Quark and Glue Structure in Nucleon

- Quark spin from anomalous Ward identity
- πN σ term and strangeness
- Comparison of cost among twisted mass, clover, DWF and overlap fermions.

χ QCD Collaboration

All Hands Meeting

BNL, Apr. 29-30, 2016

2+1 flavor DWF configurations (RBC-UKQCD)

La ~ 4.5 fm
m_π ~ 170 MeV

32^3 x 64, a =0.137 fm

↓ ($O(a^2)$ extrapolation)

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→

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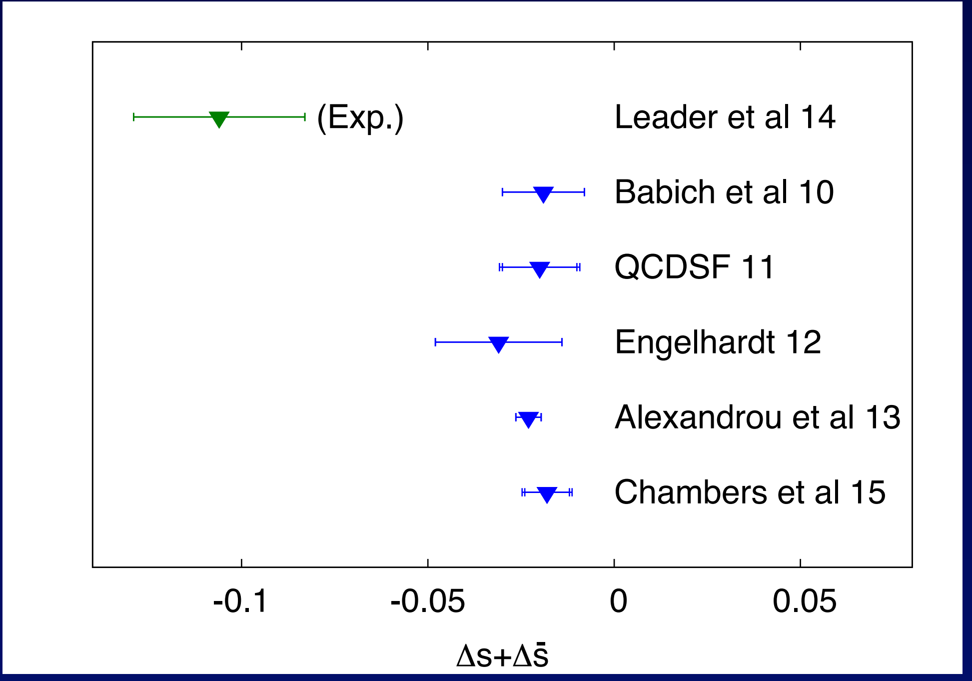
Strange quark spin $\Delta s + \Delta \bar{s}$



Quark Spin from Anomalous Ward Identify

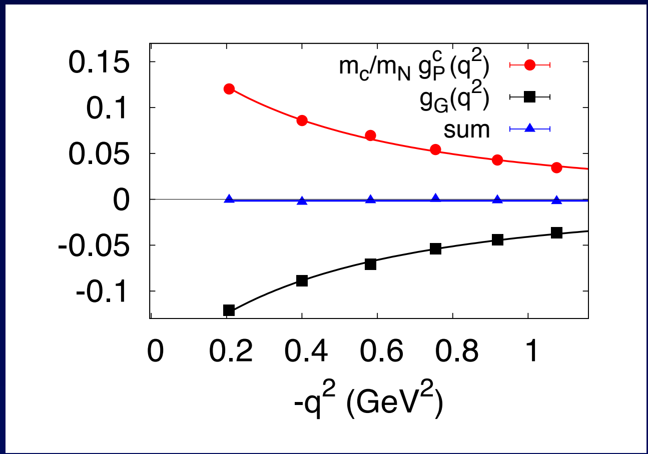
- Calculation of the axial-vector in the DI is very noisy
- Instead, try AWI $\partial_\mu A_\mu^0 = i2mP + \frac{iN_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$

$$\kappa_A \langle p',s | A_\mu | p,s\rangle = \lim_{q\to 0} \frac{i|s|}{\vec{q}\cdot\vec{s}} \langle p',s | 2\sum_{f=1}^{N_f} m_f \vec{q}_f i\gamma_5 q_f + 2iN_f q | p,s\rangle$$

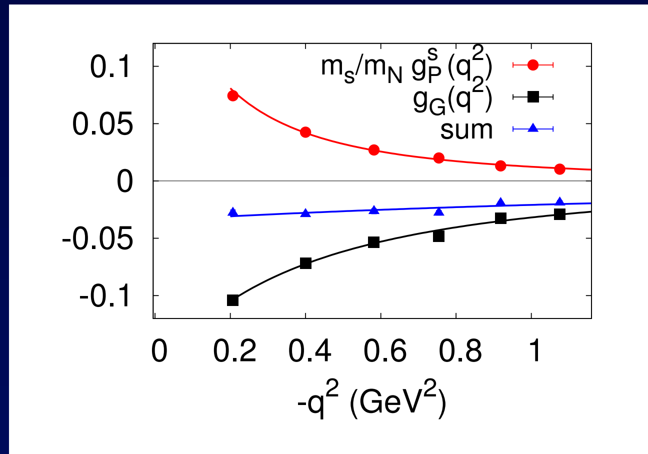
  - Overlap fermion --> mP is RGI ($Z_m Z_P=1$)
  - Overlap operator for $q(x) = -1/2 \, \mathrm{Tr}\gamma_5 D_{ov}(x,x)$ is RGI.
  - P is totally dominated by small eigenmodes.
  - q(x) from overlap is exponentially local and captures the high modes from A_μ^0.
  - Direct check the origin of `proton spin crisis'.

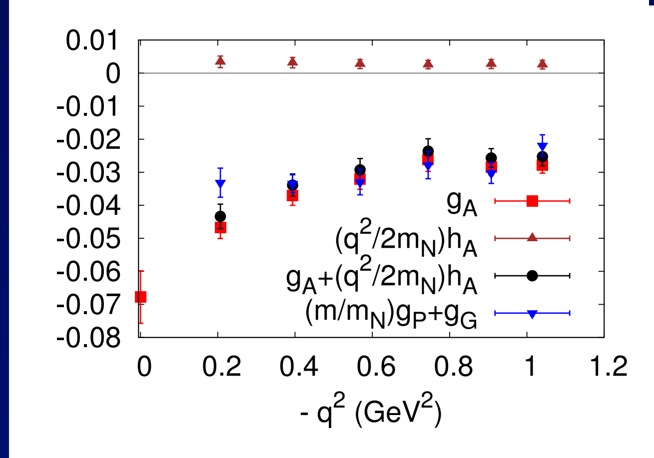
Disconnected Insertion

Charm

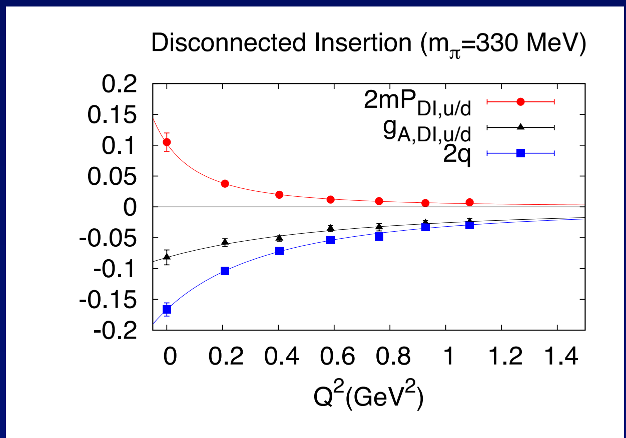


Strange



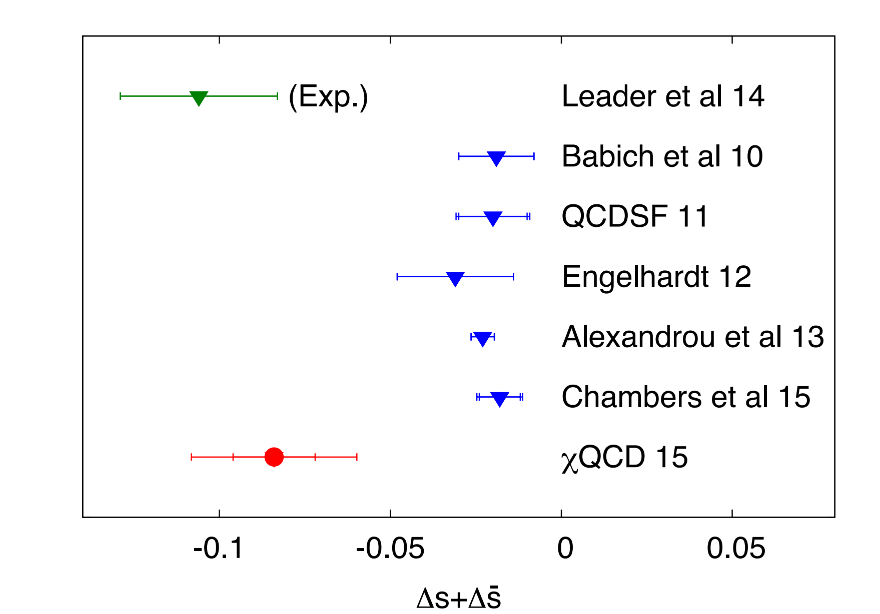


u/d



$$2M_N\kappa_A g_A^0(q^2)+q^2\kappa_{h_A}h_A^0(q^2)$$
$$=2mg_P^0(q^2)+2M_N g_G(q^2)$$

Quark Spin Decomposition



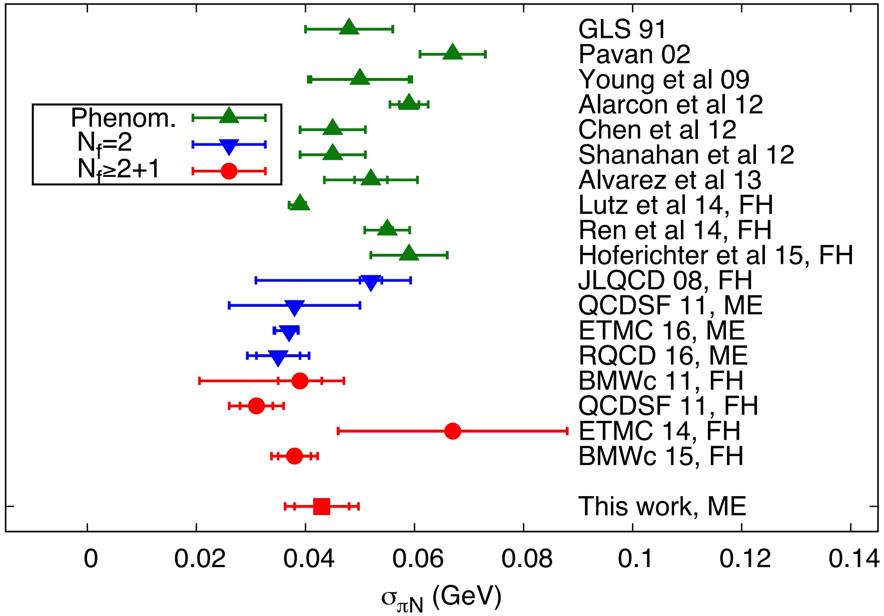
triangle anomaly

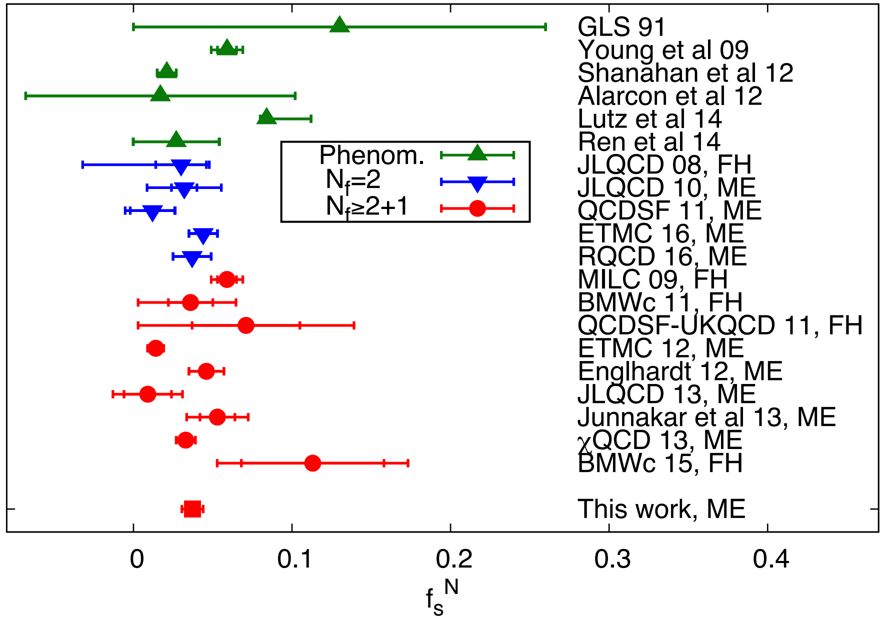
Check with conserved current

g_A^0	m_π=140 MeV
Δu+Δd (CI)	~ 0.57(3)
Δc	~0
Δs	-0.08(2)
Δu(DI)=Δd(DI)	-0.17(3)
g_A^0	~ 0.17

$\sigma_{\pi N}$

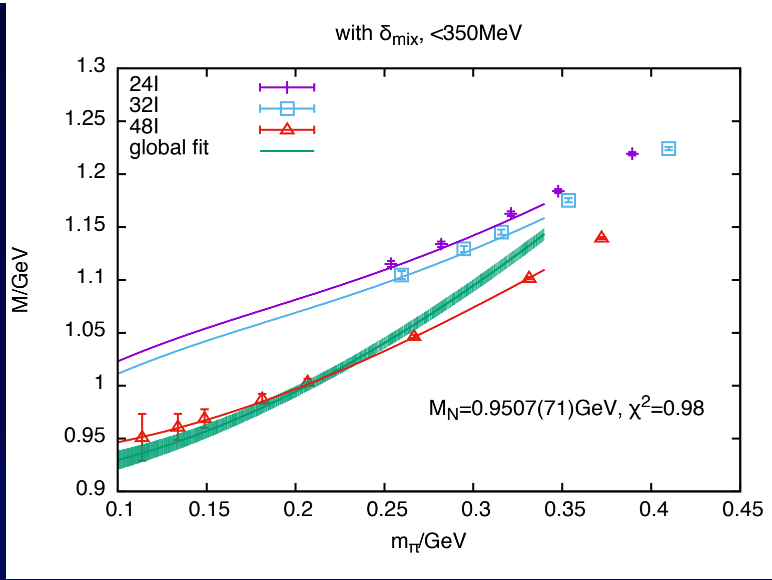
$$f_s^N = \frac{\sigma_{sN}}{m_N}$$

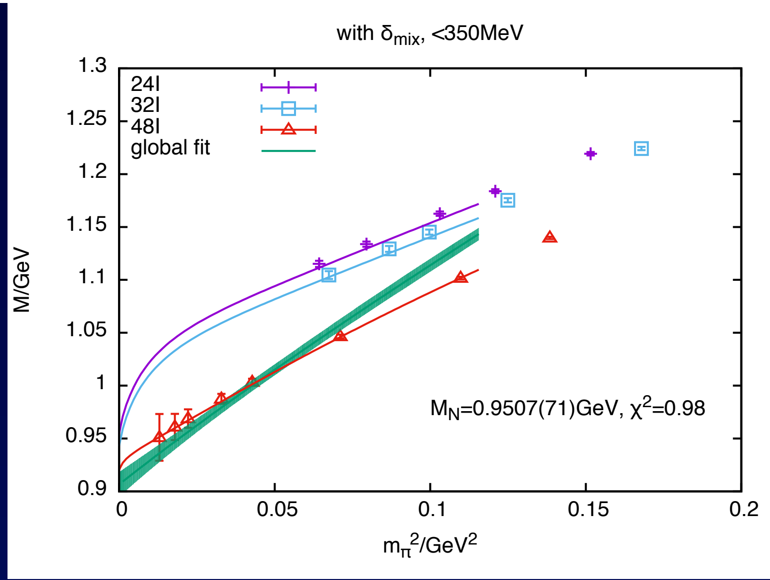




arXiV:1511.09089

$$\sigma_{sN} = \sigma_{sN}^0 + c_1 m_{\pi,vv}^2 + c_2 m_{\pi,vs}^2 + c_3 a^2 + c_4 e^{-m_\pi L}$$





m_N (134) = 961(12) MeV, m_N(global = 950.7(7.1) MeV

Cost comparison

$$\frac{\text{Clover (a09m130)}}{\text{Overlap (48I)}} = \frac{(84768 + 7064 \times 3)}{(81 \times 32)}(\text{inver}) \times \frac{1}{44}(\text{time}) \times \begin{cases} (\frac{0.011}{0.013})^2 (\text{var})(\text{unitary}) = 0.66 \\ (\frac{0.011}{0.007})^2 (\text{var})(\text{multi-mass}) = 2.3 \end{cases}$$

$$\frac{\text{DWF (48I)}}{\text{Overlap (48I)}} = \frac{(640 + 20 \times 3)}{(81 \times 32)}(\text{inver}) \times \frac{1}{3}(\text{time}) \times (\frac{0.075}{0.013})^2 (\text{var}) = 3.0$$

$$\frac{\text{Clover (BMW)}}{\text{Overlap (24I,32I,48I)}} = ? \quad \frac{13000}{81+200+300}(\text{conf}), \ \frac{40}{32}(\text{inver}), \ \frac{1}{10(?)}(\text{time}), \ (\frac{0.017}{0.007})^2 (\text{var})$$

m_N(BMW) = 929(16)(7) MeV

Results:

Compare to the SNR of Twistedmass+Clover

$48^3 x96$

The results of TM+C come from arXiv: 1507.04936

	m_π	N_cfg	Inverstions	Measurements	Method
Overlap	133 MeV	81	5+4+8+12=29	0.4k(80k)	SSM+LMSS
TM+C	131 MeV	96	16*(1+3*8)=400	1.5k	Sequential

With the factor 4^3x3 (points in the volume), the measurements would be 80k.

	OV, 0.91 fm	**OV, 1.14 fm**	**OV, 1.37 fm**	**TMC, 0.9fm**	**TMC, 1.08 fm**	**TMC 1.26 fm**
g_A^3	1.133(15)	1.150(25)	1.233(66)	1.158(16)	1.162(30)	1.242(57)
g_S^3	0.72(8)	0.93(17)	0.782(41)	0.55(18)	1.18(34)	2.20(54)
g_S^0(CI)	6.80(15)	7.23(33)	7.77(70)	6.46(27)	7.84(48)	8.93(86)
$\langle x \rangle_{u-d}$	0.214(9)	0.194(11)	0.208(27)	0.248(9)	0.218(15)	0.208(24)
$\langle x \rangle_{u+d(CI)}$	0.519(11)	0.456(15)	0.400(36)	0.645(13)	0.587(18)	0.555(63)

Cost Comparison with TSM (1601.01624)

Fermion	a (fm)	Conf.	Source	Inversion	time	LMS	Error
TSM (CI)	0.093	96	16	16*(8*3)	1	0	x%
Overlap (CI)	0.112	81	5	5+4+8+12	~10	25%	x%

Fermion	a (fm)	Conf.	Source	Loop	time	Error
TSM (strange)	0.093	1800	100	300	1	12%
Overlap (strange)	0.112	81	32	32 (or 4)	~10	40%/17% (global)

CI

$$\frac{\text{TM (48)}}{\text{Overlap (48I)}} = \frac{96}{81}(\text{conf})\times\frac{384}{29}(\text{inver})\times\frac{1}{10}(\text{time})\times\frac{1}{1.25}(\text{LMS})\times\frac{x}{x}(\text{var}) = 1.3$$

DI (scalar)

$$\frac{\text{TSM (48)}}{\text{Overlap (48I)}} = \frac{1800}{81}(\text{conf})\times\frac{100+300}{32+32}(\text{inver})\times\frac{1}{10}(\text{time})\times\begin{cases}(\frac{0.12}{0.40})^2(\text{var})(\text{unitary}) = 1.3\\(\frac{0.12}{0.17})^2(\text{var})(\text{global}) = 6.9\end{cases}$$

Cost Comparison with Clover for g_s^3 (1602.07737)

Fermion	a (fm)	Conf.	Source	Sink	tsep	time	LMS	Error
Clover (CI)	0.081	400	100	4	4	1	0	10%
Overlap (CI)	0.083	300	1	3	3	~ 44	8%	13%

Cost Comparison with DWF for g_A (C14-08-11.3)

Fermion	a (fm)	Conf.	Source	Sink	tsep	time	LMS	Error
DWF (CI)	0.114	20	32+1	2	4	1	0	9.7%
Overlap (CI)	0.114	81	5	4/8/12	3	~ 3	25%	2.2%

$$\frac{\text{DWF (48I)}}{\text{Overlap (48I)}} = \frac{20}{81}\text{(conf)} \times \frac{(32+1*3)\ \text{(source)} \times (2*3)\ \text{(sink-tsep)}}{5\ \text{(source)} + (4+8+12)\ \text{(sink-tsep)}} \times \frac{1}{3}\ \text{(time)} \times \frac{1}{1.25}\text{(LMS)} \times \left(\frac{0.097}{0.022}\right)^2 \text{(var)} = 9.0$$

2+1 flavor DWF configurations (RBC-UKQCD)

La ~ 4.5 fm
m_π ~ 170 MEV

32^3 x 64, a =0.137 fm

($O(a^2)$ extrapolation)

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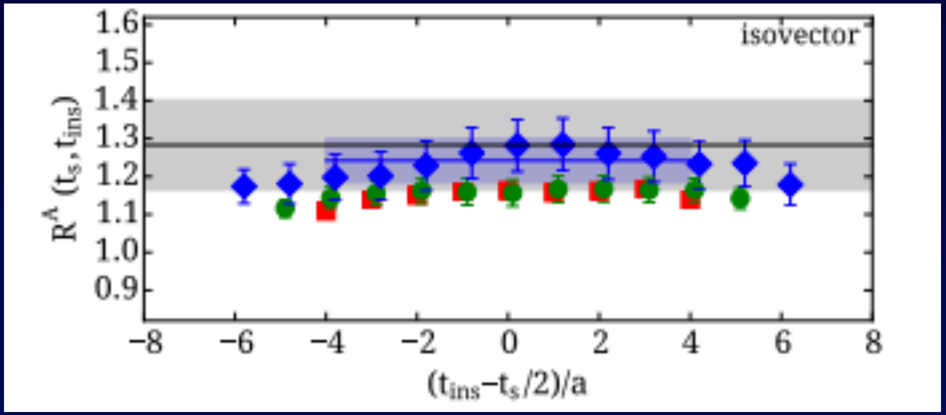
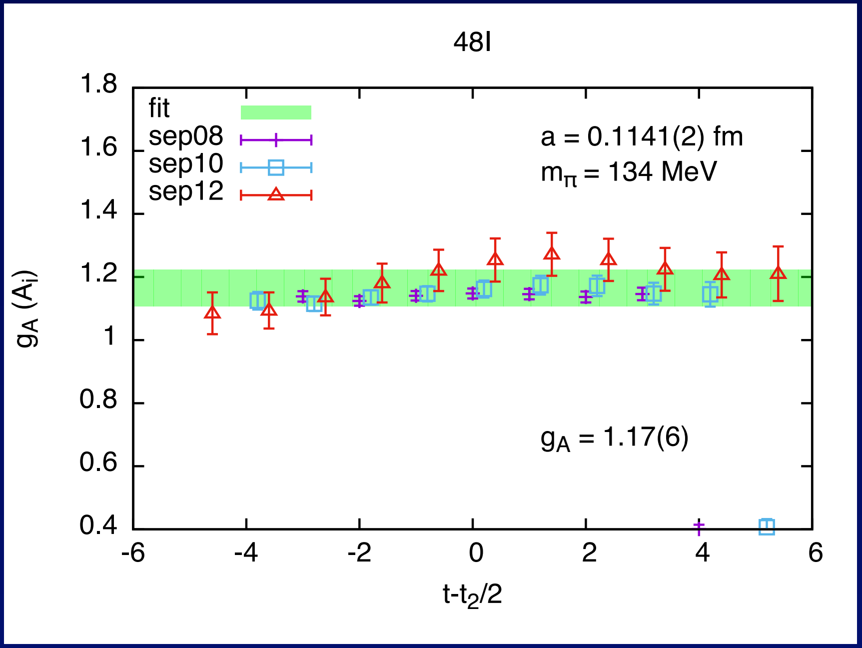
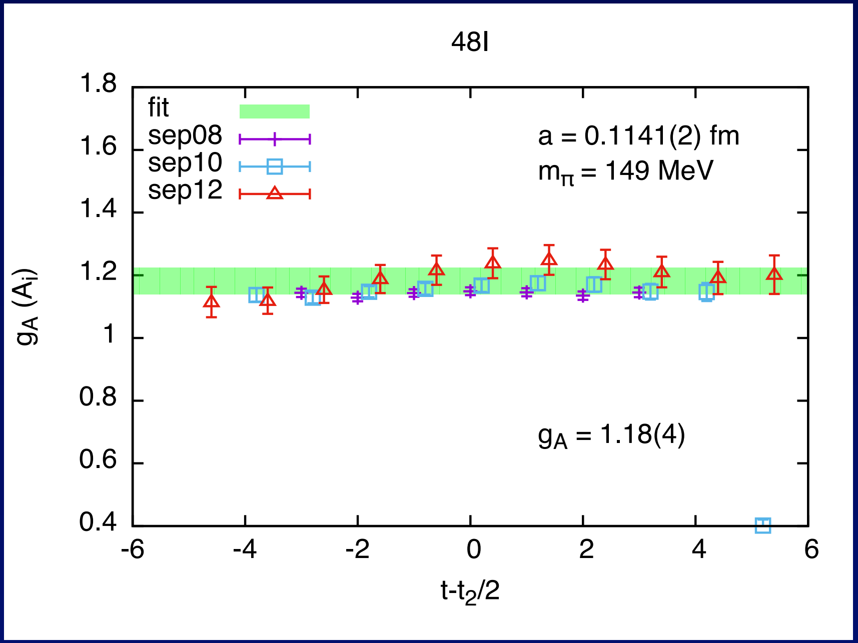
Why not use DWF?

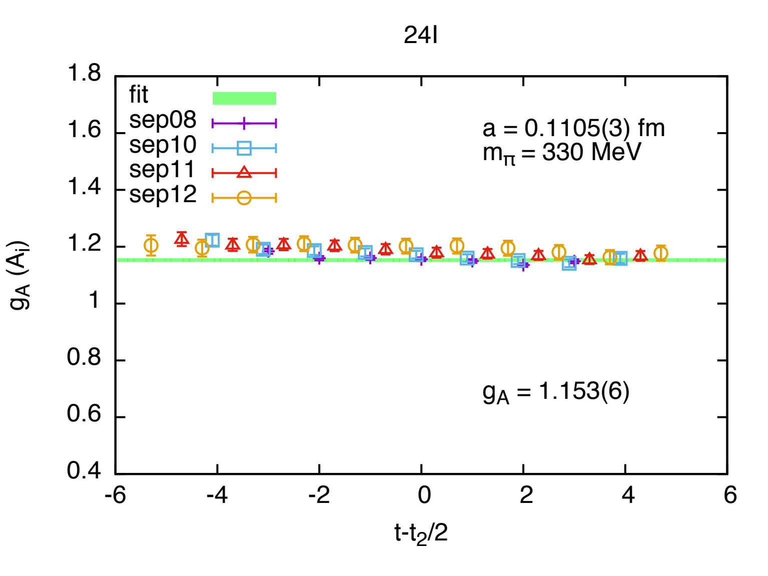
- DWF needs 2 L_s time of storage and memory.
  For 1000 pairs of eigenmodes:
  1.4 TB (overlap)/ conf → 43 TB (DWF)/conf
  110 TB (overlap) → 3.5 PB (DWF) total
- Multi-mass + deflation possible for overlap, because eigenvectors are mass independent. Helps global fit.
- For the existing DWF configurations: limited reach for heavy quark, oscillatory behavior at short time limits excited state study and possibly quark loop studies.

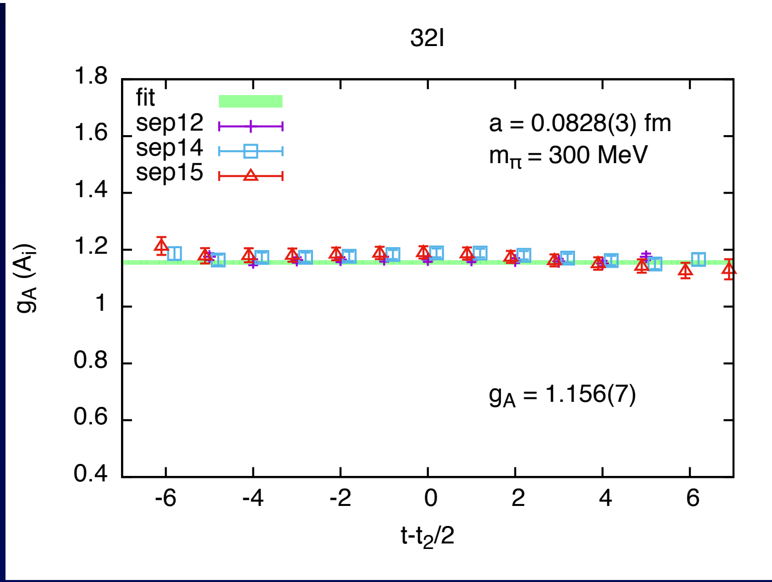
ETMC (1507.04836)

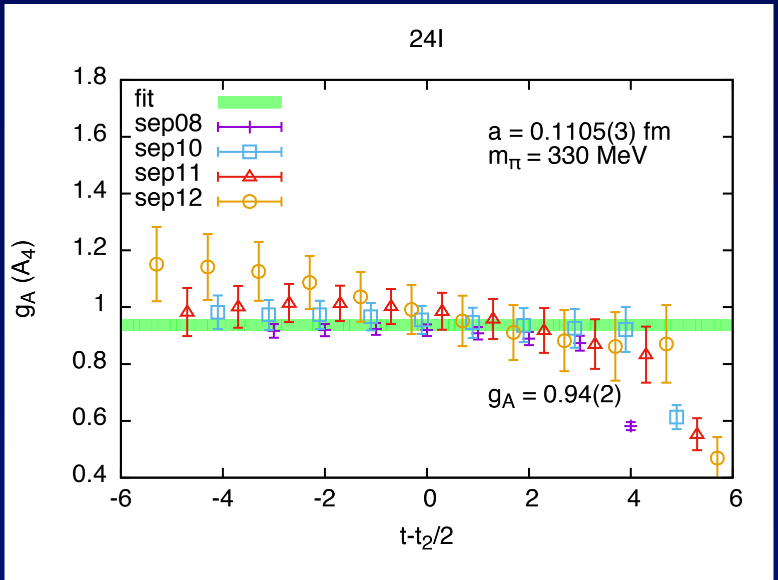


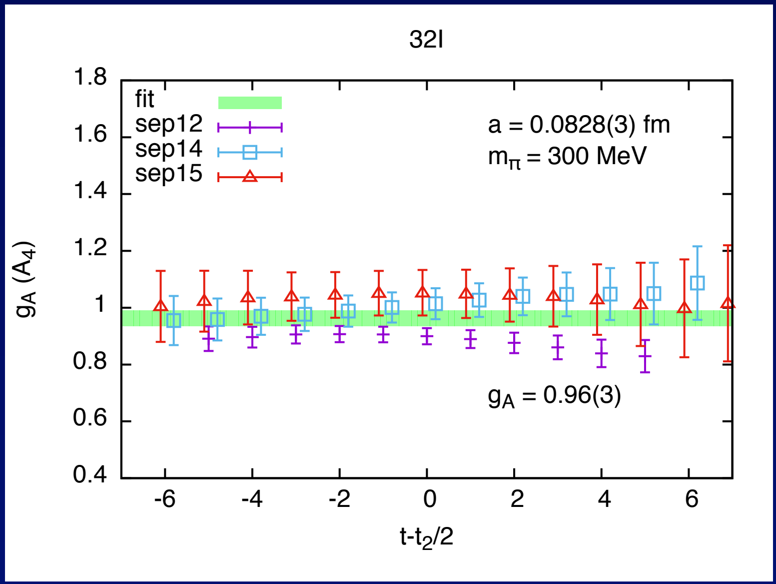


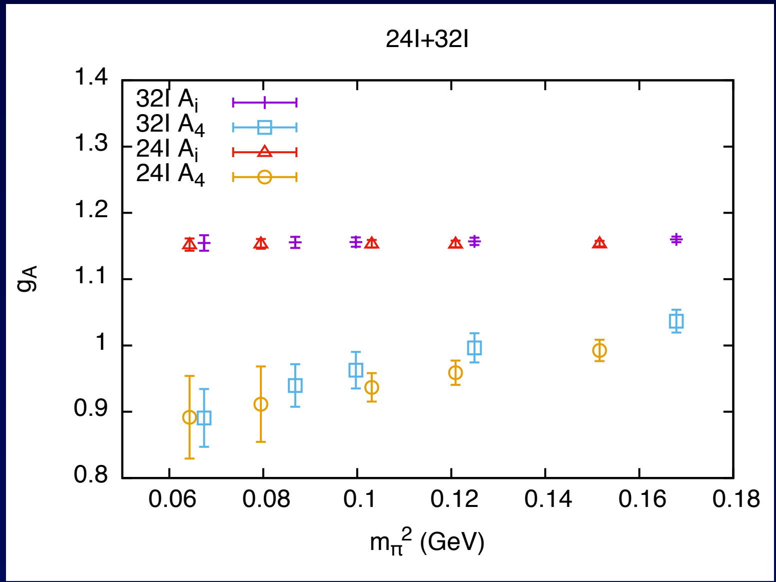


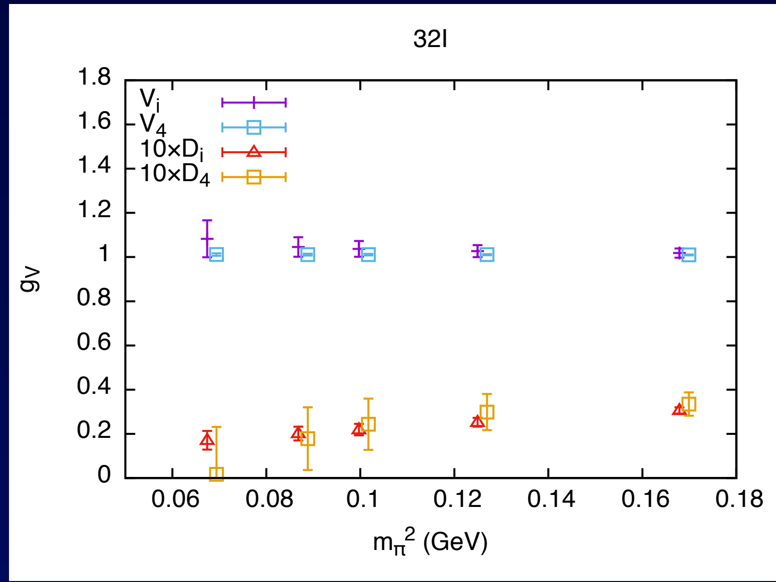

24I
fit
sep08
sep10
sep11
sep12
a = 0.1105(3) fm
m_π = 330 MeV
g_A = 1.153(6)
g_A (A_i)
$t-t_2/2$

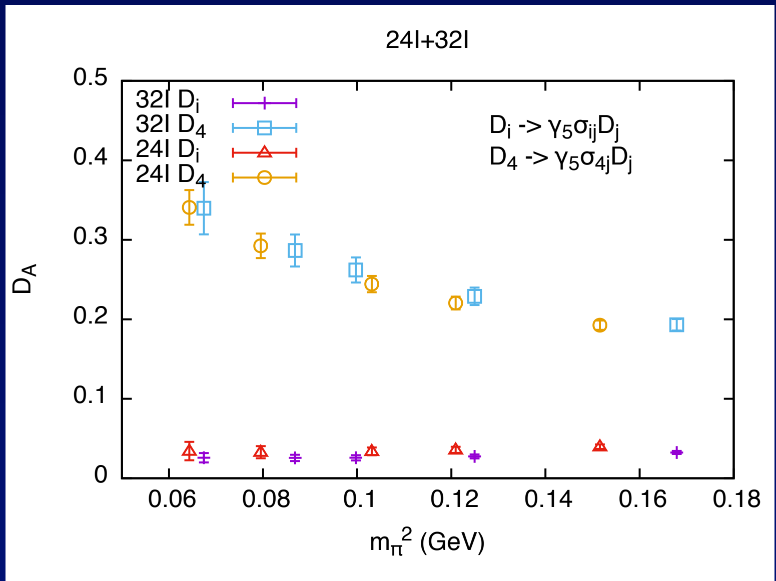

32I
fit
sep12
sep14
sep15
a = 0.0828(3) fm
m_π = 300 MeV
g_A = 1.156(7)
g_A (A_i)
$t-t_2/2$


24I
fit
sep08
sep10
sep11
sep12
a = 0.1105(3) fm
m_π = 330 MeV
g_A = 0.94(2)
g_A (A_4)
$t-t_2/2$


32I
fit
sep12
sep14
sep15
a = 0.0828(3) fm
m_π = 300 MeV
g_A = 0.96(3)
g_A (A_4)
$t-t_2/2$







g_A(24I) = 1.153(6) → g_A(imp) = 1.188(7)

g_A(32I) = 1.156(7) → g_A(imp) = 1.177(9)

g_A(48I) (134 MeV) = 1.17(6)
g_A(48I) (149 MeV) = 1.18(4)