# Non-Gaussian cumulants of conserved charge fluctuations

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BROOKHAVEN NATIONAL LABORATORY

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USQCD All Hands Meeting 2016.

# Outline

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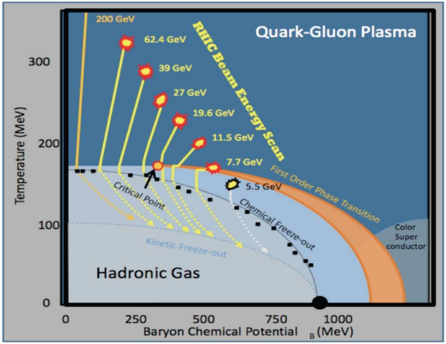
# QCD Phase Diagram: Status

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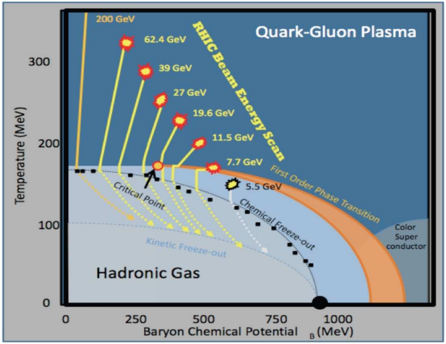


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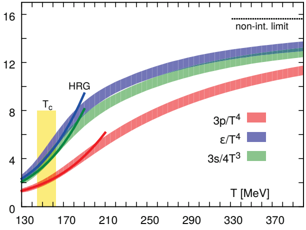
- Themes: Existence of critical end-point, hydrodynamic modeling of the QCD medium formed in the experiments to understand experimental results.

# Goals of our project

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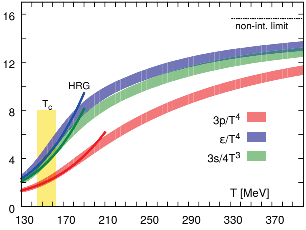


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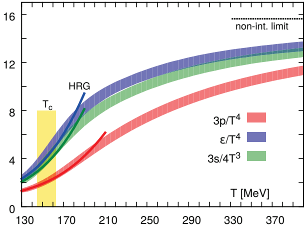


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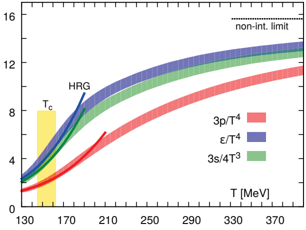


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- Can we understand the critical behaviour due to the light quarks in the crossover region?

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- $\chi_6^B$ can already constrain QCD pressure in the regime approximated by Hadron Resonance gas model.

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- Extending to higher orders?
  1. Matrix inversions increasing with the order
  2. Delicate cancellation between a large number of terms for higher order QNS.

# Outline

# A new method to introduce $\mu$

- The staggered fermion matrix used at finite $\mu$ [Hasenfratz, Karsch ,83]

$$
\begin{aligned}
D(\mu)_{xy} &= \sum_{i=1}^{3} \eta_i(x) \left[ U_i^\dagger(y)\delta_{x,y+\hat{i}} - U_i(x)\delta_{x,y-\hat{i}} \right] \\
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- One can also add $\mu$ coupled to the conserved number density as in the continuum.

$$D(0)_{xy} - \frac{\mu a}{2}\eta_4(x)\left[ U_4^\dagger(y)\delta_{x,y+\hat{4}} + U_4(x)\delta_{x,y-\hat{4}} \right] .$$

# Pros and Cons

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  $$D'' = D''' = D''''... = 0$$
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- In Exp method: counter terms already at the Lagrangian level. We use this method for $\chi_n^B$, $n = 2, 4$.

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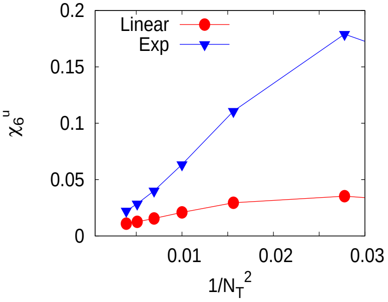
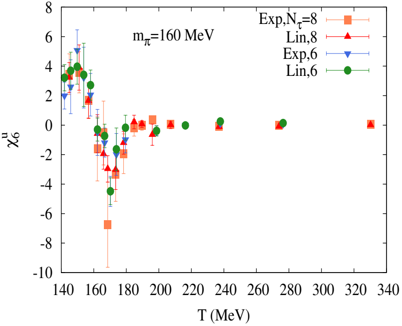
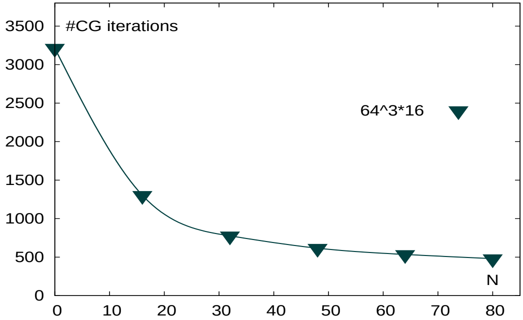


# Speeding up the inversions with deflation

- Calculating explicitly the lowest eigenvalues improves performance of the fermion inverter
  $D^{-1}|R\rangle = \sum_{i=1}^{N} 1/\lambda_i|\psi_i\rangle\langle\psi_i|R\rangle$+CG Inversion.
- We have developed highly optimized codes based on Ritz and Lanczos algorithms for CPU's and GPU respectively.
- Current volumes $N_s = 4N_\tau$, already approaches a plateau for $N = 80$ for $T \sim 145$ MeV. Typical $N = 192 - 256$.



# Performance of our codes

- We group random vectors for a single gauge configuration $\rightarrow$ use of Multiple right hand sides for Conjugate Gradient Inversion increases arithmetic intensity. [O. Kaczmarek, C. Schmidt, P. Steinbrecher, M. Wagner, 14]

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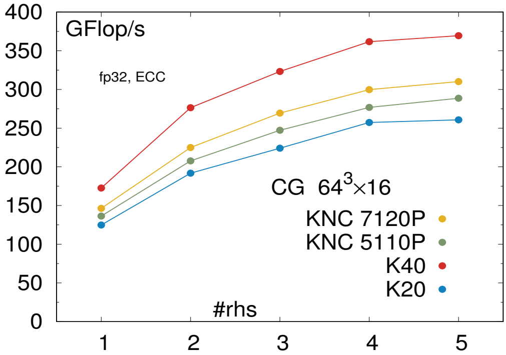
- Currently highly optimized codes for Intel Knights Corner extended to Knights Landing. [Mainly led by Patrick Steinbrecher, graduate student since 15]



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# New data analyzed 2015-16

| | $32^3 \times 8$ | | $24^3 \times 6$ | |
|---|---|---|---|---|
| $T$ [MeV] | $\beta$ | # analyzed | $\beta$ | # analyzed |
| 135 | 6.245 | 104420 | 5.980 | 68000 |
| 140 | 6.285 | 104480 | 6.015 | 120790 |
| 145 | 6.315 | 107480 | 6.045 | 120770 |
| 150 | 6.354 | 108030 | 6.080 | 30080 |
| 155 | 6.390 | 108580 | 6.120 | 23546 |
| 160 | 6.423 | 119290 | 6.150 | 31164 |
| 165 | 6.445 | 122340 | 6.170 | 20000 |
| 170 | 6.474 | 141780 | 6.200 | 138470 |
| 175 | 6.500 | 142960 | 6.225 | 125280 |

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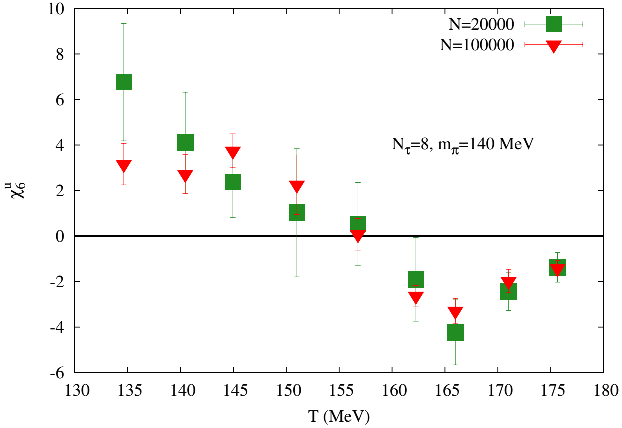
- Results with physical quark mass
- Deflation + Multiple Right hand side technique+ special care of noisy operators $\rightarrow$ a speedup of 30 allowed for analysis of extensive set of configurations.

# Main Outcome : Sixth order cumulants

- $\chi_6^u$ signal improved with the statistics $N$. We observe the negative dip just above $T_c$ $\rightarrow$ signal of $O(4)$ criticality?



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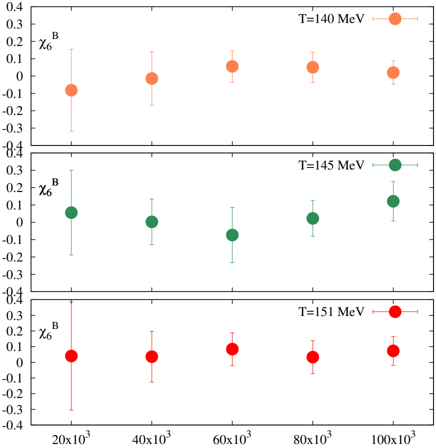
- Improvement visible already in $\chi_6^B$ at the lowest temperatures by increasing number of configurations,
- We aim to increase statistics needed to reduce errors by a factor two.



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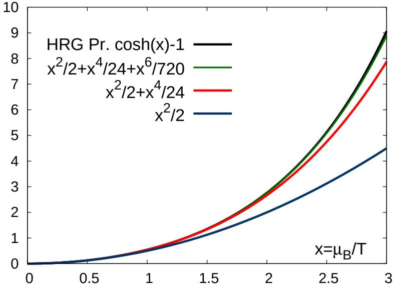
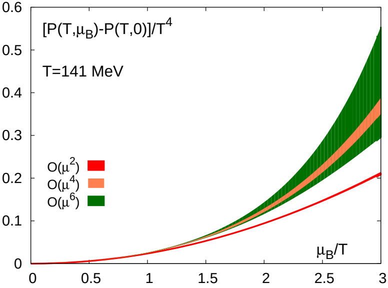
- Improve errors on our current data to observe this
  $\rightarrow$ increase statistics twofold this year.



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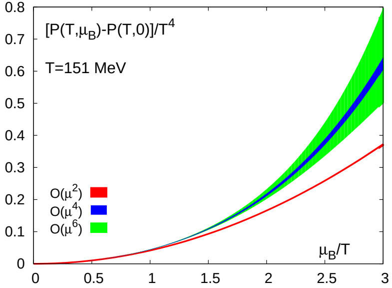
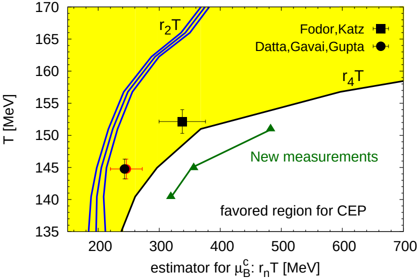
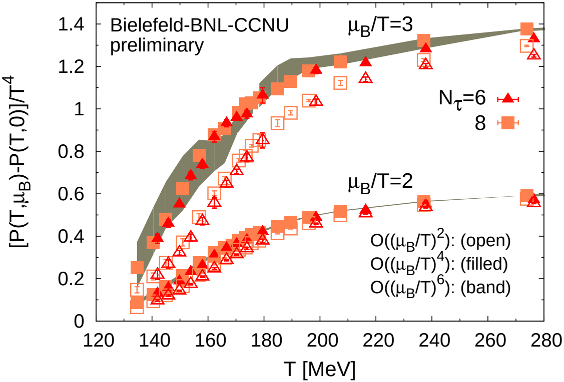
- Near critical point all terms in the Taylor expansion nearly equal $\rightarrow$ need to improve the errors!
- Our data gives a preliminary bound on the location of critical point from radius of convergence estimates, $r_{2n} \equiv \sqrt{2n(2n-1)\left|\frac{\chi_{2n}^B}{\chi_{2n+2}^B}\right|}$.

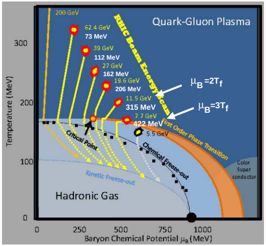


# EoS away from criticality

- The pressure for $T > 160$ MeV which is an important input for the hydrodynamic modeling of the plasma already constrained by $\chi_B^6$ even for highest $\mu_B/T$.



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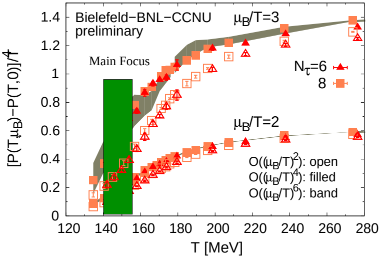


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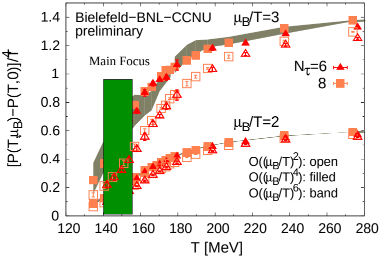


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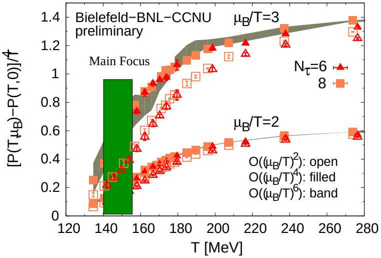


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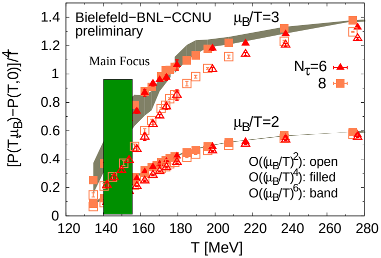


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- Higher order cumulants will also help in bracketing the possible QCD critical end-point which is one of the focus of BES-II experiments.