An Unbiased Global Analysis of the Proton's Elastic Form Factors

G_E & G_M

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The Form Factors

F(q) = ?

- Proton current is parametrized by general form factors

$$J_\gamma^\mu \quad = \quad \bar{u}_N\left(p'\right)\Gamma_\gamma^\mu(q)u_N(p)$$

- Only asymptotically constrained by theory
  - Need Experimental data to understand further

The Form Factors

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$$J_\gamma^\mu = \bar{u}_N(p')\,\Gamma_\gamma^\mu(q)u_N(p)$$

$$\Gamma_\gamma^\mu(q) = \gamma^\mu F_1(Q^2) + \frac{i\sigma^{\mu v} q_v}{2M} F_2(Q^2)$$

- Only asymptotically constrained by theory
  - Need Experimental data to understand further

Observables

$\frac{d\sigma}{d\Omega}$

- Connect observables to Form Factors

$$G_E(Q^2) = F_1 - \frac{Q^2}{4M^2} F_2$$
$$G_M(Q^2) = F_1 + F_2$$

- **LT:** OPE Cross Section:

- **PT:** Polarized Cross Sections:

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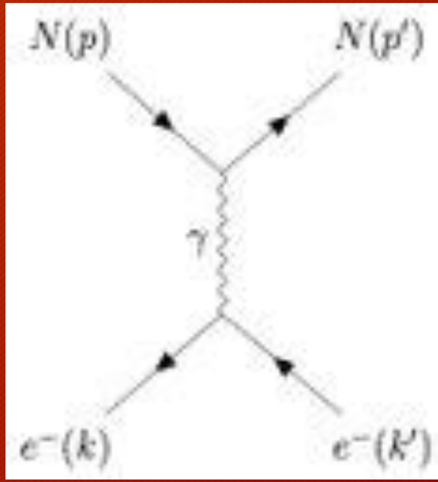

- **LT**: OPE Cross Section:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\mathrm{OPE}} \propto \varepsilon G_E^2 + \frac{Q^2}{4M^2}G_M^2$$

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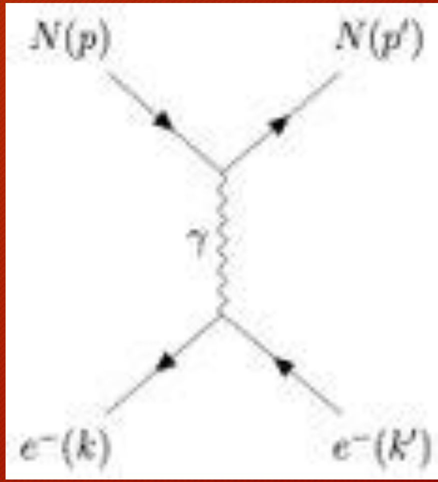

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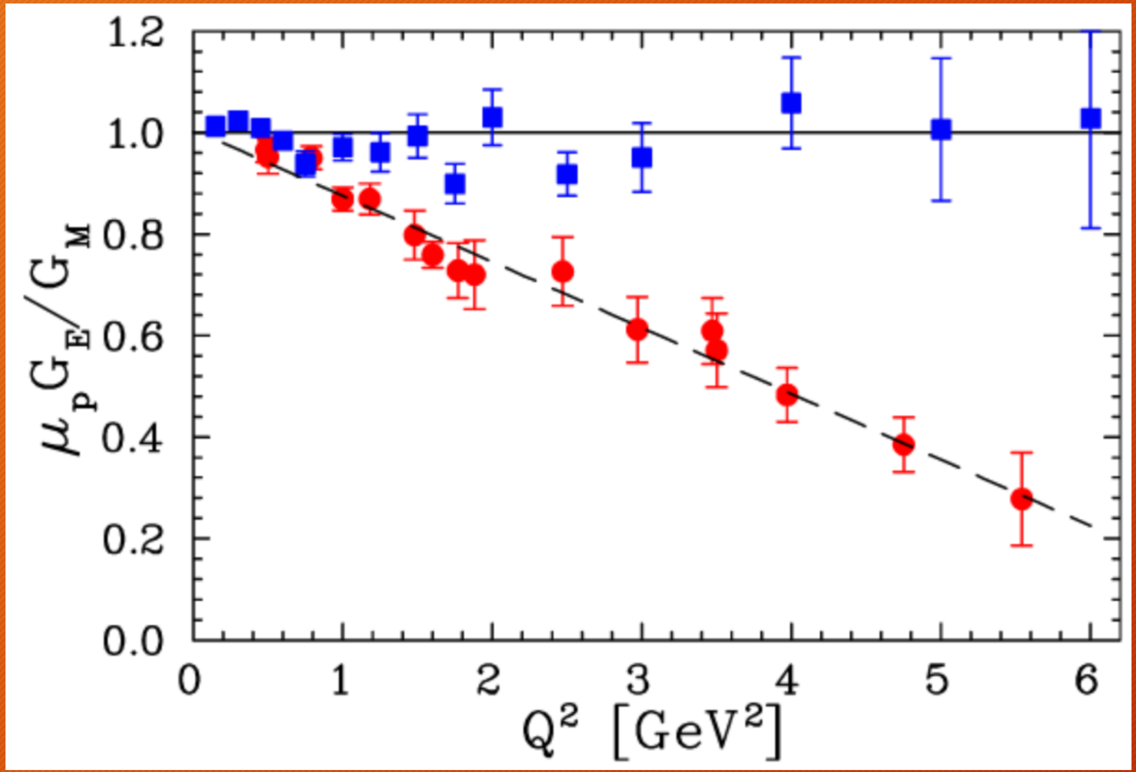
- **PT**: Polarized Cross Sections:

$$\frac{d\sigma^{(T)}}{d\Omega} \propto G_E G_M \qquad \frac{d\sigma^{(L)}}{d\Omega} \propto G_M^2$$

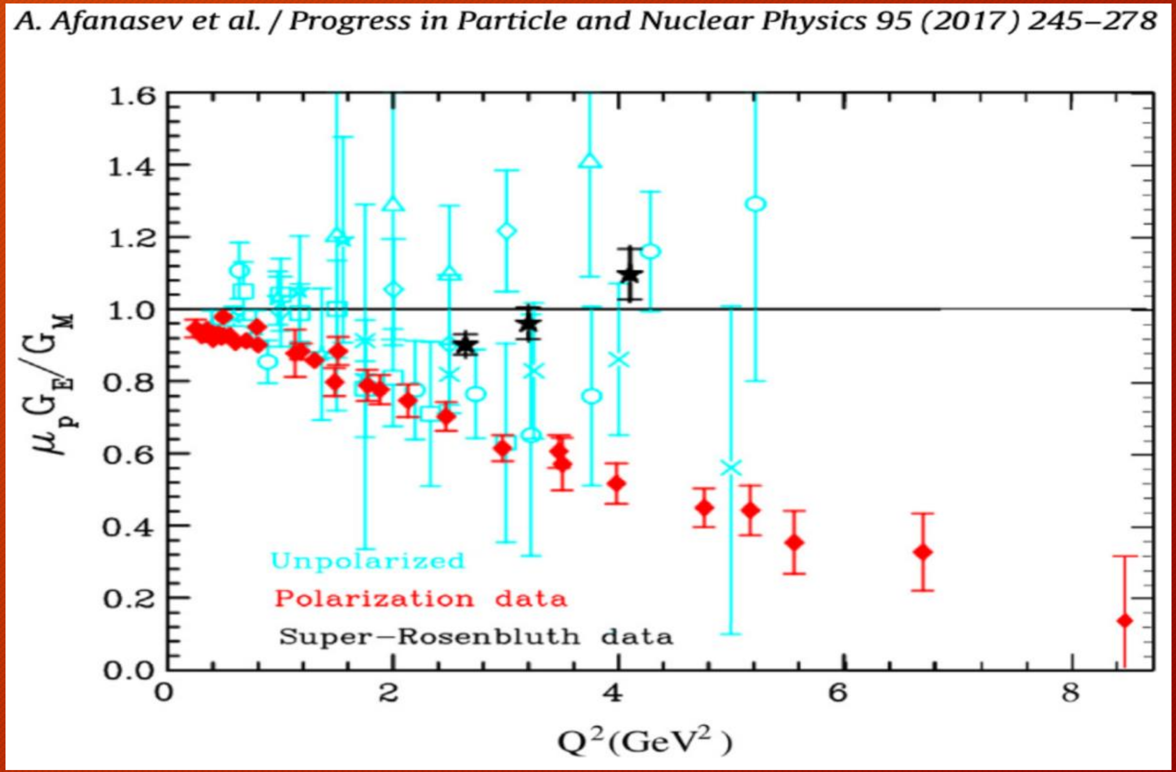
Two Independent Form Factor Ratios

LT ≠ PT

Significant Disagreement!



Single Experiment

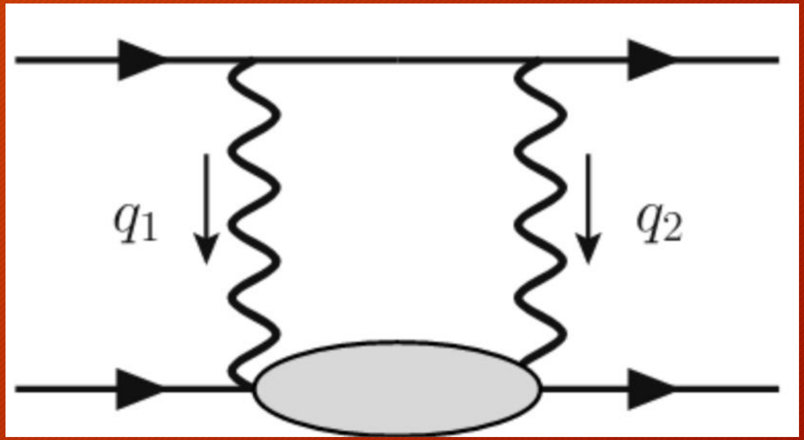


Several Experiments

Probable Causes of Discrepancy

LT ≠ PT

- Two Photon Exchange Corrections
  - Enter CS Multiplicatively



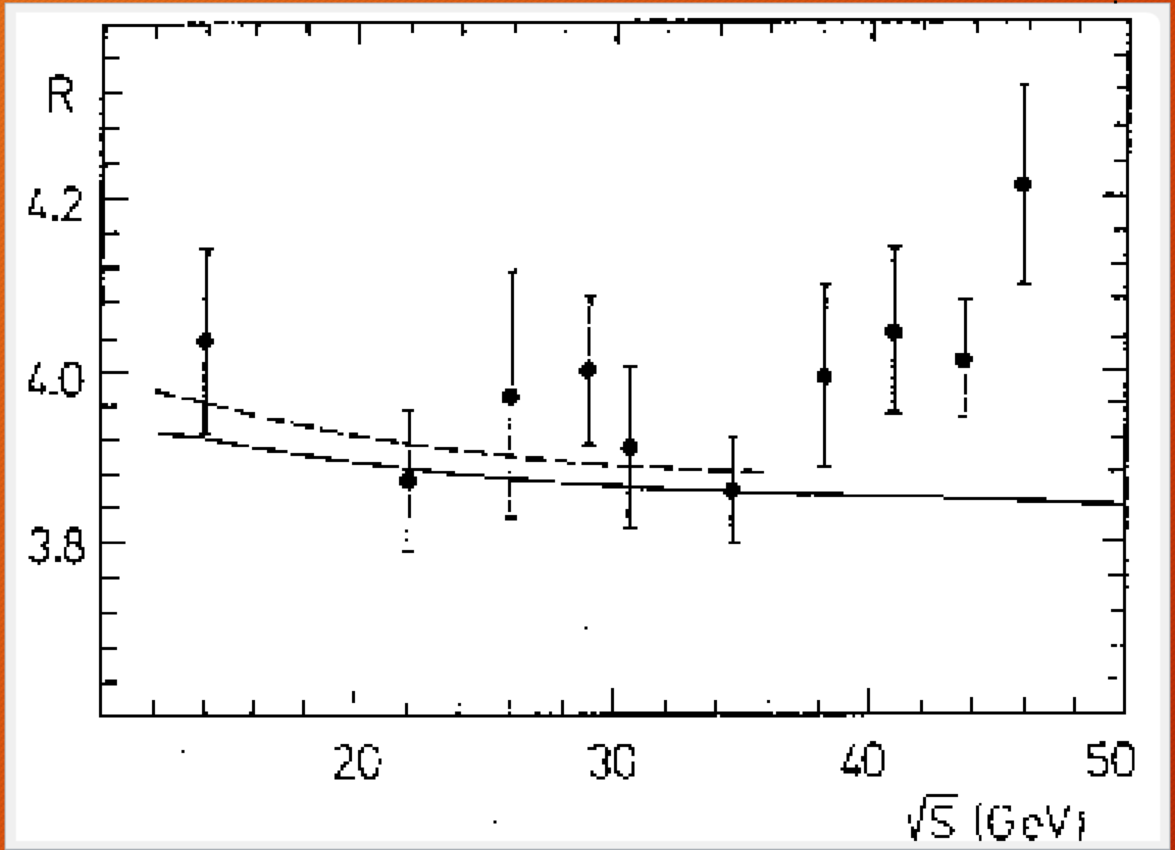
- Multiplicative Uncertainty
  - Correlates Whole Experiment

$$y_i \cdot (1 \pm \Delta n_i)$$

Multiplicative Uncertainty – How?

Δn

- Improper treatment leads misleading fits
  - "Peelle's Pertinent Puzzle" @ Cello



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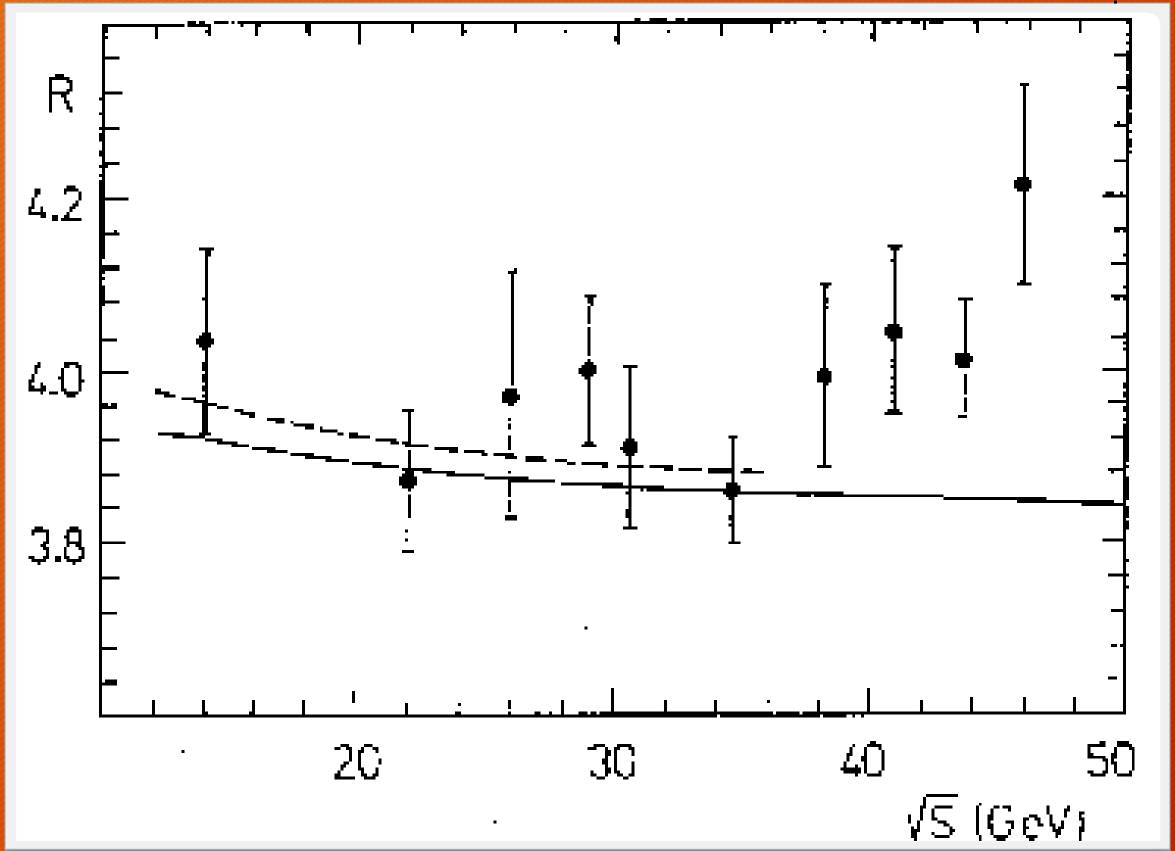

$$y_i \cdot (1 \pm \Delta n_i)$$

The chi can lie!

Traditional Fitting and The Penalty Trick

n_i

- Chi-square comes from Gaussian

$$P\left(y_1, y_2, \ldots, y_N | M_i\right) = \prod_{i=1}^{N} \frac{1}{\sqrt{2\pi}\Delta y_i} e^{-\frac{1}{2}(y_i - M_i)^2/(\Delta y_i)^2}$$

$$\chi^2(\boldsymbol{\alpha}) = \sum_{i=1}^{N} \frac{\left(y_i - M_i\right)^2}{\Delta y_i^2}$$

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- Penalty Trick
  - Scaling Factors
  - Biased

$$\sum_{i=1}^{N} \left[\frac{\left(n_i - 1\right)^2}{\left(\Delta n_i\right)^2} + \sum_{j=1}^{N_i} \frac{\left(n_i \cdot y_{ij} - M_{ij}\right)^2}{\left(\Delta y_{ij}\right)^2} \right]$$

Blueprint – The t_0 Method

t_0

$$y_i \cdot (1 \pm \Delta n_i)$$

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t_0

$$y_i \cdot (1 \pm \Delta n_i)$$

Propagate

$$\sum_{i=1}^{\mathcal{N}} \left[\sum_{j=1}^{N_i} \frac{(y_{ij} - \mathrm{M}_{ij})^2}{(\Delta y_{ij})^2 + (y_{ij}\Delta n_i)^2} \right]$$

↑Data↑

Blueprint — The t_0 Method

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↑Model↑

$$\sum_{i=1}^{\mathcal{N}} \left[\sum_{j=1}^{N_i} \frac{(y_{ij} - \mathrm{M}_{ij})^2}{(\Delta y_{ij})^2 + \left(\hat{\mathrm{M}}_{ij} \Delta n_i\right)^2} \right]$$

↑Guess↑

Blueprint — The t_0 Method

t_0

- 1) Aforementioned iterative guess

- 2) Monte-Carlo replica averaging
  - Best model is average of replica best fits (non-linearity causes issues)

$$F_{\text{best}}(Q^2;\alpha) = F^i_{\text{avg}}$$

Extending the t_0 Method — IMF Covariance Matrix

IMF

- Covariance Matrix Ambiguous

$$\begin{pmatrix} (\Delta y_{i1})^2 & 0 \\ 0 & (\Delta y_{i2})^2 \end{pmatrix} + \left(\hat{\mathrm{M}}_{ij}\Delta n_i\right)^2$$

$$\begin{pmatrix} (\Delta y_{i1})^2 + \left(\hat{\mathrm{M}}_{i1}\Delta n_i\right)^2 & \left(\hat{\mathrm{M}}_{i?}\Delta n_i\right)^2 \\ \left(\hat{\mathrm{M}}_{i?}\Delta n_i\right)^2 & (\Delta y_{i2})^2 + \left(\hat{\mathrm{M}}_{i2}\Delta n_i\right)^2 \end{pmatrix}$$

Extending the t_0 Method — IMF Covariance Matrix

IMF

- Covariance Matrix Ambiguous
- "Model Outer Product"

$$\begin{pmatrix} (\Delta y_{i1})^2 & 0 \\ 0 & (\Delta y_{i2})^2 \end{pmatrix} + \left(\hat{\mathrm{M}}_{ij}\Delta n_i\right)^2$$

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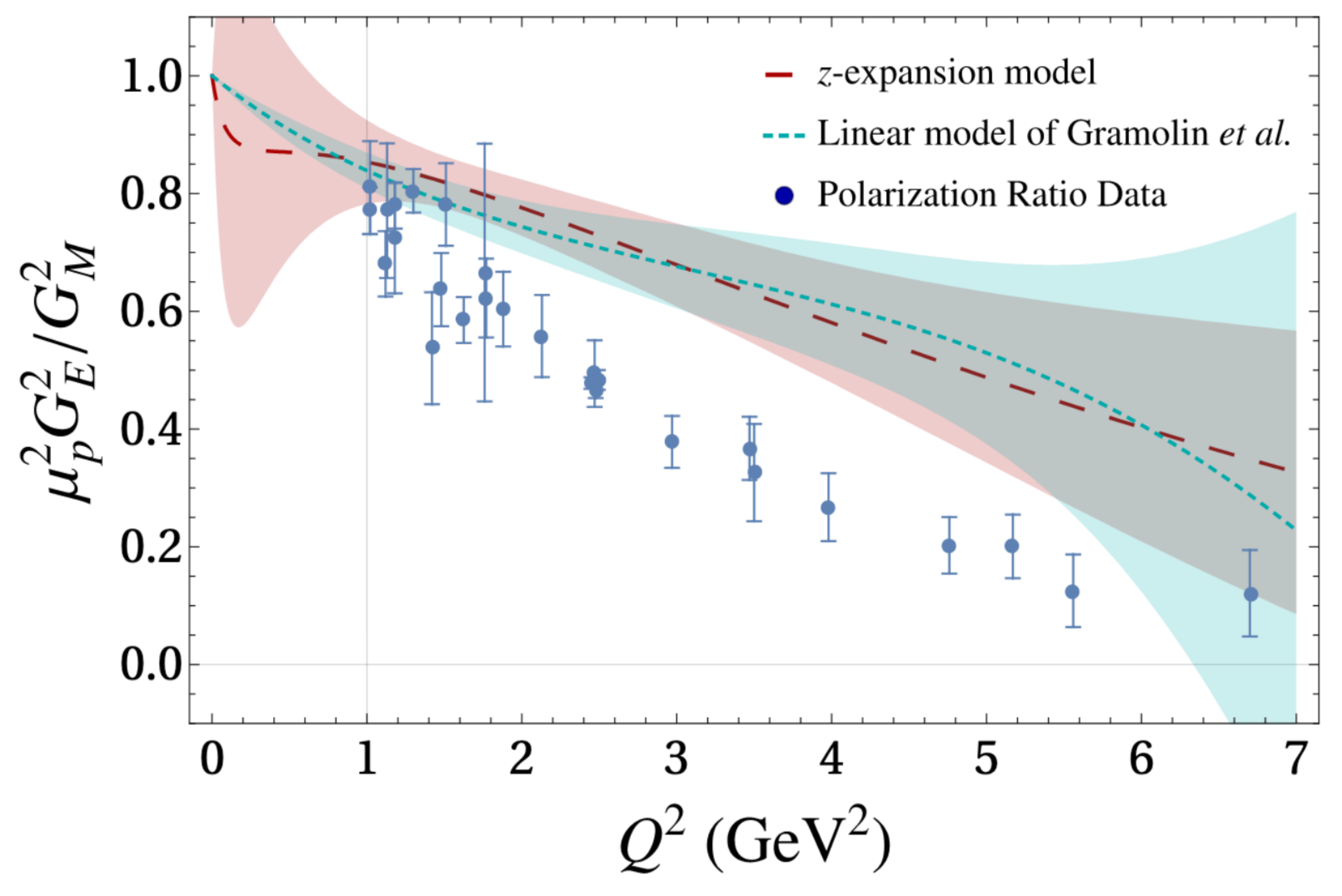
Penalty Trick vs. IMF Method

n_i vs. t_0

- Very Similar Results
  - Penalty Trick is still a good estimator

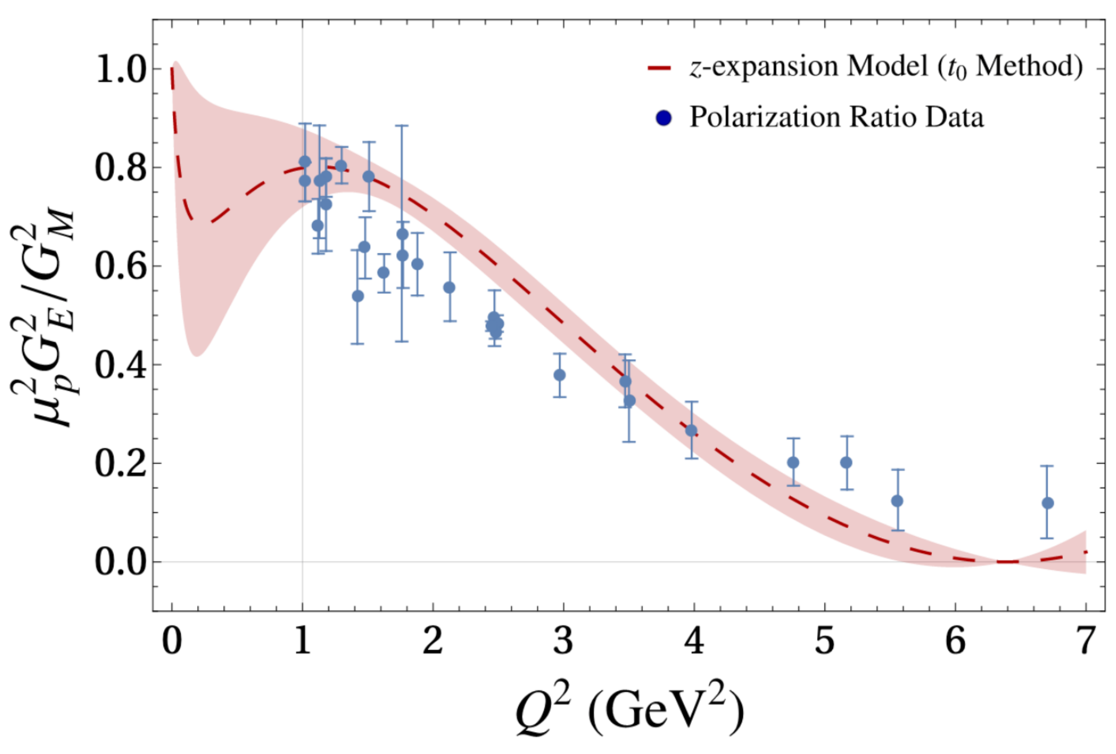
- Main takeaways
  - LT still not equal to PT
  - Fitted normalizations are merely a crutch

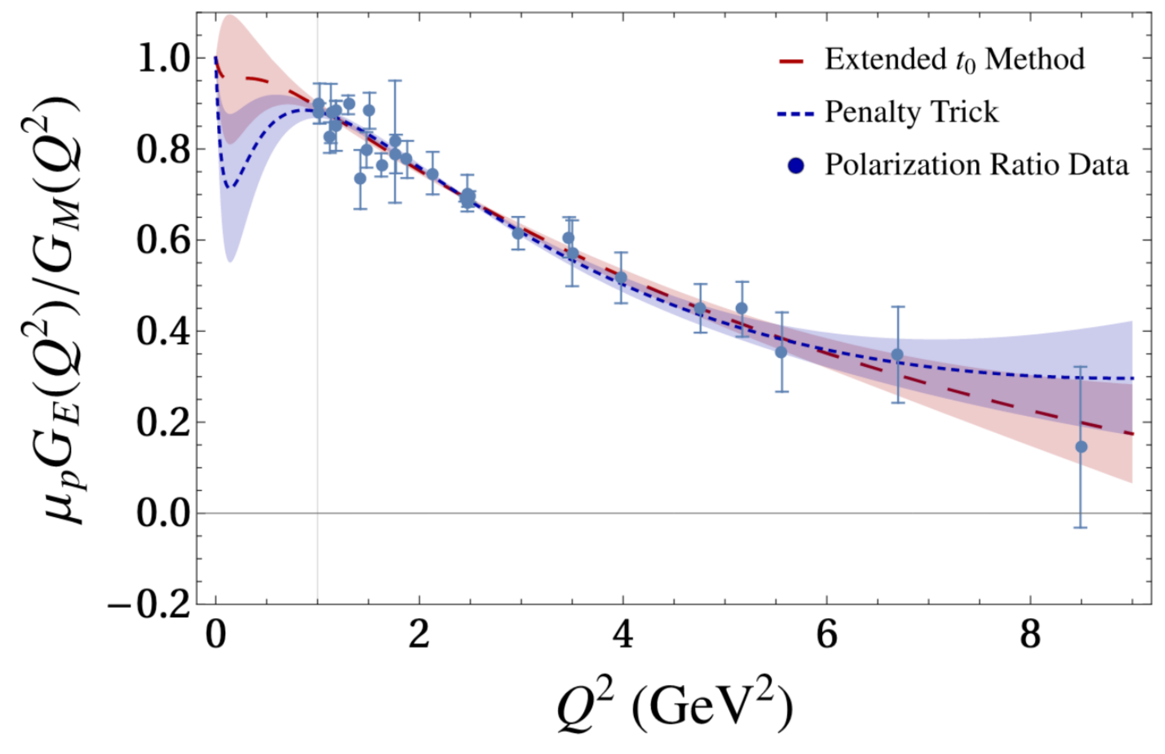


Form Factor Ratio Discrepancy Still At Large

LT ≠ PT

- Need Two-Photon-Exchange Corrections
  - Unbiased Fit To Corrected Data: 14.5% increase chi-square





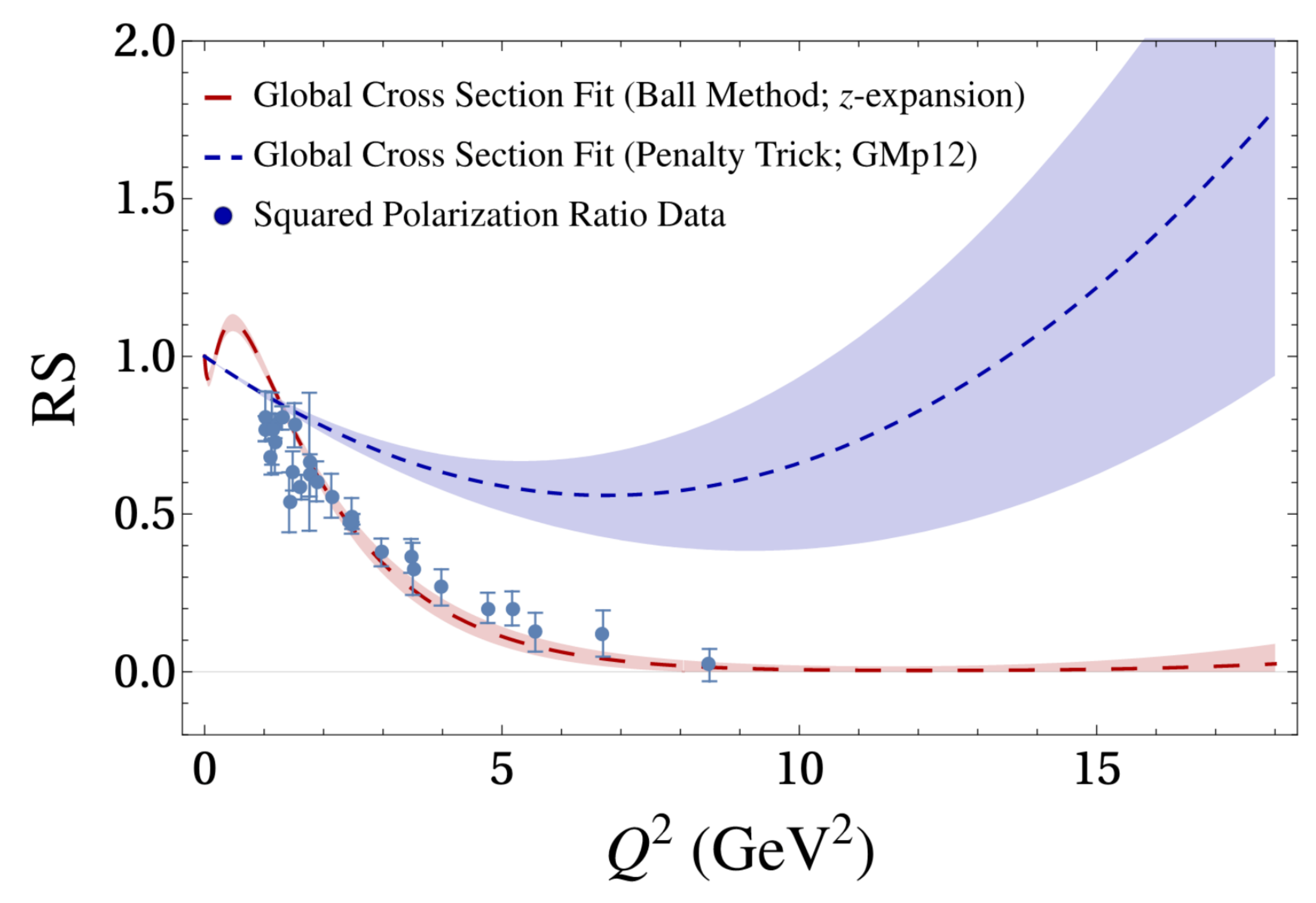
  - Improvements can be made to TPE

A Curious 'Coincidence'

?

- If one treats Normalization Error as point-to-point error:

- Is multiplicative error being overestimated?



Summary

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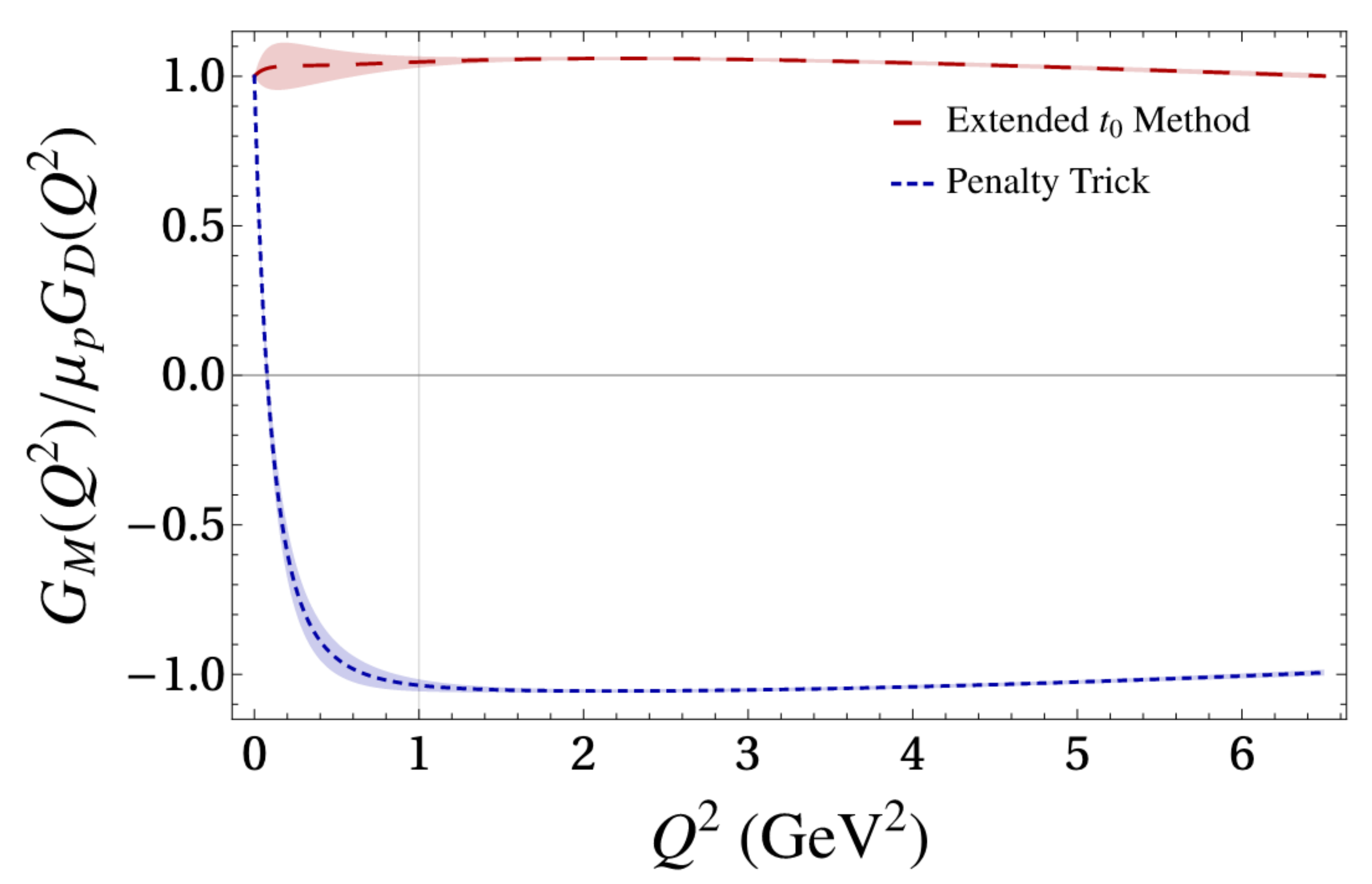
- New method for unbiased fitting of non-linear models
- Current TPE Corrections help significantly close gap, room for improvement
- Scale Uncertainties are likely being over-estimated
- Normalization Factors are Merely a Crutch
- Future Work
  - Perform updated Low-Q^2 data as in Bernaur (2014)
    - Does the updated fitting procedure effect proton radius?

Question Time

Perils of Non-linear fitting

- Non-linear fits need a lot of supervision



Extending the t_0 Method — Non-Linear Models

- Iterative parameter search only good for Linear models
  - Only when model is linear: Average of models is the model of averaged parameters
- Non-Linear Models
  - Use average parameters and hope for convergence (works surprisingly well)
  - If necessary, can use L^2 norm to find 'closest' model to average model

$$\int_{x_{\min}}^{x_{\max}} \left(F\left(x, \boldsymbol{\alpha}\right) - \bar{f}_i\right)^2 \, \mathrm{d}x$$

Multi-Experiment Rosenbluth Extraction

- Without considering full covariance matrix Rosenbluth Extractions are not useful