Muon $g-2$ Hadronic Vacuum Polarization from 2+1+1 flavors of sea quarks using the HISQ action

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April 29, 2016

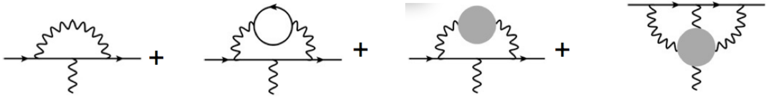
Motivation

The muon anomalous magnetic moment is currently measured to a precision of around a half a part per million, with a similar error quoted for the theory prediction.

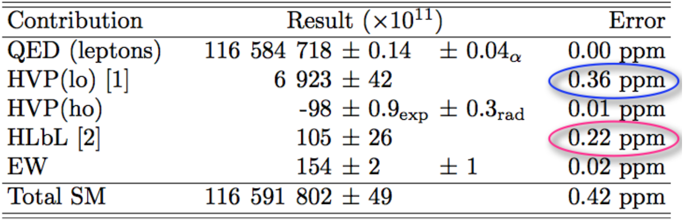
$$a_\mu^{\mathrm{exp}} = 116\,592\,089(54)(33) \times 10^{-11}, \quad a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 287(80) \times 10^{-11} [3.6\sigma] \tag{1}$$

The experimental value deviates from the Standard Model prediction by 3-4 σ making it an interesting thing to study. Errors are completely dominated by hadronic contributions. There are two types: Hadronic Vacuum Polarization (HVP) and Hadronic Light-by-Light (HLbL).

Muon $g-2$



QED(4 loops) + EW (2 loops) + HVP + HLbL



Contribution	Result ($\times 10^{11}$)		Error
QED (leptons)	116 584 718 ± 0.14	$\pm\ 0.04_{\alpha}$	0.00 ppm
HVP(lo) [1]	6 923 ± 42		0.36 ppm
HVP(ho)	-98 $\pm\ 0.9_{\text{exp}}$	$\pm\ 0.3_{\text{rad}}$	0.01 ppm
HLbL [2]	105 ± 26		0.22 ppm
EW	154 ± 2	± 1	0.02 ppm
Total SM	116 591 802 ± 49		0.42 ppm

Hadronic Vacuum Polarization

The HVP contribution can be obtained by combining perturbative QCD with experimental data for e^+e^- inclusive scattering into hadrons, or with decays of the τ lepton into hadrons.

The current precision in HVP from experiment is 0.6%, though the scatter between different experimental methods is more that this. This is due mainly to different treatments of the experimental results, especially whether or not one includes a data set from the BaBar experiment (BaBar radiative return data at low $\sqrt{x}$) Thus a lattice calculation at the 1% precision would already be interesting. HPQCD has produced a result with a 2% total error. We are now targeting the leading systematic errors in an effort to reduce this.

Improvements to sub-percent precision would require QED effects to be included.

HVP on the lattice

Method of Blum, '02:

$$a_{\mu,\mathrm{HVP}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 f(q^2) 4\pi^2 [\Pi(0) - \Pi_V(q^2)] \tag{2}$$

One must calculate on the lattice the renormalized vacuum polarization function $\hat{\Pi}(q^2) \equiv \Pi(q^2) - \Pi(0)$.

The integrand peaks at $q^2 \sim \mathscr{O}(m_\mu^2)$.

The standard method requires a calculation of $\hat{\Pi}(q^2)$ at $q^2 > 0$ and an extrapolation to zero. This leads to large uncertainties.

HPQCD Method

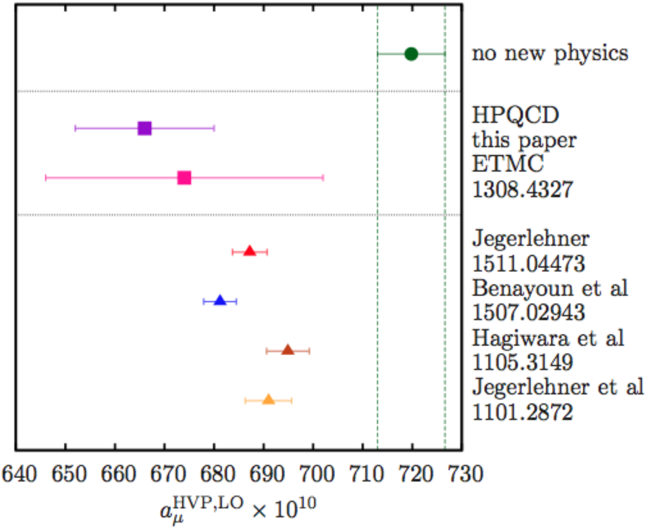
For spatial currents at zero spatial momentum:

$$G_{2n} \equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^i(\vec{x},t) j^i(0) \rangle = (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 \hat{\Pi}(q^2)|_{q^2=0}. \qquad (3)$$

$$\hat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j} \Pi_j, \qquad \Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!}. \qquad (4)$$

Time moments of the correlator give the derivatives of $q^2 = 0$ of $\hat{\Pi}$. $\hat{\Pi}(q^2)$ is replaced with its [2,2] Padé approximant derived from Π_j. This allows one to reach high momenta for the q^2 integration, which is done numerically. The result converges rapidly as one includes more Padé terms. (Chakraborty, 2014)

HPQCD result



(arXiv:1601.03071)

Error budget

Table: Error budget for the connected contributions to the muon anomaly a_μ from vacuum polarization of u/d quarks.

	$a_\mu^{\mathrm{HVP,LO}}(u/d)$
QED corrections:	1.0%
Isospin breaking corrections:	1.0%
Staggered pions, finite volume:	0.7%
Valence m_ℓ extrapolation:	0.4%
Monte Carlo statistics:	0.4%
Padé approximants:	0.4%
$a^2 \to 0$ extrapolation:	0.3%
Z_V uncertainty:	0.4%
Correlator fits:	0.2%
Tuning sea-quark masses:	0.2%
Lattice spacing uncertainty:	$< 0.05\%$
Total:	1.8%

Longer Range Plan

Table: Planned ensembles for use in this multi-year project.

a(fm)	$L^3 \times T$	number of configs	resources	Theory
≈ 0.15	$32^3 \times 48$	2000	6 M J/psi core-hours	QCD
≈ 0.12	$48^3 \times 64$	1000	12 M J/psi core-hours	QCD
≈ 0.09	$64^3 \times 96$	1000	37 M Mira core-hours	QCD
≈ 0.06	$96^3 \times 192$	1000	210 M Mira core-hours	QCD
≈ 0.15	$32^3 \times 48$	500	0.07 M K40 GPU hours	QCD+qQED
≈ 0.12	$48^3 \times 64$	500	0.28 M K40 GPU hours	QCD+qQED
≈ 0.09	$64^3 \times 96$	500	64 M Mira core-hours	QCD+qQED

USQCD Resource Request for 2015-16

Table: Computing allocated in last year's proposal.

a(fm)	$L^3 \times T$	number of configs	J/psi or Mira core-hrs	Theory
≈ 0.15	$32^3 \times 48$	2000	6 million J/psi core-hrs	QCD
≈ 0.12	$48^3 \times 64$	1000	12 million J/psi core-hrs	QCD
≈ 0.09	$64^3 \times 96$	1000	37 million Mira core-hrs	QCD

USQCD Resource Request for 2016-17

Table: Computing resource request for this year's proposal. We do not require configuration generation.

a(fm)	$L^3 \times T$	# of configs	Resource	Theory
≈ 0.15	$32^3 \times 48$	500	0.07 M K40 GPU hours	QCD+qQED
≈ 0.12	$48^3 \times 64$	500	0.28 M K40 GPU hours	QCD+qQED
≈ 0.06	$96^3 \times 192$	300	64 M Mira core-hours	QCD

Responses to the SPC

1. The SPC would like to have a complete report on the progress on this project over the last year. We understand that significant resources have been devoted to this project already.

So far we have used 11 M mira core-hours of Incite time and 3.4 M J/psi core hours on Fermilab clusters. This is about 30% of our Incite time and 17% of our Fermilab cluster time. The time on mira has gone towards an analysis of the the 0.09 fm physical mass ensemble, and the time on clusters has gone towards generating new ensembles with better tunings of the bare quark masses at 0.12 fm. The delay in using our time has been to test the implementation of variance reduction methods in order to reduce the cost of the project.

Responses to the SPC

2. The SPC would like you to explore possible coordination and collaboration with the other two g-2 proposals. The g-2 calculation is a project that requires significant resources and thus USQCD should make sure it is done in the most efficient way, using all the expertise available and without duplication of effort.

While we agree that duplication of effort (i.e. multiple collaborations calculating the same quantity with the same method, lattice actions, and gauge-field ensembles) is not a good use of USQCD resources, we do not believe that this is the current situation with g-2 HVP. Our calculation of the muon g-2 HVP uses a different method (time-moments of current-current correlation functions) than the other USQCD calculations being pursued by Aubin et al. and RBC, and is therefore complementary.

Because the theoretical value of g-2 is of such critical importance for interpreting the experiment as a test of the Standard Model, any lattice calculation will need independent confirmation. We believe that our approach has several advantages over the other calculations and will ultimately lead to the most precise determination of g-2 HVP.

There is no room for sharing the cost of computing propagators, because all three projects use different valence actions.

Responses to the SPC

3. The SPC would like to know why you are not using more sophisticated variance reduction methods. Have you done any tests and comparisons of methods and found that your approach is optimal?

We are using random wall sources with the truncated solver method (TSM), in order to reduce the cost of the analysis. We find that the TSM likely buys us a factor of 2, but we are currently tracking down a discrepancy between different versions of the code that may undermine this conclusion. We also hope to test the use of an eigenvalue solver in conjunction with the TSM (i.e. what is usually referred to as all-mode averaging) in order to further reduce the cost of the propagator generation. An eigenvalue solver has recently been integrated into the MILC code that would be suitable for this purpose.

Responses to the SPC

4. With the new resources at JLab being as yet unspecified, we would like to know if you are in a position to use them efficiently if they are a) cpu, b) GPU, c) KNL. If you are not, that is fine, but it will help in our allocation decisions to know this information from every proposal.

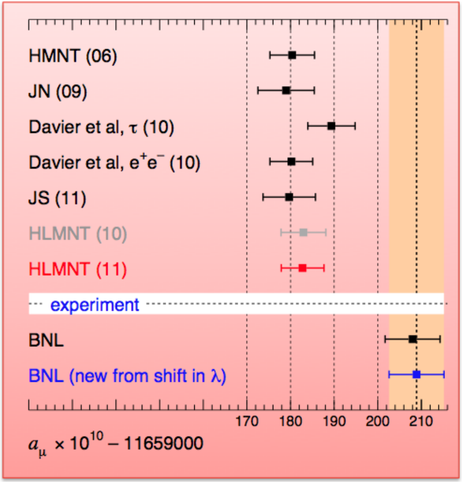
If the new hardware is CPU based, we expect to be able to make efficient use of it using the MILC code as it should be possible to run benchmarks prior to acquisition and we presume that will have been done.

If the new hardware is GPU based, we expect to be able to make efficient use for propagator solvers as the QUDA code runs well for both staggered and Wilson/Clover. Codes that require many contractions might require additional work and we have sometimes run in a mixed environment where propagators are saved and contractions are done on separate CPU resources.

If the new hardware is KNL based, we cannot predict how efficient our usage will be. We have been working for quite some time to produce efficient Xeon Phi code; however, on KNC we were never able to run successfully on multiple chips because of MPI issues. We will continue efforts to explore and improve performance on Xeon Phi, but without experience on a multichip system, we would have to say the we do not know if we are in a position to make efficient use of such a system.

Back-up Slides

Comparison


HMNT (06)
JN (09)
Davier et al, τ (10)
Davier et al, e⁺e⁻ (10)
JS (11)
HLMNT (10)
HLMNT (11)
experiment
BNL
BNL (new from shift in λ)
170
180
190
200
210
$a_\mu \times 10^{10} - 11659000$