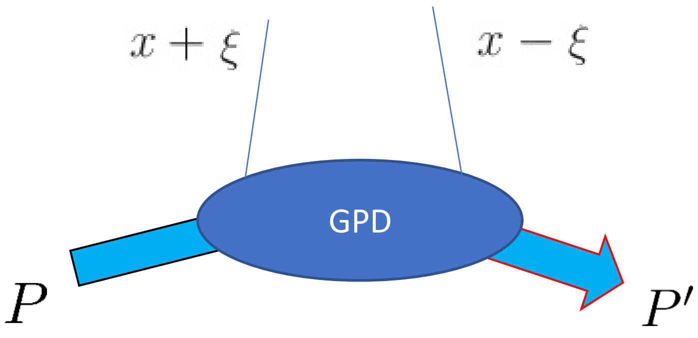
Exclusive reactions at low-x: GPD or color dipole?

Yoshitaka Hatta

BNL & RIKEN BNL

Workshop on a 2nd detector for the EIC, Temple University, May 17-19, 2023

Generalized Parton Distribution



Multidimensional tomography is one of the main scientific goals of the EIC.

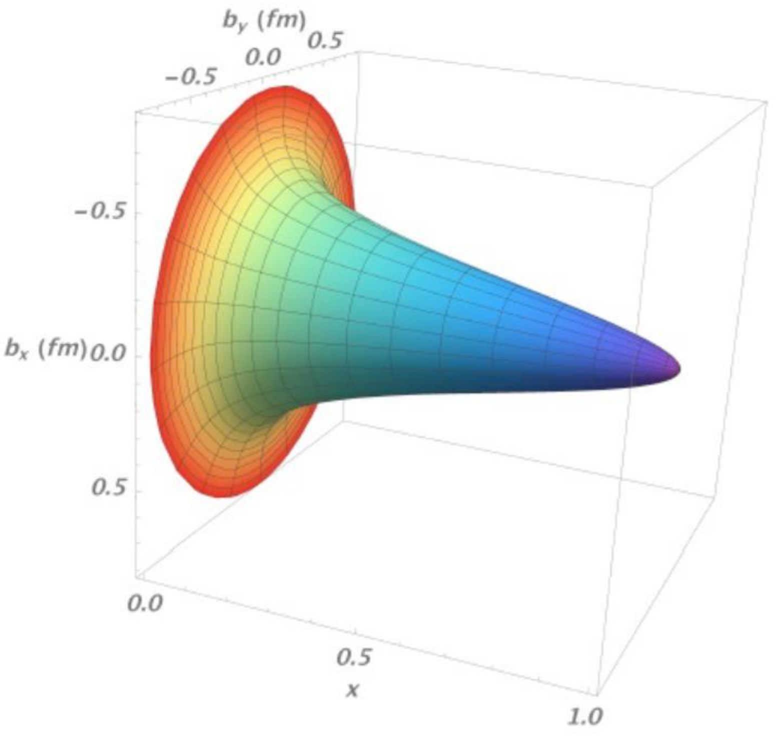
3D partonic imaging encoded in generalized parton distributions (GPDs)

$$f(x) \to f(x, \xi, t) \qquad \xi = \frac{P^+ - P'^+}{P^+ + P'^+} \quad \text{skewness}$$

$$t = (P' - P)^2 \approx -\Delta_\perp^2$$

Fourier transform $\Delta_\perp \to b_\perp$

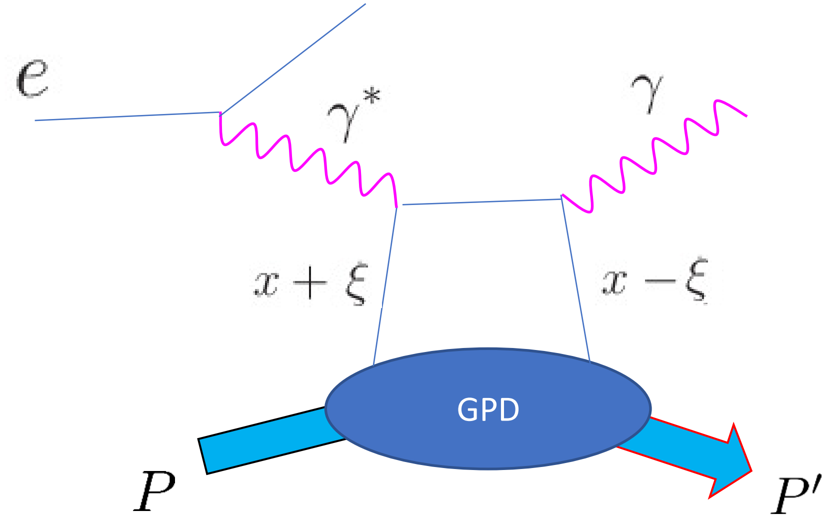
Distribution of partons in impact parameter space



First moment → elemag/axial form factors

Second moment → gravitational form factors

Deeply Virtual Compton Scattering



Factorization proof   Collins, Freund (1998); Ji, Osborne (1998)

$$i\int d^4y e^{iqy}\langle P'|T\{J^\mu(y)J^\nu(0)\}|P\rangle$$

$$= -(g^{\mu+}g^{\nu-}+g^{\nu+}g^{\mu-}-g^{\mu\nu})\int\frac{dx}{2}\left(\frac{1}{x+\xi-i\epsilon}+\frac{1}{x-\xi+i\epsilon}\right)H_q(x,\xi,\Delta)\bar{u}(P')\gamma^+u(P)+\cdots$$

Compton form factor

Theory developments

2-loop coefficient functions (singlet, unpol) Braun, Ji, Schenleber (2022)
3-loop evolution equation (nonsinglet) Braun, Manashov, Moch, Strohmaier (2017)

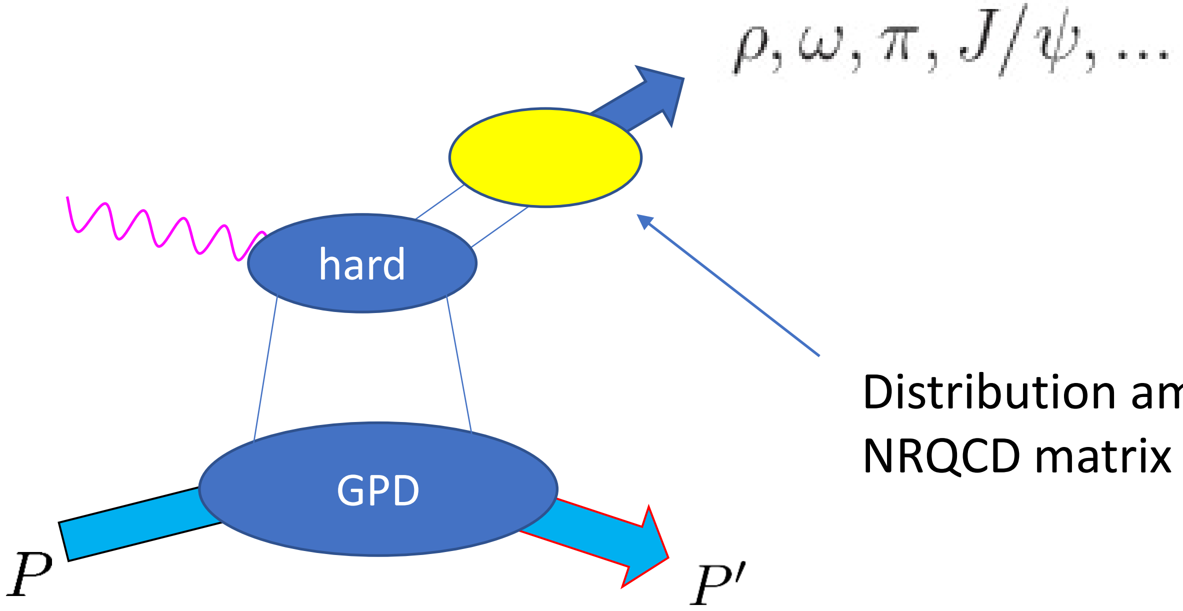
Connection between GPD and chiral/trace anomalies Bhattacharya, YH, Vogelsang (2022) preprint today

In principle, the ingredients for NNLO global analysis will be ready in near future
In practice, complete NLO global analysis is not achieved yet (but close).

Deeply Virtual Meson Production (DVMP)

QCD factorization when $Q^2 \to \infty$ — Collins, Frankfurt, Strikman (1996)

$M_{QQ} \to \infty$ — Ivanov, Schafer, Szymanowski, Krasnikov (2004)



Distribution amplitude for light mesons
NRQCD matrix element for heavy quarkonia

$$\phi_\pi(z) = \frac{-i}{f_\pi}\int \frac{dx^+}{2\pi} e^{-i(1-z)p^- x^+} \langle \pi(p)|\bar{\psi}(0)\gamma_5\gamma^- \frac{\tau^3}{2}\psi(x^+)|0\rangle$$

Small-x and GPD: general remarks

Not many discussions in the literature, the two communities usually don’t talk to each other…

Diehl (2003 review paper, Section 4.4)
Balitsky, Kuchina (2000), Goeke, Guzey, Siddikov (2007); YH, Xiao, Yuan (2017); YH, Zhou (2022)

At high energy, gluon GPDs are most important.

Amplitude dominantly imaginary, sensitive to GPDs at $x = \xi$

$$\int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H_g(x, \xi, t) \sim i\pi H_g(\xi, \xi, t)$$

In the context of GPDs, it is more correct to speak of “small- ξ “

Assume weak dependence on skewness $H_g(x, \xi, t) \approx H_g(x, 0, t)$

In the eikonal approximation, $\xi \approx 0$

Unpolarized gluon GPDs

helicity-flip

$$\delta_{ij} \int \frac{dz^-}{2\pi \bar{P}^+} e^{ix\bar{P}^+ z^-} \langle P' | F_a^{+i}(-z/2) F_a^{+j}(z/2) | P \rangle = \frac{1}{2\bar{P}^+} \bar{u}(P') \left(H_g \gamma^+ + E_g \frac{i\sigma^{+\nu}\Delta_\nu}{2m_N} \right) u(P)$$

nucleon helicity
non-flip

open indices

$$\frac{1}{P^+} \int \frac{d\zeta^-}{2\pi} e^{ixP^+\zeta^-} \langle p' | F^{+i}(-\zeta/2) F^{+j}(\zeta/2) | p \rangle$$
$$= \frac{\delta^{ij}}{2} x H_g(x, \Delta_\perp) + \frac{x E_{Tg}(x, \Delta_\perp)}{2M^2} \left(\Delta_\perp^i \Delta_\perp^j - \frac{\delta^{ij} \Delta_\perp^2}{2} \right) + \cdots$$

gluon transversity GPD
→ photon helicity-flip → $\cos 2(\phi_{P'} - \phi_l)$ asymmetry in DVCS

Polarized gluon GPD → sub-eikonal corrections needed. No study in the small-x region so far.

Spin and orbital angular momentum at small-x?

Ji sum rule

$$J_g = \frac{1}{2}\int_0^1 dx\; x\,[H_g(x,\xi) + E_g(x,\xi)]$$

OAM distribution in Jaffe-Manohar sum rule YH, Yoshida (2012)

$$\mathcal{L}_g(x) = x\int_x^1 \frac{dx'}{x'}(H_g(x') + E_g(x')) - x\int_x^1 \frac{dx'}{x'^2}\Delta G(x') + \cdots$$

Is there a significant contribution from small-x in spin sum rules?

$$H_g(x,0) = G(x) \sim \frac{1}{x^{1+\alpha(Q^2)}}$$

$\alpha(Q^2) \sim 0.3$ in the pQCD regime (from HERA)

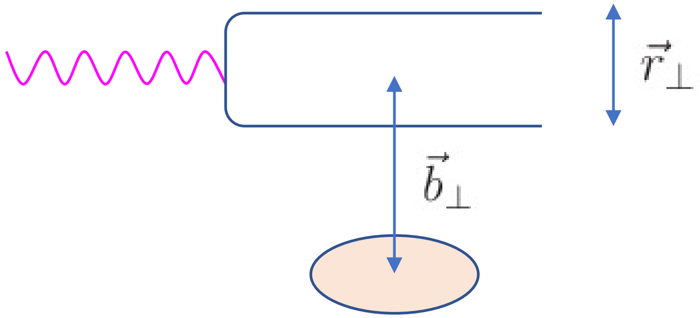
Small-x region likely important for H_g

What about E_g ?

Prejudice: nucleon helicity-flip amplitudes are suppressed at high energy (small-x)

Color dipole amplitude at small-x

→ talk by Cyrille



$$S_x(\vec{b}_\perp, \vec{r}_\perp) = \left\langle \frac{1}{N_c} \mathrm{Tr}\, U\left(\vec{b}_\perp - \frac{\vec{r}_\perp}{2}\right) U^\dagger \left(\vec{b}_\perp + \frac{\vec{r}_\perp}{2}\right) \right\rangle_x$$

You can find it in almost all papers on modern CGC/saturation literature.

Direct connection to gluon Wigner distribution at small-x   YH, Xiao, Yuan (2016)

$$xW(x, \vec{q}_\perp, \vec{b}_\perp) \approx \frac{2N_c}{\alpha_s} \int \frac{d^2\vec{r}_\perp}{(2\pi)^2} e^{i\,\vec{q}_\perp \cdot \vec{r}_\perp} \left(\frac{1}{4}\vec{\nabla}_b^2 - \vec{\nabla}_r^2 \right) S_x(\vec{b}_\perp, \vec{r}_\perp)$$

Color dipole → Mother distribution of all unpol gluon GPDs

Fourier transform → gluon GTMD

$$F_x(q_\perp, \Delta_\perp) = F_0(|q_\perp|, |\Delta_\perp|) + 2\cos 2(\phi_{q_\perp} - \phi_{\Delta_\perp}) F_\epsilon(|q_\perp|, |\Delta_\perp|) + \cdots$$

Elliptic Wigner

H_g, E_{Tg} in the color dipole picture

YH, Xiao, Yuan (2017)

See also, Goeke, Guzey, Siddikov (2007)

$$\frac{1}{P^+}\int\frac{d\zeta^-}{2\pi}e^{ixP^+\zeta^-}\langle p'|F^{+i}F^{+j}|p\rangle \approx$$

$$= \frac{2N_c}{\alpha_s}\left(\frac{\delta^{ij}}{2}\int d^2q_\perp q_\perp^2 F_0 + \frac{1}{\Delta_\perp^2}\left(\Delta_\perp^i\Delta_\perp^j - \frac{\delta^{ij}\Delta_\perp^2}{2}\right)\int d^2q_\perp q_\perp^2 F_\epsilon\right)$$

$$xH_g(x,\Delta_\perp) = \frac{2N_c}{\alpha_s}\int d^2q_\perp q_\perp^2 F_0\,,$$

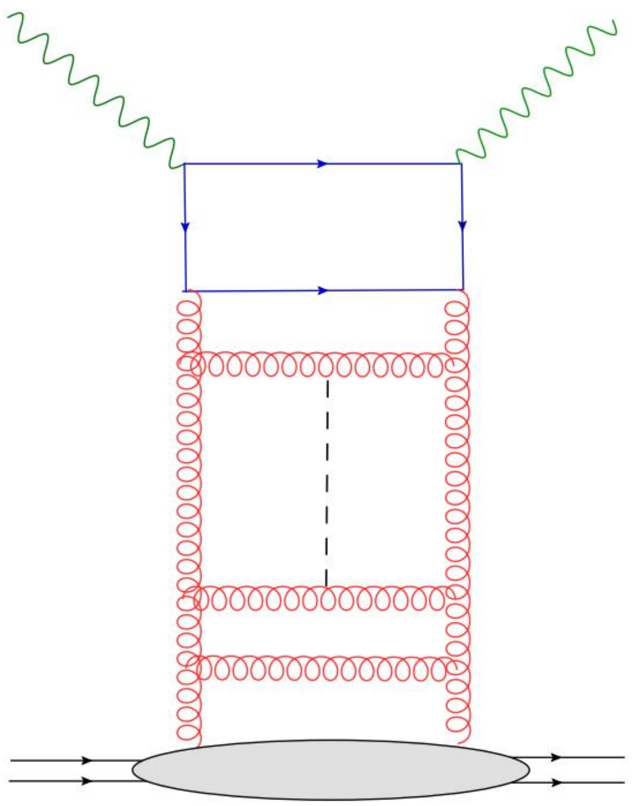
$$xE_{Tg}(x,\Delta_\perp) = \frac{4N_cM^2}{\alpha_s\Delta_\perp^2}\int d^2q_\perp q_\perp^2 F_\epsilon$$

Elliptic Wigner : Mother distribution of the gluon transversity GPD

Small-x evolution of $H_g(x)$

$$xH_g(x,\Delta_\perp) = \frac{2N_c}{\alpha_s}\int d^2q_\perp q_\perp^2 F_0 \quad \sim \left(\frac{1}{x}\right)^{4\ln 2\bar{\alpha}_s}$$

Satisfies the Balitsky-Lipatov-Kuraev-Fadin (BFKL) equation at small-x



At even smaller-x, Balitsky-Kovchegov (BK) equation → gluon saturation

$$\partial_\tau S(\vec{x},\vec{y}) = \int \frac{d^2\vec{z}}{2\pi}\mathcal{M}_{xy}(\vec{z})\left(\langle S(\vec{x},\vec{z})S(\vec{z},\vec{y})\rangle - S(\vec{x},\vec{y})\right)$$

Gluon GPD $E_g(x)$ at small-x

Nucleon helicity non-flip

$$xH_g(x) = xG(x) = \int d^2k_\perp \mathcal{G}(x, k_\perp)$$

BFKL equation

Nucleon helicity flip

$$xE_g(x) = \int d^2k_\perp \mathcal{E}(x, k_\perp)$$

$$\sim \left(\frac{1}{x}\right)^{??}$$

??

Introduce $k_\perp$ dependence in GPD → GTMD

Recent developments in GTMD help us to solve the problem

Color dipole for transversely polarized proton

→ parametrization in terms of GTMDs

YH, Zhou (2022)

$$S(b_\perp, r_\perp) = \langle P' S_\perp | \frac{1}{N_c} \mathrm{tr} U\left(b_\perp - \frac{r_\perp}{2}\right) U\left(b_\perp + \frac{r_\perp}{2}\right) | P S_\perp \rangle$$

F.T. →

$$\frac{\pi g^2}{2N_c k_\perp^2}\left[f_{1,1} - i\frac{k_\perp \times S_\perp}{M^2}\left(\frac{k_\perp \cdot \Delta_\perp}{M^2} f_{1,2} + i g_{1,2}\right) + i\frac{\Delta_\perp \times S_\perp}{2M^2}(2f_{1,3} - f_{1,1})\right]$$

From BK, one can derive coupled equations $f_{1,1}, f_{1,2}, f_{1,3}$

$$xE_g(x) = \int d^2k_\perp \left[-f_{1,1}(k_\perp) + 2f_{1,3}(k_\perp) + \frac{k_\perp^2}{M^2} f_{1,2}(k_\perp)\right]$$

$$\sim \left(\frac{1}{x}\right)^{4\ln 2\bar{\alpha}_s}$$

BFKL Pomeron behavior, the same as unpol gluon PDF!

Exhibits gluon saturation

DVCS in the color dipole picture

YH, Xiao, Yuan (2017)

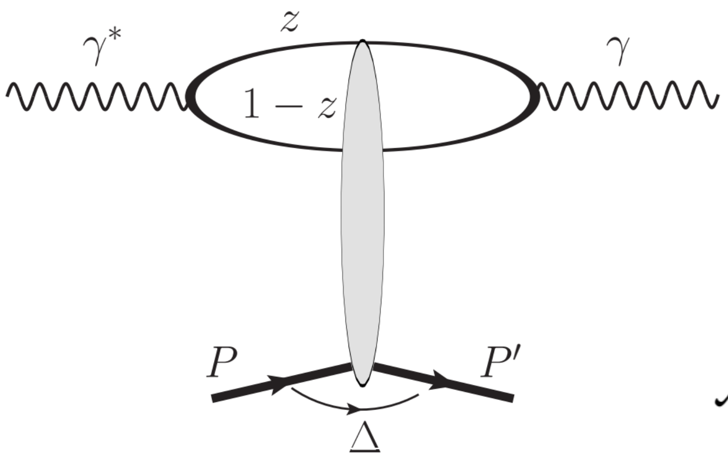
See also Kowalski, Motyka, Watt (2006)
Goeke, Guzey, Siddikov (2007)

Helicity non-flip

Photon helicity flip

$$\frac{d\sigma(ep \to e'\gamma p')}{dx_B dQ^2 d^2\Delta_\perp} = \frac{\alpha_{em}^3}{\pi x_{Bj} Q^2}\left\{\left(1 - y + \frac{y^2}{2}\right)(\mathcal{A}_0^2 + \mathcal{A}_2^2) + 2(1-y)\mathcal{A}_0\mathcal{A}_2\cos(2\phi_{\Delta l}) + (2-y)\sqrt{1-y}(\mathcal{A}_0 + \mathcal{A}_2)\mathcal{A}_L\cos\phi_{\Delta l} + (1-y)\mathcal{A}_L^2\right\}.$$

L→ T transition



$$\mathcal{A}_0 = -\sum_q \frac{e_q^2 N_c}{\pi}\int dz d^2q_\perp d^2q_{1\perp}\frac{(z^2 + (1-z)^2)q_{1\perp}\cdot k_\perp}{q_{1\perp}^2(k_\perp^2 + \epsilon_q^2)}F_x(q_\perp, \Delta_\perp)$$

Color dipole (GTMD)

$$\mathcal{A}_2 = -2\sum_q e_q^2 N_c \int dz d\alpha d^2q_\perp \frac{z(1-z)\alpha}{\alpha\tilde{q}_\perp^2 + \epsilon_q^2}\frac{2(\tilde{q}_\perp\cdot\Delta_\perp)^2 - \tilde{q}_\perp^2\Delta_\perp^2}{\Delta_\perp^2}F_x(q_\perp, \Delta_\perp)$$

Collinear limit $Q \gg q_\perp$

$$\mathcal{A}_0 = -\sum_q \frac{e_q^2 N_c}{\pi} \int dz d^2 q_\perp d^2 q_{1\perp} \frac{(z^2 + (1-z)^2) q_{1\perp} \cdot k_\perp}{q_{1\perp}^2 (k_\perp^2 + \epsilon_q^2)} F_x(q_\perp, \Delta_\perp)$$

$$\rightarrow \quad \sum_q \frac{2\pi e_q^2 \alpha_s}{Q^2} \int \frac{d^2 k'_\perp}{(2\pi)^2} \frac{1}{k'^2_\perp} x H_g(x)$$

$$\mathcal{A}_2 = -2 \sum_q e_q^2 N_c \int dz d\alpha d^2 q_\perp \frac{z(1-z)\alpha}{\alpha \tilde{q}_\perp^2 + \epsilon_q^2} \frac{2(\tilde{q}_\perp \cdot \Delta_\perp)^2 - \tilde{q}_\perp^2 \Delta_\perp^2}{\Delta_\perp^2} F_x(q_\perp, \Delta_\perp)$$

$$\rightarrow \quad -\sum_q \frac{e_q^2 N_c}{Q^2} \int d^2 q_\perp q_\perp^2 F_\epsilon(q_\perp, \Delta_\perp) = -\frac{e_q^2 \alpha_s \Delta_\perp^2}{4 Q^2 M^2} E_{Tg}(x, \Delta_\perp)$$

Consistent with the GPD description

Caveat: Reproduces only the imaginary part of the Compton form factor

see however, Mehtar-Tani, talk at DIS2023

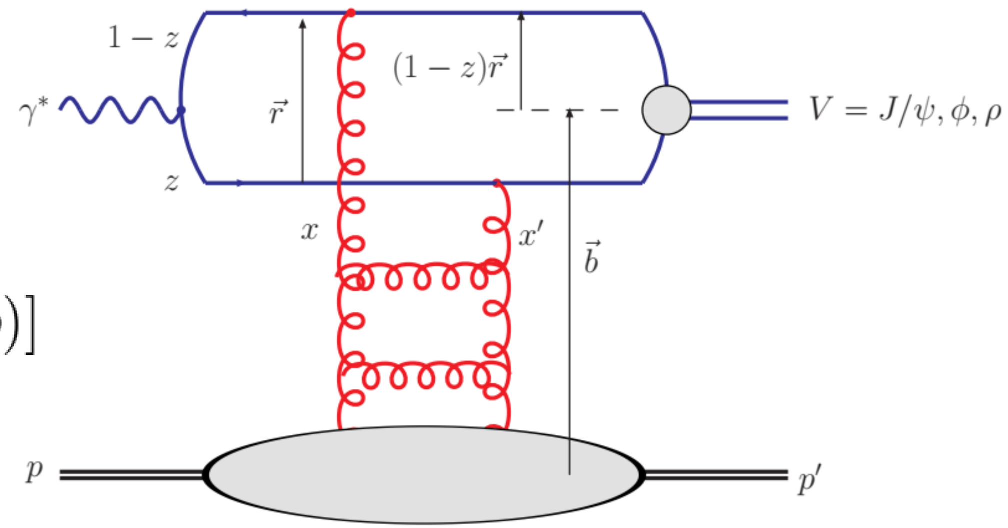
Vector meson production in the color dipole picture

Long history, vast literature.

Nikolaev, Zakharov (1994); Munier, Stasto, Mueller (2001); Kowalski, Motyka, Watt (2006)

$$\mathcal{A}_{T,L}^{\gamma^* p \to Ep}(x, Q, \Delta)$$

$$= \ \mathrm{i} \int \mathrm{d}^2\boldsymbol{r} \int_0^1 \frac{\mathrm{d}z}{4\pi} \int \mathrm{d}^2\boldsymbol{b} \ (\Psi_E^* \Psi)_{T,L} \ \mathrm{e}^{-\mathrm{i}\boldsymbol{b}\cdot\boldsymbol{\Delta}} \ 2[1 - S(x, r, b)]$$



phenomenological meson `wavefunction' $\Psi^V(r, z)$

Dipole S-matrix

Different from meson DA

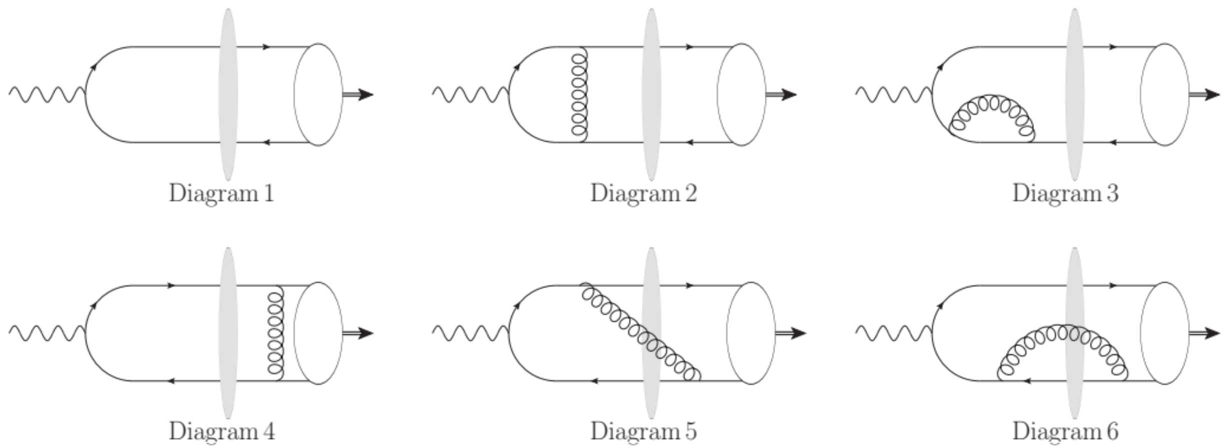
→ Difficult to compare with GPD approaches

Vector meson production at NLO

Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon (2017)

Connect color dipole approach to GPD factorization with meson DA

$$\mathcal{A}^{\eta}_{LO} \equiv -\frac{e_V\, f_V\, \varepsilon_\beta}{N_c} \int_0^1 dx\, \varphi\left(x, \mu_F\right) \int \frac{d^d \vec{p}_1}{(2\pi)^d} \frac{d^d \vec{p}_2}{(2\pi)^d}$$
$$\times\, (2\pi)^{d+1}\, \delta\left(p_V^+ - p_\gamma^+\right) \delta\left(\vec{p}_V - \vec{p}_\gamma - \vec{p}_1 - \vec{p}_2\right)$$
$$\times\, \Phi_0^\beta\left(x, \vec{p}_1, \vec{p}_2\right) \left[\mathrm{Tr}(U_1^\eta U_2^{\eta\dagger}) - N_c\right] \left(\vec{p}_1, \vec{p}_2\right).$$



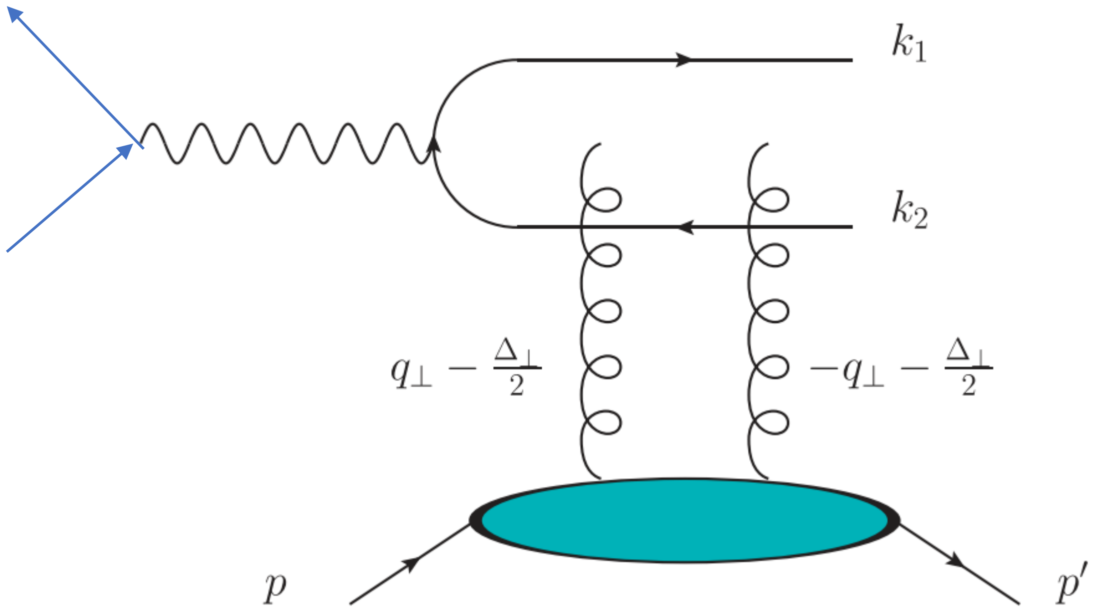
Divergences absorbed into the evolution of meson DA (ERBL equation) and the JIMWLK equation for the dipole amplitude. → factorization works at 1-loop level

Nonzero k_T cuts off the endpoint singularities. Saturation momentum Q_s provides a hard scale.
→ Applicable to a larger class of observables

Exlucive dijet at small-x

→ talk by Feng

Color dipole



Relative jet momentum $k_{1\perp} - k_{2\perp}$ provides an additional handle, probing the intrinsic $k_\perp$ distribution of color dipole → GTMD (Wigner)

YH, Xiao, Yuan (2016)

GPD

Braun, Ivanov (2009) unpolarized
Ji, Yuan, Zhao (2016) single spin asymmetry
Bhattacharya, Boussarie, YH (2022) double spin asymmetry

In GPD approach, $k_\perp$ dependence enters at twist-3,

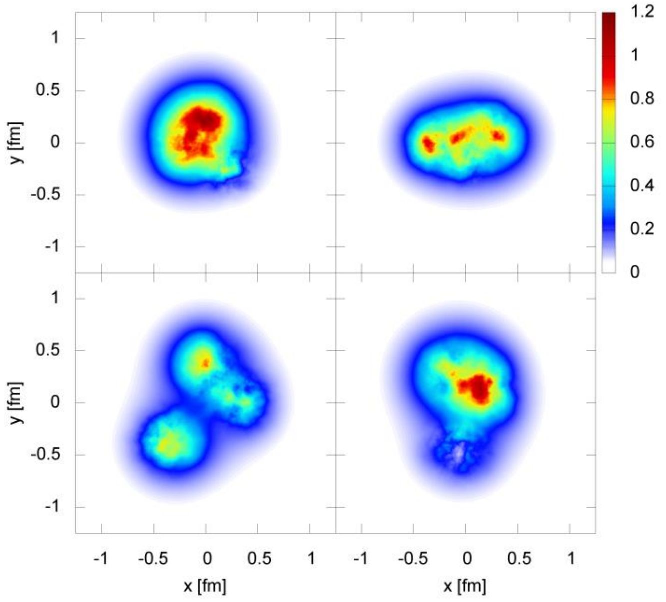
In particular, spin asymmetries can probe the gluon orbital angular momentum

$$d\sigma^{h_p h_l} \sim h_p h_l \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \mathcal{H}_g \mathcal{L}_g^*$$

Incoherent vector meson production

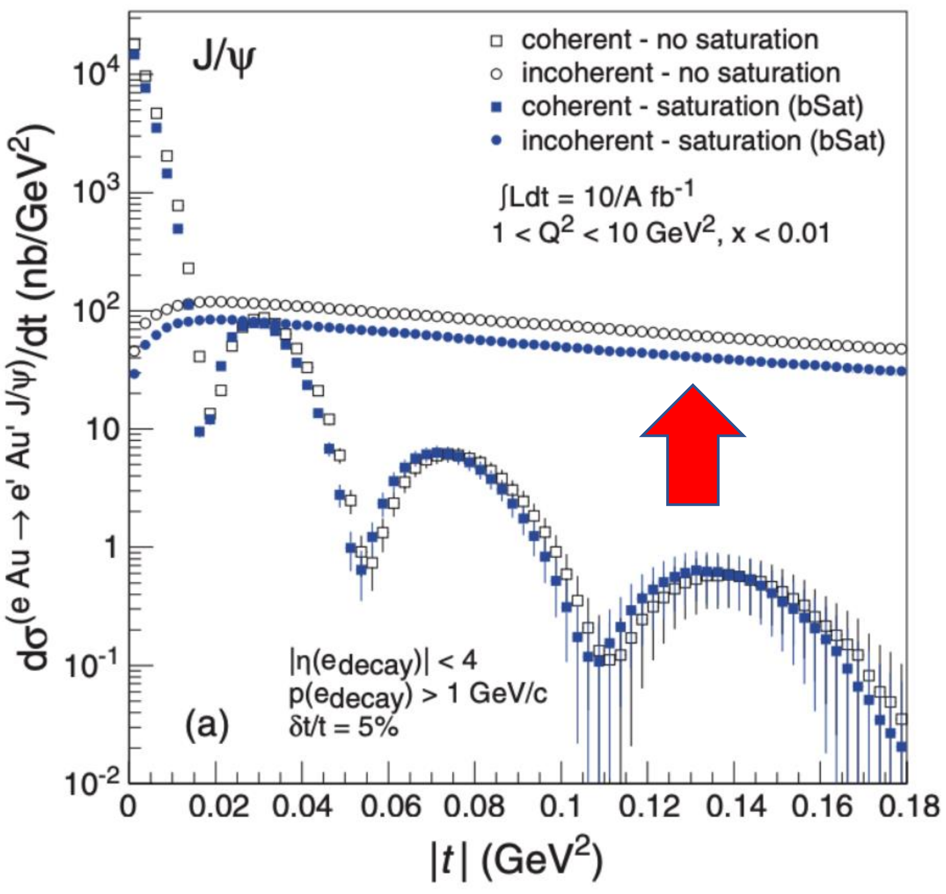
Color dipole formalism versatile,
exclusive process is just one application.

inclusive,
diffractive
SIDIS
SSA, gluon Sivers (longitudinal spin is a challenge)
nuclear targets,…
geometric fluctuations ←→ GPD: average profile



Mantysaari, Schenke (2016)

Toll, Ulrich, PRC **87** (13) 0249



Conclusions

- Two approaches to exclusive processes
  moderate energy→ GPD factorization,
  very high energy → color dipole, kt factorization
- Historically, the two communities rarely interacted.
- Some advances from the small-x side. More interaction desirable in future