Spin decomposition in Charmonium

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χQCD **collaboration**

Outline

① **Background and introduction**

② **Lattice calculation of quark spin and gluon total angular momentum**

③ **Renormalization**

④ **Summary**

Spin decomposition in QCD

- **Jaffe-Manohar decomposition** R. Jaffe and A. Manohar, NPB 337, 509 (1990)

$$J = \frac{1}{2}\sum_q \Delta_q + \boxed{\sum_q \mathscr{L}_q + \Delta_G + L_G}$$

Gauge dependent

Quark spin

Quark orbital angular momentum (OAM)

Gluon spin

Gluon OAM

- **Ji's decomposition**

X. Ji, Phys. Rev. Lett. 78, 610 (1997)

$$J = \frac{1}{2}\sum_q \Delta_q + \sum_q L_q + J_g$$

Gauge invariant

quark spin

Quark orbital angular momentum (OAM)

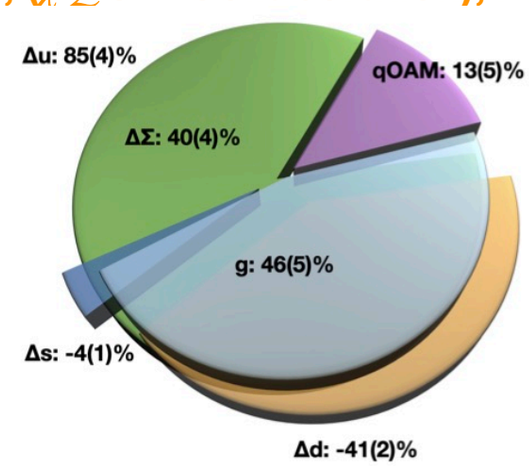
Gluon spin+OAM

Gauge indenpendent gluon spin

X. Chen, et.al, PRL 100, (2008)
X. Ji, J. Zhang and Y. Zhao, PRL, 111 (2013)
Y. Yang, et. al (χQCD Collaboration), PRL. 118 (2017)

Nucleon spin decomposition

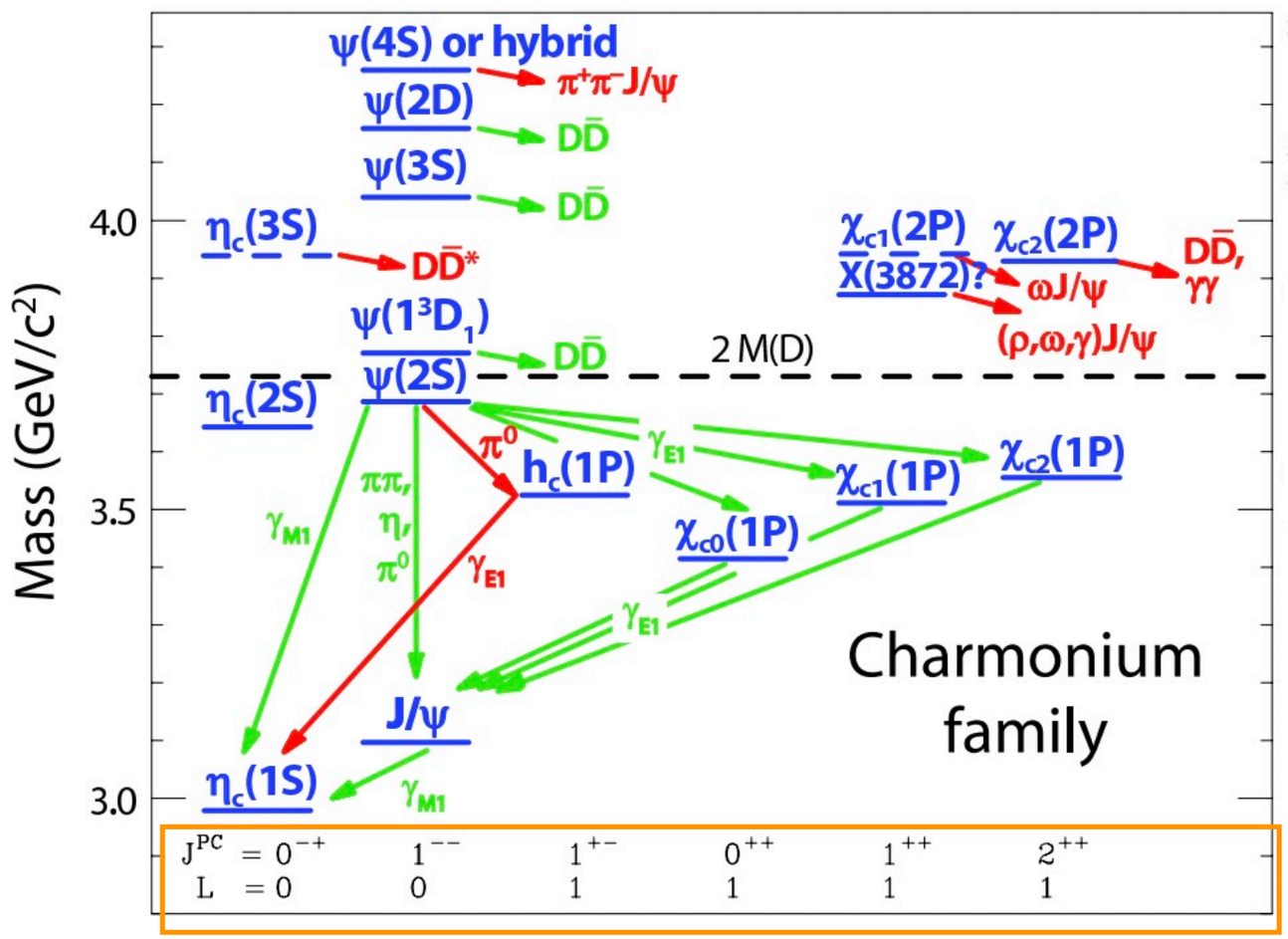
C. Alexandrou, et. al ,PRL. 119 (2017)
G. Wang, et. al, (χQCD Collaboration), *PRD* 106 (2022)



Charmonium

Known charmonium states and candidates

Rev.Mod.Phys. 80 (2008)



Different quantum numbers

Quantum numbers n	L	J^{PC}	$n^{2S+1}L_J$	Name	Mass (MeV)	Width (MeV[a])
1	0	0^{-+}	1^1S_0	$\eta_c(1S)$	2980.4±1.2	25.5±3.4
1	0	1^{--}	1^3S_1	J/ψ	3096.916±0.011	93.4±2.1 keV
1	1	0^{++}	1^3P_0	$\chi_{c0}(1P)$	3414.76±0.35	10.4±0.7
1	1	1^{++}	1^3P_1	$\chi_{c1}(1P)$	3510.66±0.07	0.89±0.05
1	1	2^{++}	1^3P_2	$\chi_{c2}(1P)$	3556.20±0.09	2.06±0.12
1	1	1^{+-}	1^1P_1	$h_c(1P)$	3525.93±0.27	<1

When the mass is below two D meson threshold, decay width is suppressed.

We assume they are stable particles in this calculation

Charmonium mass decomposition

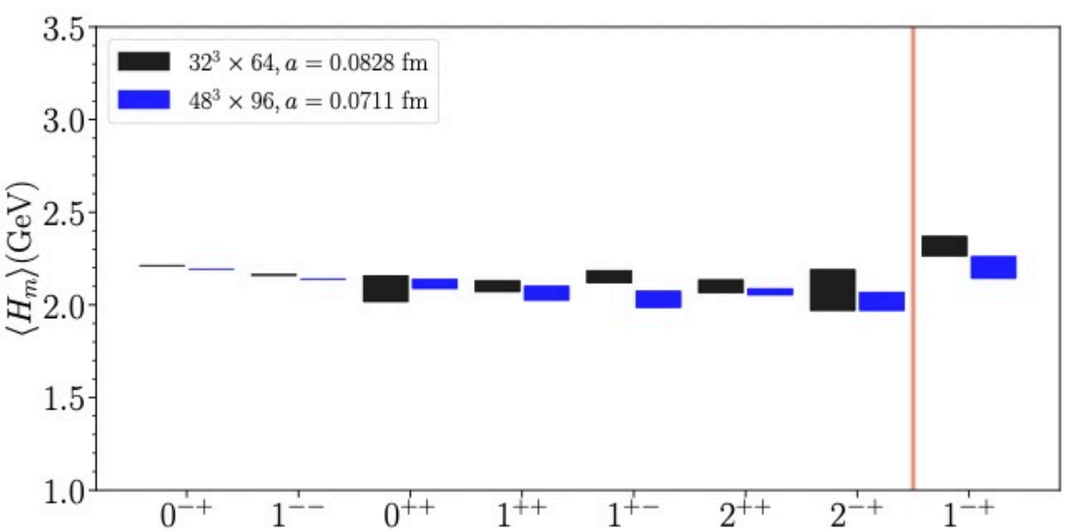
W. Sun, et. al, (χQCD collaboration) PRD 103 (2021)

- **Mass decomposition of charmonium**

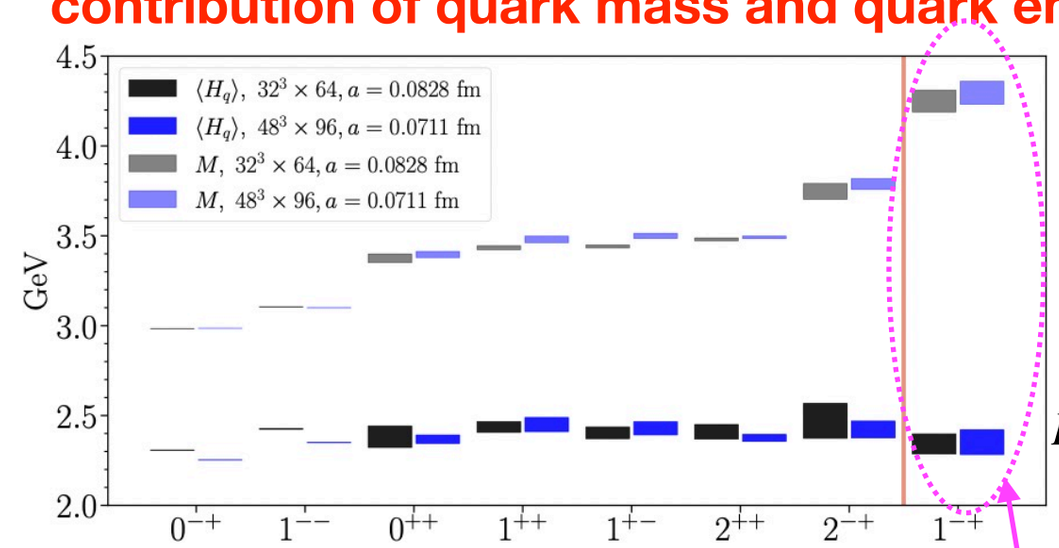
$$M_H = T^{00} = H_E + H_m + H_g + \frac{1}{4}(H_a^q + H_a^g)$$

Quark energy	$H_E = \sum_{u,d,s\ldots} \int d^3x\, \overline{\psi}(\vec{D}\cdot\vec{\gamma})\psi,$
Quark mass	$H_m = \sum_{u,d,s\cdots} \int d^3x\, m\overline{\psi}\psi,$
Gluon energy	$H_g = \int d^3x\, \frac{1}{2}(B^2 - E^2).$
Quark anomaly	$H_m^\gamma = \sum_{u,d,s\cdots} \int d^3x\, \gamma_m m\overline{\psi}\psi.$
Gluon anomaly	$H_g^a = \int d^3x\, \frac{-\beta(g)}{g}(E^2 + B^2),$

Quark mass contribution



The comparison of hadron mass with the total contribution of quark mass and quark energy



Hadron mass

$H_q = H_E + H_m$

The contribution of quark part is similar in different charmonium state.

Larger contribution from gluon part in 1^{-+} channel.

Spin decomposition of the ground state charmonium in quark model

- **The quantum number of charmonium (J^{PC})**

exotic state

J=1	1^{--}	1^{+-}	1^{++}	1^{-+}
J=2	2^{--}	2^{-+}	2^{++}	2^{+-}

$$\bar{Q}Q$$

$$P = (-1)^{L+1}$$

$$C = (-1)^{L+S}$$

- **Spin decomposition in quark model**

1^{--}	S=1	L=0
1^{+-}	S=0	L=1
1^{++}	S=1	L=1

1.Are the predictions of quark model comparable with QCD?

2.How about the contribution of gluon?

3.What is the spin structure of exotic states?

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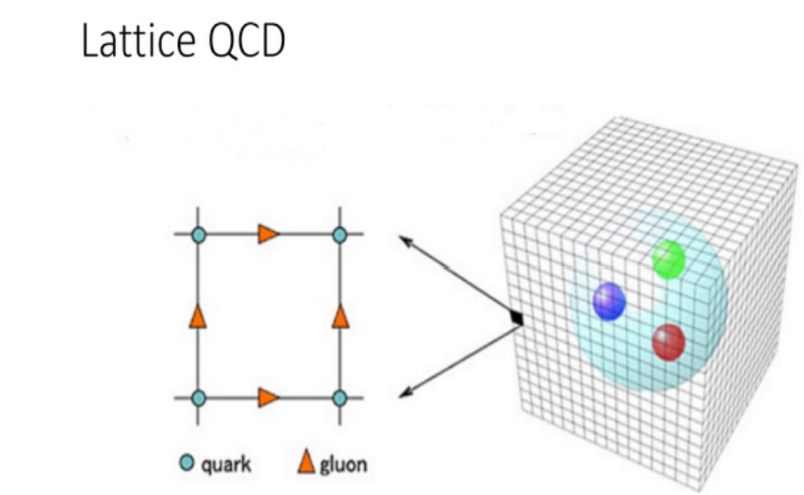
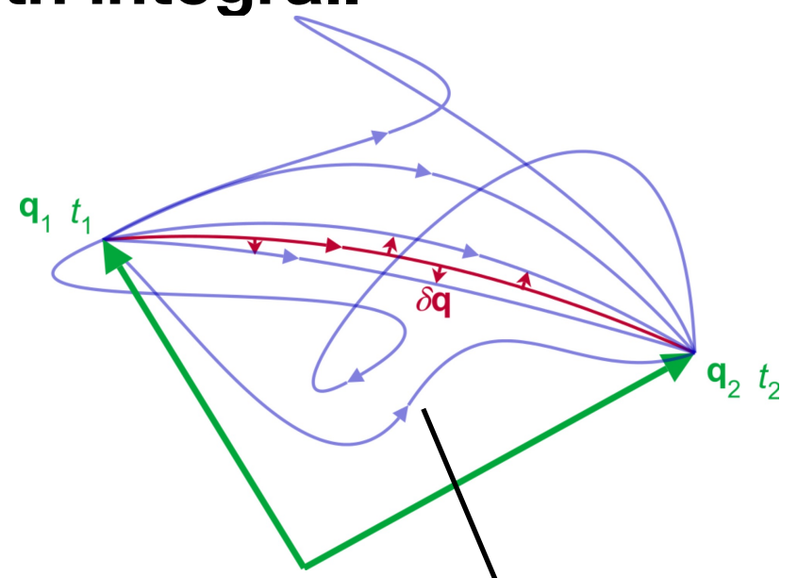
Lattice QCD

- **In lattice QCD method, the correlation functions are non-perturbatively calculated using path integral.**





Discretization the QCD action in Euclidean space

Configurations (specific path)

The configurations are distributed according to $\frac{e^{-S[U]}}{\int D[U]e^{-S[U]}}$

$$\langle O \rangle = \frac{\int D[U]e^{-S[U]}O[U]}{\int D[U]e^{-S[U]}} \approx \frac{1}{N}\sum_i O[U_i]$$

Lattice setup

- **We choose overlap fermion (chiral fermion) as valence quark**

The Dirac operator of overlap fermion satisfies Ginsparg-Wilson relation

$$D_{ov}\gamma_5 + \gamma_5 D_{ov} = aD_{ov}\gamma_5 D_{ov}$$

Chiral transformation on finite lattice

$$\psi' \to exp(i\epsilon(\gamma_5 - \frac{a}{2}\gamma_5 D_{ov}))\psi$$

$$\bar{\psi}' \to \bar{\psi}\, exp(i\epsilon(\gamma_5 - \frac{a}{2}D_{ov}\gamma_5))$$

$\longrightarrow$ **Chiral symmetry**

$$\bar{\psi}' D\psi' = \bar{\psi} D\psi$$

- **The information of gauge ensemble**

ensemble	$L^3 \times T$	a (fm)	m_π (MeV)	$m_c a$	$N_{\rm cfg}$
32I	$32^3 \times 64$	0.0828(3)	300	0.493	305

Charmonium operator

- **Meson interpolation operator for different J^{PC} charmonium**

meson	J^{PC}	operator	mass(GeV) [PDG]
$J/\psi(^3S_1)$	1^{--}	γ_i	3.097
$\chi_{c1}(^3P_1)$	1^{++}	$\gamma_5\gamma_i$	3.511
$h_c(^1P_1)$	1^{+-}	$\gamma_4\gamma_5\gamma_i$	3.525
$\chi_{c2}(^3P_2)$	2^{++}	$\lvert\epsilon_{ijk}\rvert\gamma_j D_k$	3.556
—	1^{-+}	$\epsilon_{ijk}\gamma_j B_k$	—

Quark spin in charmonium (spin one)

Quark spin operator $$O_{\Sigma_q} = \sum_x \bar{q}\gamma_z\gamma_5 q(x)$$

Spin basis

$$V_+ = \frac{1}{\sqrt{2}}(V_x + iV_y)$$

$$V_0 = V_z$$

$$V_- = \frac{1}{\sqrt{2}}(V_x - iV_y)$$

$$\langle V_+ | O_{\Sigma_q} | V_+ \rangle = -\langle V_- | O_{\Sigma_q} | V_- \rangle \neq 0$$

$$\langle V_0 | O_{\Sigma_q} | V_0 \rangle = 0$$

$$\langle V_x | \bar{q}\gamma_z\gamma_5 q | V_y \rangle = -\langle V_y | \bar{q}\gamma_z\gamma_5 q | V_x \rangle \neq 0$$

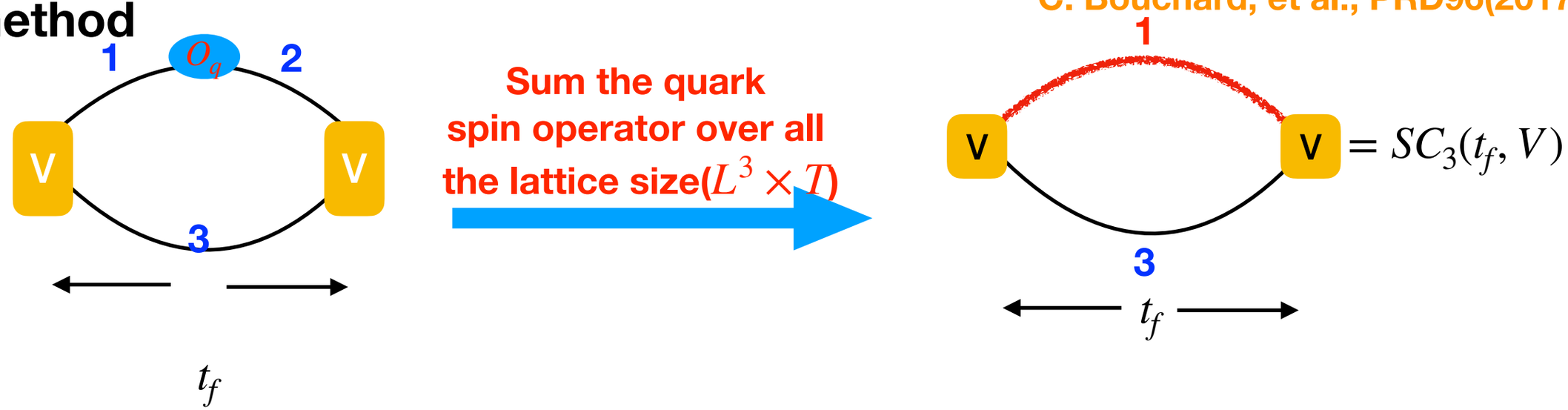
$$\langle V_z | \bar{q}\gamma_z\gamma_5 q | V_z \rangle = 0$$

We calculate these matrix elements from Lattice QCD

Calculation of the quark spin in the hardon

- The quark spin contribution can be obtained by the ratio of connected 3pt correlation function to 2pt correlation function. Since we need calculate the quark spin in different hadron states (1^{--},1^{+-}...), a better choice is using summed current sequential method

C. Bouchard, et al., PRD96(2017)



- The difference between the ratio at adjacent time slice

$$R(t_f, O) = \frac{\langle SC_3(t_f, O)\rangle}{\langle C_2(t_f)\rangle} - \frac{\langle SC_3(t_f - 1, O)\rangle}{\langle C_2(t_f - 1)\rangle} = \langle H|O|H\rangle + \mathcal{O}(e^{-\delta m t_f}),$$

Matrix element at the ground state

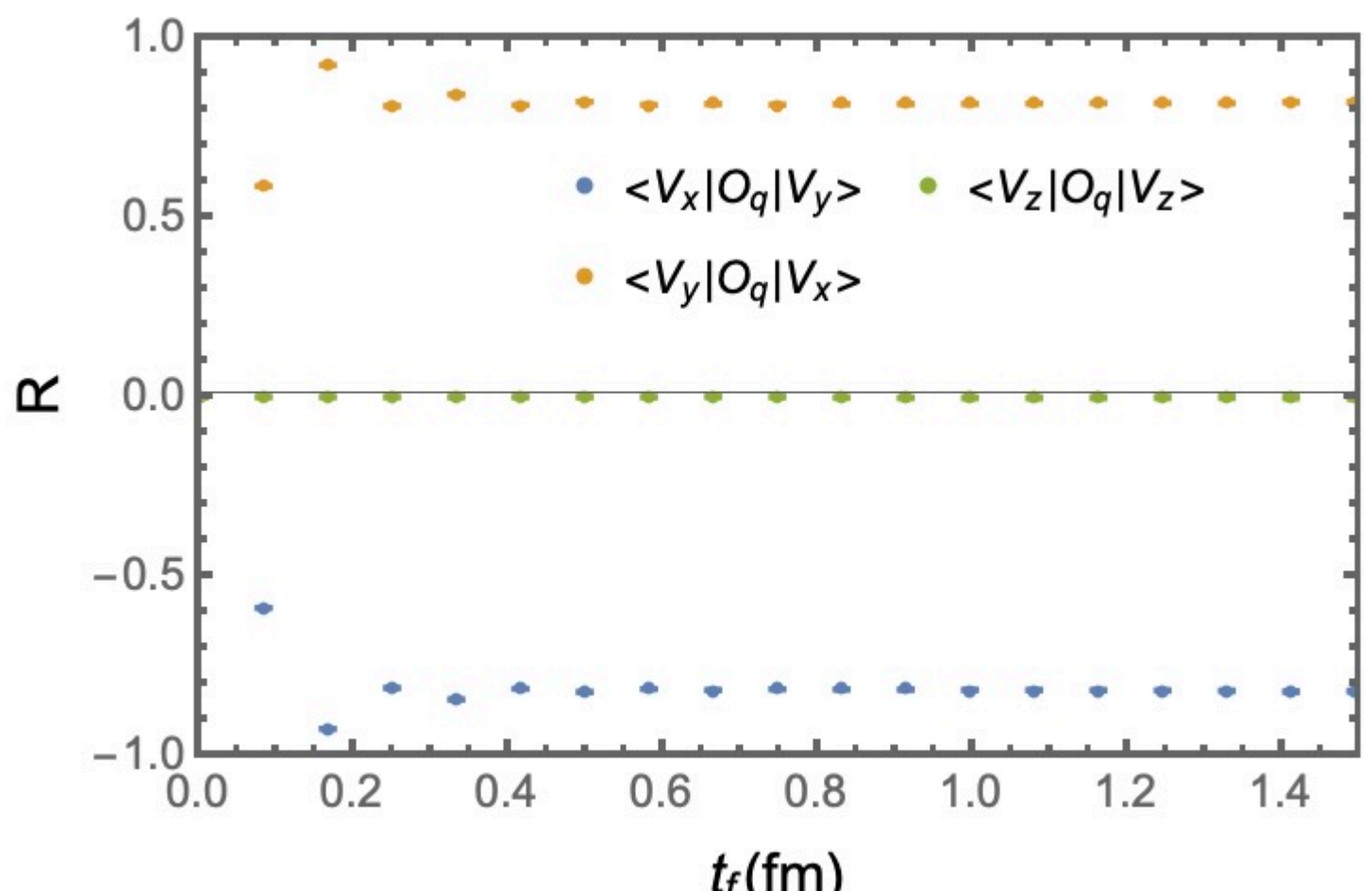
The contamination of excited state

Lattice results

- **The ratio of three point correlation function to two point correlation function**

$$R(t_f, O) = \frac{\langle SC_3(t_f, O)\rangle}{\langle C_2(t_f)\rangle} - \frac{\langle SC_3(t_f - 1, O)\rangle}{\langle C_2(t_f - 1)\rangle} = \langle H|O|H\rangle + \mathcal{O}(e^{-\delta m t_f}),$$

Matrix element at the ground state **The contamination of excited state**



Quark spin in charmonium (spin two)

- **The irreducible representation of of spin-2 charmonium can be converted to the spin basis** $S_M^{J=2} = \langle 1,m_1; 1,m_2 | J, M\rangle \bar{\psi}\gamma_{m_1} D_{m_2}\psi$

Irreducible representation (2^{++})

J. Dudek, et. al, PRD 77 (2008) 034501

T_2^x	$\lvert\epsilon_{1jk}\rvert\gamma^j D^k/\sqrt{2}$
T_2^y	$\lvert\epsilon_{2jk}\rvert\gamma^j D^k/\sqrt{2}$
T_2^z	$\lvert\epsilon_{3jk}\rvert\gamma^j D^k/\sqrt{2}$
E^x	$Q_{1jk}\gamma^j D^k$
E^y	$Q_{2jk}\gamma^j D^k$

Spin basis

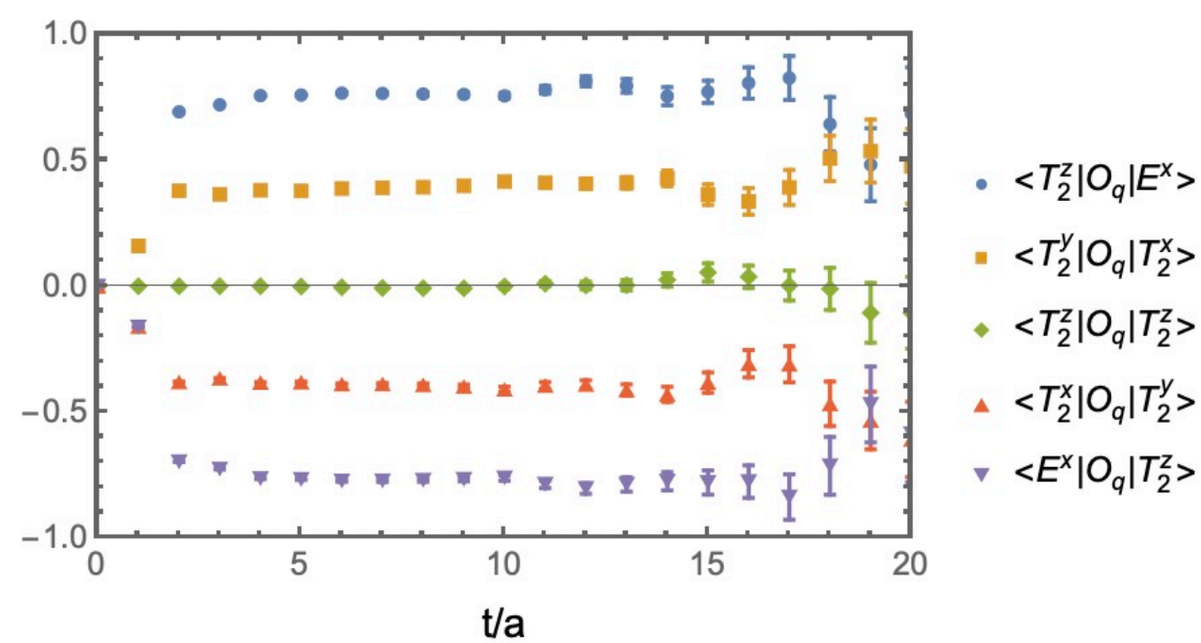
$V_{J_z=2}$	$(iT_2^z + E^x)/\sqrt{2}$
$V_{J_z=1}$	$(iT_2^y + T_2^x)/\sqrt{2}$
$V_{J_z=0}$	E^y
$V_{J_z=-1}$	$(-iT_2^y + T_2^x)/\sqrt{2}$
$V_{J_z=-2}$	$(-iT_2^z + E^x)/\sqrt{2}$

$\mathbb{Q}_{111} = \frac{1}{\sqrt{2}};\ \mathbb{Q}_{122} = -\frac{1}{\sqrt{2}};\ \mathbb{Q}_{211} = -\frac{1}{\sqrt{6}};$

$\mathbb{Q}_{222} = -\frac{1}{\sqrt{6}};\ \mathbb{Q}_{233} = \frac{2}{\sqrt{3}}.$

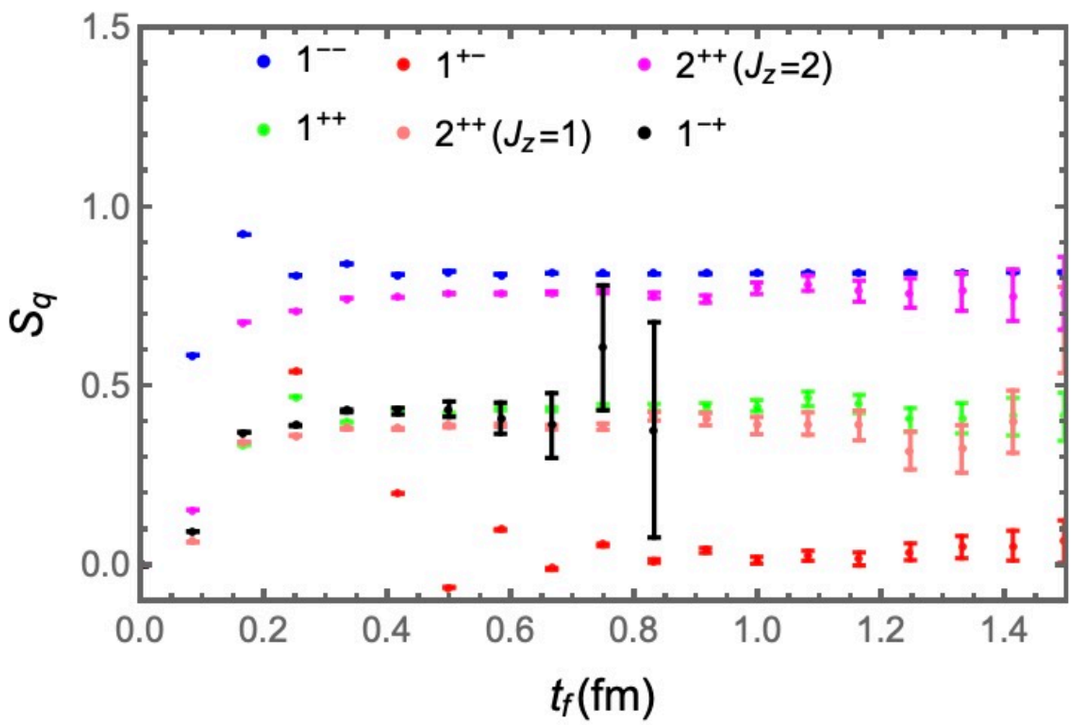
$$\langle V_{J_z=2} | O_{\Sigma_q} | V_{J_z=2}\rangle = 2\langle V_{J_z=1} | O_{\Sigma_q} | V_{J_z=1}\rangle$$

$$\boxed{\langle E^x | O_{\Sigma_q} | T_2^z\rangle = 2\langle T_2^x | O_{\Sigma_q} | T_2^y\rangle}$$



Quark spin in different charmonium

- **The contribution of quark spin to the different charmonium spin**



The plateau at large t_f corresponds to the ground state matrix elements

Comparison with the prediction of quark model

1.The contribution of quark spin is very small in 1^{+-} (p wave), but dominantly contributes to the spin of 1^{--} (s wave) .

2. The quark spin in 1^{++} and $2^{++}(J_z = 1)$ is very close since they belong to same spin triplet (L=1, S=1) state.

3. The quark spin in 1^{--} and $2^{++}(J_z = 2)$ is also similar.

4. The quark spin contributes half spin of 1^{-+} exotic state.

Total angular momentum of gluon

- **Gravitational form factor (GFFs) of vector meson**

$$T_g^{\mu\nu} = F^{a,\mu\eta} F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4} g^{\mu\nu} F^{a,\kappa\eta} F^{a,}{}_{\kappa\eta} \, .$$

M. Polyakov and B. Sun, PRD 100 (2019)

$$\begin{aligned}
\langle p', \sigma' | \hat{T}^a_{\mu\nu}(x) | p, \sigma \rangle \;=\; & \Big[2P_\mu P_\nu \Big(-\epsilon'^* \cdot \epsilon \, A_0^a(t) + \frac{\epsilon'^* \cdot P \, \epsilon \cdot P}{m^2} A_1^a(t) \Big) \\
& + 2 \left[P_\mu (\epsilon'^*_\nu \, \epsilon \cdot P + \epsilon_\nu \, \epsilon'^* \cdot P) + P_\nu (\epsilon'^*_\mu \, \epsilon \cdot P + \epsilon_\mu \, \epsilon'^* \cdot P) \right] J^a(t) \\
& + \frac{1}{2} (\Delta_\mu \Delta_\nu - g_{\mu\nu} \Delta^2) \Big(\epsilon'^* \cdot \epsilon \, D_0^a(t) + \frac{\epsilon'^* \cdot P \, \epsilon \cdot P}{m^2} D_1^a(t) \Big) \\
& + \Big[\frac{1}{2} (\epsilon_\mu \epsilon'^*_\nu + \epsilon'^*_\mu \epsilon_\nu) \Delta^2 - (\epsilon'^*_\mu \Delta_\nu + \epsilon'^*_\nu \Delta_\mu) \, \epsilon \cdot P \\
& + (\epsilon_\mu \Delta_\nu + \epsilon_\nu \Delta_\mu) \, \epsilon'^* \cdot P - 4 g_{\mu\nu} \, \epsilon'^* \cdot P \, \epsilon \cdot P \Big] E^a(t) \\
& + \Big(\epsilon_\mu \epsilon'^*_\nu + \epsilon'^*_\mu \epsilon_\nu - \frac{\epsilon'^* \cdot \epsilon}{2} g_{\mu\nu} \Big) m^2 \, \bar{f}^a(t) \\
& + g_{\mu\nu} \Big(\epsilon'^* \cdot \epsilon \, m^2 \, \bar{c}_0^a(t) + \epsilon'^* \cdot P \, \epsilon \cdot P \, \bar{c}_1^a(t) \Big) \Big] \, e^{i(p'-p)x} \, ,
\end{aligned}$$

$$\Delta = p' - p$$

$$P = \frac{p + p'}{2}$$

$$\bar{f}^q + \bar{f}^g = 0$$

- **The relation between angular momentum and GFFs**

$$J^i = \epsilon^{ijk} \int d^3x T^{0k}(x) x^j$$

Total angular momentum = $J^a(0) + \dfrac{\bar{f}^a(0)}{2}$

Extraction of angular momentum operator

- **We need combine different polarization choices of of initial and final mesons to extract the form factors related to the total angular momentum**

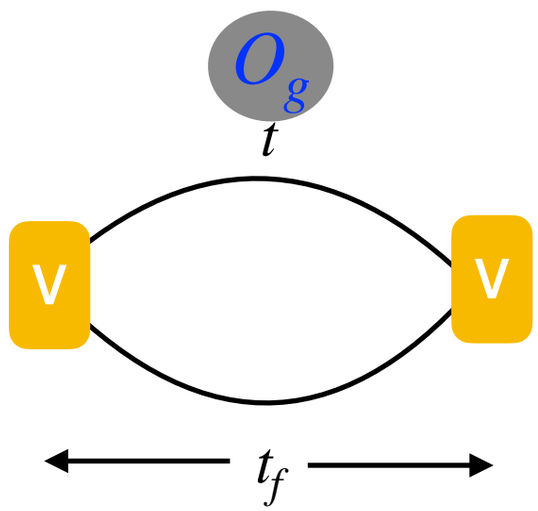
Total angular momentum of gluon $J_G = J^g(0) + \frac{\bar{f}^g(0)}{2}$

	Initial state	Final state	Matrix elements
Frame I	$p = (0,0,0,m)$ $\epsilon = (0,0,1,0)$	$p' = (0,0,q,E)$ $\epsilon' = (0,1,0,0)$	$\langle p', \sigma_y' \| T_{4y} \| p, \sigma_z \rangle = -\frac{(E+m)q}{2} J^g(t) + \frac{(E-m)q}{2} E^g(t)$
Frame II	$p = (0,0,0,m)$ $\epsilon = (0,0,1,0)$	$p' = (0,q,0,E)$ $\epsilon' = (0,\frac{E}{m},0,\frac{q}{m})$	$\langle p', \sigma_y' \| T_{4z} \| p, \sigma_z \rangle = \frac{(E+m)q}{2} J^g(t) + \left[\frac{\Delta^2 q}{2m} + \frac{(E-m)q}{2}\right] E^g(t) + mq\bar{f}^g(t)$
Frame III	$p = (0,0,0,m)$ $\epsilon = (0,1,0,0)$	$p' = (0,0,q,E)$ $\epsilon' = (1,0,0,0)$	$\langle p', \sigma_x' \| T_{xy} \| p, \sigma_y \rangle = \frac{\Delta^2}{2} E^g(t) + m^2 \bar{f}^g(t)$

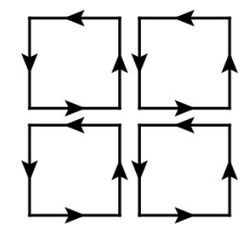
Calculation of the gluon total angular momentum (AM)

- **For the gluon AM, the 3pt correlation function can be described as**

$C_3(t_f, t) =$

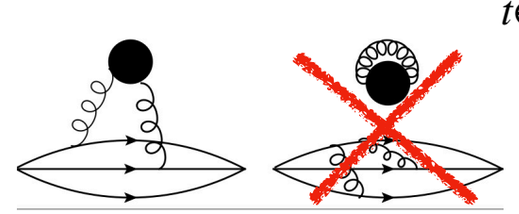


$F^{\mu\nu} \propto$



$$= \left(C_2(t_f) - \langle C_2(t_f)\rangle\right)\left(\sum_{t\in(o,t_f)} O_g(t) - \langle O_g(t)\rangle\right)$$

t: timeslice of O_g



- **We can extract the matrix element of gluon condensate at the ground state through difference between the ratio at adjacent time slice**

$$\tilde{R}(t_f, \tilde{O}) = \frac{\sum_{t_f > t > 0}\langle C_3(t_f, t, \tilde{O})\rangle}{\langle C_2(t_f)\rangle} - \frac{\sum_{t_f - 1 > t > 0}\langle C_3(t_f - 1, \tilde{O})\rangle}{\langle C_2(t_f - 1)\rangle} = \langle H|\tilde{O}|H\rangle + \mathcal{O}(e^{-\delta m t_f}),$$

Matrix element at the ground state

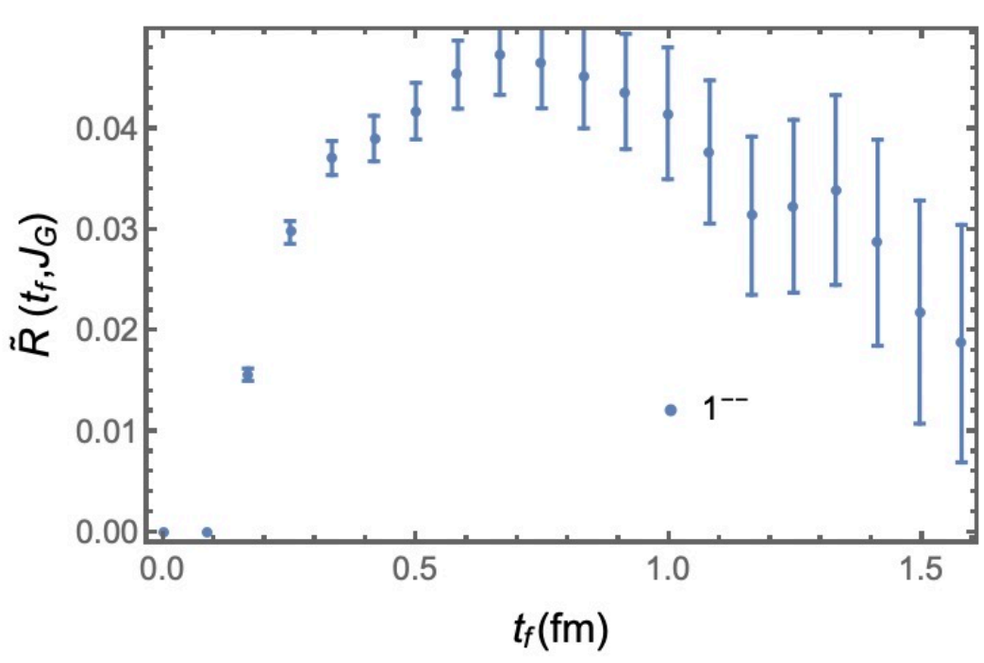
The contamination of excited state

Numerical results of ratio of gluon angular momentum (Bare matrix)

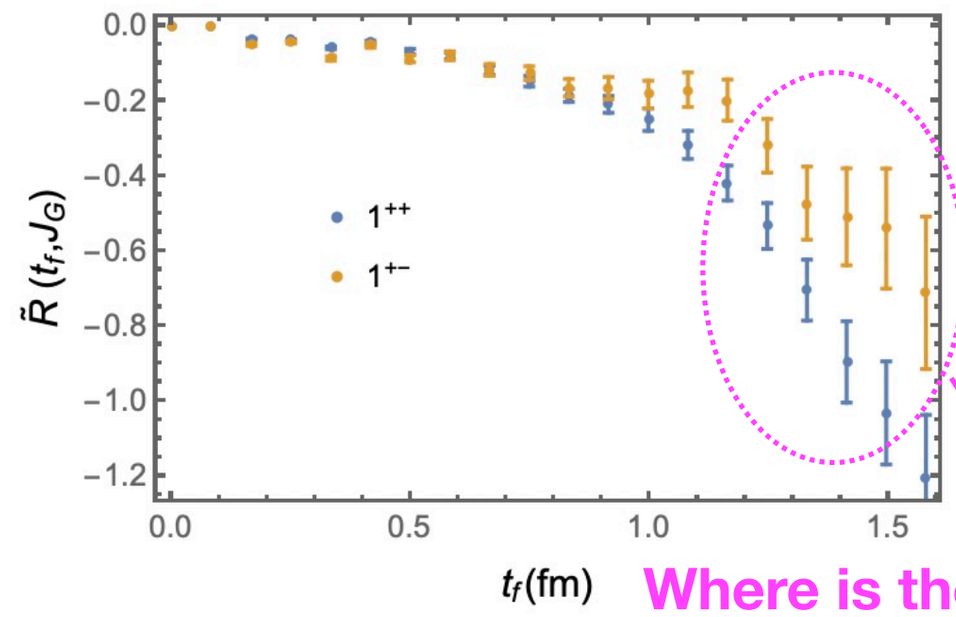
- **The numerical results of gluon angular momentum**

$$\tilde{R}(t_f,\tilde{O}) = \frac{\sum_{t_f>t>0}\langle C_3(t_f,t,\tilde{O})\rangle}{\langle C_2(t_f)\rangle} - \frac{\sum_{t_f-1>t>0}\langle C_3(t_f-1,\tilde{O})\rangle}{\langle C_2(t_f-1)\rangle} = \langle H|\tilde{O}|H\rangle + \mathcal{O}(e^{-\delta m t_f}),$$

Results of 1^{--} channel



Results of 1^{++} and 1^{+-} channel



Where is the plateau?

There is plateau for the gluon operator of 1^{--} channel, but not for the 1^{++} and 1^{--} channels.

Operator mixing

Boost for 1^{+-} operator: $\bar{\psi}\gamma_5\gamma_4\gamma^x\psi \to \alpha\bar{\psi}\gamma_5\gamma_4\gamma^x\psi + \beta\bar{\psi}\gamma_4\gamma^y\psi$ **mix with 1^{--}**

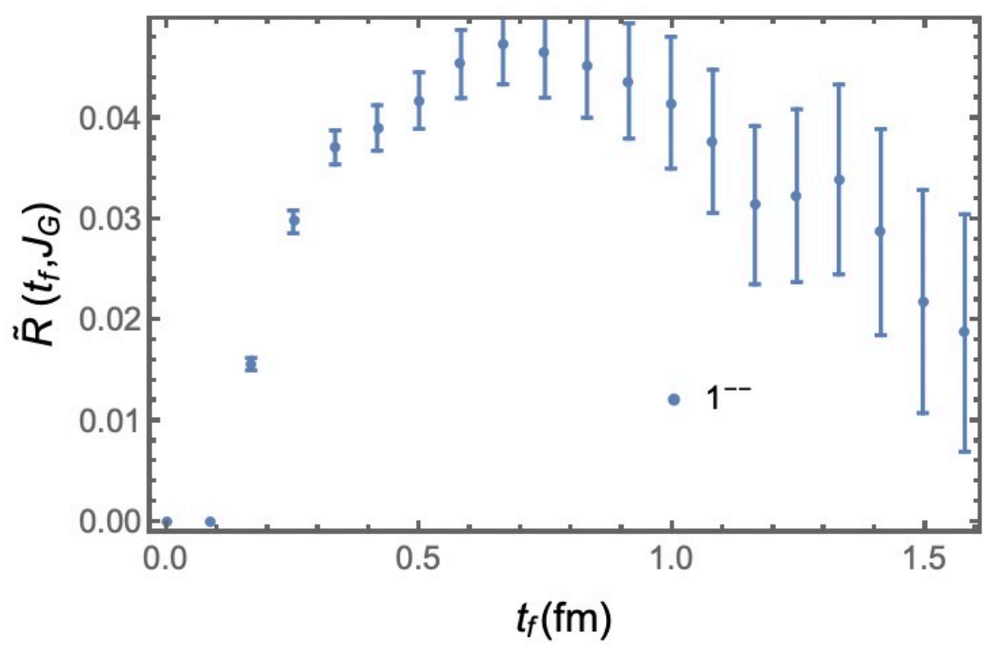
	Initial state	Final state	Operator mixing
Frame I	$p = (0,0,0,m)$ $\epsilon = (0,0,1,0)$	$p' = (0,0,q,E)$ $\epsilon' = (0,1,0,0)$	$\langle p', \sigma_y' \vert T_{4y} \vert p, \sigma_z \rangle = -\frac{(E+m)q}{2} J^g(t) + \frac{(E-m)q}{2} E^g(t)$ $\approx -\frac{(E+m)q}{2} J^g(t) + O(q^2)$
Frame II	$p = (0,0,0,m)$ $\epsilon = (0,0,1,0)$	$p' = (0,q,0,E)$ $\epsilon' = (0,\frac{E}{m},0,\frac{q}{m})$	$\langle p', \sigma_y' \vert T_{4z} \vert p, \sigma_z \rangle = \frac{(E+m)q}{2} J^g(t) + \left[\frac{\Delta^2 q}{2m} + \frac{(E-m)q}{2}\right] E^g(t) + mq\bar{f}^g(t)$ $\approx \frac{(E+m)q}{2} J^g(t) + mq\bar{f}^g(t) + O(q^2)$
Frame III	$p = (0,0,0,m)$ $\epsilon = (0,1,0,0)$	$p' = (0,0,q,E)$ $\epsilon' = (1,0,0,0)$	$\langle p', \sigma_x' \vert T_{xy} \vert p, \sigma_y \rangle = \frac{\Delta^2}{2} E^g(t) + m^2\bar{f}^g(t)$ $\approx m^2\bar{f}^g(t) + O(q^2)$

Boost for 1^{++} operator: $\bar{\psi}\gamma_5\gamma^i\psi \to \alpha\bar{\psi}\gamma_5\gamma^i\psi + \beta\bar{\psi}\gamma_5\gamma^4\psi$ **mix with 0^{-+}**

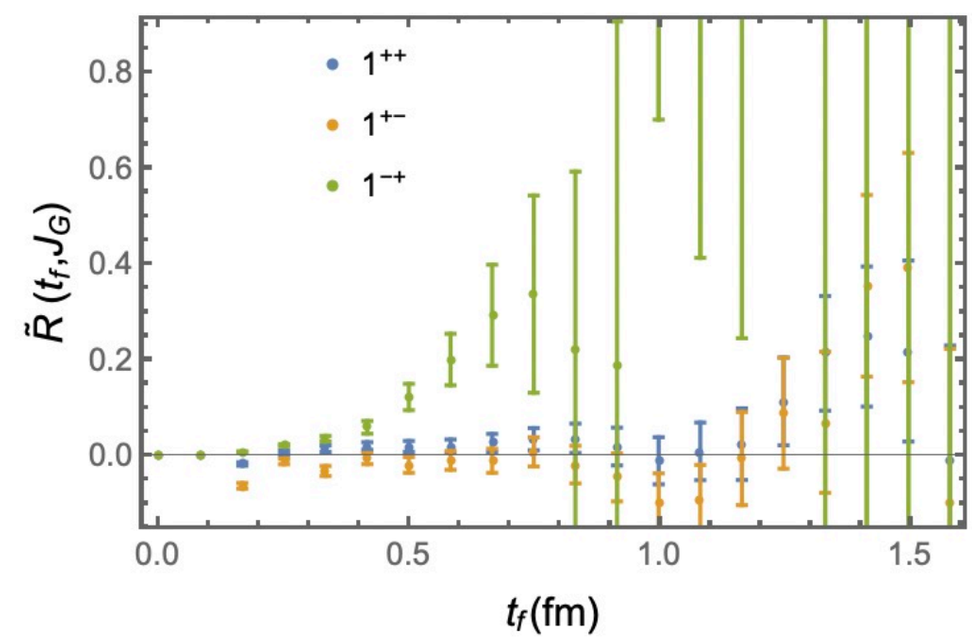
Numerical results without operator mixing

- **The numerical results of gluon angular momentum**

Results of 1^{--} channel



Results of 1^{++} ,1^{+-} and 1^{-+} channel



The bare matrix elements of gluon angular momentum operator in charmonium are very small, except 1-+ channel.

The results of bare matrix elements

- **Join fit of the two point and three point correlation functions**

$$C_2(t_f) = B_0 e^{-M_H t_f}(1 + B_1 e^{-\delta m t_f})$$

M_H **: Ground state mass**

3pt forQuark spin $SC_3^q(t_f) = e^{-M_H t_f}(B_0 t_f \langle S_q \rangle_H + B_2 e^{-\delta_m t_f} + B_3 t_f e^{-\delta_m t_f} + B_4)$

$\langle S_q \rangle_H$ **: Quark spin in the Ground state**

3pt for Gluon AM $SC_3^g(t_f) = e^{-M_H t_f}(B_0 t_f \langle J_g \rangle_H + D_2 e^{-\delta_m t_f} + D_3 t_f e^{-\delta_m t_f} + D_4)$

$\langle J_g \rangle_H$ **: Gluon AM in the Ground state**

Channel	$M_H(GeV)$	$\langle S_q \rangle$	$\langle J_g \rangle$
1^{--}	3.087(01)	0.82(1)	0.043(06)
1^{+-}	3.467(06)	0.04(1)	-0.022(35)
1^{++}	3.449(05)	0.45(4)	0.032(23)
1^{-+}	4.276(70)	0.43(10)	0.253(120)
$2^{++}(J_z = 1)$	3.512(05)	0.43(7)	0.032(23)

Outline

① **Background and introduction**

② **Lattice calculation of quark spin and gluon total angular momentum**

③ **Renormalization**

④ **Summary**

Normalization of axial current operator

F. He, et. al, (χQCD Collaboration), *PRD* 106 (2022)

- **Normalization of axial current operator (Z_A) can be obtained through partially conserved axial current relation**

$$Z_A \partial_\mu A_\mu = 2 Z_m Z_p m_q P,$$

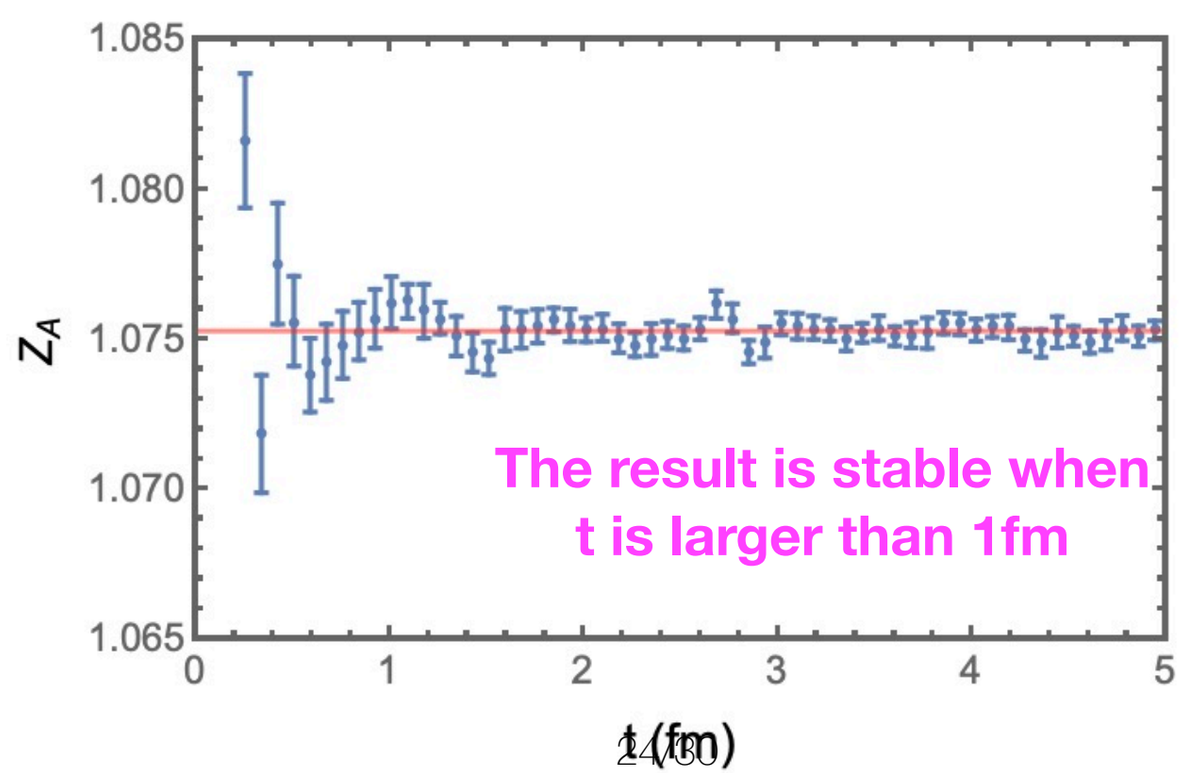
Chiral fermion $Z_m Z_p = 1$

Pseudoscalar operator

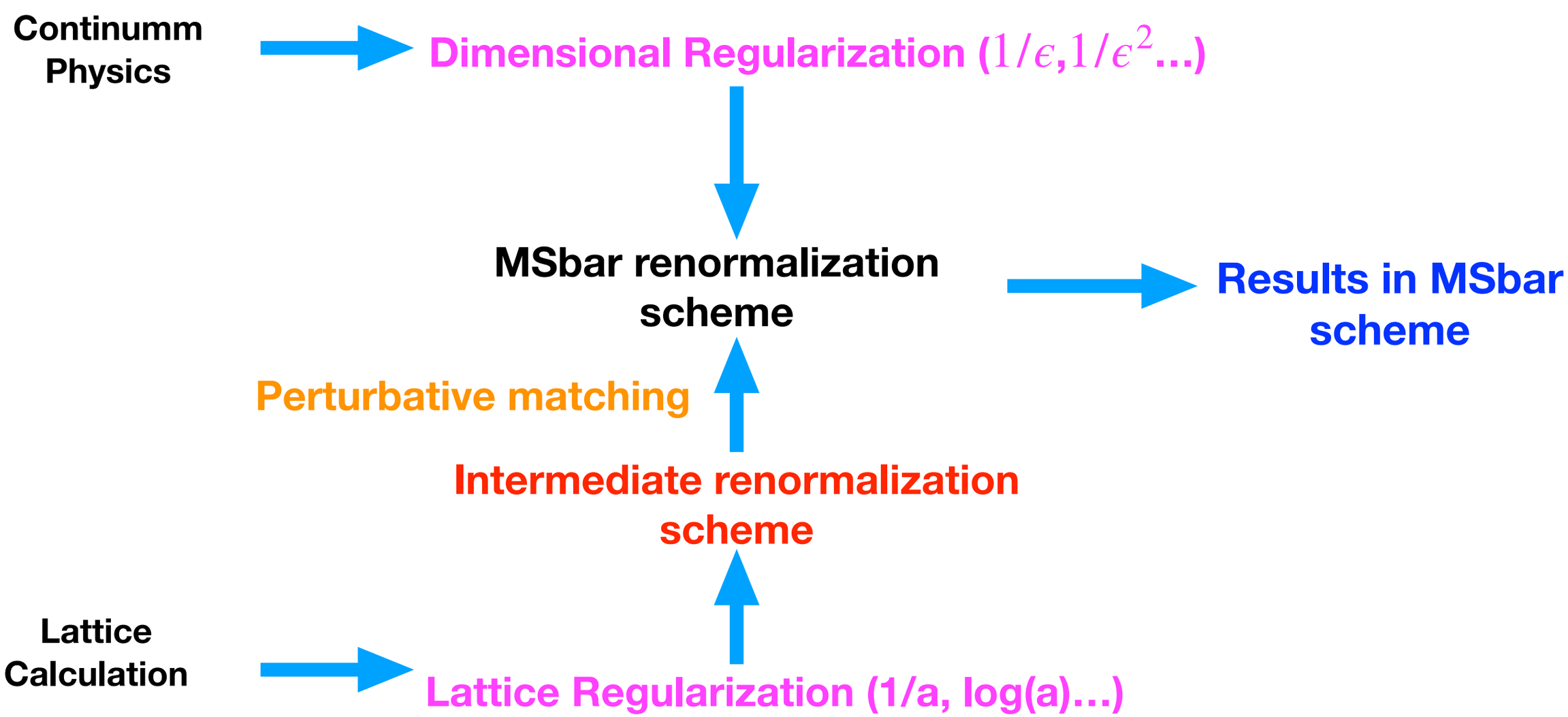
$$Z_A = \frac{2m_q \sum_{\vec{x}} \langle \Omega | P(t,\vec{x}) P^\dagger(0) | \Omega \rangle}{\sum_{\vec{x}} \langle \Omega | \partial_\mu A_\mu(t,\vec{x}) P^\dagger(0) | \Omega \rangle} |_{t\to\infty}.$$

$$Z_A = \frac{4am_q C_{PP}(t)}{C_{A_4P}(t-a) - C_{A_4P}(t+a)} |_{a\to 0},$$

Axial vector operator



Renormalization in continuum physics and Lattice



RI/MOM renormalization scheme

G. Martinelli, C. Pittori, C. T. Sachrajda, M. Testa and A. Vladikas, NPB 445 (1995)

- **Regularization independent momentum subtraction scheme (RI/MOM)**

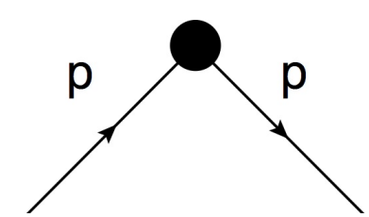
Renormalization condition for quark propagator

Bare quark propagator $i\,S_0^{-1}(p) = \not{p}\,\Sigma_1(p) - m_0\,\Sigma_2(p)$

$$Z_q^{RI}\Sigma_1(p) = 1$$

Renormalization condition for amputated green function

$$\frac{Z_O^{RI}}{Z_q^{RI}}\Lambda_O(p) = \Lambda_O^{tree}$$



Bare matrix elements in the external off-shell state

Tree level result

Perturbative matching

G. Martinelli, C. Pittori, C. T. Sachrajda, M. Testa and A. Vladikas, NPB 445 (1995)

	RI/MOM scheme	$\overline{MS}$ scheme
Dimensional regularization	$Z_q^{D,RI}(p/\mu)\Sigma_1^D(p/\mu) = 1$	$Z_q^{D,\overline{MS}}(p/\mu)\Sigma_1^D(p/\mu) = 1 + O(\alpha_s) + \ldots$
Lattice regularization	$Z_q^{L,RI}(pa)\Sigma_1^L(pa) = 1$	$Z_q^{L,\overline{MS}}(\mu a)\Sigma_1^D(pa) = 1 + O(\alpha_s) + \ldots$

$$Z_q^{L,\overline{MS}}(\mu a) = \boxed{Z_q^{L,RI}(pa)} \times \boxed{\frac{Z_q^{D,\overline{MS}}(\mu)}{Z_q^{D,RI}(p/\mu)}}$$

$$Z_O^{L,\overline{MS}}(\mu a) = \boxed{Z_O^{L,RI}(pa)} \times \boxed{\frac{Z_O^{D,\overline{MS}}(\mu)}{Z_O^{D,RI}(p/\mu)}}$$

Lattice calculable

Perturbative matching coefficient, obtained using dimreg.

Gluon self energy

Y. Yang, M. Glatzmaier, K. Liu and Y. Zhao,1612.02855

- **Renormalization of gluon self energy using the lattice regularization and under RI/MOM scheme**

$$Z_L^{RI} = 1 + \frac{g^2}{16\pi^2}[(\frac{2N_f}{3} - N_c)\log(a^2p^2) + \underline{N_f B_G^f - N_c B_G^L}] + O(g^4),$$

Depend on lattice action

- **Renormalization of gluon using dimensional regularization and under RI/MOM and MSbar scheme can be written as**

$$Z_D^{\overline{MS}} = 1 - \frac{g^2}{16\pi^2}[(\frac{2N_f}{3} - N_c)\frac{1}{\epsilon}] + O(g^4),$$

$$Z_D^{RI} = 1 - \frac{g^2}{16\pi^2}[(\frac{2N_f}{3} - N_c)(\frac{1}{\epsilon} + \log(\mu^2/p^2)) + \frac{10N_f}{9} - N_c B_G^D(z)] + O(g^4),$$

- **The Renormalization constants with the lattice regularization and under MSbar scheme** $Z_L^{\overline{MS}} = \frac{Z_D^{\overline{MS}}}{Z_D^{RI}} Z_L^{RI}(a,\mu)$

$$= 1 + \frac{g^2}{16\pi^2}[(\frac{2N_f}{3} - N_c)\log(a^2\mu^2) + N_f(\frac{10}{9} + B_G^f) - N_c(B_G^L + B_G^D)] + O(g^4)$$

Renormalization of energy momentum tensor

- **The renormalization condition for EMT in RI/MOM scheme**

S. Capitani and G. Rossi, Nucl. Phys. B433, 351 (1995),

$$\overline{T}_Q^{\{\mu,\nu\}R} = Z_{QQ}\overline{T}_Q^{\{\mu,\nu\}B} + Z_{QG}\overline{T}_G^{\{\mu,\nu\}B}$$

$$\overline{T}_G^{\{\mu,\nu\}R} = Z_{GQ}\overline{T}_Q^{\{\mu,\nu\}B} + Z_{GG}\overline{T}_G^{\{\mu,\nu\}B}$$

$$\begin{aligned}
\langle Q|\overline{\mathcal{T}}_Q^{\{\mu\nu\},R}|Q\rangle|_{p_\nu=0,p^2=\mu_R^2} &= \frac{\gamma_\nu p_\mu}{2}, \\
\langle G,\rho|\overline{\mathcal{T}}_Q^{\{\mu\nu\},R}|G,\tau\rangle|_{p^2=\mu_R^2} &= 0, \\
\langle Q|\overline{\mathcal{T}}_G^{\{\mu\nu\},R}|Q\rangle|_{p^2=\mu_R^2} &= 0, \\
\langle G,\rho|\overline{\mathcal{T}}_G^{\{\mu\nu\},R}|G,\tau\rangle|_{\rho=\tau\neq\mu,\nu,p_\rho=0,p^2=\mu_R^2} &= -2p_\mu p_\nu
\end{aligned}$$

$$Z_{ij}^{L,\overline{MS}}(\mu a) = (Z^{D,\overline{MS}(\mu)}/Z^{D,RI}(\mu/p))_{ik}Z_{kj}^{L,RI}(pa)$$

- **The result of RCs on 32I gauge ensemble ($\overline{MS}(2GeV)$)**

Y. Yang, et. al (χQCD Collaboration), PRL. 121 (2018)

Symbol	Z_{QQ}	δZ_{QQ}	Z_{QG}	Z_{GQ}	Z_{GG}
32I	1.25(0)(2)	0.008(2)(2)	0.000(14)	0.18(2)(2)	1.10(4)(5)

The mixing between quark and gluon is not small.

Results

- **The contributions of quark spin and gluon angular momentum in different charmonium states**

$$S_q^R = Z_A S_q^B \qquad \begin{pmatrix} S_q + L_q \\ J_g \end{pmatrix}^R = \begin{pmatrix} Z_{qq} & Z_{qg} \\ Z_{gq} & Z_{gg} \end{pmatrix} \begin{pmatrix} S_q + L_q \\ J_g \end{pmatrix}^B \qquad L_q^R + S_q^R + J_g^R = 1$$

$Z_{gq} = 0.18(2)$

	S_q^R (Quark Spin)	L_q^R (Quark OAM)	J_g^R (Gluon AM)
$1^{--}(S=1, L=0)$	0.88(2)	-0.05(3)	0.17(3)
$1^{+-}(S=0, L=1)$	0.04(4)	0.86(6)	0.10(4)
$1^{++}(S=1, L=1)$	0.49(5)	0.40(6)	0.11(4)
$1^{-+}(S=?, L=?)$	0.46(11)	0.17(18)	0.37(14)
$2^{++}(S=1, L=1)$	0.46(8)	0.43(9)	0.11(4)

Renormalized results of gluon AM is larger than bare results due to the mixing with quark operator.

Summary

- **We studied spin decomposition in different charmonium state. The contribution of quark spin is compatible with the prediction of quark model.**

- **Though the bare matrix elements of gluon spin in charmonium is very small, the renormalized results are large due to the mixing with quark operator.**

- **The quark spin contributes to half of spin of exotic 1^{-+} state, the contribution of gluon AM in 1^{-+} channel is larger than that in other channels.**

Thank you for your attention!