Polarized jet anisotropy

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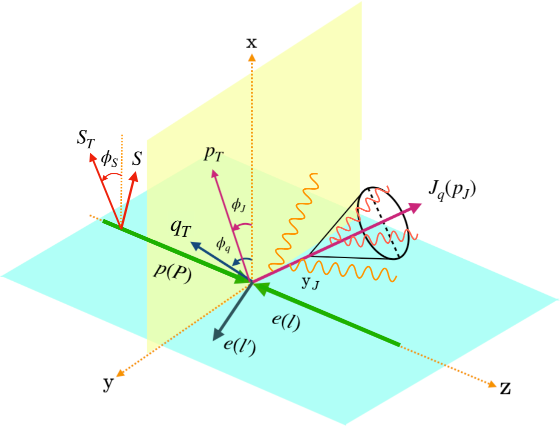
Back-to-back electron-jet production in $ep^\uparrow$ collision

- p_J, p_T and y_J: jet momentum, transverse momentum and rapidity,
- S and S_T: polarization and transverse polarization of the incoming proton,
- q_T: transver momentum imbalance, $\boldsymbol{q}_T = \boldsymbol{l}'_T + \boldsymbol{p}_T$,
- ϕ_{q_T}, ϕ_J and ϕ_{S_T}: azimuthal angle of q_T, jet and S_T.

[Liu, Ringer, Vogelsang and Yuan, 2018; Arratia, Kang, Prokudin and Ringer, 2020; Kang, Lee, Shao and Zhao, 2021]



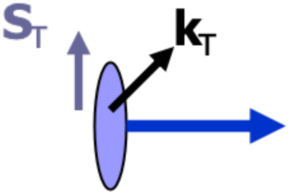
plot by Fanyi Zhao

TMD factorization

In small q_T limit, TMD factorization [Kang, Lee, Shao and Zhao, 2021] gives:

$$\frac{d\sigma^{e+p^\uparrow\to e+\text{jet}+X}}{d^2\boldsymbol{p}_T\, dy_J\, d\phi_J\, d^2\boldsymbol{q}_T} = \hat{\sigma}_0 \sum_q e_q^2 H(Q,\mu) \int \frac{d^2\boldsymbol{b}}{(2\pi)^2} e^{-\imath \boldsymbol{b}\cdot\boldsymbol{q}_T} x\widetilde{f}_{1T,q/p}^{\perp(1)}(x,\boldsymbol{b},\mu,\zeta)$$
$$\times\, \imath m_p \epsilon_{\alpha\beta} S_T^\alpha b^\beta \mathcal{J}_q(p_T R,\mu) S_{\text{global}}(\boldsymbol{b},\mu) S_{cs}(\boldsymbol{b},R,\mu),$$

Sivers effect: transverse momentum distribution of unpolarized quark in a transversely polarized proton.



plot from Feng Yuan

Angular dependence in soft function

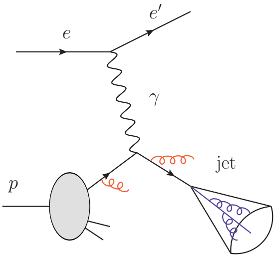
The angular dependence in S_{global} and S_{cs} are both **even** [Buffing, Kang, Lee and Liu 2018]:

$$S_{\text{global}} = 1 + \frac{\alpha_s}{2\pi} C_F \left(2 \ln(-2\imath \cos(\phi_{bJ})) \ln\left(\frac{\mu^2}{\mu_b^2}\right) + \cdots \right),$$

$$S_{cs} = 1 - \frac{\alpha_s}{2\pi} C_F \left(2 \ln(-2\imath \cos(\phi_{bJ})) \ln\left(\frac{\mu^2}{\mu_b^2 R^2}\right) + 2 \ln^2(-2\imath \cos(\phi_{bJ})) + \cdots \right),$$

where $\phi_{bJ} = \phi_b - \phi_J$.

The presence of jet breaks the azimuthal symmetry [plot from Arratia, Kang, Prokudin and Ringer, 2020].



Angular dependence from spin

In $\imath m_p \epsilon_{\alpha\beta} S_T^\alpha b^\beta$, we can project the vector $\boldsymbol{b}$ onto $\boldsymbol{q}_T$:

$$\imath m_p \epsilon_{\alpha\beta} S_T^\alpha b^\beta = \imath m_p \epsilon_{\alpha\beta} S_T^\alpha b \Big(\hat{q}_T^\beta \cos(\phi_{q_T b}) + \hat{q}_{T\perp}^\beta \sin(\phi_{q_T b}) \Big),$$

the cosine term will give us the Sivers structure function:

$$F_{UT} \sim m_p S_T \sin(\phi_{S_T} - \phi_{q_T}) \int \frac{\mathrm{d}^2 \boldsymbol{b}}{(2\pi)^2} \imath b \cos(\phi_{q_T b}) S_{\text{global}} S_{sc} \times \cdots$$

Jacobi-Anger expansion

The angular dependence is non-trivial, espicially with the extra factor of $\imath b\cos(\phi_{q_T b})$:

$$\imath b\cos(\phi_{bq_T})e^{-\imath bq_T\cos\left(\phi_{bq_T}\right)} = -\frac{\mathrm{d}e^{-\imath bq_T\cos\left(\phi_{bq_T}\right)}}{\mathrm{d}q_T}$$
$$= b\left(J_1 - \sum_{n=1}^{\infty}(-\imath)^n(J_{n-1}-J_{n+1})\cos(n\phi_{bq_T})\right),$$

where we used the derivative of Bessel functions: $\frac{\mathrm{d}J_n(z)}{\mathrm{d}z} = \frac{1}{2}(J_{n-1}(z) - J_{n+1}(z))$.

Change of variable

We would like to relate ϕ_{bq_T} to $\phi_{q_T J}$, which is measured in the experiment. Thus we write:

$$\phi_{q_T b} = \phi_{q_T J} - \phi_{bJ},$$

where $\phi_{q_T J} = \phi_{q_T} - \phi_J$. Hence

$$\begin{aligned} \cos(n\phi_{q_T b}) &= \cos(n\phi_{q_T J} - n\phi_{bJ}) \\ &= \cos(n\phi_{q_T J})\cos(n\phi_{bJ}) + \sin(n\phi_{q_T J})\sin(n\phi_{bJ}). \end{aligned}$$

and the sine terms vanish as they are odd.

dσ with angular expansion

Putting pieces together, we get:

$$\begin{aligned}
\mathrm{d}\sigma &= m_p S_T \sin(\phi_{S_T} - \phi_{q_T})\hat{\sigma}_0 \sum_q e_q^2 H(Q,\mu) \\
&\times \int \frac{b\,\mathrm{d}b\,\mathrm{d}\phi_{bJ}}{(2\pi)^2} \left(bJ_1 - b\sum_{n=1}^{\infty} (-\imath)^n (J_{n-1} - J_{n+1}) \cos(n\phi_{bJ}) \cos(n\phi_{q_T J}) \right) \\
&\times x\widetilde{f}_{1T,q/p}^{\perp(1)}(x, \mathbf{b}, \mu, \zeta) \mathcal{J}_q(p_T R, \mu) S_{\mathrm{global}}(\mathbf{b}, \mu) S_{cs}(\mathbf{b}, R, \mu).
\end{aligned}$$

Angular average of the soft function

We can now define the angular average of soft function as in the unpolarized case:

$$\overline{S}_q = \frac{1}{2\pi}\int S_{\text{global}} S_{cs}\,\mathrm{d}\phi_{bJ}\,,$$

$$S_q^{\langle\cos(n\phi_{bJ})\rangle} = \frac{1}{2\pi}\int S_{\text{global}} S_{cs}\cos(n\phi_{bJ})\,\mathrm{d}\phi_{bJ}\,,$$

and $\mathrm{d}\sigma$ can be written as (**non-trivial angular dependence**):

$$\mathrm{d}\sigma = m_p S_T \sin(\phi_{S_T} - \phi_{q_T})\hat{\sigma}_0 \sum_q e_q^2 H(Q,\mu)\int \frac{b\,\mathrm{d}b}{2\pi} x\widetilde{f}_{1T,q/p}^{\perp(1)}(x,b,\mu,\zeta)\mathcal{J}_q(p_T R,\mu)$$
$$\times\left(bJ_1\overline{S}_q - b\sum_{n=1}^{\infty}(-\imath)^n(J_{n-1} - J_{n+1})S_q^{\langle\cos(n\phi_{bJ})\rangle}\cos(n\phi_{q_T J})\right).$$

Jet anisotropy

We can define the azimuthal anisotropy:

$$A^{\langle\cos(n\phi_{bJ})\rangle} \sim \frac{\widetilde{f}_{1\,T,q/p}^{\perp(1)} \otimes \mathcal{J}_q \otimes S_q^{\langle\cos(n\phi_{bJ})\rangle}}{\widetilde{f}_{1\,T,q/p}^{\perp(1)} \otimes \mathcal{J}_q \otimes \overline{S}_q}.$$

where the angular average of soft function are:

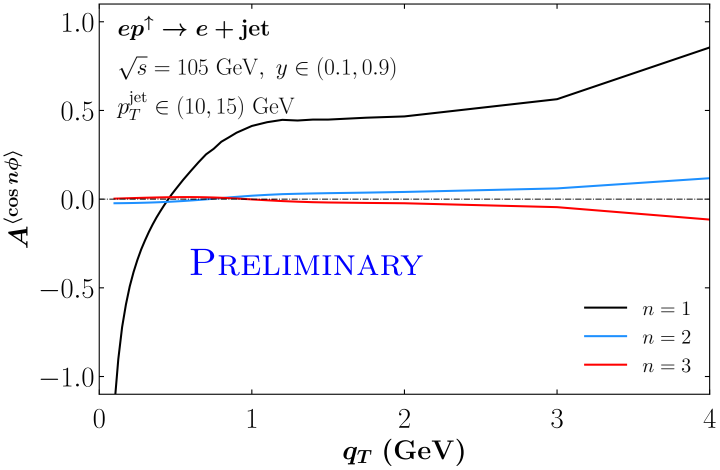
$$\overline{S}_q = 1 + \frac{\alpha_s C_F}{2\pi}\left(2y_J \ln\left(\frac{\mu^2}{\mu_b^2}\right) + \ln\left(R^2\right)\ln\left(\frac{\mu^2}{\mu_b^2}\right) - \frac{\ln^2\left(R^2\right)}{2}\right),$$

$$-\imath S_q^{\langle\cos(\phi_{bJ})\rangle} = \frac{\alpha_s C_F}{2\pi}\left(-2\ln\left(R^2\right) + 4(\ln(4)-1)\right),$$

$$(-\imath)^2 S_q^{\langle\cos(2\phi_{bJ})\rangle} = \frac{\alpha_s C_F}{2\pi}\left(-\ln\left(R^2\right) - 1\right),$$

$$(-\imath)^3 S_q^{\langle\cos(3\phi_{bJ})\rangle} = \frac{\alpha_s C_F}{2\pi}\left(\frac{-2}{3}\ln\left(R^2\right) + \frac{4}{9}(\ln(64)-7)\right),\cdots$$

$A^{\langle\cos(n\phi_{bJ})\rangle}$



Thank you for attention!

In collaboration with Zhongbo Kang and Fanyi Zhao.

Bessel functions