Odderon Mechanism for Transverse Single Spin Asymmetry in Wandzura – Wilczek Approximation

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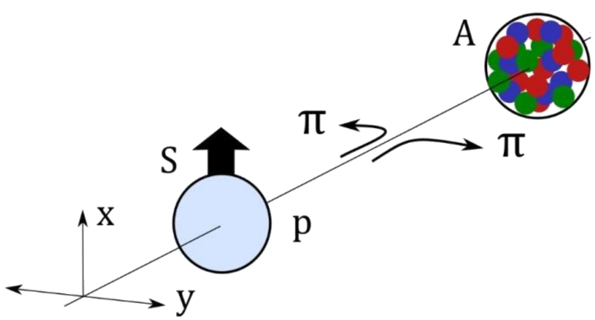
School: The 2023 CFNS-CTEQ Summer School on the Physics of the Electron-Ion Collider

S. Benić, D. Horvatić, A. Kaushik and E. A. Vivoda, Phys. Rev D **106**, 114025 (2022).

Transverse Single Spin Asymmetry

- Left-right asymmetry of produced particles in collisions involving polarized hadrons

- Known to be largest in forward region → small x effects in target



Author: Dr. Sanjin Benić.

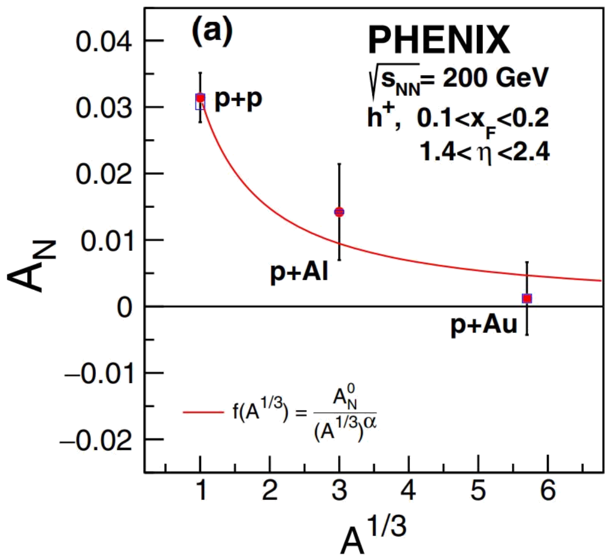
$$A_N \equiv \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$

$$A_N \sim \sin(\varphi_h - \varphi_S)$$

- **Spin vector comes with a factor of i, so to make cross section real, one has to find another factor of i from diagrams!**

A.V. Eferemov and O.V. Teryaev, Sov. J. Nucl. Phys. **36**, 140 (1982).

TSSA in pA (data):

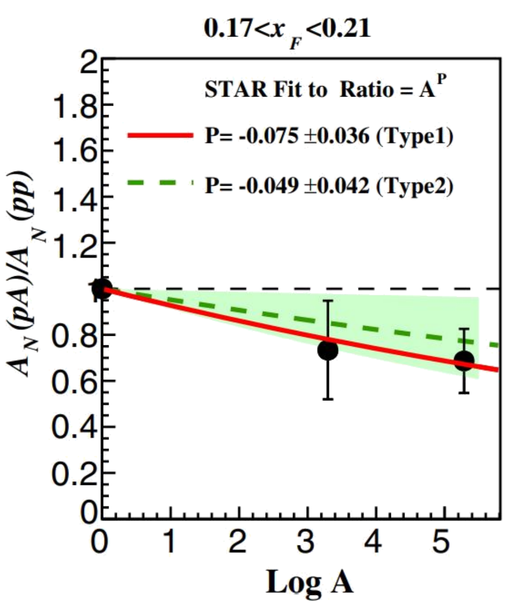


$$A_N \sim A^{-\frac{1}{3}}$$

PHENIX Collaboration, C. Aidala et. al., Phys.Rev.Lett. **123**, 122001 (2019).

$1.8 < P_{hT} < 7.0$ GeV (integrated)

$0.004 \leq x \leq 0.1$



$$A_N \sim A^{-0.027 \pm 0.005}$$

Star Collaboration, J. Adam et. al., Phys.Rev.D. **103**, 072005 (2021).

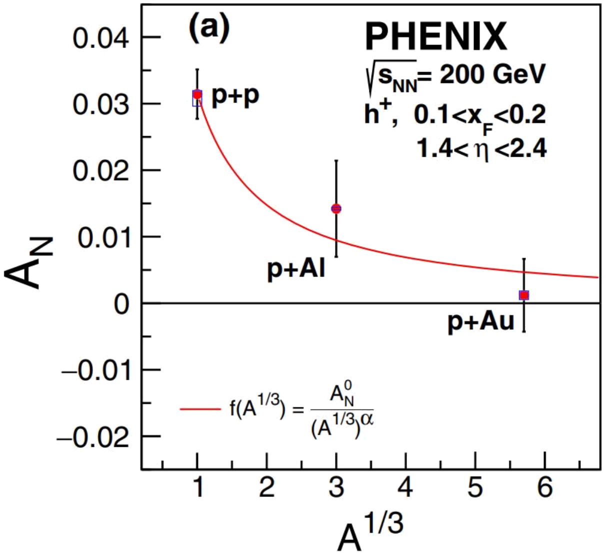
$1.5 < P_{hT} < 2.0$ GeV

$2.0 < P_{hT} < 3.0$ GeV…

$x < 0.005$

TSSA in pA (data):

A dependance!



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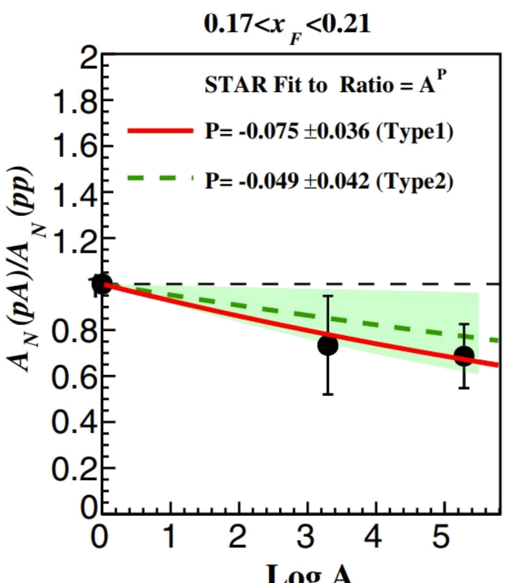

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$x < 0.005$

CGC in pA collisions:

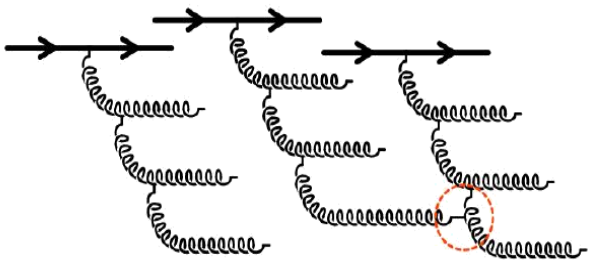
- Small x in nuclei (**Color Glass Condensate** (CGC) framework)
  - ➢ Effects of small-x and spin physics combined!
  - ➢ Small x gluons dominate in the description of collision

C SU(3) charge

G $\Delta x^+ \propto x$ (spin glass)

C Lot of gluons

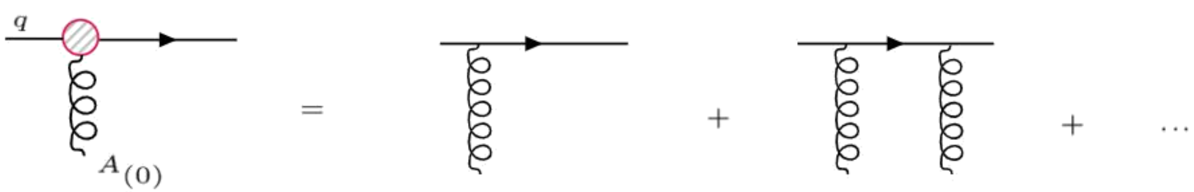
Classicall YM equations!



F. Gelis, 1211.3327

$$\Gamma_{gg\to g} \propto \frac{x}{Q_s^2} \longrightarrow$$ Q_s- Saturation scale

$$(Q_s^A)^2 = A^{\frac{1}{3}}\left(Q_s^p\right)^2 \longrightarrow$$ Nuclei saturate at higher x!



$$\propto (V(x_\perp) - 1)$$

Wilson line

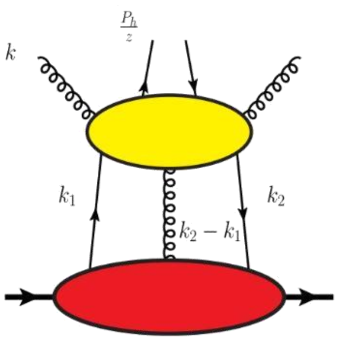
ETQS mechanism in pA

Y. Hatta, B.-W. Xiao, S. Yoshida and F. Yuan, Phys. Rev. D **94**, 054013 (2016).

- Hybrid approach:
  - Eferemov-Teryaev-Qiu-Sterman (ETQS) functions for projectile
  - Unintegrated gluon distribution for target

J. W. Qiu and G.F. Sterman, Phys. Rev. Lett. **67**, 2264 (1991).

A.V. Efremov and O.V. Teryaev , Phys. Lett. B **150**, 383 (1985).



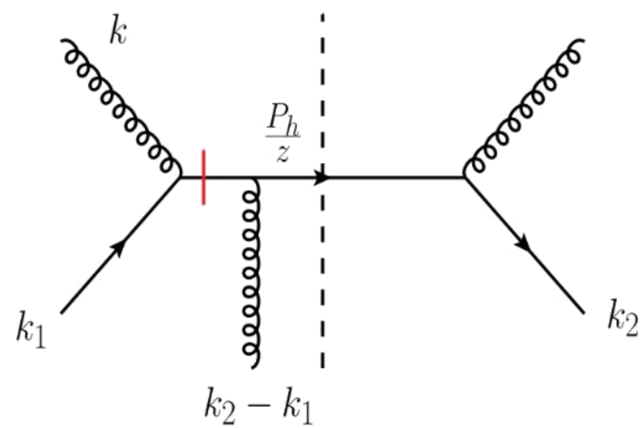
$$\frac{d^3\Delta\sigma(p^\uparrow A \to hX)}{dy_h d^2P_{hT}} = \epsilon^{\alpha\beta} P_{h\alpha} S_{T\beta} \int_{x_F} \frac{dz}{z^2} D_{h/q}(z) G_F(x_p, x_p) \otimes F(x_g, P_{hT}/z)$$

$$G_F \propto \langle \psi F \bar{\psi} \rangle$$

$$\frac{d\sigma^{\rm SGP}}{dy_h d^2P_{hT}} = \frac{\pi M x_F}{2(N_c^2-1)} \epsilon^{\alpha\beta} S_{T\beta} \int_{x_F}^1 \frac{dz}{z^3} D(z) \left\{ -\frac{1}{(P_{hT}/z)^2} \times \frac{\partial}{\partial P_h^\alpha/z} \left(\frac{P_{hT}^2}{z^2} F(x_g, P_{hT}/z) \right) G_F(x,x) + \frac{2P_{h\alpha}/z}{(P_{hT}/z)^2} F(x_g, P_{hT}/z) x \frac{d}{dx} G_F(x,x) \right\}$$

Dominant term in forward region

SGP = Soft-gluon pole:



$$\frac{A_N^{pp}}{A_N^{pA}} \approx 1 \longrightarrow$$ Phenix data can't be explained

Odderon mechanism for TSSA

- **Odderon** = imaginary part of **dipole distribution function**

$$\mathcal{S}(\boldsymbol{x}_\perp, \boldsymbol{x}'_\perp) \equiv \frac{1}{N_C} \mathrm{tr}\langle V(\boldsymbol{x}_\perp) V^\dagger(\boldsymbol{x}'_\perp)\rangle \qquad \mathcal{S}(\boldsymbol{x}_\perp, \boldsymbol{x}'_\perp) \equiv \mathcal{P}(\boldsymbol{x}_\perp, \boldsymbol{x}'_\perp) + i\mathcal{O}(\boldsymbol{x}_\perp, \boldsymbol{x}'_\perp)$$

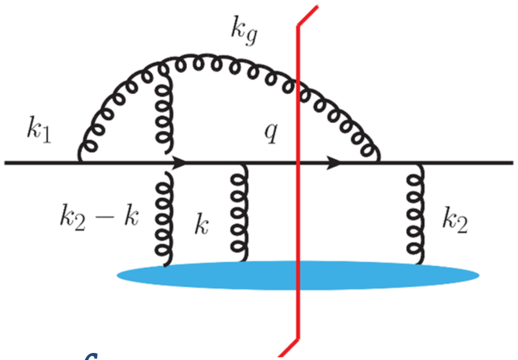
- Wilson lines come from the CGC propagator

- Odderon can supply necessary phase!

- Calculated at parton level (qA collisions)

Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D **86**, 034028 (2012).

- We need interference diagrams to get non-zero contributions to TSSA



$$\boldsymbol{r}_\perp = \boldsymbol{x}_\perp - \boldsymbol{y}_\perp$$

$$\boldsymbol{b}_\perp = \frac{1}{2}(\boldsymbol{x}_\perp + \boldsymbol{y}_\perp)$$

$$\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp = \frac{1}{2}(\boldsymbol{r}_\perp + \boldsymbol{r}'_\perp)$$

$$E_q \frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^3 q} \propto i\alpha \int_{\boldsymbol{k}_\perp \boldsymbol{k}_{2\perp}} \int_{\boldsymbol{r}_\perp \boldsymbol{b}_\perp \boldsymbol{r}'_\perp}$$

$$[\mathcal{P}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)\mathcal{O}(\boldsymbol{r}'_\perp, \boldsymbol{b}'_\perp) - \mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)\mathcal{P}(\boldsymbol{r}'_\perp, \boldsymbol{b}'_\perp)]\, \mathcal{H}(\boldsymbol{r}_\perp, \boldsymbol{r}'_\perp, \boldsymbol{S}_\perp)$$

Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D **86**, 034028 (2012).

- Result: $A_N \propto A^{-\frac{7}{6}}$

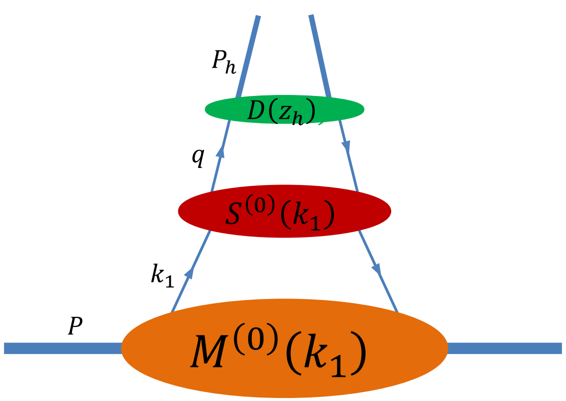
- What will change when we go from partonic to hadronic level?
- What is responsible PDF?

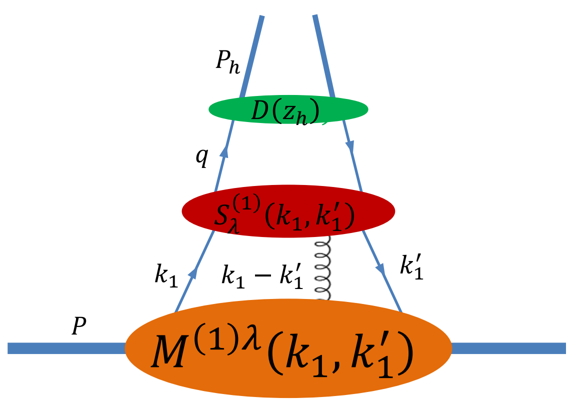
Polarized cross section in pA collisions

S. Benić, Y. Hatta, H. Li, D.J. Yang,
Phys. Rev. D **100**, 094027 (2019).

- All order formula for twist-3 cross section:

$$
\begin{aligned}
E_h \frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^3 P_h} = \frac{1}{2(2\pi)^3}\int \frac{\mathrm{d}z_h}{z_h^2} D(z_h) \Bigg\{ & \frac{M_N}{2}\int \mathrm{d}x_p\, g_T(x_p)\,\mathrm{Tr}\left[\gamma_5 \not{S}_\perp S^{(0)}(p_1)\right] + \qquad \text{Intrinsic} \\
& \frac{M_N}{2}\int \mathrm{d}x_p\, g_{1T}^{(1)}(x_p)\,\mathrm{Tr}\left[\gamma_5 \not{P}_p S_\perp^\lambda \left(\frac{\partial S^{(0)}(k_1)}{\partial k_{1\perp}^\lambda}\right)_{k_1=p_1}\right] + \qquad \text{Kinematical} \\
& \frac{iM_N}{4}\int \mathrm{d}x_p \mathrm{d}x_p'\,\mathrm{Tr}\left[\left(\not{P}_p \epsilon^{\bar{n}n\lambda S_\perp}\frac{G_F(x_p,x_p')}{x_p - x_p'} + i\gamma_5 \not{P}_p S_\perp^\lambda \frac{\tilde{G}_F(x_p,x_p')}{x_p - x_p'}\right) S_\lambda^{(1)}(x_p P_p, x_p' P_p)\right]\Bigg\} \qquad \text{Dynamical}
\end{aligned}
$$





$$
g_T(x) = \int_x^1 \frac{dx'}{x'}\Delta_q(x') + \cdots
$$

Helicity quark PDF

Wandzura – Wilczek approximation

$$x g_T(x) \approx g_{1T}^{(1)}(x)$$

- Neglect all genuine twist-3 contributions

$$E_h \frac{\mathrm{d}\Delta\sigma}{\mathrm{d}^3 P_h} \simeq \frac{1}{2(2\pi)^3} \frac{M_N}{2} \int \frac{\mathrm{d}z_h}{z_h^2} D(z_h) \int dx_p g_T(x_p) \times \left(S_\perp^\lambda \frac{\partial}{\partial k_{1\perp}^\lambda} \mathrm{tr}[\gamma_5 \not{k}_1 S^{(0)}(k_1)] \right)_{k_1 = p_1}$$

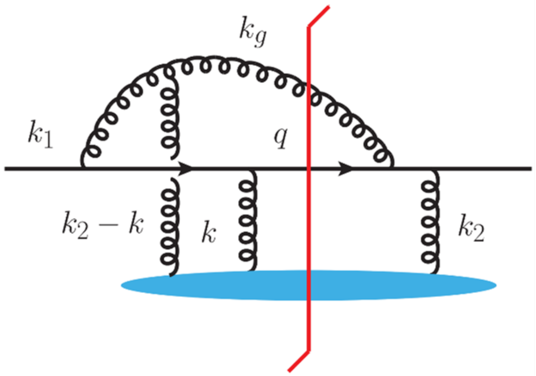
- This is our MASTER formula

- $S^{(0)}(\mathrm{k}_1)$ is calculated in perturbation theory

- First two contributions
  1. $q \to q$ Real contribution (integration over final state gluon)
  2. $q \to q$ Imaginary contribution

$q \to q$ Real contribution

- We need interference diagrams:



- Extraction of $S^0(k_1)$:

$$S^{(0)}(k_1) = \frac{q^+}{P_p^+}\frac{g^2 C_F}{4q^+k_g^+}\int_{\boldsymbol{k}_{g\perp}\boldsymbol{k}_\perp \boldsymbol{k}'_\perp}\int_{\boldsymbol{x}_\perp \boldsymbol{x}'_\perp \boldsymbol{y}_\perp \boldsymbol{y}'_\perp} \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\perp\cdot \boldsymbol{x}_\perp} \mathrm{e}^{\mathrm{i}(\boldsymbol{k}_{2\perp}-\boldsymbol{k}_\perp)\cdot \boldsymbol{y}_\perp}$$

$$\times\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}'_\perp\cdot \boldsymbol{x}'_\perp} \mathrm{e}^{-\mathrm{i}(\boldsymbol{k}_{2\perp}-\boldsymbol{k}'_\perp)\cdot \boldsymbol{y}'_\perp} d_{\mu\mu'}(k_g)\Big[\mathcal{S}(\boldsymbol{x}_\perp,\boldsymbol{x}'_\perp)\bar{T}_q^{\mu'} \not{q}\, T_q^\mu + \boxed{\mathcal{S}_{qqg}(\boldsymbol{x}'_\perp,\boldsymbol{x}_\perp,\boldsymbol{y}'_\perp)\bar{T}^{\mu'}_{qg}(\boldsymbol{k}'_\perp)\not{q}\, T^\mu_q}$$

$$+ \boxed{\mathcal{S}_{qqg}(\boldsymbol{x}'_\perp,\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\bar{T}^{\mu'}_q \not{q}\, T^\mu_{qg}(\boldsymbol{k}_\perp)} + \mathcal{S}_{qgqg}(\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\bar{T}^{\mu'}_{qg}(\boldsymbol{k}'_\perp)\not{q}\, T^\mu_{qg}(\boldsymbol{k}_\perp)\Big]$$

S. Benić, D. Horvatić, A. Kaushik and E. A. Vivoda, Phys. Rev D **106**, 114025 (2022).

- After final state gluon integration (and due to C-parity) only interference terms survive, so the trace takes following form:

$$
\begin{aligned}
&\mathrm{tr}\left[\gamma_5 \not{k}_1 S^{(0)}(k_1)\right] \\
&\qquad = \frac{q^+}{P_p^+} g^2 C_F \int_{\boldsymbol{k}_{g\perp}\boldsymbol{k}_\perp \boldsymbol{k}'_\perp} \int_{\boldsymbol{x}_\perp \boldsymbol{x}'_\perp \boldsymbol{y}_\perp \boldsymbol{y}'_\perp} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp} e^{i(\boldsymbol{k}_{2\perp}-\boldsymbol{k}_\perp)\cdot\boldsymbol{y}_\perp} e^{-i\boldsymbol{k}'_\perp\cdot\boldsymbol{x}'_\perp} e^{-i(\boldsymbol{k}_{2\perp}-\boldsymbol{k}'_\perp)\cdot\boldsymbol{y}'_\perp} \\
&\qquad \times \left[-\mathcal{S}_{qqg}(\boldsymbol{x}'_\perp, \boldsymbol{x}_\perp, \boldsymbol{y}'_\perp)\mathcal{H}(\boldsymbol{k}'_\perp, \boldsymbol{k}_{1\perp}) + \mathcal{S}_{qqg}(\boldsymbol{x}'_\perp, \boldsymbol{x}_\perp, \boldsymbol{y}_\perp)\mathcal{H}(\boldsymbol{k}_\perp, \boldsymbol{k}_{1\perp})\right]
\end{aligned}
$$

$$
\mathcal{H}(\boldsymbol{k}_\perp, \boldsymbol{k}_{1\perp}) \equiv \frac{1}{4q^+k_g^+} d_{\mu\mu'}(k_g)\mathrm{Tr}[\gamma_5 \not{k}_1 \bar{T}_q^{\mu} \not{q} \bar{T}_{qg}^{\mu'}(\boldsymbol{k}_\perp)]
$$

$$
\begin{aligned}
&\mathcal{S}_{qqg}(\boldsymbol{x}'_\perp, \boldsymbol{x}_\perp, \boldsymbol{y}'_\perp) \\
&\qquad = \frac{1}{2C_F N_C}\left(N_C^2 \mathcal{S}(\boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)\mathcal{S}(\boldsymbol{x}_\perp, \boldsymbol{y}'_\perp) - \mathcal{S}(\boldsymbol{x}_\perp, \boldsymbol{x}'_\perp)\right)
\end{aligned}
$$

Dipole distribution

S. Benić, D. Horvatić, A. Kaushik and E. A. Vivoda, Phys. Rev D **106**, 114025 (2022).

- Hard factor is easy to calculate:

$$\mathcal{H}(\boldsymbol{k}_\perp, \boldsymbol{k}_{1\perp}) = 4i(\bar{z}+1)\frac{\boldsymbol{v}_{1\perp} \times \boldsymbol{v}_{2\perp}}{\boldsymbol{v}_{1\perp}^2 \boldsymbol{v}_{2\perp}^2}$$

→ Manifestly finite

$$\mathrm{tr}[\gamma_5 \not{k}_1 S^{(0)}(k_1)] = ig^2 N_C \frac{q^+}{P_p^+} \int_{\boldsymbol{k}_{2\perp}\boldsymbol{k}_\perp} \int_{\boldsymbol{x}_\perp \boldsymbol{x}'_\perp \boldsymbol{y}_\perp} \mathrm{e}^{i\boldsymbol{k}_\perp \cdot (\boldsymbol{x}_\perp - \boldsymbol{y}_\perp)} \mathrm{e}^{-i\boldsymbol{k}_{2\perp}\cdot(\boldsymbol{x}'_\perp - \boldsymbol{y}_\perp)}$$

$$\times [\mathcal{P}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)\mathcal{O}(\boldsymbol{x}'_\perp, \boldsymbol{y}_\perp) - \mathcal{O}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp)\mathcal{P}(\boldsymbol{x}'_\perp, \boldsymbol{y}_\perp)]\mathcal{H}(\boldsymbol{k}_\perp, \boldsymbol{k}_{1\perp})$$

$$\boldsymbol{v}_{1\perp} \equiv \boxed{\boldsymbol{q}_\perp - \bar{z}\boldsymbol{k}_{1\perp}} - \bar{z}\boldsymbol{k}_{2\perp}$$

$$\boldsymbol{v}_{2\perp} \equiv \boxed{\boldsymbol{q}_\perp - \bar{z}\boldsymbol{k}_{1\perp}} - \boldsymbol{k}_\perp$$

Same vectors!

S. Benić, D. Horvatić, A. Kaushik and E. A. Vivoda, Phys. Rev D **106**, 114025 (2022).

There is no polarized cross section!

- To get a usual sine modulation we need a **reference vector**:

$$q_\perp \times S_\perp$$

- Our proxy for $S_\perp$is $k_{1\perp}$ (because of the derivative)

- In our calculation $q_\perp$ and $k_{1\perp}$ appear in unique combination:

$$q_{1\perp} = q_\perp - \bar{z} k_{1\perp}$$

➢There is no polarized cross section in this channel!

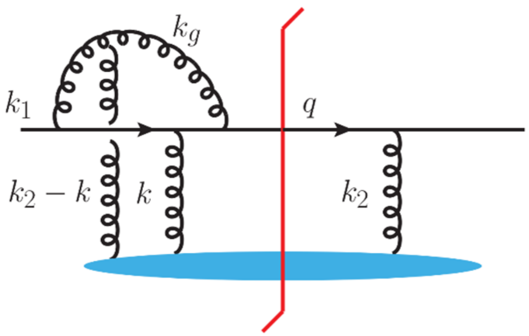
$$\mathbf{d\Delta\sigma_{\mathit{WW}} = 0}$$

S. Benić, D. Horvatić, A. Kaushik and E. A. Vivoda, Phys. Rev D **106**, 114025 (2022).

$q \to q$ Virtual contribution

- Interference with leading order amplitude:

$y = \frac{z}{\bar{z}}$



$$\boldsymbol{v}_{1\perp} \equiv y\boldsymbol{q}_\perp - \boldsymbol{k}_{g\perp}$$

$$\boldsymbol{v}_{2\perp} \equiv \boldsymbol{k}_\perp + \bar{y}\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{g\perp} - \boldsymbol{q}_\perp$$

- Same manipulations as before lead to:

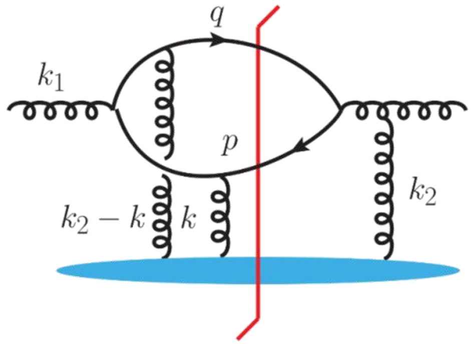
$$\mathcal{H}(\boldsymbol{k}_\perp, \boldsymbol{k}_{1\perp}) = -4i(\bar{y}+1)\frac{\boldsymbol{v}_{1\perp} \times \boldsymbol{v}_{2\perp}}{\boldsymbol{v}_{1\perp}^2 \boldsymbol{v}_{2\perp}^2}$$

Integration over virtual gluon
transverse momentum:

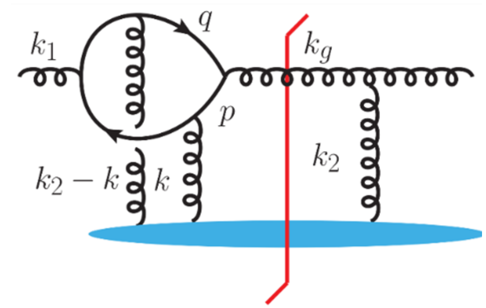
$$\mathbf{d\Delta}\boldsymbol{\sigma}_{\boldsymbol{WW}} = \mathbf{0}$$

S. Benić, D. Horvatić, A. Kaushik and E. A. Vivoda, Phys. Rev D **106**, 114025 (2022).

Gluon initiated channels



$g \to q\bar{q}$



$g \to g$

- There is no $g \to gg$ contribution because adjoint Wilson lines are real

- In Wandzura – Wilczek approximation there is no TSSA in above channels

$$\mathbf{d\Delta\sigma_{WW} = 0}$$

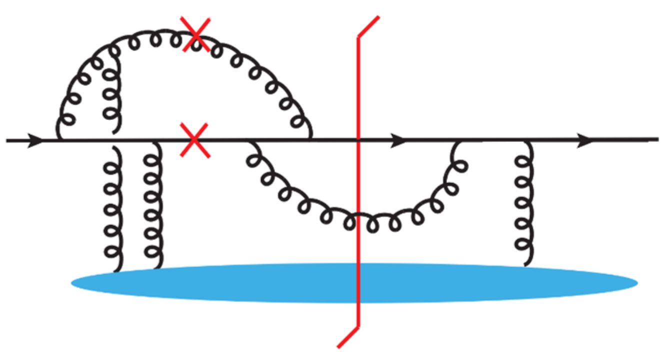
S. Benić, D. Horvatić, A. Kaushik and E. A. Vivoda, Phys. Rev D **106**, 114025 (2022).

Is there any odderon contribution?

1. Going beyond WW approximation
   1. Taking the real distribution in target (pomeron) and the phase from the cut of the propagator
   2. Principal value of the propagator and phase from the odderon

$$\frac{1}{k^2 + i\epsilon} = P\frac{1}{k^2} - i\pi\delta(k^2)$$

2. NNLO -> competing mechanisms (lensing vs. odderon)



Conclusions:

- Odderon mechanism for TSSA on hadron level
- Wandzura-Wilczek approximation (intrinsic and kinematical contribution)
- There is no TSSA in LO and NLO
- NNLO, beyond WW??

Thank you!