

Small-x Weizsäcker-Williams gluon distribution at NLO

Farid Salazar (UCLA/LBL)

California EIC UC Collaboration Meeting
Jan 28th, 2023

Based on

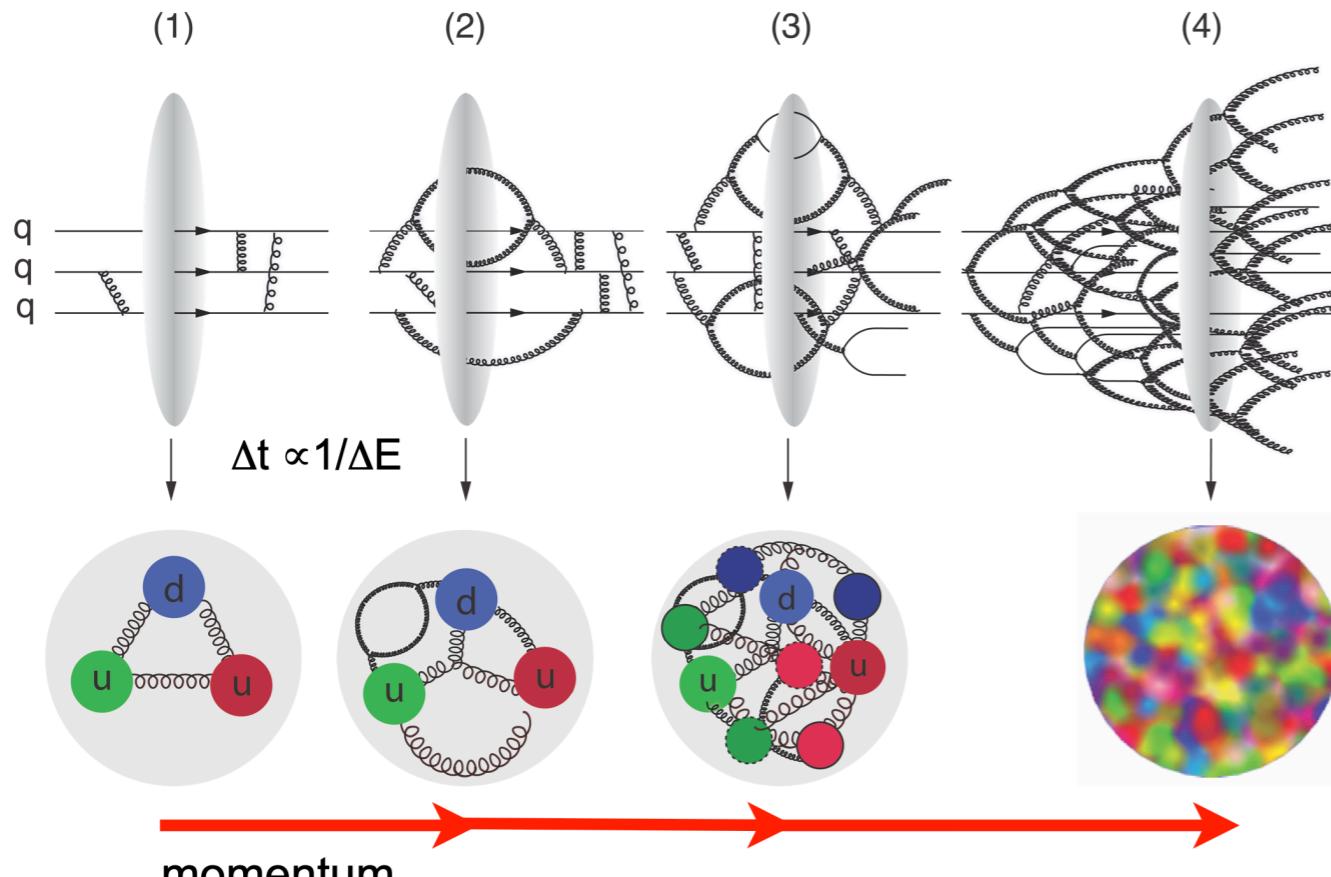
- (1) [2108.06347](#) [*JHEP 11 (2021) 222*]
- (2) [2208.13872](#) [*JHEP 11 (2022) 169*]
- (3) [2302.XXXX](#)

In collaboration with

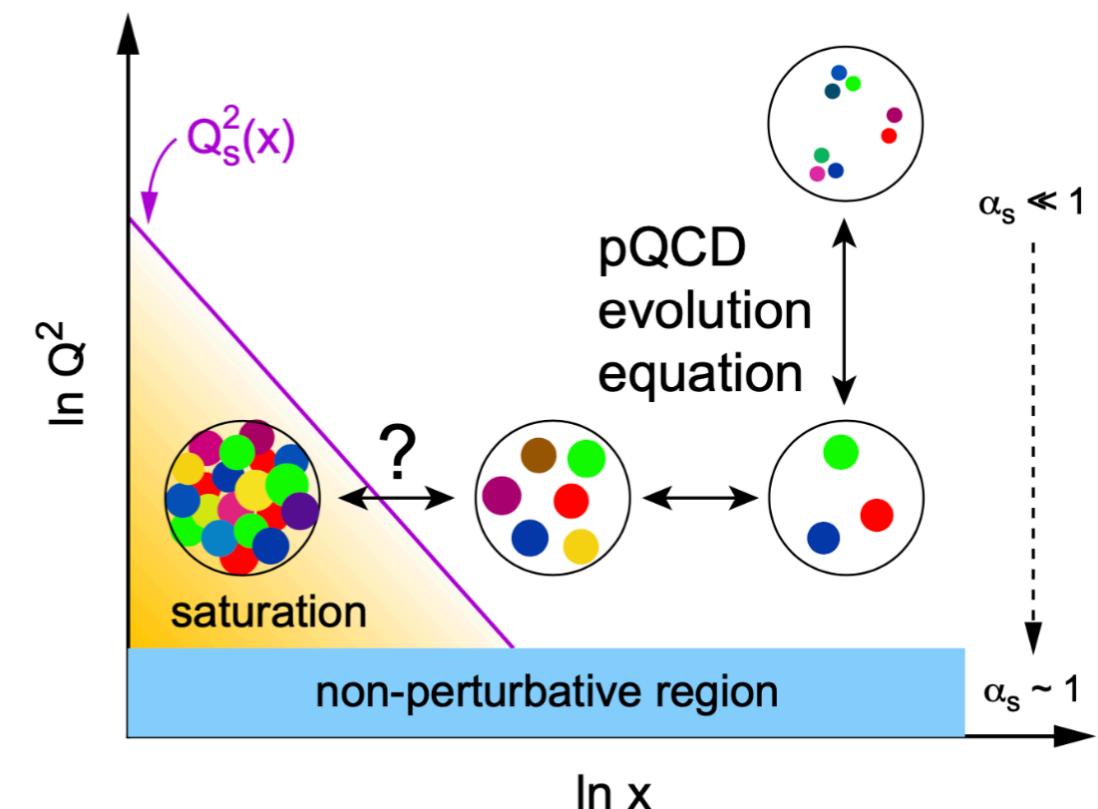
Paul Caucal (Nantes)
Björn Schenke (BNL)
Tomasz Stebel (Jagiellonian)
Raju Venugopalan (BNL)

Gluon saturation

The Color Glass Condensate (CGC) an effective theory for high-energy QCD



Artwork: T. Ullrich



Emergence of an energy and nuclear specie dependent momentum scale (saturation scale)

Multiple scattering (higher twist effects)

Non-linear evolution equations (BK/JIMWLK)

$$Q_s^2 \propto A^{1/3} x^{-\lambda} \quad x \propto 1/s$$

For a review see Mining gluon saturation at colliders. FS, A. Morreale (Universe 2021)

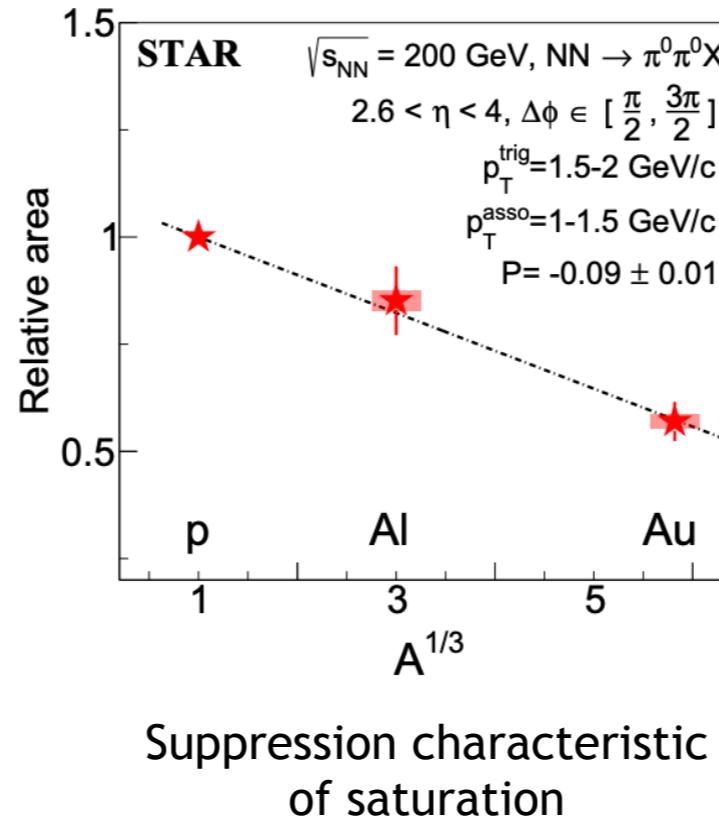
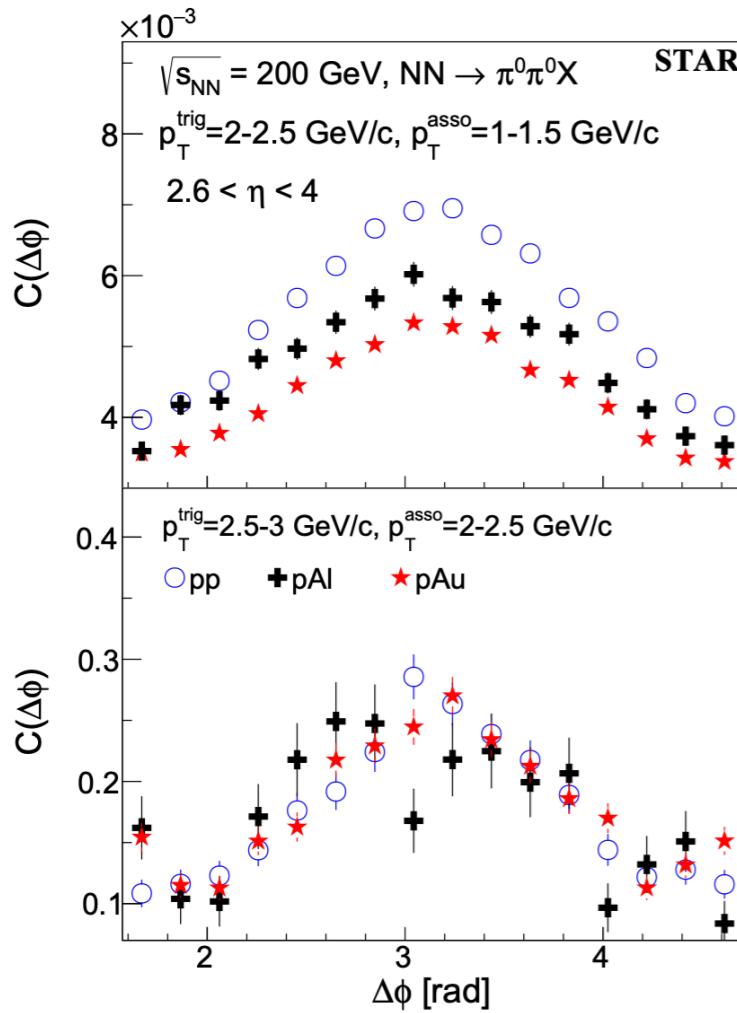
Gluon saturation

Dihadron azimuthal correlations at RHIC

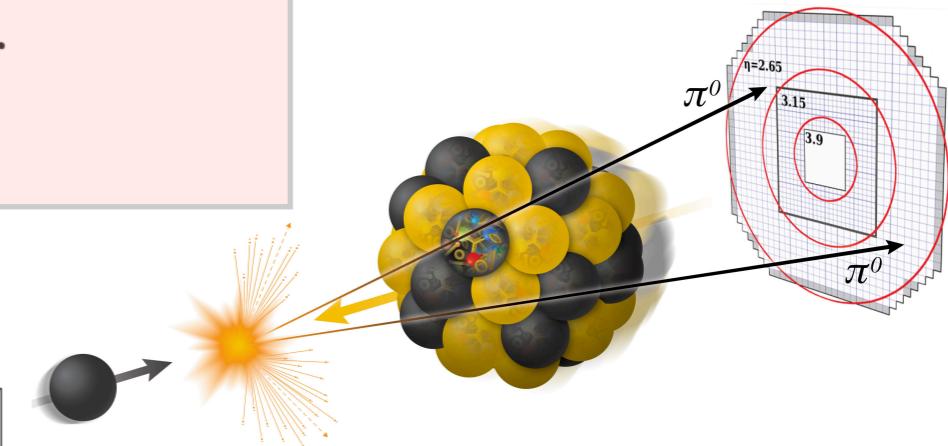
PHYSICAL REVIEW LETTERS 129, 092501 (2022)

Evidence for Nonlinear Gluon Effects in QCD and Their Mass Number Dependence at STAR

STAR Collaboration



$$Q_s^2 \propto A^{1/3}$$



Xiaoxuan Chu and Elke Aschenauer



HIT Seminar on Feb 7th!

What about dihadron production at the EIC?

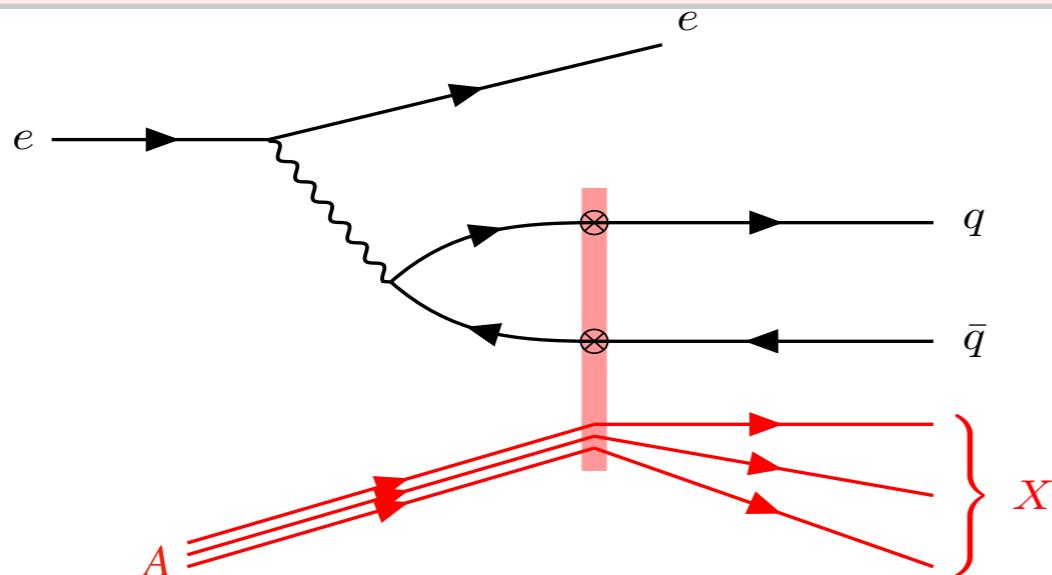
Gluon saturation

Dijet azimuthal correlations in DIS

PHYSICAL REVIEW D 67, 074019 (2003)

From deep inelastic scattering to proton-nucleus collisions in the color glass condensate model

François Gelis Jamal Jalilian-Marian



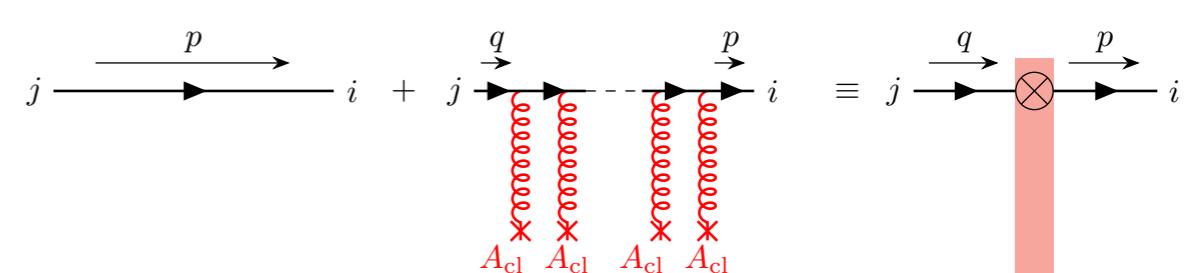
Unpolarized differential cross-section:

$$\frac{d\sigma^{\gamma_\lambda^* + A \rightarrow q\bar{q} + X}}{d^2 k_{1\perp} d^2 k_{2\perp} d\eta_1 d\eta_2} \propto \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot (\mathbf{x}_\perp - \mathbf{x}'_\perp)} e^{-i\mathbf{k}_{2\perp} \cdot (\mathbf{y}_\perp - \mathbf{y}'_\perp)} \\ \times \langle \Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{y}'_\perp, \mathbf{x}'_\perp) \rangle_Y \mathcal{R}^\lambda(\mathbf{x}_\perp - \mathbf{y}_\perp, \mathbf{x}'_\perp - \mathbf{y}'_\perp)$$

Correlators of Wilson lines = multiple scattering + non-linear evolution

Numerical evaluation is challenging,
done for the first time in 2020

Effective interaction of quark with CGC:



$$V_{ij}(\mathbf{x}) = P \exp \left\{ ig \int dx^- A_{cl}^{+,a}(\mathbf{x}, x^-) t^a \right\}$$

$q\bar{q}$ interaction with nucleus

γ^* splitting to $q\bar{q}$

PHYSICAL REVIEW LETTERS 124, 112301 (2020)

Multigluon Correlations and Evidence of Saturation from Dijet Measurements at an Electron-Ion Collider

Heikki Mäntysaari^{1,2,*}, Niklas Mueller,^{3,†}, Farid Salazar^{3,4,‡}, and Björn Schenke^{3,§}

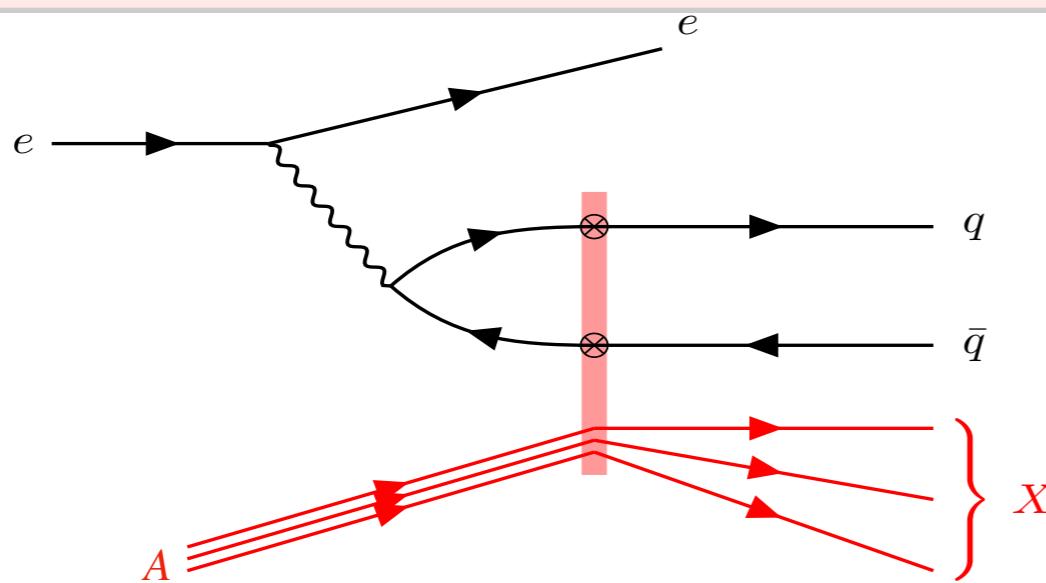
CGC and TMD correspondence

Small- x Weizsäcker-Williams distribution

PHYSICAL REVIEW D 83, 105005 (2011)

Universality of unintegrated gluon distributions at small x

Fabio Dominguez,¹ Cyrille Marquet,² Bo-Wen Xiao,^{3,4} and Feng Yuan^{4,5}



Dijets are produced back-to-back (in transverse place)
CGC cross-section factorizes:

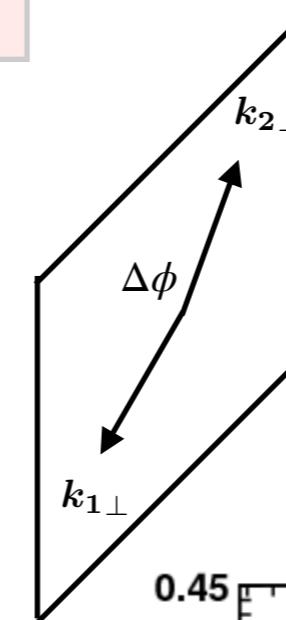
$$d\sigma^{\gamma^* + A \rightarrow q\bar{q} + X} \sim \mathcal{H}^{ij}(Q, \mathbf{P}_\perp) \alpha_s G_Y^{ij}(\mathbf{k}_\perp)$$

Perturbatively
calculable

WW gluon TMD

Typical momentum of WW $\langle k_\perp \rangle \sim Q_s$

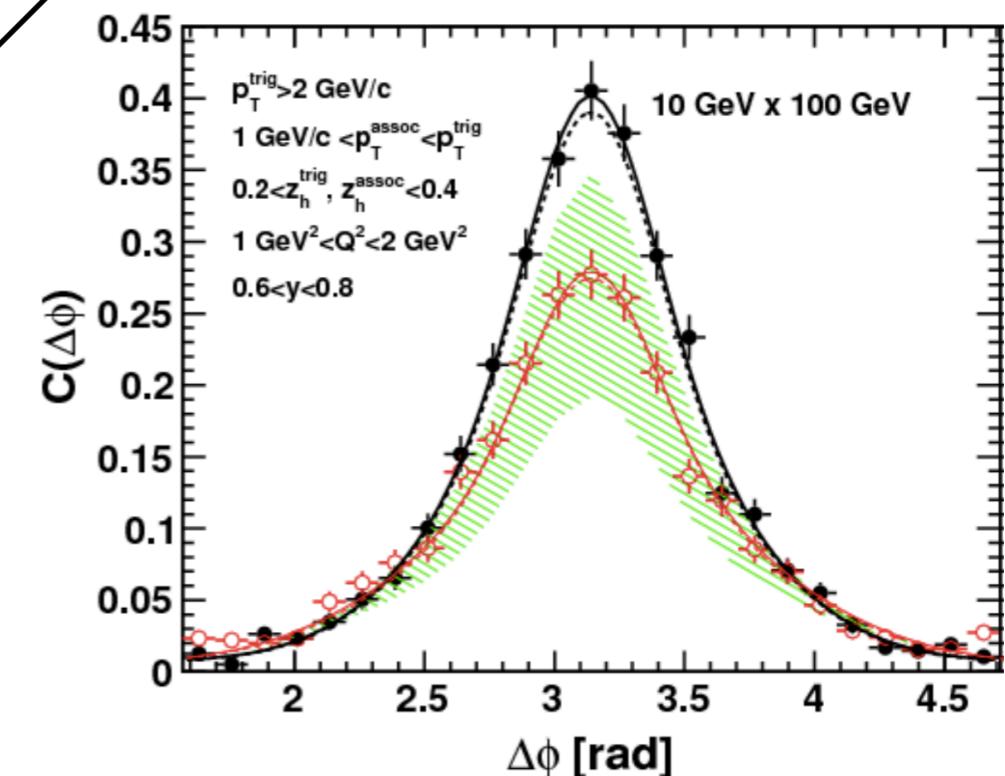
Nuclear dependence $Q_s^2 \propto A^{1/3}$



$$\mathbf{k}_\perp = \mathbf{k}_{1\perp} + \mathbf{k}_{2\perp}$$

$$\mathbf{P}_\perp = z_2 \mathbf{k}_{1\perp} - z_1 \mathbf{k}_{2\perp}$$

$\Delta\phi$ distribution sensitive to
gluon saturation Q_s via WW



Zheng, Aschenauer, Lee, Xiao (2014)

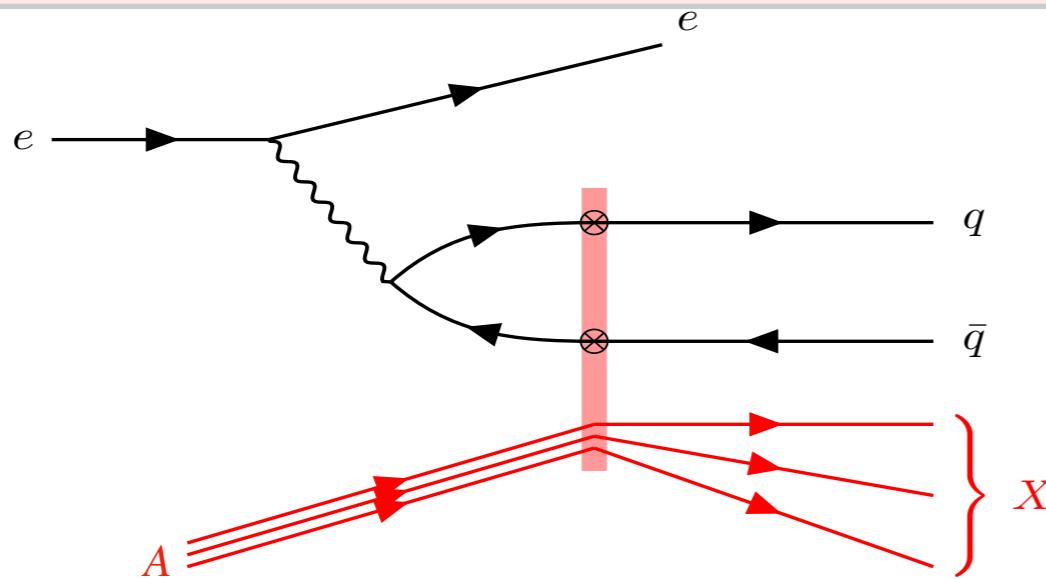
CGC and TMD correspondence

Small-x Weizsäcker-Williams distribution

PHYSICAL REVIEW D 85, 045003 (2012)

Linearly polarized gluon distributions in the color dipole model

Fabio Dominguez,¹ Jian-Wei Qiu,^{2,3} Bo-Wen Xiao,⁴ and Feng Yuan⁵



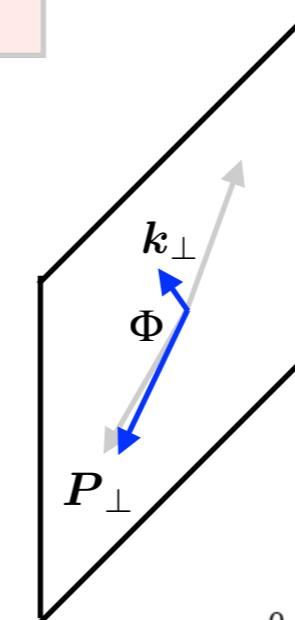
Dijets are produced back-to-back (in transverse place)
CGC cross-section factorizes:

$$d\sigma^{\gamma^* + A \rightarrow q\bar{q} + X} \sim \mathcal{H}^{ij}(Q, P_\perp) \alpha_s G_Y^{ij}(k_\perp)$$

Perturbatively calculable WW gluon TMD

$$\langle \cos(2\Phi) \rangle \propto h_Y^0(k_\perp)/G_Y^0(k_\perp)$$

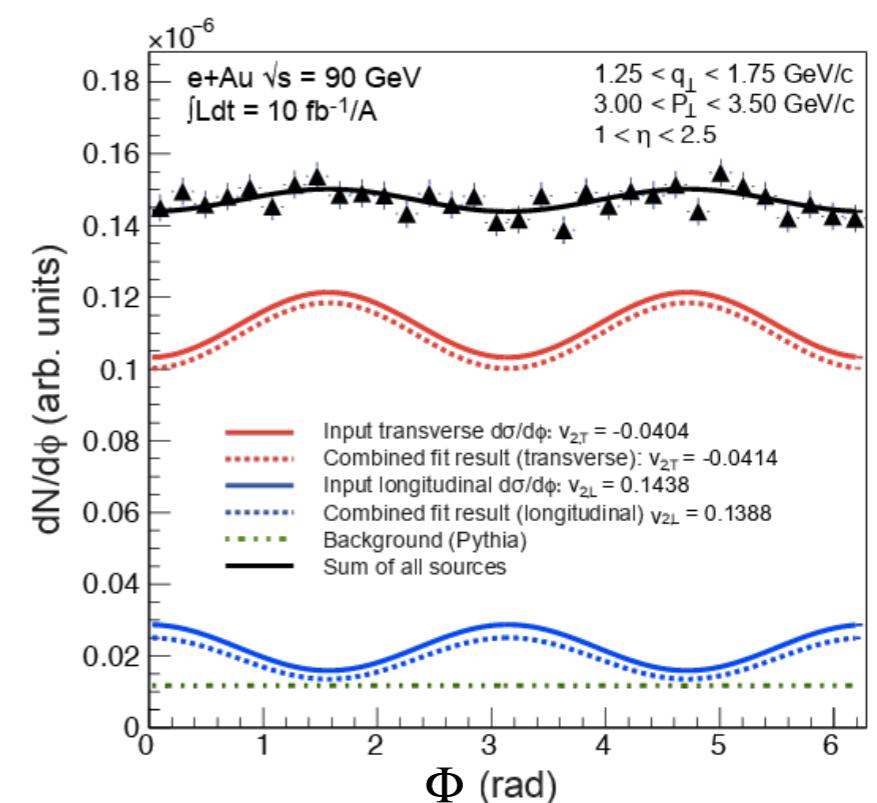
Ratio linearly polarized to unpolarized WW



$$k_\perp = k_{1\perp} + k_{2\perp}$$

$$P_\perp = z_2 k_{1\perp} - z_1 k_{2\perp}$$

Φ distribution sensitive to
linearly polarized WW gluon TMD



Dumitru, Skokov, Ullrich (2018)

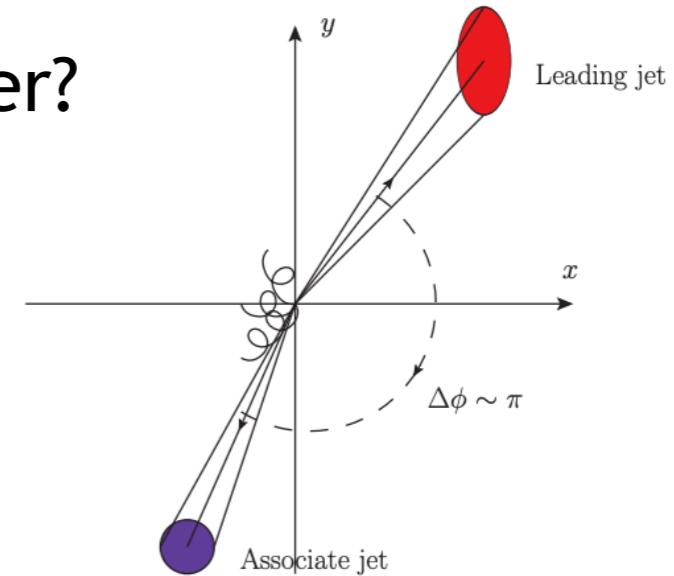
CGC and TMD correspondence

Does correspondence hold at next-to-leading order?

PHYSICAL REVIEW D 88, 114010 (2013)

**Sudakov double logarithms resummation in hard processes
in the small- x saturation formalism**

A. H. Mueller,¹ Bo-Wen Xiao,² and Feng Yuan³



Conjecture: joint (soft) Sudakov + small- x resummation

$$d\sigma^{\gamma_\lambda^* + A \rightarrow q\bar{q} + X} \propto \mathcal{H}(Q, \mathbf{P}_\perp) \int \frac{d^2 \mathbf{b}_\perp}{(2\pi)^2} e^{-i\mathbf{q}_\perp \cdot \mathbf{b}_\perp} \alpha_s \tilde{G}_Y^0(\mathbf{b}_\perp) e^{-S_{\text{Sud}}(\mathbf{b}_\perp, \mathbf{P}_\perp)}$$

Perturbative
Sudakov factor:

$$S_{\text{Sud}}(\mathbf{b}_\perp, Q) = \int_{\mu_b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[A \log \left(\frac{Q^2}{\bar{\mu}^2} \right) + B \right]$$

Soft gluon emissions



Change profile of
azimuthal correlations

Can we derive these results more rigorously? Can we obtain finite NLO pieces?

CGC and TMD correspondence

Full CGC calculation at NLO



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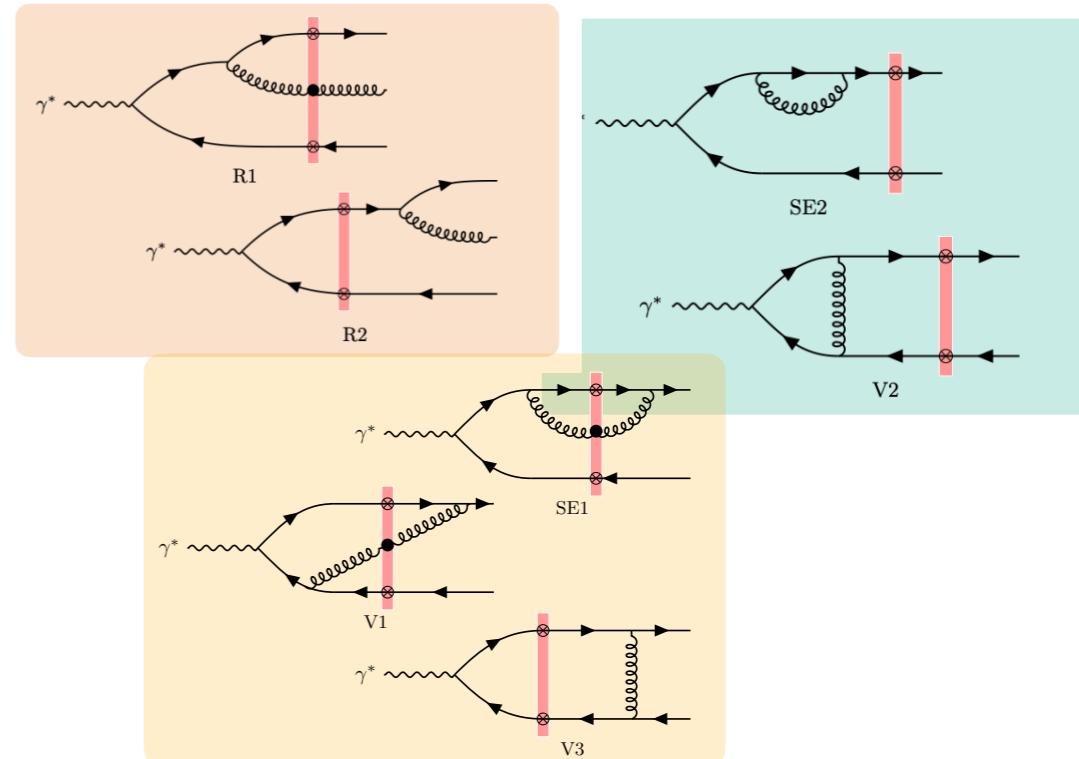
Dijet impact factor in DIS at next-to-leading order in the Color Glass Condensate

Paul Caucal,^a Farid Salazar^{a,b,c} and Raju Venugopalan^a

$$d\sigma_{\text{NLO}} = d\tilde{\sigma}_0 \ln \left(\frac{z_f}{z_0} \right) + \int_0^z \frac{dz_g}{z_g} [d\tilde{\sigma}_{\text{NLO}} - d\tilde{\sigma}_0 \Theta(z_f - z_g)]$$

Large log at small x

impact factor



$$\begin{aligned} \frac{d\sigma_{\text{NLO}}^\lambda}{d^2\mathbf{k}_{1\perp} d\eta_1 d^2\mathbf{k}_{2\perp} d\eta_2} \Big|_{\text{LLx}} &= \frac{\alpha_{\text{em}} e_f^2 N_c}{(2\pi)^6} \delta(1 - z_q - z_{\bar{q}}) \int d\mathbf{X}_\perp \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \ln \left(\Lambda_f^- / \Lambda^- \right) \\ &\times \frac{\alpha_s N_c}{4\pi^2} \left\langle \int d^2\mathbf{z}_\perp \left\{ \frac{\mathbf{r}_{xy}^2}{\mathbf{r}_{zx}^2 \mathbf{r}_{zy}^2} (2D_{xy} - 2D_{xz}D_{zy} + D_{zy}Q_{y'x',xz} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xy}D_{y'x'}) \right. \right. \\ &+ \frac{\mathbf{r}_{x'y'}^2}{\mathbf{r}_{zx}^2 \mathbf{r}_{zy'}^2} (2D_{y'x'} - 2D_{y'z}D_{zx'} + D_{zx'}Q_{xy,y'z} + D_{y'z}Q_{xy,zx'} - Q_{xy,y'x'} - D_{xy}D_{y'x'}) \\ &+ \frac{\mathbf{r}_{xx'}^2}{\mathbf{r}_{zx}^2 \mathbf{r}_{zx'}^2} (D_{zx'}Q_{xy,y'z} + D_{xz}Q_{y'x',zy} - Q_{xy,y'x'} - D_{xx'}D_{y'y}) \\ &+ \frac{\mathbf{r}_{yy'}^2}{\mathbf{r}_{zy}^2 \mathbf{r}_{zy'}^2} (D_{y'z}Q_{xy,zx'} + D_{zy}Q_{y'x',xz} - Q_{xy,y'x'} - D_{xx'}D_{y'y}) \\ &+ \frac{\mathbf{r}_{xy'}^2}{\mathbf{r}_{zx}^2 \mathbf{r}_{zy'}^2} (D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{zx'}Q_{xy,y'z} - D_{zy}Q_{y'x',zy}) \\ &+ \left. \left. \frac{\mathbf{r}_{x'y}^2}{\mathbf{r}_{zx}^2 \mathbf{r}_{zy}^2} (D_{xx'}D_{y'y} + D_{xy}D_{y'x'} - D_{y'z}Q_{xy,zx'} - D_{xz}Q_{y'x',zy}) \right\} \right\rangle_Y \end{aligned}$$

Small- x evolution of dipole and quadrupole!

JIMWLK LL Hamiltonian
acting on LO cross-section

Renormalization of
Wilson line operators

CGC and TMD correspondence

Full CGC calculation at NLO



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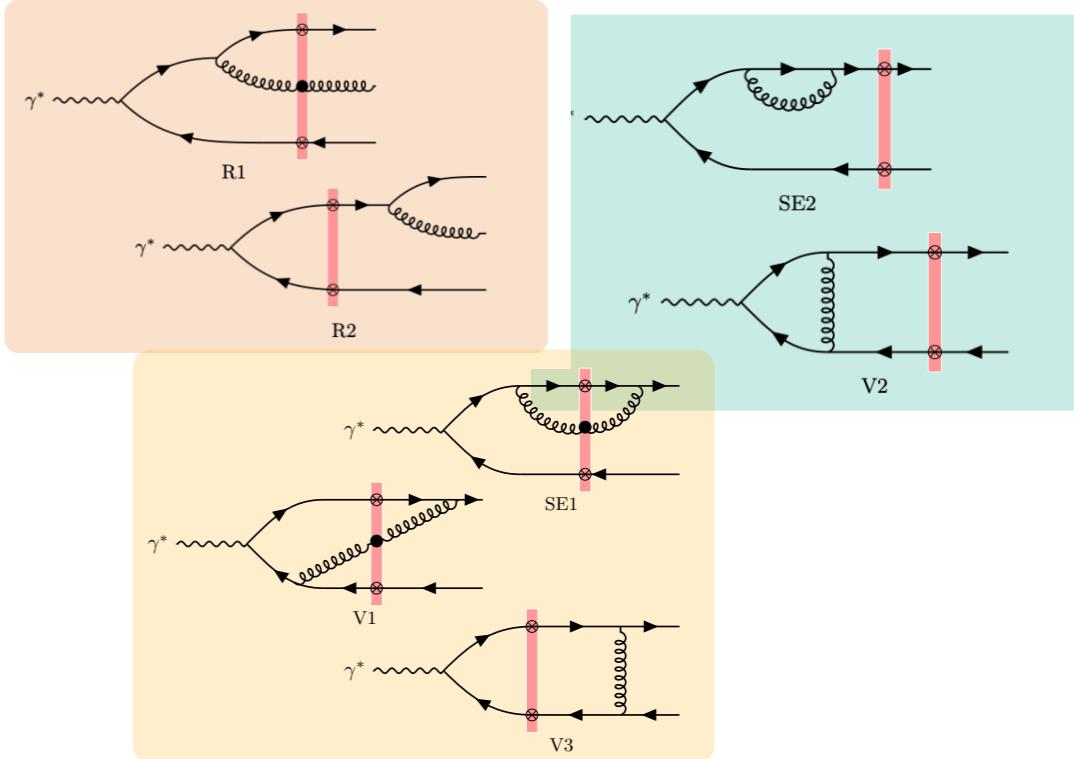
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Dijet impact factor in DIS at next-to-leading order in the Color Glass Condensate

Paul Caucal,^a Farid Salazar^{a,b,c} and Raju Venugopalan^a

$$d\sigma_{\text{NLO}} = d\tilde{\sigma}_0 \ln \left(\frac{z_f}{z_0} \right) + \int_0^z \frac{dz_g}{z_g} [d\tilde{\sigma}_{\text{NLO}} - d\tilde{\sigma}_0 \Theta(z_f - z_g)]$$

Large log at small x
impact factor



$$d\sigma_{R_2 \times R_2, \text{sud2}} = \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \\ \times C_F \Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \times \frac{\alpha_s}{\pi} \int_0^1 \frac{d\xi}{\xi} [1 - e^{-i\xi \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'}}] \ln \left(\frac{\mathbf{k}_{1\perp}^2 \mathbf{r}_{xx'}^2 R^2 \xi^2}{c_0^2} \right)$$

$$d\sigma_{\text{sud1}} = \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \times \frac{\alpha_s}{\pi} \\ \times \left\{ C_F \Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \left[\ln \left(\frac{z_f}{z_1} \right) \ln \left(\frac{\mathbf{r}_{xx'}^2}{|\mathbf{r}_{xy}| |\mathbf{r}_{x'y'}|} \right) + \ln \left(\frac{z_f}{z_2} \right) \ln \left(\frac{\mathbf{r}_{yy'}^2}{|\mathbf{r}_{xy}| |\mathbf{r}_{x'y'}|} \right) \right] \right\}$$

$$d\sigma_{V,\text{no-sud},\text{NLO}_3} = \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int d^3 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} 8z_1^3 z_2^3 Q^2 K_0(\bar{Q} r_{x'y'}) \\ \times \frac{\alpha_s}{\pi} \int_0^{z_1} \frac{dz_g}{z_g} \left\{ K_0(\bar{Q} V_3 r_{xy}) \left[\left(1 - \frac{z_g}{z_1} \right)^2 \left(1 + \frac{z_g}{z_2} \right) (1 + z_g) e^{i(\mathbf{P}_\perp + z_g \mathbf{q}_\perp) \cdot \mathbf{r}_{xy}} K_0(-i\Delta V_3 r_{xy}) \right. \right. \right. \\ \left. \left. \left. - \left(1 - \frac{z_g}{2z_1} + \frac{z_g}{2z_2} - \frac{z_g^2}{2z_1 z_2} \right) e^{i \frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy}} \mathcal{J}_\odot \left(\mathbf{r}_{xy}, \left(1 - \frac{z_g}{z_1} \right) \mathbf{P}_\perp, \Delta V_3 \right) \right] \right. \\ \left. + K_0(\bar{Q} r_{xy}) \ln \left(\frac{z_g P_{\perp} r_{xy}}{c_0 z_1 z_2} \right) + (1 \leftrightarrow 2) \right\} \Xi_{\text{NLO},3}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) + c.c.$$

$$d\sigma_{V,\text{no-sud},\text{LO}} = \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \\ \times \frac{\alpha_s C_F}{\pi} \left\{ -\frac{3}{4} \ln \left(\frac{\mathbf{k}_{1\perp}^2 \mathbf{k}_{2\perp}^2 \mathbf{r}_{xy}^2 \mathbf{r}_{x'y'}^2}{c_0^4} \right) - 3 \ln(R) + \frac{1}{2} \ln^2 \left(\frac{z_1}{z_2} \right) + \frac{11}{2} + 3 \ln(2) - \frac{\pi^2}{2} \right\}$$

$$d\sigma_{V,\text{no-sud},\text{other}}^{\lambda=L} = \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} 8z_1^3 z_2^3 Q^2 K_0(\bar{Q} r_{x'y'}) \int_0^{z_1} \frac{dz_g}{z_g} \\ \times \frac{\alpha_s}{\pi} \int \frac{d^2 \mathbf{z}_\perp}{\pi} \left\{ \frac{1}{\mathbf{r}_{zx}^2} \left[\left(1 - \frac{z_g}{z_1} + \frac{z_g^2}{2z_1^2} \right) e^{-i \frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{zx}} K_0(Q X_V) - \Theta(z_f - z_g) K_0(\bar{Q} r_{xy}) \right] \Xi_{\text{NLO},1} \right. \\ \left. - \frac{1}{\mathbf{r}_{zx}^2} \left[\left(1 - \frac{z_g}{z_1} + \frac{z_g^2}{2z_1^2} \right) e^{-i \frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy}} K_0(\bar{Q} r_{xy}) - \Theta(z_f - z_g) e^{-i \frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy}} K_0(\bar{Q} r_{xy}) \right] C_F \Xi_{\text{LO}} \right. \\ \left. - \frac{\mathbf{r}_{zx} \cdot \mathbf{r}_{zy}}{\mathbf{r}_{zx}^2 \mathbf{r}_{zy}^2} \left[\left(1 - \frac{z_g}{z_1} \right) \left(1 + \frac{z_g}{z_2} \right) \left(1 - \frac{z_g}{2z_1} - \frac{z_g}{2(z_2 + z_g)} \right) e^{-i \frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{zx}} K_0(Q X_V) \right. \right. \\ \left. \left. - \Theta(z_f - z_g) K_0(\bar{Q} r_{xy}) \right] \Xi_{\text{NLO},1} + (1 \leftrightarrow 2) \right\} + c.c..$$

$$d\sigma_{R_2 \times R'_2, \text{sud2}} = \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \\ \times \Xi_{\text{NLO},3}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \times \frac{(-\alpha_s)}{\pi} \int_0^1 \frac{d\xi}{\xi} [1 - e^{-i\xi \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy'}]} \ln \left(\frac{\mathbf{P}_\perp^2 \mathbf{r}_{xy}^2 \xi^2}{z_2^2 c_0^2} \right) \\ + \Xi_{\text{NLO},3}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \left[\ln \left(\frac{z_1}{z_f} \right) \ln \left(\frac{\mathbf{r}_{xy}^2}{|\mathbf{r}_{xy}| |\mathbf{r}_{x'y'}|} \right) + \ln \left(\frac{z_2}{z_f} \right) \ln \left(\frac{\mathbf{r}_{y'x'}^2}{|\mathbf{r}_{xy}| |\mathbf{r}_{x'y'}|} \right) \right]$$

$$d\sigma_{R,\text{no-sud},\text{LO}}^{\gamma_L^* + A \rightarrow q\bar{q}g + X} = \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(3)}}{(2\pi)^8} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} (4\alpha_s C_F) \Xi_{\text{LO}}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \\ \times \frac{e^{-i\mathbf{k}_{g\perp} \cdot \mathbf{r}_{zx'}}}{(\mathbf{k}_{g\perp} - \frac{z_g}{z_1} \mathbf{k}_{1\perp})^2} \left\{ 8z_1^3 z_2^3 (1 - z_2)^2 Q^2 \left(1 + \frac{z_g}{z_1} + \frac{z_g^2}{2z_1^2} \right) K_0(\bar{Q} r_{xy}) K_0(\bar{Q} r_{x'y'}) \delta_z^{(3)} \right. \\ \left. - \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \Theta(z_1 - z_g) \delta_z^{(2)} \right\} + (1 \leftrightarrow 2)$$

$$d\sigma_{R,\text{no-sud},\text{NLO}_3}^{\gamma_L^* + A \rightarrow q\bar{q}g + X} = \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(3)}}{(2\pi)^8} \int d^2 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} (-4\alpha_s) \Xi_{\text{NLO},3}(\mathbf{x}_\perp, \mathbf{y}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp) \\ \times \frac{e^{-i \frac{z_g}{z_1} \mathbf{k}_{1\perp} \cdot \mathbf{r}_{xy'}}}{l_\perp^2} \left\{ 8z_1^2 z_2^2 (1 - z_2) (1 - z_1) Q^2 K_0(\bar{Q} r_{xy}) K_0(\bar{Q} r_{x'y'}) \left[1 + \frac{z_g}{2z_1} + \frac{z_g}{2z_2} \right] \right. \\ \times e^{-i \mathbf{l}_\perp \cdot \mathbf{r}_{xy'}} \frac{\mathbf{l}_\perp \cdot (\mathbf{l}_\perp + \mathbf{K}_\perp)}{(\mathbf{l}_\perp + \mathbf{K}_\perp)^2} \delta_z^{(3)} - \mathcal{R}_{\text{LO}}^\lambda(\mathbf{r}_{xy}, \mathbf{r}_{x'y'}) \Theta \left(\frac{c_0^2}{r_{xy}^2} \geq l_\perp^2 \geq \mathbf{K}_\perp^2 \right) \Theta(z_1 - z_g) \delta_z^{(2)} \Big\} \\ + (1 \leftrightarrow 2)$$

$$d\sigma_{R,\text{no-sud},\text{other}}^{\gamma_L^* + A \rightarrow q\bar{q}g + X} = \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(3)}}{(2\pi)^8} \int d^8 \mathbf{X}_\perp e^{-i\mathbf{k}_{1\perp} \cdot \mathbf{r}_{xx'} - i\mathbf{k}_{2\perp} \cdot \mathbf{r}_{yy'}} 8z_1^3 z_2^3 Q^2 \int \frac{d^2 \mathbf{z}_\perp}{\pi} \frac{d^2 \mathbf{z}'_\perp}{\pi} e^{-i\mathbf{k}_{g\perp} \cdot \mathbf{r}_{zz'}} \\ \times \alpha_s \left\{ - \frac{\mathbf{r}_{zx} \cdot \mathbf{r}_{z'x'}}{\mathbf{r}_{zx}^2 \mathbf{r}_{z'x'}^2} K_0(Q X_R) K_0(\bar{Q} r_{xy}) \left(1 + \frac{z_g}{z_1} + \frac{z_g^2}{2z_1^2} \right) \Xi_{\text{NLO},1}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp; \mathbf{w}'_\perp, \mathbf{y}'_\perp) \right. \\ + \frac{\mathbf{r}_{zy} \cdot \mathbf{r}_{z'y'}}{\mathbf{r}_{zy}^2 \mathbf{r}_{z'y'}^2} K_0(Q X_R) K_0(\bar{Q} r_{xy}) \left(1 + \frac{z_g}{2z_1} + \frac{z_g}{2z_2} \right) \Xi_{\text{NLO},1}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp; \mathbf{w}'_\perp, \mathbf{y}'_\perp) \\ + \frac{1}{2} \frac{\mathbf{r}_{zx} \cdot \mathbf{r}_{z'x'}}{\mathbf{r}_{zx}^2 \mathbf{r}_{z'x'}^2} K_0(Q X_R) K_0(Q X'_R) \left(1 + \frac{z_g}{z_1} + \frac{z_g^2}{2z_1^2} \right) \Xi_{\text{NLO},4}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \\ - \frac{1}{2} \frac{\mathbf{r}_{zy} \cdot \mathbf{r}_{z'y'}}{\mathbf{r}_{zy}^2 \mathbf{r}_{z'y'}^2} K_0(Q X_R) K_0(Q X'_R) \left(1 + \frac{z_g}{2z_1} + \frac{z_g}{2z_2} \right) \Xi_{\text{NLO},4}(\mathbf{x}_\perp, \mathbf{y}_\perp, \mathbf{z}_\perp; \mathbf{x}'_\perp, \mathbf{y}'_\perp, \mathbf{z}'_\perp) \\ \left. + (1 \leftrightarrow 2) + c.c. \right\} - \frac{\alpha_{\text{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^8} \alpha_s \Theta(z_f - z_g) \times \text{"slow"}$$

CGC and TMD correspondence

Back-to-back limit of CGC calculation at NLO



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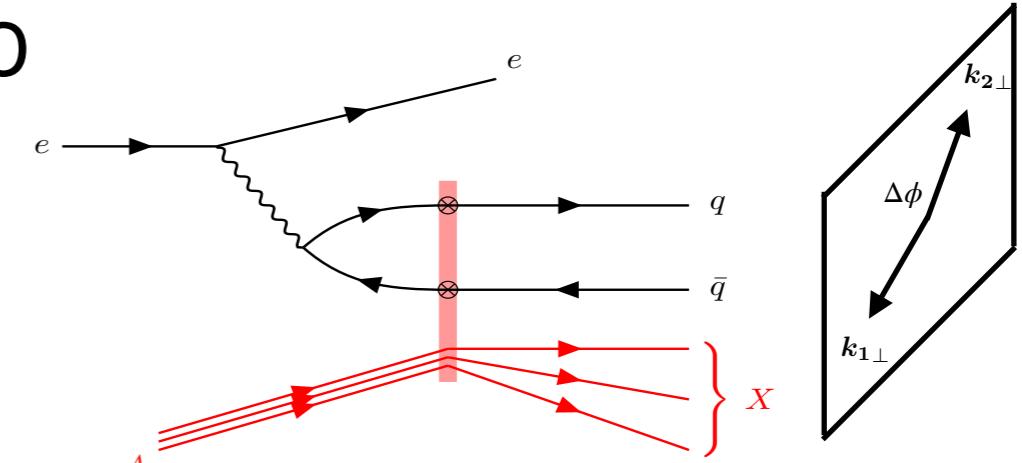
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**Back-to-back inclusive dijets in DIS at small x :
Sudakov suppression and gluon saturation at NLO**

Paul Caucal,^a Farid Salazar,^{b,c,d,e} Björn Schenke^a and Raju Venugopalan^a



$$d\sigma^{\gamma^*_\lambda + A \rightarrow q\bar{q} + X} = \mathcal{H}(Q, \mathbf{P}_\perp) \int \frac{d^2 \mathbf{b}_\perp}{(2\pi)^2} e^{-i\mathbf{k}_\perp \cdot \mathbf{b}_\perp} \alpha_s \tilde{G}_Y^0(\mathbf{b}_\perp) e^{-S_{\text{Sud}}(\mathbf{b}_\perp, \mathbf{P}_\perp)} + \mathcal{O}(\alpha_s^2)$$

Joint resummation of **small-x** and **soft gluons at finite N_c** and to **single log accuracy**

Small-x evolution for WW follows well-known BK-JIMWLK equations **amended** with a kinematic constrain to separate **small-x** and **soft gluons**

Pure $\mathcal{O}(\alpha_s^2)$

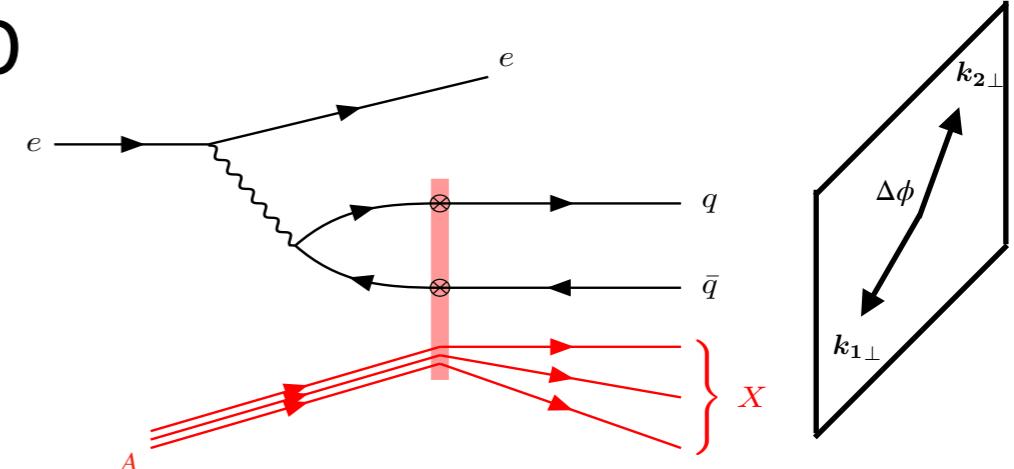
Explicit results, yet *apparently* involve complicated convolution including operators beyond WW, but **needed for precision!**

CGC and TMD correspondence

Back-to-back limit of CGC calculation at NLO

Work in progress (2203.XXXX)

In collaboration with Paul Caucal, Björn Schenke,
Tomasz Stebel, and Raju Venugopalan



Pure $\mathcal{O}(\alpha_s^2)$ contributions can be absorbed into NLO impact factor

$$d\sigma^{\gamma_\lambda^* + A \rightarrow \text{dijet} + X} \propto \mathcal{H}_{\text{NLO}}^\lambda(Q, \mathbf{P}_\perp; \mu_F; R) \int \frac{d^2 \mathbf{b}_\perp}{(2\pi)^2} e^{-i \mathbf{q}_\perp \cdot \mathbf{b}_\perp} \alpha_s \tilde{G}_Y^0(\mathbf{b}_\perp) e^{-S_{\text{Sud}}(\mathbf{b}_\perp, \mu_F)} + \mathcal{O}(\alpha_s^3)$$

fully analytic result

- The first proof of TMD factorization at NLO in small-x kinematics (modulo the non-linear evolution of the WW)?
- Fully analytic NLO impact factor (contains $\ln(Q/P_\perp)$ and dilogarithms...) in back-to-back kinematics, suitable for numerical implementation

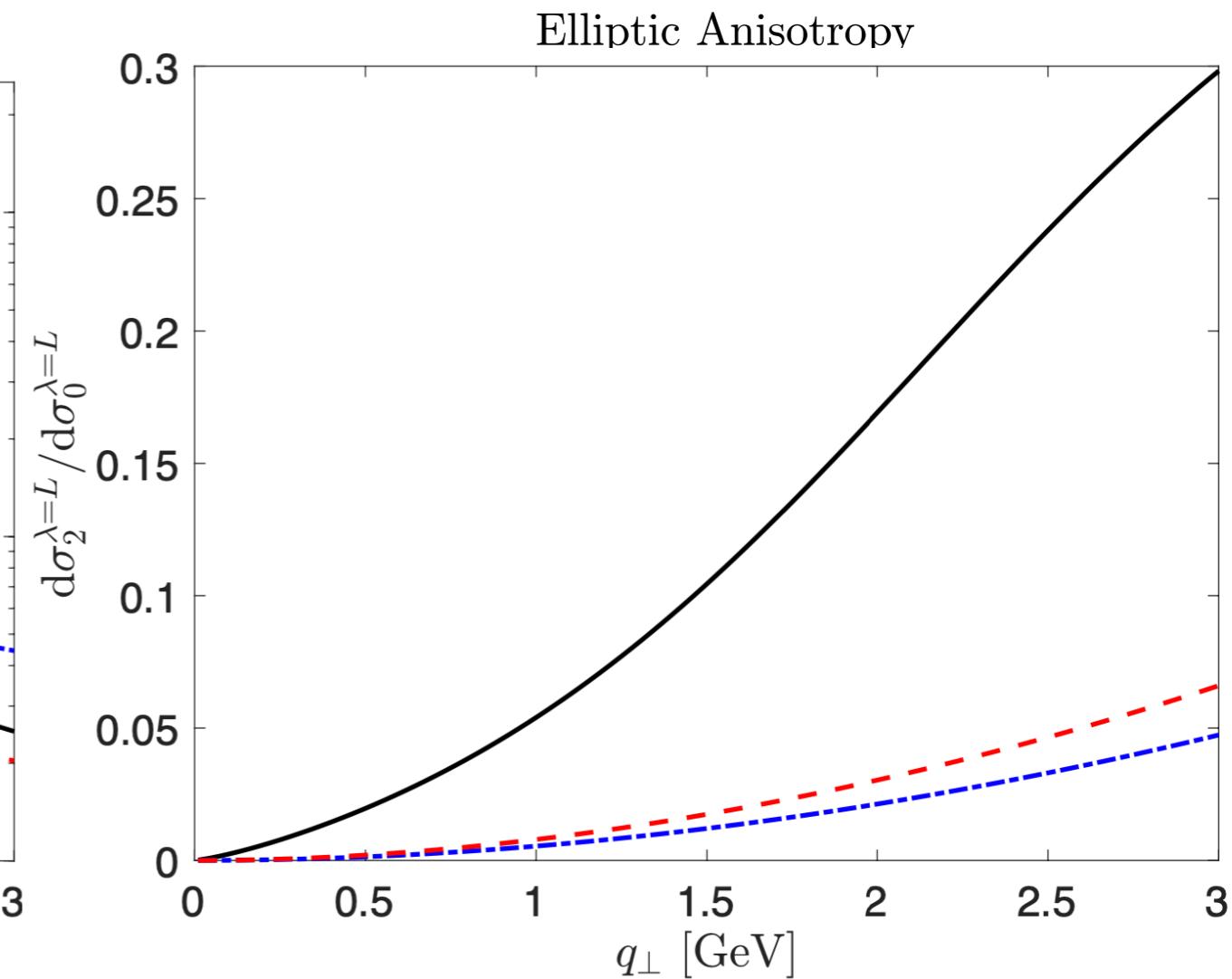
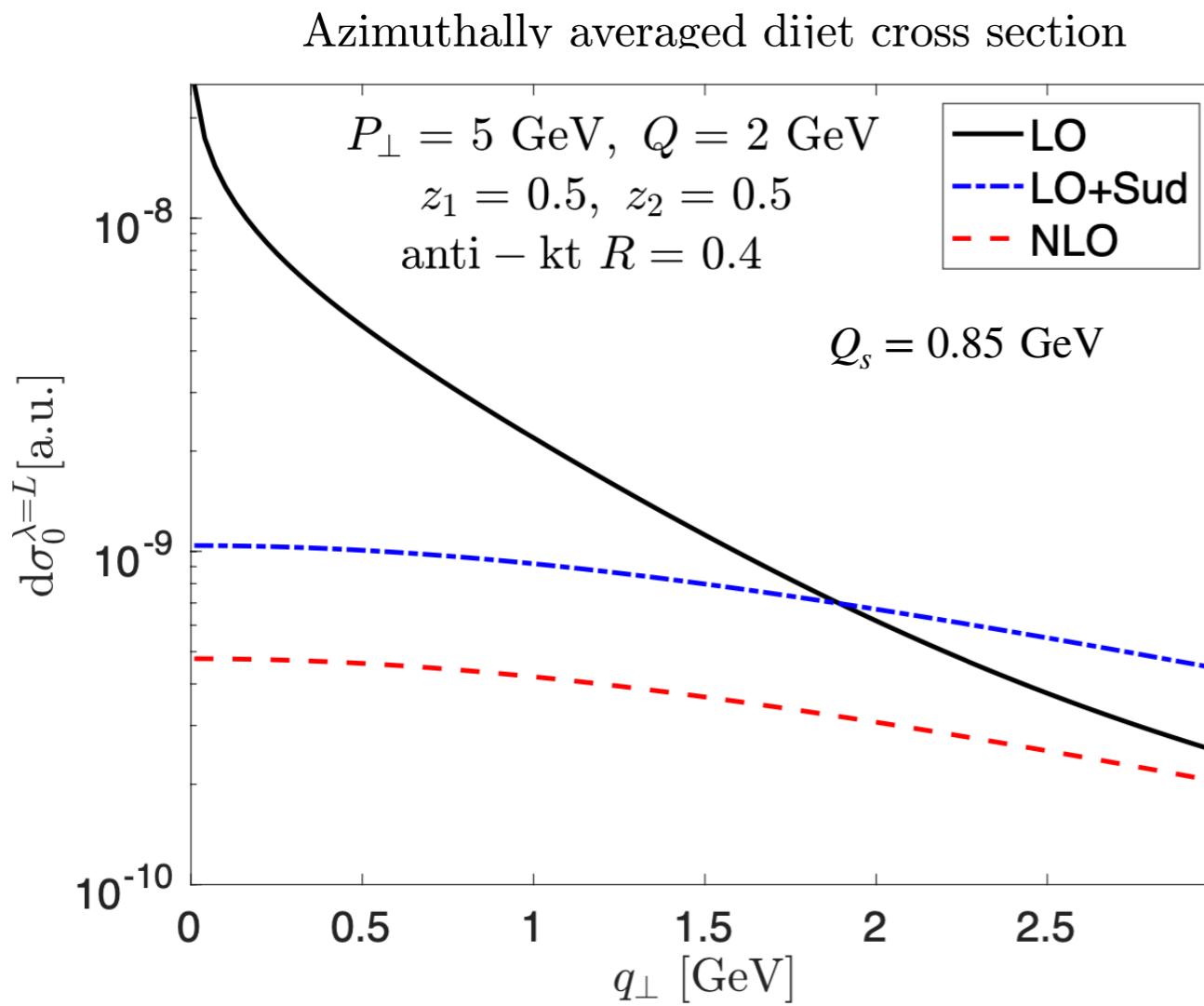
Back-to-back inclusive dijets in DIS at NLO

Numerical results teaser (only $\gamma_L^* + A \rightarrow \text{dijet} + X$)

Work in progress (2203.XXXX)

In collaboration with Paul Caucal, Björn Schenke,
Tomasz Stelbel, and Raju Venugopalan

- NLO* Include:
Sudakov double and single logs at finite N_c + NLO impact factor
- NLO* does not include:
Proper small- x evolution.



More coming soon!

Summary

Motivation:

2-particle azimuthal correlations



powerful observables to search for saturation

Results:

full NLO calculation
dijets in DIS

back-to-back limit



small-x and soft gluon resummation

NLO impact (hard) factor

preliminary numerical results

Outlook: Include kinematically constrained small-x evolution in numerical result

Other observables: Dihadrons, UPCs

Could SCET-like techniques help us promote results beyond NLO/
more complex observables?

*We choose to do these things not because
they are easy, but because ~~they are hard~~
we thought they would be easy*

On a more serious note:

*Extraordinary discoveries
require extraordinary evidence
[a lot of hard work and
collective effort]*