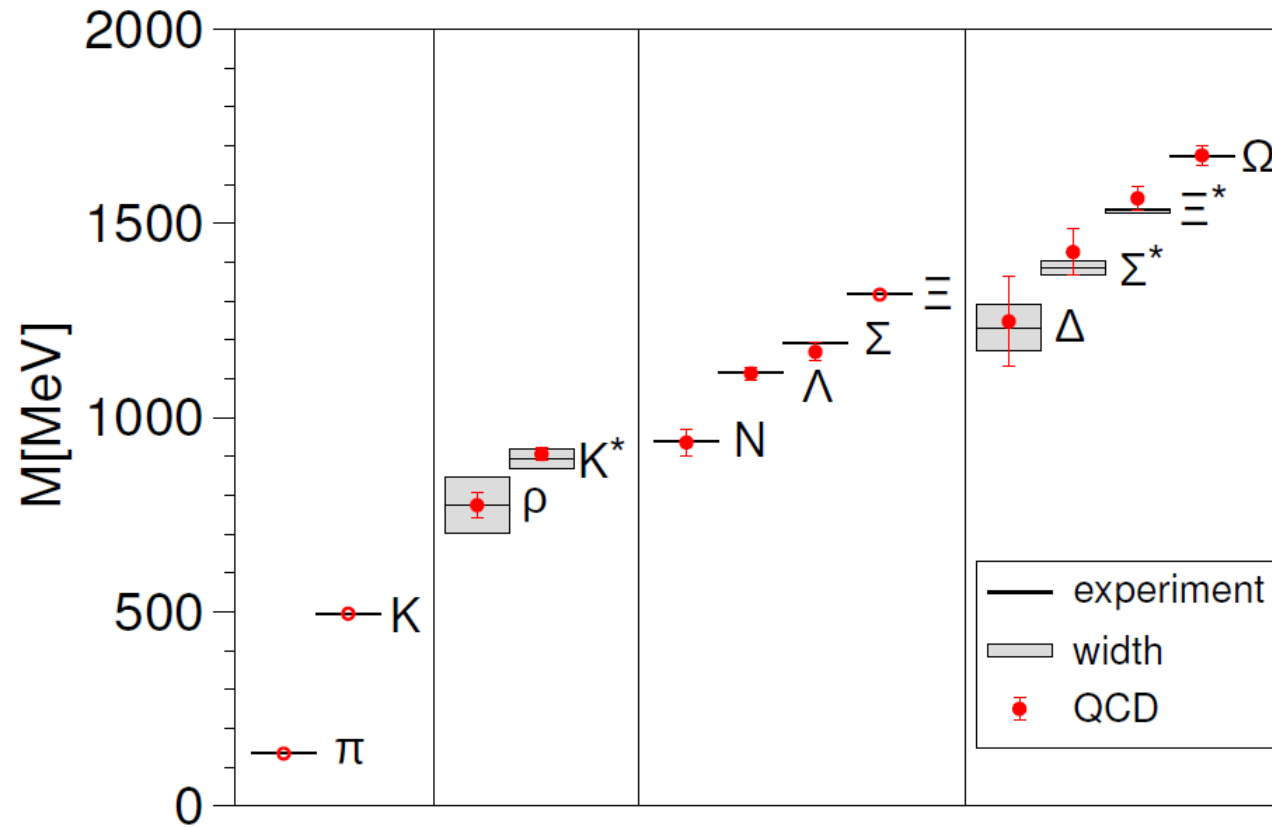
BMW Collaboration, Science (2008)



# Lattice in the News

## § Post-dictions of well known quantities

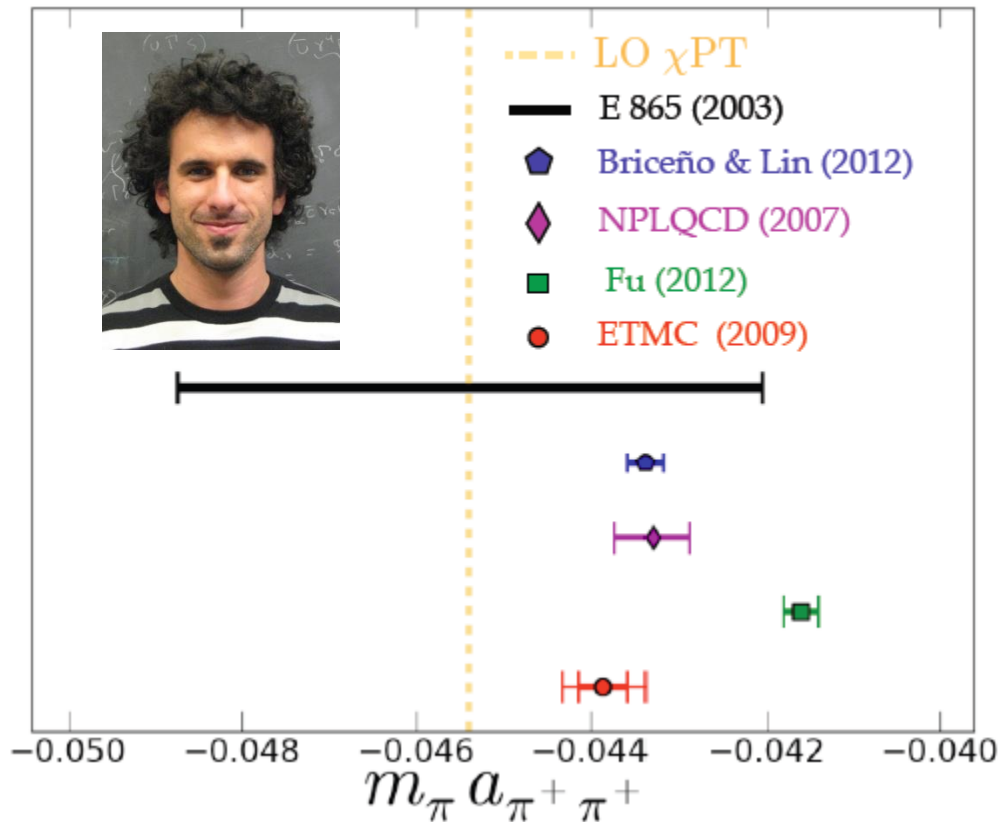
☞ Example: BMW Collaboration, Science 2008



# Successful Examples

§ Provide higher precision for known quantities

§ Make a lot of mass predictions

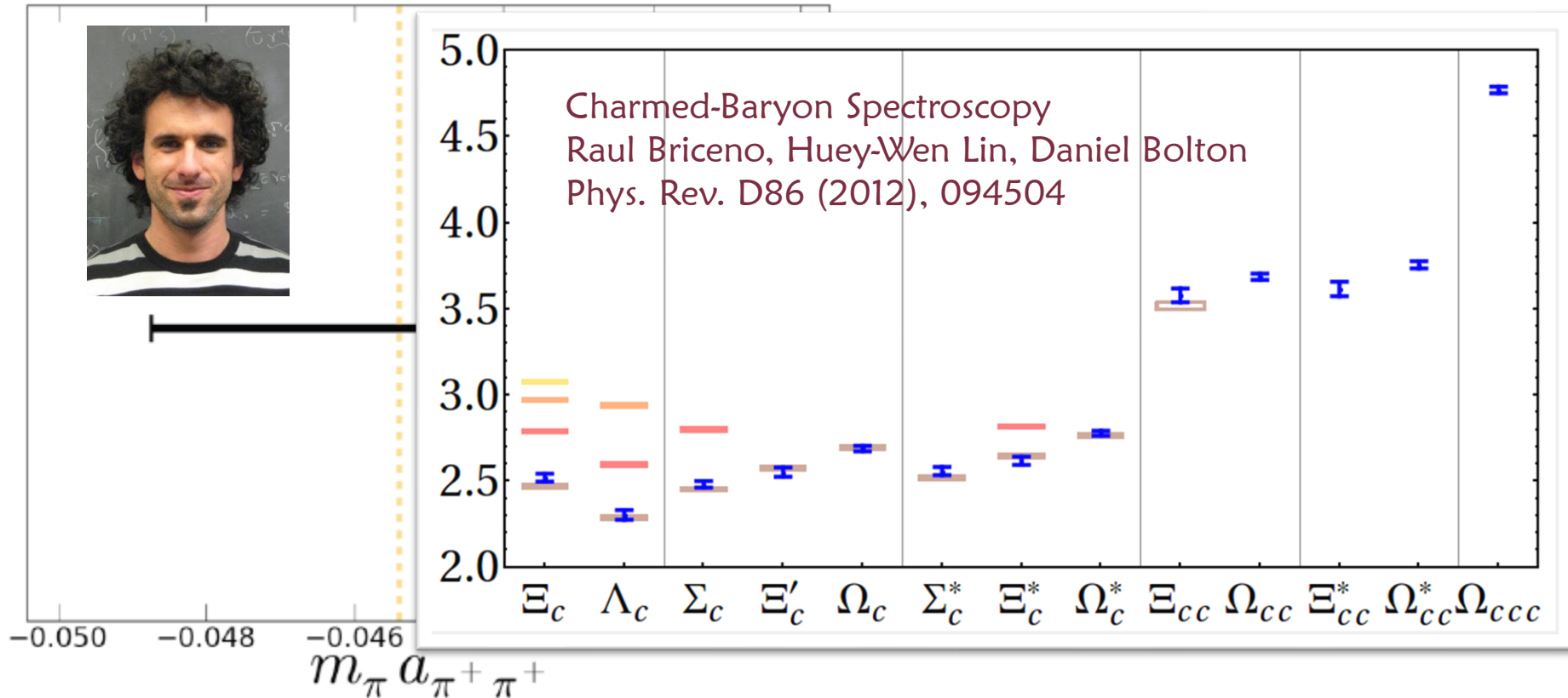




# Successful Examples

§ Provide higher precision for known quantities

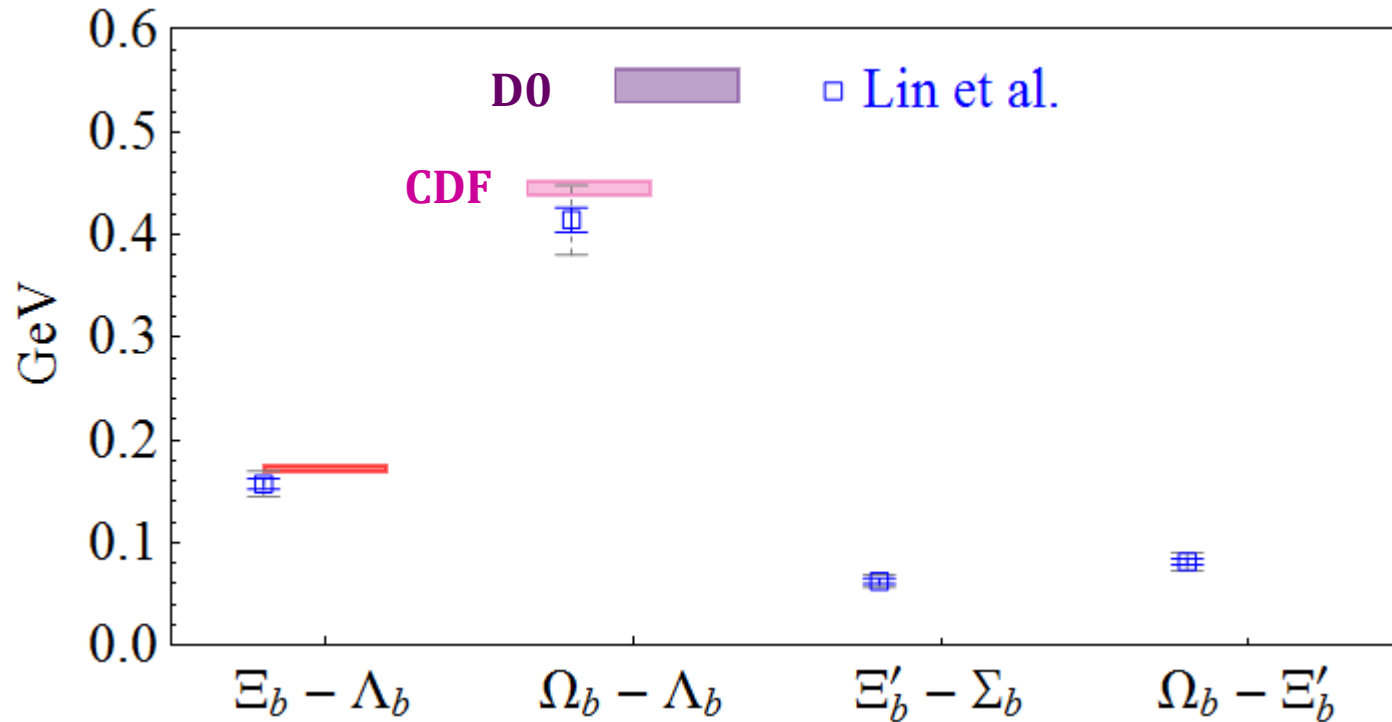
§ Make a lot of mass predictions



# Bottom Baryons

## § Inconsistency in the CDF and DØ results for $\Omega_b$ mass

↻ Our  $\Omega_b$  agrees with the CDF result

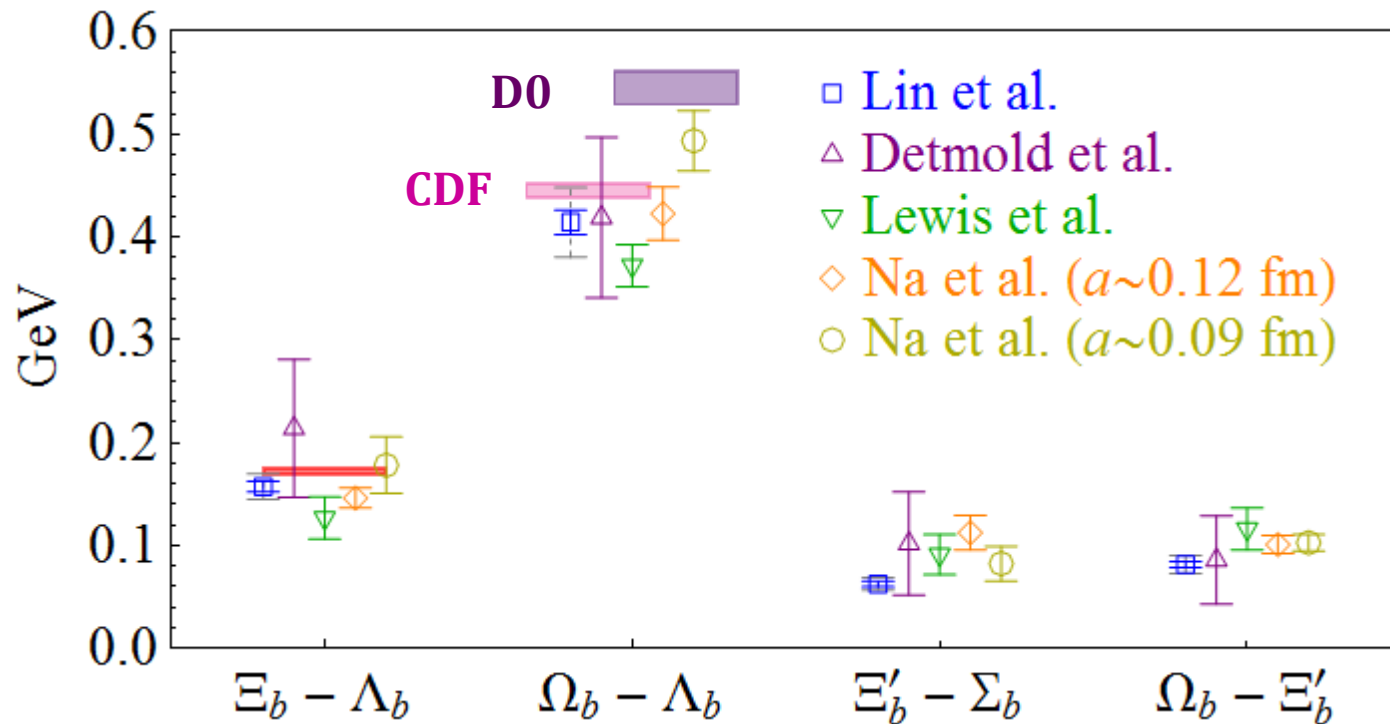


HWL, S. D. Cohen, N. Mathur, K. Orginos, Phys. Rev. D80, 054027 (2009)

# Bottom Baryons

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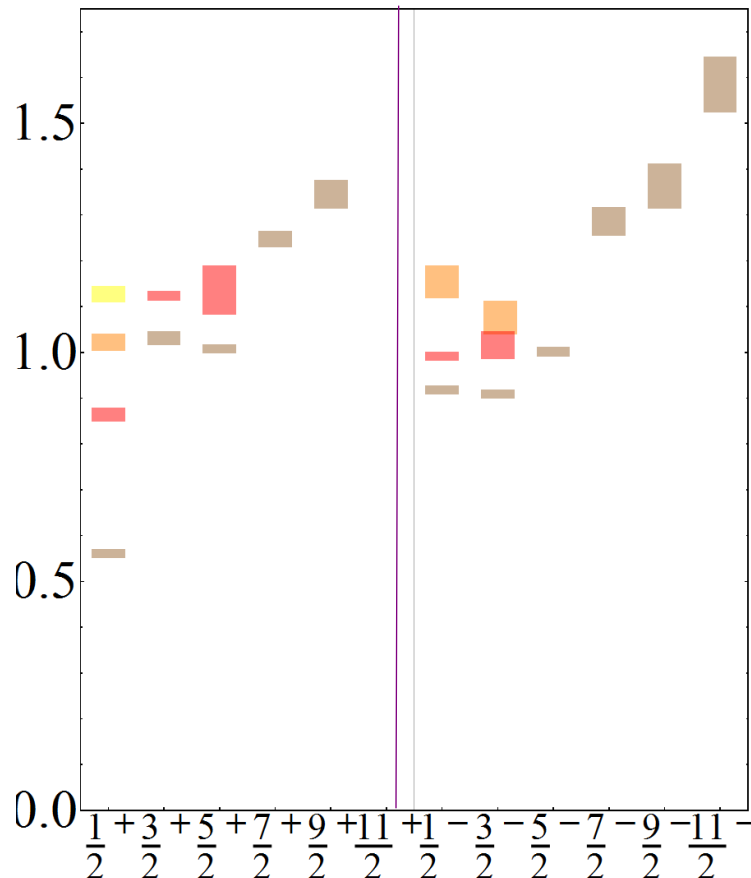
HWL, S. D. Cohen, N. Mathur, K. Orginos, Phys. Rev. D80, 054027 (2009);  
W. Detmold et al., (2008), 0812.2583[hep-lat]; R. Lewis et al., PRD79, 014502 (2009);  
H. Na et al., PoS LATTICE2008, 119 (2008).



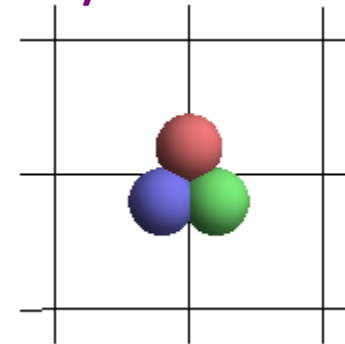
# Nucleon

§ All baryon spin states wanted:  $|J| = 1/2, 3/2, 5/2, \dots$

§ List of 4-star states



§ Only 3 distinguished states with  $J = 1/2$



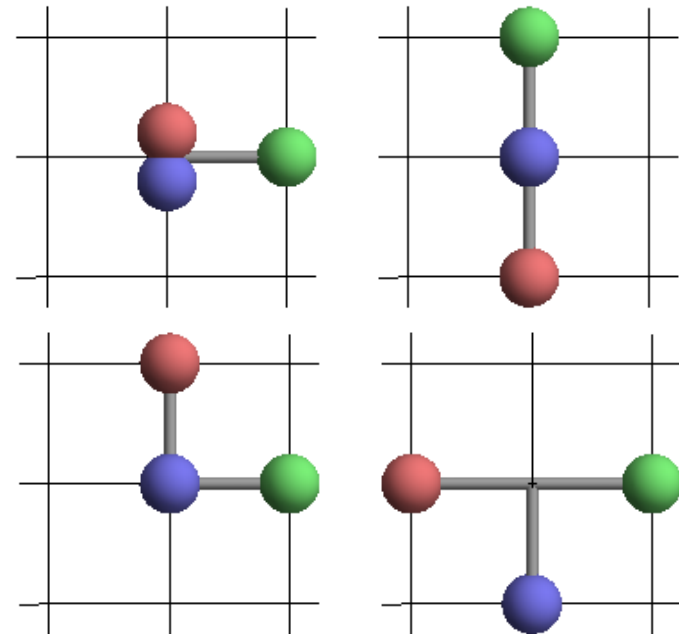
$$\begin{aligned} &\epsilon^{abc} (u^{Ta}(x) C \gamma_5 d^b(x)) u^c(x) \\ &\epsilon^{abc} (u^{Ta}(x) C d^b(x)) \gamma_5 u^c(x) \\ &\epsilon^{abc} (u^{Ta}(x) C \gamma_5 \gamma_4 d^b(x)) u^c(x) \end{aligned}$$

# Nucleon

§ Rotation symmetry is reduced  
rotation  $SO(3) \Rightarrow$  octahedral  $O_h$  group

§ Include more quark orientations

j	Irreps
$\frac{1}{2}$	$G_1$
$\frac{3}{2}$	H
$\frac{5}{2}$	$G_2 \oplus H$
$\frac{7}{2}$	$G_1 \oplus G_2 \oplus H$

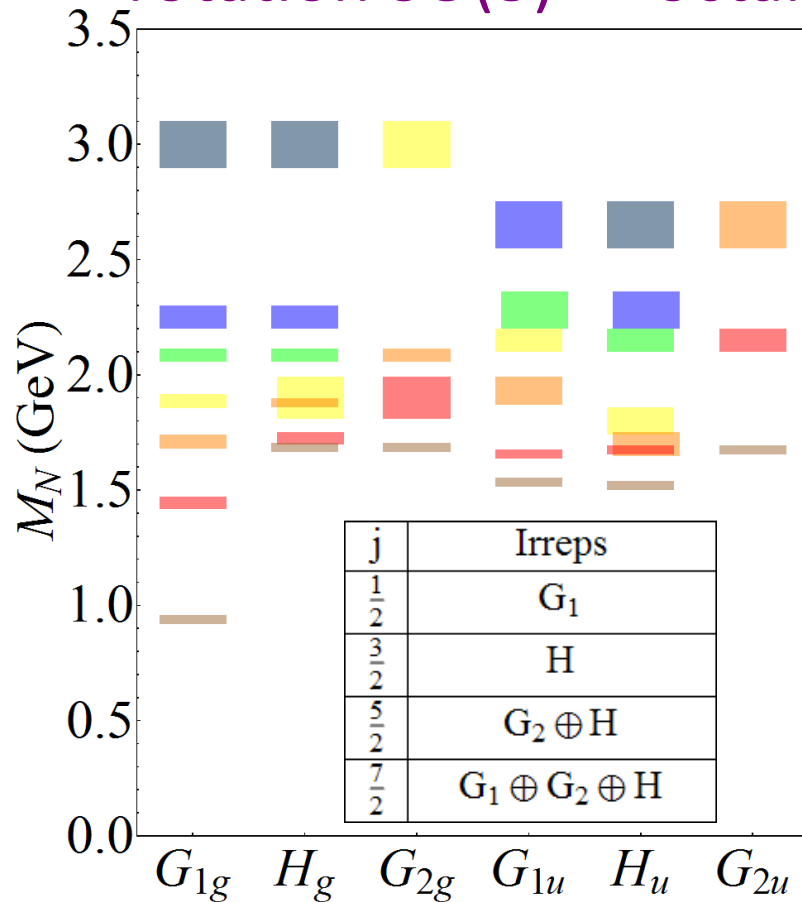


S. Basak et al., Phys. Rev. D72, 094506 (2005)

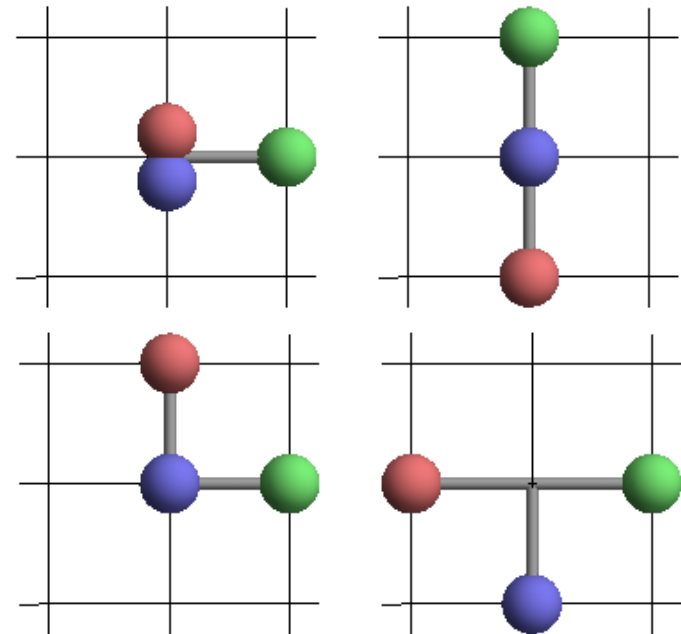
# Nucleon

§ Rotation symmetry is reduced

rotation  $SO(3) \Rightarrow$  octahedral  $O_h$  group



§ Include more quark orientations



§ More details on operators: check out [this YouTube video](#)



# Variational Method

## § Recall: still a coupling problem

lowest-mass state dominates  $C(t) = \sum_n \langle O|n\rangle \langle n|O\rangle e^{-E_n t}$

## § Decouple them:

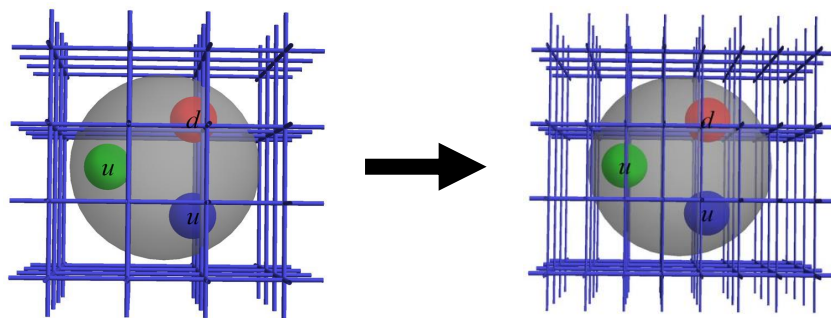
1. Construct correlator matrix  $C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0) | 0 \rangle$
2. Solve the gen. eigensystem  $C(t_0)^{-1/2} C(t) C(t_0)^{-1/2} \psi = \lambda(t, t_0) \psi$
3. Simple analysis of eigenvalues (t-dependence)

to get each excited state  $\lambda_n(t, t_0) = e^{-(t-t_0)E_n} (1 + \mathcal{O}(e^{-|\delta E|(t-t_0)}))$

C. Michael, Nucl. Phys. B 259, 58 (1985)

M. Lüscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)

## § Higher resolution (at least in time direction)



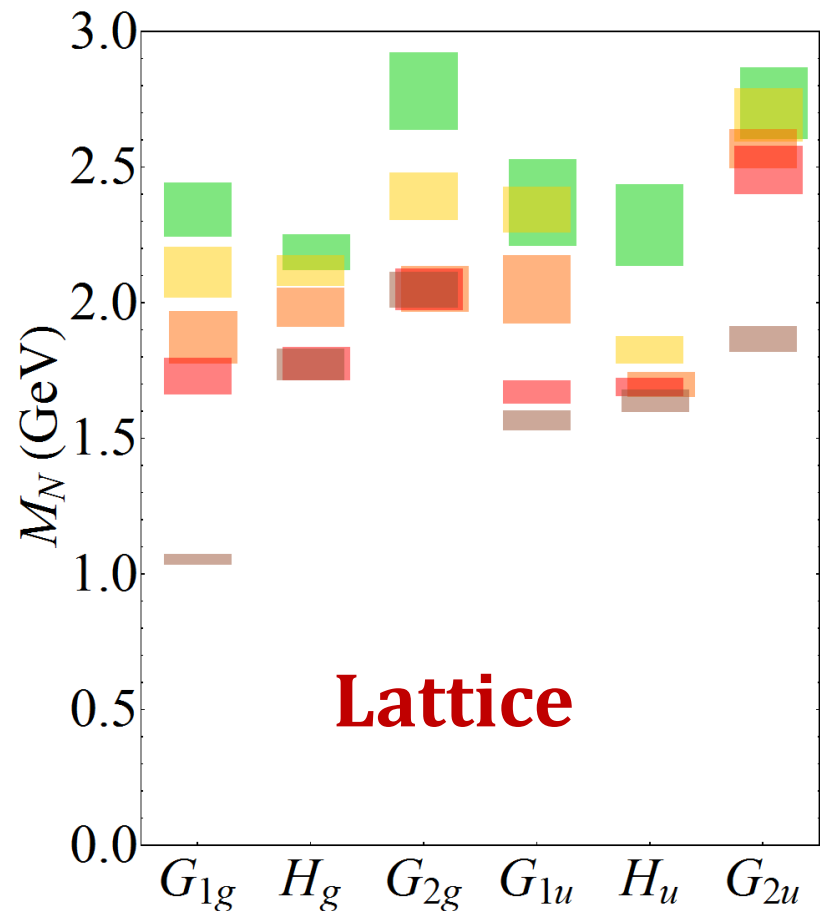
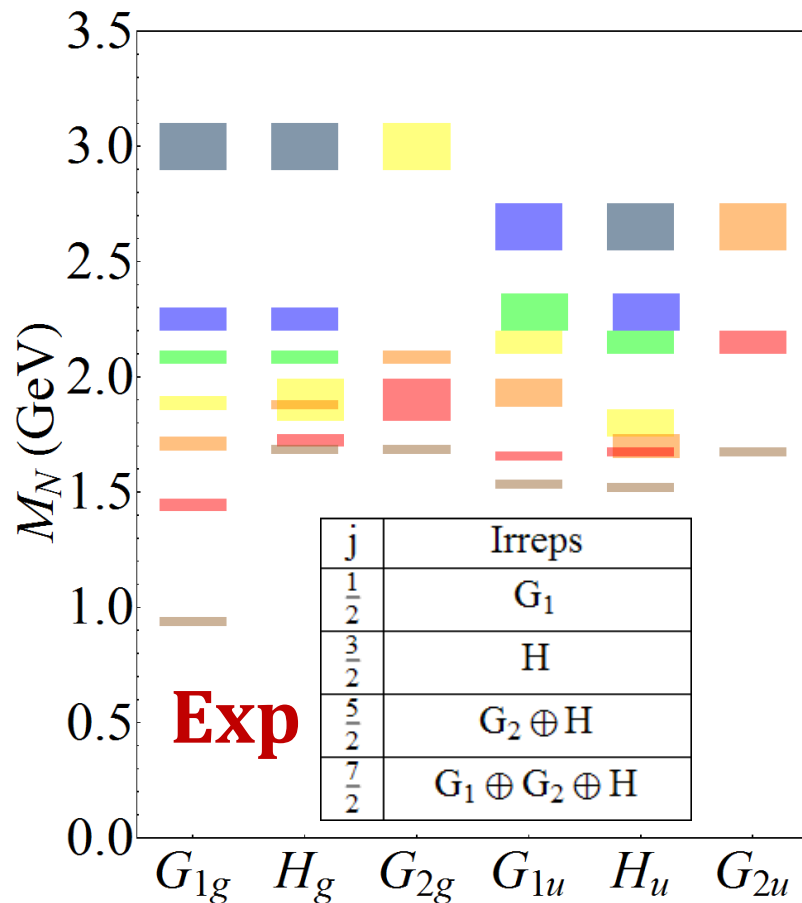
R. Edwards, B. Joo, HWL, Phys. Rev. D 78, 014505 (2008);  
HWL et al., Phys. Rev. D 79, 034502 (2009)

# $\mathcal{N}_f = 2+1$ Study: Nucleon

§  $\mathcal{N}_f = 2+1$ , anisotropic clover action

HSC, 1004.5072[hep-lat]

$V = 16^3 \times 128$ ,  $a_s \approx 0.12$  fm,  $a_s/a_t \approx 3.5$ ,  $M_\pi \approx 390$  MeV



# Less Known Case

§  $N_f = 2+1$ , anisotropic clover action

HSC, 1004.5072[hep-lat]

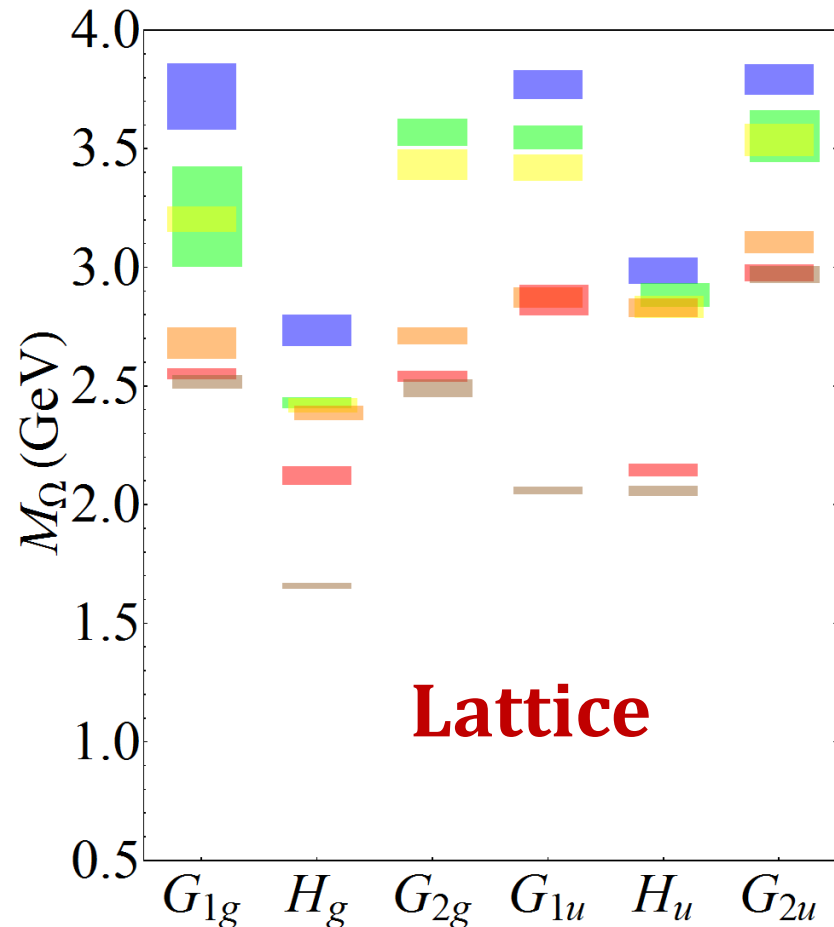
$V = 16^3 \times 128$ ,  $a_s \approx 0.12$  fm,  $a_s/a_t \approx 3.5$ ,  $M_\pi \approx 390$  MeV

$\Omega$  BARYONS ( $S = -3$ ,  $I = 0$ )

$\Omega^-$	$0(3/2^+)$	*****
$\Omega(2250)^-$	$0(?)^?$	***
$\Omega(2380)^-$		•**
$\Omega(2470)^-$		•**

**Exp**

§ Predictive power



**Lattice**



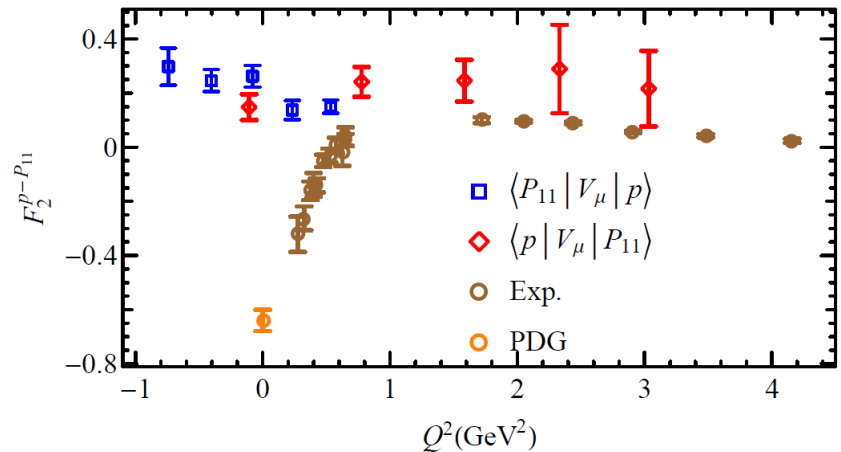
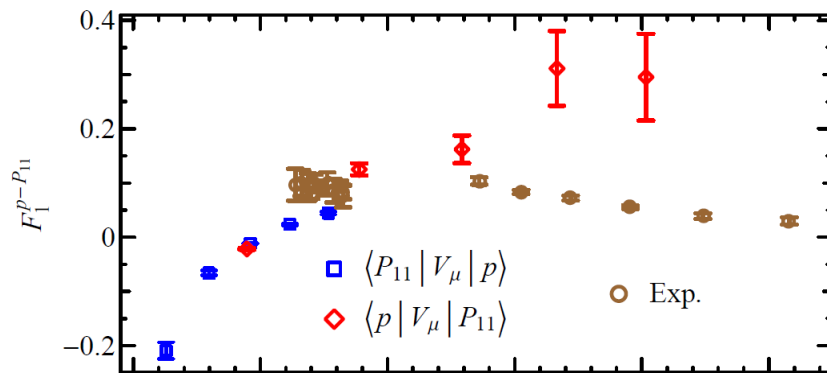
# *Backup Slides*



# Transition Form Factors

## § Challenge: Resonances and multiple-particle final states

- Neutrino scattering above the pion-production threshold, where the scattered nucleon is excited into resonances
- Inputs: need knowledge on  $N \rightarrow \Delta$  and  $N \rightarrow N^*$
- $N$ - $P_{11}$  transition EM form factors
- $\langle N_2 | V_\mu | N_1 \rangle_\mu(q) = \bar{u}_{N_2}(p') \left[ F_1(q^2) \left( \gamma_\mu - \frac{q_\mu}{q^2} \not{q} \right) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{M_{N_1} + M_{N_2}} \right] u_{N_1}(p) e^{-iq \cdot x}$

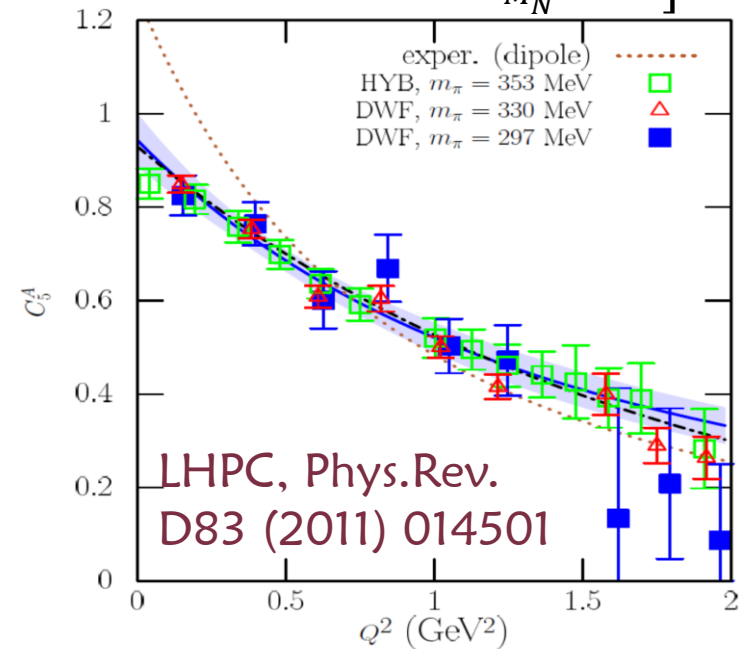
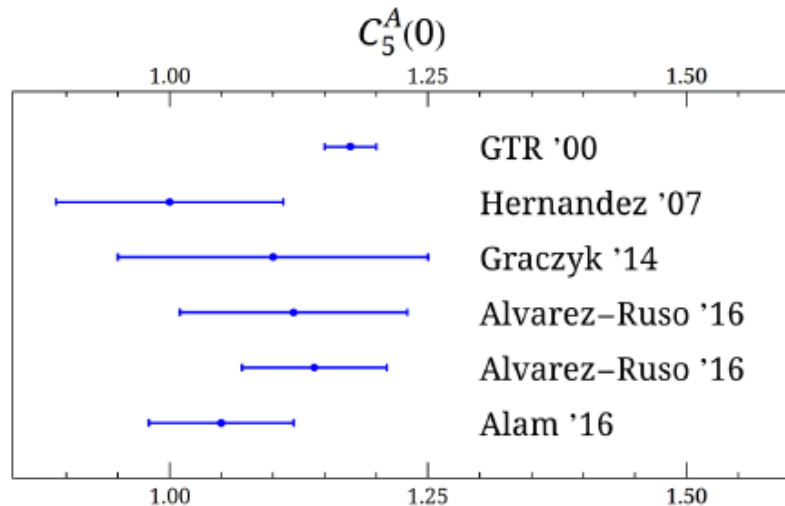


Lin et al, Phys.Rev.D78 (2008), 114508

# Transition Form Factors

## § Challenge: Resonances and multiple-particle final states

- Neutrino scattering above the pion-production threshold, where the scattered nucleon is excited into resonances
- Inputs: need knowledge on  $N \rightarrow \Delta$  and  $N \rightarrow N^*$
- $N\Delta$  axial form factors  $C_{3-6}^A$  in the transition axial current are
- $$-\mathcal{A}_{N\Delta}^\mu = \bar{u}_\alpha(p') \left[ \frac{C_3^A}{M_N} (g^{\alpha\mu} \not{q} - q^\alpha \gamma^\mu) + \frac{C_4^A}{M_N^2} (g^{\alpha\mu} q \cdot p' - q^\alpha p'^\mu) + C_5^A g^{\alpha\mu} + \frac{C_6^A}{M_N^2} q^\alpha q^\mu \right] \gamma_5 u(p)$$



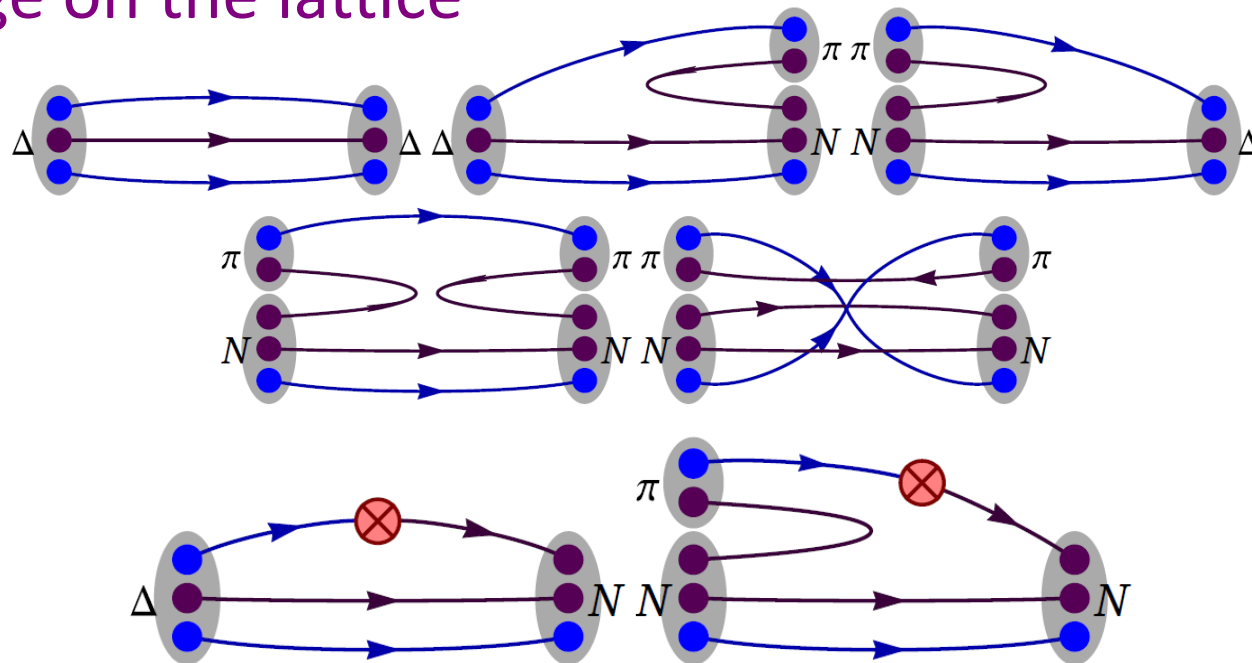


# Transition Form Factors

## § Challenge: Resonances and multiple-particle final states

- ↻ Neutrino scattering above the pion-production threshold, where the scattered nucleon is excited into resonances
- ↻ Inputs: need knowledge on  $N \rightarrow \Delta$  and  $N \rightarrow N^*$
- ↻ CAVER: Past lattice calculations have used  $E_\pi + E_N > E_{\Delta, N^*}$

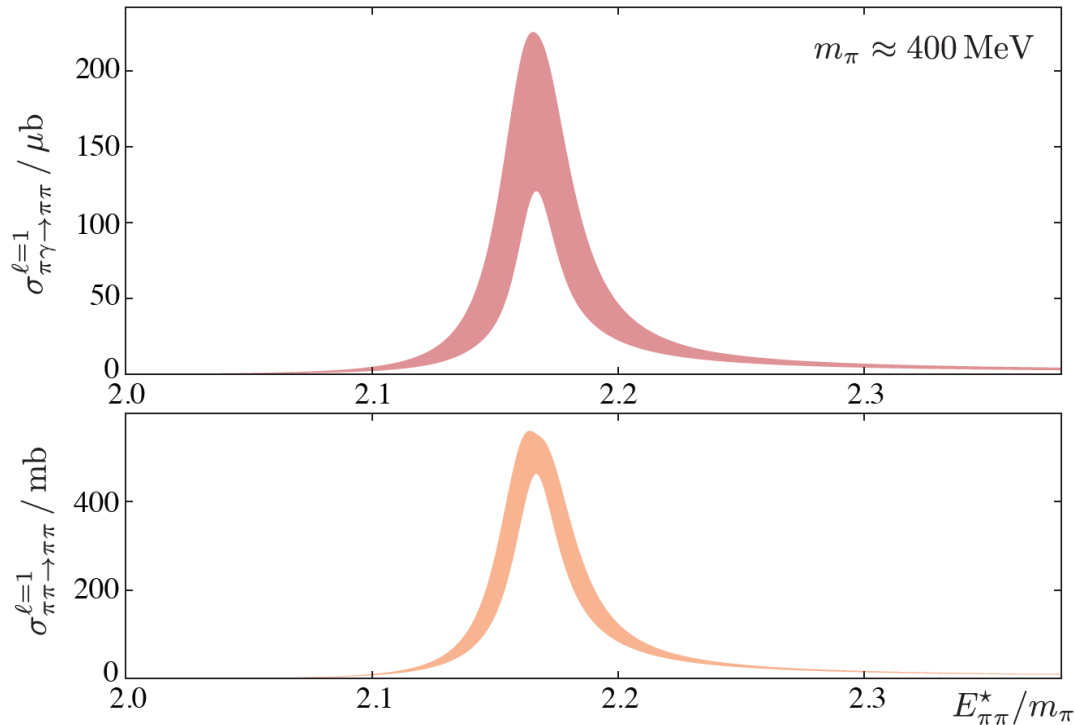
## § Challenge on the lattice



# Transition Form Factors

§ Challenge: Resonances and multiple-particle final states

↻ Simpler cases show promising results from lattice



R. Briceno et al, Phys. Rev. D92, 074509 (2015)