Analysis of the higher twist GTMD F_{31} for proton in the light-front quark-diquark model.

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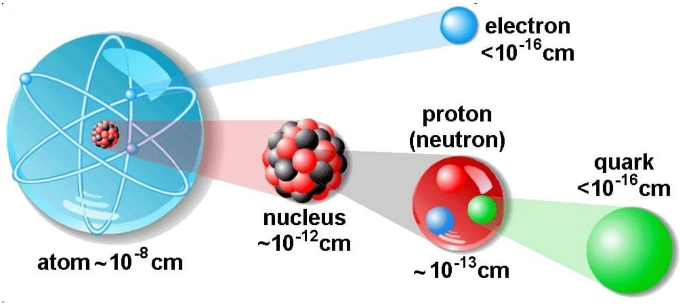
9 June, 2023

Outline

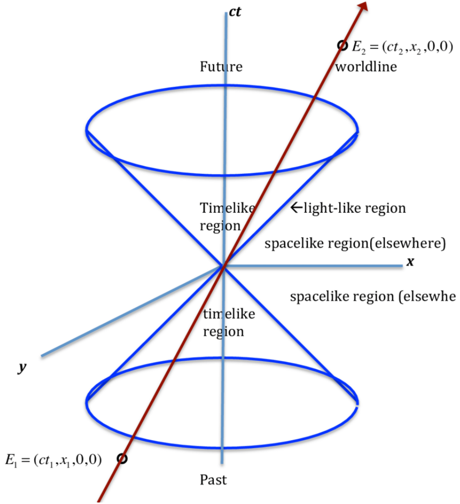
Introduction



- The theory of the strong interaction which provides the fundamental description of hadronic structure and dynamics in terms of their elementary quarks and gluons degrees of freedom is Quantum Chromodynamics (QCD).
- The foremost problem of hadron physics is to unravel the internal structure of hadron.

From Special Theory of Relativity:

- Space and time independently are not invariant quantities.
- Rather space-time is an invariant object.



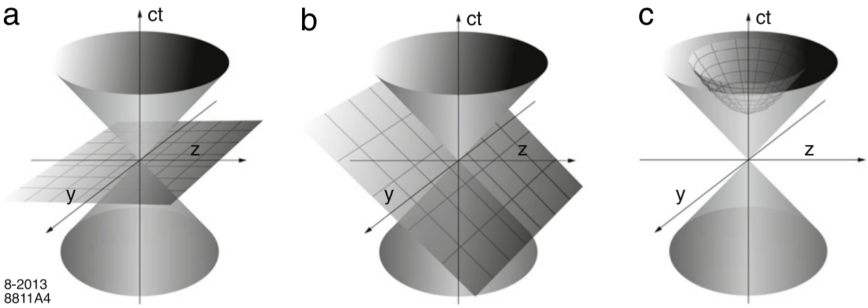


Figure 1: (a) the instant form, (b) the front form, (c) the point form.

Their initial surfaces are

a) $x^0 = 0$

b) $x^0 + x^3 = 0$

c) $x^2 = a^2 > 0, x^0 > 0$

Why Light Front?

- It is an Ideal Framework to describe theoretically the hadronic structure in terms of quarks and gluons. It can overcome many obstacles and has many advantages:
  - Simple vacuum structure ∼ vacuum expectation value is zero.
  - A dynamical system is characterized by ten fundamental quantities: energy, momentum, angular momentum and boost.

    ∼ seven out of which are kinematical. It allows unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents.
  - Dispersion Relation (for ON shell particles)

    $$k^- = \frac{(k\perp)^2+m^2}{k^+}$$

    ∼ no square root factor.

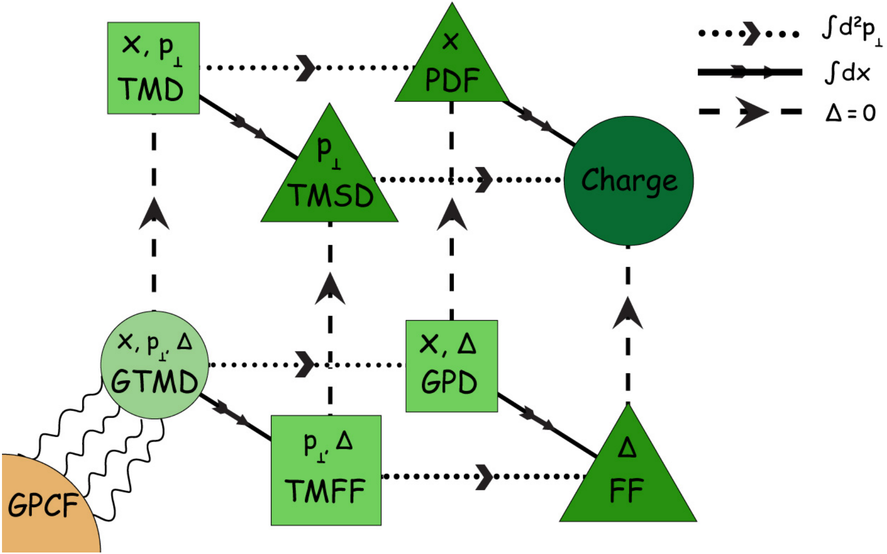
Light-Front Coordinates

- A generic four Vector x^μ in light-cone coordinates is describe as $x^\mu = (x^-, x^+, x_\perp)$.
- $x^+ = x^0 + x^3$ is called as light-front time.
- $x^- = x^0 - x^3$ is called as light-front longitudinal space variable.
- $x^\perp = (x^1, x^2)$ is the transverse variable.
- Similarly, we can define the longitudinal momentum $k^+ = k^0 + k^3$ and light-front energy $k^- = k^0 - k^3$.

Distribution Function

- The spatial distribution of charge and current in a system can be probed through elastic scattering of electrons, photons etc.
- The distribution of the constituents in momentum space can be measured through deep inelastic knock-out scattering.

Relation between GTMDs, TMDs, GPDs and PDFs



Outline

Light-Front Quark-Diquark Model I

- In this model the proton is described as an aggregate of an active quark and a diquark spectator of definite mass.
- The proton has spin-flavor $SU(4)$ structure and it has been expressed as a made up of isoscalar-scalar diquark singlet $|u\, S^0\rangle$, isoscalar-vector diquark $|u\, A^0\rangle$ and isovector-vector diquark $|d\, A^1\rangle$ states as [1, 2]

$$|P;\pm\rangle = C_S|u\, S^0\rangle^{\pm} + C_V|u\, A^0\rangle^{\pm} + C_{VV}|d\, A^1\rangle^{\pm}.$$

  Here, the scalar and vector diquark has been denoted by S and A respectively. Their isospin has been represented by the superscripts on them.
- The light-cone convention $z^{\pm} = z^0 \pm z^3$ has been used.

Light-Front Quark-Diquark Model II

- The frame is picked such that the proton's average momentum (P) and the momentum transfer (Δ) between the initial and the final state is

$$P \equiv \left(P^+, \frac{M^2 + \mathbf{\Delta}_\perp^2/4}{P^+}, \mathbf{0}_\perp\right),$$
$$\Delta \equiv \left(0, 0, \mathbf{\Delta}_\perp\right).$$

- The momentum of the smacked quark (p) and diquark (P_X) are

$$p \equiv \left(xP^+, \frac{p^2 + |\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp\right),$$
$$P_X \equiv \left((1-x)P^+, P_X^-, -\mathbf{p}_\perp\right).$$

Light-Front Quark-Diquark Model III

- The Fock-state expansion in the case of two particle for $J^z = \pm 1/2$ for the scalar diquark can be expressed as

$$|u\,S\rangle^{\pm} = \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}}\Bigg[\psi_+^{\pm(\nu)}(x,\mathbf{p}_\perp)\Bigg|+\frac{1}{2}\ s; xP^+, \mathbf{p}_\perp\Bigg\rangle + \psi_-^{\pm(\nu)}(x,\mathbf{p}_\perp)\Bigg|-\frac{1}{2}\ s; xP^+, \mathbf{p}_\perp\Bigg\rangle\Bigg],$$

  where, flavour index is $\nu = u, d$.

- $|\lambda_q\,\lambda_S\,; xP^+, \mathbf{p}_\perp\rangle$ represents the state of two particle having helicity of struck quark as λ_q and helicity of a scalar diquark as λ_S.

Light-Front Quark-Diquark Model IV

- The LFWFs for the scalar diquark are expressed as [3]

$$
\begin{aligned}
\psi_{+}^{+(\nu)}(x, \mathbf{p}_\perp) &= N_S\ \varphi_1^{(\nu)}(x, \mathbf{p}_\perp),\\
\psi_{-}^{+(\nu)}(x, \mathbf{p}_\perp) &= N_S\left(-\frac{p^1 + ip^2}{xM}\right)\varphi_2^{(\nu)}(x, \mathbf{p}_\perp),\\
\psi_{+}^{-(\nu)}(x, \mathbf{p}_\perp) &= N_S\left(\frac{p^1 - ip^2}{xM}\right)\varphi_2^{(\nu)}(x, \mathbf{p}_\perp),\\
\psi_{-}^{-(\nu)}(x, \mathbf{p}_\perp) &= N_S\ \varphi_1^{(\nu)}(x, \mathbf{p}_\perp).
\end{aligned}
$$

Here $\varphi_i^{(\nu)}(x, \mathbf{p}_\perp)$ are LFWFs and N_S is the normalization constant.

Light-Front Quark-Diquark Model V

- Similarly, Fock-state expansion in the case of two particle for the vector diquark is given as [4]

$$
\begin{aligned}
|\nu\, A\rangle^{\pm} = \int \frac{dx\, d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \Bigg[& \psi_{++}^{\pm(\nu)}(x,\mathbf{p}_\perp) \bigg| +\frac{1}{2}\ +1; xP^+, \mathbf{p}_\perp \bigg\rangle \\
& + \psi_{-+}^{\pm(\nu)}(x,\mathbf{p}_\perp) \bigg| -\frac{1}{2}\ +1; xP^+, \mathbf{p}_\perp \bigg\rangle + \psi_{+0}^{\pm(\nu)}(x,\mathbf{p}_\perp) \bigg| +\frac{1}{2}\ 0; xP^+, \mathbf{p}_\perp \bigg\rangle \\
& + \psi_{-0}^{\pm(\nu)}(x,\mathbf{p}_\perp) \bigg| -\frac{1}{2}\ 0; xP^+, \mathbf{p}_\perp \bigg\rangle + \psi_{+-}^{\pm(\nu)}(x,\mathbf{p}_\perp) \bigg| +\frac{1}{2}\ -1; xP^+, \mathbf{p}_\perp \bigg\rangle \\
& + \psi_{--}^{\pm(\nu)}(x,\mathbf{p}_\perp) \bigg| -\frac{1}{2}\ -1; xP^+, \mathbf{p}_\perp \bigg\rangle \Bigg].
\end{aligned}
$$

Here $|\lambda_q\ \lambda_D; xP^+, \mathbf{p}_\perp\rangle$ is the state of two-particle with helicity of quark being $\lambda_q = \pm\frac{1}{2}$ and helicity of vector diquark being $\lambda_D = \pm 1, 0$ (triplet).

Light-Front Quark-Diquark Model VI

- The LFWFs for the vector diquark for the case when $J^z = +1/2$ are given as

$$\psi^{+(\nu)}_{+\,+}(x,\mathbf{p}_\perp) = N_1^{(\nu)}\sqrt{\frac{2}{3}}\Big(\frac{p^1 - ip^2}{xM}\Big)\varphi_2^{(\nu)}(x,\mathbf{p}_\perp),$$

$$\psi^{+(\nu)}_{-\,+}(x,\mathbf{p}_\perp) = N_1^{(\nu)}\sqrt{\frac{2}{3}}\varphi_1^{(\nu)}(x,\mathbf{p}_\perp),$$

$$\psi^{+(\nu)}_{+\,0}(x,\mathbf{p}_\perp) = -N_0^{(\nu)}\sqrt{\frac{1}{3}}\varphi_1^{(\nu)}(x,\mathbf{p}_\perp),$$

$$\psi^{+(\nu)}_{-\,0}(x,\mathbf{p}_\perp) = N_0^{(\nu)}\sqrt{\frac{1}{3}}\Big(\frac{p^1 + ip^2}{xM}\Big)\varphi_2^{(\nu)}(x,\mathbf{p}_\perp),$$

$$\psi^{+(\nu)}_{+\,-}(x,\mathbf{p}_\perp) = 0,$$

$$\psi^{+(\nu)}_{-\,-}(x,\mathbf{p}_\perp) = 0,$$

Light-Front Quark-Diquark Model VII

- The LFWFs for the vector diquark for the case when $J^z = -1/2$ are given as

$$\psi_{+\,+}^{-(\nu)}(x, \mathbf{p}_\perp) = 0,$$
$$\psi_{-\,+}^{-(\nu)}(x, \mathbf{p}_\perp) = 0,$$
$$\psi_{+\,0}^{-(\nu)}(x, \mathbf{p}_\perp) = N_0^{(\nu)} \sqrt{\frac{1}{3}} \Big(\frac{p^1 - ip^2}{xM}\Big) \varphi_2^{(\nu)}(x, \mathbf{p}_\perp),$$
$$\psi_{-\,0}^{-(\nu)}(x, \mathbf{p}_\perp) = N_0^{(\nu)} \sqrt{\frac{1}{3}} \varphi_1^{(\nu)}(x, \mathbf{p}_\perp),$$
$$\psi_{+\,-}^{-(\nu)}(x, \mathbf{p}_\perp) = -N_1^{(\nu)} \sqrt{\frac{2}{3}} \varphi_1^{(\nu)}(x, \mathbf{p}_\perp),$$
$$\psi_{-\,-}^{-(\nu)}(x, \mathbf{p}_\perp) = N_1^{(\nu)} \sqrt{\frac{2}{3}} \Big(\frac{p^1 + ip^2}{xM}\Big) \varphi_2^{(\nu)}(x, \mathbf{p}_\perp),$$

where N_0, N_1 are the normalization constants.

Light-Front Quark-Diquark Model VIII

- Generic ansatz of LFWFs $\varphi_i^{(\nu)}(x, \mathbf{p}_\perp)$ is being adopted from the soft-wall AdS/QCD prediction [5, 6] and the parameters $a_i^\nu,\ b_i^\nu$ and δ^ν are established as [7]

$$\varphi_i^{(\nu)}(x, \mathbf{p}_\perp) = \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}\, x^{a_i^\nu}(1-x)^{b_i^\nu} \exp\left[-\delta^\nu \frac{\mathbf{p}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right].$$

Outline

Input Parameters I

- The parameters a_i^ν and b_i^ν, have been fitted at model scale $\mu_0 = 0.313$ GeV using the Dirac and Pauli data of form factors. [8, 9, 10].

ν	a_1^ν	b_1^ν	a_2^ν	b_2^ν	δ^ν
u	0.280	0.1716	0.84	0.2284	1.0
d	0.5850	0.7000	0.9434	0.64	1.0

Table 1: Values of model parameters corresponding to up and down quarks.

ν	N_S	N_0^ν	N_1^ν
u	2.0191	3.2050	0.9895
d	2.0191	5.9423	1.1616

Table 2: Values of normalization constants N_i^2 corresponding to both up and down quarks.

Input Parameters II

- The AdS/QCD scale parameter κ is chosen to be 0.4 GeV [11].
- Constituent quark mass (m) and the proton mass (M) are taken to be 0.055 GeV and 0.938 GeV sequentially.
- The coefficients C_i of scalar and vector diquarks are given as

$$\begin{aligned} C_S^2 &= 1.3872, \\ C_V^2 &= 0.6128, \\ C_{VV}^2 &= 1. \end{aligned}$$

Outline

GTMD Correlator I

GTMD Correlator

- The fully unintegrated quark-quark correlator $W^{\nu[\Gamma]}_{[\Lambda^{N_i}\Lambda^{N_f}]}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ for a spin-$\frac{1}{2}$ hadron at the fixed light-cone time $z^+ = 0$, is defined as [15]

$$W^{\nu[\Gamma]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2}\int \frac{dz^-}{(2\pi)}\frac{d^2 z_T}{(2\pi)^2} e^{ip.z}\langle P^f; \Lambda^{N_f}|\bar{\psi}^\nu(-z/2)\Gamma\mathcal{W}_{[-z/2,z/2]}\psi^\nu(z/2)|P^i; \Lambda^{N_i}\rangle\Big|_{z^+=0}$$

- $|P^i; \Lambda^{N_i}\rangle$ and $|P^f; \Lambda^{N_f}\rangle$ are the initial and final states of the proton with helicities Λ^{N_i} and Λ^{N_f}, respectively.
- The initial and final four momenta of the proton are then given by

$$P^i \equiv \left(P^+, \frac{M^2 + \mathbf{\Delta}_\perp^2/4}{P^+}, -\mathbf{\Delta}_\perp/2\right),$$
$$P^f \equiv \left(P^+, \frac{M^2 + \mathbf{\Delta}_\perp^2/4}{P^+}, \mathbf{\Delta}_\perp/2\right).$$

GTMD Correlator II

- The frame is picked such that the proton's average momentum (P) and the momentum transfer (Δ) between the initial and the final state is

$$P \equiv \left(P^+, \frac{M^2 + \mathbf{\Delta}_\perp^2/4}{P^+}, \mathbf{0}_\perp \right),$$
$$\Delta \equiv \left(0, 0, \mathbf{\Delta}_\perp \right).$$

- The momentum of the smacked quark (p) and diquark (P_X) are

$$p \equiv \left(xP^+, \frac{p^2 + |\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp \right),$$
$$P_X \equiv \left((1-x)P^+, P_X^-, -\mathbf{p}_\perp \right).$$

- The square of the total momentum transfer is $t = \Delta^2 = -\mathbf{\Delta}_\perp^2$.
- The value of Wilson line $\mathcal{W}_{[0,z]}$ is chosen to be 1.

GTMD Parameterization for proton at twist-4

$$
\begin{aligned}
W^{[\gamma^-]}_{[\Lambda^{N_i}\Lambda^{N_f}]} &= \frac{M}{2(P^+)^2}\,\bar{u}(P^f,\Lambda^{N_F})\Big[F_{3,1} + \frac{i\sigma^{i+}p_T^i}{P^+}\,F_{3,2} + \frac{i\sigma^{i+}\Delta_T^i}{P^+}\,F_{3,3} \\
&\quad + \frac{i\sigma^{ij}p_T^i\Delta_T^j}{M^2}\,F_{3,4}\Big]u(P^i,\Lambda^{N_i})\,,
\end{aligned}
$$

$$
\begin{aligned}
W^{[\gamma^-\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} &= \frac{M}{2(P^+)^2}\,\bar{u}(P^f,\Lambda^{N_F})\Big[-\frac{i\varepsilon_T^{ij}p_T^i\Delta_T^j}{M^2}\,G_{3,1} + \frac{i\sigma^{i+}\gamma_5 p_T^i}{P^+}\,G_{3,2} + \frac{i\sigma^{i+}\gamma_5\Delta_T^i}{P^+}\,G_{3,3} \\
&\quad + i\sigma^{+-}\gamma_5\,G_{3,4}\Big]u(P^i,\Lambda^{N_i})\,,
\end{aligned}
$$

$$
\begin{aligned}
W^{[i\sigma^{j-}\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} &= \frac{M}{2(P^+)^2}\,\bar{u}(P^f,\Lambda^{N_F})\Big[-\frac{i\varepsilon_T^{ij}p_T^i}{M}\,H_{3,1} - \frac{i\varepsilon_T^{ij}\Delta_T^i}{M}\,H_{3,2} + \frac{M\,i\sigma^{j+}\gamma_5}{P^+}\,H_{3,3} \\
&\quad + \frac{p_T^j\,i\sigma^{p+}\gamma_5 p_T^p}{M\,P^+}\,H_{3,4} + \frac{\Delta_T^j\,i\sigma^{p+}\gamma_5 p_T^p}{M\,P^+}\,H_{3,5} + \frac{\Delta_T^j\,i\sigma^{p+}\gamma_5\Delta_T^p}{M\,P^+}\,H_{3,6} \\
&\quad + \frac{p_T^j\,i\sigma^{+-}\gamma_5}{M}\,H_{3,7} + \frac{\Delta_T^j\,i\sigma^{+-}\gamma_5}{M}\,H_{3,8}\Big]u(P^i,\Lambda^{N_i})\,.
\end{aligned}
$$

Outline

Results

- For proton, the twist-4 GTMD $F^{\nu}_{31}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ for up quark is given as

$$
\begin{aligned}
F^{u}_{31} = \frac{1}{16\pi^3}\frac{1}{4x^2M^2}\Big(C_S^2 N_s^2 + \frac{1}{3}C_V^2\big(|N_0^u|^2 + |N_1^u|^2\big)\Big) \\
\Big[(4m^2 + 4p_\perp^2 - \Delta_\perp^2)|\varphi_1^u|^2 + (\frac{4m(1-x)\Delta_\perp^2}{xM})|\varphi_1^u||\varphi_2^u| \\
+\Big[(4m^2 + 4p_\perp^2 - \Delta_\perp^2)(p_\perp^2 - \frac{(1-x)^2}{4}\Delta_\perp^2) \\
+4(1-x)\big(p_\perp^2\Delta_\perp^2 - (p_\perp \cdot \Delta_\perp)^2\big)\Big]\frac{|\varphi_2^u|^2}{x^2M^2}\Big]
\end{aligned}
$$

- For proton, the twist-4 GTMD $F_{31}^{\nu}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ for down quark is given as

$$
\begin{aligned}
F_{31}^{d} = \frac{1}{16\pi^3}\frac{1}{4x^2M^2}\Bigg(\frac{1}{3}C_{VV}^2\Big(|N_0^d|^2 + |N_1^d|^2\Big)\Bigg) \\
\Bigg[(4m^2 + 4p_\perp^2 - \Delta_\perp^2)|\varphi_1^d|^2 + (\frac{4m(1-x)\Delta_\perp^2}{xM})|\varphi_1^d||\varphi_2^d| \\
+\Big[(4m^2 + 4p_\perp^2 - \Delta_\perp^2)(p_\perp^2 - \frac{(1-x)^2}{4}\Delta_\perp^2) \\
+4(1-x)(p_\perp^2\Delta_\perp^2 - (p_\perp \cdot \Delta_\perp)^2)\Big]\frac{|\varphi_2^d|^2}{x^2M^2}\Bigg]
\end{aligned}
$$

- The model relation of TMD $f_3^{\nu}(x, \mathbf{p}_\perp)$ with twist-2 TMD $f_1^{\nu}(x, \mathbf{p}_\perp)$ [7, 17]

$$
x^2\, f_3^{\nu}(x, \mathbf{p}_\perp) \overset{LFQDM}{=} \left(\frac{p_\perp^2 + m^2}{M^2}\right) f_1^{\nu}(x, \mathbf{p}_\perp).,
$$

[Sharma and Dahiya, IJMPA (2022)]

- Average Transverse Momentum

$$\langle \mathbf{p}_\perp^r(\Upsilon)\rangle^\nu = \frac{\int dx \int d^2p_\perp p_\perp^r \Upsilon^\nu(x,\mathbf{p}_\perp^2)}{\int dx \int d^2p_\perp \Upsilon^\nu(x,\mathbf{p}_\perp^2)}.$$

MODEL	f_3^ν (LFQDM)	f_3^ν (LFCQM)
$\langle p_\perp\rangle^u$	0.26	0.28
$\langle p_\perp\rangle^d$	0.27	0.28
$\langle p_\perp^2\rangle^u$	0.073	0.11
$\langle p_\perp^2\rangle^d$	0.078	0.11

Table 3: Comparison of average transverse momentum in units of GeV and average transverse momentum squares in units of GeV^2 for TMD $f_3^\nu(x,\mathbf{p}_\perp^2)$ in LFQDM (our model) and LFCQM [18].

[Sharma and Dahiya, IJMPA (2022)]

x and $p_\perp$ Dependence at TMD limit

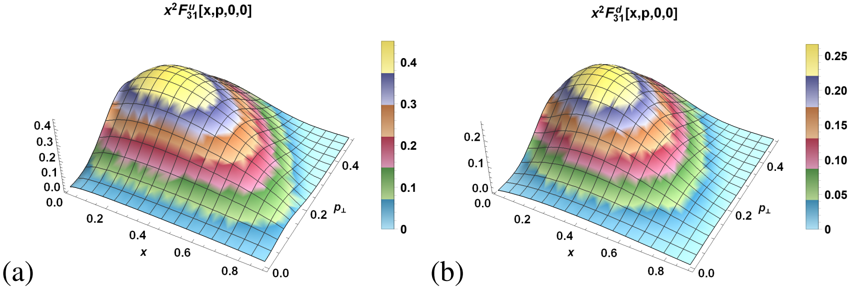
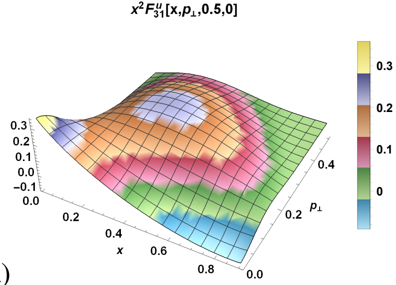
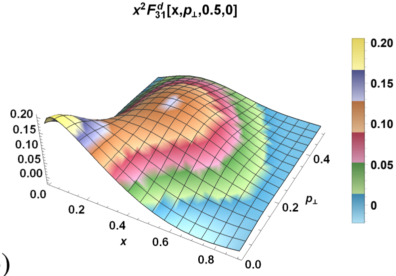


Figure 2: The GTMD $x^2 F^{\nu}_{31}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ is plotted with respect to x and $\mathbf{p}_\perp$ at $\mathbf{\Delta}_\perp = \mathbf{0}$ (i.e., at TMD limit). The left and right column correspond to u and d quarks sequentially.

x and $p_\perp$ Dependence at $\Delta_\perp = 0.5$ GeV



(a)



(b)

Figure 3: The GTMD $x^2 F^{\nu}_{31}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ is plotted with respect to x and $\mathbf{p}_\perp$ at $\mathbf{\Delta}_\perp = \mathbf{0}$ (i.e., at $\mathbf{\Delta}_\perp = 0.5$ GeV). The left and right column correspond to u and d quarks sequentially.

x and $\Delta_\perp$ Dependence

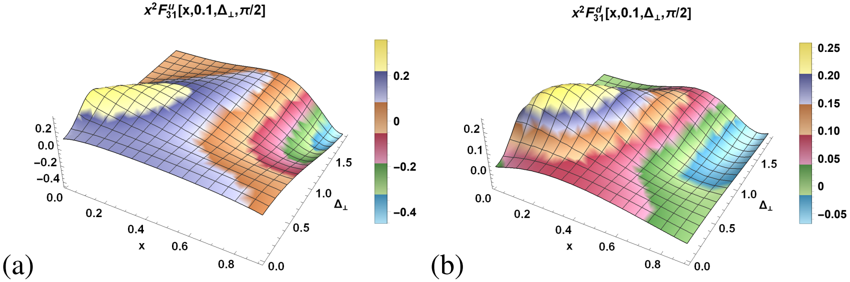


Figure 4: The GTMD $x^2F^{\nu}_{31}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ is plotted with respect to x and $\mathbf{\Delta}_\perp$ at $\mathbf{p}_\perp = 0.1$ and $\theta = \frac{\pi}{2}$. The left and right column correspond to u and d quarks sequentially.

$p_\perp$ and $\Delta_\perp$ Dependence

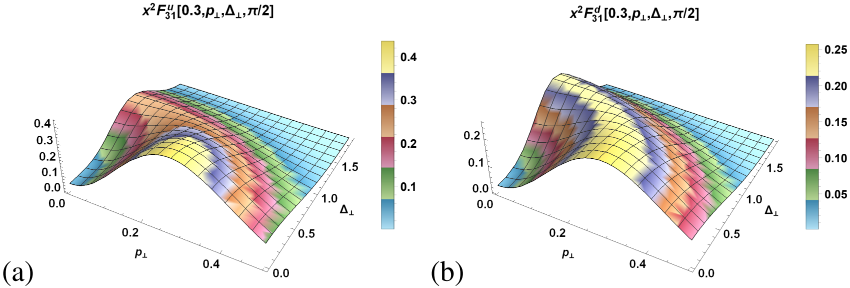


Figure 5: The GTMD $x^2F^\nu_{31}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ is plotted with respect to $\mathbf{p}_\perp$ and $\mathbf{\Delta}_\perp$ at $x = 0.3$ and $\theta = \frac{\pi}{2}$. The left and right column correspond to u and d quarks sequentially.

Outline

Summary I

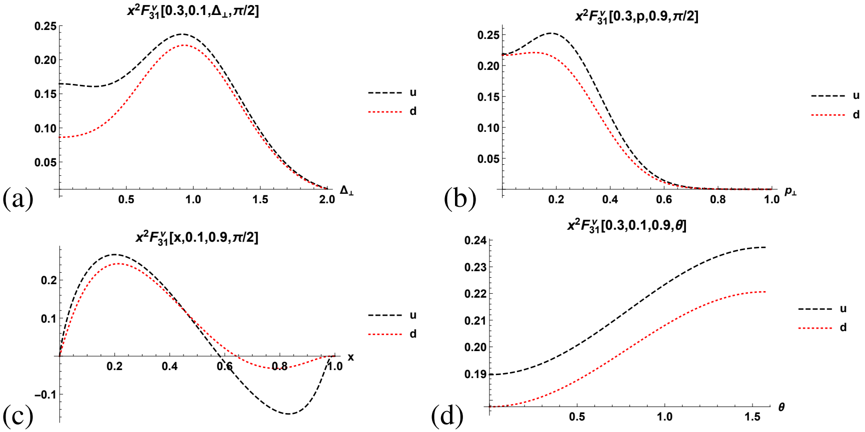


Figure 6: The GTMD $x^2F_{31}^{\nu}(x,\mathbf{p}_\perp,\mathbf{\Delta}_\perp,\theta)$ is plotted with respect to its variables one by one while keeping the other fixed.

Summary II

- The GTMD $x^2 F_{31}^{\nu}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ is plotted with respect to x and $\mathbf{p}_\perp$ at $\mathbf{\Delta}_\perp = \mathbf{0}$ (i.e., at TMD limit $x^2 f_3(x, \mathbf{p}_\perp)$). The GTMD remains positive for both u and d quarks.
- In plots of GTMD $x^2 F_{31}^{\nu}(x, \mathbf{p}_\perp, \mathbf{\Delta}_\perp, \theta)$ for both u and d quarks, it has been observed that the value diminishes
  - when $\mathbf{p}_\perp$ greater than 0.6 GeV
  - when $\mathbf{\Delta}_\perp$ greater than 1.9 GeV
  - the possibility of distribution exists only at an optimal blend of $\mathbf{p}_\perp$ and $\mathbf{\Delta}_\perp$ values.
- The GTMD F_{31} does not flip its sign on changing the quark flavour from u to d quarks.
- As the orientation between $\mathbf{p}_\perp$ and $\mathbf{\Delta}_\perp$ changes from parallel to perpendicular, the value of GTMD increases.

Thank you!

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