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Beyond single photon exchange in electron-nucleus scattering: the distorted picture

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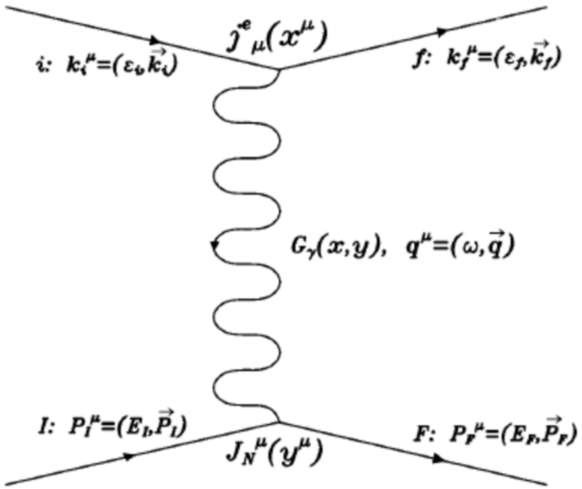
Elastic electron scattering off nuclei

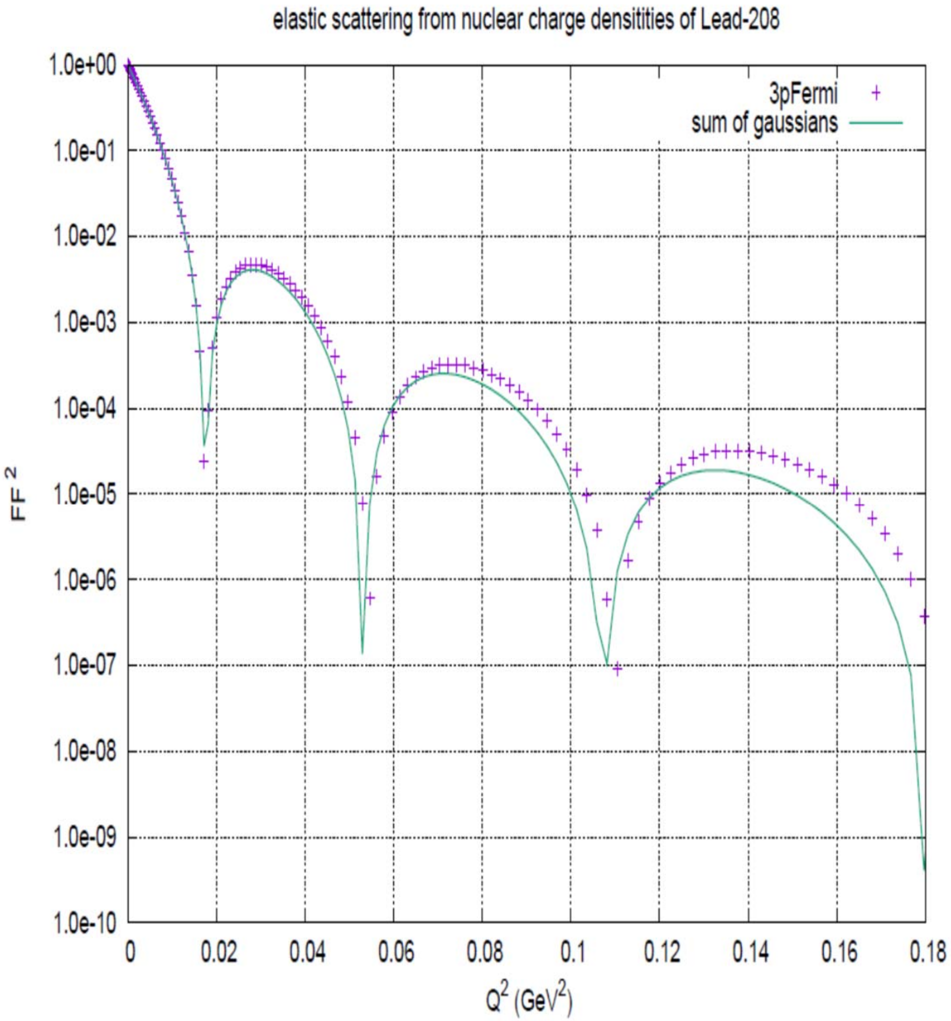
- Electron scattering off nuclei has been used to determine the charge distribution of many nucleus. It is a standard tool. Well established for spin-0 nuclei
- Plane wave plus single photon exchange approximation, that is first order Born approximation (PWBA): factorizes the Mott cross-section times the Fourier transform of the nuclear charge distribution

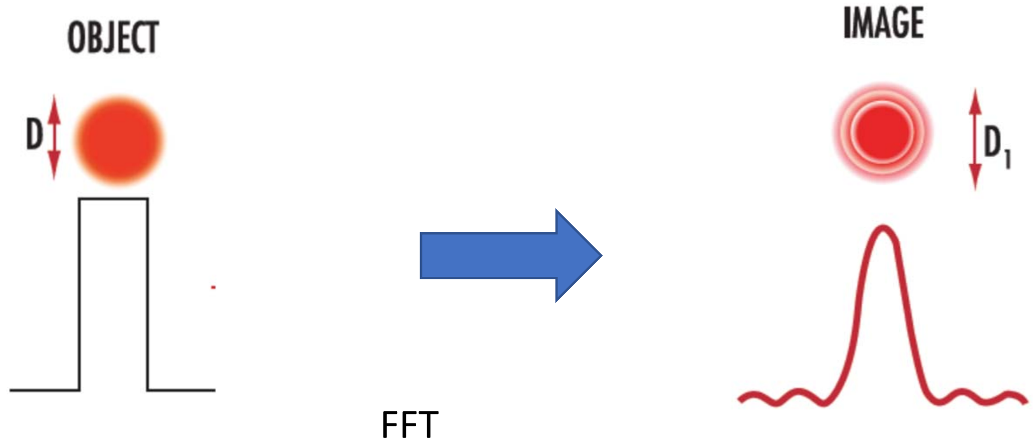


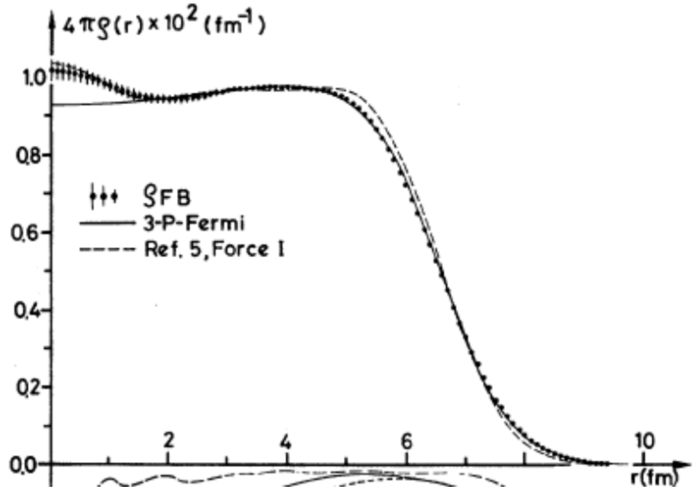
$$\frac{d\sigma}{d\Omega} = \sigma_{\mathrm{M}} \left|F_p(q)\right|^2$$

$$\sigma_{\mathrm{M}}(\theta) = \left(\frac{Ze^2}{2E}\right)^2 \frac{\cos^2(\theta/2)}{\sin^4(\theta/2)} \quad F_p(q) = \int d^3r\, j_0(qr)\rho_{ch}(r)$$

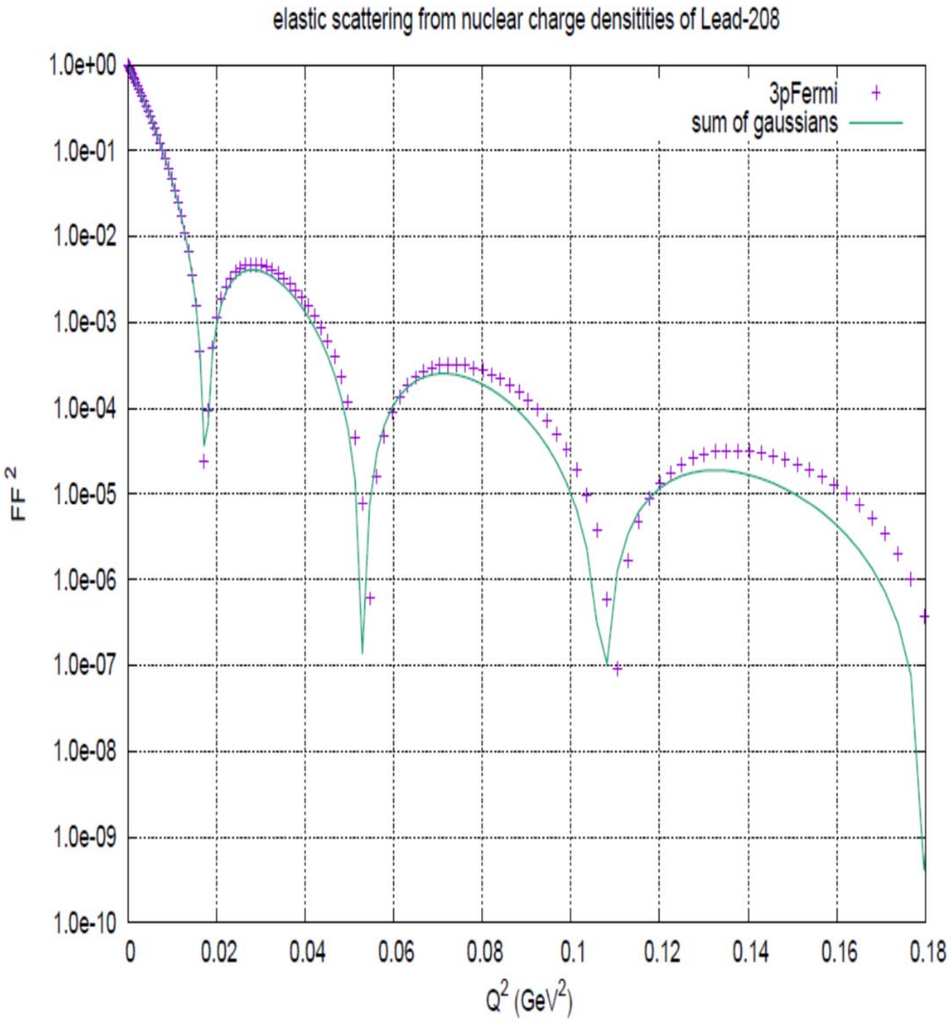
Diffractive patterns, as this is scattering from a disk with smeared borders



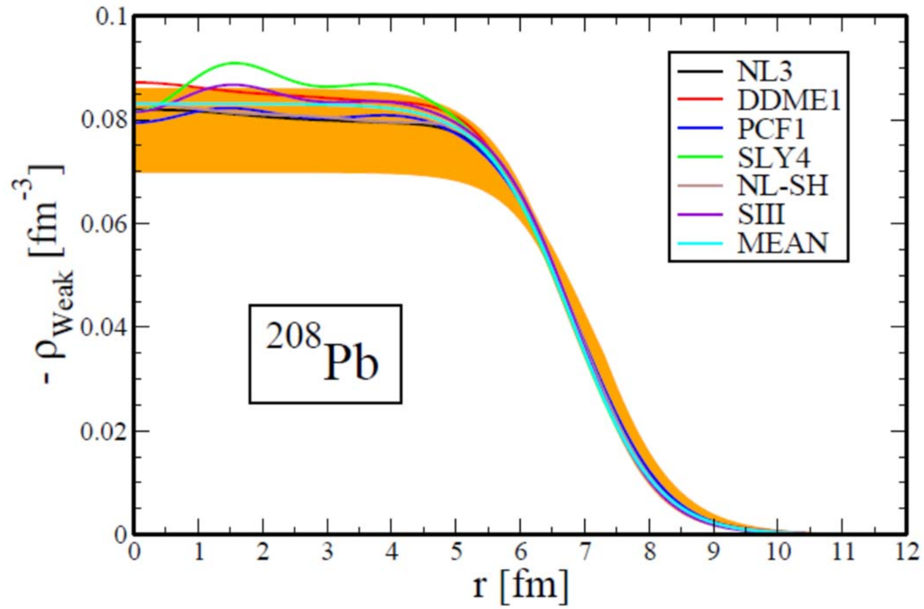




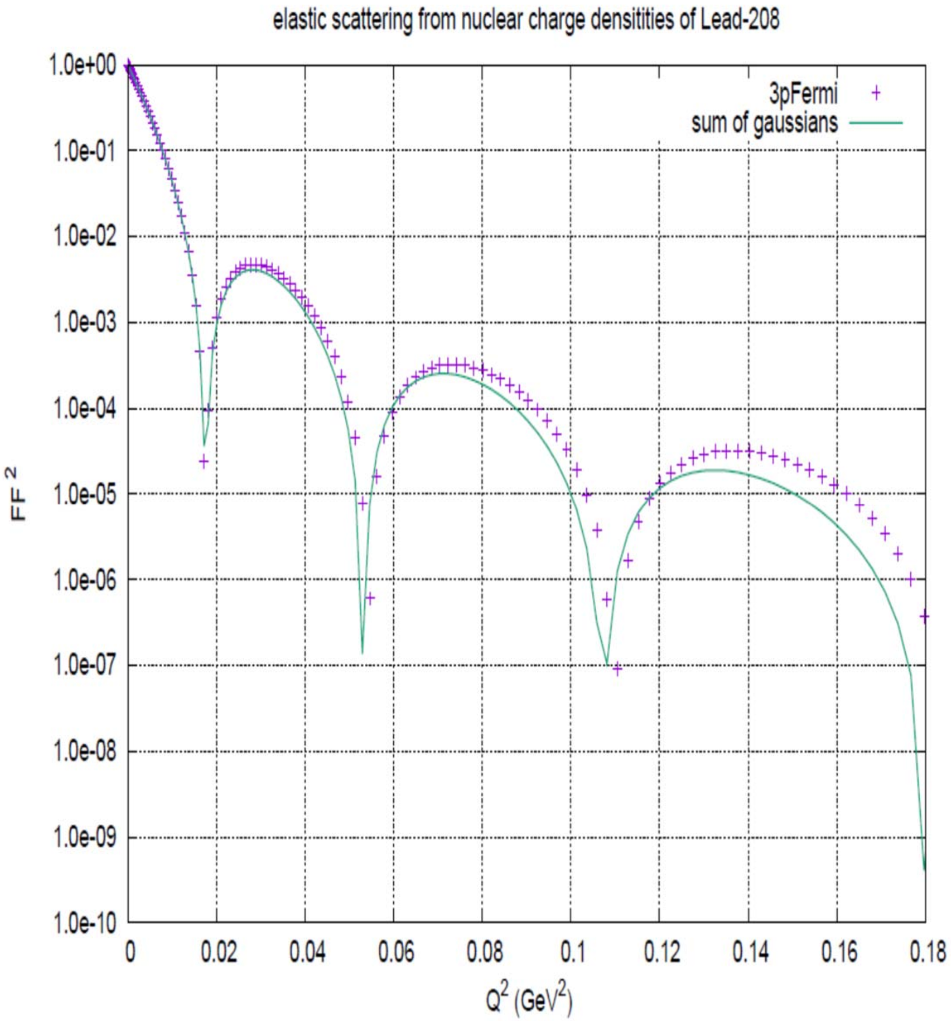
Diffractive patterns, as this is scattering from a disk with smeared borders

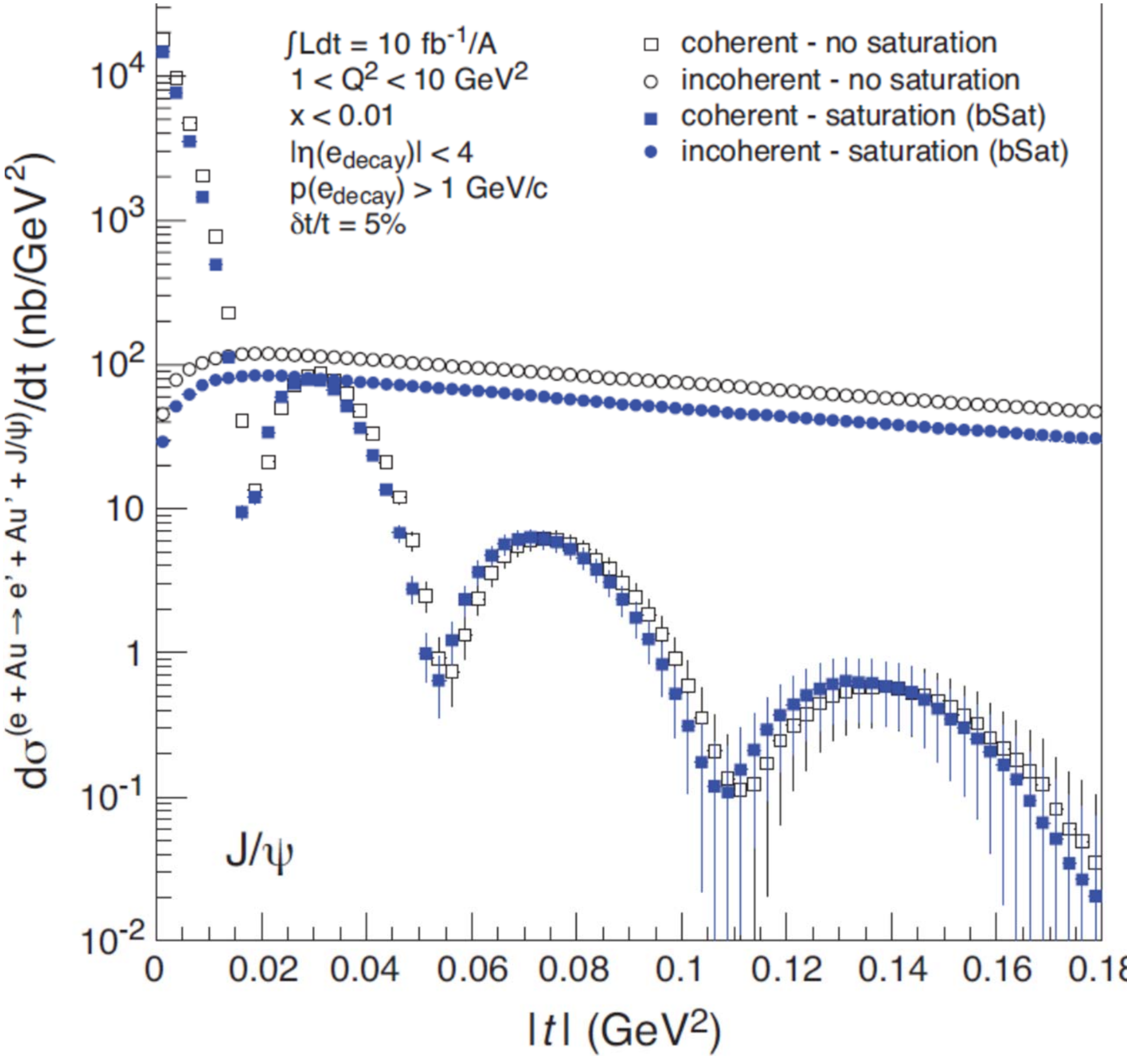


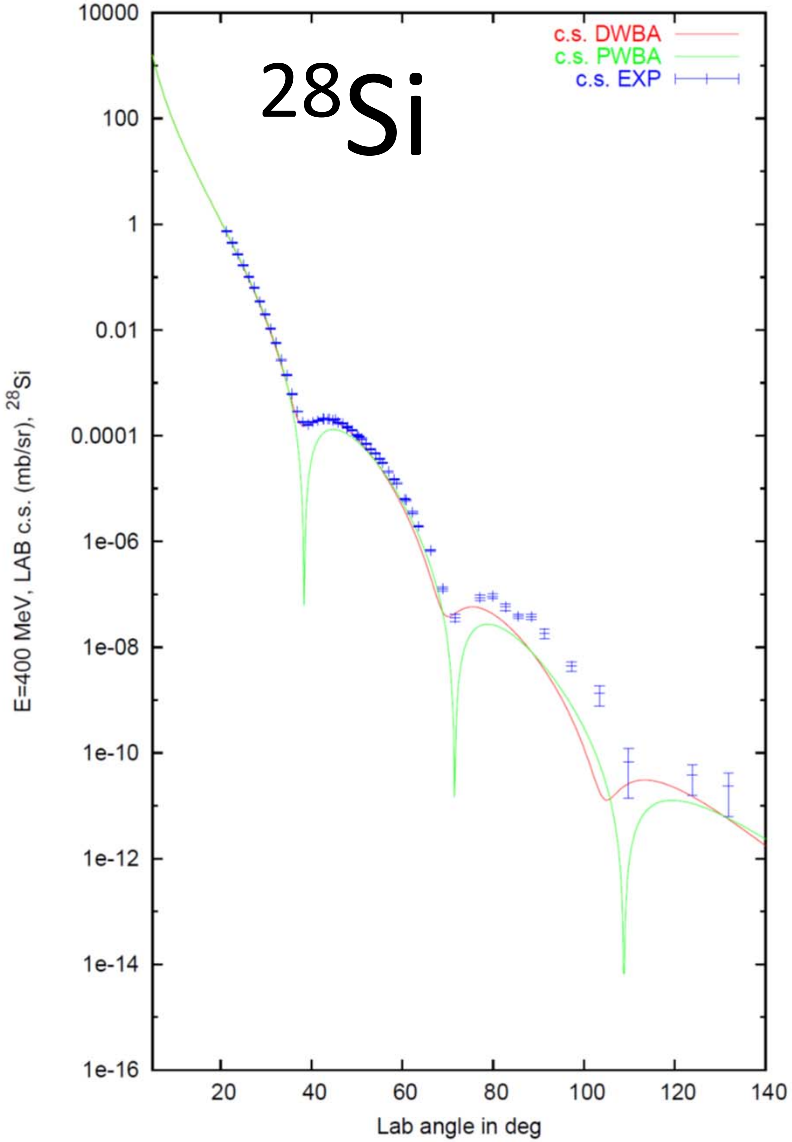
- It can be applied to elastic PV (neutral current mediated) scattering from nucleus



Diffractive patterns, will also be predicted by analogous experiments



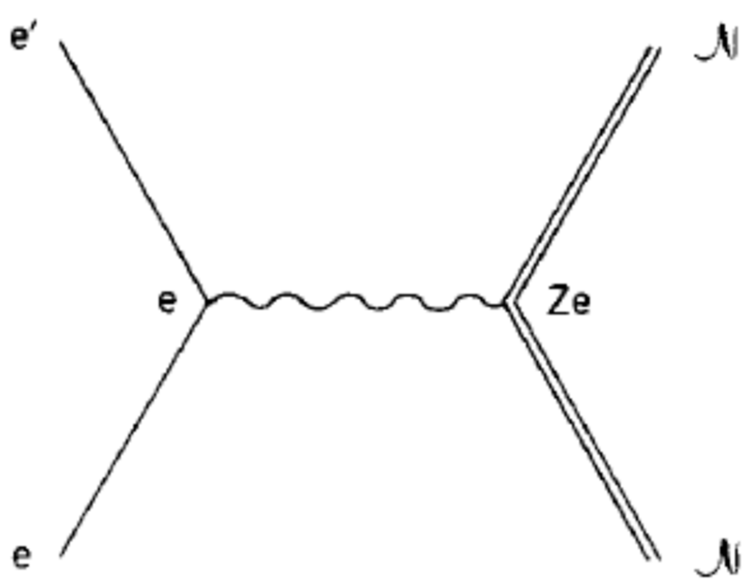




However, to compare with data the PWBA results are not good enough. The data show minima filled in and the maxima semared. The cross-section is shifted with regards to the data

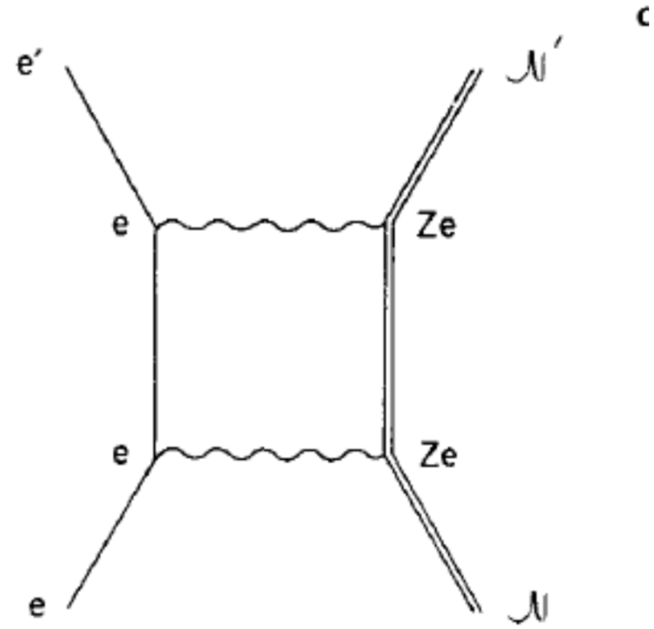
What is the problem? The nucleus has a charge Z, the coupling at the nuclear vertex is not small anymore but of order Zα/(kr). Z/α can be up to 0.65

First order BA



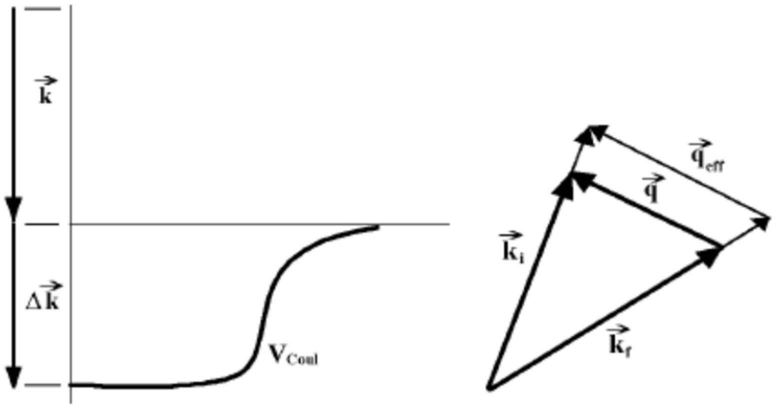
(a)

Second order BA



(b)

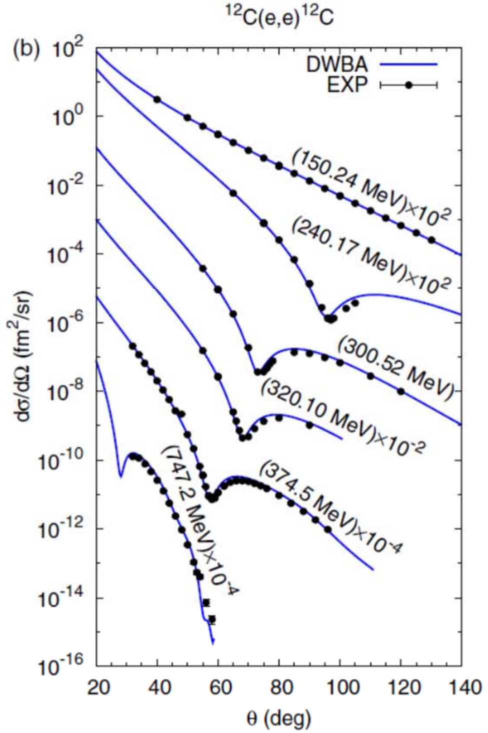
By far the most important correction to PWBA is the effect of the Coulomb potential on the incoming and outgoing electrons, which cannot be considered as free scattering states anymore



The electrons close to the nucleus have an effective momento that is larger than the asymptotic one, plus there is a focussing effect, the probability of having the electrons near the nucleus is larger than away from it.

This Coulomb effect is the same as the Fermi correction factor in beta decay.

It can be taken into account exactly by replacing the plane waves for the electrons by scattering states solutions of the Dirac equation with the Coulomb potential, keeping the single-photon exhange (first Born) approximation.
This is calles Distorted Waves Born Approximation or DWBA



DWBA results match the experimental data very well. DWBA is needed (arguably) whenever Z is larger than 2, even for large energies

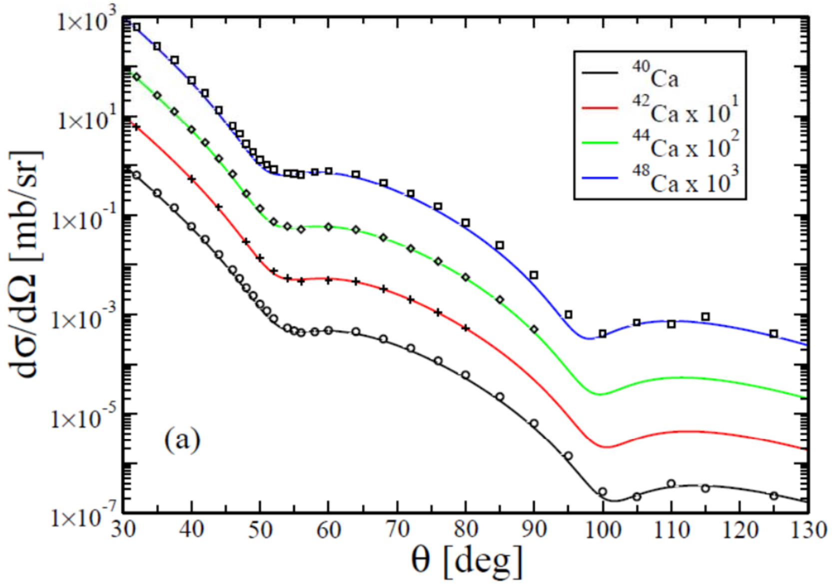
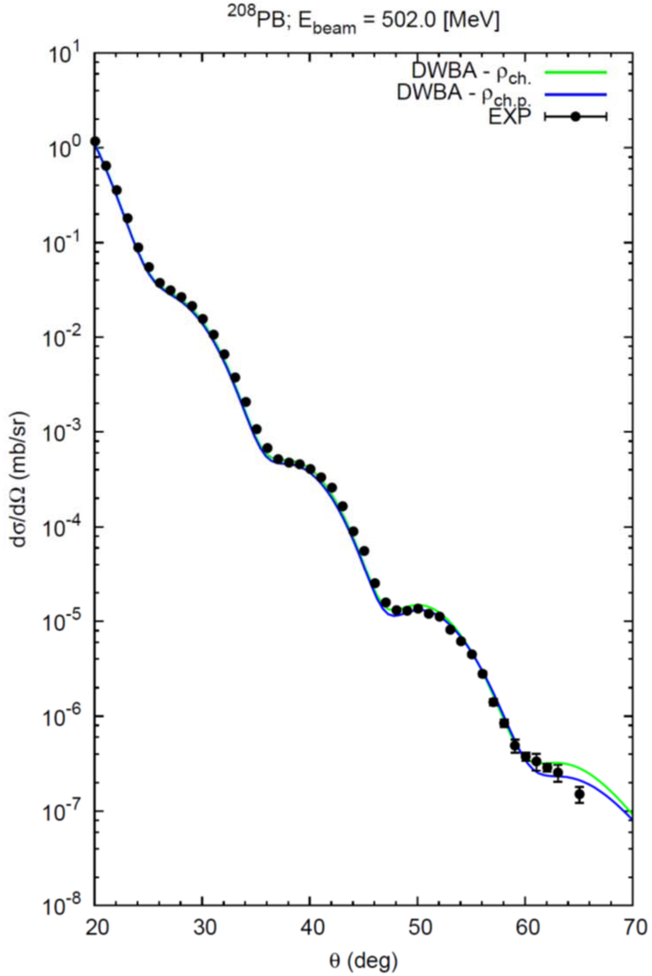
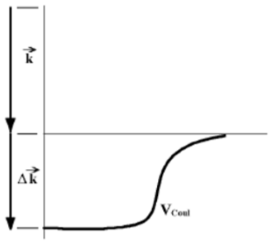


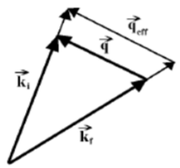
Figure 3.2: Differential cross sections for elastic electron scattering on calcium isotopes at an electron energy $\varepsilon = 250$ MeV as functions of the scattering angle θ. Experimental data from [121].



Effects of Coulomb distortion

A shift of the kinematical quantities of the electrons. This can be estimated easily and it is usually taken into account to compare theory and data, by shifting The data by the effective momentum. This goes as Zα/(rR), and thus becomes negligible for electron energies larger tan say 5 GeV.





A focusing effect, increasing the cross-section, due to the attraction of the electron by the nucleus, which causes the wave function to be enhanced in the vicinity of the nucleus. This does not get smaller as the beam energy increases.

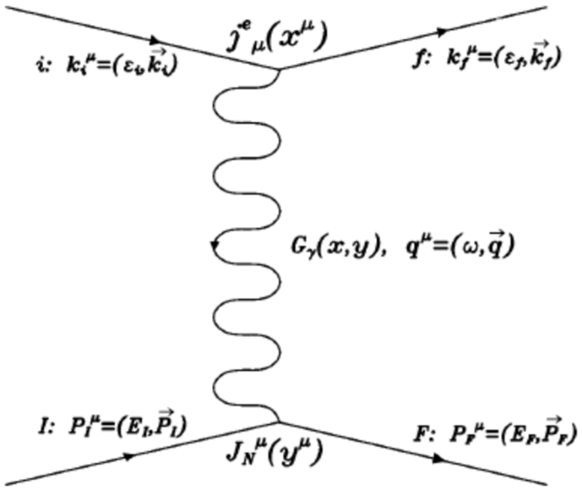
A blurring of the longitudinal-transverse separation, as the direction of the virtual photon is not determined at the vertex. This effect gets reduced as the beam energy Increases, but it will remain in place when looking for kinematics where one component (longitudinal at 180º, say) will not contribute in PWBA, while it will no be zero in DWBA.

Elastic electron scattering off nuclei in DWBA

- Single photon exchange approximation, that is first order Born approximation (BA) in a general framework

$$S_{fi} = (ie^2) \int d^4x \int d^4y \, j^e_\mu(x) G_\gamma(x,y) J^\mu_N(y)$$

$$G_\gamma(x,y) = \int \frac{d^4q}{(2\pi)^4} e^{+iq(x-y)} \frac{-1}{q^2_\mu}$$



Here if we substitute plane waves for the electrons:

$$S^{PWBA}_{fi} = +ie^2 2\pi\delta(\epsilon_i + E_A - \epsilon_f - E_F - E_{A-1}) \frac{m}{V} (\epsilon_i \epsilon_f)^{-\frac{1}{2}} \frac{-1}{q^2_\mu}$$

$$\times \; \overline{u}(\boldsymbol{k}_f, \sigma_f) \gamma^\mu u(\boldsymbol{k}_i, \sigma_i) \int d\boldsymbol{y} e^{i\boldsymbol{q}\boldsymbol{y}} J^{N,ee'p}_\mu(\boldsymbol{y})$$

(B.24

$$\boldsymbol{q} = \boldsymbol{k}_i - \boldsymbol{k}_f .$$

Elastic electron scattering off nuclei in DWBA

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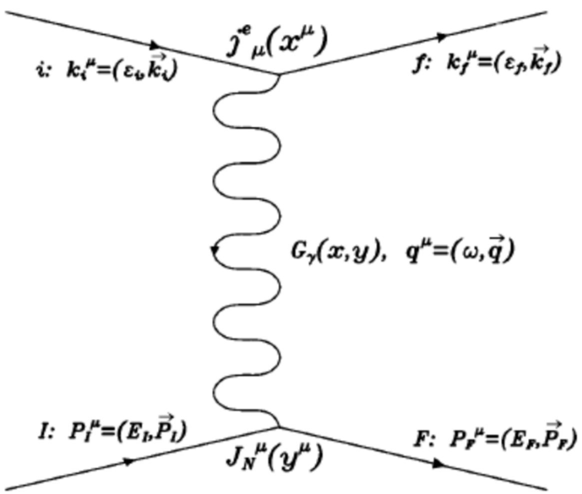


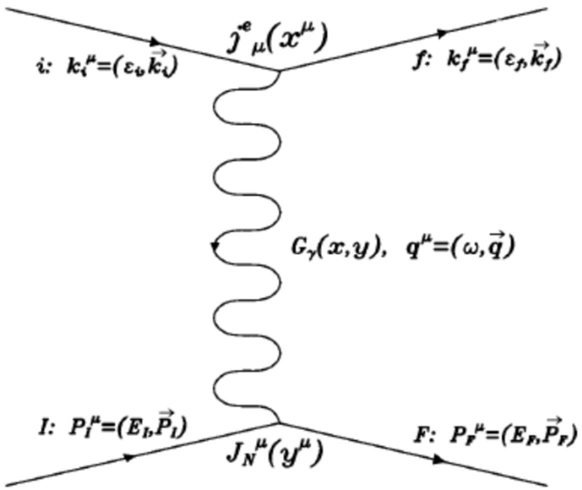
Here if we substitute plane waves for the electrons:

$$\begin{aligned} S^{PWBA}_{fi} &= +ie^2 2\pi\delta(\epsilon_i + E_A - \epsilon_f - E_F - E_{A-1}) \frac{m}{V} (\epsilon_i \epsilon_f)^{-\frac{1}{2}} \frac{-1}{q^2_\mu} \\ &\times \overline{u}(\boldsymbol{k}_f, \sigma_f) \gamma^\mu u(\boldsymbol{k}_i, \sigma_i) \int d\boldsymbol{y} e^{i\boldsymbol{q}\boldsymbol{y}} J^{N,ee'p}_\mu(\boldsymbol{y}) \qquad \text{(B.24} \end{aligned}$$

$$\boldsymbol{q} = \boldsymbol{k}_i - \boldsymbol{k}_f .$$

In DWBA, however, the matrix element is written in general as:

$$W_{if} = \int d\boldsymbol{x} \int d\boldsymbol{y} \int \frac{d\boldsymbol{q}}{(2\pi)^2} j^e_\mu(\boldsymbol{x}) e^{-i\boldsymbol{q}(\boldsymbol{x}-\boldsymbol{y})} \frac{-1}{q^2_\mu} J^\mu_N(\boldsymbol{y})$$

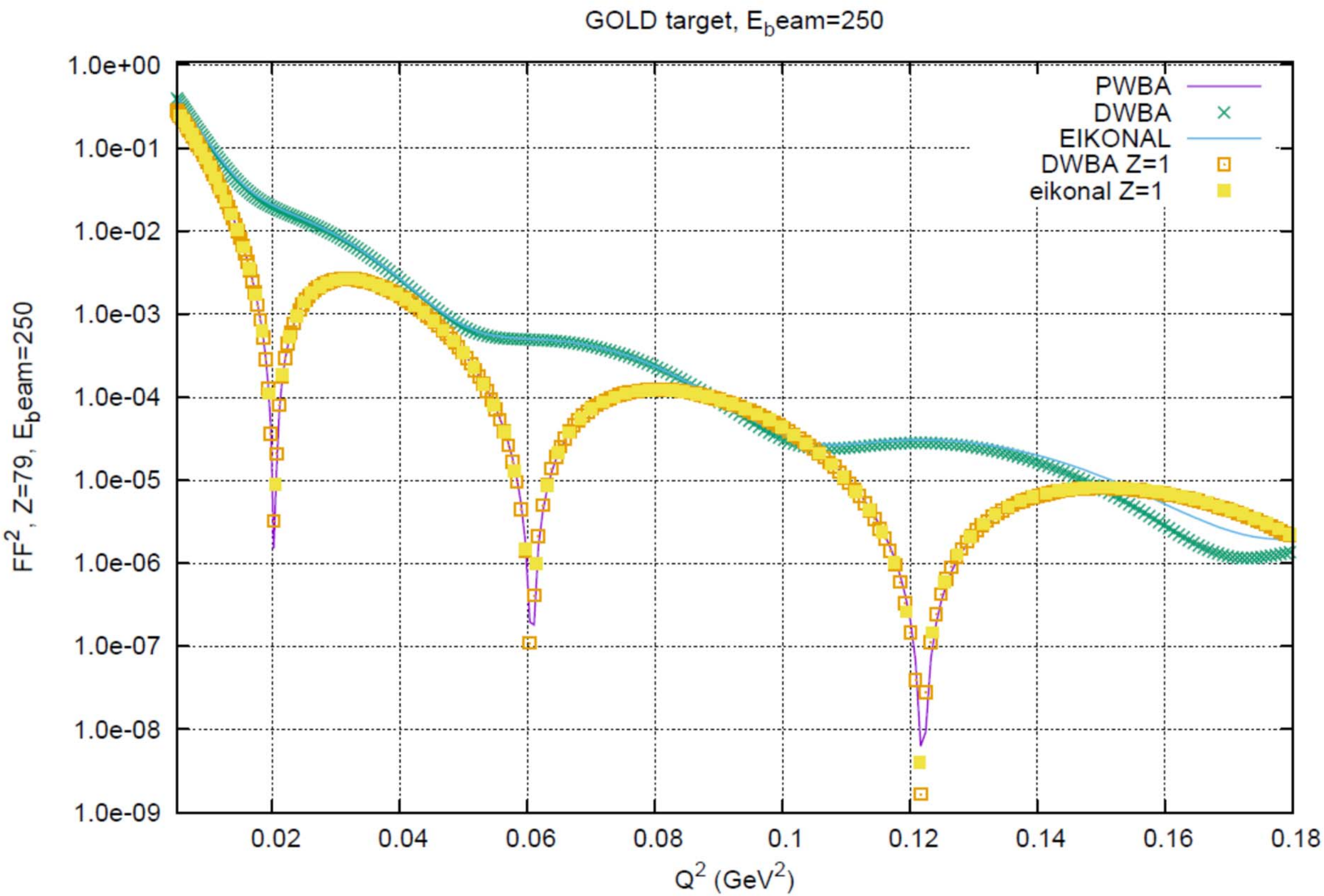


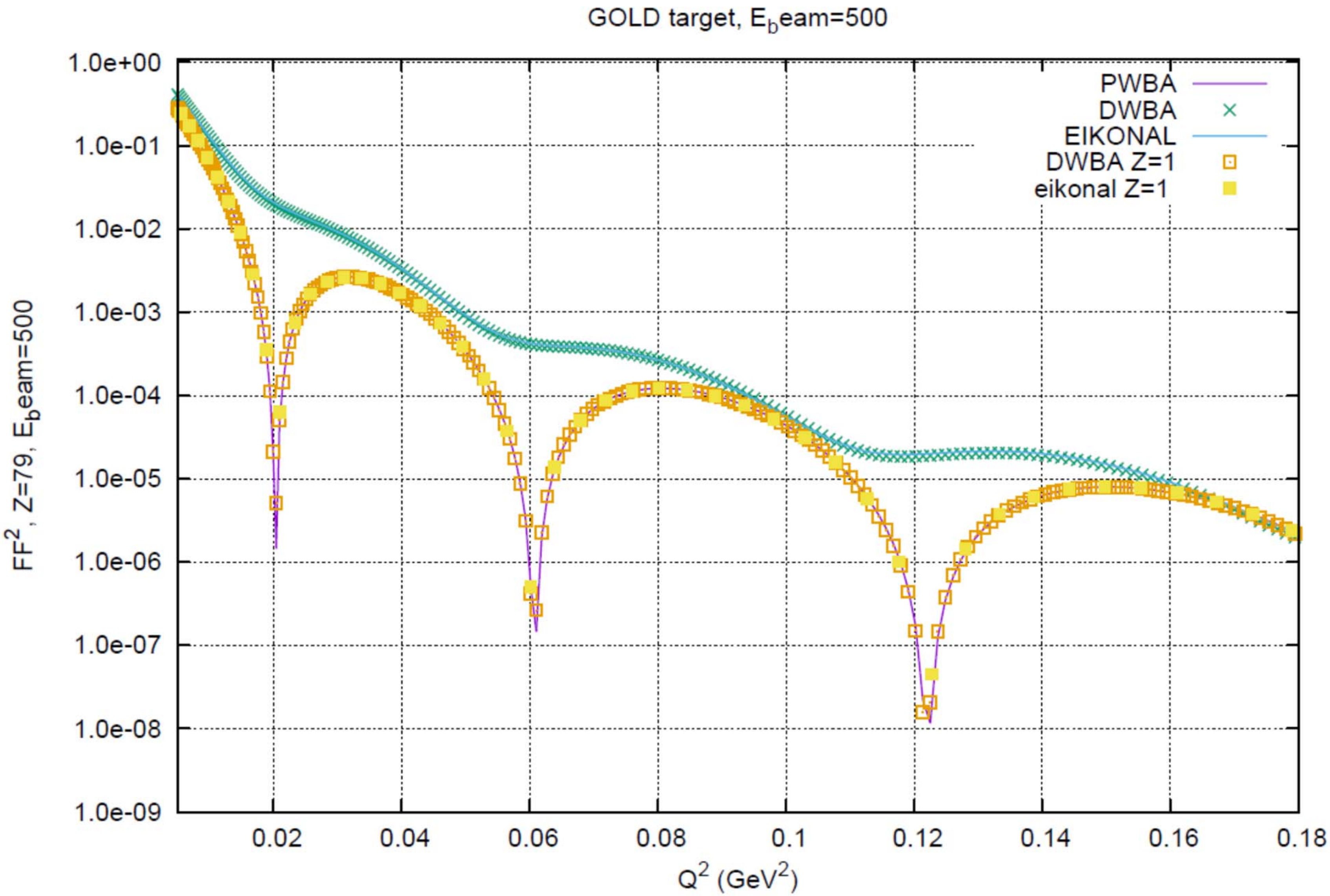
The calculation is more involved, and not only because we have to compute the current for the electron explicitly (not a plane wave anymore).

Below say 500 MeV of energy of the electrons, we should use a partial wave calculation, solving exactly the Dirac equation.

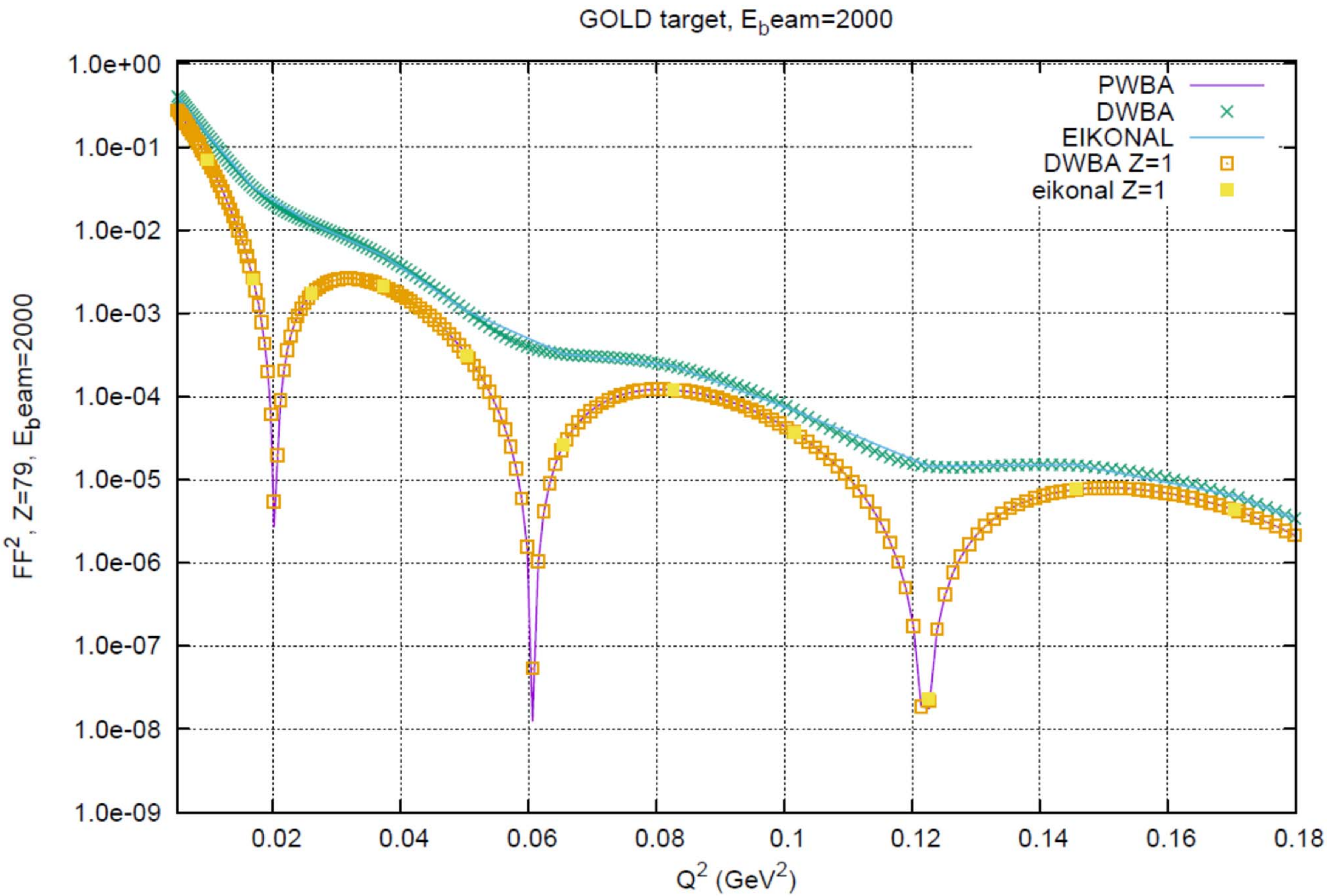
Above 500 MeV we could use one of the several prescriptions available for eikonal approximations.

Calculations are numercally more involved, but anyway precise results can be obtained. Typically if we can get the matrix element in PWBA for a range of q values, the DWBA matrix element can be obtained as an 1-dimensional integral on q of of said matrix element folded in with the (non plane wave) electron current.
It can be done in different ways, with advantages and disavantages

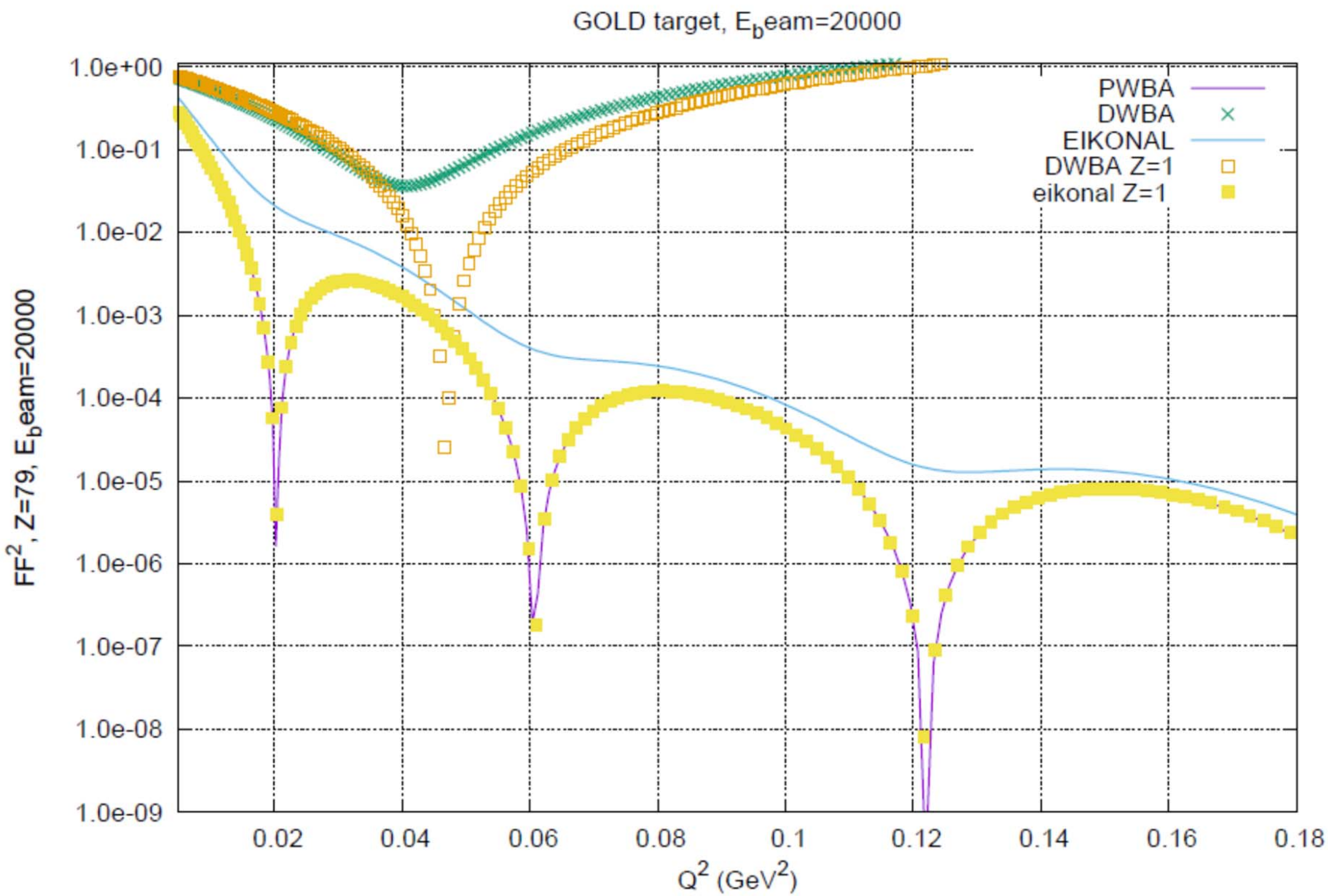

GOLD target, E_beam=250
FF^2, Z=79, E_beam=250
Q^2 (GeV^2)
PWBA
DWBA
EIKONAL
DWBA Z=1
eikonal Z=1
1.0e+00
1.0e-01
1.0e-02
1.0e-03
1.0e-04
1.0e-05
1.0e-06
1.0e-07
1.0e-08
1.0e-09
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18



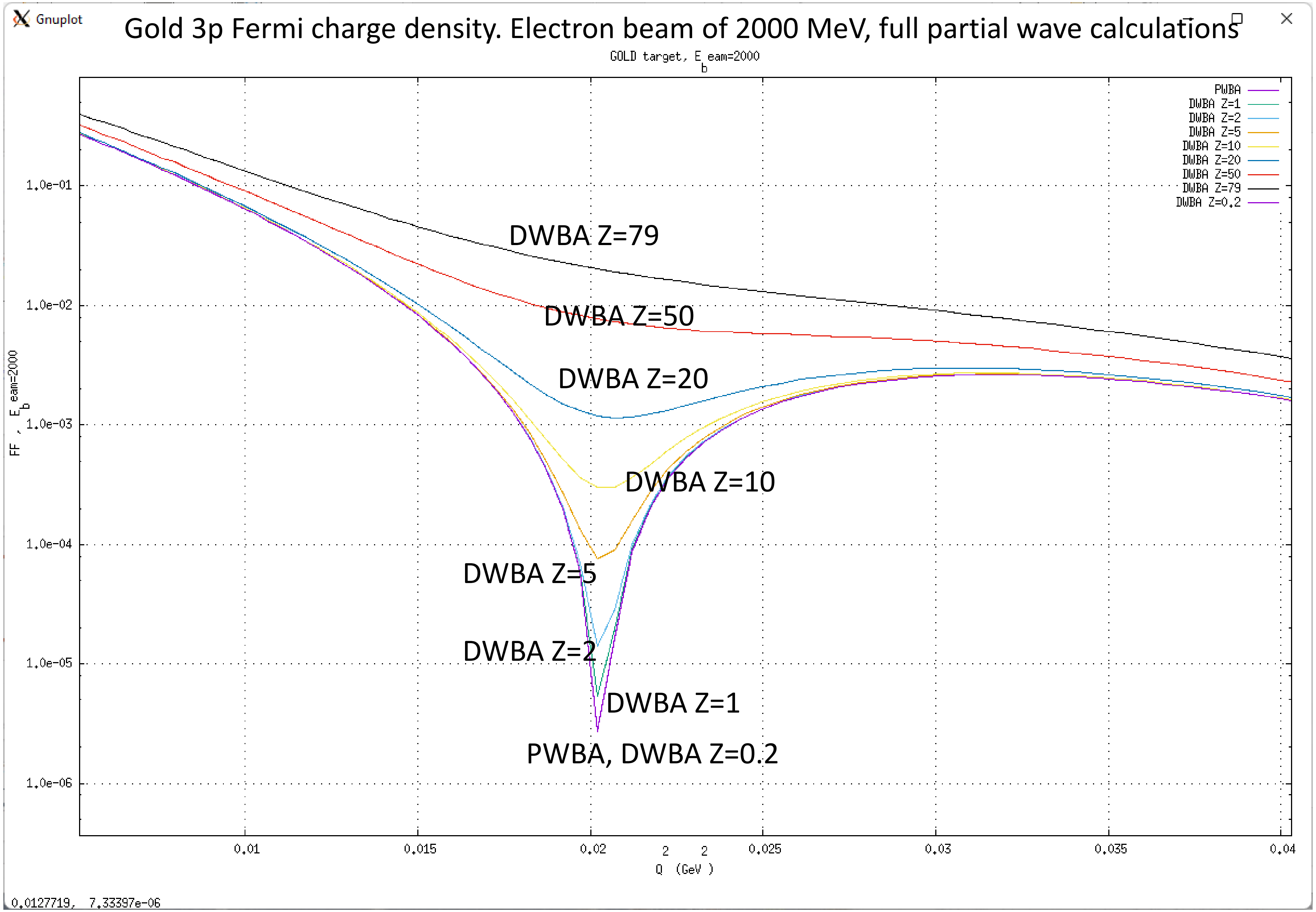
GOLD target, E_beam=500
PWBA
DWBA
EIKONAL
DWBA Z=1
eikonal Z=1
FF^2, Z=79, E_beam=500
Q^2 (GeV^2)
1.0e+00
1.0e-01
1.0e-02
1.0e-03
1.0e-04
1.0e-05
1.0e-06
1.0e-07
1.0e-08
1.0e-09
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18



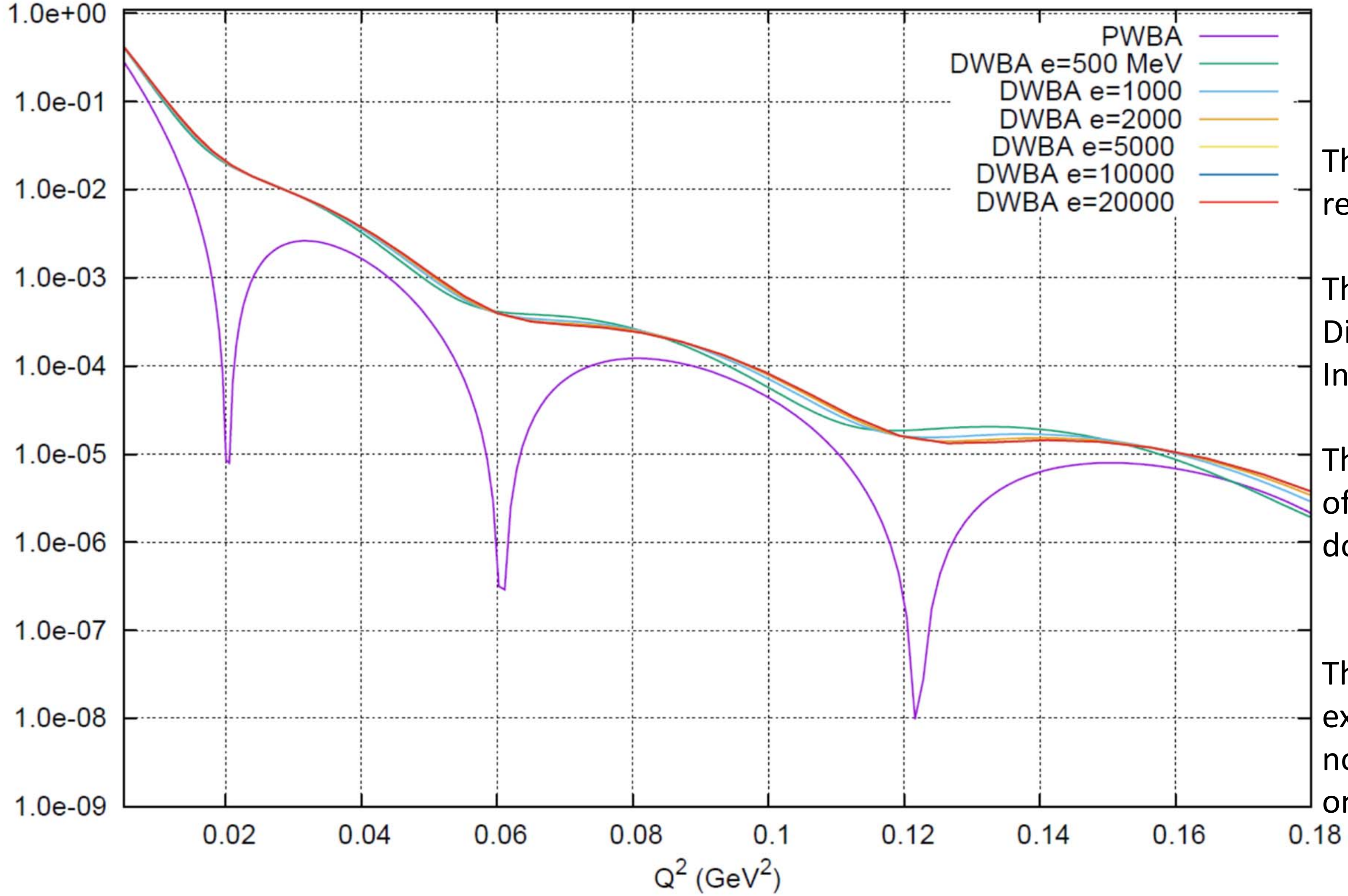
GOLD target, E_beam=2000
PWBA
DWBA
EIKONAL
DWBA Z=1
eikonal Z=1
FF^2, Z=79, E_beam=2000
Q^2 (GeV^2)
1.0e+00
1.0e-01
1.0e-02
1.0e-03
1.0e-04
1.0e-05
1.0e-06
1.0e-07
1.0e-08
1.0e-09
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18



GOLD target, E_beam=20000
FF^2, Z=79, E_beam=20000
Q^2 (GeV^2)
PWBA
DWBA
EIKONAL
DWBA Z=1
eikonal Z=1
1.0e+00
1.0e-01
1.0e-02
1.0e-03
1.0e-04
1.0e-05
1.0e-06
1.0e-07
1.0e-08
1.0e-09
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18



Gnuplot
Gold 3p Fermi charge density. Electron beam of 2000 MeV, full partial wave calculations
GOLD target, E_beam=2000
PWBA
DWBA Z=1
DWBA Z=2
DWBA Z=5
DWBA Z=10
DWBA Z=20
DWBA Z=50
DWBA Z=79
DWBA Z=0.2
1.0e-01
1.0e-02
1.0e-03
1.0e-04
1.0e-05
1.0e-06
FF , E_beam=2000
DWBA Z=79
DWBA Z=50
DWBA Z=20
DWBA Z=10
DWBA Z=5
DWBA Z=2
DWBA Z=1
PWBA, DWBA Z=0.2
0.01
0.015
0.02
0.025
0.03
0.035
0.04
Q² (GeV²)
0.0127719, 7.33397e-06



The shift of Q (effective Q) gets reduced with beam energy

The focusing effect does not Dissappear nor gets reduced with Increasing electron energy!!!!

This is due to the nature of the Coulomb potential, which does not go to zero fast enough

This is also the reason why the expansions in power of Zα/k are not very practical, they converge only asymptotically

Coulomb distortions and CS (DPVM) at the EIC

The matrix element would be very similar to the one portrayed here. It will show Coulomb focusing, even at the very high energies of the electrons. The effect of Coulomb distortions would be very easily computed with a simple eikonal calculation. It will not preclude comparison to data

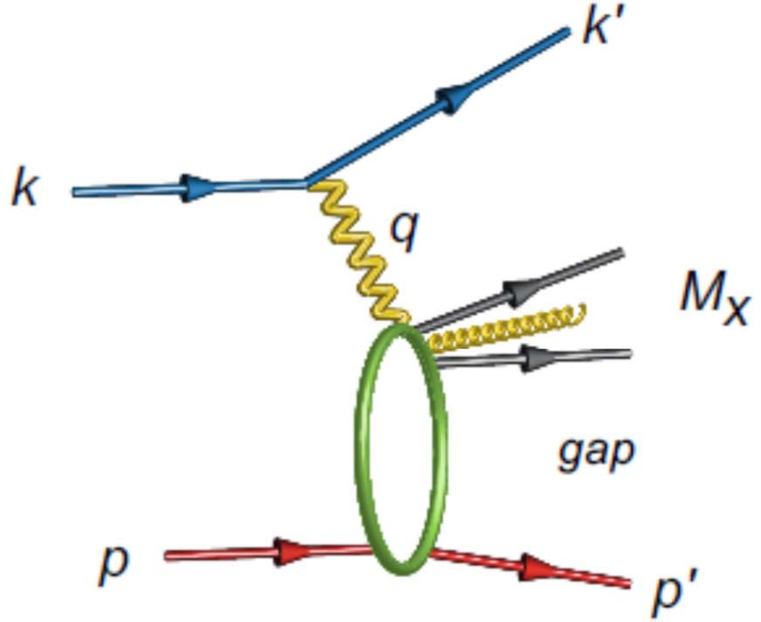


Fig. 33. Kinematic quantities for the description of a diffractive event.

Summary and conclusions

- Coulomb distortions are intrinsic to electron scattering for nuclei with Z>1. Comparison to data requires introducing Coulomb distortion in the theoretical calculation. There is no conceptual difficulty, even if the calculations lose the extremely appealing simplicity of PWBA. Anyway, it is just a technological concern, but the technology is solid and very mature.

- There are some sizeable effects of Coulomb distortion that remain even at very high beam energies.

- Introduction of Coulomb distortion as a simple 1-dimensional convolution of PWBA calculations is possible within a general framework, after the work of Verlee van der Sluys from the Ghent group in the early 90's.