Investigation of $B \to K\pi\ell^+\ell^-$ decays with lattice QCD

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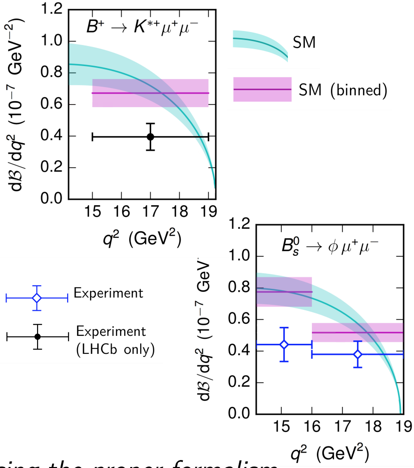
Why ...

- Flavor Changing Neutral Currents

$$b \to s\ \ell^+\ell^-$$

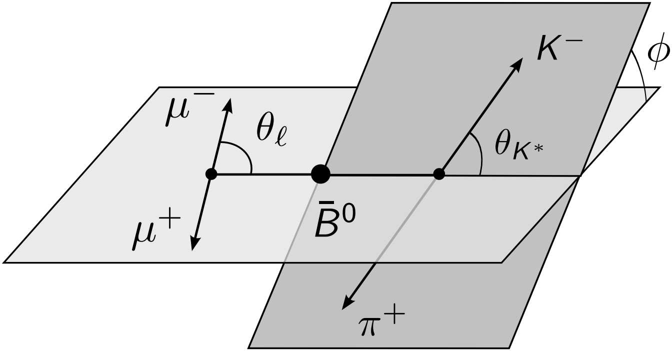
probe for Physics Beyond the SM
- discrepancy between experiment and theory at high q^2
- sign of New Physics?
- effect of $K^*(892)$ strong decay?

[Meinel et al. PRL 2014]



study Hadronic effects in $b \to s\ \ell^+\ell^-$ using the proper formalism

[Briceño et al. PRD 2015, Lellouch & Lüscher CMP 2001]


K^-
ϕ
μ^-
θ_ℓ
θ_{K^*}
μ^+
$\bar{B}^0$
π^+

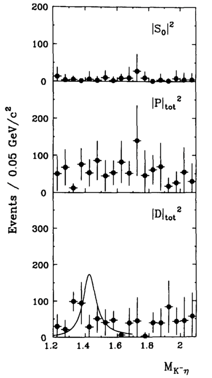
NEW:

treat $K^*(892)$ as a strong resonance

$B \to K\pi\ell^+\ell^-$ vs. $B_s \to K\bar{K}\ell^+\ell^-$

$B \to K\pi\ell^+\ell^-$:

- $K^*(892)$ dominant, $\Gamma = 50$ MeV
- $K\pi$ elastic up to $K\eta$ threshold
- $K\eta$ couples weakly to $K^*(J^P = 1^-)$



No structure in p-wave

[Aston et al. PLB 1988]

Caution: [Wilson et al. PRD 2015]

$B_s \to K\bar{K}\ell^+\ell^-$:

- $\phi(1020)$ dominant resonance
- coupling to $\pi\pi\pi$ channel

$\phi(1020)$ MASS	1019.461 ± 0.019 MeV
$\phi(1020)$ WIDTH	4.266 ± 0.031 MeV

	Mode	Fraction (Γ_i / Γ)
Γ_1	K^+K^-	$(48.9 \pm 0.5)\%$
Γ_2	$K_L^0\ K_S^0$	$(34.2 \pm 0.4)\%$
Γ_3	$\rho\pi + \pi^+\pi^-\pi^0$	$(15.32 \pm 0.32)\%$

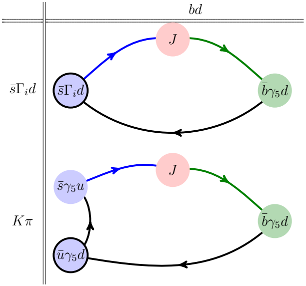
- two coupled channels:
  1. $K\bar{K}$
  2. $\pi\pi\pi$
- Lellouch-Lüscher formalism only for two body final states

[Briceño et al. PRD 2015, Lellouch & Lüscher CMP 2001]

… A brief description …

$$\langle K\pi(\vec{p}_{K\pi})|J_{weak}(\vec{q})|B(\vec{p_B})\rangle$$
$$\vec{q} = \vec{p}_B - \vec{p}_{K\pi}$$

- single hadron operators K^*,B
- multi hadron operators $K\pi(\vec{p}_{K\pi})$
- J_{weak} has the form $\bar{b}\Gamma s$
  $\Gamma = \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}$



- variationally improved correlators for matrix elements
  similar methods used in HadSpec $\pi\gamma \to \pi\pi$ study [Briceño et al. PRL 2016]
- matrix element FV effects:
  [Briceño et al. PRD 2015]

$$\frac{|\langle K\pi|J_{weak}|B\rangle_{IV}|^2}{|\langle K\pi|J_{weak}|B\rangle_{FV}|^2} \propto \frac{\partial \delta_P}{\partial E_{K\pi}} + \frac{\partial \phi_P}{\partial E_{K\pi}}$$

- form factors from matrix element (p-wave $K\pi$)

$$\gamma_\mu \to V(q^2, s_{K\pi}),$$
$$\gamma_5\gamma_\mu \to A_{0,1,2}(q^2, s_{K\pi}),$$
$$\sigma_{\mu\nu} \to T_{1,2,3}(q^2, s_{K\pi})$$

$K\pi$ scattering

- isospin $I = 1/2$, $J^P = 1^-, 0^+$
- spectrum: multi hadron approach
- use rest and moving frames:

$$p_{\vec{K\pi}} = \vec{p}_K + \vec{p}_\pi$$

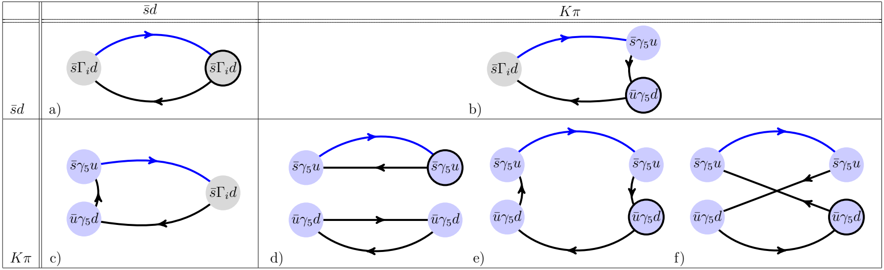
- moving frame symmetries: $J^P = 1^-, 0^+$ mix as $m_K \neq m_\pi$

- phases: Lüscher method

$$\cot\delta + \cot\phi^{\vec{p}_{K\pi}} = 0$$

$$\cot\phi^{\vec{p}_{K\pi}} \propto \sum_{lm} \alpha_{lm} \mathcal{Z}_{lm}^{\vec{p}_{K\pi}}(E_{K\pi})$$

- $J^P = 1^-$ phase: BW model
- $J^P = 0^+$ phase: [Wilson et al. PRD 2015]



... our Setup ...

Lattice

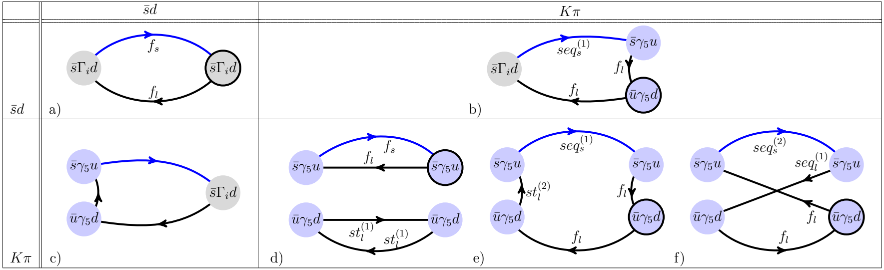
K. Orginos et al.
large scale effort with $N_f = 2+1$ Clover fermions on isotropic lattices

Label	$N_s^3 \times N_t$	$a\,(\mathrm{fm})$	L (fm)	m_π^{est} (MeV)	m_K^{est} (MeV)
C13	$32^3 \times 96$	0.114	3.65	317	530
D6	$48^3 \times 96$	0.080	3.84	190	500
D7	$64^3 \times 128$	0.080	5.12	190	500

good lattice for our project:

- non-polynomial FV effects very small: $e^{-m_\pi L} \approx 0.3\%$
- m_π low enough: K^* is unstable

- relativistic heavy quark action for c, b quarks [El-Khadra et al. PRD 1997]
- quark mass and anisotropy tuned to B_s, D_s physical mass and speed of light



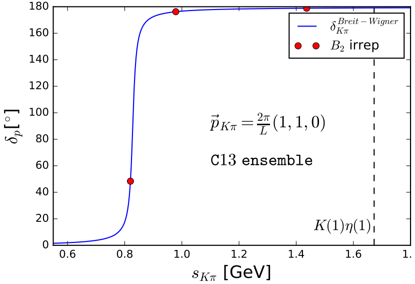
propagator type	$N_{\text{prop.}}$
f_l	1
f_s	1
$seq_l^{(1)}$	27
$st_l^{(1)}$	1
$st_l^{(2)}$	1
$seq_s^{(1)}$	27
$seq_s^{(2)}$	27

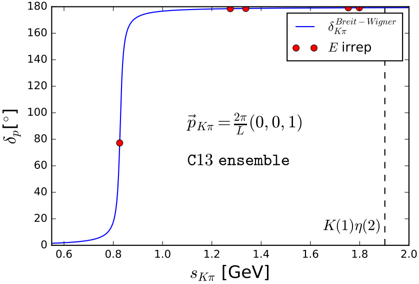
$\vec{p}_{K\pi}$	$(LG^{\vec{p}_{K\pi}})$	$\Lambda^{\vec{p}_{K\pi},r}$	spin decom.
$(0,0,0)$	(O_h)	T_1	$J=1,3,\ldots$
$(0,0,1)$	C_{4v}	E	$J=1,2,\ldots$
$(0,1,1)$	C_{2v}	B_1	$J=1,2,\ldots$
$(0,1,1)$	C_{2v}	B_2	$J=1,2,\ldots$
$(1,1,1)$	C_{3v}	E	$J=1,2,\ldots$
$(0,0,1)$	C_{4v}	A_2	$J=0,1,\ldots$
$(0,1,1)$	C_{2v}	B_3	$J=0,1,\ldots$
$(1,1,1)$	C_{3v}	A_2	$J=0,1,\ldots$

In irreps without S and P wave mixing:

- 8 (moving) frames at 4 $|\vec{p}_{K\pi}|$
- 3×3 correlation matrix per irrep
- in certain cases 4×4

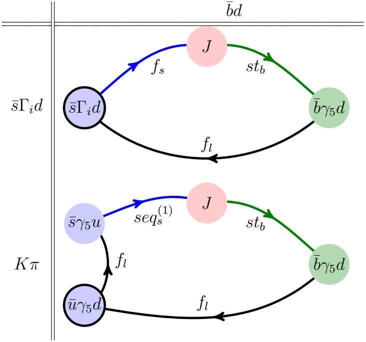
- 2-3 energies per irrep
- $K\eta$ levels far away... stay below





In irreps with S and P wave mixing:

- stay below $K\eta$ threshold (1.1 GeV)
- use knowledge on δ_P from other irreps



- additional stochastic propagator with heavy flavor
- 2 momenta for B meson

- variationally optimized 3-point correlation function
- matrix element at several $s_{K\pi}$ on single irrep

At little additional computational cost:

- change b to c
  $D \to K\pi\ell\nu$
- change b to s
  $K^*(892) \to K\gamma$

Software

QLUA (USQCD software)

- on-the-fly propagators with multigrid
- all contractions
- propagators cheaper than contractions
- low I/O
- NERSC project

We already developed the code:

- GEVP
- zeta function
- zeta function derivative
- Lüscher analysis
- 3-pt analysis

Calculation part	computational cost [J/ψ core hours]
propagators	7.5 million
2-point correlation matrices	3.2 million
3-point correlation functions	22.1 million
sum	32.8 million

storage	
2TB of tape storage	12 000
500GB of HDD	20 000

... to conclude.

Conclusion

- determine the effect of unstable hadrons in the $B \to K\pi\ell^+\ell^-$
- investigate the S and P wave mixing in $B \to K\pi\ell^+\ell^-$ final state

USQCD Intensity Frontier

... calculate the new, more computationally demanding, matrix elements that are needed for the interpretation of planned (and in some cases old) experiments ...

Thank you :)

With the new resources at JLab being as yet unspecified, we would like to know if you are in a position to use them efficiently if they are a) cpu, b) GPU, c) KNL. If you are not, that is fine, but it will help in our allocation decisions to know this information from every proposal.

The code to generate the propagators, the 2-point and 3-point correlation functions is written in QLUA, which is a scripting language based on LUA. It works as an interface to the USQCD software libraries such as QDP, QLA, QIO etc. In a current project at NERSC we are using this framework on CPUs, where it reaches good performance.

In principle the background software behind QLUA works on KNL as well, however certain parts of the code (e.g. the clover inverters) are not ported to OpenMP and thus perform less than optimally on KNL. So in conclusion, in our project we can only utilize CPU's efficiently.

Would it make sense to look at $B_s \to K\bar{K}\ell^+\ell^-$ near the ϕ resonance? You would potentially avoid the problem with the $K\eta$ state?

While the decay $B_s \to K\bar{K}l^+l^-$ probes very similar physics as $B \to K\pi\ell^+\ell^-$ its study using the multi-hadron formalism on the lattice is much more difficult than the decay we are proposing. The $\phi(1020)$ meson, that appears as a resonance in the $K\bar{K}$ final state, is a narrow resonance with the decay width of approximately 5 MeV, which can decay both to $K\bar{K}$ ($BR \approx 85\%$) as well as $\pi\rho(\to \pi\pi)$ ($BR \approx 15\%$).
So the $K\bar{K}$ scattering in the region near the $\phi(1020)$ resonance suffers from the coupling to $\pi\rho(\to \pi\pi)$ - a three particle channel. Because the ρ in this channel is quite wide it affects the entire $K\bar{K}$ invariant mass region we would be investigating in the $B_s \to K\bar{K}l^+l^-$ decay.

Would it make sense to look at $B_s \to K\bar{K}\ell^+\ell^-$ near the ϕ resonance? You would potentially avoid the problem with the $K\eta$ state?

While we would avoid the potential problem with the $K\eta$ channel by studying $B_s \to K\bar{K}l^+l^-$, the $K\bar{K}$ channel opens up problems that are significantly more challenging than the $K\eta$ channel. In our view the $B \to K\pi\ell^+\ell^-$ channel is a safer and more logical choice, as the $K\eta$ channel coupling has been demonstrated to be small by both experiment and the HadSpec Collaboration.

Finally, note that the single hadron treatment of the decay $B_s \to \phi(1020)l^+l^-$ is likely already a good approximation, given that the $\phi(1020)$ is a much narrower resonance than the $K^*(892)$.

Is it necessary to do all of the weak operator matrix elements at the present time? Are there opportunities to postpone some parts of the calculation? Would a partial calculation give you the ability to test the methodology and provide initial results in a kinematic regime where a subset of operators dominate?

The contributions of the tensor current matrix element are suppressed at high q^2 relative to the contributions of the vector and axial vector current matrix elements, but are nevertheless crucial for phenomenological applications in Flavor Physics. In the longer term we envision additional calculations at lighter pion masses, to make contact with experiment. For a 1-to-1 comparison with experiment, the tensor current matrix element must be included.

We can already address the questions concerning the effects of multi-hadron states individually for each current, but it is computationally more efficient to perform a lattice QCD calculation for all three currents at the same time. Should it become necessary to postpone some parts of the calculation due to limited resources, we could simply run on fewer configurations.

Is it necessary to do all of the weak operator matrix elements at the present time? Are there opportunities to postpone some parts of the calculation? Would a partial calculation give you the ability to test the methodology and provide initial results in a kinematic regime where a subset of operators dominate?

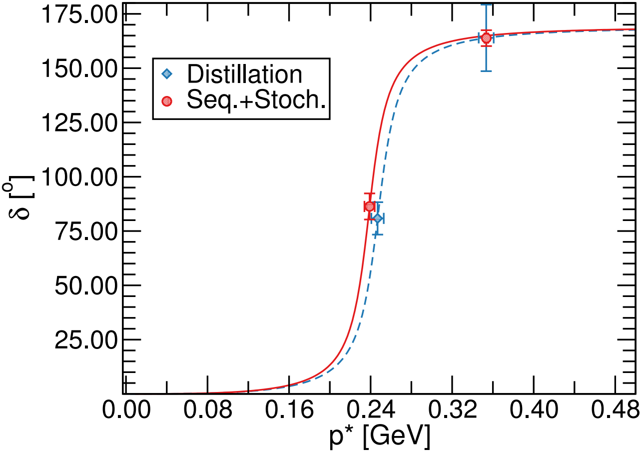
Concerning the methodology involved in the Lüscher analysis of the 2-point function and the Lellouch-Lüscher analysis of the 3-point function, we are currently involved in a project at NERSC where we are studying the decay $B \to \pi\pi\ell\nu$ where we are currently developing and tuning the methodology.

How important will be verifying the volume dependence as well as using boosted systems? The Lellouch-Lüscher analysis leaves exponential volume effects unaccounted for. At what level will these be important?

The lattice we are working on has the spatial extent $L \approx 3.65$ fm. Together with a pion mass of $m_\pi = 317$ MeV, this brings the order of the non-polynomial finite volume effects to $e^{-\frac{m_\pi L}{\hbar c}} = 0.28\%$. These effects will be much smaller than the remainder of the error budget and can be in our opinion safely neglected.

The use of boosted frames will be crucial in determining the $K\pi$ scattering phase shifts and we already consider all boosted frames that can be built from propagators with momenta $(\{-1,0,1\},\{-1,0,1\},\{-1,0,1\})$.

BACKUP I - distillation vs. our method



Comparison of the $\pi\pi$ $I = 1$ p-wave phase shift between distillation and our method, at a comparable computational cost.