INTRODUCTION TO SMEFT



S. Dawson, BNL CTEQ, 2023 Lecture 2



Complaints, suggestions to dawson@bnl.gov

MOMENTUM DEPENDENT OPERATORS CHANGE KINEMATIC DISTRIBUTIONS

• Typically quite small effects:

 $\mathcal{O}\left(\frac{p_T^2}{\Lambda^2}\right)$

- Couplings constrained to give correct rate for ggH
- Look in tails of distributions (Is theory valid?)





Schlaffer, Spannowsky, Takeuchi, Weiler, Wymant, arxiv:1405.4295

IDENTIFY SMEFT COEFFICIENTS

- Is the ttH coupling the Standard Model coupling?
- Non-SM contributions change rate/distributions





- Observation of gluon fusion production of Higgs at expected rate doesn't mean Higgs has SM ttH coupling
- Need ttH production
- High luminosity will pin down coupling

*tH production is sensitive to the sign of the coupling





HIGGS DECAYS

- Example: $H \rightarrow b\bar{b}$ $\frac{\Gamma(H \rightarrow b\bar{b})}{\Gamma(H \rightarrow b\bar{b}) \mid_{SM}} = (1 + \Delta\kappa_b)^2$ $\Delta\kappa_b = \frac{1}{\sqrt{2}G_F\Lambda^2} \left(C_{H\Box} - \frac{C_{HD}}{4} - C_{Hl}^{(3)} + \frac{C_{ll}^1}{2} - \frac{C_{dH}}{2^{3/4}m_b\sqrt{G_F}} \right)$ From normalizing From change in relation between G_F and v
 - Is this just a fancy way of writing the κ 's?

 $O_{dH} = Y_d(\phi^{\dagger}\phi)\overline{q}_L\phi d_R$

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Feynman rules: <u>1704.03888</u>

CONSIDER $H \rightarrow ZZ$

• $H \rightarrow Zff$

$$\frac{\Gamma(H \to Z f \overline{f})}{\Gamma(H \to Z f \overline{f}) \mid_{SM}} = 1 + \frac{1}{\sqrt{2}G_F \Lambda^2} \left[c_k - \underbrace{.97}_{ZZ} \right]$$

c_{ZZ} generate momentum dependent contributions

• EFT can capture off-shell effects (not just a κ)

$$c_k = \frac{C_{HD}}{2} + 2C_{H\Box} + C_{ll} - 2C_{Hl}^{(3)}$$

$$c_{ZZ} = \frac{M_W^2}{M_Z^2} C_{HW} + (1 - \frac{M_W^2}{M_Z^2}) C_{HB} + \frac{M_W}{M_Z} \sqrt{1 - \frac{M_W^2}{M_Z^2}} C_{HWB}$$

These operators have derivatives

- EFT has more information than total rate
- q² is fermion pair invariant mass squared $\frac{d\Gamma}{dq^2} \mid_{EFT} = \frac{d\Gamma}{dq^2} \mid_{SM} \left[1 + \frac{1}{\sqrt{2}G_F\Lambda^2} c_k \right] + \frac{G_F q^2}{\Lambda^2} c_{ZZ}(...)$
- Integrate up to q_{cut}
- $(G_F q^2 / \Lambda^2) f(q_{cut})$ is coefficient of c_{ZZ}

 κ formalism does not capture this momentum dependence present in the SMEFT

Off- Shell Decay, $H \rightarrow Z$ ff .1.4 .1.6

CONNECTING HZZ SMEFT WITH HZZ κ CALCULATIONS

• Define an effective g_{HZZ} coupling such that

 $\frac{\Gamma(H \to ZZ) \mid_{SMEFT}}{\Gamma(H \to ZZ) \mid_{SM}} = (1 + \Delta g_{HZZ})^2 \sim 1 + 2\Delta g_{HZZ}$

• Hides SMEFT complications, but they are still there

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EXTRACTION OF HIGGS COUPLINGS AT FUTURE COLLIDERS

- Plot assumes no BSM (ie. Higgs width is sum of SM decay widths)
- Significant improvements at future colliders
- Differences between e⁺e⁻ colliders due to different L assumptions



Snowmass Higgs report, 2209.07510

*Next include electroweak precision observables and diboson production in SMEFT framework

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CALCULATE AT ONE LOOP IN SMEFT

- Theory is renormalizable order by order in $1/\Lambda^2$ expansion
 - Drop terms of $O(I/\Lambda^4)$
- Wilson coefficients renormalized in \overline{MS} : $C_i(\mu) = C_i^0 \frac{1}{32\pi^2\epsilon}\gamma_{ij}C_j$
 - Tree level relationships changed from SM
 - Renormalization counterterms have $1/\Lambda^2$ contributions

 $G_{\mu} = \frac{1}{\sqrt{2}\Lambda^2} \bigg(2C_{Hl}^3 - C_{ll} \bigg) + \frac{1}{\sqrt{2}v_0^2} (1 + \Delta r) \quad * \Delta r \text{ in SMEFT different from SM}$

• Idea is that including SMEFT@NLO gives a window into the dependence on operators that don't contribute at tree level

NLO RESULTS FOR $H \rightarrow \gamma \gamma$

- Use α, M_Z, G_μ as input parameters
 - Using M_W , M_Z , G_μ as inputs changes coefficients of C_{II} , $C_{HI}^{(3)}$, C_{HD}

$$\mu_{\gamma\gamma} = 1 + \left[-40C_{HB} - 13C_{HW} + 22C_{HWB} \right] \checkmark \sim 5\% \text{ shifts from tree level values}$$

$$\left[-.9C_W + .12C_{H\Box} - .2C_{HD} \right] \checkmark \text{Limited from VV production}$$
New $\longrightarrow +.03C_{uH} - 1.2C_{uW} - 2C_{uB}$

$$\left[-.36C_{Hl}^{(3)} + .18C_{ll} \right] \checkmark \text{G}_{\mu}$$

Note NLO results involve many coefficients

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*Coefficients evaluated at scale Λ =I TeV

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NON-SM COUPLINGS IN EFFECTIVE COUPLING LANGUAGE

- Effective Z/W-fermion couplings
- (neglect RHW couplings and RH υ couplings since they don't interfere with the SM)

$$\begin{split} L =& 2M_Z \sqrt{\sqrt{2}G_{\mu}} Z_{\mu} \Big\{ \left[g_L^{Zu} + \delta g_L^{Zu} \right] \overline{u}_L \gamma_{\mu} u_L + \left[g_L^{Zd} + \delta g_L^{Zd} \right] \overline{d}_L \gamma_{\mu} d_L + \left[g_R^{Zu} + \delta g_R^{Zu} \right] \overline{u}_R \gamma_{\mu} u_R \\ &+ \left[g_R^{Zd} + \delta g_R^{Zd} \right] \overline{d}_R \gamma_{\mu} d_R + \left[g_L^{Ze} + \delta g_L^{Ze} \right] \overline{e}_L \gamma_{\mu} e_L + \left[g_L^{Z\nu} + \delta g_L^{Z\nu} \right] \overline{\nu}_L \gamma_{\mu} \nu_L \\ &+ \left[g_R^{Ze} + \delta g_R^{Ze} \right] \overline{e}_R \gamma_{\mu} e_R \Big\} + \frac{\overline{g}_2}{\sqrt{2}} \Big\{ W_{\mu} \Big[(1 + \delta g_L^{Wq}) \overline{u}_L \gamma_{\mu} d_L \Big] + W_{\mu} \Big[(1 + \delta g_L^{Wl}) \overline{\nu}_L \gamma_{\mu} e_L \Big] + h.c. \Big\} \end{split}$$

• SU(2) invariance requires

 $egin{aligned} \delta g_L^{Wq} = & \delta g_L^{Zu} - \delta g_L^{Zd} \ \delta g_L^{Wl} = & \delta g_L^{Z
u} - \delta g_L^{Ze} \end{aligned}$

7 new parameters + M_W

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RH W couplings do not interfere with SM

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CHANGES TO FERMION-GAUGE COUPLINGS

	Warsaw Basis	
δg_L^{Zu}	$-\frac{v^2}{2\Lambda^2} \left(\mathcal{C}_{Hq}^{(1)} - \mathcal{C}_{Hq}^{(3)} \right) + \frac{1}{2} \delta g_Z + \frac{2}{3} \left(\delta s_W^2 - s_W^2 \delta g_Z \right) $	
δg_L^{Zd}	$-\frac{v^2}{2\Lambda^2} \left(\mathcal{C}_{Hq}^{(1)} + \mathcal{C}_{Hq}^{(3)} \right) - \frac{1}{2} \delta g_Z - \frac{1}{3} \left(\delta s_W^2 - s_W^2 \delta g_Z \right)$	
$\delta g_L^{Z\nu}$	$-\frac{v^2}{2\Lambda^2} \left(\mathcal{C}_{Hl}^{(1)} - \mathcal{C}_{Hl}^{(3)} \right) + \frac{1}{2} \delta g_Z$	
δg_L^{Ze}	$-\frac{v^2}{2\Lambda^2} \left(\mathcal{C}_{Hl}^{(1)} + \mathcal{C}_{Hl}^{(3)} \right) - \frac{1}{2} \delta g_Z - \left(\delta s_W^2 - s_W^2 \delta g_Z \right)$	
δg_R^{Zu}	$-\frac{v^2}{2\Lambda^2}\mathcal{C}_{Hu}+\frac{2}{3}\left(\delta s_W^2-s_W^2\delta g_Z\right)$	
δg_R^{Zd}	$-\frac{v^2}{2\Lambda^2}\mathcal{C}_{Hd} - \frac{1}{3}\left(\delta s_W^2 - s_W^2\delta g_Z\right)$	
δg_R^{Ze}	$-rac{v^2}{2\Lambda^2}\mathcal{C}_{He}-\left(\delta s_W^2-s_W^2\delta g_Z ight)$	
δg_L^{Wq}	$rac{v^2}{\Lambda^2} \mathcal{C}_{Hq}^{(3)} + c_W^2 \delta g_Z + \delta s_W^2$	
δg_L^{Wl}	$\frac{v^2}{\Lambda^2} \mathcal{C}_{Hl}^{(3)} + c_W^2 \delta g_Z + \delta s_W^2$	
δg_Z	$-\frac{v^2}{\Lambda^2}\left(\delta v + \frac{1}{4}\mathcal{C}_{HD}\right)$	1
δv	${\cal C}_{Hl}^{(3)}-rac{1}{2}{\cal C}_{ll}$	
δs_W^2	$-\frac{v^2}{\Lambda^2}\frac{s_W c_W}{c_W^2 - s_W^2} \left[2s_W c_W \left(\delta v + \frac{1}{4}\mathcal{C}_{HD}\right) + \mathcal{C}_{HWB}\right]$	

Expansion in v^2/Λ^2 Scale is fixed to M_Z

- Change in relationship between input parameters: $g_Z = G_F M_Z$
- Change in relationship between v and G_F

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WAND Z POLE OBSERVABLES

- Fit to 14 data points—inputs are G_{μ},M_Z,α

 $M_W, \Gamma_W, \Gamma_Z, \sigma_h, A_{l,FB}, A_{b,FB}, A_{c,FB}, A_b, A_c, A_l, R_l, R_b, R_c$

• Tree level expressions depend on (in Warsaw basis)

 $C_{ll}, C_{HWB}, C_{Hu}, C_{Hq}^{(3)}, C_{Hq}^{(1)}, C_{Hl}^{(3)}, C_{Hl}^{(1)}, C_{He}, C_{HD}, C_{Hd}$

• Tree level observables depend on 8 combinations of operators $M_W, \delta g_L^{Zu}, \delta g_L^{Zd}, \delta g_L^{Z\nu}, \delta g_L^{Ze}, \delta g_R^{Zu}, \delta g_R^{Zd}, \delta g_R^{Ze}$ \Rightarrow 2 blind directions (resolved by other measurements) S. Dawson

FITS ARE STRAIGHTFORWARD

- Compute observables in SMEFT: $O_i = O_i^{SM} + \delta O_i$
- Use most accurate SM theory
- Do χ^2 fit to data
- Operators contributing to LEP observables strongly restricted

Measurement	Experiment	"Best" theory
$\Gamma_Z(\text{GeV})$	2.4952 ± 0.0023	2.4945 ± 0.0006
$\sigma_h(nb)$	41.540 ± 0.037	41.491 ± 0.008
R_l	20.767 ± 0.025	20.749 ± 0.009
R_b	0.21629 ± 0.00066	0.21586 ± 0.0001
R_c	0.1721 ± 0.0030	0.17221 ± 0.00005
A_l	0.1465 ± 0.0033	0.1472 ± 0.0004
A_c	0.670 ± 0.027	0.6679 ± 0.0002
A_b	0.923 ± 0.020	0.92699 ± 0.00006
$A_{l,FB}$	0.0171 ± 0.0010	0.0162 ± 0.0001
$A_{b,FB}$	0.0992 ± 0.0016	0.1023 ± 0.0003
$A_{c,FB}$	0.0707 ± 0.0035	0.0737 ± 0.0003
$A_l(SLD)$	0.1513 ± 0.0021	0.1472 ± 0.0004
$\sin^2 \theta_{l,eff}$	0.23179 ± 0.00035	0.23150 ± 0.00006
$M_W({ m GeV})$	80.379 ± 0.012	80.359 ± 0.006
$\Gamma_W(\text{GeV})$	2.085 ± 0.042	2.0904 ± 0.0003

LEP FITS

- Strong cancellations possible
- Can't just fit to one operator
- Contributions from different operators vary greatly
- At NLO, dependence on many operators

$$\begin{split} \delta M_W^{LO} &= \frac{v^2}{\Lambda^2} \bigg\{ -29.827 \mathcal{C}_{Hl}^{(3)} + 14.914 \mathcal{C}_{ll} - 27.691 \mathcal{C}_{HD} - 57.479 \mathcal{C}_{HWB} \bigg\} \\ \delta \Gamma(Z \to b\bar{b})^{LO} &= \frac{v^2}{\Lambda^2} \bigg\{ -0.1400 \mathcal{C}_{Hd} - 1.0242 \mathcal{C}_{Hl}^{(3)} + 0.8498 \mathcal{C}_{Hq}^{(1)} + 0.8498 \mathcal{C}_{Hq}^{(3)} + 0.5121 \mathcal{C}_{ll} \\ &\quad -0.2561 \mathcal{C}_{HD} - 0.3361 \mathcal{C}_{HWB} \bigg\} \\ \delta R_l^{LO} &= \frac{v^2}{\Lambda^2} \bigg\{ -4.978 \mathcal{C}_{Hd} + 33.673 \mathcal{C}_{He} - 45.688 \mathcal{C}_{Hl}^{(1)} - 49.393 \mathcal{C}_{Hl}^{(3)} + 13.39 \mathcal{C}_{Hq}^{(1)} + 47.041 \mathcal{C}_{Hq}^{(3)} \\ &\quad +6.637 \mathcal{C}_{Hu} + 1.853 \mathcal{C}_{ll} - 0.926 \mathcal{C}_{HD} - 4.532 \mathcal{C}_{HWB} \bigg\} \end{split}$$

Z AND W POLE OBSERVABLES

• 10 operators contribute at tree level

\mathcal{O}_{ll}	$(\bar{l}_L \gamma_\mu l_L)(\bar{l}_L \gamma^\mu l_L)$	\mathcal{O}_{HWB}	$(H^{\dagger}\tau^{a}H)W^{a}_{\mu\nu}B^{\mu\nu}$	\mathcal{O}_{HD}	$\left(H^{\dagger}D^{\mu}H\right)^{*}\left(H^{\dagger}D_{\mu}H\right)$
\mathcal{O}_{He}	$(H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\overline{e}_{R}\gamma^{\mu}e_{R})$	\mathcal{O}_{Hu}	$(H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\overline{u}_{R}\gamma^{\mu}u_{R})$	\mathcal{O}_{Hd}	$(H^{\dagger}i \overleftrightarrow{D}_{\mu} H)(\overline{d}_R \gamma^{\mu} d_R)$
$\mathcal{O}_{\phi q}^{(3)}$	$(H^{\dagger}i\overleftrightarrow{D}^{a}_{\mu}H)(\bar{q}_{L}\tau^{a}\gamma^{\mu}q_{L})$	$\mathcal{O}_{Hq}^{(1)}$	$(H^{\dagger}i\overleftrightarrow{D}_{\mu}H)(\bar{q}_{L}\tau^{a}\gamma^{\mu}q_{L})$	$\mathcal{O}_{Hl}^{(3)}$	$(H^{\dagger}i \overset{\leftrightarrow}{D}{}^a_{\mu} H)(\bar{l}_L \tau^a \gamma^{\mu} l_L)$
$\mathcal{O}_{Hl}^{(1)}$	$(H^{\dagger}i \overleftrightarrow{D}_{\mu} H)(\bar{l}_L \tau^a \gamma^{\mu} l_L)$				

$$\alpha \Delta S = 4c_W s_W \frac{v^2}{\Lambda^2} C_{HWB}$$
$$\alpha \Delta T = -\frac{v^2}{2\Lambda^2} C_{HD}$$

С 0.500 0.100 LEP@LO LEP@NLO 0.050 ILC@LO ILC@NLO FCC@LO 0.010 FCC@NLO 0.005 Cau C_{eD} Ced C_{el}(3) C_{al}(1) G C ... (1)

> Improving these fits is a major motivation for future e+e- colliders

DIBOSON PRODUCTION IS OLD STORY

• Sensitive to variations of Zff and $Z(\gamma)WW$ couplings



No growth with energy in SM

- Individual contributions grow with energy
- Cancellations keep amplitudes from growing at high energy in SM

Changing gauge or fermion couplings spoils cancellation

Question: Who cares if cross sections grow with energy?

REAL WORLD CONSEQUENCES

- Probing the Weak Boson Sector in e⁺e⁻ →W⁺W (Hagiwara, Peccei, Zeppenfeld, Hikasa, 1987)
- At that time the structure of the 3 gauge boson interactions had not been verified experimentally



SMEFT CONTRIBUTIONS SPOIL THIS CANCELLATION!



CAN'T JUST FIT HIGGS COUPLINGS

Operators that contribute to VVV vertices and Higgs-VV vertices



Leads to concept of global fits

PRACTICAL PROBLEMS

$$L \to L_{SM} + \Sigma_i \frac{C_{6i}}{\Lambda^2} O_{6i} + \Sigma_i \frac{C_{8i}}{\Lambda^4} O_{8i} + \dots$$

- SMEFT
 - **1EFT** $A^2 \sim |A_{SM} + \frac{A_6}{\Lambda^2} + \dots |^2 \sim A_{SM}^2 + \frac{A_{SM}A_6}{\Lambda^2} + \frac{A_6^2}{\Lambda^4} + \dots$
- Problem is that $(A_6)^2$ terms are the same order as A_8 terms that we have dropped
- If I only keep A_6/Λ^2 terms and drop $(A_6/\Lambda^2)^2$, the cross section is not guaranteed to be positive
- Corrections are O(s/ Λ^2) or O(v²/ Λ^2)
- Information from distributions

COUNTING LORE

$$\sigma \sim g_{SM}^2 (A_{SM})^2 + g_{SM} g_{BSM} A_{SM} A_6 \frac{s}{\Lambda^2}$$

$$+ g_{BSM}^2 (A_6)^2 \frac{s^2}{\Lambda^4} + g_{SM} g_{BSM} A_{SM} A_8 \frac{s^2}{\Lambda^4}$$

$$\int$$
Same order of magnitude if $g_{SM} \sim g_{BSM}$
(Dim-6)² could dominate if $g_{BSM} >> g_{SM}$

Dimension-6 quadratic expansion can be valid for strongly interacting theory

OBVIOUS PROBLEM WHEN TRUNCATING THE SERIES WITH THE INTERFERENCE PIECE

• One proposal for dealing with this issue is to put a cut on the maximum energy where the SMEFT is assumed to be valid



ASSUMPTIONS CREEPING IN

- Single parameter fit to WW/WZ/WH/ZH
- For linear fit, throw out points with negative cross section
- Fit assumes SM efficiencies in each bin (not necessarily true)
- Fit ignores flavor



<u>2003.07862</u>

IS IT ALL THE LAST BIN?

• Fit results depend on cut on maximum energy



Plots successively remove high m_T bins

MANY GLOBAL FITS TO DATA

Take home message:

- Precision on operators varies over many order of magnitude
- Exact values of couplings sensitive to assumptions
- Rates (with QCD) can be generated automatically
- Compare linear and quadratic contributions



2105.00006

Challenge to experimental collaborations to include data outside of Higgs sector

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MANY NEW FITS

- Fits have different assumptions, different sets of data included
- Probing TeV scale new physics



Fit includes top, Higgs, EWPO, diboson
Compare results with and without top data
C~I: New physics generated at tree level
C~I/(4π)²: New physics generated at loop level

At dim-6 only sensitive to C/Λ^2

Ellis, Madigan, Mimasu, Sanz, You, 2012.02779

DREAM OF SMEFT FITTERS

- Fit to data \rightarrow non-zero coefficients
- Pattern of coefficients \rightarrow UV model
- Lots of theory holes to fill in this program





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THE HIGGS INVERSE PROBLEM

- If we measure non-zero SMEFT coefficients, can we determine the high scale model?
- In simple models (ie I new massive particle, whose interactions are described in terms of a single parameter) the particles that can contribute to dimension-6 operators have been categorized long ago <u>1711.10391</u>
- Dimension-6 contributions only sensitive to C/Λ^2



NUCLEAR PHYSICS AND SMEFT



- Familiar SMEFT modification of Z vertices and introduction of 4-fermion operators
- All the same issues as Higgs physics!

2306.05564

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	Joint EIC	Joint LHeC	Joint FCCeh	EW diboson, Higgs, and top data
$C_{\varphi D}$	[-3.8, 3.8]	[-0.019, 0.019]	[-0.013, 0.013]	[-1.6, 0.81]
$\frac{\Lambda}{\sqrt{C_{rD}}}$	0.51	7.2	8.8	0.91
$C_{\varphi WB}$	[-9.9, 9.9]	[-0.098, 0.098]	[-0.034, 0.034]	[-0.36, 0.73]
$\frac{\Lambda}{\sqrt{C_{rWB}}}$	0.32	3.2	5.4	1.4
$C^{(1)}_{\varphi q}$	[-38., 38.]	[-0.40, 0.40]	[-0.39, 0.39]	[-0.27, 0.18]
$\frac{\Lambda}{\sqrt{C_{eq}^{(l)}}}$	0.16	1.6	1.6	2.1
$C_{\varphi q}^{(3)}$	[-4.1, 4.1]	[-0.11, 0.11]	[-0.031, 0.031]	[-0.11, 0.012]
$\frac{\Lambda}{\sqrt{C_{rel}^{2}}}$	0.49	3.1	5.7	4.1
$C_{\varphi u}$	[-38., 38.]	[-0.51, 0.51]	[-0.45, 0.45]	[-0.63, 0.25]
$\frac{\Lambda}{\sqrt{C_{rn}}}$	0.16	1.4	1.5	1.5
$C_{\varphi d}$	[-84., 84.]	[-0.82, 0.82]	[-0.71, 0.71]	[-0.91, 0.13]
$\frac{\Lambda}{\sqrt{C_{rd}}}$	0.11	1.1	1.2	1.4
$C_{\varphi\ell}^{(1)}$	[-18., 18.]	[-0.094, 0.094]	[-0.060, 0.060]	[-0.19, 0.41]
$\frac{\Lambda}{\sqrt{C_{\varphi}^{(l)}}}$	0.23	3.3	4.1	1.8
$C_{\varphi \ell}^{(3)}$	[-4.1, 4.1]	[-0.060, 0.060]	[-0.022, 0.022]	[-0.13, 0.055]
$\frac{\Lambda}{\sqrt{C_{\mu}^{ch}}}$	0.49	4.1	6.7	3.3
$C_{\varphi e}$	[-5.7, 5.7]	[-0.16, 0.16]	[-0.046, 0.046]	[-0.41, 0.79]
$\frac{\Lambda}{\sqrt{C_{re}}}$	0.42	2.5	4.6	1.3
Cu	[-7.7, 7.7]	[-0.039, 0.039]	[-0.026, 0.026]	[-0.084, 0.02]
$\frac{\Lambda}{\sqrt{C_{er}}}$	0.36	5.1	6.2	4.4

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