Entanglement (and thermalization?) in pair creation

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C2QA

Thermalization, from Cold Atoms to Hot Quantum Chromodynamics

Plan

“Real-time non-perturbative dynamics of jet production in Schwinger model: quantum entanglement and vacuum modification”

with D. Frenklakh, K. Ikeda, D. Kharzeev, V. Korepin, S. Shi, K. Yu, PRL 131 (2023) 2, 021902

Work in progress

Plan

“Real-time non-perturbative dynamics of jet production in Schwinger model: quantum entanglement and vacuum modification”

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Work in progress

Motivation

Real-time dynamics $1+1$ models

Learn about physics

Opportunity for co-design quantum comp.

Schwinger model

Electromagnetism in 1 dimension

Schwinger model

Electromagnetism in 1 dimension

Full fledged quantum field theory

Simulable in the near future (?)

Solved in some limit ($m \to 0$)

Schwinger model

Electromagnetism in 1 dimension

$\nabla E = \rho$

Full fledged quantum field theory

Confines $V(x) \propto x$

Simulable in the near future (?)

Highly non-trivial vacuum

Solved in some limit ($m \to 0$)

Schwinger model

Electromagnetism in 1 dimension

$\nabla E = \rho$

Full fledged quantum field theory

Confines $V(x) \propto x$

Simulable in the near future (?)

Highly non-trivial vacuum

Solved in some limit ($m \to 0$)

Use-case/testbed $\longleftrightarrow$ Learn new physics (dynamics)

Word of caution

Not QCD, only toy model ($1D$, no dynamical gluons)

↓

Need to ask reasonable questions

Only qualitative predictions

Dynamical string breaking

Look at string breaking

En.$\sim m + m + \alpha l_1$

Dynamical string breaking

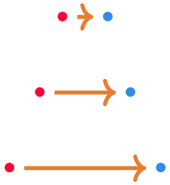
Look at string breaking

En.$\sim$ $m + m + \alpha l_1$

En.$\sim$ $m + m + \alpha l_2$

Dynamical string breaking

Look at string breaking



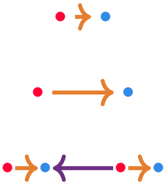
En.$\sim m + m + \alpha l_1$

En.$\sim m + m + \alpha l_2$

En.$\sim m + m + \alpha l_3$

Dynamical string breaking

Look at string breaking



En. $\sim m + m + \alpha l_1$

En. $\sim m + m + \alpha l_2$

~~En. $\sim m + m + \alpha l_3$~~

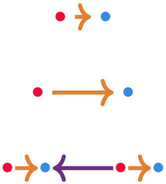
$\downarrow$

En. $\sim m + m + m + m$

when $\alpha l_3 > 2m$

Dynamical string breaking

Look at string breaking



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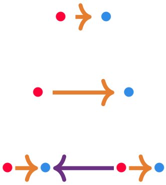
En.$\sim m + m + m + m$

when $\alpha l_3 > 2m$

Screen field by creating particles!

Dynamical string breaking

Look at string breaking

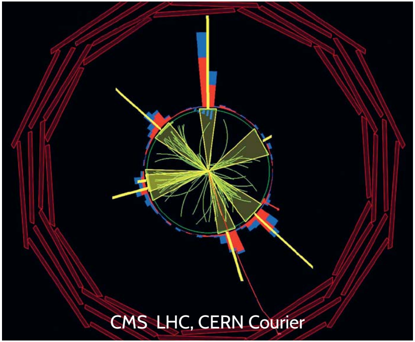


En.$\sim m + m + \alpha l_1$

En.$\sim m + m + \alpha l_2$

Screen field by creating particles!

Motivation: QCD jets



Our set-up

$$H(t) = \int \mathrm{d}x \left[\frac{1}{2} E^2 + \hat{\bar{\psi}} \left(-i\gamma^1 \partial_1 + g A^1 \gamma_1 + m \right) \hat{\psi} + j^1_{ext}(t) A_1 \right]$$

Our set-up

Electric field

$$H(t) = \int \mathrm{d}x \left[\frac{1}{2} E^2 + \hat{\bar{\psi}} \left(-i\gamma^1 \partial_1 + gA^1 \gamma_1 + m \right) \hat{\psi} + j^1_{ext}(t) A_1 \right]$$

Our set-up

Electric field

Vector potential

$$H(t) = \int \mathrm{d}x \left[\frac{1}{2} E^2 + \hat{\bar{\psi}} \left(-i\gamma^1 \partial_1 + g A^1 \gamma_1 + m \right) \hat{\psi} + j^1_{ext}(t) A_1 \right]$$

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Fermion

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Electric field

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Fermion

External charges: $j^1_{ext}(t) = g\left(\delta(x+t) + \delta(x-t)\right)\theta(t)$

2 point charges moving apart at speed of light

Our set-up

Electric field

Vector potential

$$H(t) = \int \mathrm{d}x \left[\frac{1}{2}E^2 + \hat{\bar{\psi}}\left(-i\gamma^1\partial_1 + gA^1\gamma_1 + m\right)\hat{\psi} + j^1_{ext}(t)A_1\right]$$

Fermion

External charges: $j^1_{ext}(t) = g\left(\delta(x+t) + \delta(x-t)\right)\theta(t)$

2 point charges moving apart at speed of light

Idea: • Find $|vac\rangle_{t<0}$

• Compute $|\psi(t)\rangle = e^{-i\int_0^t \mathrm{d}t' H(t')}|vac\rangle_{t<0}$

Our set-up

Electric field

Vector potential

$$H(t) = \int \mathrm{d}x \left[\frac{1}{2} E^2 + \hat{\bar{\psi}} \left(-i\gamma^1 \partial_1 + gA^1 \gamma_1 + m \right) \hat{\psi} + j^1_{ext}(t) A_1 \right]$$

Fermion

External charges: $j^1_{ext}(t) = g\left(\delta(x+t) + \delta(x-t)\right)\theta(t)$

2 point charges moving apart at speed of light

Idea:
- Find $|vac\rangle_{t<0}$
- Compute $|\psi(t)\rangle = e^{-i\int_0^t \mathrm{d}t' H(t')}|vac\rangle_{t<0}$

see also [74, Casher, Kogut, Susskind], [12,13, Kharzeev, Loshaj], [14, Berges, Hebenstreit]

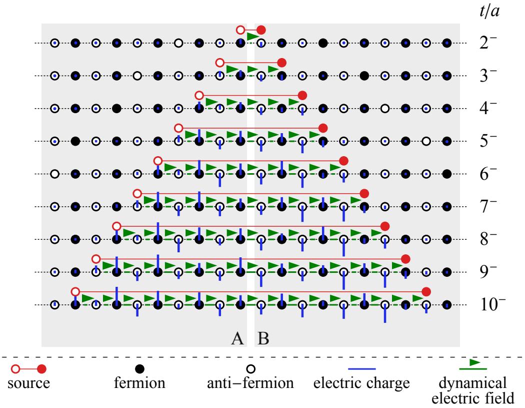
In practice

- Staggered fermions χ_n
- Integrate out E: $\partial_1 E = \rho + \rho_{ext}$
- (Map to non-local spin chain)

$$H(t) = H_\pm + H_{ZZ} + H_Z(t)$$
$$H_\pm = \frac{1}{4a} \sum_{n=1}^{N-1} (X_n X_{n+1} + Y_n Y_{n+1})$$
$$H_{ZZ} = \frac{ag^2}{4} \sum_{n=1}^{N-1} \sum_{m=1}^{n} \sum_{k=1}^{m-1} Z_m Z_k, \quad H_Z = \sum_{n=1}^{N} f(n) Z_n$$

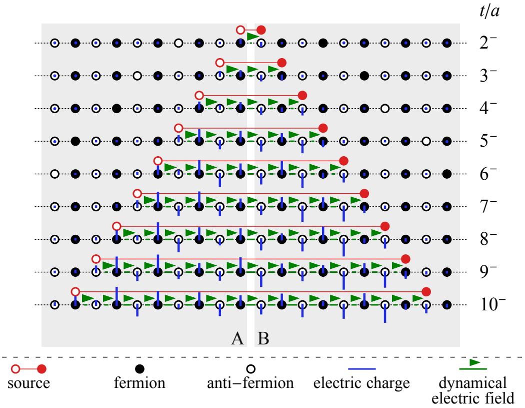
- Use exact diagonalization
- Compute observables $\bar{O}(t) = \langle \psi(t) | O | \psi(t) \rangle$

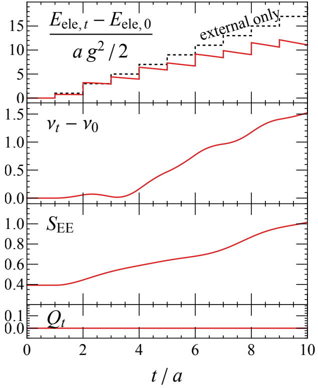
Results, $m = 0.25, g = 0.5, a = 1$



Results, $m = 0.25, g = 0.5, a = 1$

ν : $\langle\bar{\psi}\psi\rangle$, S_{EE} : entanglement entropy A/B

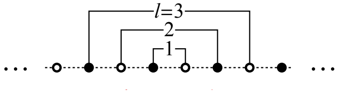




Is entanglement manifest in correlations $\leftrightarrow$ measurable?

Correlation

1) Look at $\langle \Delta\nu_{N/2+l+1}(t)\Delta\nu_{N/2-l}(t)\rangle, \quad \Delta\nu_n = \bar{\psi}\psi|_n(t) - \bar{\nu}$



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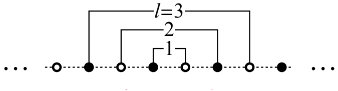
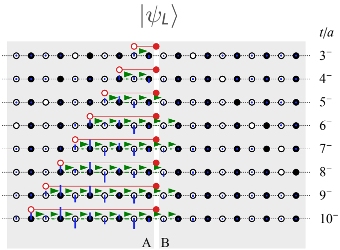
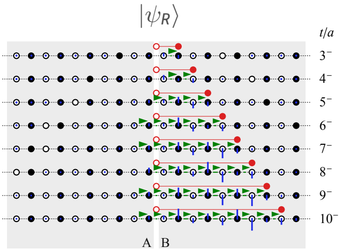


2) Compare to uncorrelated reference case

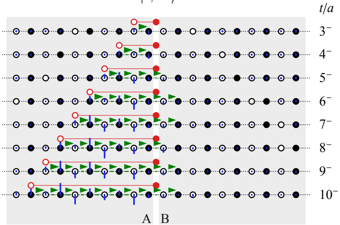




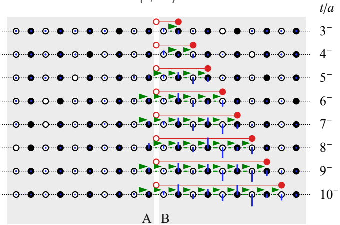
Correlation

2) Compare to uncorrelated reference case

$|\psi_L\rangle$



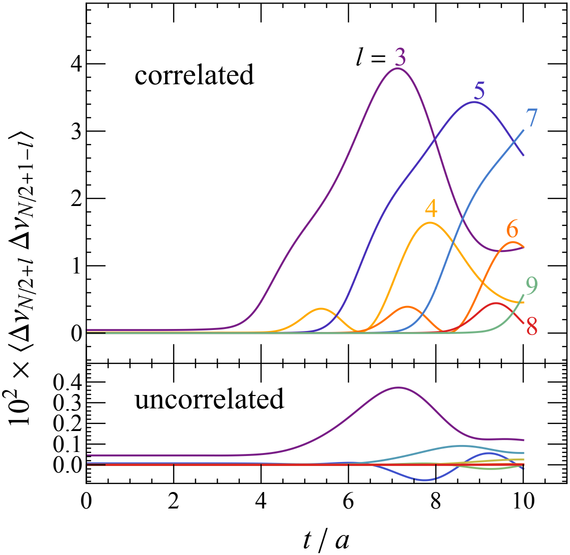
$|\psi_R\rangle$



$$|\psi_{ref}\rangle = |\psi_L\rangle + e^{i\phi}|\psi_R\rangle$$

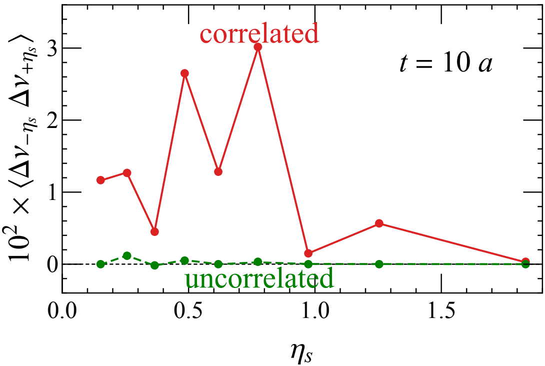
Random uniform phase

$$\langle\langle\psi_{ref}|O|\psi_{ref}\rangle\rangle \equiv \int \langle\psi_{ref}|O|\psi_{ref}\rangle \frac{\mathrm{d}\varphi}{2\pi} = \frac{\langle\psi_{\mathrm{L}}|O|\psi_{\mathrm{L}}\rangle}{2} + \frac{\langle\psi_{\mathrm{R}}|O|\psi_{\mathrm{R}}\rangle}{2}$$



correlated
l = 3
5
7
4
6
9
8
uncorrelated
0
1
2
3
4
0.4
0.3
0.2
0.1
0.0
0
2
4
6
8
10
t / a
$10^2 \times \langle \Delta\nu_{N/2+l}\, \Delta\nu_{N/2+1-l} \rangle$

For exp. $\rightarrow$ spatial rapidity $\eta_s \equiv \mathrm{arctanh}\,\frac{x}{t}$



Next steps

Finite temperature

Thermalization/ETH

Next steps

Finite temperature

Thermalization/ETH

Tensor networks

Need of larger systems

↓

Use matrix product states (MPS)

Tensor networks

Need of larger systems

↓

Use matrix product states (MPS)

→

Ground state: efficient (DMRG)

Time-evolution: inefficient

Tensor networks

Need of larger systems

↓

Use matrix product states (MPS)

→

Ground state: efficient (DMRG)

Time-evolution: inefficient

Can still do a lot! (TDVP)

Tensor networks

Need of larger systems

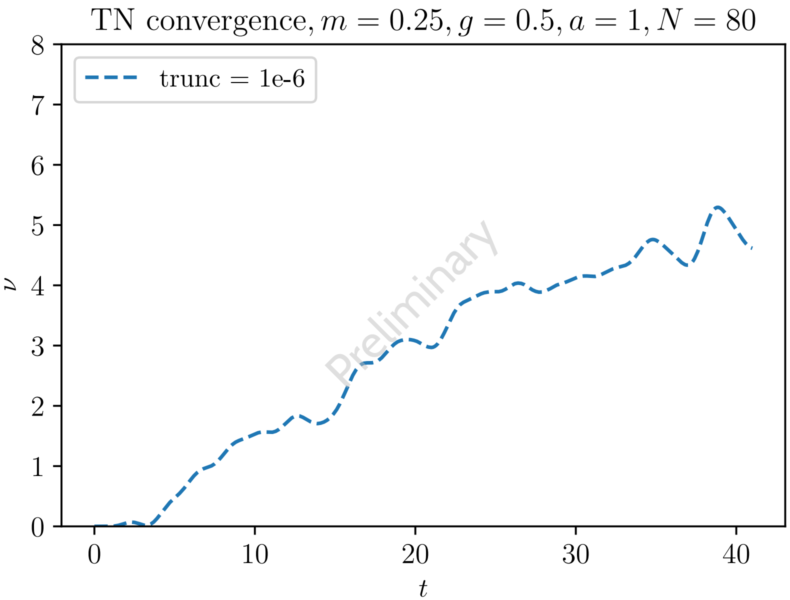
Use matrix product states (MPS)

Ground state: efficient (DMRG)

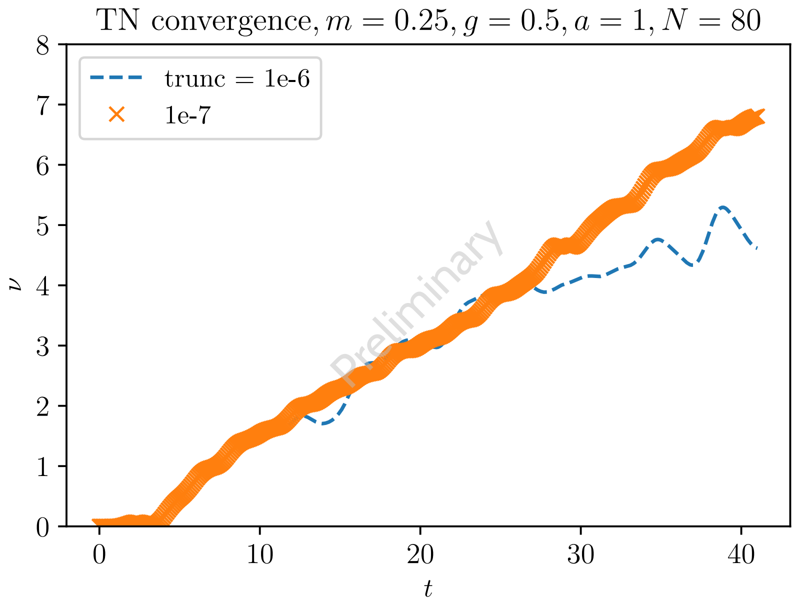
Time-evolution: inefficient

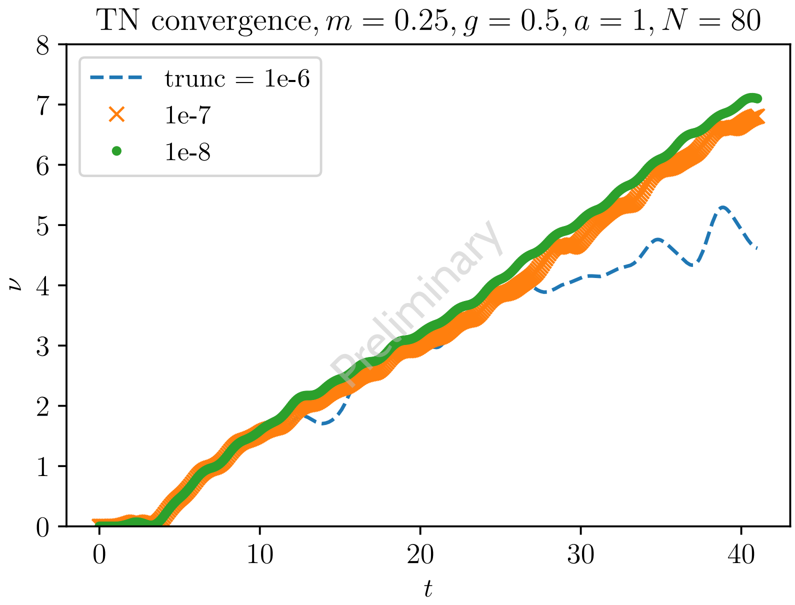
Can still do a lot! (TDVP)

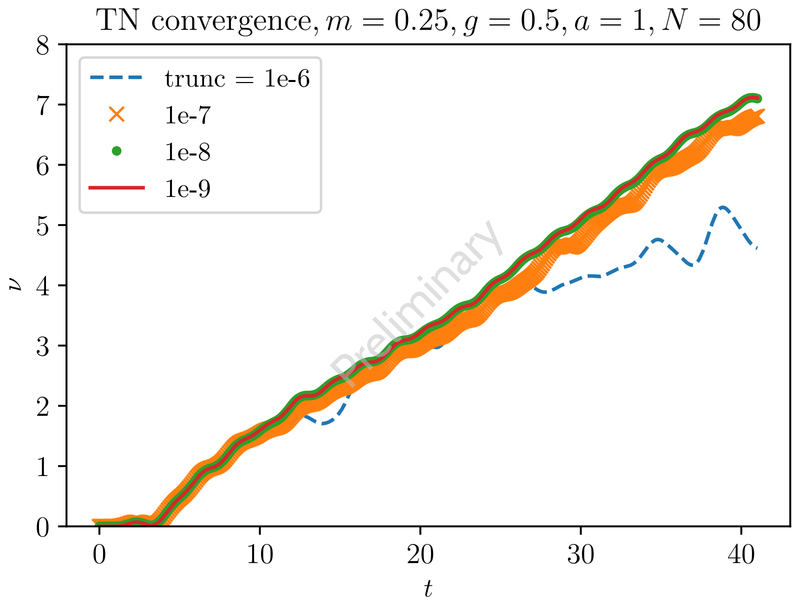
Shoutout to [Fishman, Stoudenmire, ITensors]



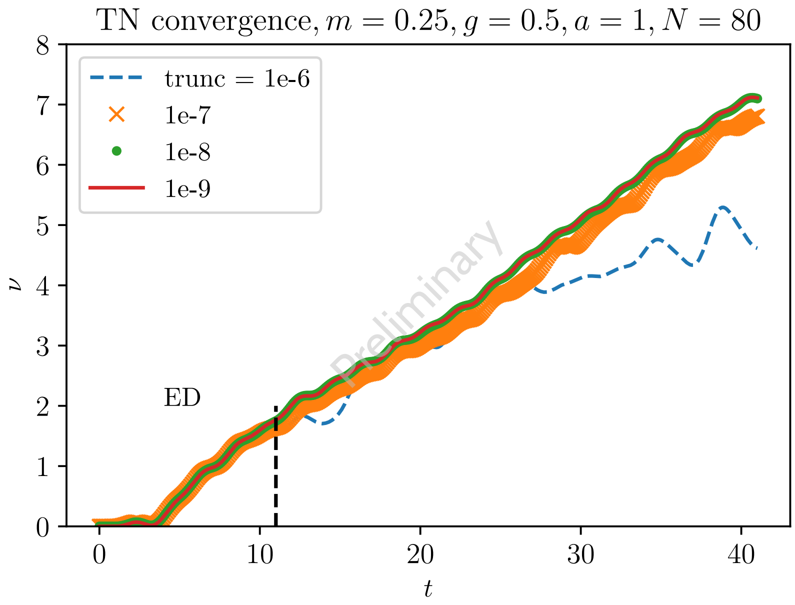
TN convergence, $m = 0.25, g = 0.5, a = 1, N = 80$
trunc = 1e-6
Preliminary
ν
t
0
1
2
3
4
5
6
7
8
0
10
20
30
40


TN convergence, $m = 0.25, g = 0.5, a = 1, N = 80$
trunc = 1e-6
1e-7
Preliminary
ν
t
0
1
2
3
4
5
6
7
8
0
10
20
30
40

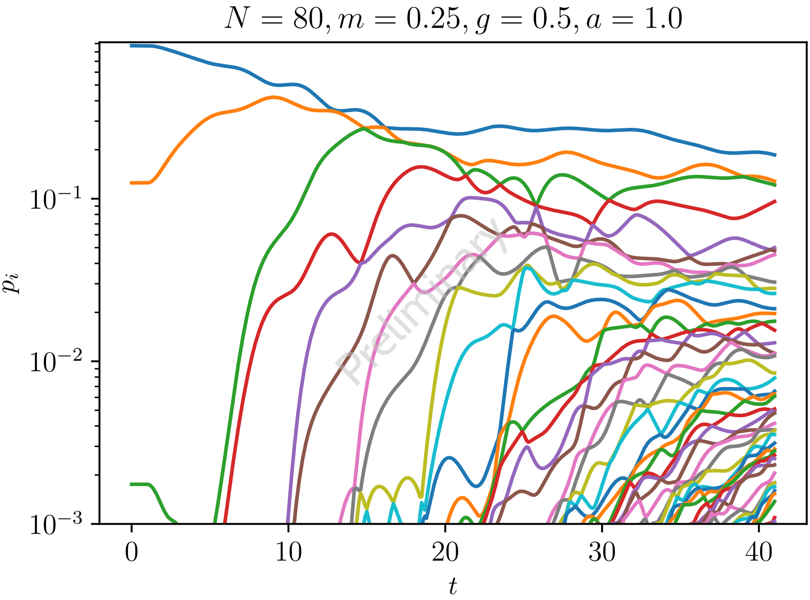

TN convergence, $m = 0.25, g = 0.5, a = 1, N = 80$
trunc = 1e-6
1e-7
1e-8
Preliminary
ν
t
0
10
20
30
40
0
1
2
3
4
5
6
7
8



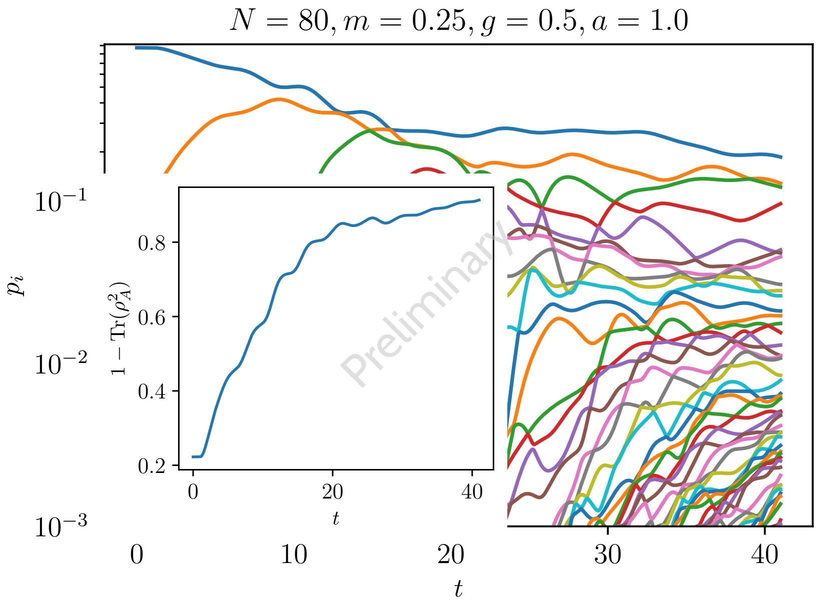
TN convergence, $m = 0.25, g = 0.5, a = 1, N = 80$
trunc = 1e-6
1e-7
1e-8
1e-9
Preliminary
ν
t
0
1
2
3
4
5
6
7
8
0
10
20
30
40



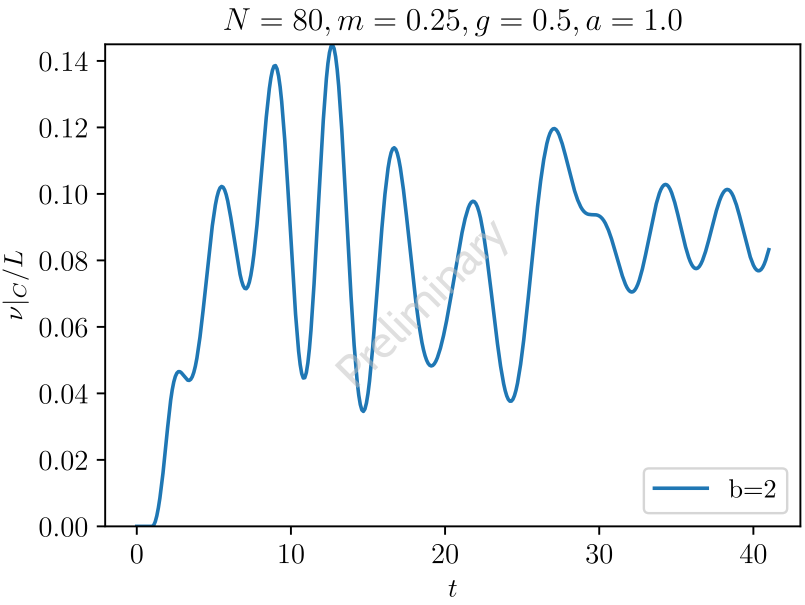
TN convergence, $m = 0.25, g = 0.5, a = 1, N = 80$
trunc = 1e-6
1e-7
1e-8
1e-9
ED
Preliminary
ν
t
0
10
20
30
40
1
2
3
4
5
6
7
8



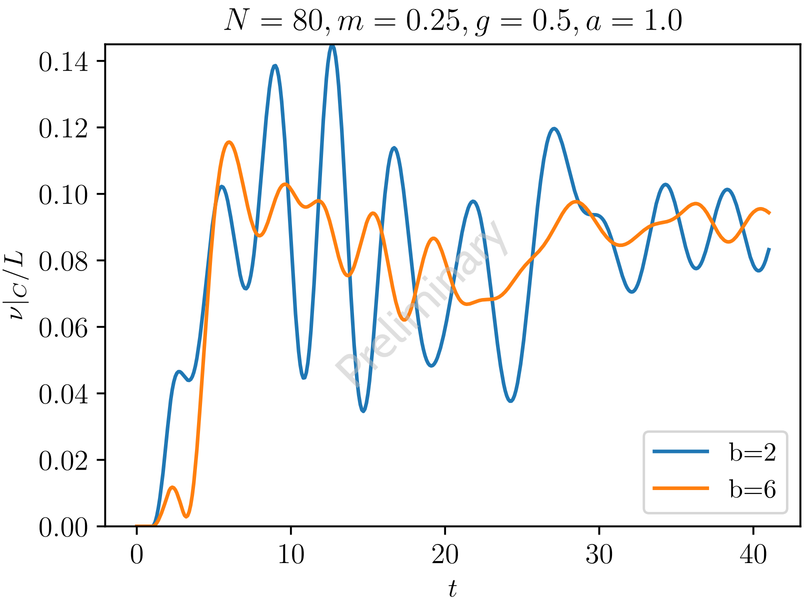
$N = 80, m = 0.25, g = 0.5, a = 1.0$
p_i
10^{-1}
10^{-2}
10^{-3}
0
10
20
30
40
t
Preliminary



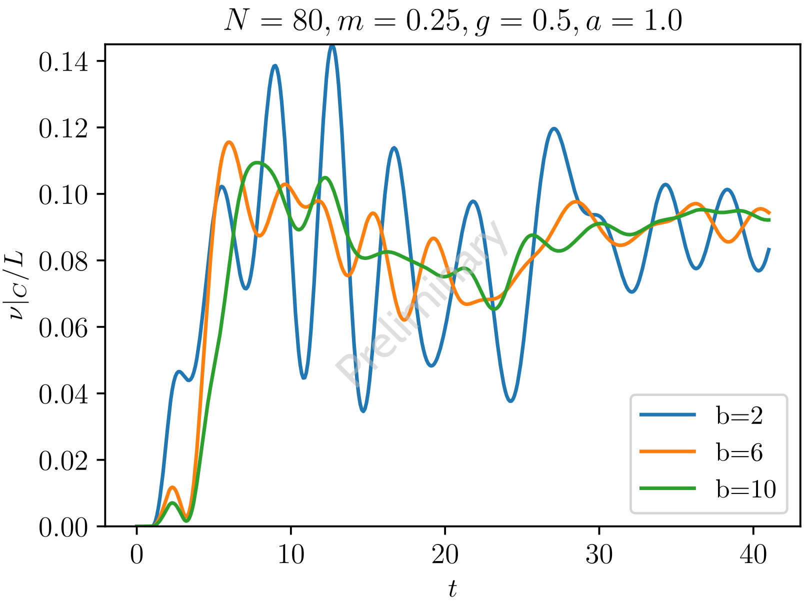
$N = 80, m = 0.25, g = 0.5, a = 1.0$
p_i
10^{-1}
10^{-2}
10^{-3}
0
10
20
30
40
t
$1 - \mathrm{Tr}(\rho_A^2)$
0.2
0.4
0.6
0.8
0
20
40
t
Preliminary



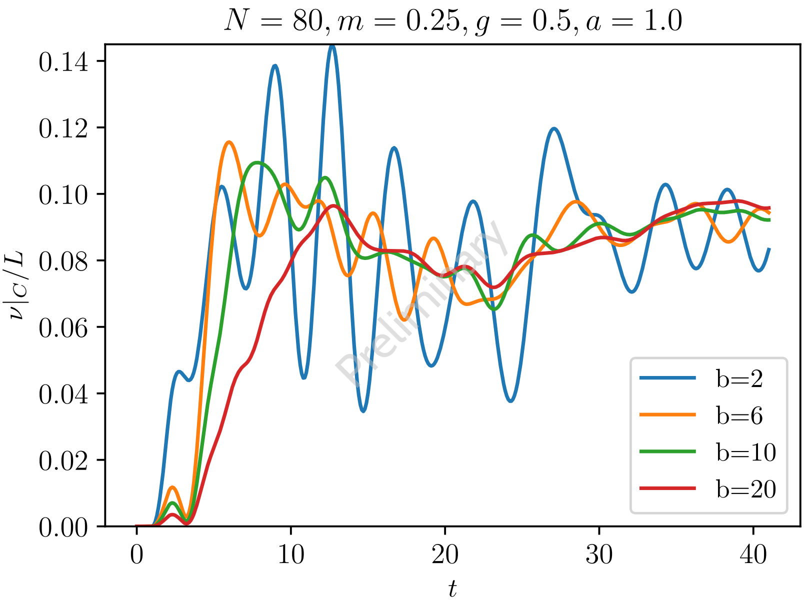
$N = 80, m = 0.25, g = 0.5, a = 1.0$
$\nu|_C/L$
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0
10
20
30
40
t
b=2
Preliminary



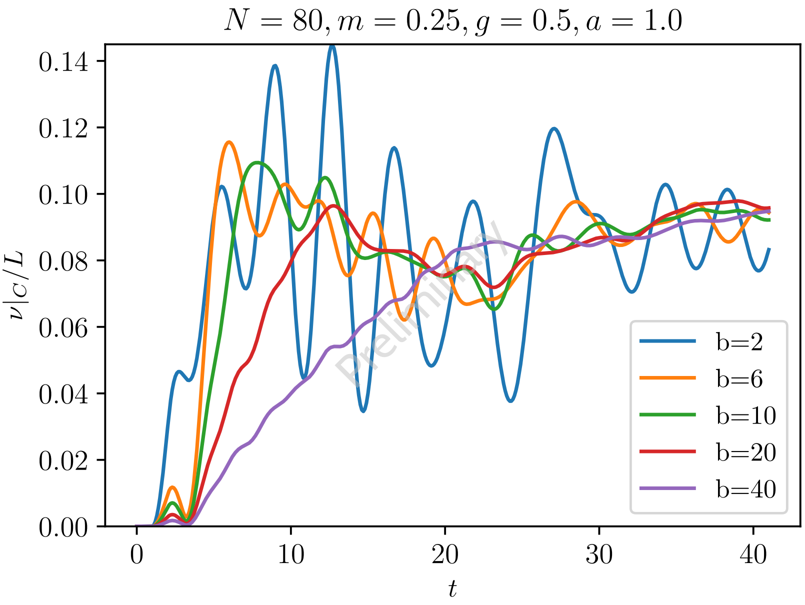
$N = 80, m = 0.25, g = 0.5, a = 1.0$
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
$\nu|_C/L$
0
10
20
30
40
t
b=2
b=6
Preliminary



$N = 80, m = 0.25, g = 0.5, a = 1.0$
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
$\nu|_C/L$
0
10
20
30
40
t
b=2
b=6
b=10
Preliminary



$N = 80, m = 0.25, g = 0.5, a = 1.0$
$\nu|_C/L$
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0
10
20
30
40
t
b=2
b=6
b=10
b=20
Preliminary



$N = 80, m = 0.25, g = 0.5, a = 1.0$
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
$\nu|_C/L$
0
10
20
30
40
t
b=2
b=6
b=10
b=20
b=40
Preliminary

Outlooks/questions

How to better quantify this?

Compare to canonical simulations?

Symmetry resolved spectrum on C?

Summary

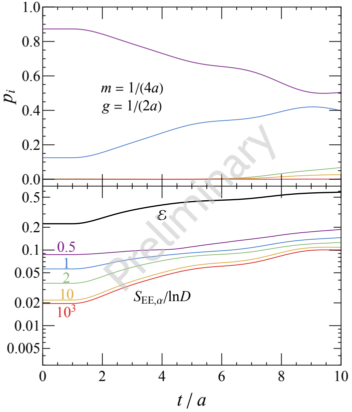
- Schwinger model can still teach us some physics
- Direct observation of quantum properties of string breaking
- Suggests enhanced correlations at low/mid rapidities in jet production
- Hints of thermalization

Thank you!

Trailer #1: entanglement spectrum

Entanglement spectrum: $\{p_i\}$, e-values of ρ_A

$$S_{\text{Rényi},\alpha} \equiv \frac{\ln \mathrm{tr}(\rho_A^\alpha)}{1-\alpha} \qquad \mathcal{E} \equiv \frac{1-\mathrm{tr}\rho_A^2}{1-1/D} = \frac{1-\sum_{i=1}^D p_i^2}{1-1/D} \,.$$



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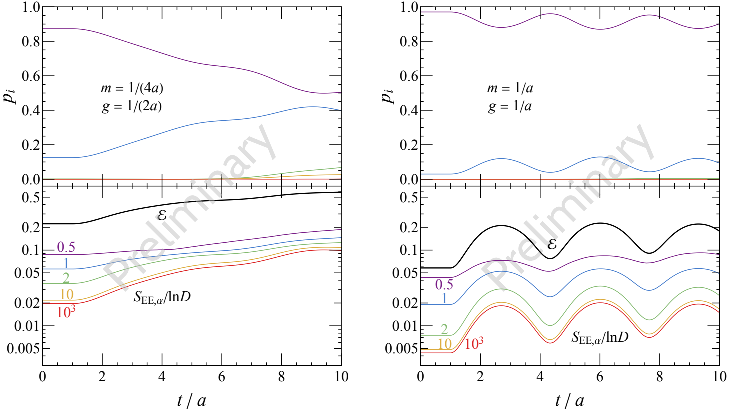
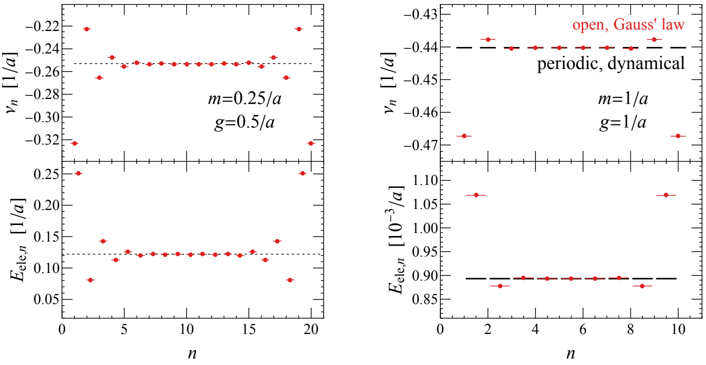
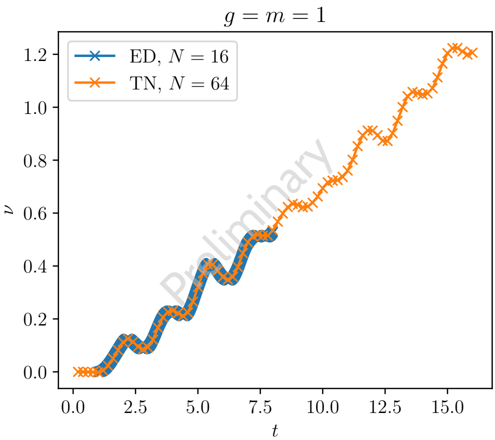


Trailer #1: entanglement spectrum

Boundary effects



Trailer #2: TN



$g = m = 1$
ED, $N = 16$
TN, $N = 64$
Preliminary
1.2
1.0
0.8
0.6
0.4
0.2
0.0
ν
0.0
2.5
5.0
7.5
10.0
12.5
15.0
t