Reconstructing Parton Structure from Lattice QCD

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Parton Structure

Wigner Distribution/
Generalized Transverse Momentum
Distribution (GTMD)

$F(x, b_T, k_T)$

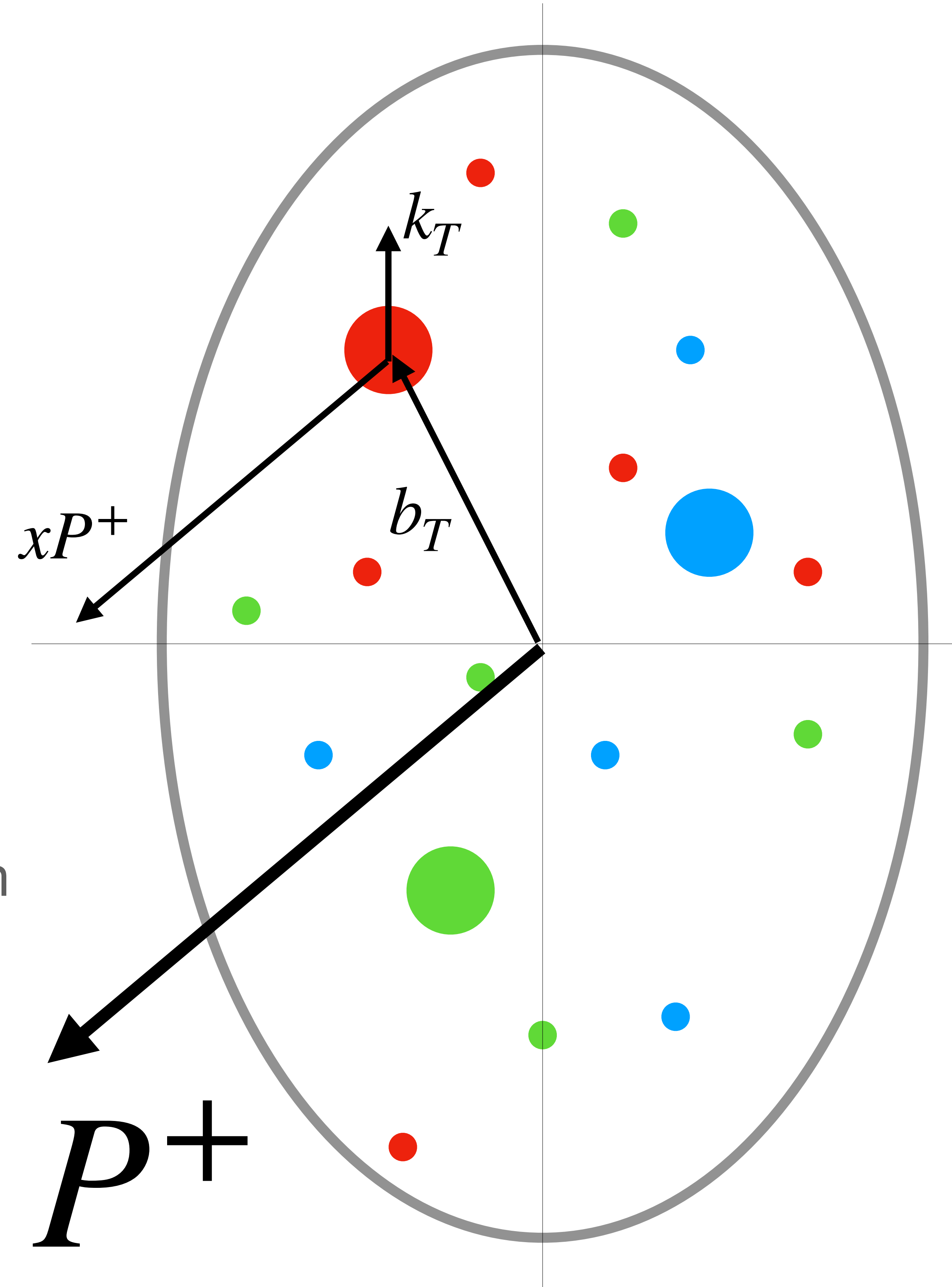
$\int d^2 b_t$ → Transverse Momentum Distribution (TMD) $f(x, k_T)$

$\int d^2 k_t$ → Generalized Parton Distribution (GPD) $f(x, b_T)$

Transverse Momentum Distribution (TMD): $\int d^2 k_t$ → PDF

Generalized Parton Distribution (GPD): $\int d^2 b_t$ → PDF

Parton Distribution Function (PDF)
$f(x)$

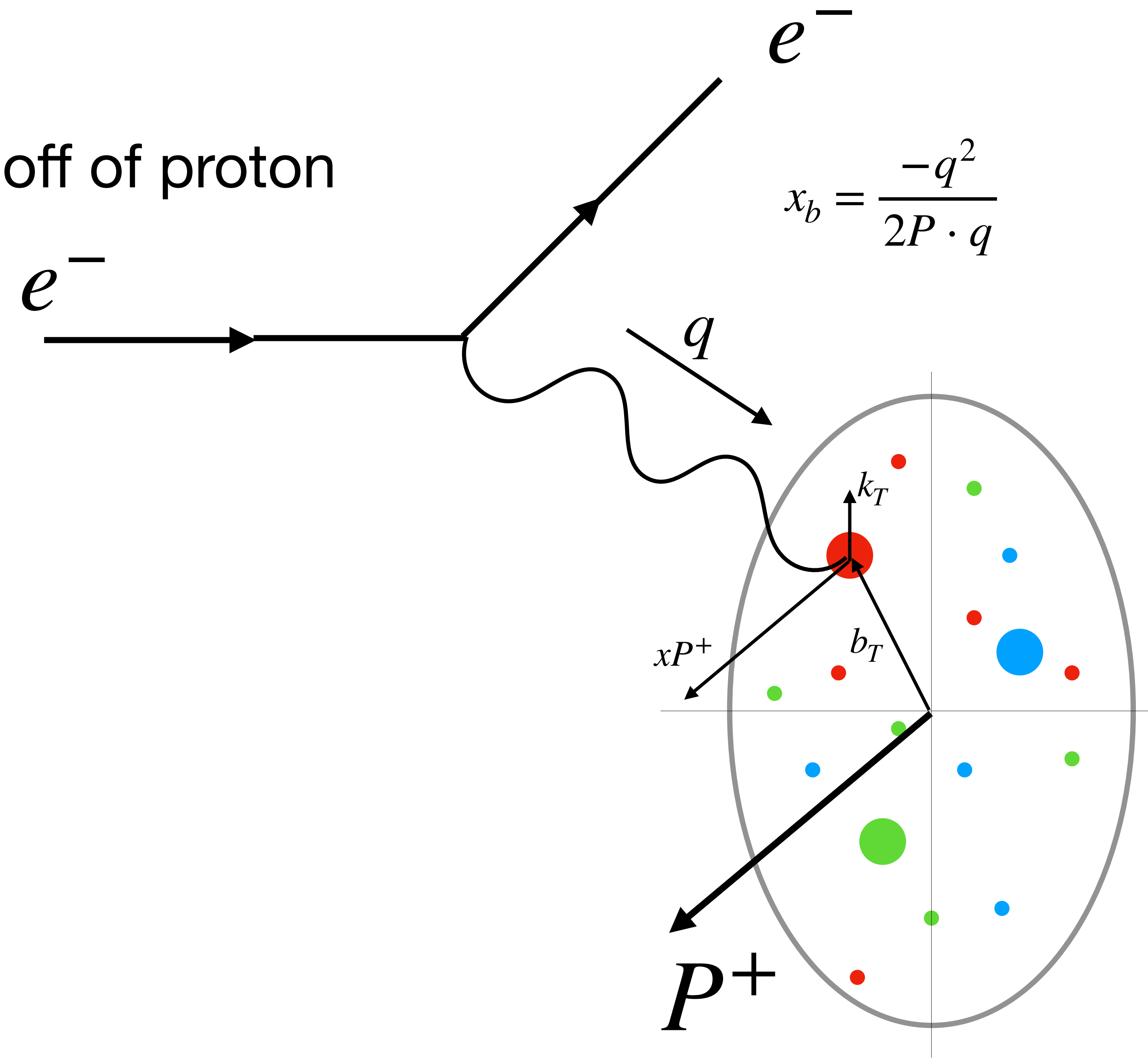


(Almost true) Probabilistic Interpretation:
Probability of interaction with parton with momentum xP^+

Partons from Experiments

- Deep Inelastic Scattering
    - Hard Scattering of electron off of proton

$$Q^2 = -q^2 \gg \Lambda^2_{\mathrm{QCD}}$$



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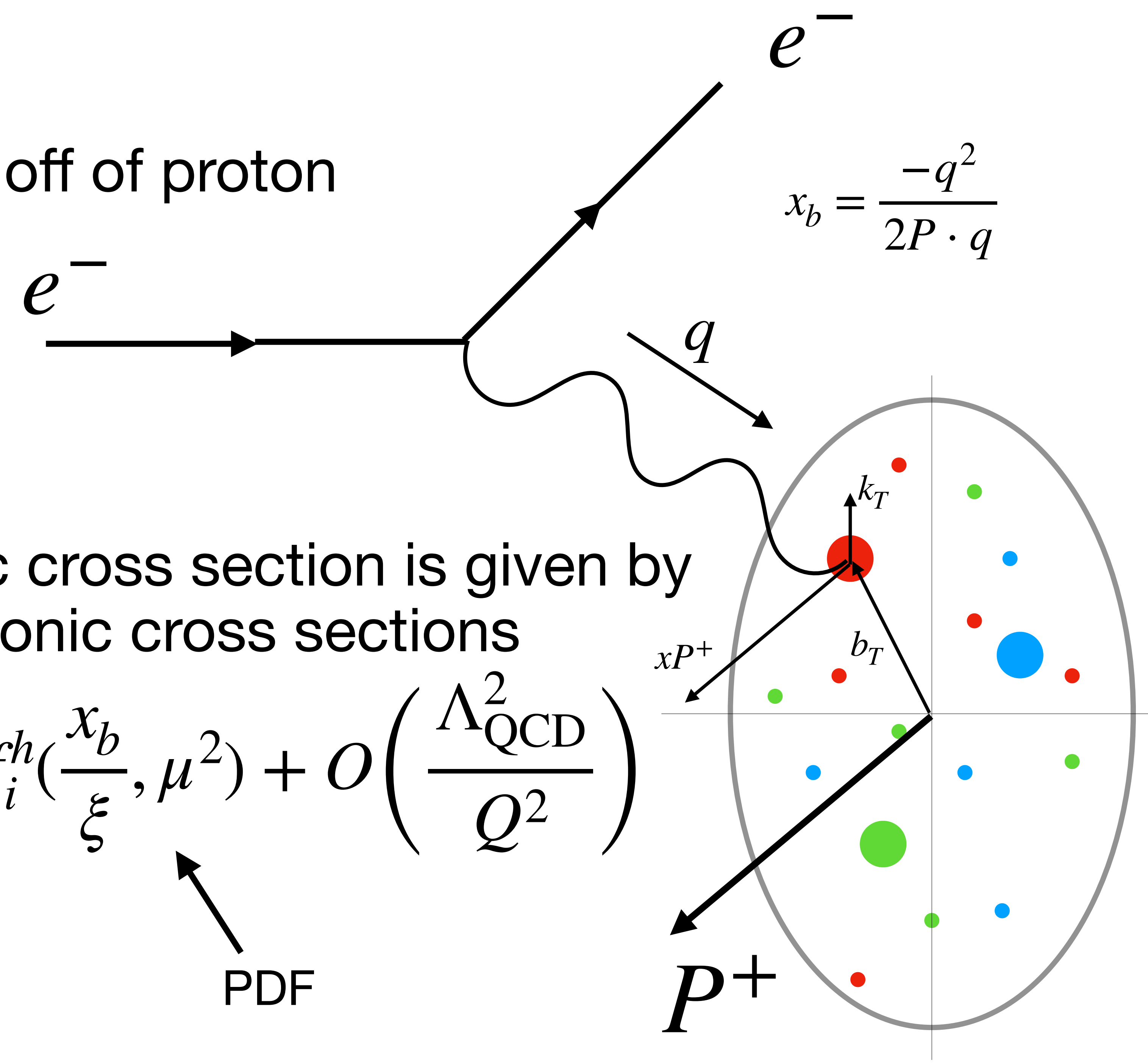

- **QCD Factorization:** Hadronic cross section is given by convolution of PDFs with partonic cross sections

$$F_2^h(x_b, Q^2) = \sum_i \int_{x_b}^1 d\xi \, F_2^i(\xi, \frac{\mu^2}{Q^2}) f_i^h(\frac{x_b}{\xi}, \mu^2) + O\left(\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}\right)$$

Hadron Structure Function

PDF

Parton Structure Function

Partons from Experiments

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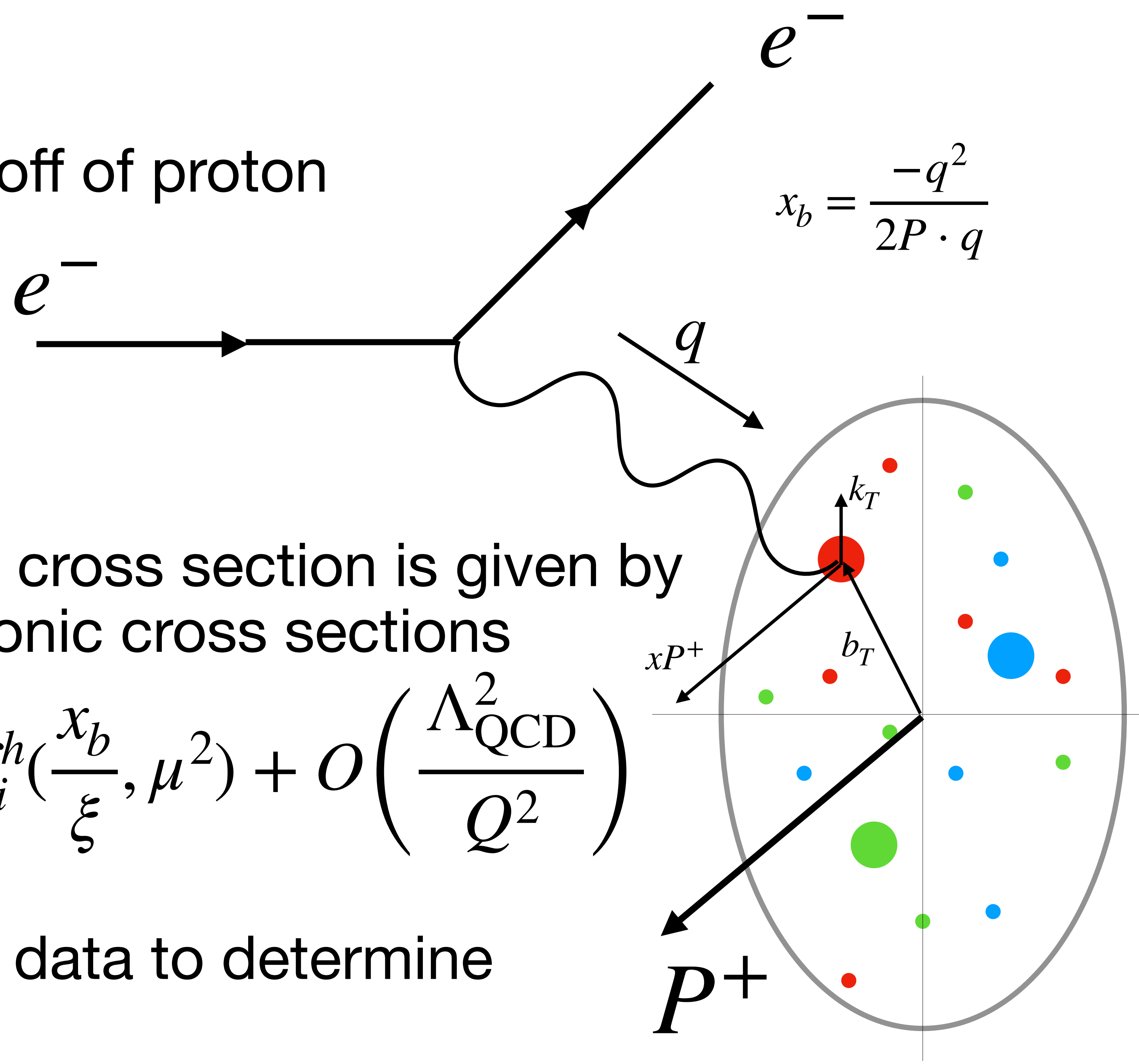

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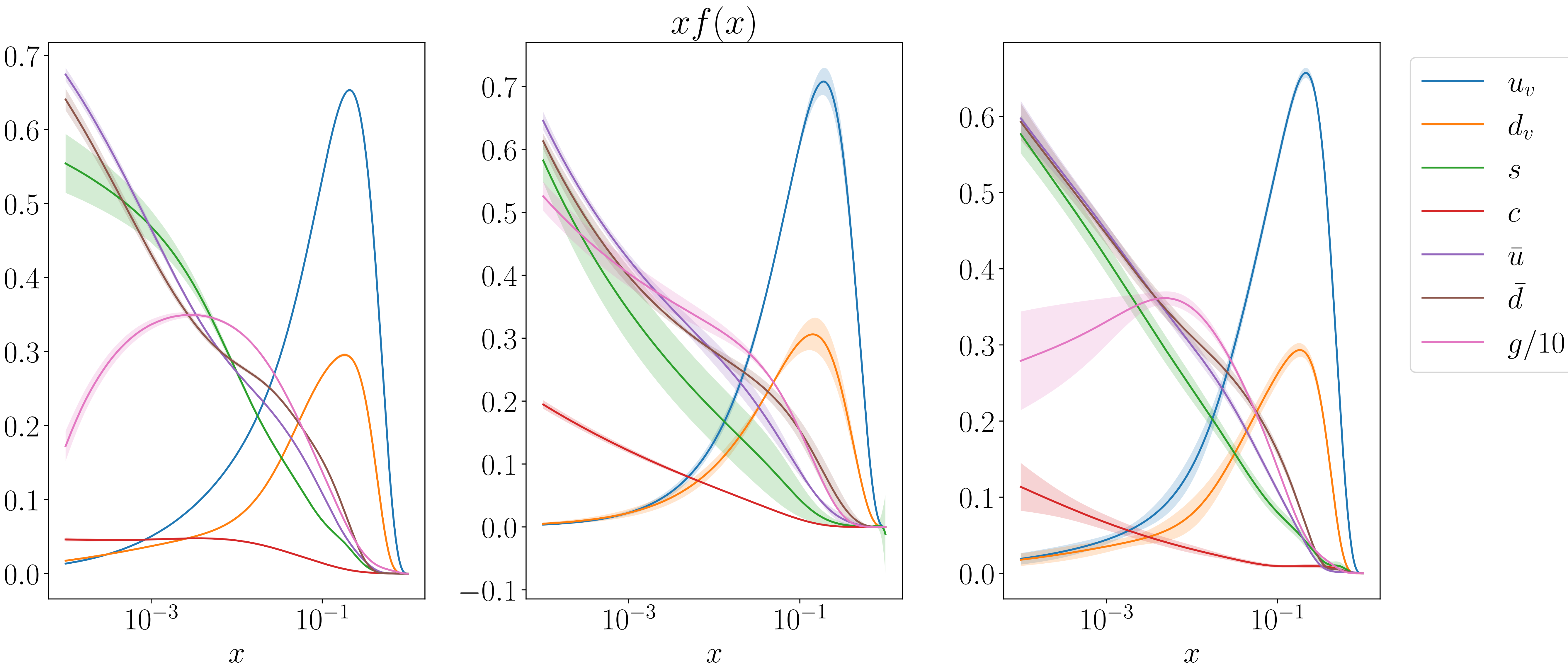
- Global fits: Use global collider data to determine universal PDFs

- PDFs interpolate between Parton process and Hadronic processes

Phenomenological Fits

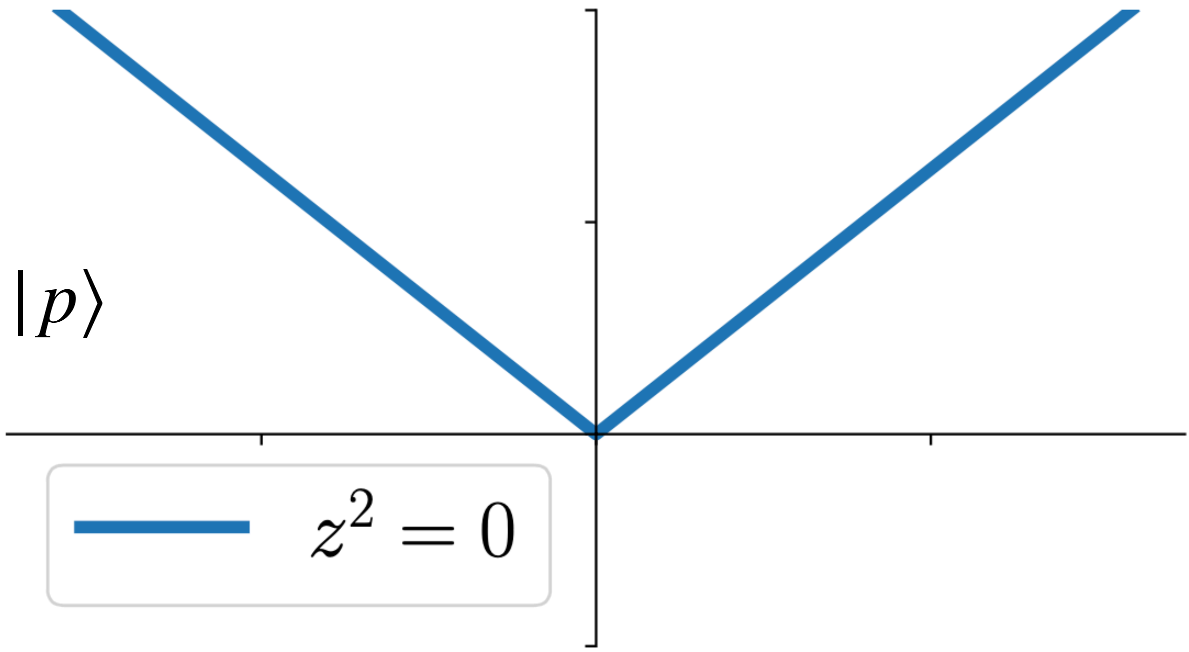
- **Nucleon unpolarized PDFs** from analysis of global experimental data
  - Parameterize PDF with fixed form $x^a(1-x)^b(1+\ldots)$ or NN(x)



(Left) MSHT20 Eur. Phys. J. C 81 (2021) 4, 341. (Center) JAM20 Phys. Rev. D 104 (2021) 1, 016015.
(Right) NNPDF4.0 Eur. Phys. J. C 82 (2022) 5, 428

Parton Distributions and the Lattice

Minkowski



- Parton Distributions are defined by operators with light-like separations

$$f(x,\mu^2) = P^+ \int dz\_ e^{ixP^+z_-} \langle p \,|\, \bar{q}(z\_)\Gamma W(z\_;0) q(0) \,|\, p\rangle$$

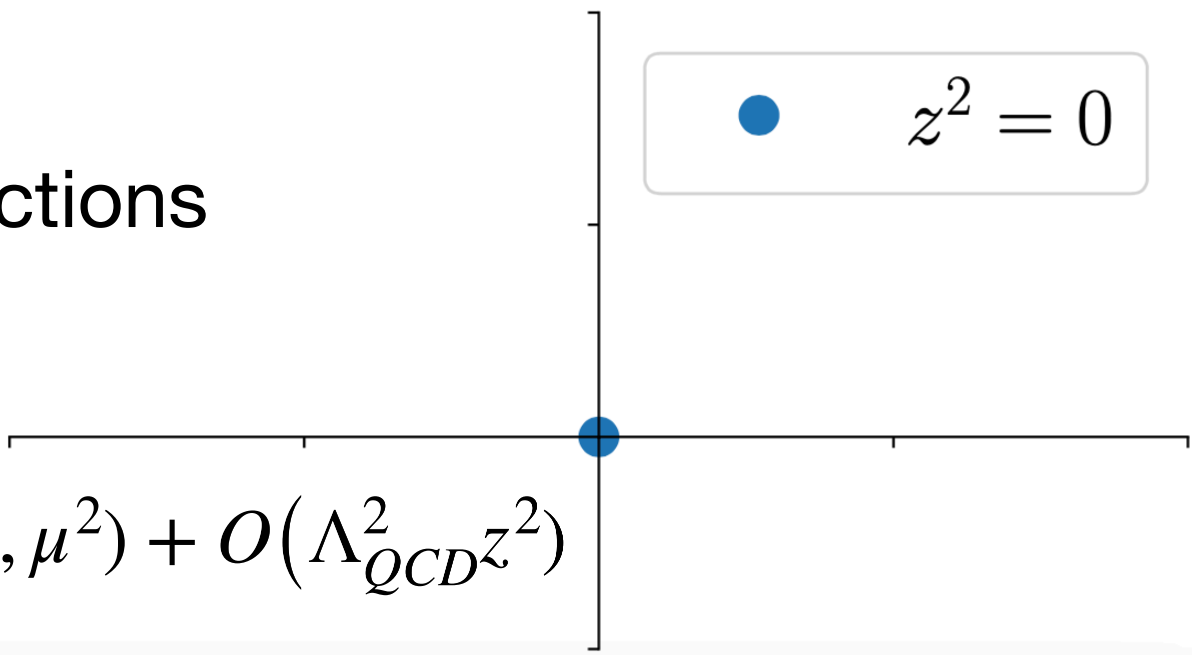
$$\Gamma = \gamma^+, \gamma^+\gamma^5, \sigma^{+i}\gamma^5$$

- Γ chooses spin (a)symmetry

- Use space-like separations

X. Ji *Phys Rev Lett* 110 (2013) 262002

Euclidean



$$O_\Gamma^{\mathrm{WL}}(z) = \bar{\psi}(z)\Gamma W(z;0)\psi(0) \qquad z^2 \neq 0$$

- Factorization analogous to cross sections

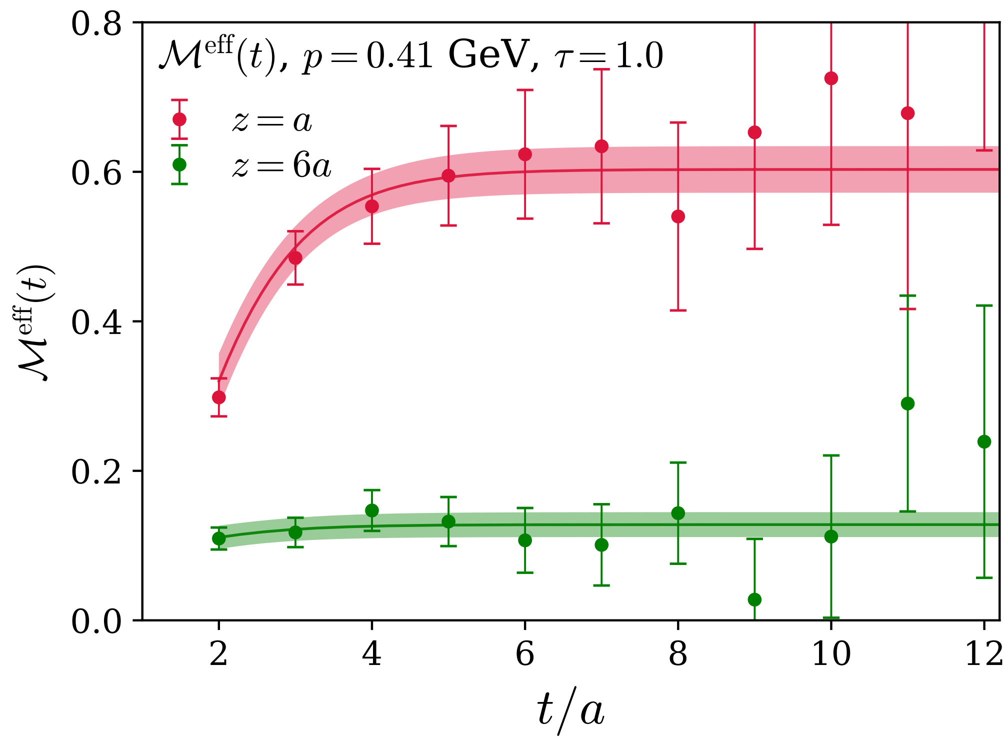
A. Radyushkin Phys Rev D 96 (2017) 3, 034025

$$\mathfrak{M}(\nu, z^2) = \langle p \,|\, O_\Gamma^{WL}(z) \,|\, p\rangle = \int dx\, C(x\nu; \mu^2 z^2) f(x,\mu^2) + O\left(\Lambda_{QCD}^2 z^2\right)$$

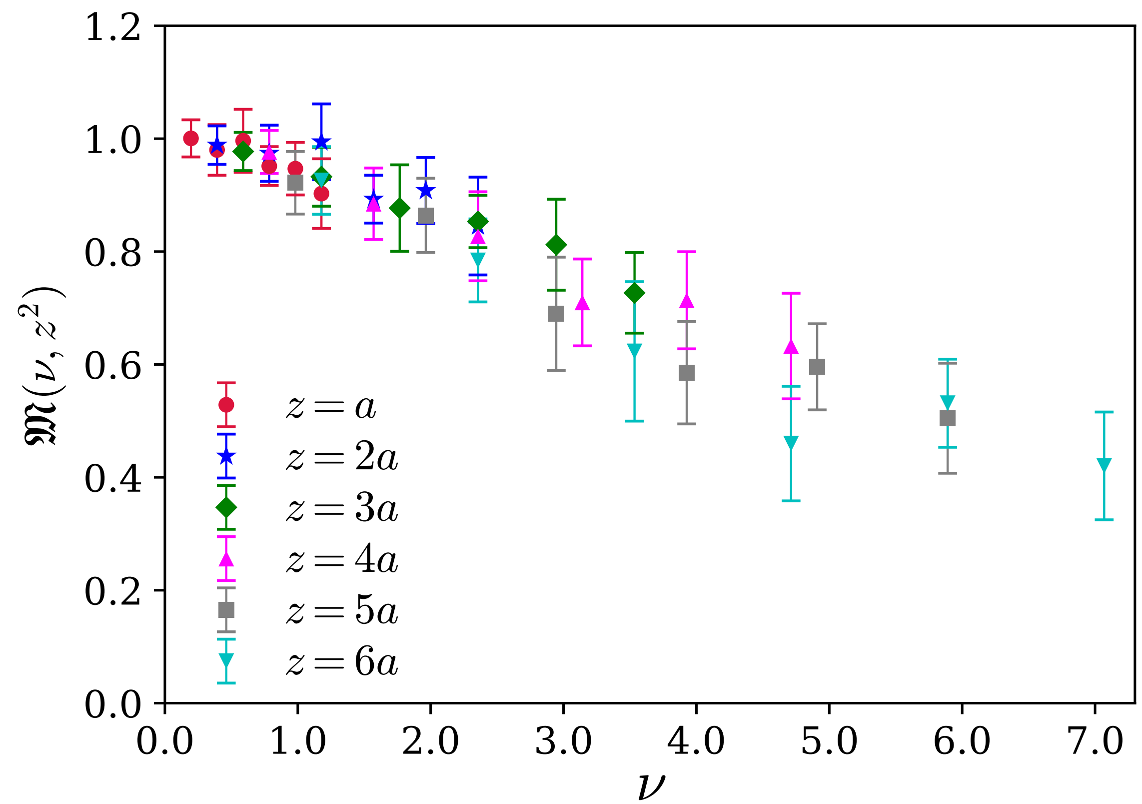
$$\nu = p \cdot z$$

From Lattice QCD to PDFs

Lattice Correlation Functions



Hadron Matrix Elements



- Correlators are series of exponentials (Euclidean space)
- Model and remove subdominant at large time
- Common procedure in LQCD hadronic studies

Unpolarized Gluon PDF

T. Khan, R. Sufian, JK, C. Monahan, C. Egerer, B. Joo, W. Morris, K. Orginos, A. Radyushkin, D. Richards, E. Romero, S. Zafeiropoulos
PRD 104 (2021) 9, 094516

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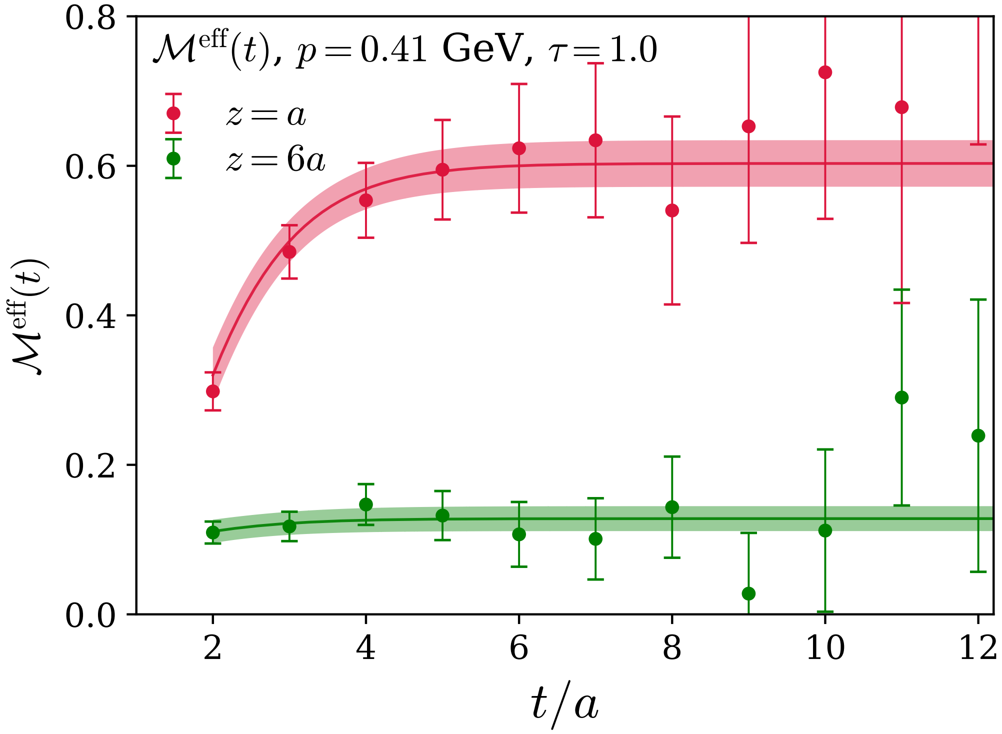

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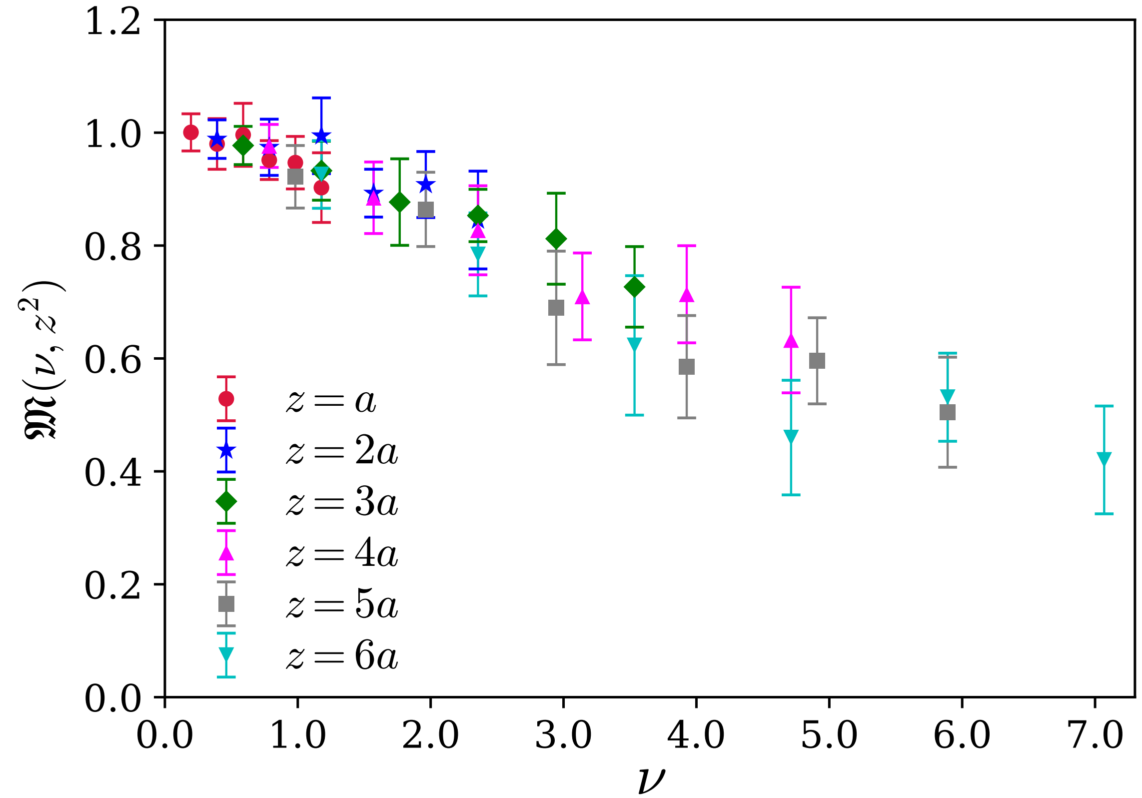
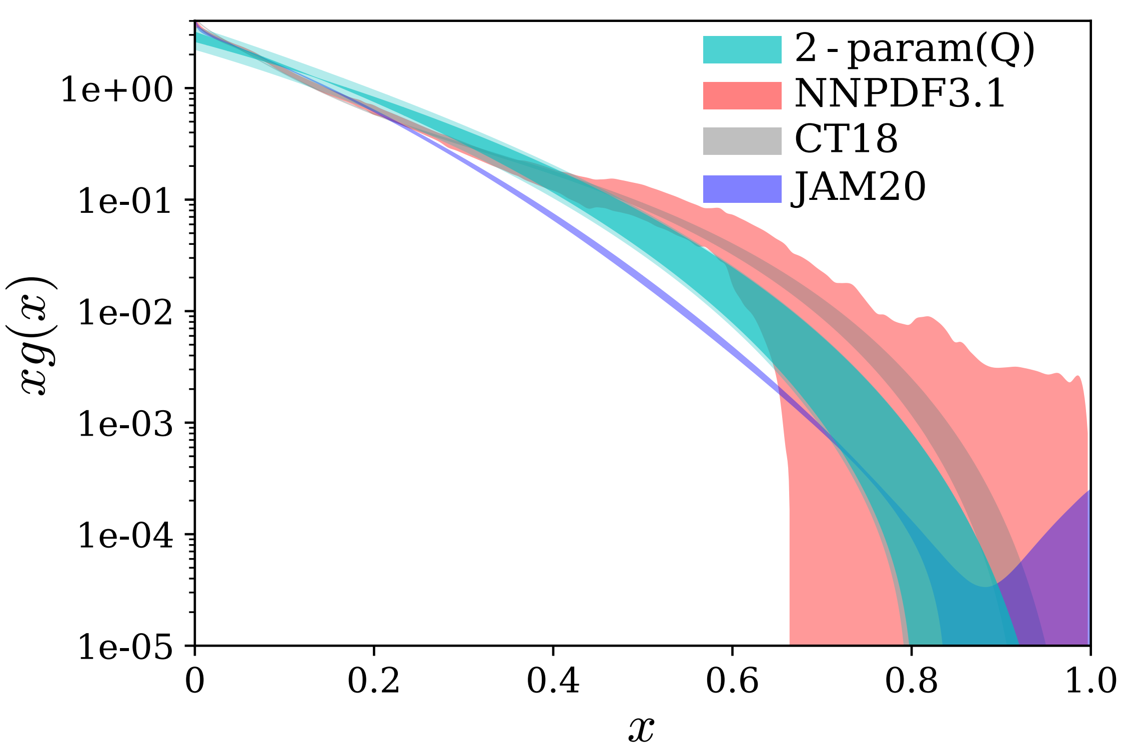


Parton Distributions



- Incomplete information gives integral inverse problem

$$xg(x) = x^a(1-x)^b / B(a+1, b+1)$$

- To more accurately infer PDF, we need larger ν

Unpolarized Gluon PDF

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Inverse Problems for Parton Physics

- **Structure Functions**

$$F_2(x, Q^2) = \sum_i \int_x^1 d\xi\, C(\xi, \frac{\mu^2}{Q^2})\, q(\frac{x}{\xi}, \mu^2)$$

- **LaMET (on the lattice)**

$$M(p_z, z) = \int_{-\infty}^{\infty} dy\, e^{iyp_z z}\, \tilde{q}(y, p_z^2)$$

- **pseudo-Distributions / Good Lattice Cross Sections**

$$\mathfrak{M}(\nu, z^2) = \int_{-1}^{1} dx\, C(x\nu, \mu^2 z^2)\, q(x, \mu^2)$$

- **Local Matrix elements / HOPE / OPE-without-OPE**

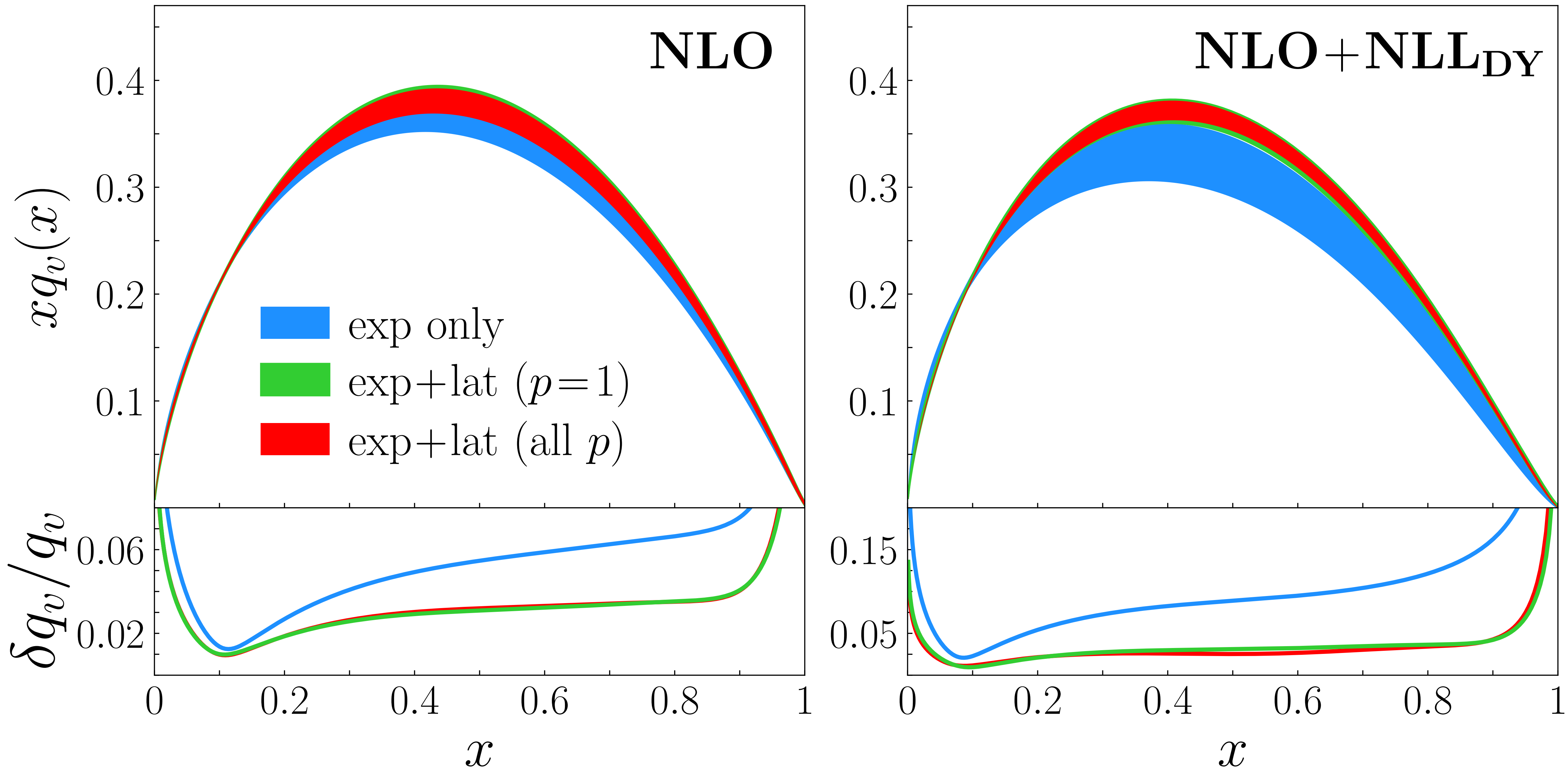
$$a_n(\mu^2) = \int_{-1}^{1} dx\, x^{n-1}\, q(x, \mu^2)$$

- **Hadronic Tensor**

$$\tilde{W}_{\mu\nu}(\tau) = \int d\nu\, e^{-\nu\tau}\, W_{\mu\nu}(\nu)$$

Experiment Meets the Lattice

- Simultaneously fit Lattice and Experimental pion PDF data
- Each gives unique information complementing each other



P. Barry, C. Egerer, JK, W. Melnitchouk, C. Monahan, K. Orginos, JW. Qiu, D. Richards, N. Sato, R. Sufian, S. Zafeiropoulos, *Phys. Rev. D* 105 (2022) 11, 114051

Inverse Problems for pseudo-PDFs

- Limited range of z and p cannot approach $\nu \to \infty$ to integrate inverse

$$q(x) = \int_0^\infty d\nu \, C^{-1}(x\nu)\mathfrak{M}(\nu)$$

JK, K. Orginos, A. Rothkopf, S. Zafeiropoulos JHEP 04 (2019) 057

Inverse Problems for pseudo-PDFs

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- Forward integral to an ill posed matrix equation

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$$\mathfrak{M}(\nu) = \int_0^1 dx \, C(x\nu) \, q(x) \to [\mathbf{C}][\mathbf{q}]$$

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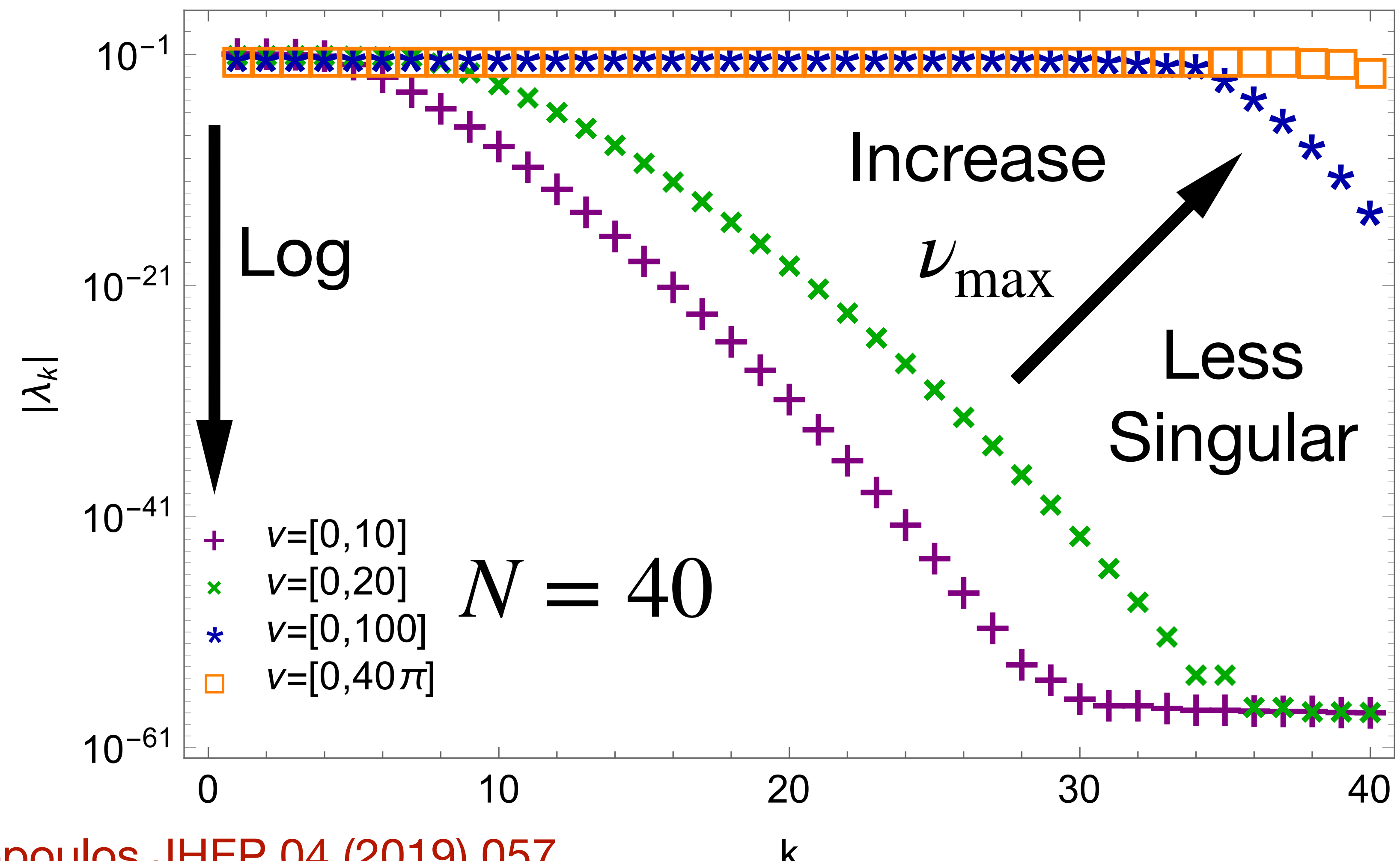

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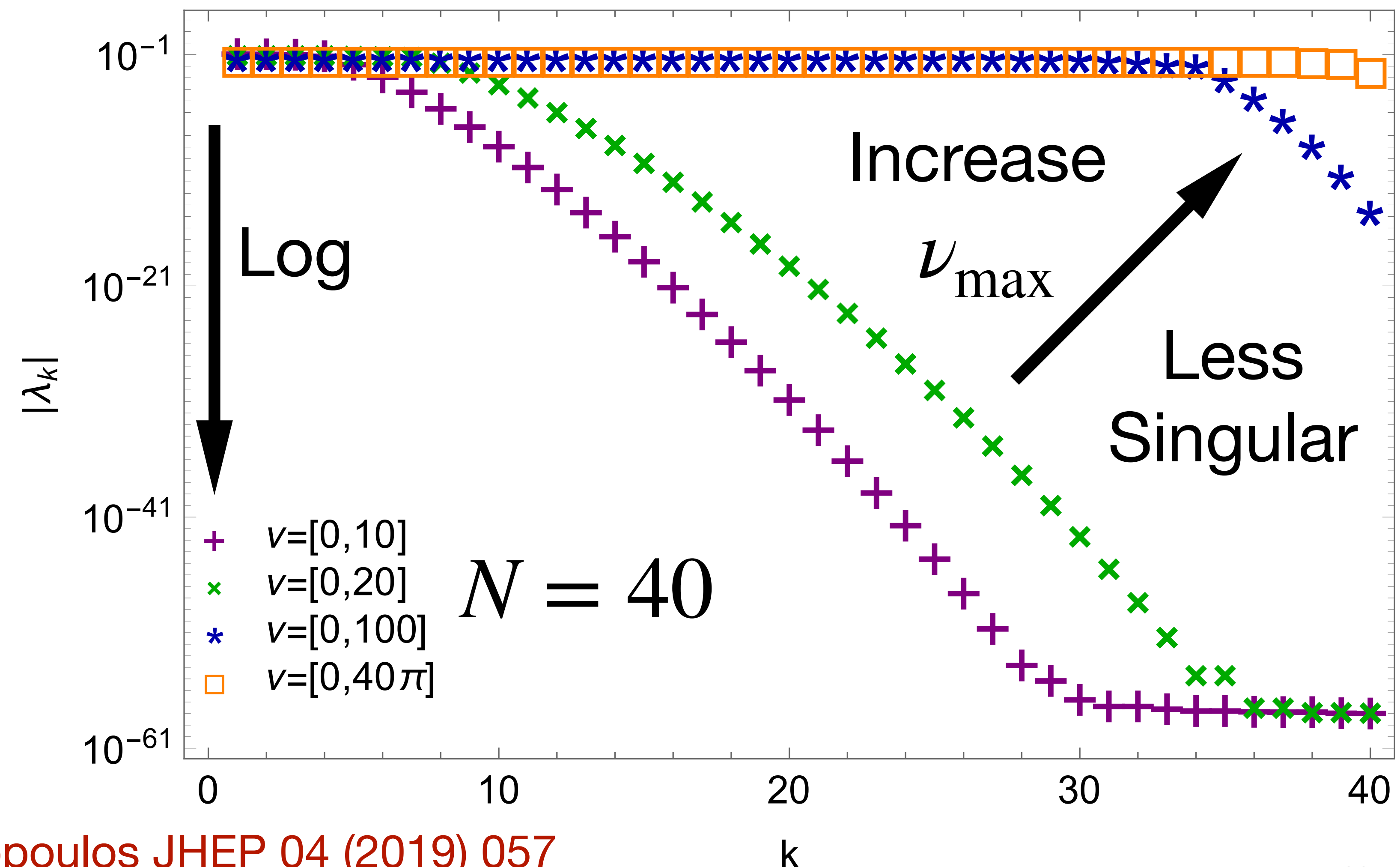
- Must be regulated by additional information
  - Restricted functional form
  - Constraints on the PDF or parameters
  - Assumptions of smoothness, continuity, ….

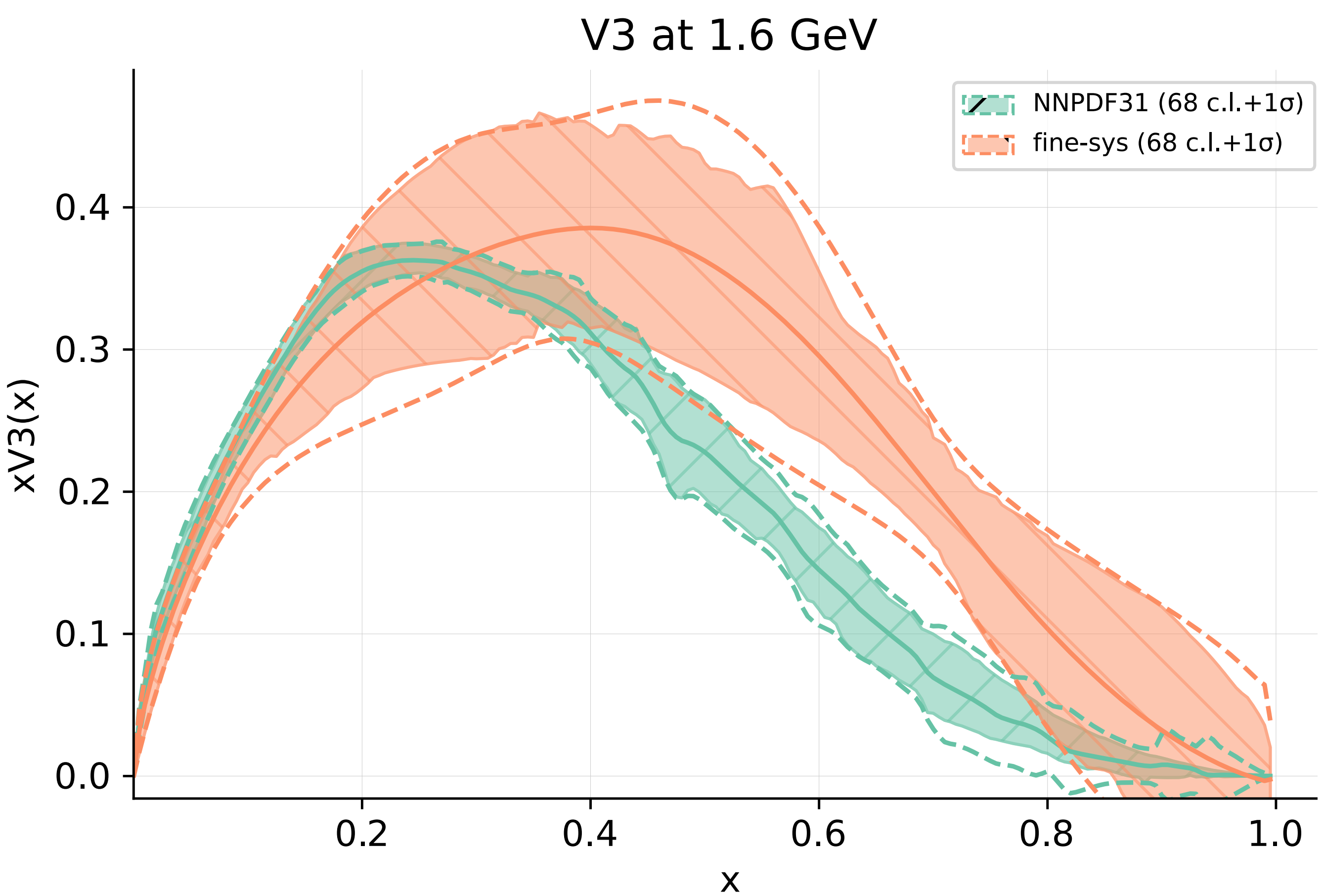


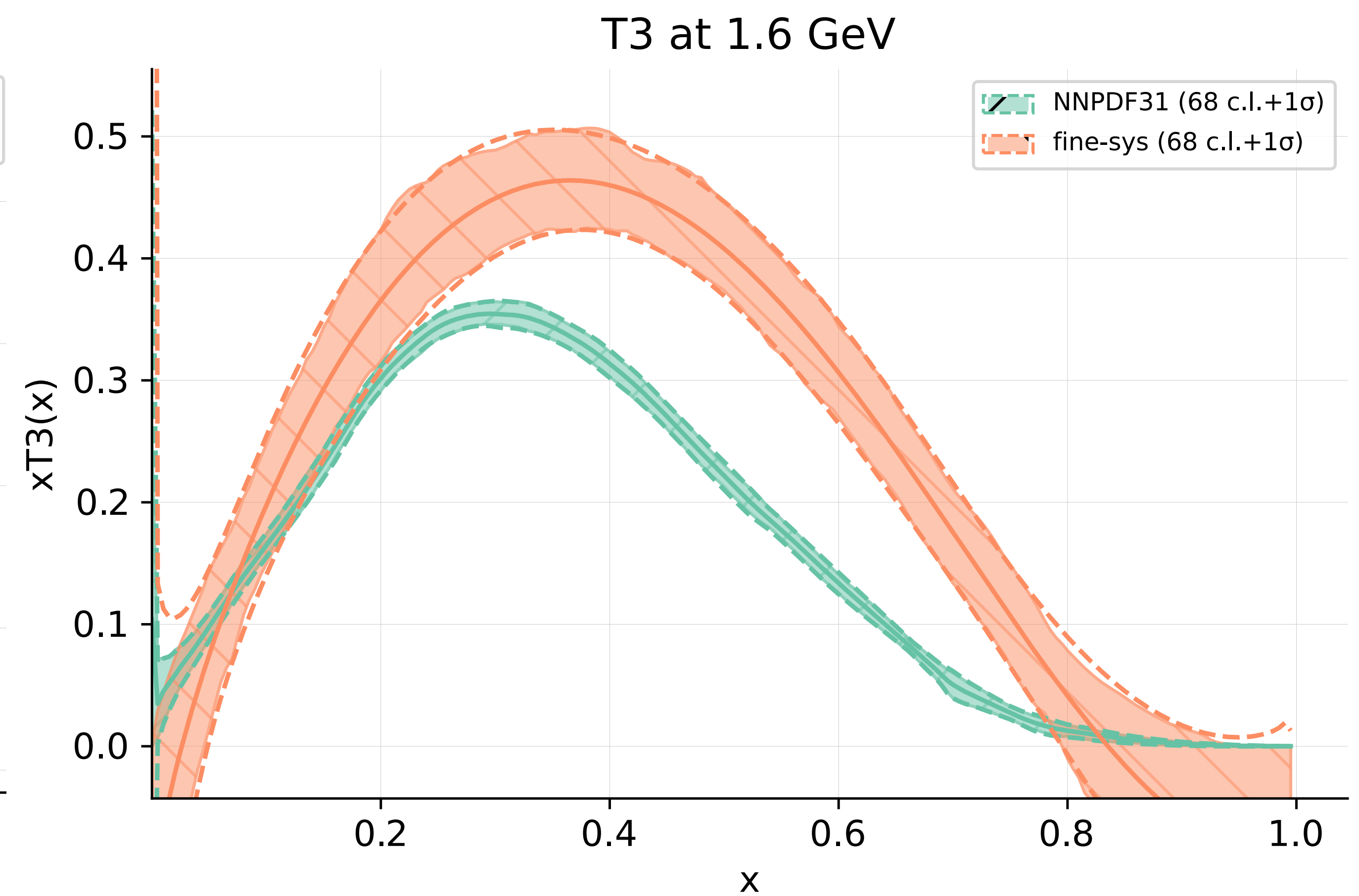
JK, K. Orginos, A. Rothkopf, S. Zafeiropoulos JHEP 04 (2019) 057

Neural Networks for Inverse Problems

- NNPDF approach to define PDF with NN of geom 2-5-3-1

L. Del Debbio, T. Giani, JK, K. Orginos, A. Radyushkin, S. Zafeiropoulos JHEP 02 (2021) 138





$V_3 = q_- = q_v \quad T_3 = q_+$

- Higher dimensional distributions will need larger networks
  - Sivers asymmetry and TMDs with NNs I. Fernando, D. Keller PRD 108 (2023) 5, 054007

Bayesian Reconstruction

- Neural Networks may be obtuse for how prior information is added
- Take advantage of single dimension and limited range
- Approximate PDF by its value on a grid and interpolate for integrals
- Maximize the posterior distribution

$$P\left[q\,|\,\mathfrak{M},I\right] \propto P\left[\mathfrak{M}\,|\,q,I\right] P\left[q\,|\,I\right]$$

- Add prior information to regulate the inverse problem

$$P\left[q\,|\,I\right] \propto \exp[-S(q)]$$

Y. Burnier and A. Rothkopf (2013) 1307.6106

Shannon-Jaynes entropy

$$S(q) = \alpha \int_0^1 dx \left(q(x) - m(x) - q(x)\log(\frac{q(x)}{m(x)}) \right)$$

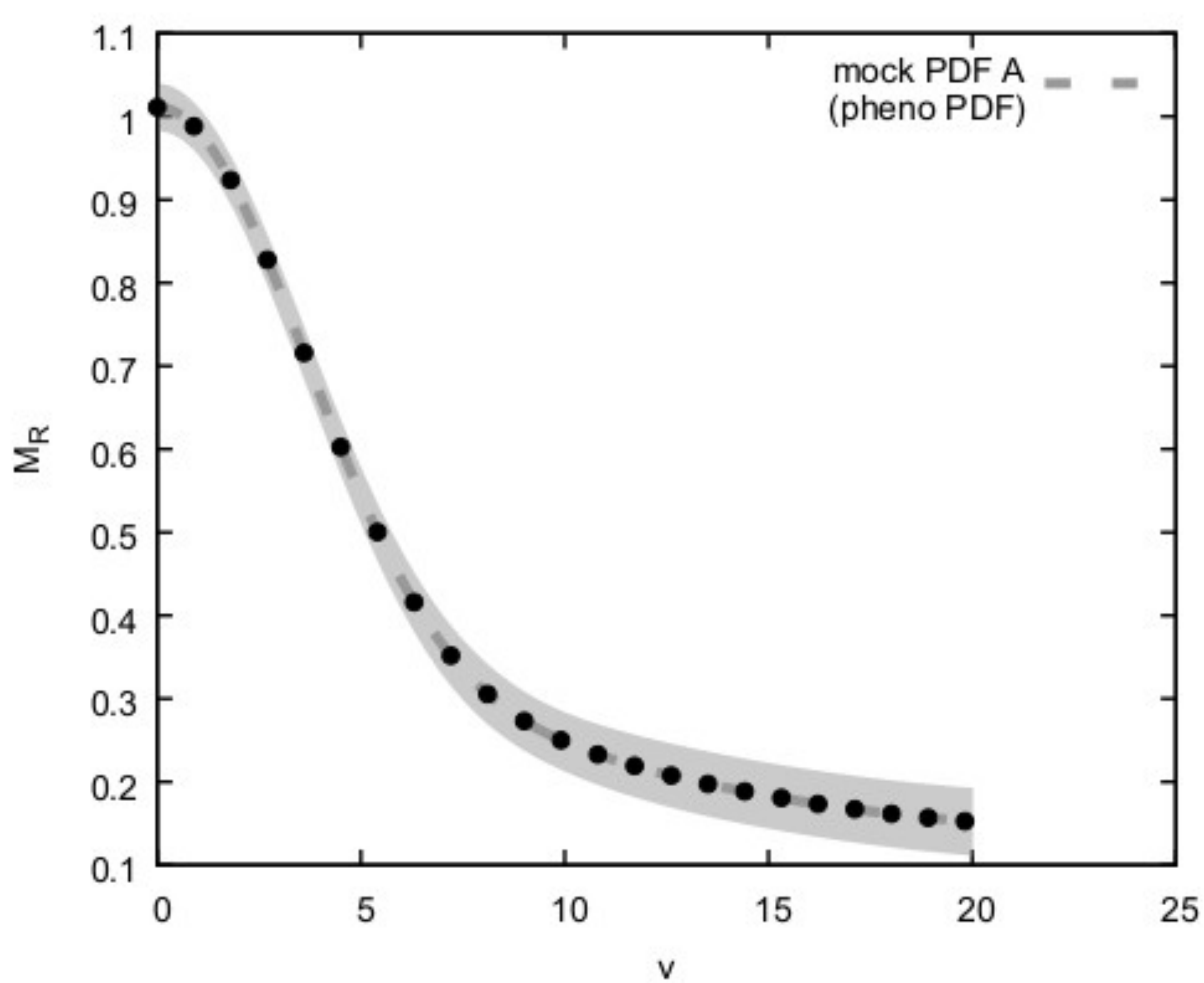
Burnier-Rothkopf

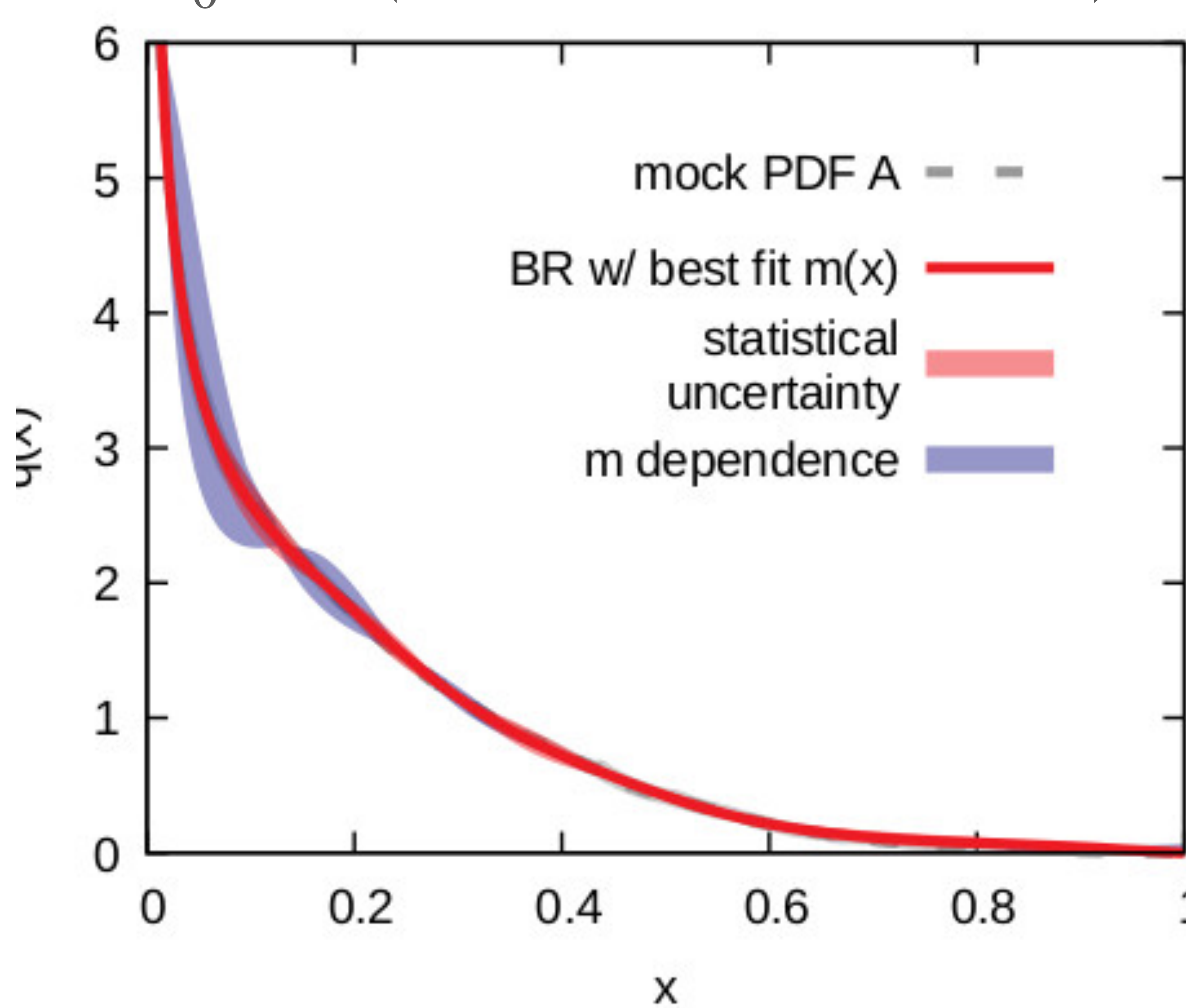
$$S(q) = \alpha \int_0^1 dx \left(1 - \frac{q(x)}{m(x)} + \log(\frac{q(x)}{m(x)}) \right)$$

Study on Mock Data

- Made Fake data sets to study dependence on maximum ν
  JK, K. Orginos, A. Rothkopf, S. Zafeiropoulos JHEP 04 (2019) 057
- Current range of data can well reconstruct large x

$$S(q) = \alpha \int_0^1 dx \left(1 - \frac{q(x)}{m(x)} + \log(\frac{q(x)}{m(x)}) \right)$$





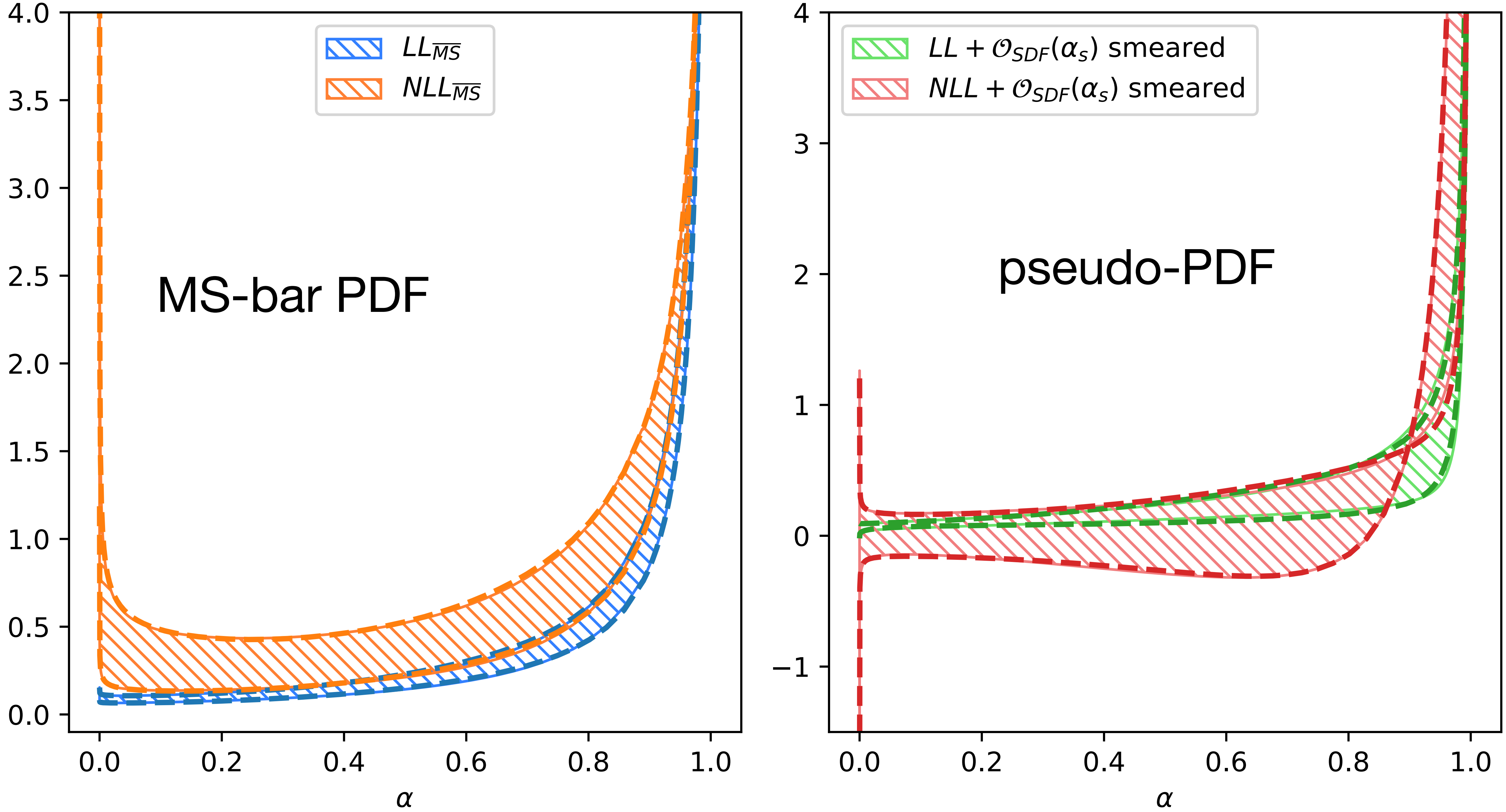
Evolution of parton distributions

- Standard DGLAP evolution $$\mu^2 \frac{d}{d\mu^2} q(x,\mu^2) = \int_x^1 dy\, P_{qq}(y) q(\frac{x}{y},\mu^2)$$
  - Parton model: Splitting of partons into smaller x
- MSbar Step Scaling function $$q(x,\mu^2) = \int_x^1 dy\, \mathscr{E}(y,\mu^2,\mu_0^2) q(\frac{x}{y},\mu_0^2)$$
  - Discretized version of evolution
- pseudo-PDF Step scaling $$\mathfrak{M}(\nu,z^2) = \int_0^1 d\alpha\, \Sigma(\alpha,z^2,z_0^2)\mathfrak{M}(\alpha\nu,z_0^2)$$
  - Evolution kernel can be determined from lattice data

$$\mathscr{E}(\mu^2,\mu_0^2) = C^{-1}(\mu^2 z^2) \otimes \Sigma(z^2,z_0^2) \otimes C(\mu_0^2 z_0^2)$$

Evolution of parton distributions

- Perturbative evolution from ~700 MeV (0.282 fm) to ~1GeV (0.188 fm)
- Errors from varying scale by factor of 2

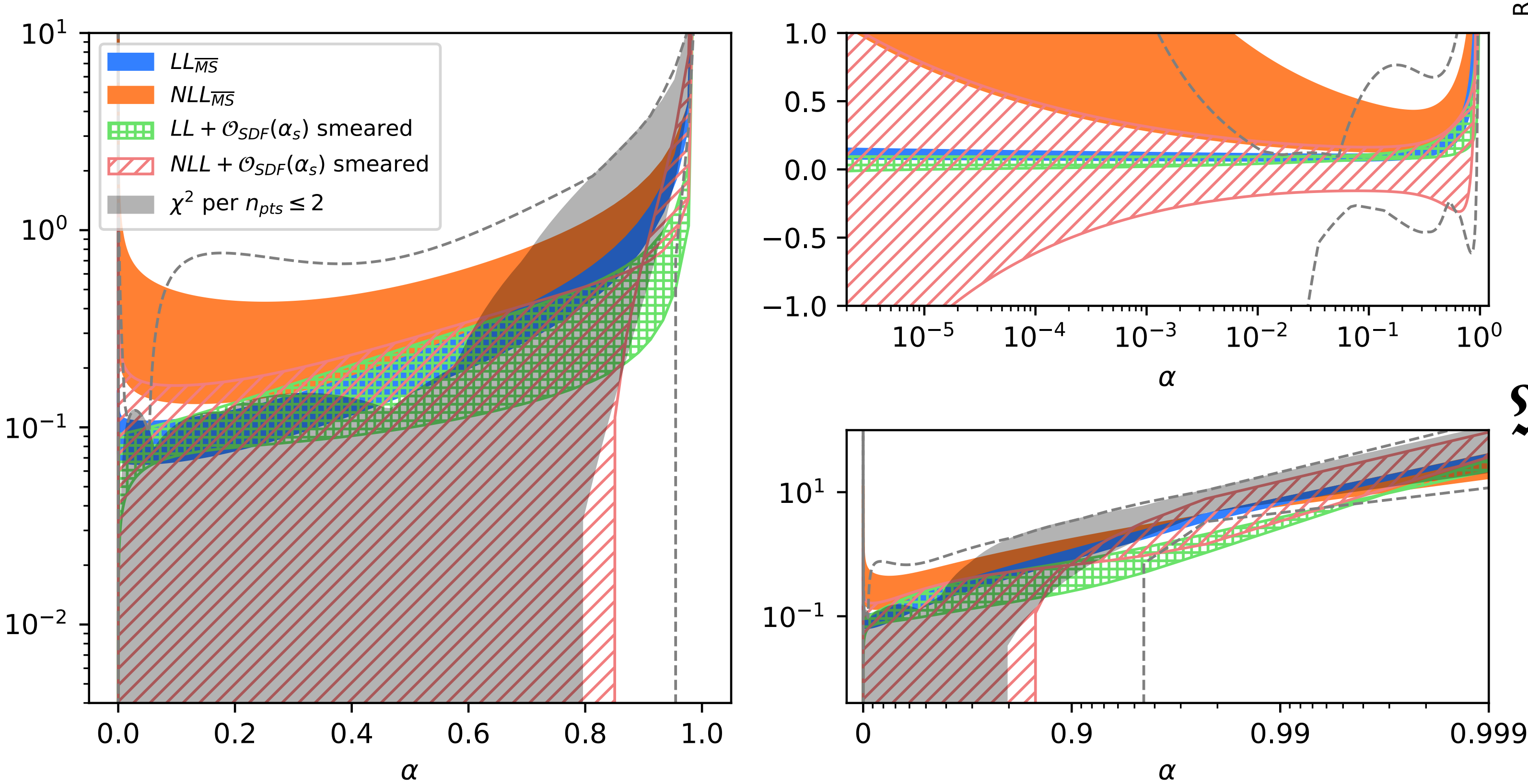


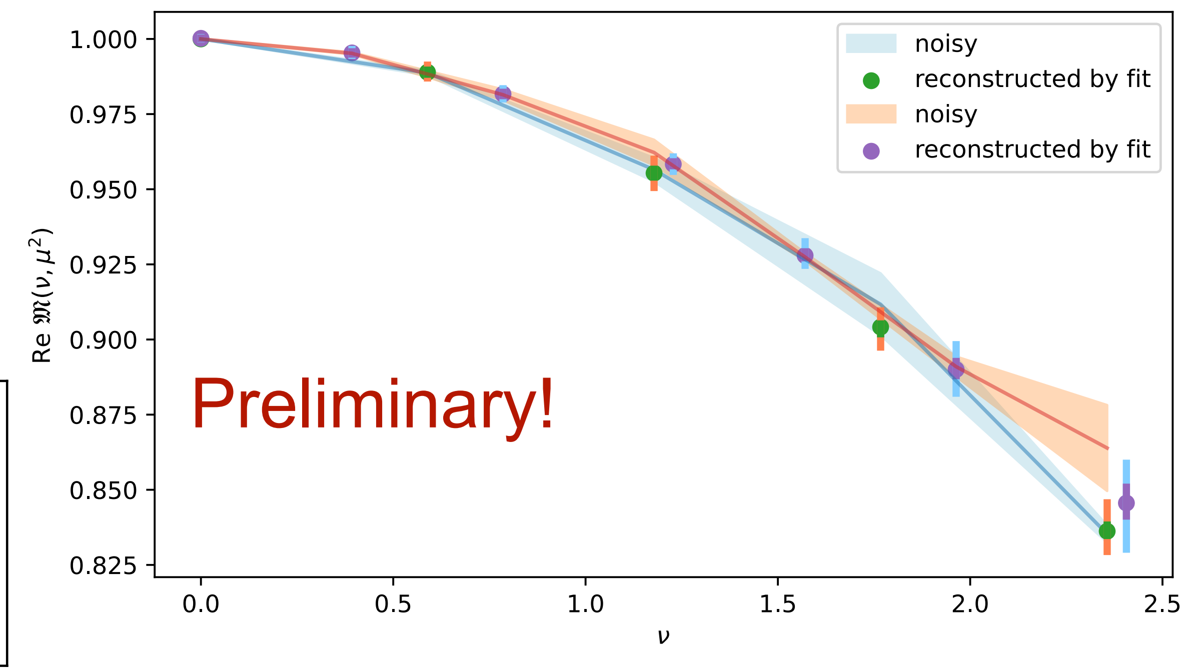
Step Scaling from the lattice

- Requires data in same range of ν and different z

- Model Function

$$\Sigma(\alpha) = A\alpha^{-\delta}(1 + r\alpha) + B(-\ln(\alpha))^{-\eta}\ln^2(1-\alpha) + \sigma\alpha(1-\alpha)$$





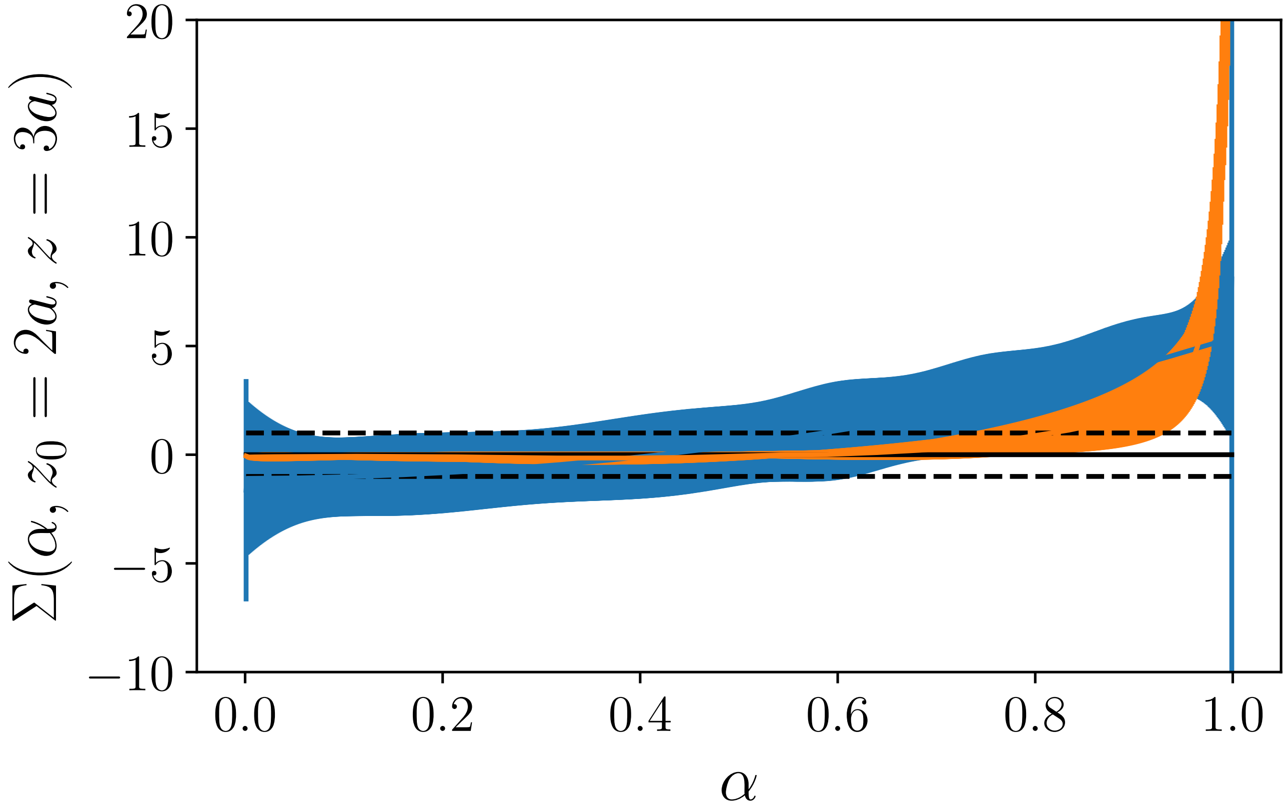
$$\mathfrak{M}(\nu, z^2) = \int_0^1 d\alpha\, \Sigma(\alpha, z^2, z_0^2)\mathfrak{M}(\alpha\nu, z_0^2)$$

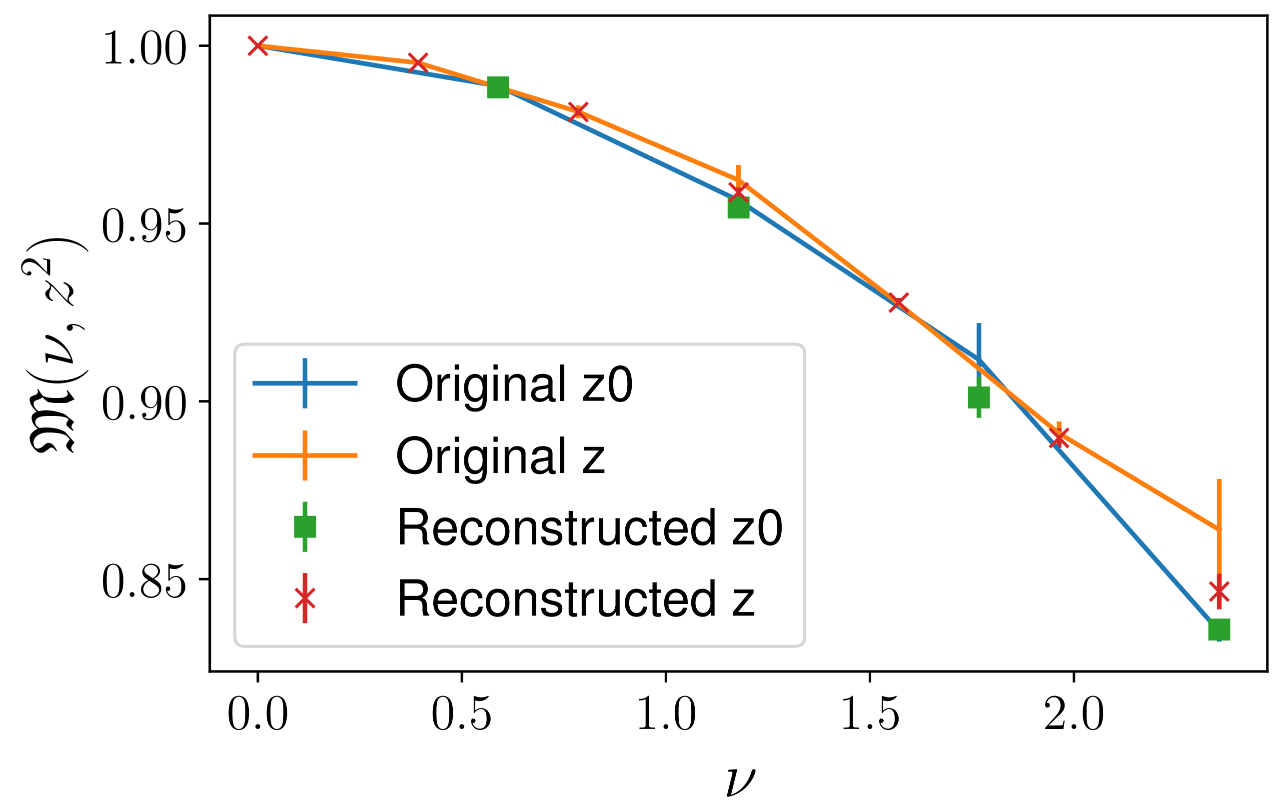
- Catch: Requires assumption of leading twist dominance and ranges of ν are limited
  - Need very large volumes and very fine lattices to do right.

Bayesian Reconstruction

- Use different BR priors to study model dependencies
- First prior with easily understood biases
  - Quadratic Difference Ratio (QDR) $S(\Sigma) = u \int_0^1 d\alpha \, \frac{\left(\Sigma(\alpha) - h(\alpha)\right)^2}{\sigma(\alpha)^2}$

Preliminary!





$$u = 1 \quad h(\alpha) = 0 \quad \sigma(\alpha) = 1$$

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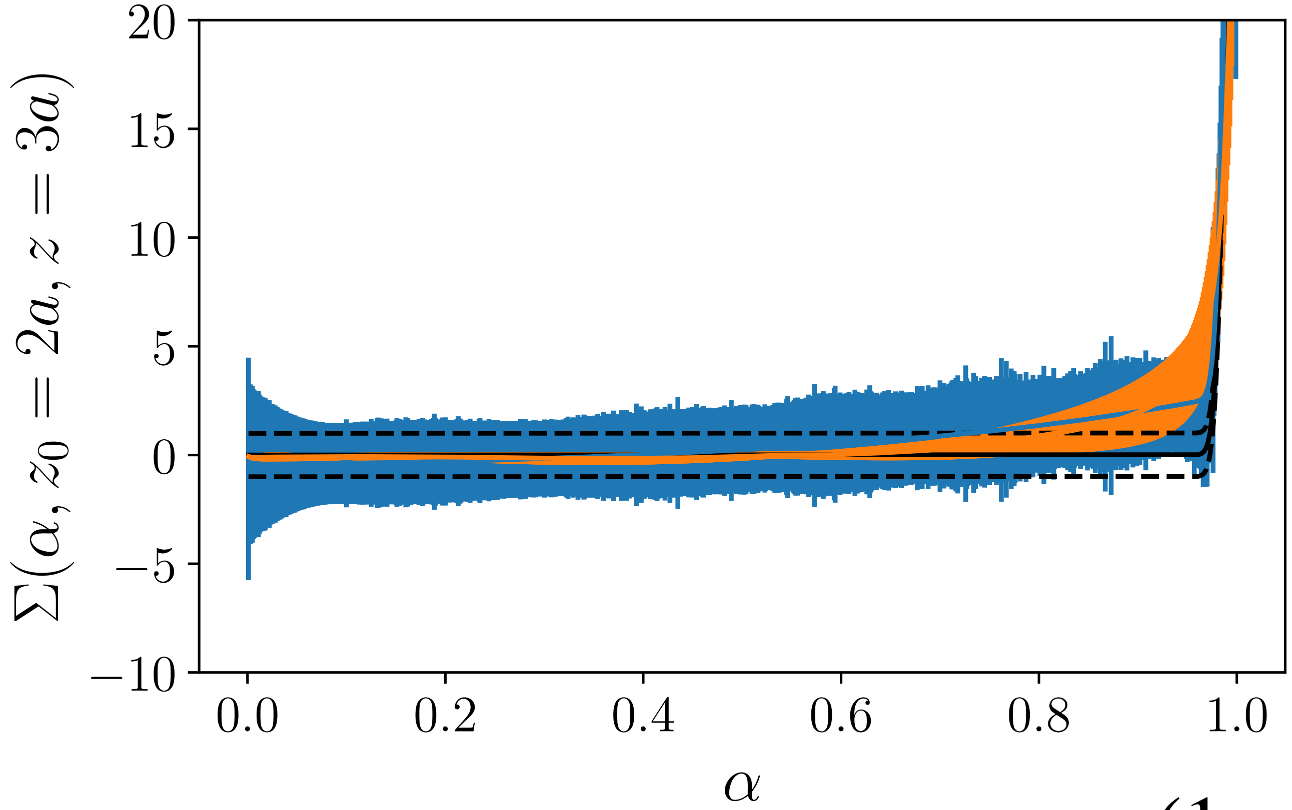
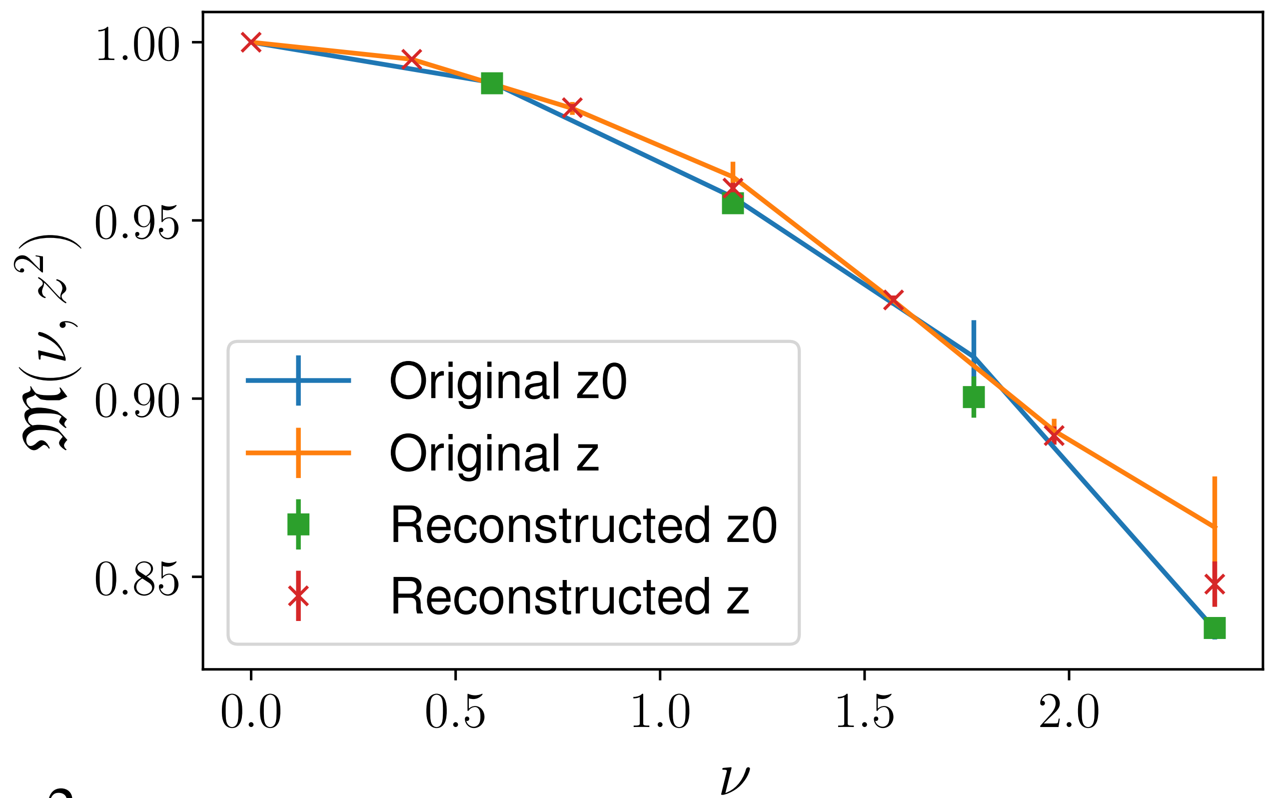




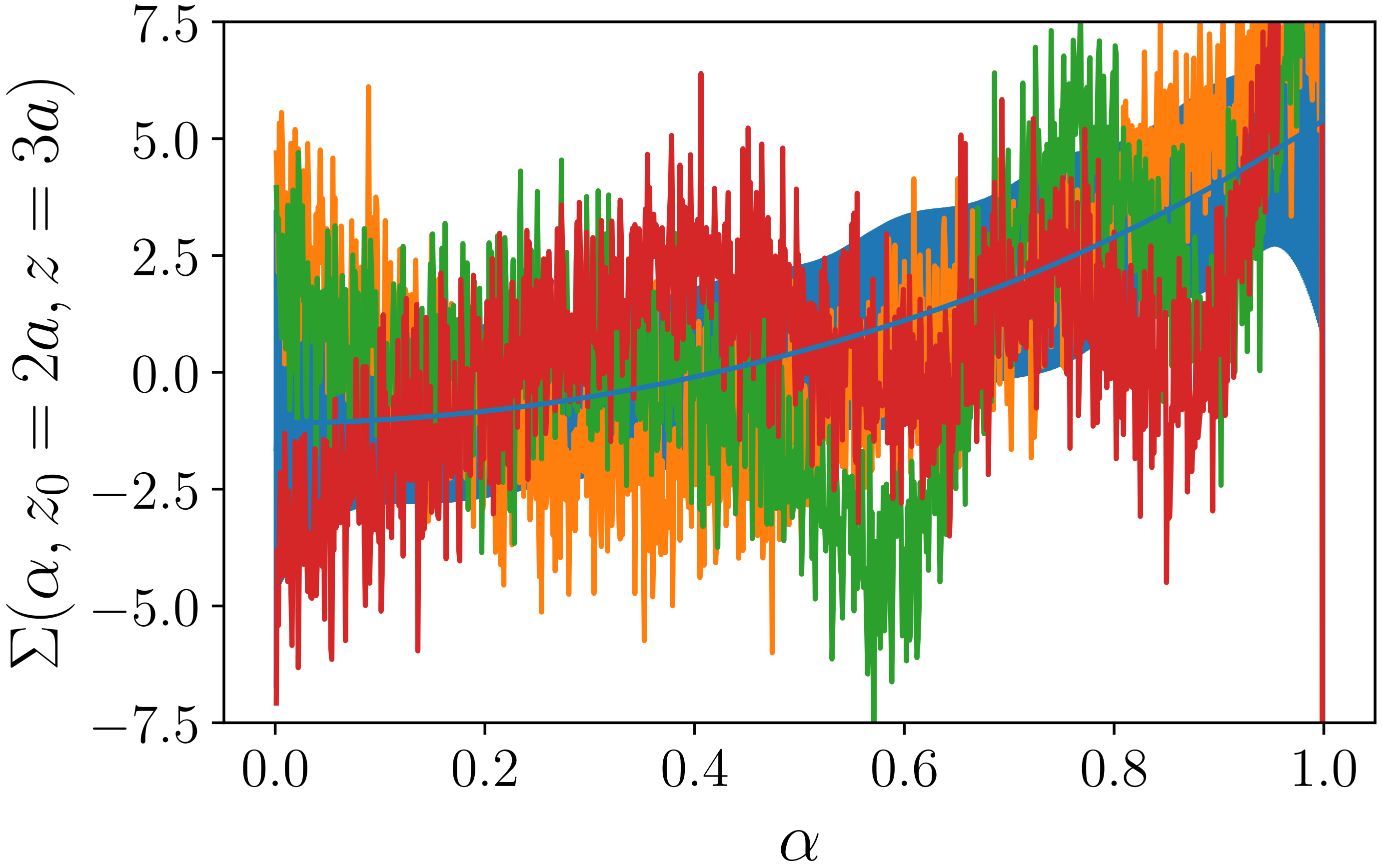
$$u = 1 \quad h(\alpha) = \exp\left(-\frac{(1-\alpha)^2}{w^2}\right)/(w\sqrt{2\pi}) \quad \sigma(\alpha) = 1$$

$$w = 0.01$$

“I’m sorry, Nature hates Wiggles”

-A. Radyushkin

- Characteristic curves from fit
- QDR has no correlations between neighbors
- Need better priors!



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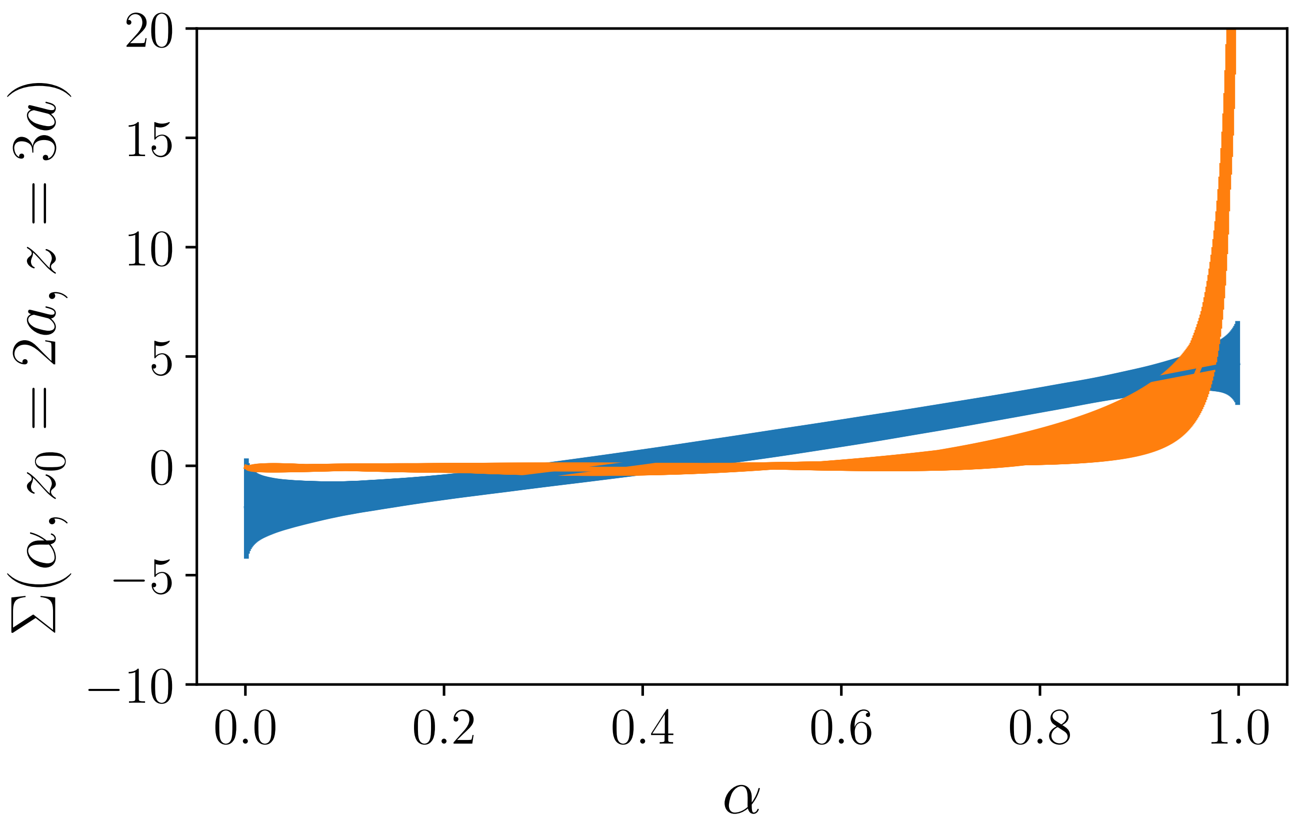
- Use different BR priors to study model dependencies
- Can we remove the wiggles?
  - A smoothing prior

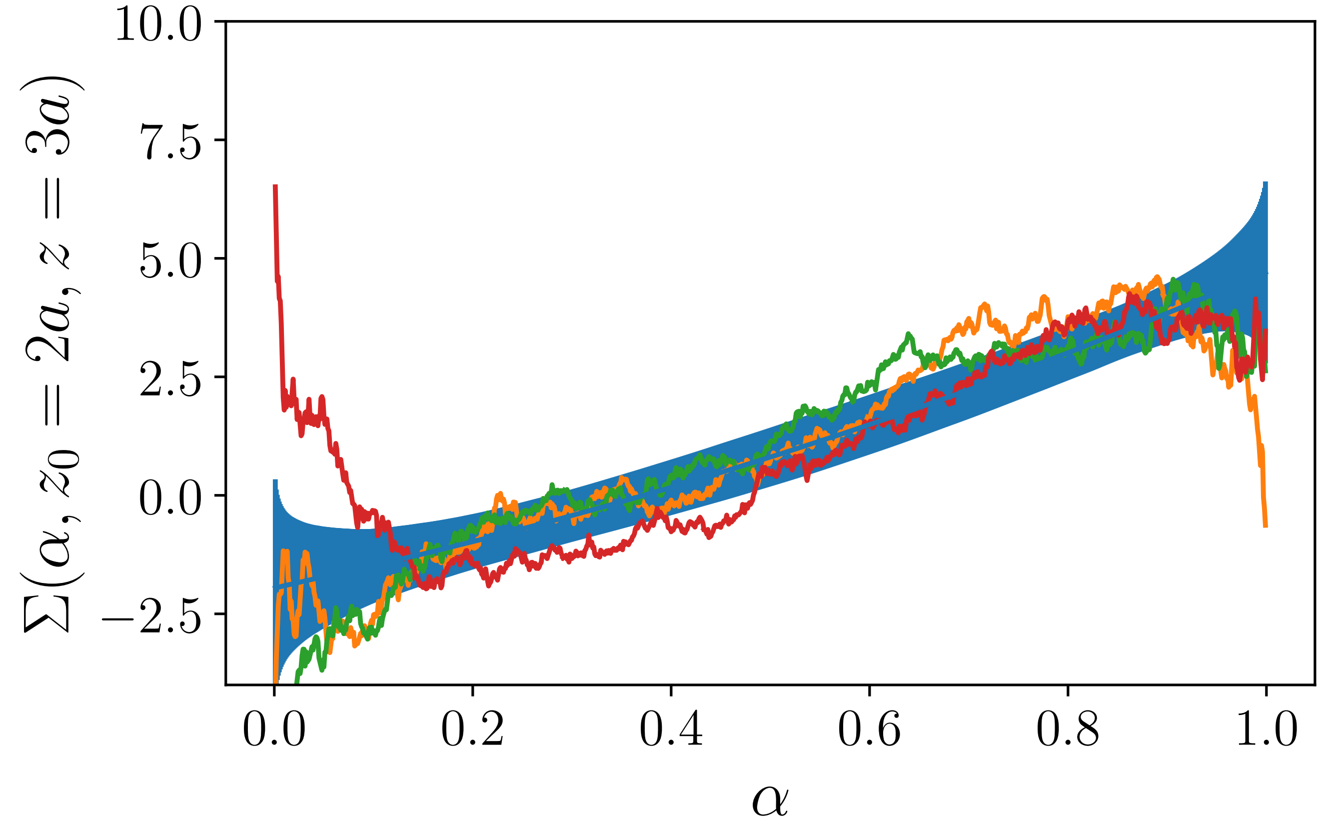
$$S(\Sigma) = u \int_0^1 d\alpha\, \alpha(1-\alpha) \left(\frac{\partial \Sigma}{\partial \alpha} \right)^2$$

- Set u too large and it forces a flat result.

Preliminary!

$u = 1$





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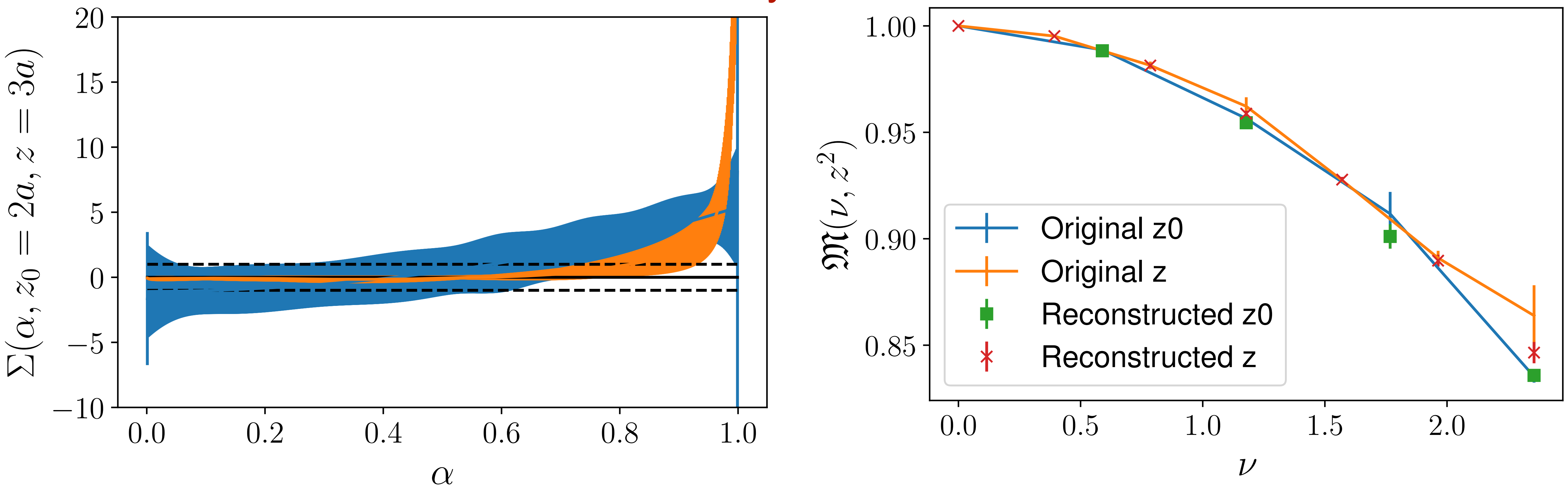

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Conclusions

- Inferring parton distributions is major experimental and lattice effort
- Lattice data can fit PDFs or Step Scaling functions
- Combined analyses have great potential to get better results than either in isolation
    - Future combined analysis can remove shadow GPD problem
- Bayesian Reconstruction can be used to reproduce 1D function under the integrals
    - Dedicated study of evolution and priors in the BR is underway
    - Requires comparing/averaging different priors