# QCD + QED studies using Twist-Averaging

Christoph Lehner (BNL)

April 30, 2016 – USQCD AHM

# QCD + QED studies using Twist-Averaging

Xu Feng, Taku Izubuchi, Luchang Jin, Christoph Lehner (PI)

Project homepage:
`http://quark.phy.bnl.gov/~clehner/usqcd/ta/`

Collaborators: Tom Blum, Norman Christ, Chulwoo Jung, Amarjit Soni

(RBC collaboration)

**Abstract**

We propose a continuation of last year's QCD+QED proposal using twist-averaging. While last year's proposal focused on optimizing the strategy to control finite-volume effects in QCD+QED simulations, this year's proposal addresses the computation of the QED pion mass splitting $\Delta m_\pi$, QED corrections to $f_\pi$, as well as QED corrections to the hadronic vacuum polarization contribution to $(g-2)_\mu$ at physical pion mass. A minimal set of carefully chosen propagators will allow for the simultaneous computation of all quantities. **We request 33.2 Mio Jpsi-equivalent core-hours on the Fermilab cluster pi0.** We are also asking for 76 TB temporary disk space that will be shared with the HVP proposal by RBC/UKQCD or 3 Mio Jpsi-equivalent core-hours.

Motivation:

As lattice QCD calculations reach percent and sub-percent precision, the computation of $\mathcal{O}\left((m_u - m_d)/\Lambda_{\mathrm{QCD}}\right)$ and $\mathcal{O}(\alpha_{\mathrm{QED}})$ corrections is essential to match current and future precision requirements.

This proposal:

Compute at physical pion mass

- QED mass-corrections ($\Delta m_\pi$),
- QED corrections to the $(g-2)_\mu$ HVP contribution, and
- charged meson decays in QCD+QED

within a uniform framework that is optimized for statistical uncertainties, re-use of costly propagators, and the control of finite-volume errors.

## QED corrections to the $(g-2)_\mu$ HVP contribution:

| Contribution | Central Value $\times 10^{10}$ | Uncertainty $\times 10^{10}$ |
|---|---|---|
| QED | 11 658 471.895 | 0.008 |
| EW | 15.4 | 0.1 |
| **HVP** (Leading-order) | 692.3 | **4.2** |
| HVP (Higher-order) | -9.84 | 0.06 |
| **HLbL** | 10.5 | **2.6** |
| Total SM prediction | 11 659 180.3 | 4.9 |
| BNL E821 result | 11 659 209.1 | 6.3 |
| Fermilab E989 target | | $\approx$ **1.6** |

FNAL E989 may have early results around 2018

Lattice status:

**HLbL** Our progress: PRL114 (2015) 012001, PRD93 (2016) 014503, about 10% stat.err., $a \to 0$ and disc. in progress, $V \to \infty$ to be done

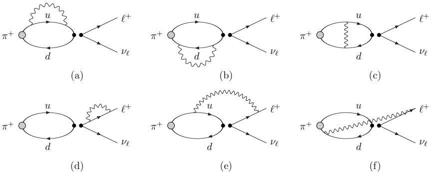
**HVP connected** at $\delta a_\mu^{\mathrm{HVP,C}} \approx 7 \times 10^{-10}$ in 2016/2017 (HPQCD arXiv:1601.03071 and our preliminary results, see proposal Blum)

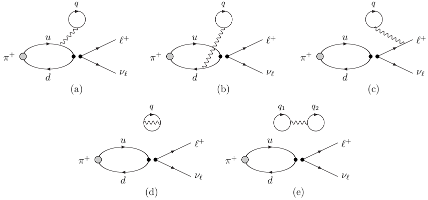
**HVP disc.** at $\delta a_\mu^{\mathrm{HVP,D}} \approx 4 \times 10^{-10}$ in 2015 (our result in arXiv:1512.09054), methodology allows for factor 4 improvement

**HVP QED corrections** not yet computed, estimated at $\delta a_\mu^{\mathrm{HVP,E/M}} = \mathcal{O}(10 \times 10^{-10})$; target in this proposal, **timely calculation needed**

# Commonalities of target observables:

Reminder – diagrams for charged meson decay:



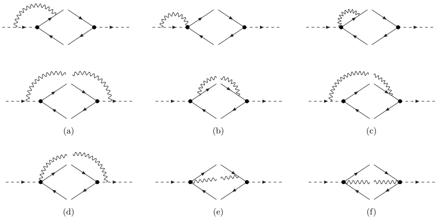




Diagrams for pion mass-splitting and QED corrections to the $a_\mu$ HVP contribution are essentially a subset of the above diagrams with different spinor structure at the operators.

This proposal: Sample random source positions for point source propagators and let the source be two opposing vertices in the connected QCD four-point functions. Same propagators can also be used for all three-point, two-point, and disconnected contributions.

## Differences in infrared:

Charged meson decays have IR divergences that need to be regulated and canceled against soft photon emission:




Divergences can be computed in EFT Carrasco et al. 2015

The $1/L$ and $1/L^2$ corrections for mass shift are universal and can be subtracted exactly; the remaining error in $\text{QED}_L$ regulator ($\vec{k}=0$ subtraction) is O(1.5%) for our setup.

For QED corrections to $a_\mu$ HVP we have no analytic knowledge of $1/L^n$ corrections at this point.

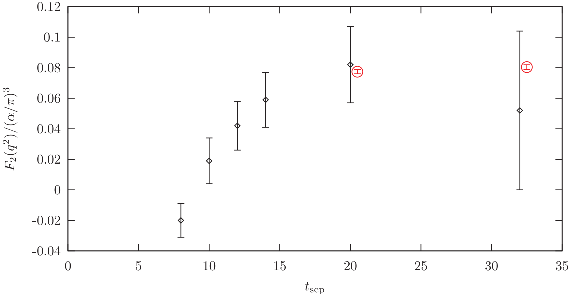
This proposal: factor QCD and QED in a way that we can trivially change the QED infrared regulator offline. Use also improved twist-averaged $\text{QED}_\infty$ regulator to suppress finite-volume errors (C.L. lattice 2015).

Propagator strategy: We randomly sample $M$ four-dimensional positions of point-source propagators on $N$ configurations. We use local vector currents to couple the photons. The propagator source positions serve as vertex/operator positions. The use of local currents modifies but does not complicate our renormalization procedure.

In this random sampling strategy we can

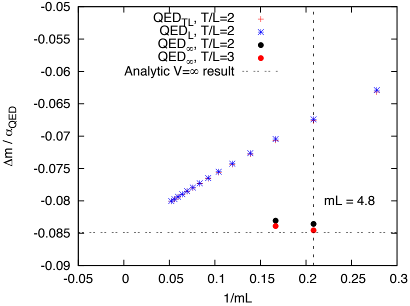
- optimize the probability distribution from which we draw to reduce statistical noise,
- combine different pairs of point-source propagators to effectively increase our statistical sample from $O(M)$ to $O(M^2)$ samples of the diagram, and
- expose the quark-photon vertex positions explicitly such that we can use different QED infrared regulators offline (such as by re-combining with different photon propagators; for multiple quark loops an additional subtraction step is needed). This gives additional insight into, e.g., errors associated with the finite simulation volume.

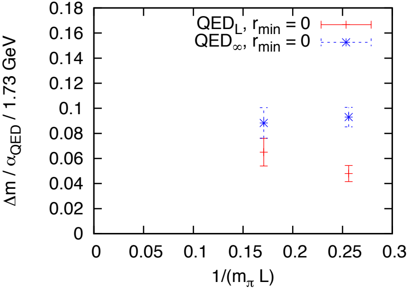
The importance sampling and "$M^2$" strategy was recently successfully used in our HLbL calculation (L. Jin et al. 2015) to reduce the statistical noise for the HLbL at same cost by more than an order of magnitude:



The improved QED$_\infty$ regulator can be used to reduce finite-volume errors in general QCD+QED simulations in the same way that the 2003 PRL by Blum did for the QCD HVP contribution.

Numerical tests confirm this for the QED mass shift both in scalar QED (left) and the mass-shift at heavier pion mass in QCD (right):




A more detailed discussion is given in my 2015 lattice talk that is linked in the project homepage.

## Ensembles and cost

We plan to compute on 30 configurations of our $48^3$ ($a^{-1} = 1.73$ GeV) and $64^3$ ($a^{-1} = 2.36$ GeV) Mobius DWF ensembles at physical pion mass. We use an AMA/zMobius strategy to reduce our computational cost significantly. Our request for CPU hours on pi0 (need the 128 GB/node memory) is:

| | |
|---|---|
| 12 sloppy $48^3$ solves on 192 pi0 nodes | 440 seconds |
| 12 exact $48^3$ solves on 192 pi0 nodes | 2200 seconds |
| Number of configurations | 30 |
| Number of sloppy solves per configuration | $150 \times 12$ |
| Number of exact solves per configuration | $15 \times 12$ |
| Cost of $64^3$ run divided by cost of $48^3$ run | 3.16 |
| Total computational cost on $48^3$ for sloppy solves in Mio Jpsi-core hours | 5.3 |
| Total computational cost on $48^3$ for exact solves in Mio Jpsi-core hours | 2.7 |
| Total computational cost on $64^3$ for sloppy solves in Mio Jpsi-core hours | 16.7 |
| Total computational cost on $64^3$ for exact solves in Mio Jpsi-core hours | 8.5 |
| Total request | 33.2 Mio Jpsi-core hours |

We also request 76 TB temporary disk space to buffer eigenvector data and 252 TB tape storage to save the propagators for future re-use (cost of generating the sloppy propagators on pi0 is about a factor of 80 more than reading them from disk).

We are able to leverage our HLbL calculation at Argonne (ALCC) that generated zMobius/Mobius eigenvectors for the $48^3$ and $64^3$ lattices. The production setup of transferring these eigenvectors from Argonne to FNAL is well-tested in the current allocation and produced our HVP disconnected results and the HVP connected preliminary results shown by Blum. We are ready to start this calculation on July 1st!

# SPC questions and answers

**1) What is your expected precision for the QED corrections for the three types of quantities you plan to calculate?**
We expect a total uncertainty of O(10-20%) for all three quantities.

**2) In your proposal you state that you were still testing your perturbative QED code. What is the current status of these tests, and do you still expect that your code will be ready by the start of the allocation year?**
We expect all tests to be completed before July 1st, however, even if unforeseen delays were to occur, we are ready to start running on July 1st: Our solve time dominates propagator IO time by more than a factor of 50 such that starting with (the well-tested) propagator generation on July 1st and defering parts of the contractions until later in the year would not introduce a noticeable overhead.

**3) To clarify your plans for storing the propagators, since you are requesting only temporary disk space in your proposal, are you planning to archive the propagators using non-USQCD resources?**
The absence of tape storage request in our submitted proposal document was an unfortunate oversight. We need 252 Tbyte of tape storage for permanent storage of propagators.

Thank you