Mont Carlo Efficient phase space generator

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Outline

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Introduction

- Phase space integration is essential for collider experiments calculations, however, the phase space of the final-state particles is huge and of a variable number of dimensions, for large numbers of partons it is impossible to integrate the Phase space due to the effects of inelasticity and multiplicity,

- These effects need to be modeled and incorporated into computer simulations using Monte Carlo techniques.

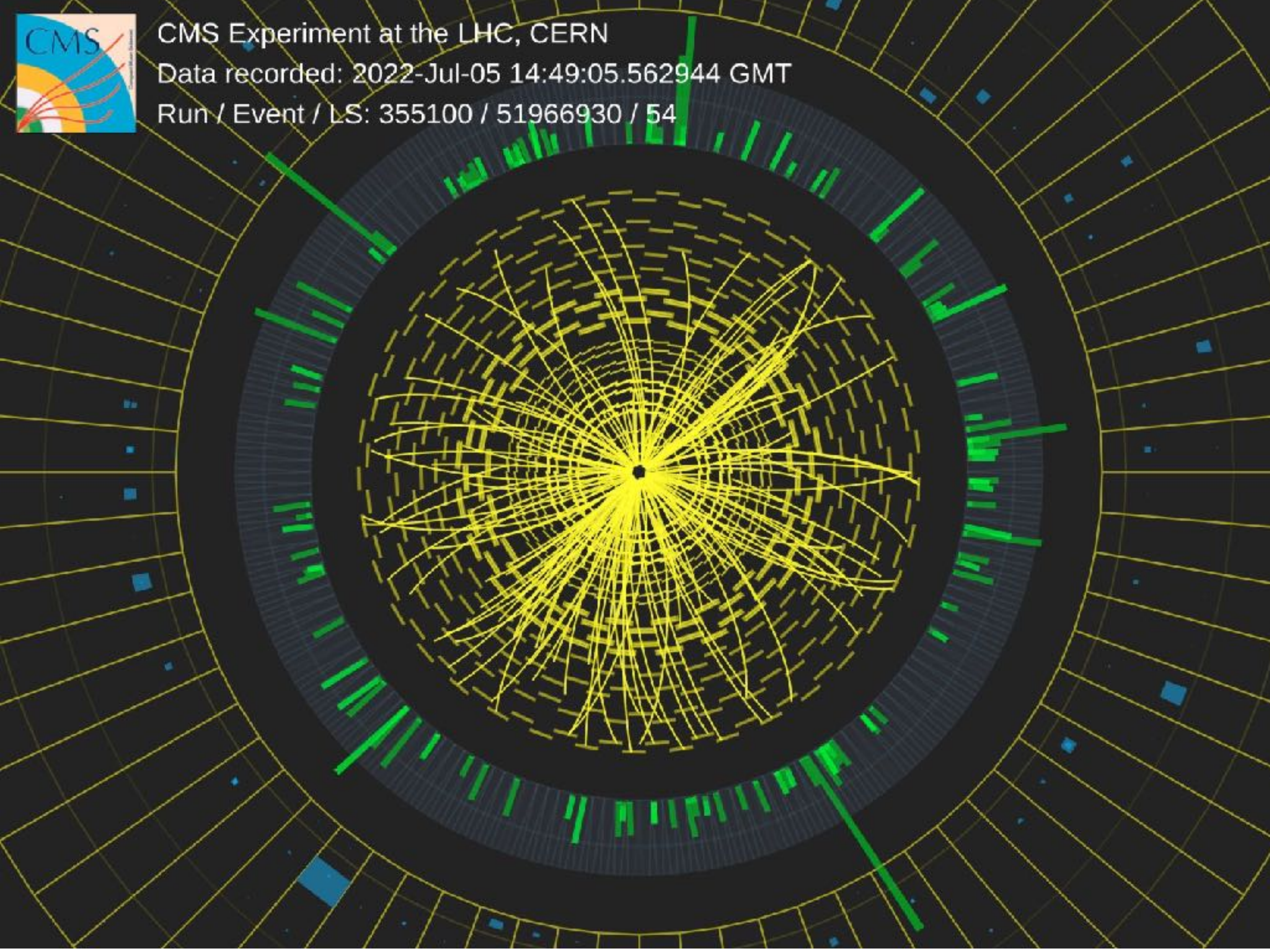
CMS Experiment at the LHC, CERN
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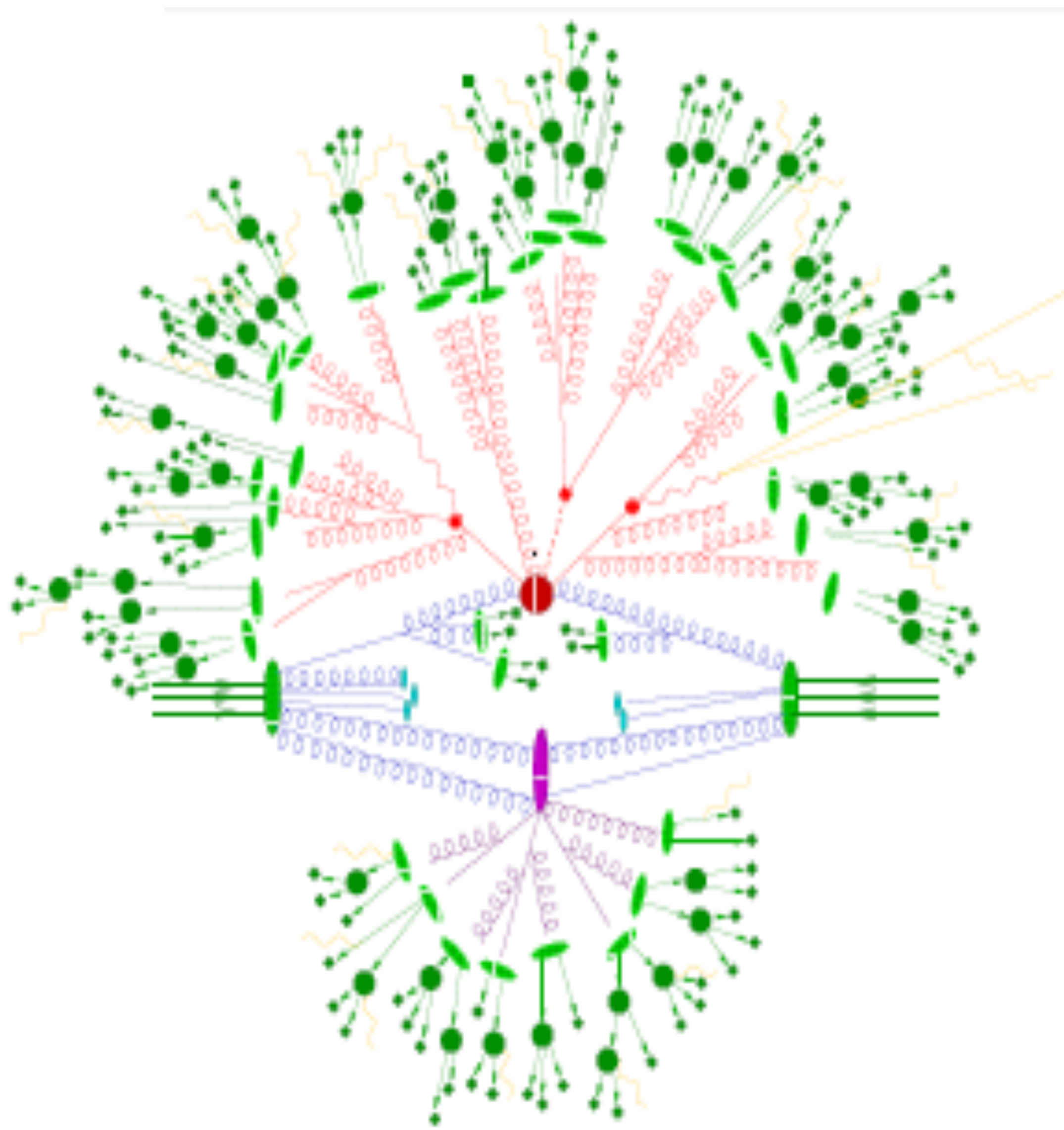
Image F. Krauss



- We will concentrate on hadron collisions, and how to involve the QCD calculation right and we will focus on the ISR radiations

- Initial state radiation ISR arises because incoming charged particles can radiate before entering the hard process.The branching of these partons terminates when they collide to initiate the hard subprocess.

DGLAP Functions in the Initial State Radiation

We need to consider the distribution of momenta of the colliding partons inside the protons as well as the different contributing quark flavors, characterized by the parton density functions

$$f_{a\to bc}(t,z)=\frac{\alpha_s}{2\pi}\frac{1}{t}P_{a\to bc}(z)\quad\frac{1}{z}\frac{f_a(t,x')}{f_a(t,x')}\theta(z_{min}<z<z_{max})$$

The DGLAP splitting functions as the underlying probability.

We have 4 Splitting kernels and 7 Splitting functions

The générique form of the DGLAP in the Initial state radiation

$$f_b(t,z)=\frac{\alpha_s}{2\pi}\frac{1}{t}\sum_{a,b}P_{a\to bc}(z)\quad\frac{1}{z}\frac{f_a(t,x')}{f_a(t,x')}\theta(z_{min}<z<z_{max})$$

$$P_{q\to qg}(z) = C_F\frac{1+z^2}{1-z}$$

$$P(z)_{q\to gq}=C_F\frac{1+(1-z)^2}{z}$$

$$P_{g\to q\bar{q}}(z) = T_R\left[z^2+(1-z)^2\right]$$

$$P_{g\to gg}(z) = C_A\left[\frac{1-z}{z}+\frac{z}{1-z}+z(1-z)\right]$$

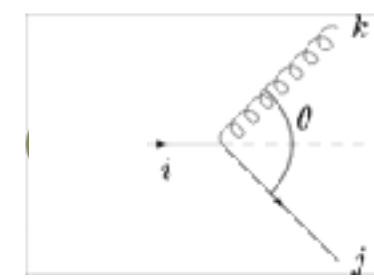
QCD Radiations: Collinear and Soft singularities

We treat the regions where the emission of QCD radiation is enhanced, collinear parton splitting or soft (low-energy) gluon emission.

$$f_{a\to bc}(t,z) = \frac{\alpha_s}{2\pi}\,\frac{1}{t}\,P_{a\to bc}(z)\quad \frac{1}{z}\,\frac{f_a(t,x')}{f_a(t,x')}\,\theta(z_{min}<z<z_{max})$$

In the collinear limit the cross section for a process factorizes

This expression is singular as $t \longrightarrow 0$ (Collinear)



Soft gluons (low energy) come from all over the event.

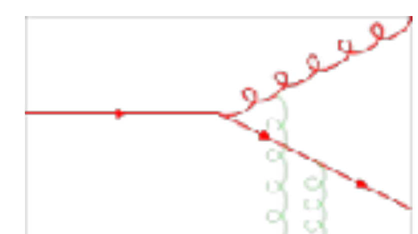
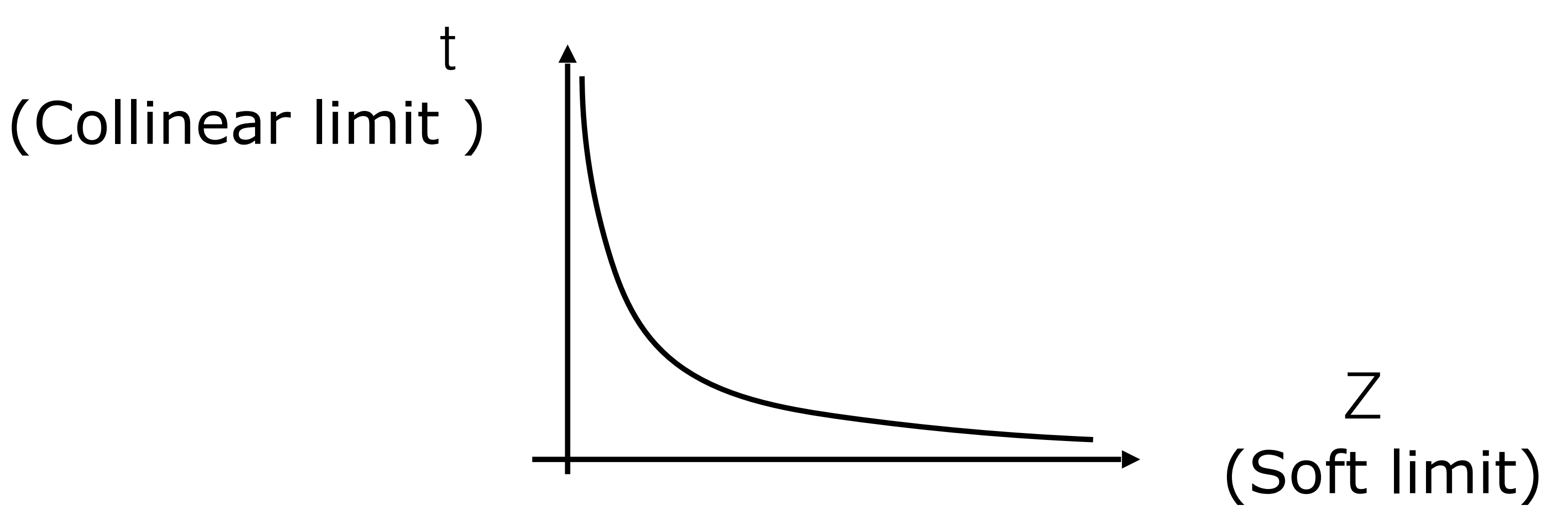


Image P. Richardson



Splitting Probability in the Initial State Radiation

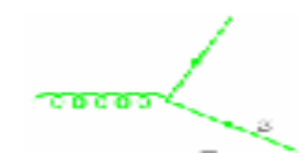
- $q, \bar{q}$, g Splitting Probability

$$P_b(x;t,z,\varphi) = \sum_{a,b} f_{a\to bc}(t,z)\,\Delta_b(x;t,t_{max})$$

- Sudakov form factor : the probability of evolving from a high scale to the cut-off with no real emission

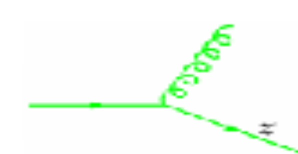
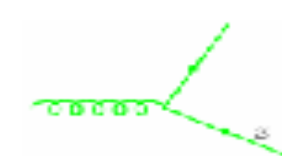
$$\Delta_b(x;t,t_{max}) = exp\left\{-\int_t^{t_{max}} dt' \int dz\, f_a(t,z)\right\}$$

- Quark splitting probability

$$P_q(x;t,z) = [f_{qqg}(t,z) + f_{gq\bar{q}}(t,z)]\,\Delta_{qqg}(x;t,t_{max})\Delta_{gq\bar{q}}(x;t,t_{max})$$

- Antiquark splitting probability

$$P_{\bar{q}}(x;t,z) = [f_{\bar{q}\bar{q}g}(t,z) + f_{g\bar{q}q}(t,z)]\,\Delta_{\bar{q}\bar{q}g}(x;t,t_{max})\Delta_{g\bar{q}q}(x;t,t_{max})$$

- Gluon splitting probability



$$P_g(x;t,z) = [f_{ggg}(t,z) + f_{qgq}(t,z) + f_{\bar{q}g\bar{q}}(t,z)]\,\Delta_{ggg}(x;t,t_{max})\Delta_{qgq}(x;t,t_{max})\Delta_{\bar{q}g\bar{q}}(x;t,t_{max})$$

Probability Distribution

Difficult to distribute according to these functions all the way to t = 0. Instead :
Distribute according to P(x,t,z) for $t > t_{IR}$,
and according to a flat distribution for $t < t_{IR}$

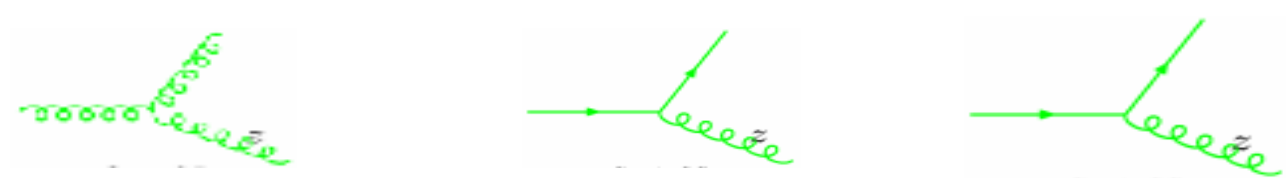
- Quark distribution

$$\int_{t_{IR}}^{t_{max}} \int_{z_{min}}^{z_{max}} dtdzP_g(x;t,z) = 1-[\Delta_{ggg}(x;t_{IR},t_{max})\Delta_{qgq}(x;t_{IR},t_{max})\Delta_{\bar{q}g\bar{q}}(x;t_{IR},t_{max})]$$

- Antiquark distribution

$$\int_{t_{IR}}^{t_{max}} \int_{z_{min}}^{z_{max}} dtdzP_{\bar{q}}(x;t,z) = 1-[\Delta_{\bar{q}\bar{q}g}(x;t_{IR},t_{max})\Delta_{g\bar{q}q}(x;t_{IR},t_{max})]$$

- Gluon Quark distribution



$$\int_{t_{IR}}^{t_{max}} \int_{z_{min}}^{z_{max}} dtdzP_q(x;t,z) = 1-[\Delta_{qqg}(x;t_{IR},t_{max})\Delta_{gq\bar{q}}(x;t_{IR},t_{max})]$$

Monte Carlo Calculation: Distribution according to the grids

- The variance can be reduced by a change of variables that "flattens" the integrand Using the Jacobian $Jac(x(r),t(s,r),z(s,r,v))$.

- Write the phase space volume as an n-dimensional hypercube with volume 1

$$\int_{x_{min}}^{x_{max}}\int_{t_{IR}}^{t_{max}}\int_{z_{min}}^{z_{max}} dxdtdz\, P_b(x;t,z) = \int_0^1\int_0^1\int_0^1 drdsdv\, Jac(x(r),t(s,r),z(s,r,v))P_b(x(r),t(s,r),z(s,v,r))$$

- This is a function with less variance than P(x,t,z) itself, then the error will be reduced by distributing points uniformly in r, s, v space

$$G_{ijk}=\frac{dxdtdz}{drdsdv}(x_{ijk}\ ,t_{ijk}\ ,z_{ijk})P(x_{ijk}\ ,t_{ijk}\ ,z_{ijk})$$

Histories and Signatures

- Every grid is representing one history , in total 6 signatures: each signature rely on one or more than one history

- $\bar{q}\, q \to zg$: $\bar{q}\,(q \to qg) \to zg + (\bar{q} \to \bar{q}\, g)q \to zg$
- $q\, \bar{q} \to zg$: $q\,(\bar{q} \to \bar{q}\, g) \to zg + (q \to q\, g)\bar{q} \to zg$
- $qg \to zq$: $q\,(g \to \bar{q}\, q) \to zq$
- $\bar{q}\, g \to z\bar{q}$: $\bar{q}(g \to q\, \bar{q}) \to z\bar{q}$
- $gq \to zq$: $(g \to \bar{q}\, q)q \to zq$
- $g\bar{q} \to z\bar{q}$: $(g \to q\, \bar{q})\, \bar{q} \to z\, \bar{q}$

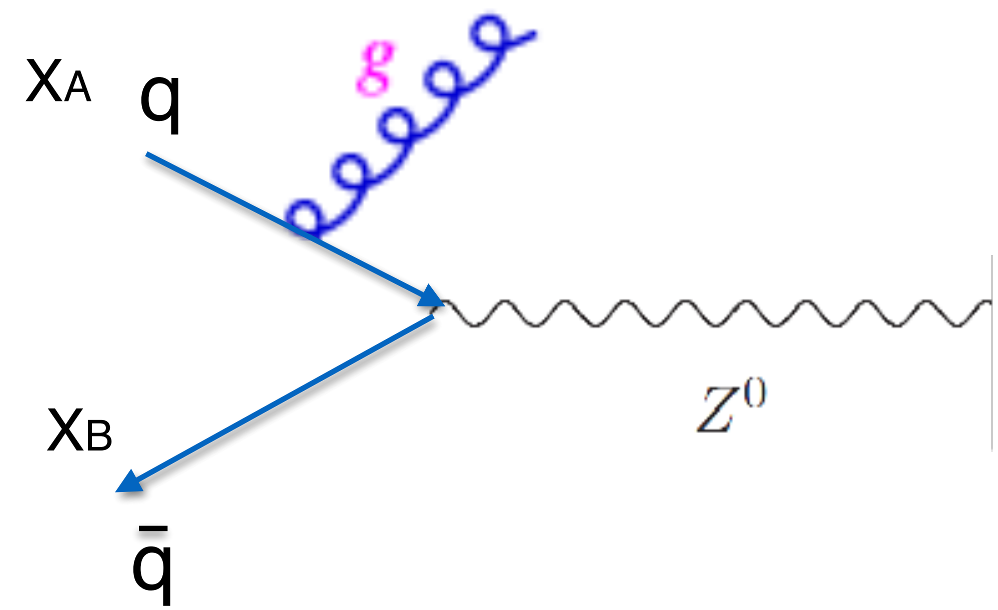
History= Feynman diagram of signature

Signature= incoming and outgoing particles in one interaction

$q\ \bar{q}$ —>gz

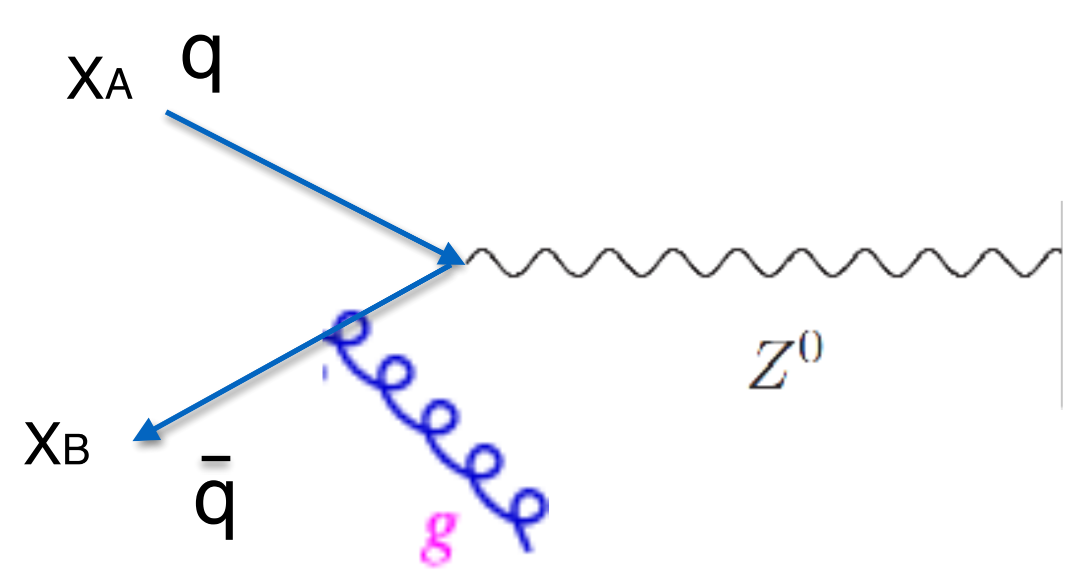
=

(q—>q g)$\bar{q}$ —>zg



+

q ($\bar{q}$—>$\bar{q}$ g) —>zg



Tests of the grids

A/ All the grids calculated for one type of particles are summing up to one :

$P_q = P_{g \to q \bar{q}} + P_{q \to q g}$

$P_{\bar{q}} = P_{g \to \bar{q} q} + P_{\bar{q} \to \bar{q} g}$

$P_g = P_{g \to g g} + P_{q \to g q} + P_{\bar{q} \to g \bar{q}}$

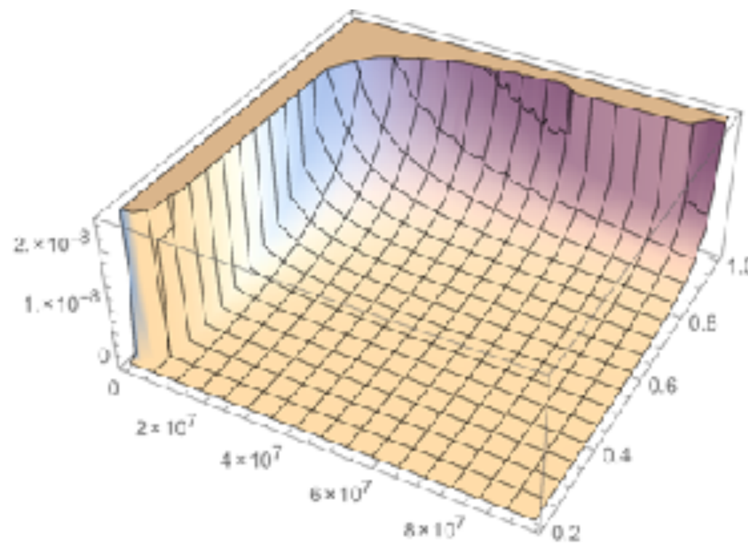
in total we have 31 grid, considering the flavor of the parton entering into the hard process there are only 11 grid 5P$\bar{q}$, 5Pq, 1Pg

B/ Plot all the piecewise functions, which is the gridded version of P(t, z, x) in the same plot, the results were showing a big agreement between all the plots.

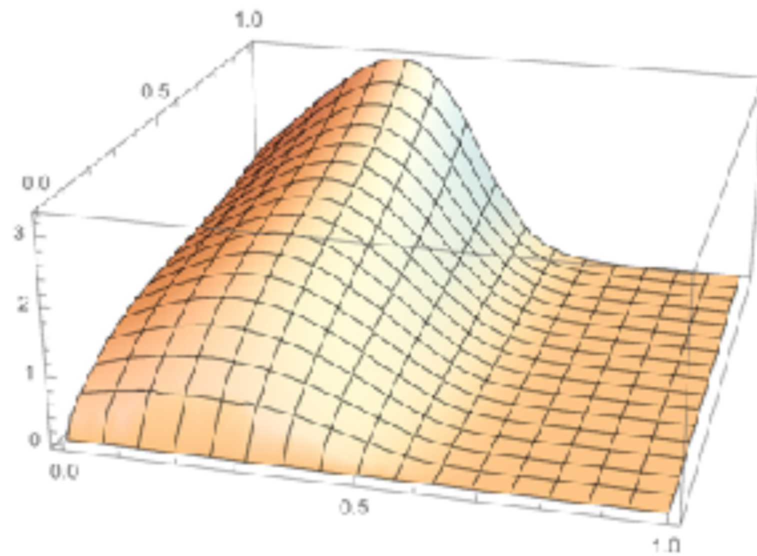
Up Quark distribution function

for all the 11 grids

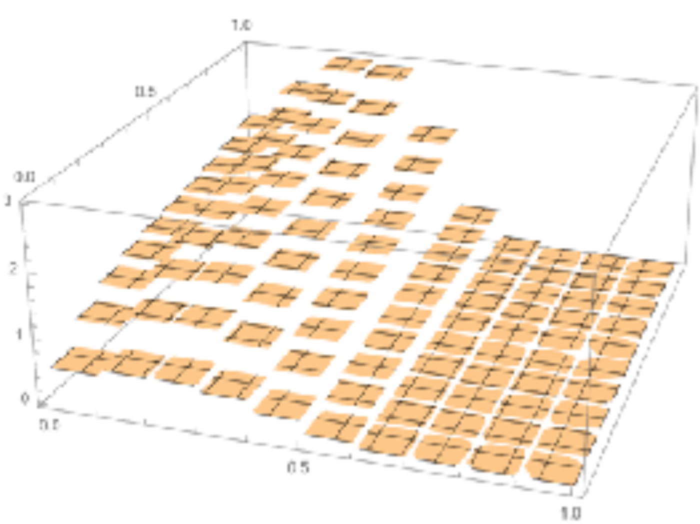
$P_b(x;t,z)$

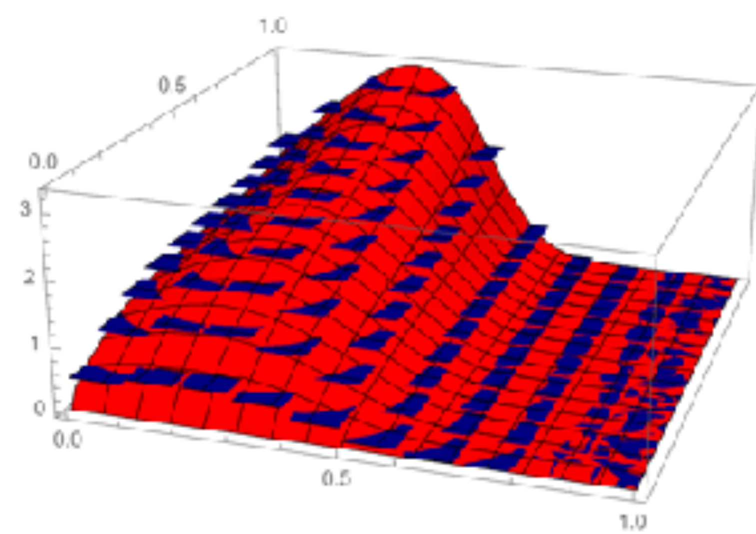


$Jac(x(r),t(s,r),z(s,r,v))P_b(x(r),t(s,r),z(s,v,r))$



- We have five grids for the Quark distribution





-

G_{ijk}

MC Phase space integration

To integrate the phase space we pick up the flavor of a splitting based on the individual grids , MC Algorithm in C++

- Simulate the distribution function thru the grid using MC method by generating N combinations of s and v
- Determine {Smin, Smax, Vmin, Vmax} the interval of integration
- Count the number of points that fall into this range (how many combinations of {s, v} fall in the defined interval)
- Check the code by integrating the probability distribution over the grid without MC and do comparison.
- Introduced the calculation of the error

Conclusion

- We are aiming to develop a phase space generator that distribute phase space points according to the singular limit of QCD using the DGLAP evolution equation as the underlying distribution functions.

- The large dimensionality of the phase space makes the Monte Carlo integration the method of choice.

- The output can then be given to a general-purpose event generator for showering and hadronization

- The main power of this approach comes from choosing the distribution according to the QCD radiation and splitting functions.