Hadronic contributions to the muon *g*-2 using staggered fermions

Christopher Aubin, Tom Blum, Maarten Golterman, Santi Peris, Cheng Tu

Fordham University, University of Connecticut, San Francisco State University, Universitat Autònoma de Barcelona

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Outline

Current status of muon g-2

Previous allocation period results

Current proposal and response to SPC

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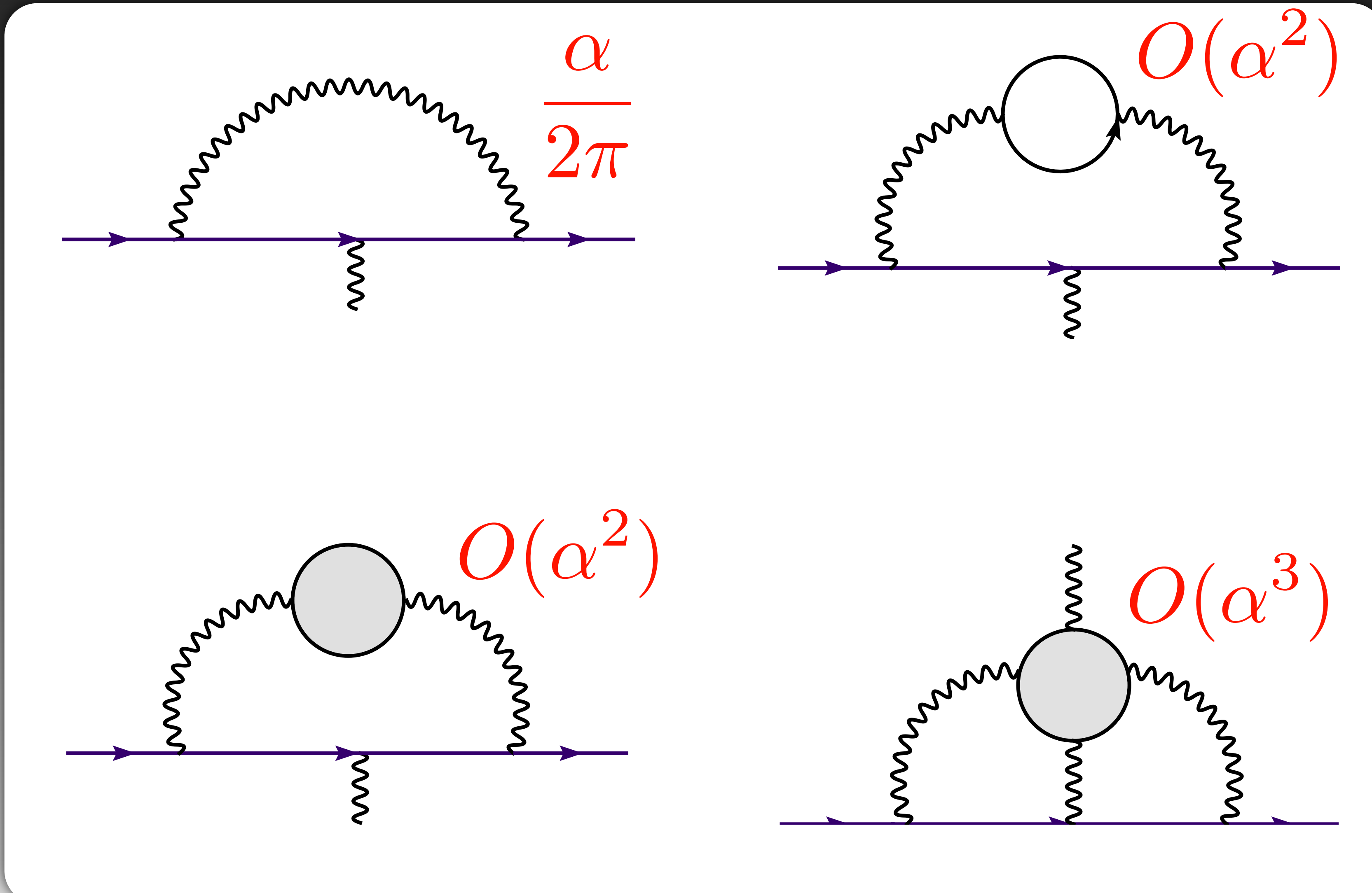
Muon (g-2)/2

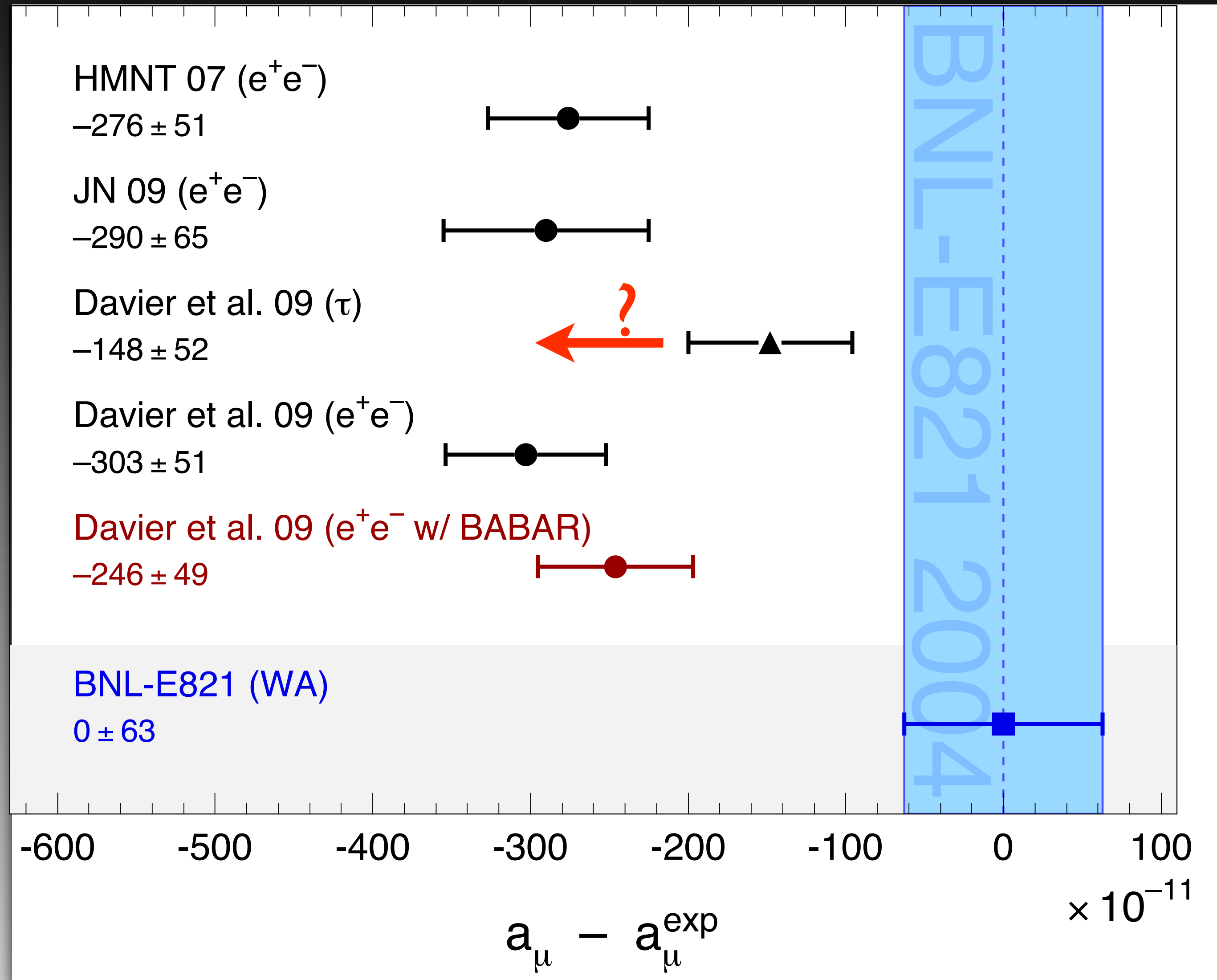


Leading hadronic contribution

Light-by-light contribution

Current status (w/o Lattice results)

Jegerlehner & Szafron, EPJ C



from M. Davier, arXiv:1001.2243

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Currently studied by several lattice groups

[19] C. Aubin and T. Blum, Phys. Rev. **D75**, 114502 (2007), hep-lat/0608011.

[20] A. Juttner and M. Della Morte, PoS **LAT2009**, 143 (2009), 0910.3755.

[21] M. Della Morte, B. Jager, A. Juttner, and H. Wittig, PoS **LATTICE2012**, 175 (2012), 1211.1159.

[22] F. Burger *et al.*, PoS **LATTICE2013**, 301 (2013), 1311.3885.

[23] E. B. Gregory *et al.*, (2013), 1311.4446.

[24] C. Aubin *et al.*, (2015), 1512.07555.

[25] B. Chakraborty *et al.*, (2015), 1512.03270.

[26] T. Blum *et al.*, (2015), 1512.09054.

This is by no means an exhaustive list, and doesn't include some of the USQCD groups proposing to work on this.

Lattice Calculation

$$\Pi_{\mu\nu}(q) \equiv \int d^4x \; e^{iqx} \langle J_\mu(x) J_\nu(0) \rangle = \left(q^2 \delta_{\mu\nu} - q_\mu q_\nu\right) \Pi(q^2)$$

$$f(q^2) = m_\mu^2 q^2 Z^3(q^2) \, \frac{1 - q^2 Z(q^2)}{1 + m_\mu^2 q^2 Z^2(q^2)} \, ,$$

$$Z(q^2) = \left(\sqrt{(q^2)^2 + 4 m_\mu^2 q^2} - q^2 \right) / (2 m_\mu^2 q^2)$$

$$a_\mu^{\mathrm{LO,HVP}} = \lim_{q^2_{max} \to \infty} a_\mu^{\mathrm{LO,HVP}}[q^2_{max}] \, ,$$

$$a_\mu^{\mathrm{LO,HVP}}[q^2_{max}] = 4\alpha^2 \int_0^{q^2_{max}} dq^2 \, f(q^2) \, \hat{\Pi}(q^2) \, ,$$

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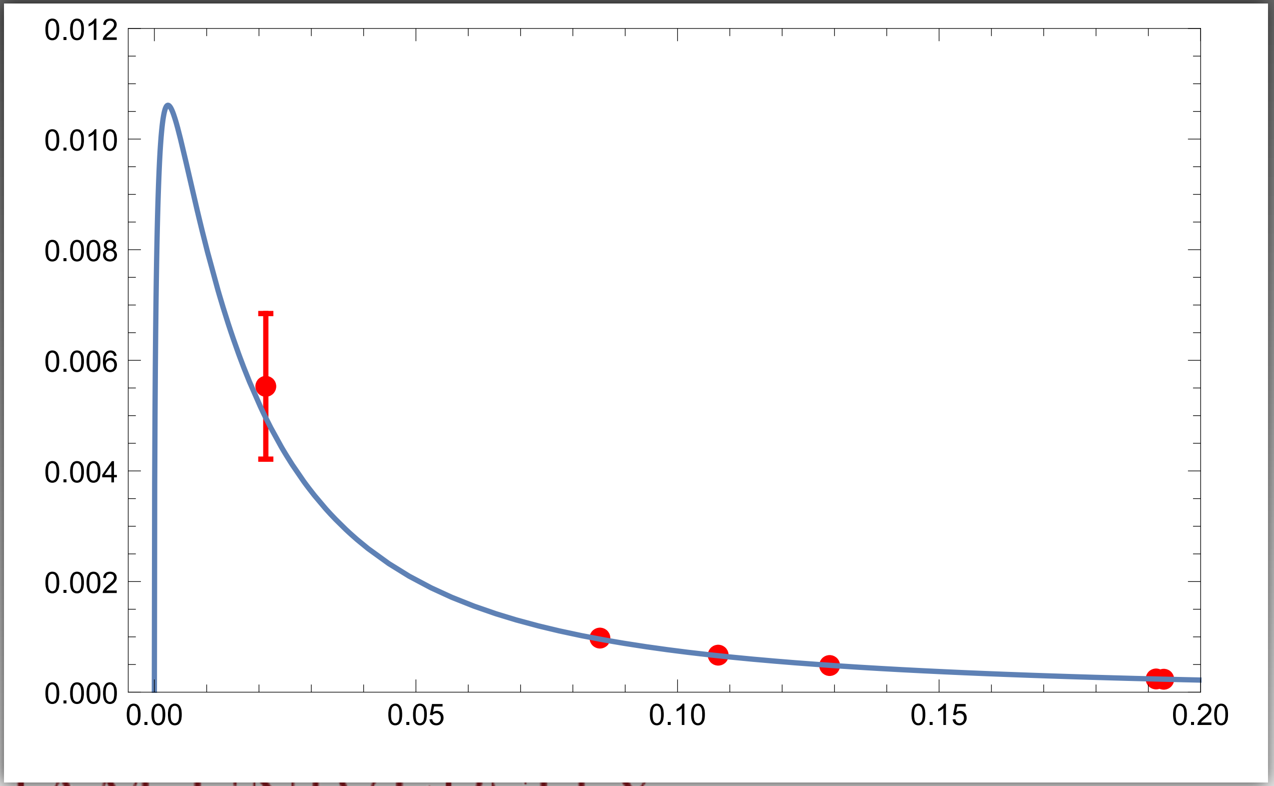
2015-2016 Results

Using the Asqtad data we published a systematic study of the finite volume effects (PRD93 054508, 2016)



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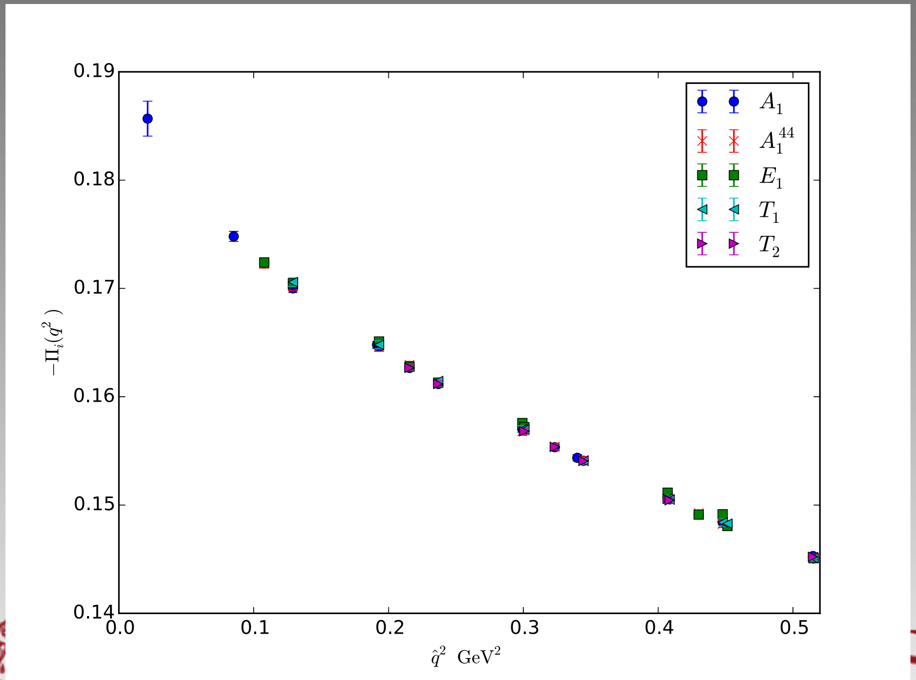
2015-2016 Results

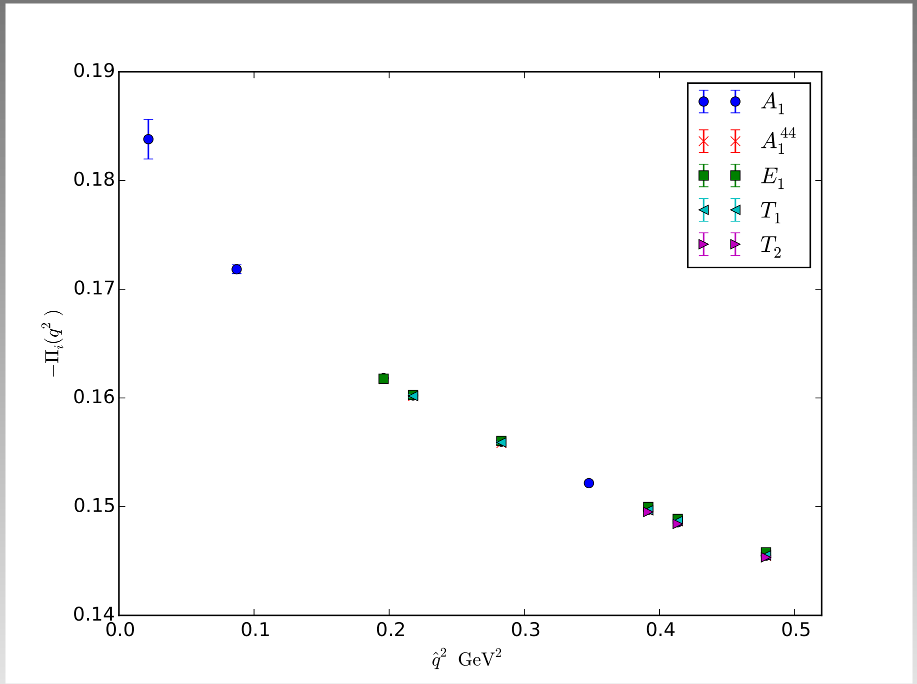
With AMA approach, we generated data:
a~0.06fm Asqtad ensembles, m_π ~220MeV
a~0.06fm HISQ ensembles, m_π ~300 MeV.

Currently running on the superfine HISQ ensemble at the physical pion mass (to be finished before the end of the cycle).

Asqtad, *64³x144, a~0.06fm*



HISQ, *48³x144, a~0.06fm*



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2015-2016 Results

On the lattice no rotation symmetry so the VP tensor is written in terms of five different irreps of the cubic group:

$$
\begin{aligned}
A_1&: \quad \sum_i \overline{\Pi}_{ii} = (3q^2 - \vec{q}^{\,2})\overline{\Pi}_{A_1} \ , \\
A_1^{44}&: \quad \overline{\Pi}_{44} = (\vec{q}^{\,2})\overline{\Pi}_{A_1^{44}} \ , \\
T_1&: \quad \overline{\Pi}_{4i} = -(q_4 q_i)\overline{\Pi}_{T_1} \ , \\
T_2&: \quad \overline{\Pi}_{ij} = -(q_i q_j)\overline{\Pi}_{T_2} \ , \ i \neq j \ , \\
E&: \quad \overline{\Pi}_{ii} - \sum_i \overline{\Pi}_{ii}/3 = (-q_i^2 + \vec{q}^{\,2}/3)\overline{\Pi}_E
\end{aligned}
$$

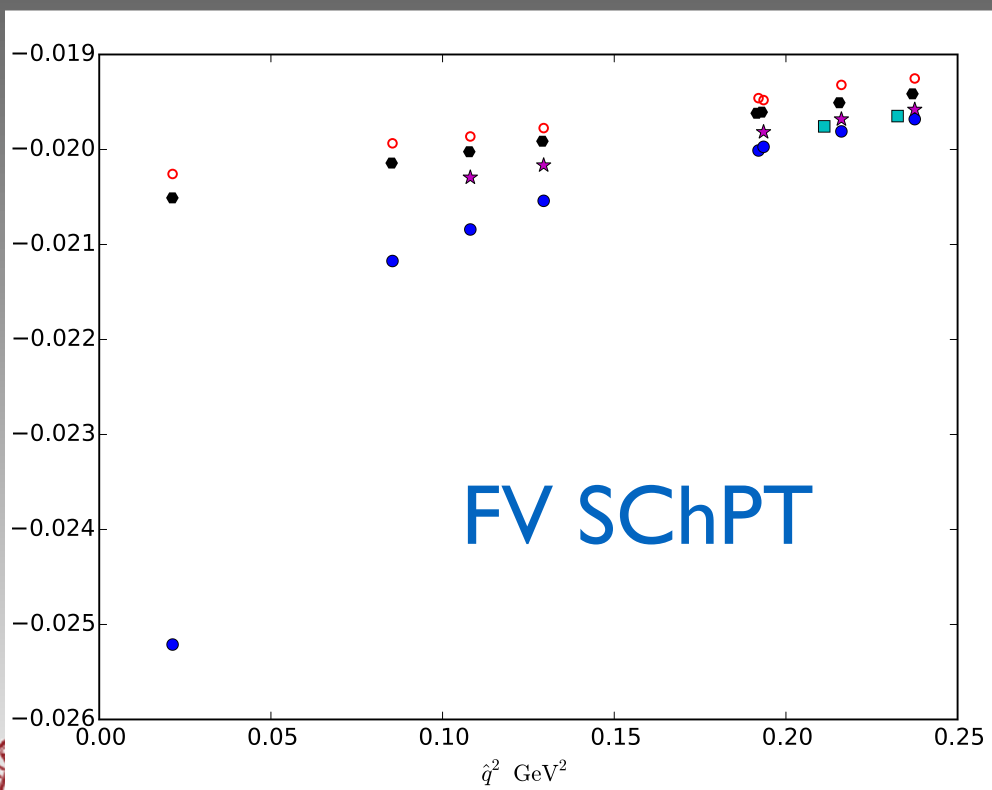
As the VP tensor does not necessarily vanish at zero momentum we consider the subtracted/projector tensor as well:

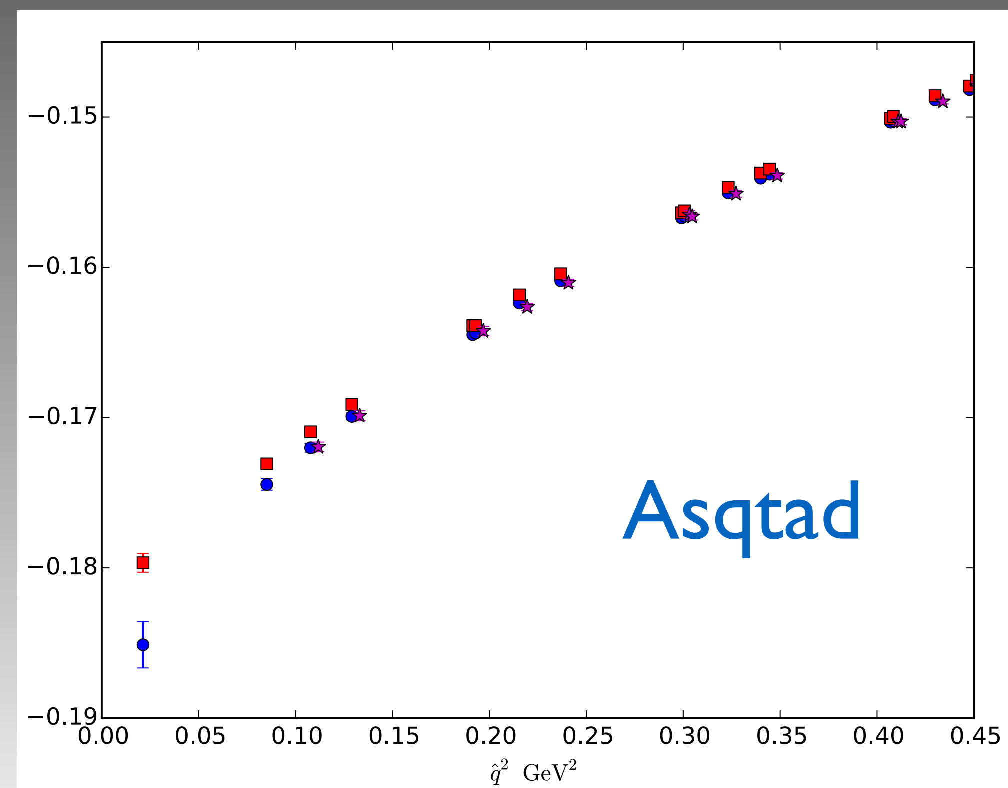
$$
\begin{aligned}
\overline{\Pi}_{\mu\nu}(q) &= \sum_{\kappa\lambda} P^T_{\mu\kappa} \left(\Pi_{\kappa\lambda}(q) - \Pi_{\kappa\lambda}(0)\right) P^T_{\lambda\nu} \\
&= \Pi_{\mu\nu}(q) - P^T_{\mu\nu}\Pi_s(0) - P^T_{\mu 4}P^T_{4\nu}\left(\Pi_4(0) - \Pi_s(0)\right) \\
P^T_{\mu\nu}(q) &= \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \ ,
\end{aligned}
$$

2015-2016 Results

Using ChPT to examine the FV effects:

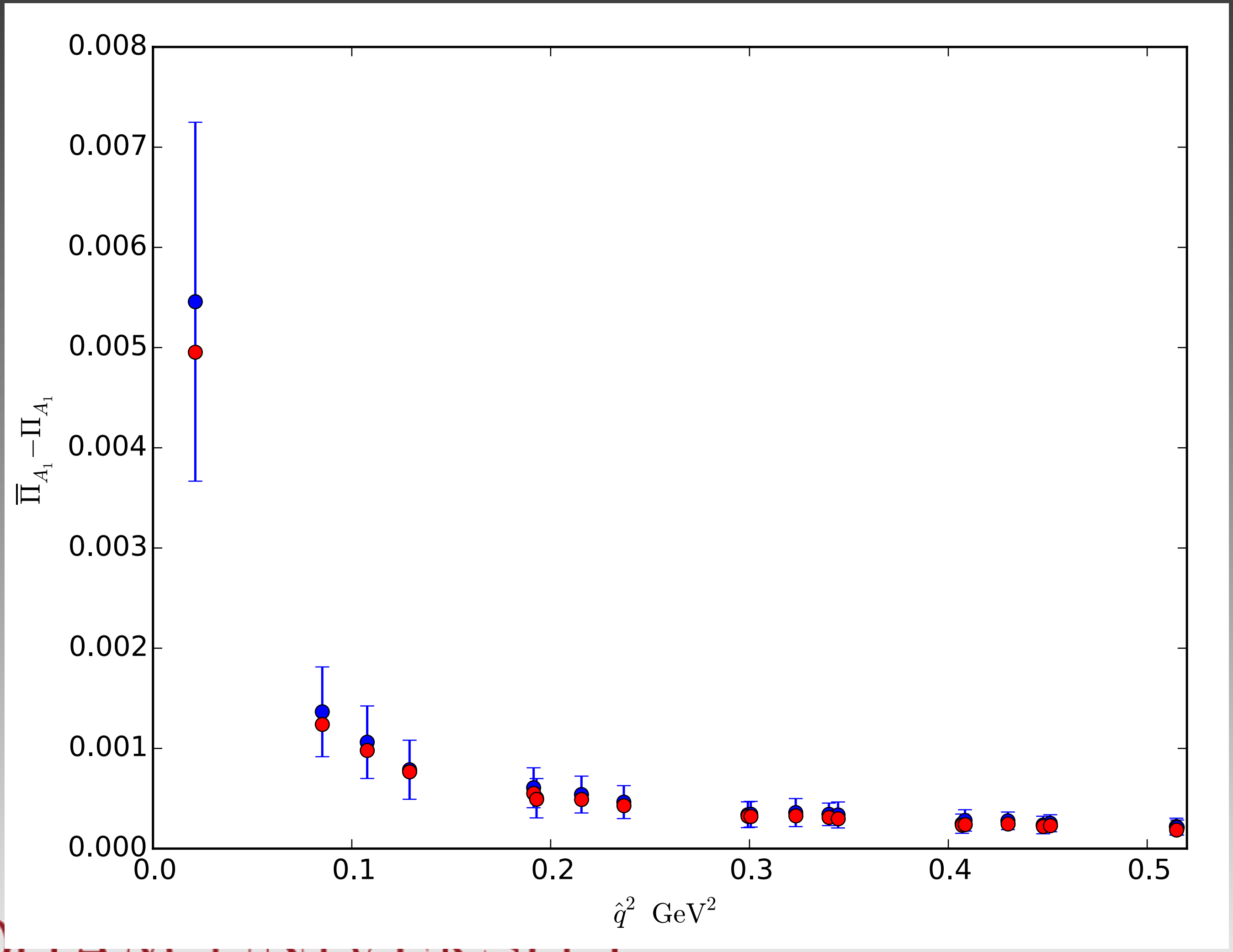
$$\Pi_{\mu\nu}(q) = \frac{10}{9} e^2 \frac{1}{L^3 T} \sum_p \Bigg[\frac{4 \sin\left(p + q/2\right)_\mu \sin\left(p + q/2\right)_\nu}{(2\sum_\kappa (1 - \cos p_\kappa) + m_\pi^2)(2\sum_\kappa (1 - \cos(p+q)_\kappa) + m_\pi^2)} - \delta_{\mu\nu} \left(\frac{2 \cos p_\mu}{(2\sum_\kappa (1 - \cos p_\kappa) + m_\pi^2)} \right) \Bigg],$$





2015-2016 Results

Comparison of Asqtad data & SChPT of the difference between the subtracted and unsubtracted A_1 irrep:



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2015-2016 Results

Extracting the *g*-2 for different irreps gives results that differ by 10-15%:

[0,1] Padé fit:

$$a_{\mu,A_1}^{\mathrm{LO,HVP}}[0.1\ \mathrm{GeV}^2] = 6.8(4) \times 10^{-8} ,$$
$$a_{\mu,A_1^{44}}^{\mathrm{LO,HVP}}[0.1\ \mathrm{GeV}^2] = 7.5(3) \times 10^{-8} .$$

Quad. conformally mapped poly:

$$a_{\mu,A_1}^{\mathrm{LO,HVP}}[0.1\ \mathrm{GeV}^2] = 6.8(4) \times 10^{-8} ,$$
$$a_{\mu,A_1^{44}}^{\mathrm{LO,HVP}}[0.1\ \mathrm{GeV}^2] = 7.9(4) \times 10^{-8} .$$

Remaining issues

The primary concern regarding systematics is with regards to the finite volume effects.

When those are under control then we will then apply our various model-independent fits (Padés, conformal polynomials) to extract *g*-2.

Our primary focus for the upcoming year is a fully systematic study of the FV effects to hopefully remove them using SChPT.

Proposal

2+1+1-flavor MILC HISQ configurations

HISQ Ensembles					
a (fm)	m_l/m_s	m_π (MeV)	$L^3 \times T$	$m_\pi L$	J/psi core-hrs ($\times 10^6$)
≈ 0.12	1/10	218	$24^3 \times 64$	3.2	0.27
≈ 0.12	1/10	217	$32^3 \times 64$	4.2	0.64
≈ 0.12	1/10	217	$40^3 \times 64$	6.3	1.26
≈ 0.12	1/27	134	$48^3 \times 64$	3.9	2.17
≈ 0.09	1/27	128	$64^3 \times 96$	3.7	7.73
≈ 0.06*	1/27	136	$96^3 \times 192$	4.0	26.1

We plan to use the direct-double subtraction method [32,33] and a hybrid AMA-A2A approach (RBC).

The times above correspond to 50 configurations on each ensemble (3000 eigenvectors on the smaller volume, 2000 eigenvectors on the larger volume)

We'll do 4 sets of exact+sloppy CG inversions per configuration

Questions from SPC

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1) What is the expected precision for the currently proposed calculation? What is the long-term goal?

Sub-percent statistical errors using AMA-A2A and DDS

If SChPT can be used to correct lattice data: 1-3% FV errors.

Long-term goal: <1% error

2) How well do you expect to be able to control finite volume effects based on the study you propose here?

We hope to eliminate the FV error analyzing each irrep separately. It is unclear if this can be accomplished without a larger box size

3) Table 1 lists a $48^3\times 96$ ensemble for $a=0.12$ fm with $m_l/m_s =0.1$, however, there is no such ensemble. There is one with volume $40^3\time 64$. Is that the one you want to use and, if so, does that change your request?

This was a typo: it reduces the time by 2M J/psi core-hrs

4) In your 2015 request, you proposed to study six ensembles. They covered the lattice spacings 0.09 and 0.06 fm, with m_l/m_s = 1/5, 1/10, and 1/27. We realize you did not get all the time requested; however, we would like to know the status of your running. The current proposal states the 0.06 fm 300 MeV pion running is done, and that half of physical mass ensembles will be done by end of allocation. However, that does not seem to cover status of m_l/m_s = 1/5, 1/10, a=0.09 fm ensembles, or the a=0.06 fm, m_l/m_s = 1/10 ensemble. So, can you please provide us with a table covering all the ensembles from Tables 1 in both the current and 2015 proposals with number of configurations run, expected to be run by end of current allocation, and desired to be run in next allocation?

During the current period, our finite volume analysis showed considerable FV erects for the lowest momentum points.

We re-evaluated our short-term goals to focus on this error.

Currently we are finishing the physical point ensemble for the remainder of this year, and the remaining ensembles we planned on studying last year will not be used in the upcoming period.

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5) Can you compare your approach with that of other groups both inside USQCD and beyond?

Our priority is to understand the systematics as best as possible, as the errors on the final result for the muon *g*-2 are crucial.

As it is for other groups, our long-term goal is to obtain a reliable value for the muon *g*-2, while in the short term we are focused on reducing this uncertainties.

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6) Related, your proposal and the Laiho proposal use some of the same HISQ ensembles. Have you explored any possible cost savings available through sharing common propagators or correlation functions?

Our methods of AMA+AMA differ significantly with those of Laiho, and hence there not much to save. For example, with AMA we will use an order of magnitude fewer configurations. It may be possible to work together on generating eigenvectors, which could significantly speed up their calculation through deflation.

7) With the new resources at JLab being as yet unspecified, we would like to know if you are in a position to use them efficiently if they are a) cpu, b) GPU, c) KNL. If you are not, that is fine, but it will help in our allocation decisions to know this information from every proposal.

Our code is part of the CPS code base, and is ready to run efficiently on new cpu resources at JLAB. We could also quickly take advantage of RBC/UKQCD expertise with KNL to port our code to this architecture.

Summary

As the finite volume effects are considerable, it is important to be able to remove that systematic reliably.

With our proposal we will test explicitly how well the SChPT can be used to correct this systematic, and thus by the end of the upcoming allocation have a reliable (and hopefully small) finite volume uncertainty (1-3%).

Our request:

38.2 M J/psi core-hrs

Storage: 96 TB disk, 96 TB tape = 4.4 M J/psi eq. core-hrs.