Understanding Jet Charge

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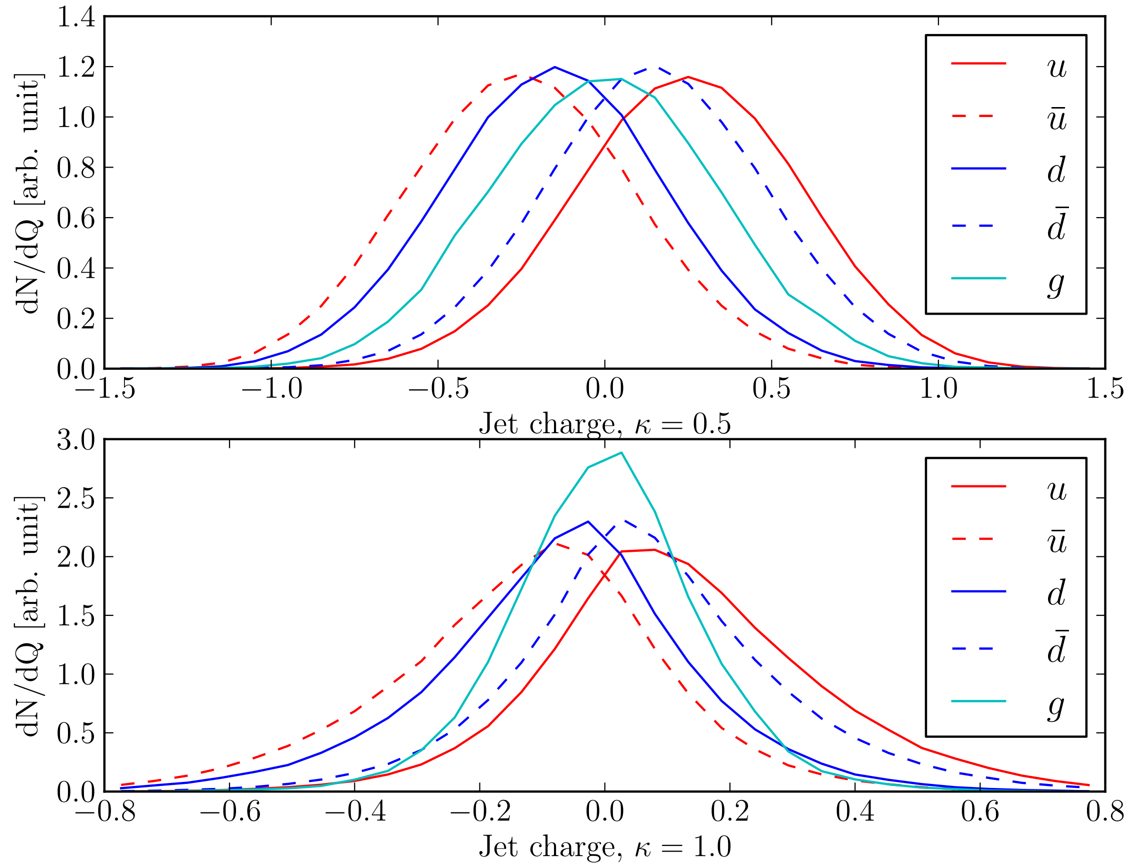
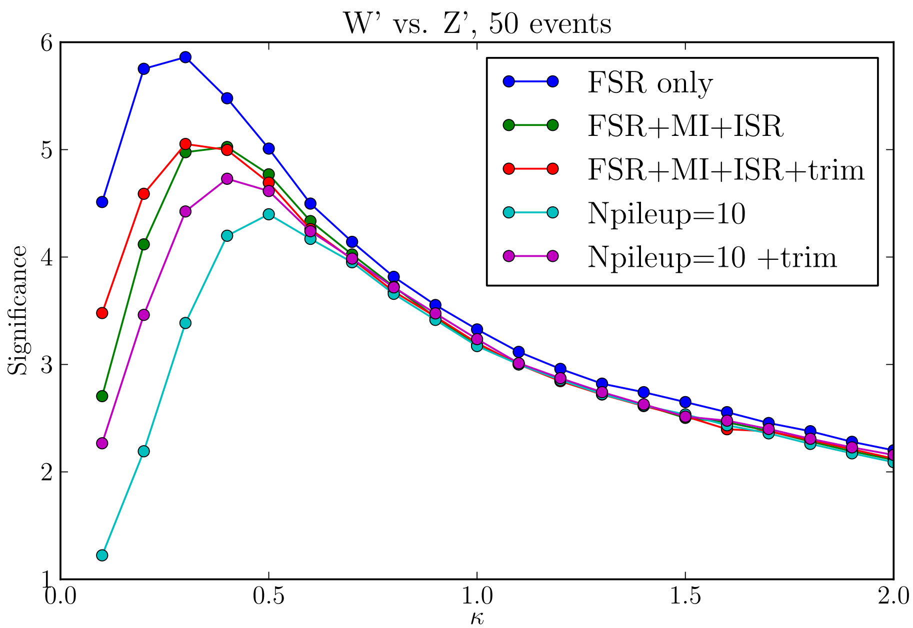
2301.09649

EIC Consortium, January 28, 2023

$$Q_\kappa \equiv \sum_{i \in J} z_i^\kappa \, Q_i$$

Feynman, Field 1977; 1209.2421

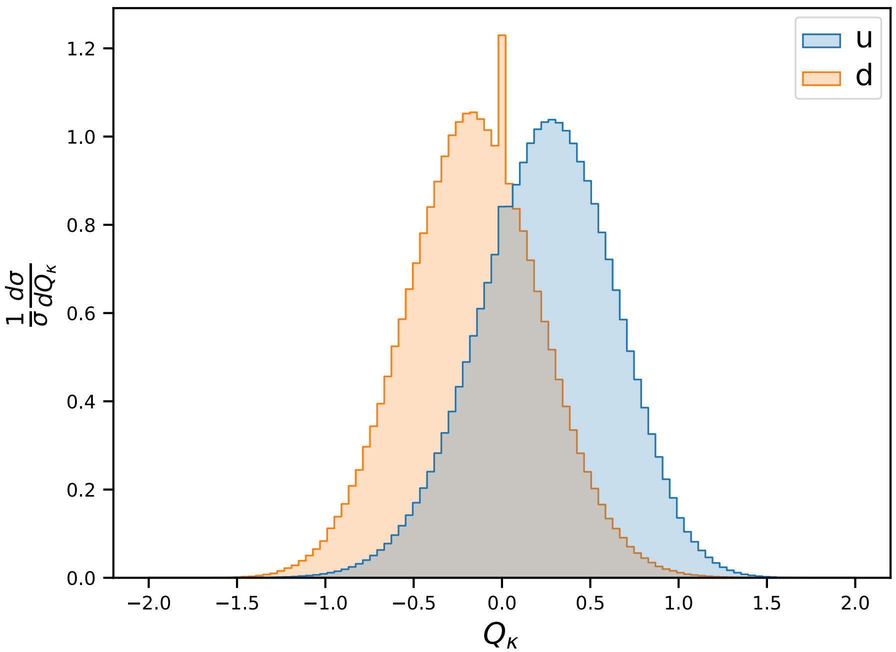




Powerful discriminant between parton flavors

Improved discrimination power as $\kappa \to 0$

Distribution narrows as κ increases



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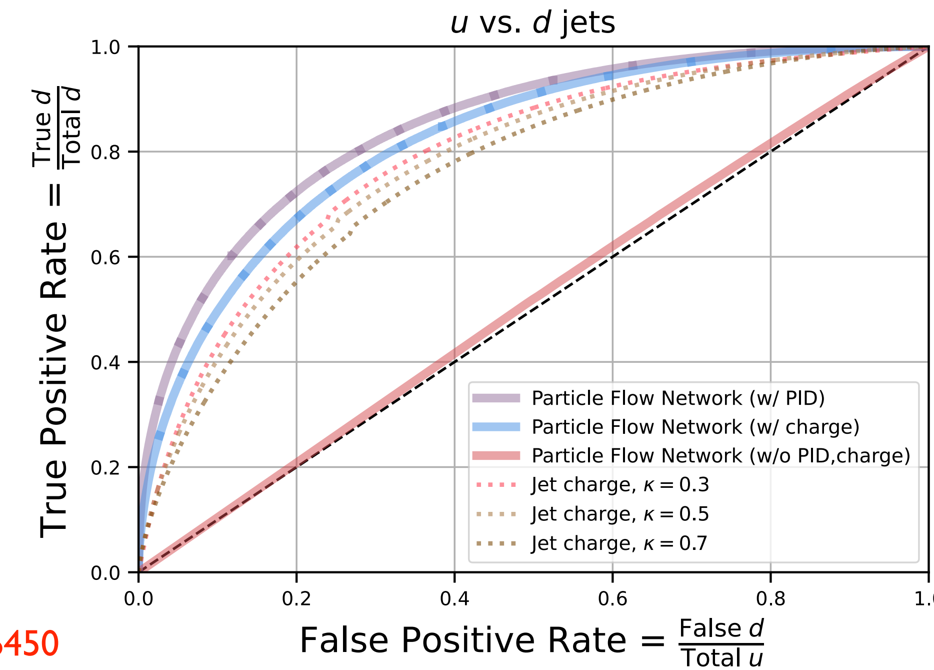

Application of jet charge to EIC physics: *u* versus *d* jet identification

Essentially no useful discrimination information exclusively in distribution of particle momenta

Jet charge is a (the only?) useful discriminant between these jets

Our Approach: Make Assumptions as Simple as Possible but No Simpler

1. Particles (hadrons) in the jet are produced though identical, independent processes.
2. The multiplicity of particles in the jet N is large.
3. The only particles are the pions: π^+, π^-, and π^0.
4. SU(2) isospin of the pions is an exact symmetry.

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Mean on u or d jets is set by fractional moment

$$\langle Q_\kappa\rangle = \left\langle \sum_{i\in J} z_i^\kappa Q_i \right\rangle = N\langle z^\kappa\rangle\langle Q\rangle = \langle z^\kappa\rangle(Q_u, Q_d)$$

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The variance is set by an energy fraction moment:

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$$p_u(Q_\kappa|N) = \frac{1}{\sqrt{2\pi \frac{2}{3} N\langle z^{2\kappa}\rangle}}\, e^{-\frac{\left(Q_\kappa - \frac{2}{3}\langle z^\kappa\rangle\right)^2}{\frac{4}{3}N\langle z^{2\kappa}\rangle}} \qquad p_d(Q_\kappa|N) = \frac{1}{\sqrt{2\pi \frac{2}{3} N\langle z^{2\kappa}\rangle}}\, e^{-\frac{\left(Q_\kappa + \frac{1}{3}\langle z^\kappa\rangle\right)^2}{\frac{4}{3}N\langle z^{2\kappa}\rangle}}$$

Let's calculate the mean/variance moments:

$$\langle Q_\kappa \rangle = Q_q \langle z^\kappa \rangle$$

$$\sigma_\kappa^2 = \frac{2}{3} N \langle z^{2\kappa} \rangle$$

$$\langle z \rangle = \int_0^1 dz\, p(z|N) = \frac{1}{N}$$

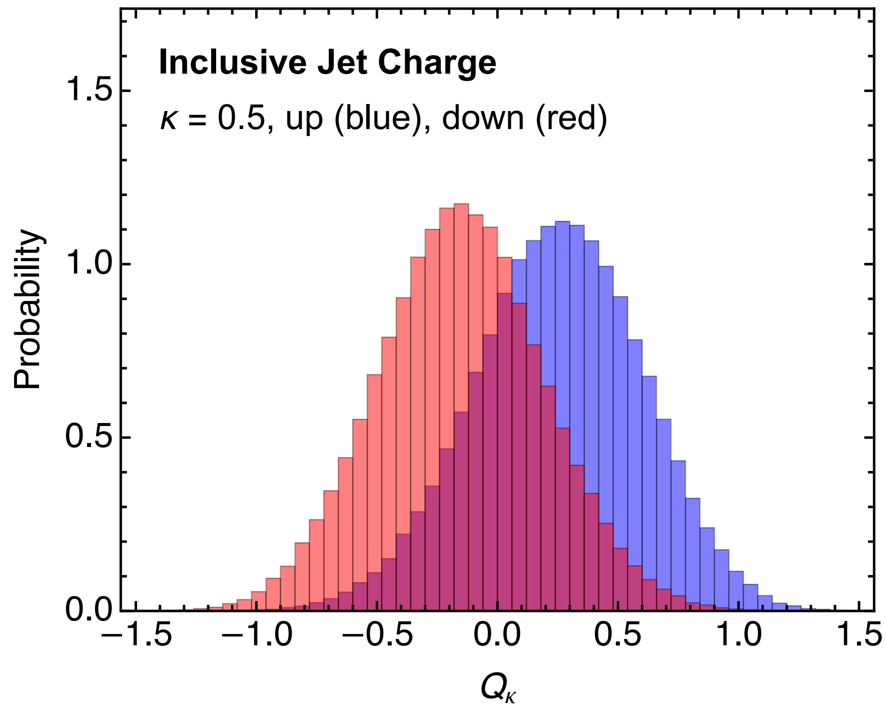
Make a central moment expansion:

$$p(z|N) = \delta\left(z - \frac{1}{N}\right) + \frac{\sigma_z^2}{2}\delta''\left(z - \frac{1}{N}\right) + \cdots$$

Fractional moments can be expressed as:

$$\langle z^\kappa \rangle = \int_0^1 dz\, z^\kappa\, p(z|N) = N^{-\kappa}\left(1 + \frac{\kappa}{2}(\kappa - 1)\sigma_z^2 N^2 + \cdots\right)$$

Optimal Parameter Predictions



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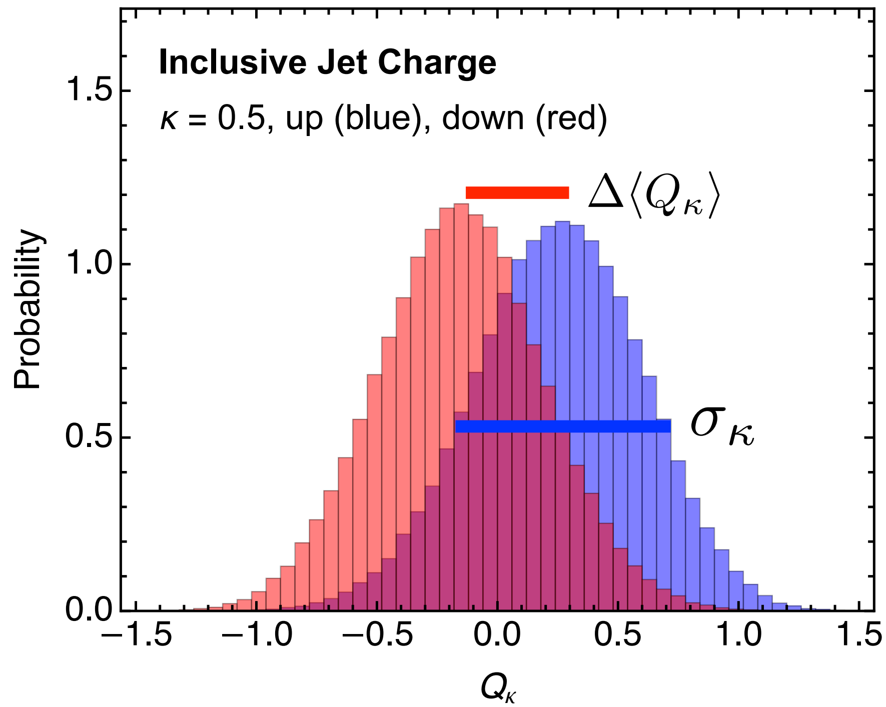
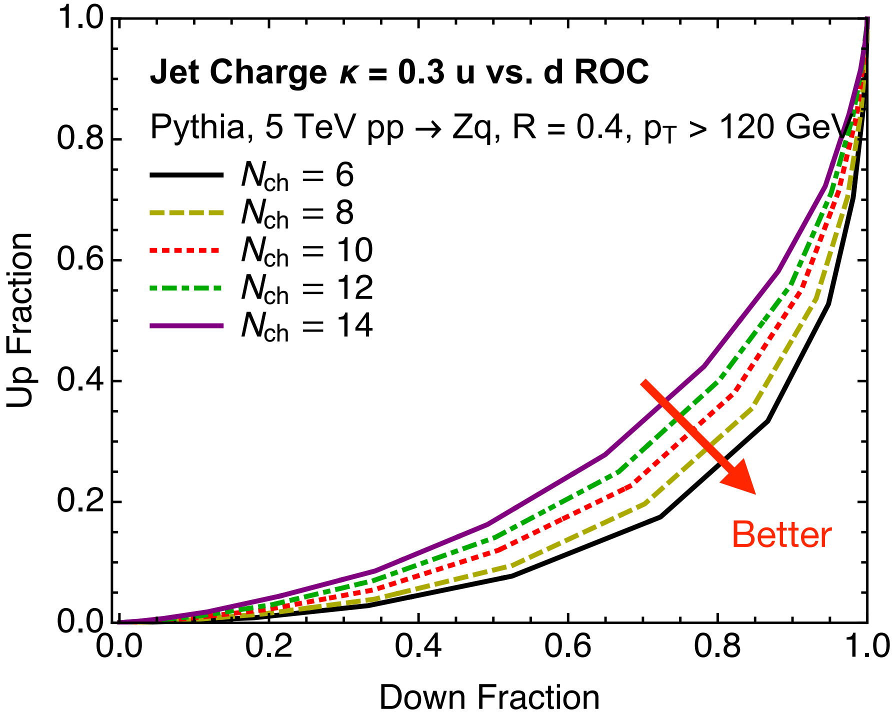


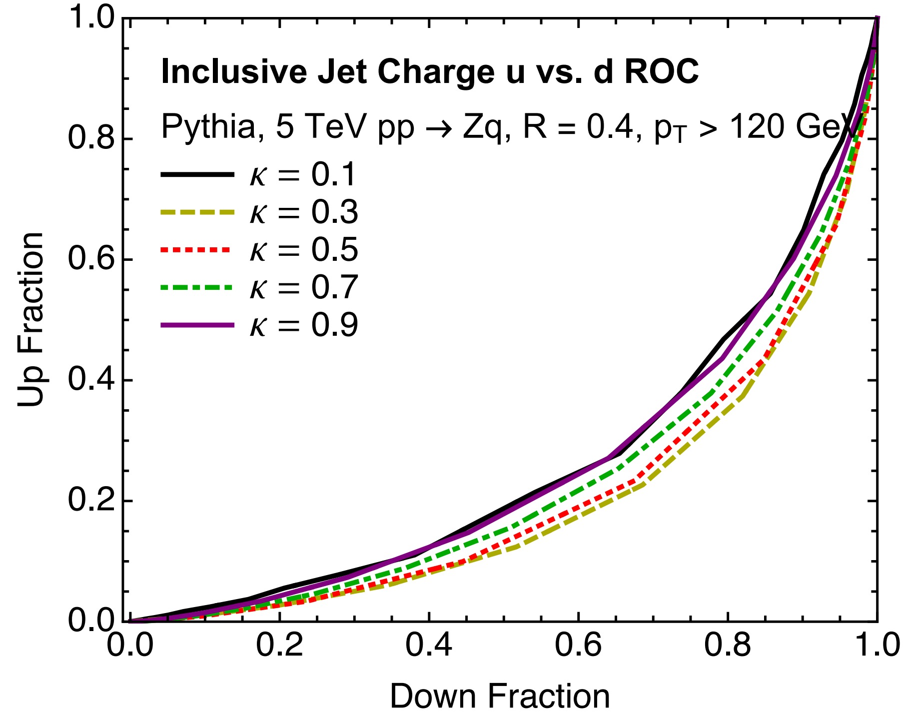
Discrimination Power is Maximized When

$$\arg\max_{\kappa,N} \frac{(\Delta\langle Q_\kappa\rangle)^2}{\sigma_\kappa^2} = \arg\max_{\kappa,N} \frac{\langle z^\kappa\rangle^2}{\frac{2}{3}N\langle z^{2\kappa}\rangle} = \arg\max_{\kappa,N} \frac{1}{\frac{2}{3}N}\left(1-\kappa^2\sigma_z^2 N^2+\cdots\right)$$

Optimal discrimination when κ and N are small

Optimal Parameter Predictions





If κ is too small, then IR contamination overwhelms jet charge

One More Thing…

Optimal Discrimination Observable by Neyman-Pearson is Log-Likelihood:

$$\mathcal{O} = \log \frac{p_u(Q_\kappa, N)}{p_d(Q_\kappa, N)} = \log \frac{p_u(Q_\kappa|N)p(N)}{p_d(Q_\kappa|N)p(N)}$$

Assuming that multiplicity distribution is identical for up and down jets (see bonus)

Just Take Ratio of Gaussian Distributions:

$$\mathcal{O} = \frac{3}{2} N^{-1+\kappa} Q_\kappa - \frac{N^{-1}}{4}$$

Not Monotonically Related to Jet Charge Q_κ

Necessarily Improve Discrimination Power by Measuring Jet Charge Differential in Multiplicity

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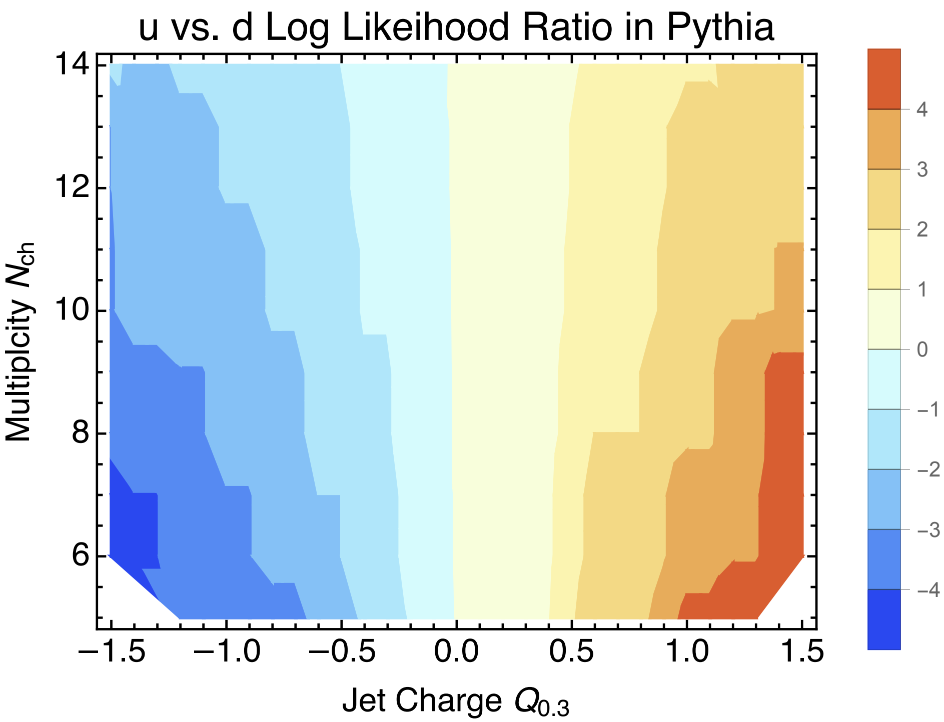
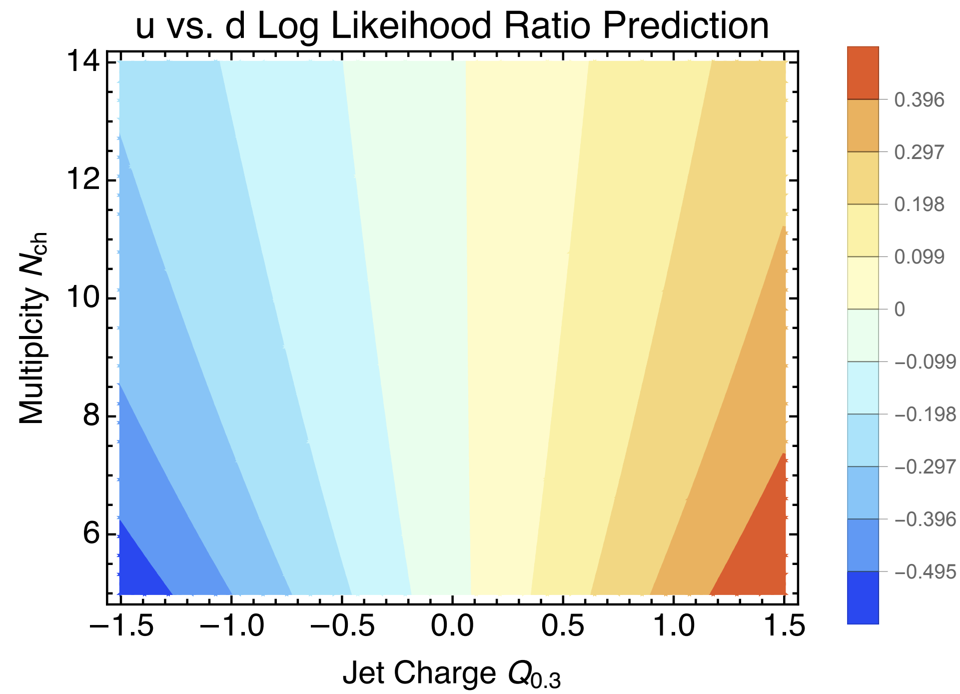
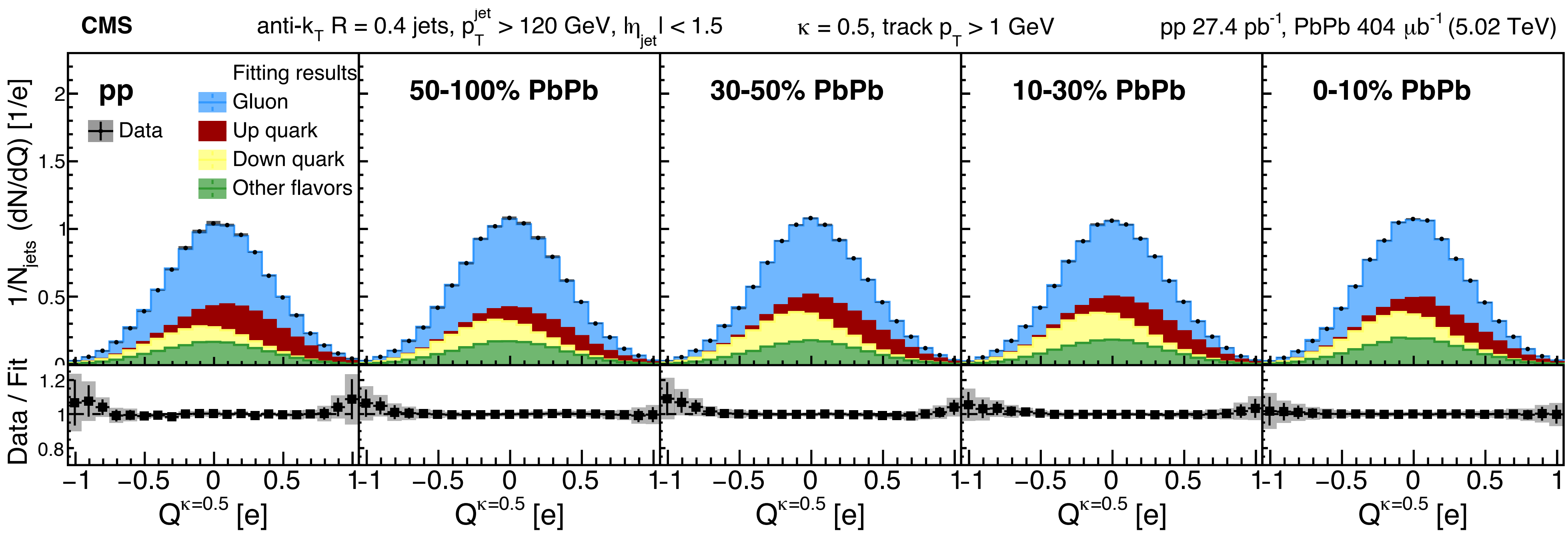




Takeaway:

Measure Jet Charge Differential in Multiplicity (and not centrality)!

Bonus



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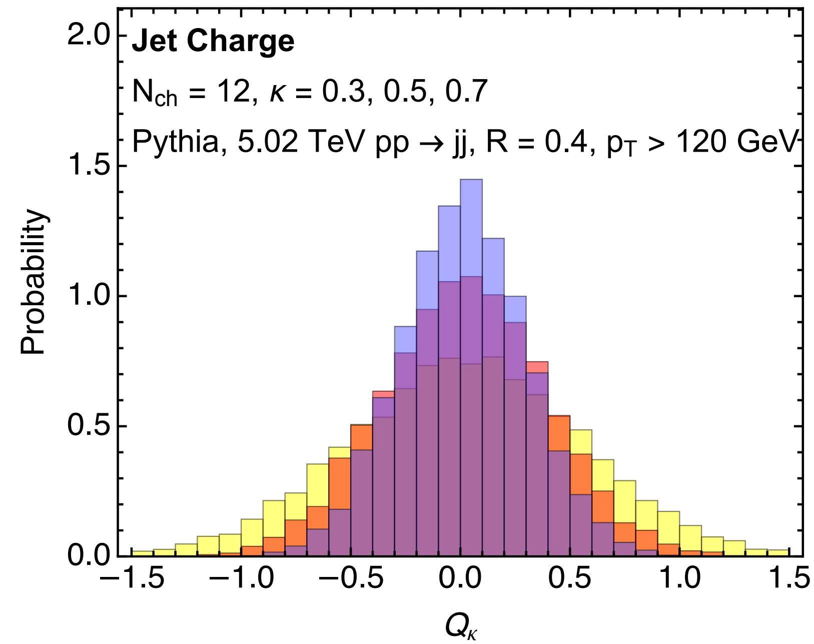
Surprisingly little medium modification to jet charge from pp to PbPb

Aren’t gluon jets quenched more than quark jets? How large is UE effect?

Can we understand this?

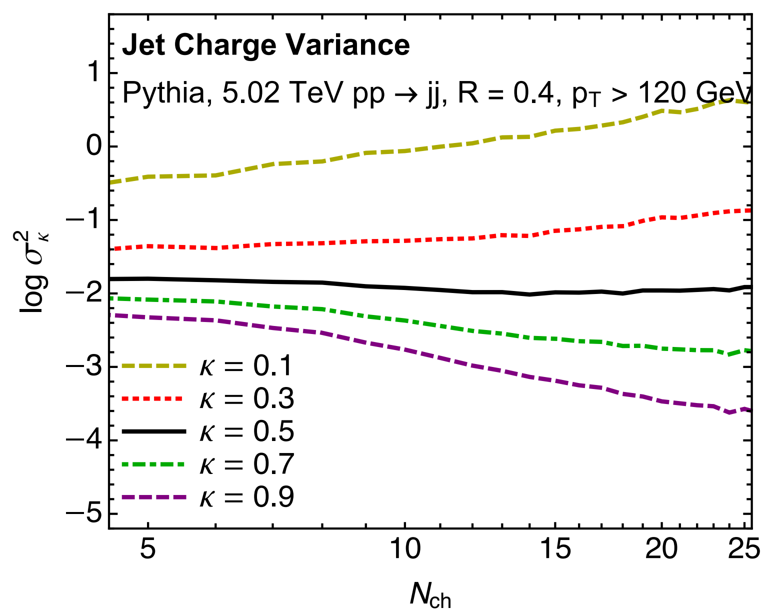
Predictions

$$\sigma_\kappa^2 = \frac{2}{3} N^{1-2\kappa} \left(1 + \kappa(2\kappa - 1)\sigma_z^2 N^2 + \cdots\right)$$



Jet charge distribution narrows as κ increases

As multiplicity *N* increases (jet pT increases), distribution widens if $\kappa < 0.5$



As multiplicity *N* increases (jet pT increases), distribution narrows if $\kappa > 0.5$

Width is independent of *N* if $\kappa = 0.5$

Up and Down Quark Jet Multiplicity Distributions