Continuum limit of hadronic vacuum polarization contributions for (g-2)$_\mu$ and inclusive τ decay analysis on the physical Möbius-DWF ensembles

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Outline I

Experiment - Standard Model Theory = difference

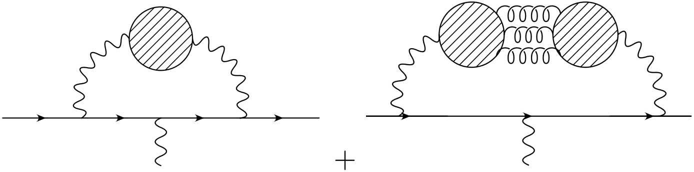
SM Contribution	Value $\pm$ Error ($\times 10^{11}$)	Ref
QED (5 loops)	116584718.951 ± 0.080	[Aoyama et al., 2012]
HVP LO	6923 ± 42	[Davier et al., 2011]
	6949 ± 43	[Hagiwara et al., 2011]
HVP NLO	-98.4 ± 0.7	[Hagiwara et al., 2011]
		[Kurz et al., 2014]
HVP NNLO	12.4 ± 0.1	[Kurz et al., 2014]
HLbL	105 ± 26	[Prades et al., 2009]
HLbL (NLO)	3 ± 2	[Colangelo et al., 2014]
Weak (2 loops)	153.6 ± 1.0	[Gnendiger et al., 2013]
SM Tot (0.42 ppm)	116591802 ± 49	[Davier et al., 2011]
(0.43 ppm)	116591828 ± 50	[Hagiwara et al., 2011]
(0.51 ppm)	116591840 ± 59	[Aoyama et al., 2012]
Exp (0.54 ppm)	116592080 ± 63	[Bennett et al., 2006]
Diff (Exp − SM)	287 ± 80	[Davier et al., 2011]
	261 ± 78	[Hagiwara et al., 2011]
	249 ± 87	[Aoyama et al., 2012]

New experiments+new theory=new physics

- Fermilab E989 early 2017, aims for 0.14 ppm
- J-PARC E34 late 2010's-2020, aims for 0.3-0.4 ppm
- Today $a_\mu(\mathrm{Expt})$-$a_\mu(\mathrm{SM}) \approx 2.9 - 3.6\sigma$
- If both central values stay the same,
  - E989 ($\sim 4\times$ smaller error) $\rightarrow \sim 5\sigma$
  - E989+new HLBL theory (models+lattice, 10%) $\rightarrow \sim 6\sigma$
  - E989+new HLBL +new HVP (50% reduction) $\rightarrow \sim 8\sigma$
- Good for discriminating models if discovery of BSM at LHC

  [Stckinger, 2013]
- Lattice calculations important to trust theory errors

Outline I



The blobs (quark loops), which represent all possible intermediate hadronic states (ρ, $\pi\pi$, ...) are not calculable in perturbation theory, but can be calculated from

- dispersion relation + experimental cross-section for $e^+e^- \to$ hadrons
- first principles using lattice QCD

Dispersive method [Bouchiat and Michel (1961); Durand (1962); ...]

The vacuum polarization (blob) is an *analytic* function.

$$\Pi(q^2) = \frac{1}{\pi}\int_0^\infty \mathrm{d}s \frac{\Im\Pi(s)}{(s-q^2)}$$

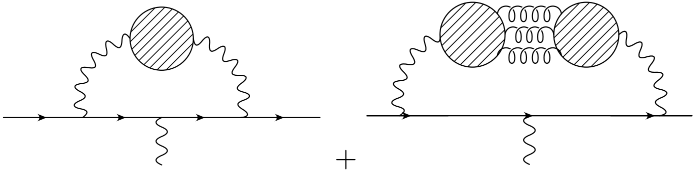
$$\sigma_{\rm total}(e^+e^- \to \mathrm{hadrons}) = \frac{4\pi^2\alpha}{s}\frac{1}{\pi}\Im\Pi(s)$$

(by the optical theorem) which leads to

$$a_\mu(\mathrm{HVP}) = \frac{1}{4\pi^2}\int_{4m_\pi^2}^\infty \mathrm{d}s\, K(s)\sigma_{\rm total}(s)$$

- $a_\mu(\mathrm{HVP}) \sim 693(4)$ (0.6% error, but largest contribution to SM value)
- $\sigma_{\rm total}(S)$ also from $\tau \to \pi^\pm\pi^0\nu$ (needs isospin correction)

Lattice QCD method [Blum, 2003, Lautrup et al., 1971]



Using lattice QCD and continuum, ∞-volume pQED

$$a_\mu(\mathrm{HVP}) = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2\, f(q^2)\, \hat{\Pi}(q^2)$$

$f(q^2)$ is known, $\hat{\Pi}(q^2)$ is subtracted HVP, $\hat{\Pi}(q^2) = \Pi(q^2) - \Pi(0)$, computed directly on the lattice

$$\begin{aligned} \Pi^{\mu\nu}(q) &= \int e^{iqx} \langle j^\mu(x) j^\nu(0) \rangle \qquad j^\mu(x) = \sum_i Q_i \bar{\psi}(x) \gamma^\mu \psi(x) \\ &= \Pi(q^2)(q^\mu q^\nu - q^2 \delta^{\mu\nu}) \end{aligned}$$

Simulation details

Gauge field ensembles generated by RBC/UKQCD collaborations

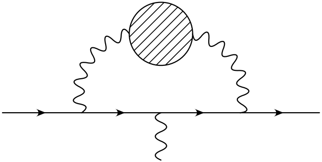
Möbius Domain wall fermions: chiral symmetry at finite a

Iwasaki Gauge action (gluons)

- Range of pion (quark) masses $m_\pi = 140, 170, 330, 420$ MeV
- Range of lattice spacings, $a = 0.144, 0.114, 0.086$ fm
- Range of lattice sizes, $L/a = 16, 24, 32, 48, 64$
- Range of lattice volumes, $(1.8)^3$, $(2.7)^3$, $(4.6)^3$, $(5.5)^3$ fm^3

Use all-mode-averaging technique [Izubuchi et al., 2013]

Quark Connected Contribution to HVP

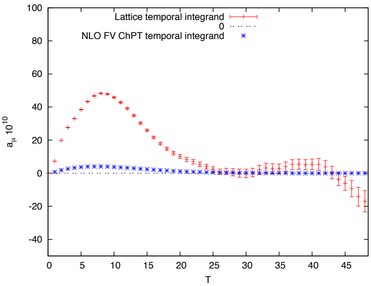


- Two orders of magnitude larger than disconnected
- Relatively harder: need (sub) percent accuracy
- Current calculations, $\gtrsim 2\%$ error [Chakraborty et al., 2016]
- Finite volume effects significant barrier [Aubin et al., 2015]
- lots of activity by many groups
- RBC/UKQCD on-going calculation at the physical point. Sub-1% stat errors appear feasible. a, FV, QED/isospin breaking effects

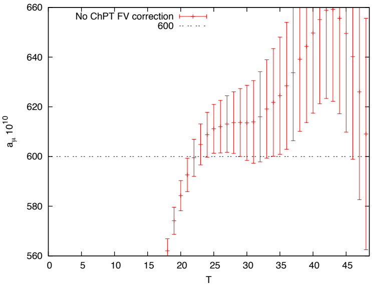
48^3 physical point Möbius-DWF ensemble, 64 configurations (separated by 40 trajectories)

$$\Pi(q^2) - \Pi(0) = \sum_t \left(\frac{\cos(qt) - 1}{q^2} + \frac{1}{2} t^2 \right) \frac{\sum_i C_{ii}(t, \vec{0})}{3}$$

a_μ integrand



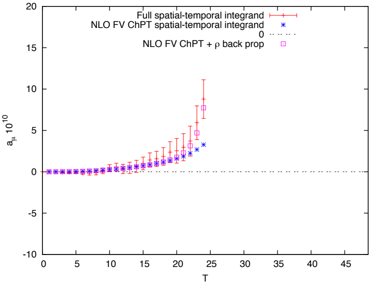
$a_\mu = \sum_t^T w(t) C(t)$



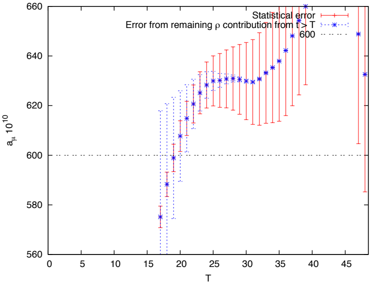
48^3 physical point Möbius-DWF ensemble, 64 configurations (separated by 40 trajectories)

FV effects from 2 π state (talk of C. Aubin, [Aubin et al., 2015])

$a_\mu(A_{144}) - a_\mu(A_1)$



$a_\mu(T) - +$ 2 pion contrib.



Cumulative sum, remaining rho contribution $(f_\rho^2 m_\rho/2)e^{-m_\rho t}$ (*)

Request for 2016-2017 AY

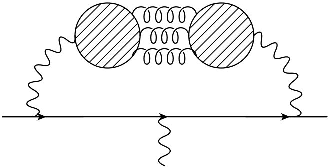
To compute on 64^3, physical point, Möbius-DWF ensemble for $a \to 0$ limit

Table: Resource request. Timings (per configuration) for the 64^3 calculation estimated from the 48^3 HVP calculation on Pi0. "exact solves" includes 1 deflated light quark propagator and 10 strange quark propagators.

quantity	core-hours
eigenvectors	310.7 K
sloppy solves	132.7 K
exact solves	30.0K
LMA	116.5 K
I/O	7.1 K
total	597.0 K

50 configs $\to$ 64.5 M JPsi core-hrs (30 sets of evecs from ALCC)

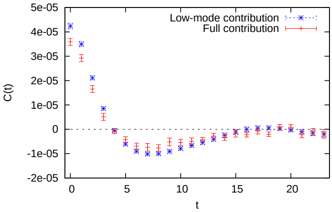
Disconnected HVP contribution to g-2

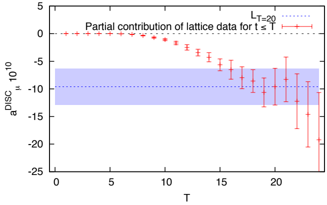


- quark-disconnected diagrams notoriously diffcult
- Expected to be small (vanishes in SU(3) limit)
- Still important to reach (sub-) percent precision
- Physical pion mass Möbius-DWF ensemble RBC/UKQCD
- use all-to-all quark propagator strategy [Foley et al., 2005], separate low and high modes of the Dirac operator (quark propagator). Treat the low modes exactly, high stochastically
- Until our recent calculation statistically unresolved
- (degenerate)light - strange difference computed directly (Mainz Group [Gulpers et al., 2014])

- Low mode separation crucial since light- strange don't cancel
- contributions above m_s suppressed
- (sparse) random sources effective for high modes

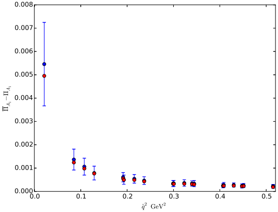
$$\Pi(q^2) - \Pi(0) = \sum_t \left(\frac{\cos(qt) - 1}{q^2} + \frac{1}{2} t^2 \right) C(t)$$

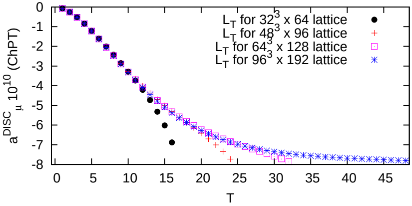




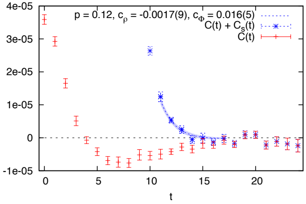
- $-(9.6 \pm 3.3) \times 10^{-10}$ or about 1.5% of total at 3 σ level

- non-zero lattice spacing: proxy strange-connected 5%
- FV, ChiPT [Aubin et al., 2015, Della Morte and Juttner, 2010]: 14.6%





- missing long distance piece 17.7%



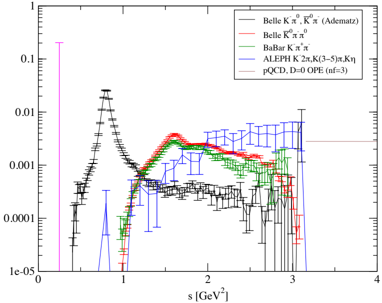
$-(9.6 \pm 3.3 \pm 2.3) \times 10^{-10}$

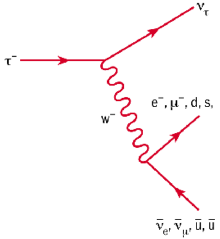
0.6 % accuracy on total HVP!

Outline I

Dispersion relation and optical theorem relate τ decay rate to the (H)VP

$\rho_{exp}(s)$



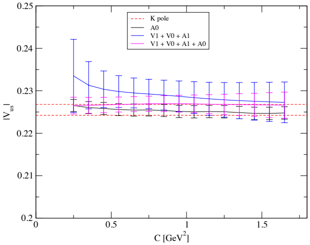


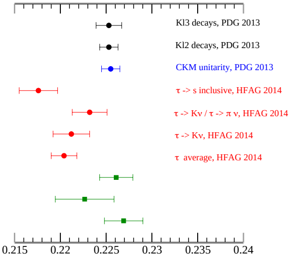
$$\sum_{k=1}^{N} \mathrm{Res}(\omega(-Q_k^2))\Pi_{\mathrm{lat}}(-Q_k^2) \quad = \quad |V_{us}|^{-2}\rho_{\mathrm{exp}} + \int_{m_\tau^2}^{\infty} ds\, \omega(s)\Pi^{\mathrm{pQCD}}(s)$$

$$(\mathrm{arbitrary}) \qquad \omega(s) \quad = \quad \frac{1}{(s+Q_1^2)(s+Q_2^2)\cdots(s+Q_N^2)}$$

Preliminary results (T. Izubuchi and H. Ohki)

$N = 4$ poles centered at C (GeV^2), spaced by 0.1 GeV^2





$N = 3\,,\ C = 0.3$

$N = 3\,,\ C = 1.00$

$N = 4\,,\ C = 1.05$

V_{us} puzzle: inclusive / exclusive differ by about $3\ \sigma$

Outline I

Questions from the SPC

1. What are the expected uncertainties in V_{us} from the tau decay analysis?

   Ans: current error is roughly 0.9%,
   expect to get $\sim$ 0.5% total error

2. Can any estimate be made of the disconnected contributions that are not part of your calculations?

   Ans: All disconnected contributions accounted for in [Blum et al., 2015], no disc. contributions in V_{us} analysis

3. Are you ready to use the new JLab resource? Ans: yes

Summary

- The muon anomalous magnetic moment provides a stringent test of the SM: ~ 3 standard deviation difference at the level of 0.5 ppm
- Physical mass, large box, ensembles + improved algorithms powerful
- Lattice QCD calculations will reduce and solidify current theory errors in time for
- Upcoming E989 measurement at Fermilab (goal 0.14 ppm)
- New analysis of V_{us} with non-PT lattice input to reduce pQCD errors, solve V_{us} puzzle

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Outline I

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