Introduction to Color Glass Condensate

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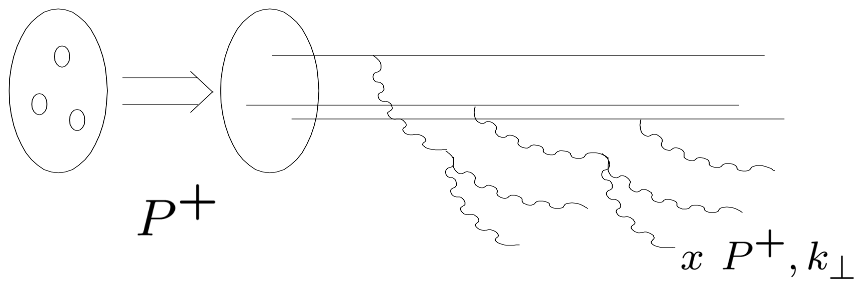
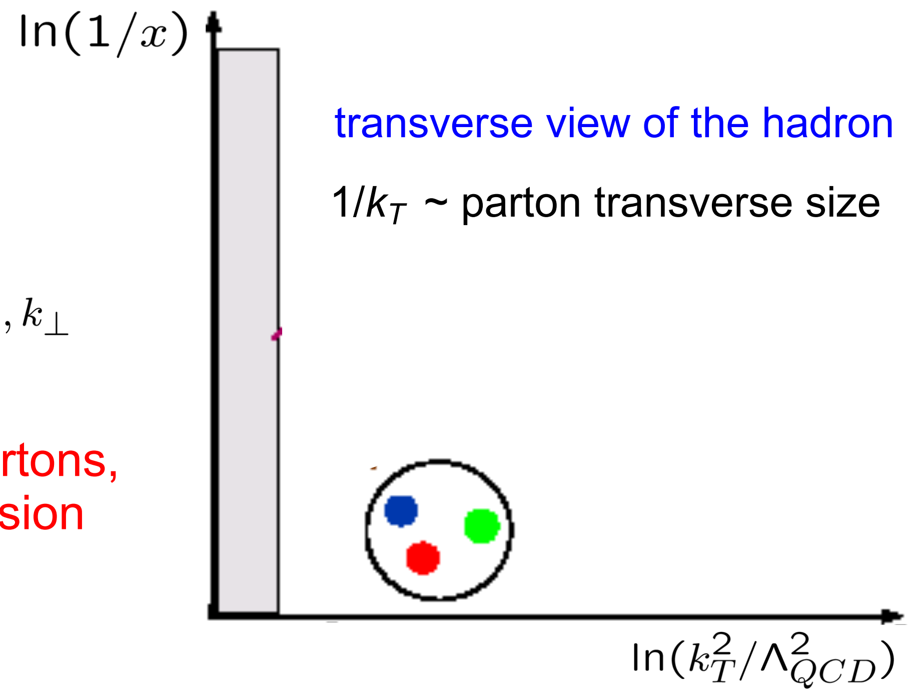
Parton saturation at small x

From independent partons...

the parton content of high-energy hadrons:





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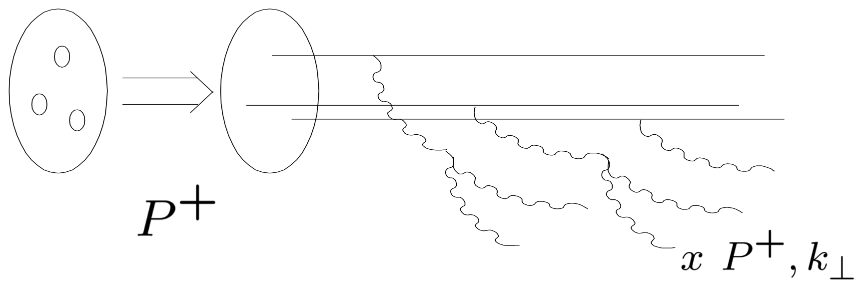
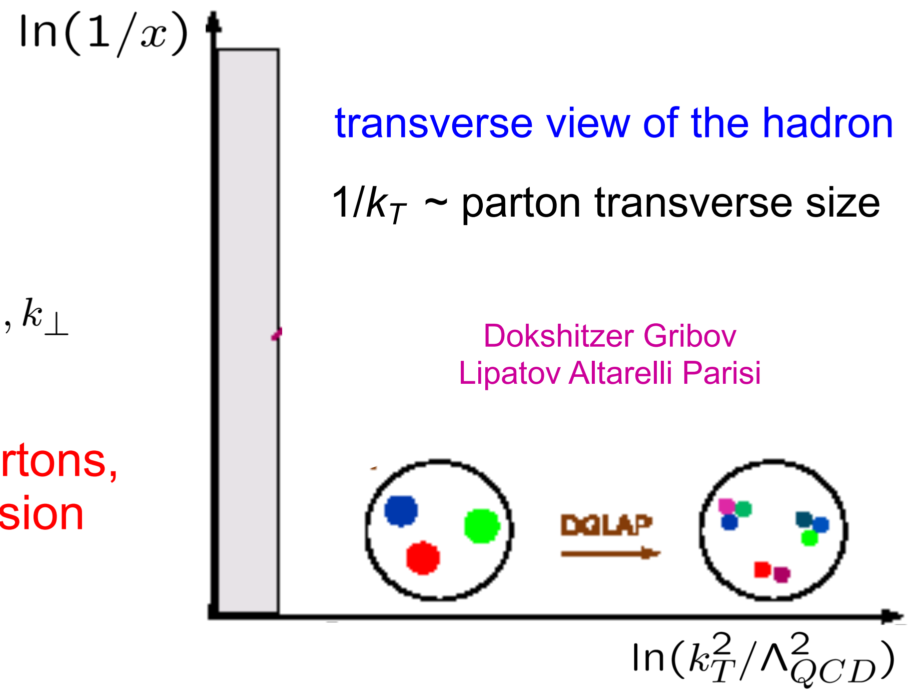




when a hadron is a dilute system of partons, they interact incoherently during a collision

standard QCD evolution: as k_T increases, the hadron gets more dilute

standard QCD factorization: probabilistic sum of partonic cross-sections

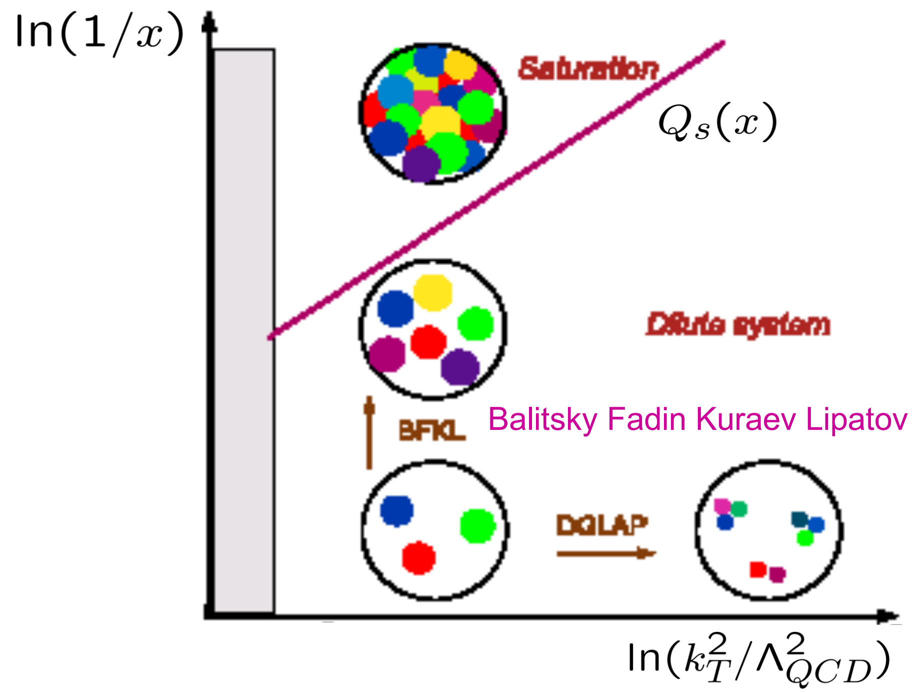
$$d\sigma_{AB\to X} = \sum_{ij} \int dx_1 dx_2\ f_{i/A}(x_1,\mu^2) f_{j/B}(x_2,\mu'^2)\ d\hat{\sigma}_{ij\to X} + \mathcal{O}\left(\Lambda_{QCD}^2/M^2\right)$$

...to collective behavior

when x gets smaller and smaller, the hadron is no longer dilute, the partons start interacting coherently

the Λ^2_{QCD}/M^2 power corrections get enhanced by $x^{-\lambda}$

for heavy-nuclei, those density effect are further amplified by $A^{1/3}$



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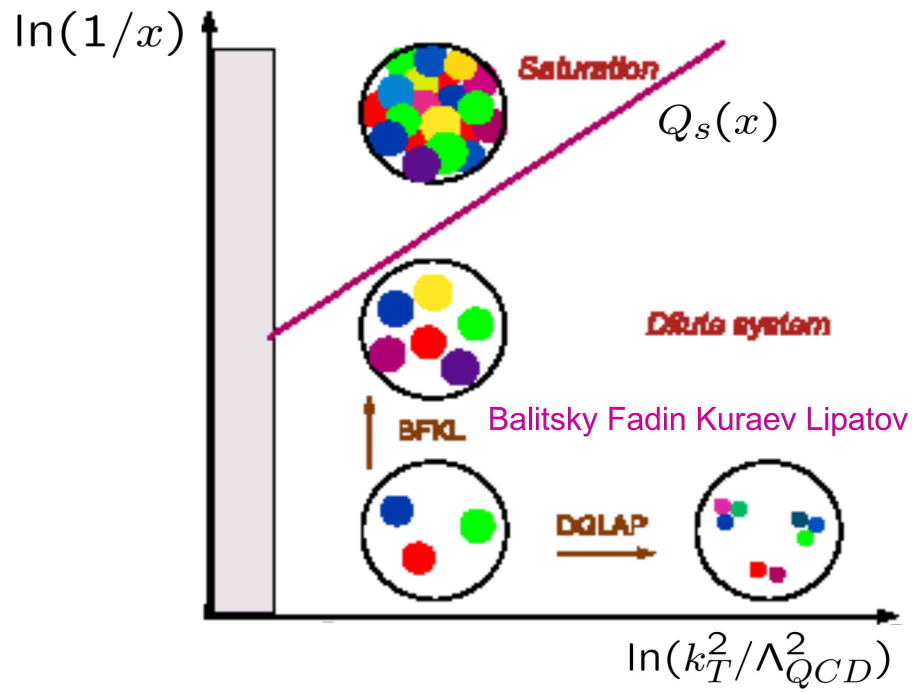


an alternate long-distance/short-distance factorization scheme is needed

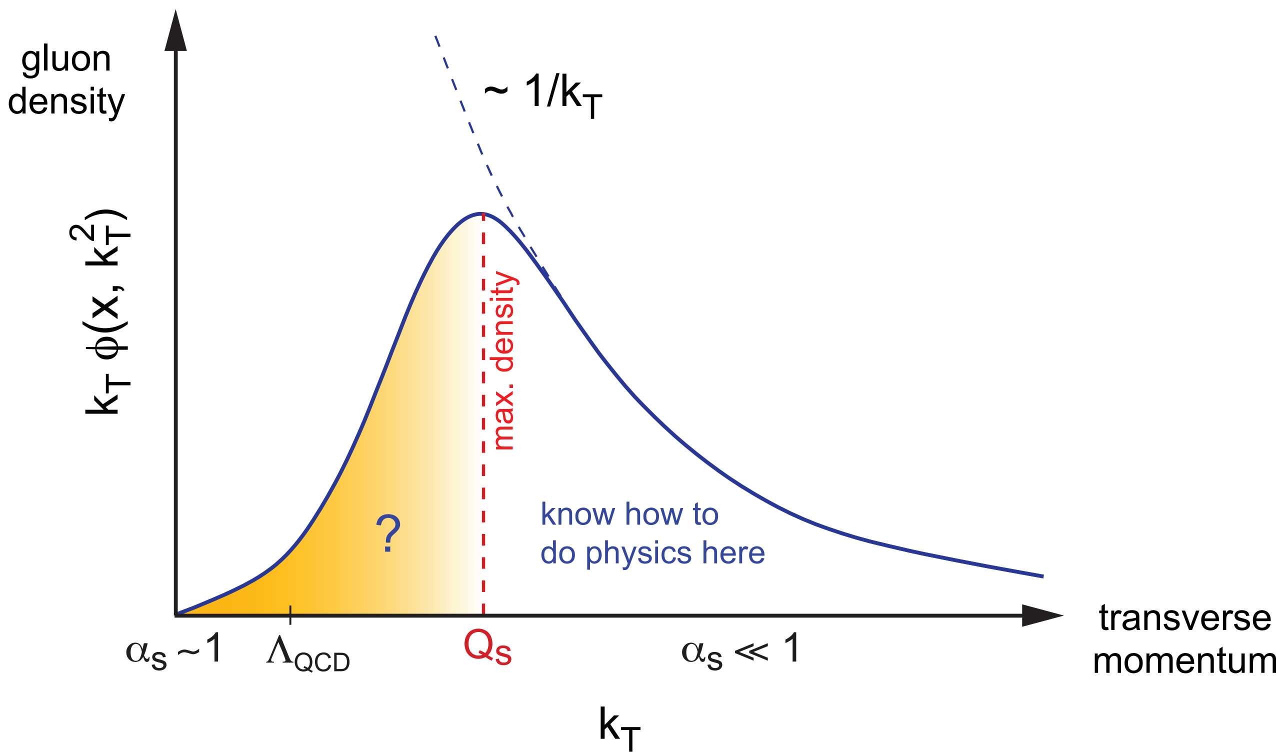
it involves effective degrees of freedom (Wilson lines, Reggeized gluons, …), new operators governed by an effective action (Color Glass Condensate, Lipatov's action, …)

→ an approximation of QCD suited to describe physics at large parton densities

The saturation scale

The saturation scale $Q_S(x)$ is the momentum scale which characterizes the transition between the dilute and dense regimes

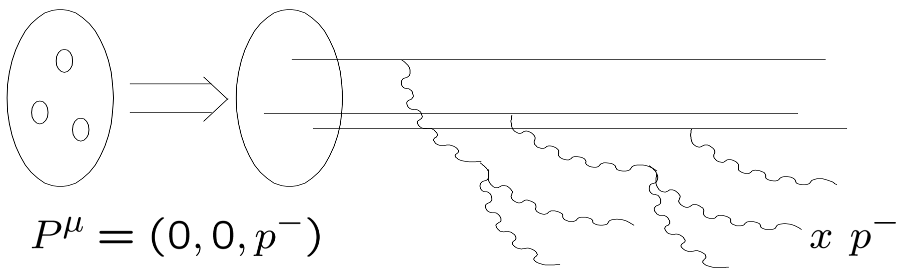
at small-x, the typical gluon transverse momentum is no more Λ_{QCD} , it is instead $Q_S(x)$



the dynamics is non-linear, but the theory stays weakly coupled $\alpha_s(Q_s) \ll 1$

Color Glass Condensate

Effective description of the hadron/nucleus



McLerran and Venugopalan (1994)

the numerous small-x gluons are responsible for a large color field

$$A^\mu = \delta^{\mu -} T^c A_c^- \sim 1/g_s$$

which can be treated as a classical field

$x\ p^-$: smallest value of longitudinal impulsion

$Y = \ln(1/x)$ is called the hadron rapidity

To describe a hadron dressed with many small-x gluons, we use an effective theory:

$$|\text{hadron}\rangle = |qqq\rangle + |qqqg\rangle + \ldots + |qqq.....ggggg\rangle$$

$$\Downarrow$$

effective wavefunction for the dressed hadron

$$|\text{hadron}\rangle = \int D\alpha\ \Phi_Y[\alpha]\ |\alpha\rangle$$

$$\alpha_c(x^+, \mathbf{x}) = A_c^-(x^+, \mathbf{x}, x^- = 0)$$

light-cone gauge $A^+ = 0$

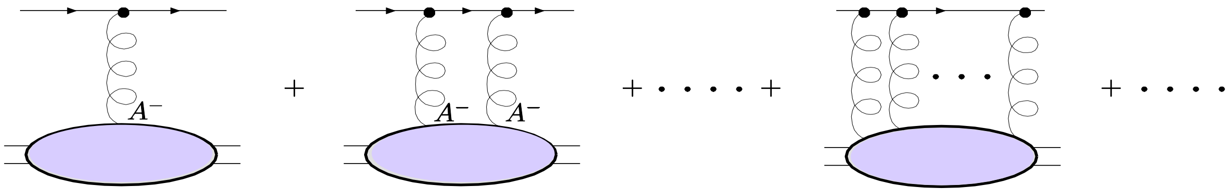
α : large color fields created by the small-x gluons $\quad \alpha_c \sim 1/g_s$

Scattering off the CGC

- this is described by Wilson lines $W_F[\alpha](\mathbf{x}) = \mathcal{P}\exp\left\{ig_s\int dx^+ T^c \alpha_c(x^+,\mathbf{x})\right\}$

α dependence kept implicit in the following

scattering of a quark:



in the CGC framework, any cross-section is determined by colorless combinations of Wilson lines $S[\alpha]$, averaged over the CGC wave function $\langle S\rangle_x = \int D\alpha\ |\Phi_x[\alpha]|^2 S[\alpha]$

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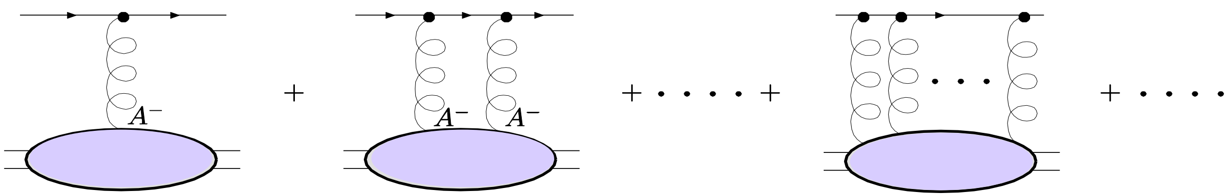

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- the 2-point function or dipole amplitude $T_{\mathbf{xy}}[\alpha] = 1 - \frac{1}{N_c} Tr(W_F^+(\mathbf{y})W_F(\mathbf{x}))$

the $q\bar{q}$ dipole scattering amplitude: $\langle T_{\mathbf{xy}}\rangle_x$

x : quark space transverse coordinate
y : antiquark space transverse coordinate

this is the most common CGC average
it determines deep inelastic scattering

Evolution of the wave function

- the JIMWLK equation   Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner

the energy evolution of cross-sections is encoded in the evolution of $\left|\Phi_x[\alpha]\right|^2$

this wave function is mainly non-perturbative, but its evolution is known

$$\frac{d}{d\ln(1/x)}\left|\Phi_x[\alpha]\right|^2 = H^{JIMWLK} \otimes \left|\Phi_x[\alpha]\right|^2$$

$\Rightarrow$ Balitsky hierarchy for Wilson lines correlators

sums both $\alpha_s^n \ln^n(1/x)$ and $g_s^n \alpha^n$

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- the BK equation Balitsky (1996), Kovchegov (1998)

the BK equation is a closed equation for $\langle T_{\mathbf{xy}} \rangle_Y$ obtained by assuming $\langle T_{\mathbf{xz}} T_{\mathbf{zy}} \rangle_Y = \langle T_{\mathbf{xz}} \rangle_Y \langle T_{\mathbf{zy}} \rangle_Y$

$$\frac{d}{dY}\langle T_{\mathbf{xy}} \rangle_Y = \bar{\alpha} \int \frac{d^2z}{2\pi} \frac{(\mathbf{x}-\mathbf{y})^2}{(\mathbf{x}-\mathbf{z})^2(\mathbf{z}-\mathbf{y})^2} \left(\langle T_{\mathbf{xz}} \rangle_Y + \langle T_{\mathbf{zy}} \rangle_Y - \langle T_{\mathbf{xy}} \rangle_Y - \langle T_{\mathbf{xz}} \rangle_Y \langle T_{\mathbf{zy}} \rangle_Y \right)$$

robust only for impact-parameter independent solutions $\langle T_{\mathbf{xy}} \rangle_Y = N_Y(r = |\mathbf{x}-\mathbf{y}|)$

r = dipole size

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- the gluon distribution in momentum space:

$$f_Y(k) = \int \frac{d^2r}{2\pi r^2}\, e^{i\mathbf{k}.\mathbf{r}} N_Y(r) \Rightarrow \frac{d}{dY}f_Y(k) = \bar{\alpha}\int\frac{dk'^2}{k'^2}\left[\frac{k'^2 f_Y(k') - k^2 f_Y(k)}{|k^2 - k'^2|} + \frac{k^2 f_Y(k)}{\sqrt{4k'^4 + k^4}}\right] - \bar{\alpha} f_Y^2(k)$$

Back to the wavefunction phase diagram

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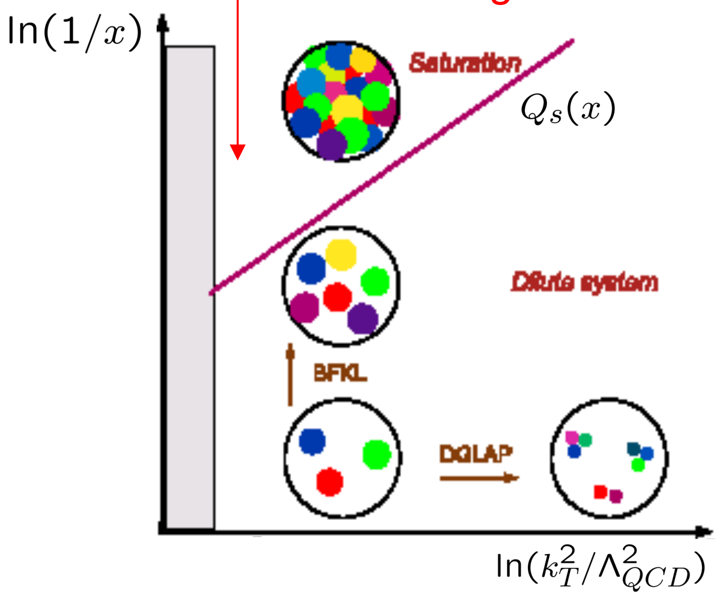
$$Y = \ln\left(\frac{1}{x}\right) \qquad \bar{\alpha} = \frac{\alpha_s N_c}{\pi}$$

non-linearity important when the gluon density becomes large



the distribution of partons as a function of x and k_T

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BFKL

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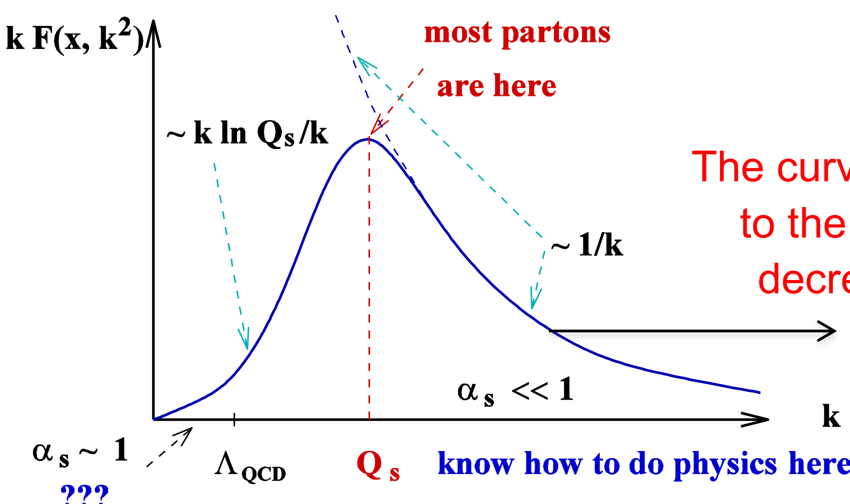
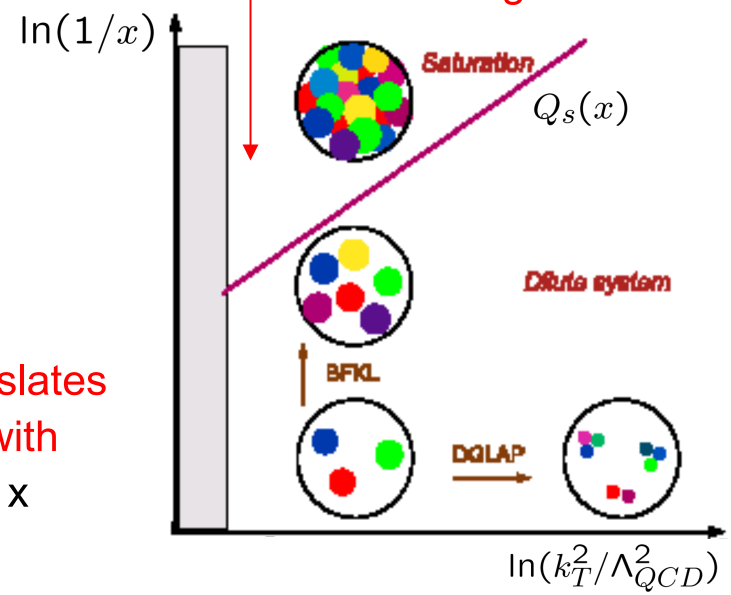
- solutions: qualitative behavior



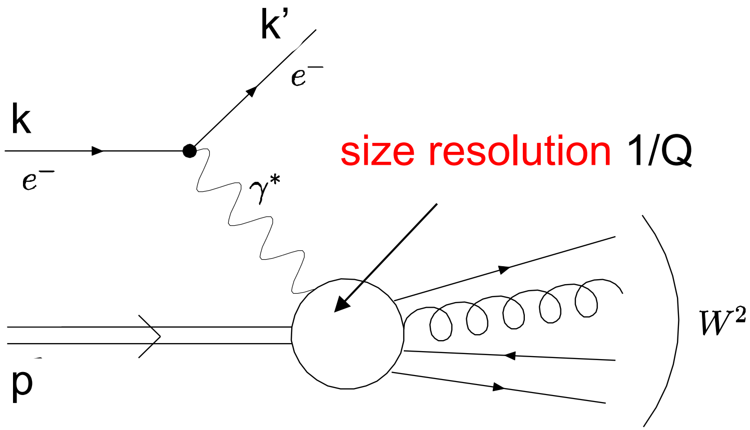
The curve translates to the right with decreasing x



the distribution of partons as a function of x and k_T

CGC in e+p(A) context

Deep inelastic scattering (DIS)



size resolution 1/Q

eh center-of-mass energy
S = $(k+P)^2$

$\gamma^* h$ center-of-mass energy
$W^2 = (k-k'+P)^2$

photon virtuality
$Q^2 = -(k-k')^2 > 0$

$$x = \frac{Q^2}{2P.(k-k')} = \frac{Q^2}{W^2 - M_h^2 + Q^2}$$

$$y = \frac{P.(k-k')}{P.k} = \frac{Q^2/x}{S - M_h^2}$$

x ~ momentum fraction of the struck parton

y ~ W^2/S

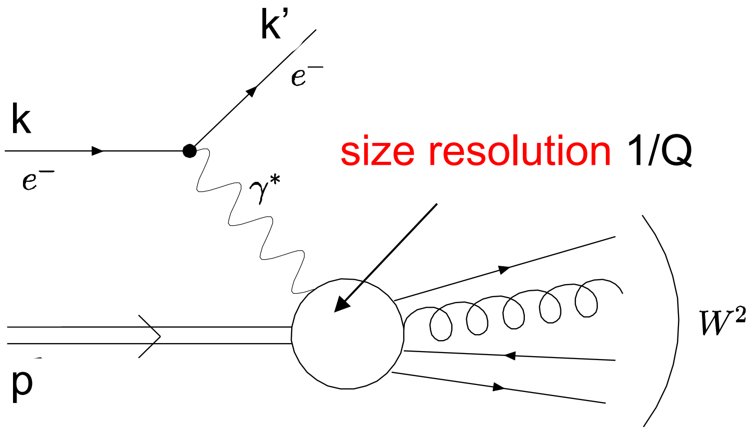
$$\frac{d^2\sigma^{eh\to eX}}{dx dQ^2} = \frac{4\pi\alpha_{em}^2}{xQ^4}\left[\left(1 - y + \frac{y^2}{2}\right)F_2(x,Q^2) - \frac{y^2}{2}F_L(x,Q^2)\right]$$

measures

quark distributions

gluon distribution

Deep inelastic scattering (DIS)



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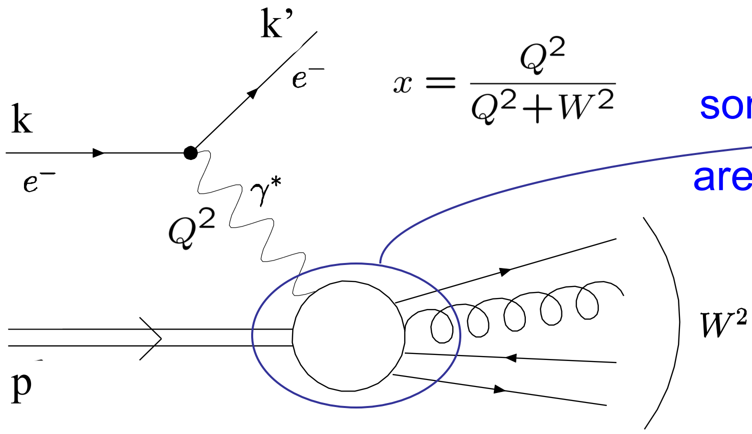
- in terms of transverse (T) and longitudinal (L) photon contributions:

$$\frac{d^2\sigma^{eh\to eX}}{dx dQ^2} = \frac{4\pi\alpha_{em}^2}{xQ^4}\left[\left(1 - y + \frac{y^2}{2}\right) F_T(x,Q^2) + (1-y) F_L(x,Q^2)\right]$$

perturbative Q^2 and F_L sensitivity (y not too close to unity)
→ restricted small x range

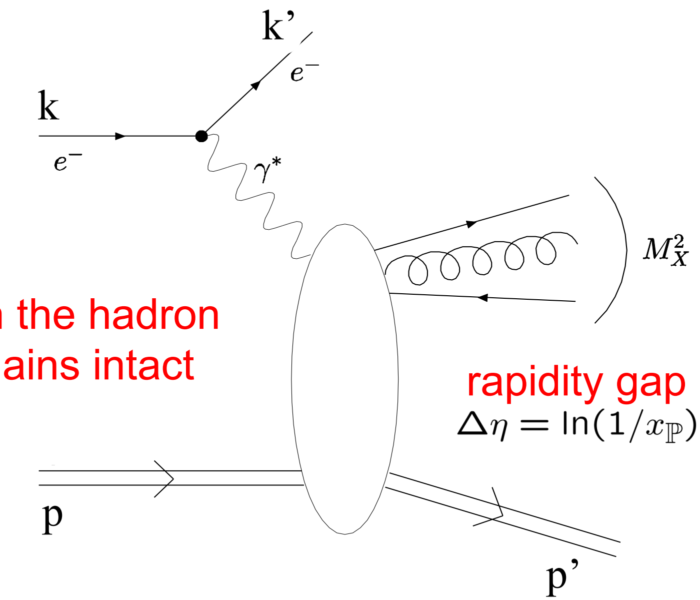
Inclusive diffraction in DIS



$$x = \frac{Q^2}{Q^2 + W^2}$$

some events are diffractive

when the hadron remains intact



rapidity gap
$\Delta\eta = \ln(1/x_{\mathbb{P}})$

diffractive mass

$M_X^2 = (p\text{-}p' + k\text{-}k')^2$

momentum transfer

$t = (p\text{-}p')^2 < 0$

$$\beta = \frac{Q^2}{2(p-p').(k-k')} = \frac{Q^2}{M_X^2 - t + Q^2}$$

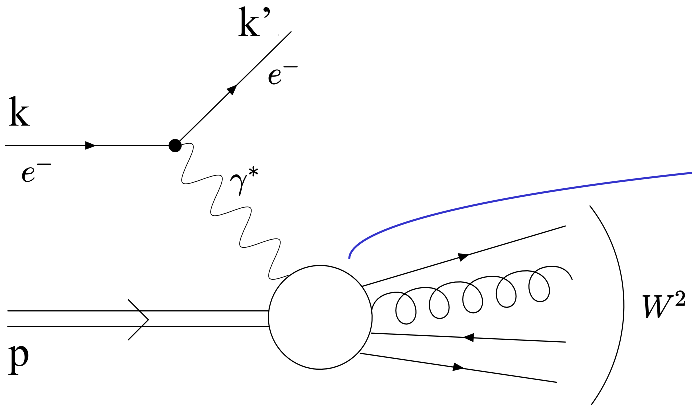
$$x_{\mathbb{P}} = x/\beta$$

momentum fraction of the exchanged object (Pomeron) with respect to the hadron

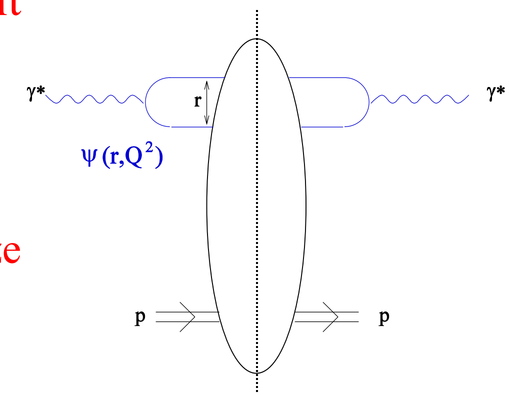
- the measured cross-section

$$\frac{d^4\sigma^{eh\to eXh}}{dx dQ^2 d\beta dt} = \frac{4\pi\alpha_{em}^2}{\beta^2 Q^4}\left[\left(1 - y + \frac{y^2}{2}\right) F_2^{D,4}(x,Q^2,\beta,t) - \frac{y^2}{2} F_L^{D,4}(x,Q^2,\beta,t)\right]$$

The dipole picture of DIS



valid in the small-x limit



r : dipole size

$$\sigma_{tot}^{\gamma^* h\to X}(x,Q^2) = \int d^2r \;|\psi(r,Q^2)|^2 \;\sigma_{q\bar{q}}(r,x)$$

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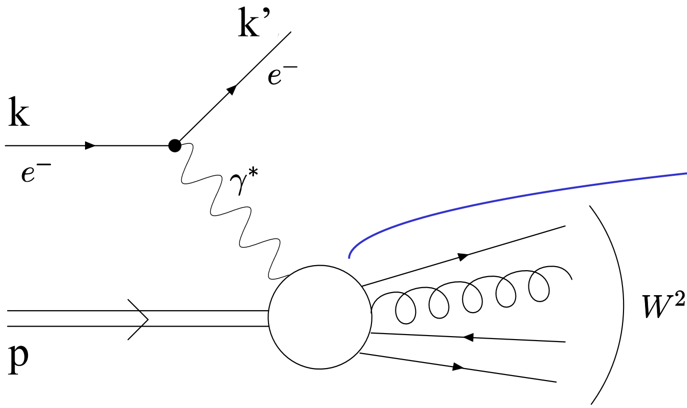

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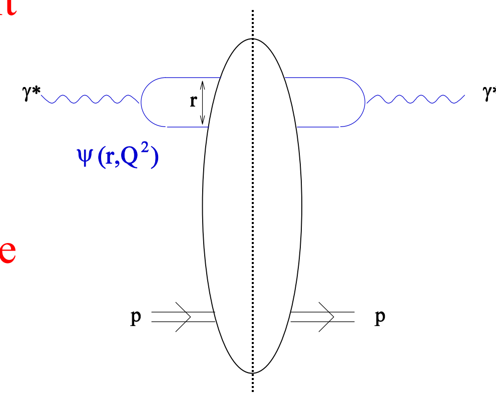

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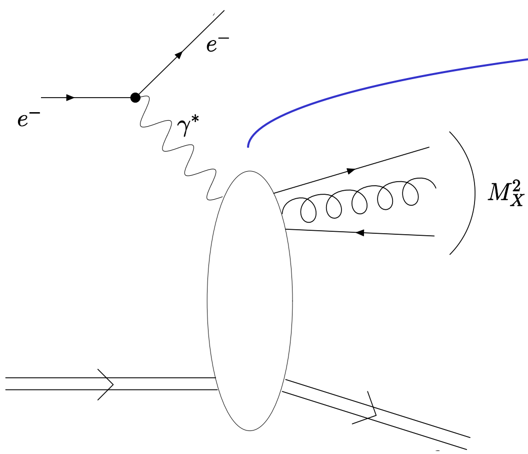
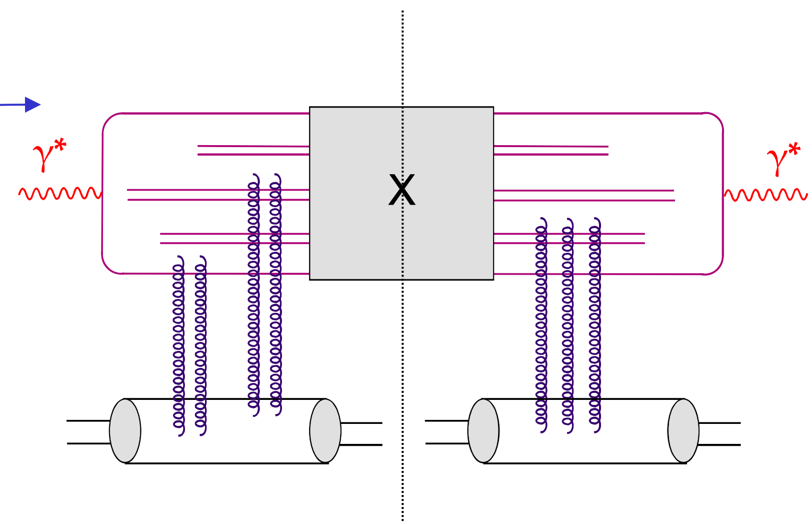
in diffraction:



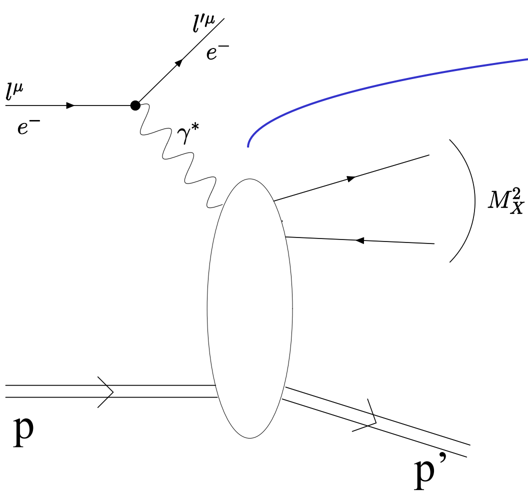


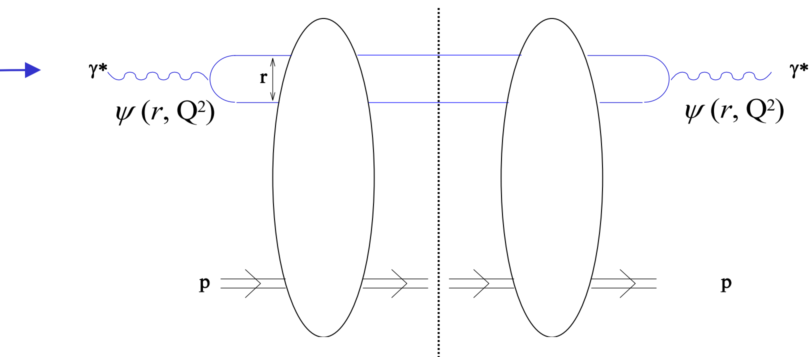
at large Nc, 1 dipole emitting N-1 gluons = N dipoles

Elastic/inelastic components

$$\sigma_{diff} = \sigma_{elas} + \sigma_{dissoc}$$

σ_{elas}: involves the quark-antiquark final state, dominant for small diffractive mass (large β)





$$\sigma^{\gamma^* h \to Xh}(x, Q^2) = \int d^2r \ |\psi(r, Q^2)|^2 \ \sigma^2_{q\bar{q}}(r, x)$$

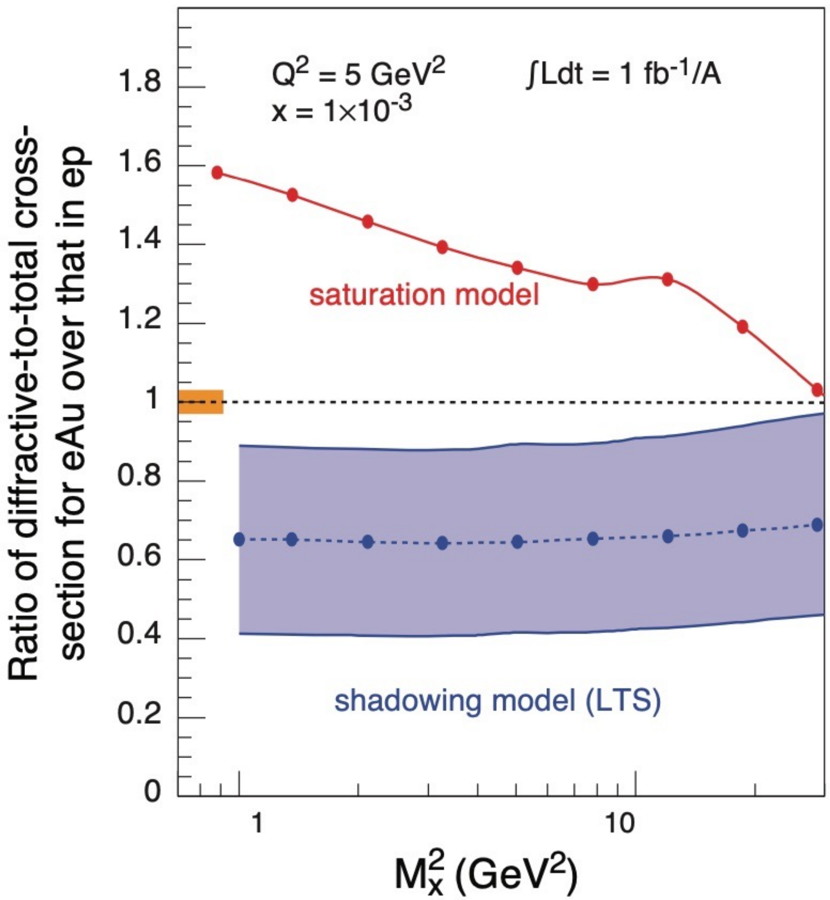
same object for inclusive
and diffractive cross-section

σ_{dissoc}: involves higher order final states: $q\bar{q}g$, …dominant for large diffractive mass (small β)

can also be expressed in the dipole picture

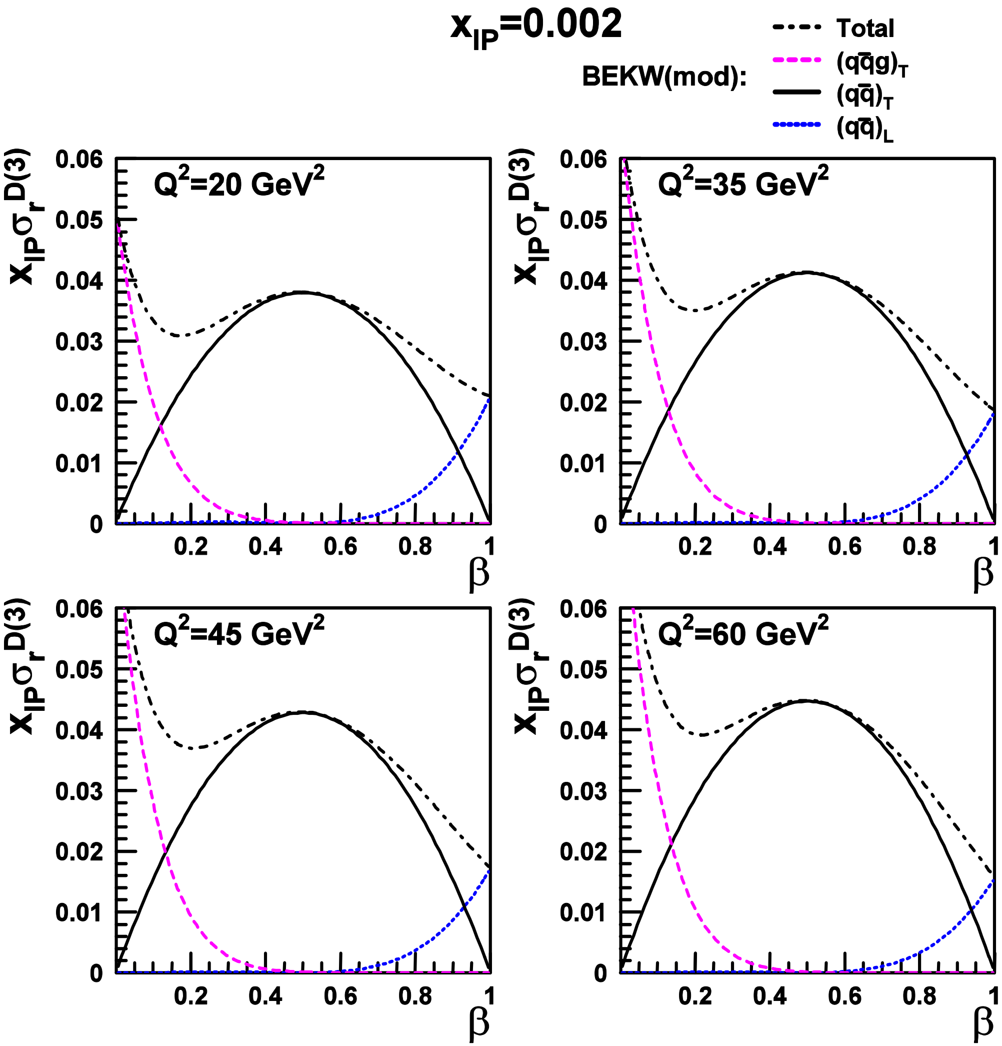
Hard diffraction in DIS

observable subject to strong non-linear effects,
even with Q^2 values significantly bigger than Q_S^2



clean and unambiguous signal of saturation

Measuring F_L^D?



Contributions of the different final states to the diffractive cross-section:

at small β: quark-antiquark-gluon

at intermediate β: quark-antiquark (T)

at large β: quark-antiquark (L)

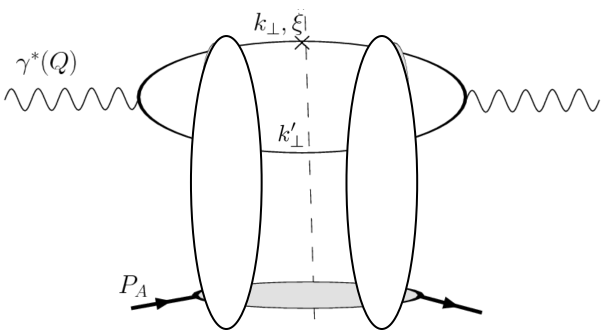
large β measurements $\Rightarrow F_L^D$

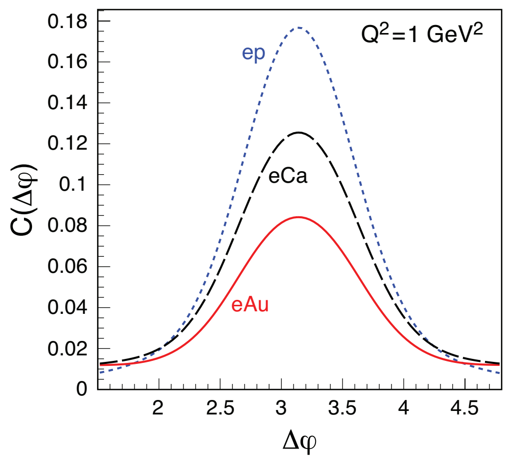
F_L^D is higher twist:

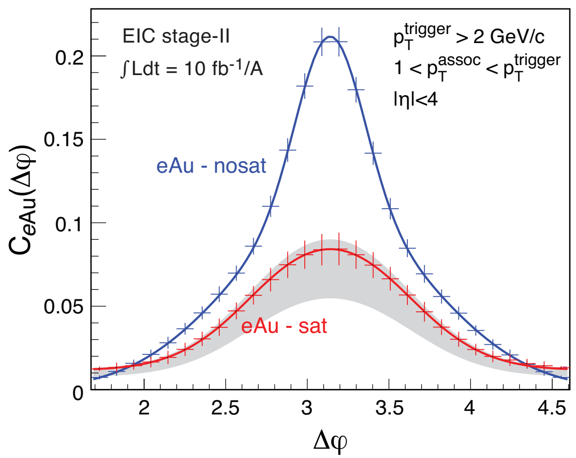
very small in collinear factorization

Di-hadrons in DIS

- away-side peak suppression in e+A vs e+p signals saturation







at the qualitative level:
similar effects as in p+A

however, the saturation signal may be blurred by large Sudakov logarithms

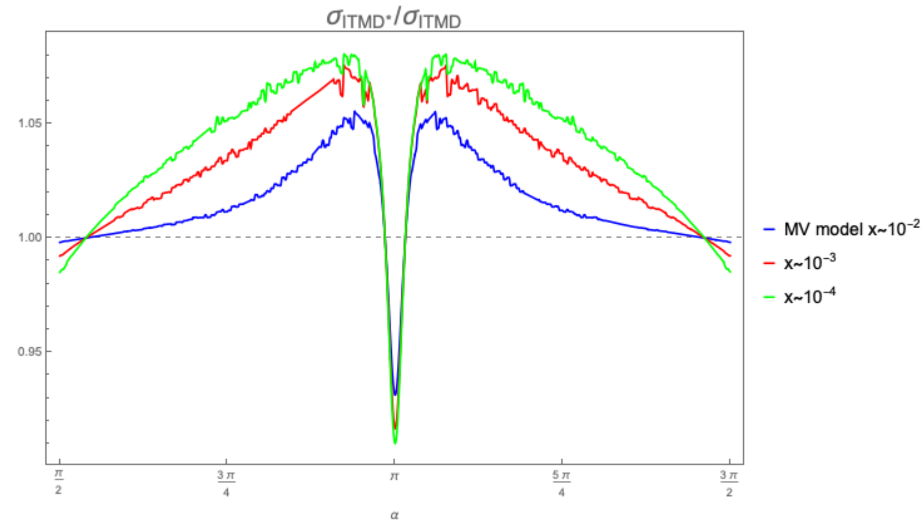
when the di-jets are initiated by a longitudinal photon,
saturation effects are large also away from the Sudakov region

Di-jets/di-hadrons in e+p/e+A

- nosat/sat ratio of the $\Delta\Phi$ distributions: Altinoluk, CM, Taels (2021)

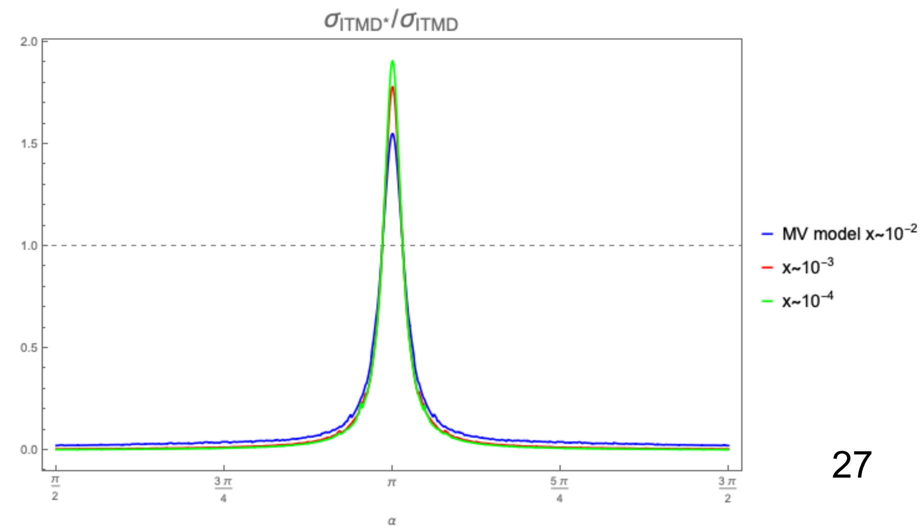
$$\frac{\text{without saturation}}{\text{with saturation}}$$

for transversely-polarized incoming photons, deviations from unity are located in the away-side peak



$$\frac{\text{without saturation}}{\text{with saturation}}$$

for longitudinally-polarized incoming photons, deviations from unity everywhere



Future Prospects

Higher-order corrections

the field of high-energy QCD has recently entered the NLO era: higher-order corrections of several kinds to be computed

- next to leading order in α_S: essential to prove factorization and assess robustness of predictions

in most cases, perturbation theory must be done in conjunction with all-order resummations of various large logarithms

- next-to-eikonal corrections: energy-suppressed but give access to spin-dependent observables

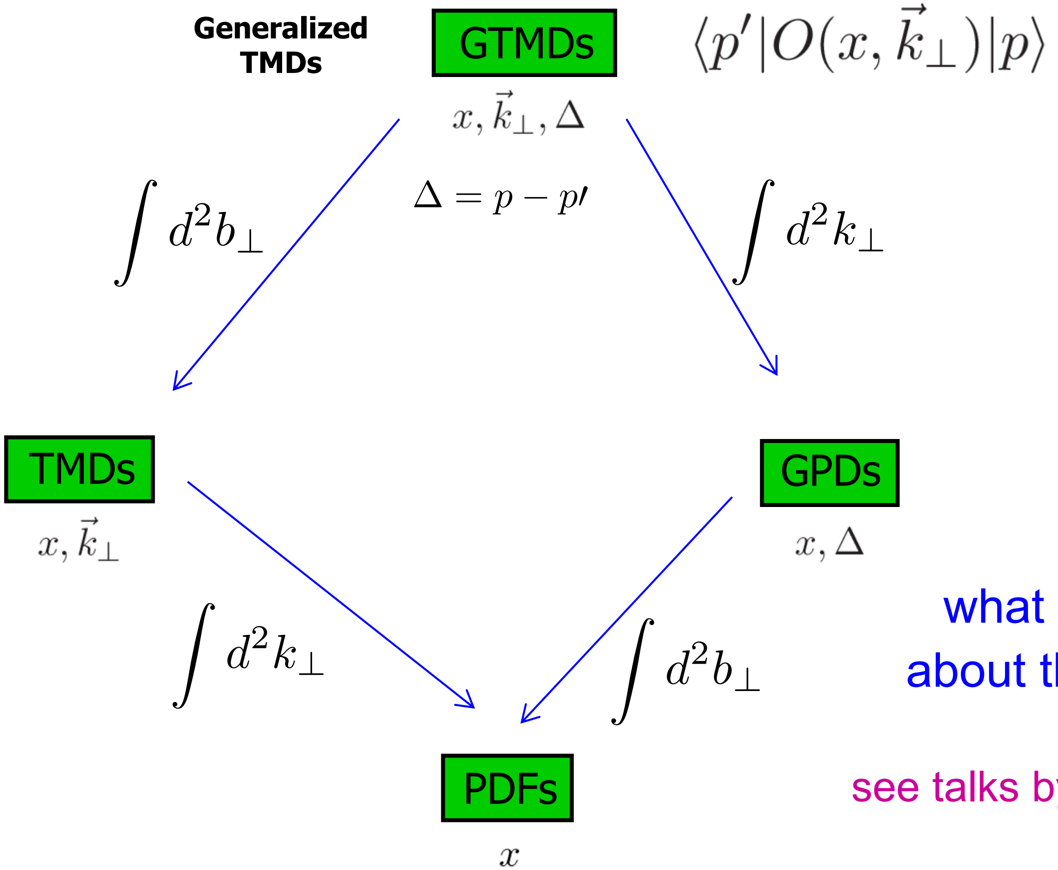
- next-to-planar corrections: going beyond the large-Nc limit

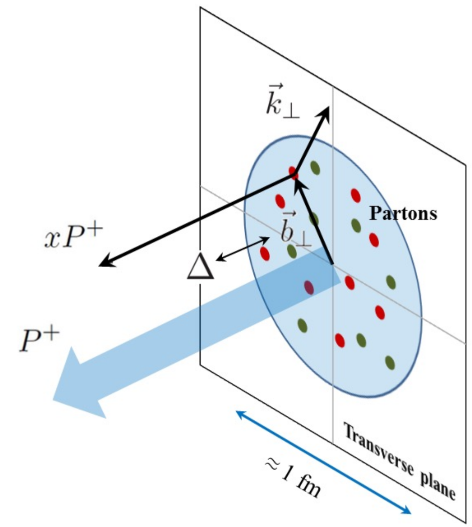
these must be addressed for less and less inclusive observables measured in experiments: exclusive and diffractive cross sections, correlation measurements, global event properties …

Hadron structure at small-x

establish the connections with the “standard” hadron-structure lore

especially important in the context of the EIC





what does small-x physics has to say
about those various parton distributions?

see talks by Y. Hatta (today) and F. Yuan (friday)

Thank you for your attention!