Quark and Glue Structure in Nucleon

- Quark spin from anomalous Ward identity
- $\pi N \sigma$ term and strangeness
- Comparison of cost among twisted mass, clover, DWF and overlap fermions.

χ QCD Collaboration

All Hands Meeting
BNL, Apr. 29-30, 2016
### 2+1 flavor DWF configurations (RBC-UKQCD)

<table>
<thead>
<tr>
<th>Lattice Size</th>
<th>a (fm)</th>
<th>(m_\pi) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(32^3 \times 64)</td>
<td>0.137</td>
<td>330</td>
</tr>
<tr>
<td>(24^3 \times 64)</td>
<td>0.115</td>
<td>295</td>
</tr>
<tr>
<td>(32^3 \times 64)</td>
<td>0.085</td>
<td>170</td>
</tr>
<tr>
<td>(48^3 \times 96)</td>
<td>0.115</td>
<td>140</td>
</tr>
<tr>
<td>(64^3 \times 128)</td>
<td>0.085</td>
<td>140</td>
</tr>
</tbody>
</table>

\(La \approx 4.5\) fm
\(m_\pi \approx 170\) MeV

\(La \approx 2.8\) fm
\(m_\pi \approx 330\) MeV

\(La \approx 2.7\) fm
\(m_\pi \approx 295\) MeV

\(La \approx 5.5\) fm
\(m_\pi \approx 140\) MeV

\((O(a^2)\) extrapolation)
Strange quark spin $\Delta s + \Delta \bar{s}$
Quark Spin from Anomalous Ward Identity

- Calculation of the axial-vector in the DI is very noisy
- Instead, try AWI

$$\partial_\mu A^{\mu 0} = i 2 m P + \frac{i N_f}{8\pi^2} G_{\mu\nu} \tilde{G}_{\mu\nu}$$

$$\kappa_A \langle p', s \mid A_\mu \mid p, s \rangle = \lim_{q \to 0} \frac{i \mid s \mid}{q \cdot \bar{s}} \langle p', s \mid 2 \sum_{f=1}^{N_f} m_f \bar{q}_f \gamma_5 q_f + 2 i N_f q \mid p, s \rangle$$

- Overlap fermion --> mP is RGI ($Z_m Z_P = 1$)
- Overlap operator for $q(x) = -1/2 \text{Tr} \gamma_5 D_{ov}(x,x)$ is RGI.
- P is totally dominated by small eigenmodes.
- q(x) from overlap is exponentially local and captures the high modes from $A^{\mu 0}$.
- Direct check the origin of `proton spin crisis'.
Disconnected Insertion

Charm

Strange

u/d

\[2M_N \kappa_A g_A^0(q^2) + q^2 \kappa_A h_A^0(q^2)\]
\[= 2mg_P^0(q^2) + 2M_N g_G(q^2)\]
Quark Spin Decomposition

\[
\begin{align*}
\Delta s^+ &\approx 0.57(3) \\
\Delta s^- &\approx -0.08(2) \\
\Delta u^+(\text{CI}) &\approx 0.17(3) \\
\Delta c &\approx 0 \\
g_A^0 &\approx 0.17 \\
m_n = 140 \text{ MeV}
\end{align*}
\]

Check with conserved current

triangle anomaly
\[ \sigma_{\pi N} = \sigma_{sN}^0 + c_1 m_{\pi,\rho}^2 + c_2 m_{\pi,\omega}^2 + c_3 a^2 + c_4 e^{-m_{\pi} L} \]

\[ f_s^N = \frac{\sigma_{sN}}{m_N} \]
Cost comparison

\[
\frac{\text{Clover (a09m130)}}{\text{Overlap (48I)}} = \frac{(84768 + 7064 \times 3)}{(81 \times 32)} \text{(inver)} \times \frac{1}{44} \text{(time)} \times \begin{bmatrix} 0.011 \\ 0.013 \end{bmatrix}^2 \text{(var)(unitary)} = 0.66
\]

\[
\frac{\text{DWF (48I)}}{\text{Overlap (48I)}} = \frac{(640 + 20 \times 3)}{(81 \times 32)} \text{(inver)} \times \frac{1}{3} \text{(time)} \times \left(\frac{0.075}{0.013}\right)^2 \text{(var)} = 3.0
\]

\[
\frac{\text{Clover (BMW)}}{\text{Overlap (24I, 32I, 48I)}} = \frac{13000}{81 + 200 + 300} \text{(conf)}, \frac{40}{32} \text{(inver)}, \frac{1}{10(?)} \text{(time)}, \left(\frac{0.017}{0.007}\right)^2 \text{(var)}
\]

\[m_N = 961(12) \text{ MeV}, \quad m_N \text{(global) = 950.7(7.1) MeV}\]
Results:

Compare to the SNR of Twistedmass+Clover

The results of TM+C come from arXiv: 1507.04936

<table>
<thead>
<tr>
<th>Method</th>
<th>( m_{\pi} )</th>
<th>( N_{\text{cfg}} )</th>
<th>Inversions</th>
<th>Measurements</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overlap</td>
<td>133 MeV</td>
<td>81</td>
<td>5+4+8+12=29</td>
<td>0.4k(80k)</td>
<td>SSM+LMSS</td>
</tr>
<tr>
<td>TM+C</td>
<td>131 MeV</td>
<td>96</td>
<td>16*(1+3*8)=400</td>
<td>1.5k</td>
<td>Sequential</td>
</tr>
</tbody>
</table>

With the factor \( 4^3 \times 3 \) (points in the volume), the measurements would be 80k.

<table>
<thead>
<tr>
<th></th>
<th>( g_A^3 )</th>
<th>( g_s^3 )</th>
<th>( g_s^0(\text{CI}) )</th>
<th>( \langle X \rangle_{u-d} )</th>
<th>( \langle X \rangle_{u+d(\text{CI})} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>OV, 0.91 fm</td>
<td>1.133(15)</td>
<td>0.72(8)</td>
<td>6.80(15)</td>
<td>0.214(9)</td>
<td>0.519(11)</td>
</tr>
<tr>
<td>OV, 1.14 fm</td>
<td>1.150(25)</td>
<td>0.93(17)</td>
<td>7.23(33)</td>
<td>0.194(11)</td>
<td>0.456(15)</td>
</tr>
<tr>
<td>OV, 1.37 fm</td>
<td>1.233(66)</td>
<td>0.782(41)</td>
<td>7.77(70)</td>
<td>0.208(27)</td>
<td>0.400(36)</td>
</tr>
<tr>
<td>TMC, 0.9 fm</td>
<td>1.158(16)</td>
<td>0.55(18)</td>
<td>6.46(27)</td>
<td>0.248(9)</td>
<td>0.645(13)</td>
</tr>
<tr>
<td>TMC, 1.08 fm</td>
<td>1.162(30)</td>
<td>1.18(34)</td>
<td>7.84(48)</td>
<td>0.218(15)</td>
<td>0.587(18)</td>
</tr>
<tr>
<td>TMC 1.26 fm</td>
<td>1.242(57)</td>
<td>2.20(54)</td>
<td>8.93(86)</td>
<td>0.208(24)</td>
<td>0.555(63)</td>
</tr>
</tbody>
</table>
## Cost Comparison with TSM (1601.01624)

<table>
<thead>
<tr>
<th>Fermion</th>
<th>a (fm)</th>
<th>Conf.</th>
<th>Source</th>
<th>Inversion</th>
<th>time</th>
<th>LMS</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSM (CI)</td>
<td>0.093</td>
<td>96</td>
<td>16</td>
<td>16*(8*3)</td>
<td>1</td>
<td>0</td>
<td>x%</td>
</tr>
<tr>
<td>Overlap (CI)</td>
<td>0.112</td>
<td>81</td>
<td>5</td>
<td>5+4+8+12</td>
<td>~10</td>
<td>25%</td>
<td>x%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fermion</th>
<th>a (fm)</th>
<th>Conf.</th>
<th>Source</th>
<th>Loop</th>
<th>time</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>TSM (strange)</td>
<td>0.093</td>
<td>1800</td>
<td>100</td>
<td>300</td>
<td>1</td>
<td>12%</td>
</tr>
<tr>
<td>Overlap (strange)</td>
<td>0.112</td>
<td>81</td>
<td>32</td>
<td>32 (or 4)</td>
<td>~10</td>
<td>40%/17% (global)</td>
</tr>
</tbody>
</table>

\[ \frac{\text{TM (48)}}{\text{Overlap (48I)}} = \frac{96}{81} \times \frac{384}{29} \times \frac{1}{10} \times \frac{1}{1.25} \times \frac{x}{x} = 1.3 \]

\[ \frac{\text{TSM (48)}}{\text{Overlap (48I)}} = \frac{1800}{81} \times \frac{100 + 300}{32 + 32} \times \frac{1}{10} \times \left\{ \frac{0.12}{0.40} \right\}^2 \text{(var)(unitary)} = 1.3 \]

\[ \frac{\text{TSM (48)}}{\text{Overlap (48I)}} = \frac{1800}{81} \times \frac{100 + 300}{32 + 32} \times \frac{1}{10} \times \left\{ \frac{0.12}{0.17} \right\}^2 \text{(var)(global)} = 6.9 \]
## Cost Comparison with Clover for $g_s^3$ (1602.07737)

<table>
<thead>
<tr>
<th>Fermion</th>
<th>a (fm)</th>
<th>Conf.</th>
<th>Source</th>
<th>Sink</th>
<th>tsep</th>
<th>time</th>
<th>LMS</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clover (CI)</td>
<td>0.081</td>
<td>400</td>
<td>100</td>
<td>4</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>10%</td>
</tr>
<tr>
<td>Overlap (CI)</td>
<td>0.083</td>
<td>300</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>~44</td>
<td>8%</td>
<td>13%</td>
</tr>
</tbody>
</table>

## Cost Comparison with DWF for $g_A$ (C14-08-11.3)

<table>
<thead>
<tr>
<th>Fermion</th>
<th>a (fm)</th>
<th>Conf.</th>
<th>Source</th>
<th>Sink</th>
<th>tsep</th>
<th>time</th>
<th>LMS</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>DWF (CI)</td>
<td>0.114</td>
<td>20</td>
<td>32+1</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>9.7%</td>
</tr>
<tr>
<td>Overlap (CI)</td>
<td>0.114</td>
<td>81</td>
<td>5</td>
<td>4/8/12</td>
<td>3</td>
<td>~3</td>
<td>25%</td>
<td>2.2%</td>
</tr>
</tbody>
</table>

\[
\frac{DWF (48I)}{Overlap (48I)} = \frac{20}{81} \times \frac{(32 + 1 \times 3) \text{ (source)} \times (2 \times 3) \text{ (sink-tsep)}}{5 \text{ (source)} + (4+8+12) \text{ (sink-tsep)}} \times \frac{1}{3} \text{ (time)} \times \frac{1}{1.25} \text{ (LMS)} \times \frac{0.097}{0.022}^2 \text{(var)} = 9.0
\]
2+1 flavor DWF configurations (RBC-UKQCD)

La ~ 4.5 fm
\(m_\pi \sim 170\) MeV

\(32^3 \times 64, a = 0.137\) fm

La ~ 2.8 fm
\(m_\pi \sim 330\) MeV

\(24^3 \times 64, a = 0.115\) fm

La ~ 2.7 fm
\(m_\pi \sim 295\) MeV

\(32^3 \times 64, a = 0.085\) fm

La ~ 5.5 fm
\(m_\pi \sim 140\) MeV

\(48^3 \times 96, a = 0.115\) fm

\(64^3 \times 128, a = 0.085\) fm

\((O(a^2)\) extrapolation)
Why not use DWF?

◆ DWF needs 2 $L_s$ time of storage and memory.
  
  For 1000 pairs of eigenmodes:
  
  1.4 TB (overlap)/ conf $\rightarrow$ 43 TB (DWF)/conf
  
  110 TB (overlap) $\rightarrow$ 3.5 PB (DWF) total

◆ Multi-mass + deflation possible for overlap, because eigenvectors are mass independent. Helps global fit.

◆ For the existing DWF configurations: limited reach for heavy quark, oscillatory behavior at short time limits excited state study and possibly quark loop studies.
$g_A(24I) = 1.153(6) \rightarrow g_A(\text{imp}) = 1.188(7)$

$g_A(32I) = 1.156(7) \rightarrow g_A(\text{imp}) = 1.177(9)$

$g_A(48I) (134 \text{ MeV}) = 1.17(6)$

$g_A(48I) (149 \text{ MeV}) = 1.18(4)$