Three dimensional structure of proton within the Basis Light-front Quantization framework

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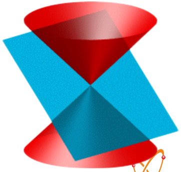

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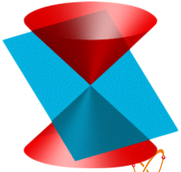

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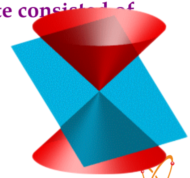

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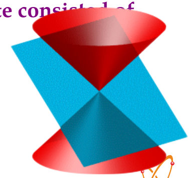
    $|\text{electron}\rangle = |e\rangle + |e\gamma\rangle + \ldots \quad |\text{proton}\rangle = |qqq\rangle + |qqqg\rangle + \ldots \quad |\text{pion}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + \ldots$

    and then define the light-front wave function (LFWF) as

$$|P,\Lambda\rangle \equiv \sum_{n,\{\lambda_i\}} \int \frac{\prod_i^{n-1}\{\mathrm{d}x_i \mathrm{d}q_i^\perp\}}{N_n(\{q_i\})} \boldsymbol{\psi^{\Lambda}_{\{\lambda_i\}}(\{q_i\})}\,|\{\lambda_i, p_i\}\rangle$$



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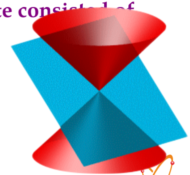
  - solving **the light-front mass eigne function** is equivalent to finding the **LFWF of the bound state**



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- **discretization and truncation** [Vary et al.,*PRC* **81**.3(2010):035205]

$$\langle\alpha|H_{\mathrm{LC}}|\beta\rangle\langle\beta|P,\Lambda\rangle = M^2\langle\alpha|P,\Lambda\rangle \qquad \langle\beta|P,\Lambda\rangle = \psi(\alpha) \qquad \psi^{\Lambda}_{\{\lambda_i\}}(\{q_i\}) \propto \sum_{\{n_i,m_i\}}\psi(\beta)\{\phi_{n_im_i}(p_i^{\perp})\}$$

d.o.f.	basis	truncations or sum rules	other elements
transverse momentum	2D-HO $\phi_{n_im_i}(p_i^{\perp})$	$\sum_i 2n_i + \lvert m_i\rvert + 1 \le N_{\max}$	energy scale of the 2D-HO b_x $H_{\mathrm{LC}} \to H_{\mathrm{LC}} + H'$
longitudinal momentum	plane wave $e^{ip_i^+x^-}$	$\sum_i k_i = K,\ \frac{k_i}{K} = x_i$	box normalization of length $2L$ $p_i^+ = \frac{k_i\pi}{L}$
spin	light-front helicity eigen state λ_i	$\sum_i \lambda_i + m_i = M_j$	
multi-particle state of the Fock space	$\lvert\alpha_i\rangle = \lvert\lambda_i, k_i, n_i, m_i\rangle$ $\lvert\alpha\rangle = \bigotimes_i \lvert\alpha_i\rangle$	Fock space truncation	

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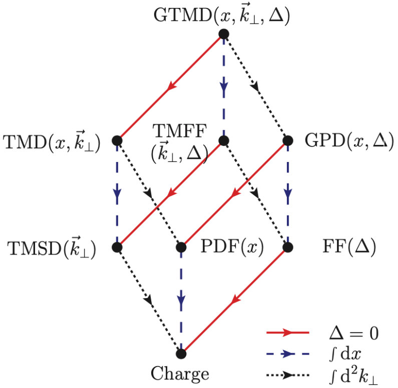

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- **QCD bound state**: [Xu et al.,*PRD* **104**.9(2021):1–23], [Lan et al.,*PRL* **122**.17(2019):1–6], [Wu et al.(in preparation)]
- **QED bound state**: [Zhao et al.,*PLB* **737**(2014):65–69], [Fu et al.:2003.11781], [Nair et al.,*PLB* **827**(2022)]
- **TMDs**: [Hu et al.,*PRD* **103**.3(2021):36005], [Hu et al.,*PLB* **833**(2022):137360], [Hu et al.(in preparation)], [Zhu et al.,*PLB* **839**(2023):137808]
- **GPDs**: [Zhang et al.(in preparation)], [Fu et al.(in preparation)]
- **DPDs**: [Peng et al.(in preparation)]

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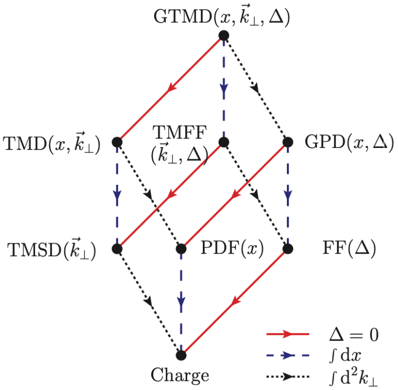
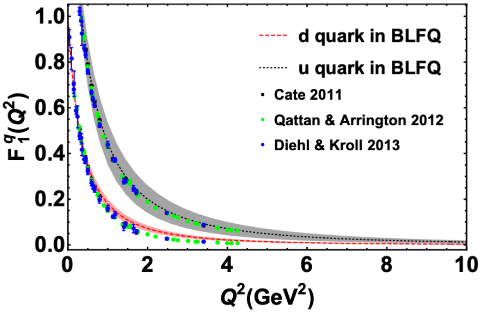
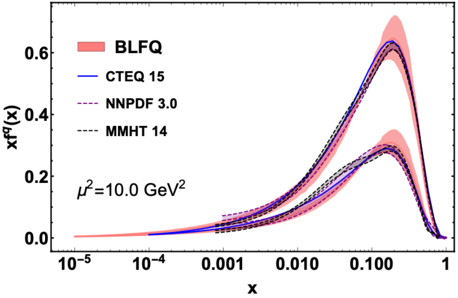


How to calculate those TMDs

- **effective Hamiltonian suitable for $|qqq\rangle$ Fock space proton** and **parameters fitted from unpolarized collinear PDFs and form factors** $\Longrightarrow$ **LFWF** of the $|qqq\rangle$ Fock space proton [Xu et al.,*PRD* **104**.9(2021):1–23]





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- $\mathcal{W} \approx \mathbb{1}$ **approximation**: T-even TMDs can be expressed by **simple overlaps of LFWF** $\Phi^{\Lambda'\Lambda}_{\lambda'_1\lambda_i} = \sum_{\lambda_2\lambda_3} \psi^{\Lambda' *}_{\lambda'_1\lambda_2\lambda_3} \psi^{\Lambda}_{\lambda_1\lambda_2\lambda_3}$, and all T-odd TMDs reduce to zero

$$f_1 = \Phi^{++}_{++} + \Phi^{++}_{--} \qquad g_{1L} = \Phi^{++}_{++} - \Phi^{++}_{--}$$

$$g_{1T} = \frac{2M}{|p^\perp|}\Phi^{+-}_{++} \qquad h_1 = \Phi^{+-}_{+-}$$

$$h^\perp_{1T} = \frac{2M^2}{(p^\perp)^2}\Phi^{+-}_{-+} \qquad h^\perp_{1L} = \frac{2M}{|p^\perp|}\Phi^{++}_{-+}$$

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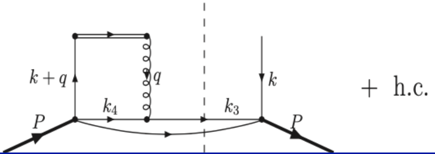
- **one-gluon exchange (OGE) approximation** [Yuan,*PLB*575.1-2(2003):45–54] [Bacchetta et al.,*PRD*78.7(2008):074010]
  - physically, non-trivial gauge link encode the **final-state interaction** between struck parton and the spectators
  - the contribution of OGE approximation to T-even TMDs are zero and contribution to T-odd TMDs are proportional to the following complex integral

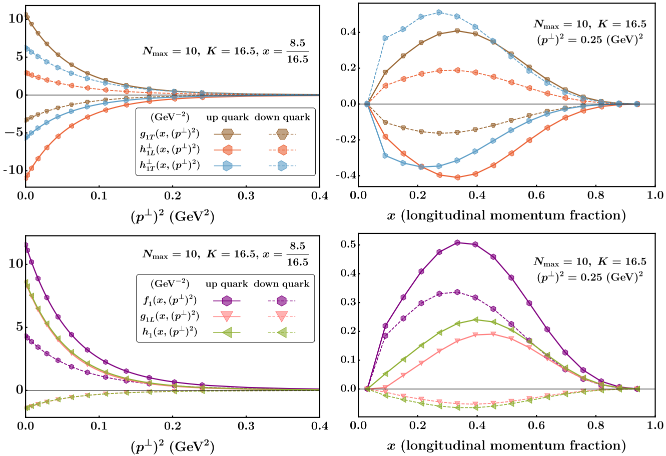
$$\int \mathrm{d}^2k'_3\mathrm{d}^2q\,\frac{1}{q^2+\lambda^2}\phi_{n'_fm'_f}(k-q,\sqrt{x_1(x_2+x_3)}b)\phi_{n'_mm'_m}(k'_3-\frac{x_2}{x_2+x_3}q,\sqrt{\frac{x_2x_3}{x_2+x_3}}b)\phi_{n_mm_m}(k'_3-q,\sqrt{\frac{x_2x_3}{x_2+x_3}}b)$$



Properties of T-even TMDs of quarks in the proton

[Hu et al., *PLB*833(2022):137360]

- qualitative properties

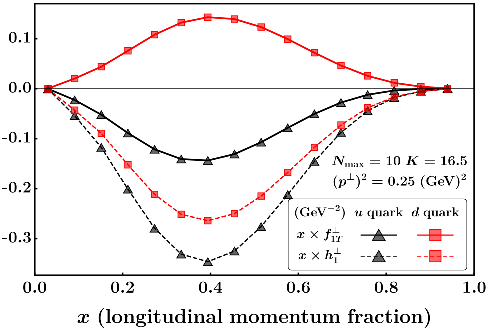


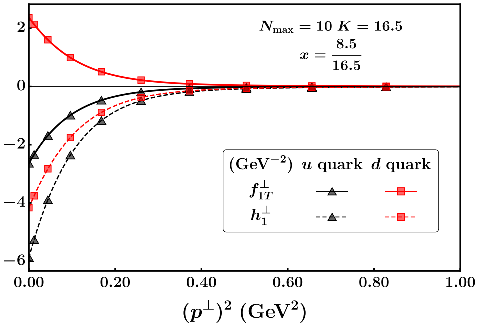
- plots at initial energy $\mu_0^2=\zeta_0=0.195\,\mathrm{GeV}^2$
- T-even TMDs of u, d quarks
- monotonically decrease in the transverse direction
- distributions analouge to $x^\alpha(1-x)^\beta$ in the longitudinal direction
- consistent with other theoretical calculations [Bastami et al., *PRD*103.1(2021):14024] [Bacchetta et al., *PRD*78.7(2008):074010] [Musch et al., *PRD*83.9(2011):1–38] [Lattice Parton Collaboration et al.:2211.02340]

Properties of T-even TMDs of quarks in the proton

[Hu et al.(in preparation)]

- Sivers function ($f_{1T}^{\perp}$) is negative for u and positive for d with similar magnitude, Boer-Mulders functions ($h_1^{\perp}$) are negative for both with larger magnitude for u than d





From quark TMDs to the Sivers asymmetry of SIDIS

[Hu et al.(in preparation)]

- Sivers asymmetry: the difference in cross section of unpolarized lepton scattered from oppositely transversely polarized proton

$$l + H \to l' + h + X \qquad A_{\mathrm{UT}}^{\sin(\phi_h - \phi_S)} \propto \frac{\mathrm{d}\sigma^{\rightarrow} - \mathrm{d}\sigma^{\leftarrow}}{\mathrm{d}\sigma^{\rightarrow} + \mathrm{d}\sigma^{\leftarrow}}$$

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- within the TMD factorization, one can express Sivers asymmetry via Sivers function $f_{1T}^{\perp}$, unpolarized TMD distribution f_1, and unpolarized TMD fragmentation D_1

$$A_{\mathrm{UT}}^{\sin(\phi_h - \phi_S)}(x, z, P_{h\perp}, Q^2) = -\frac{M \sum_q e_q^2 \int_0^\infty \frac{b^2 db}{2\pi} J_1(\frac{bP_{h\perp}}{z}) f_{1T}^{\perp q \leftarrow h_1}(x, b) D_1^{q \rightarrow h_2}(z, b)}{\sum_q e_q^2 \int_0^\infty \frac{bdb}{2\pi} J_0(\frac{bP_{h\perp}}{z}) f_1^{q \leftarrow h_1}(x, b) D_1^{q \rightarrow h_2}(z, b)}$$

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  - we have to introduce TMD evolution to account for the scale difference of various experiments
  - BLFQ framework currently cannot calculate the fragmentation function, so we use the extracted D_1 [Scimemi et al.,*JHEP* **2020**.6(2020); Zeng et al.,*PRD* **106**.9(2022):94039]

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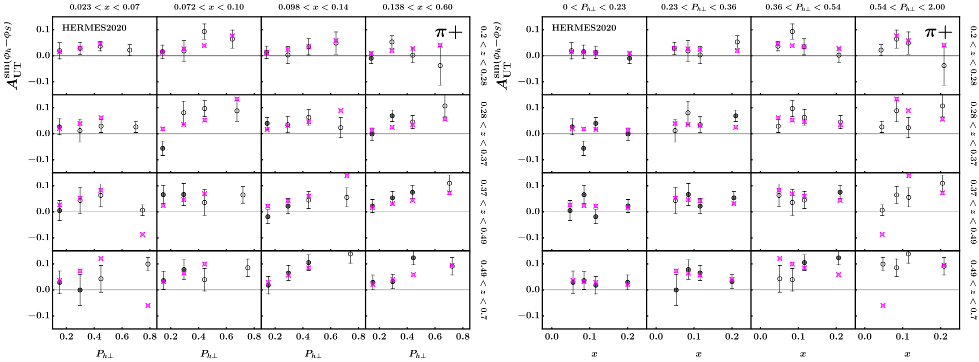

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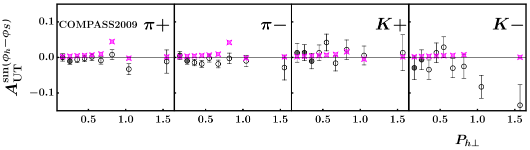

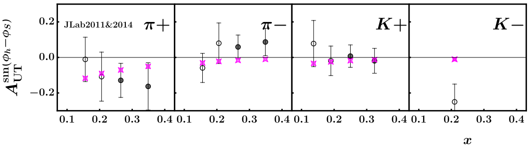

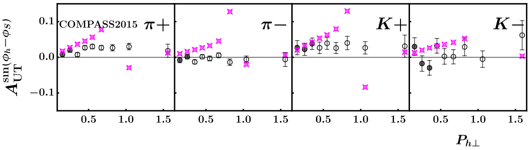

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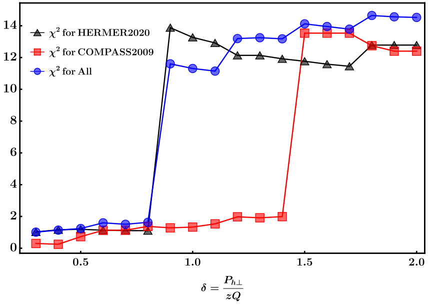

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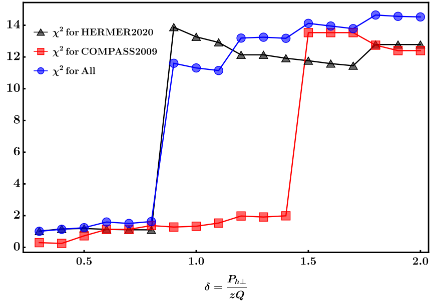
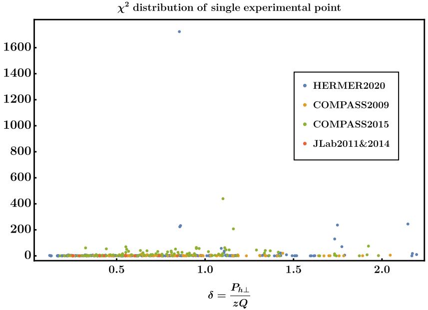
$$f_{u\leftarrow A} = c_{p\leftarrow H} f_{u\leftarrow p} + c_{n\leftarrow H} f_{u\leftarrow n} = c_{p\leftarrow H} f_{u\leftarrow p} + c_{n\leftarrow H} f_{d\leftarrow p}$$

- $\chi^2 = \frac{1}{N_{\text{point}}} \sum \frac{(\text{the}-\text{exp})^2}{\text{err}^2}$ to quantify the difference between theoretical and experimental results





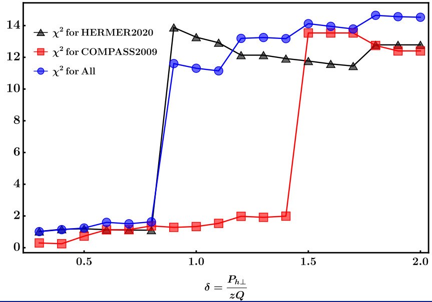
From quark TMDs to the Sivers asymmetry of SIDIS

[Hu et al.(in preparation)]

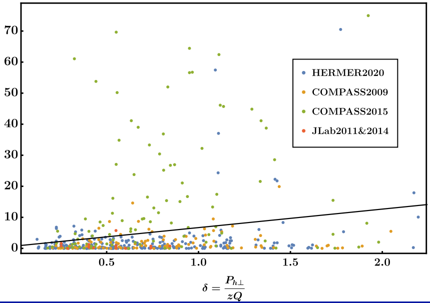
- we compare with experimental data from COMPASS2009 (H = deuteron), JLab2011&2014 (H = ^{3}He), COMPASS2015 (H = proton), HERMES2020 (H = proton) and introduce a very simple nuclear modification for $H \neq$ proton [Zeng et al.,*PRD* **106**.9(2022):94039]:

$$f_{u\leftarrow A} = c_{p\leftarrow H} f_{u\leftarrow p} + c_{n\leftarrow H} f_{u\leftarrow n} = c_{p\leftarrow H} f_{u\leftarrow p} + c_{n\leftarrow H} f_{d\leftarrow p}$$

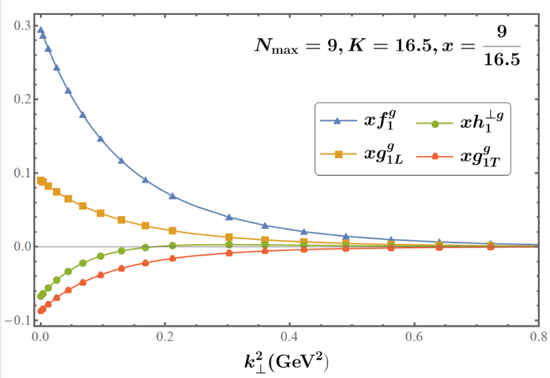
- $\chi^2 = \frac{1}{N_{\text{point}}} \sum \frac{(\text{the}-\text{exp})^2}{\text{err}^2}$ to quantify the difference between theoretical and experimental results

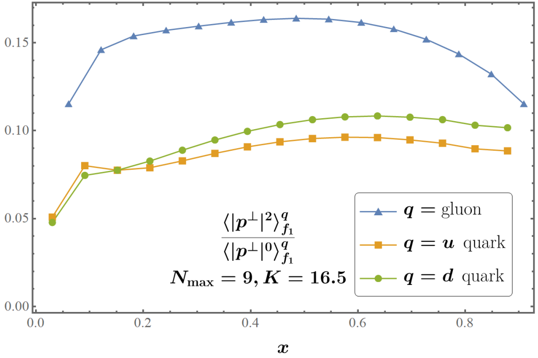


χ^2 distribution of single experimental point

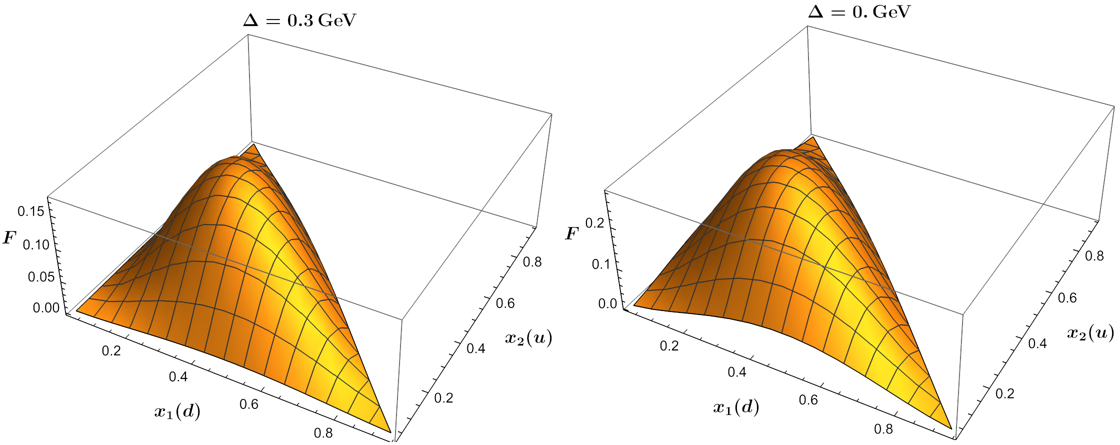


Comparison between quark and gluon TMDs in $|qqq\rangle + |qqqg\rangle$ proton





Unpolarized DPDs



Summary and outlook

- Lagrangian ⇒ light-front mass eigen equation ⇒ LFWF of the proton ⇒ parton TMDs/GPDs/DPDs/…in the proton ⇒ cross sections of high energy scattering
- start from full QCD Lagrangian, **on going projects now**
- rapidity divergency, soft factor is missing in my work, **soft factor is the VEV of a Wilson loop**
- how to calculate fragmentation function **how to deal with the unobserved X using the wave function formalism**

$$D_1 \propto \sum_X \langle 0 | \psi \mathcal{W} | H, X \rangle \langle H, X | \bar{\psi} \mathcal{W}^\dagger | 0 \rangle$$

- how to incorperate small-x physics in our framework **seem like we both use light-front quantization and LFWF/LCWF**

Thank you!