Covariant Extension of Generalized Parton Distributions using Artificial Neural Networks

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Probing the Frontiers of Nuclear Physics with AI at the EIC

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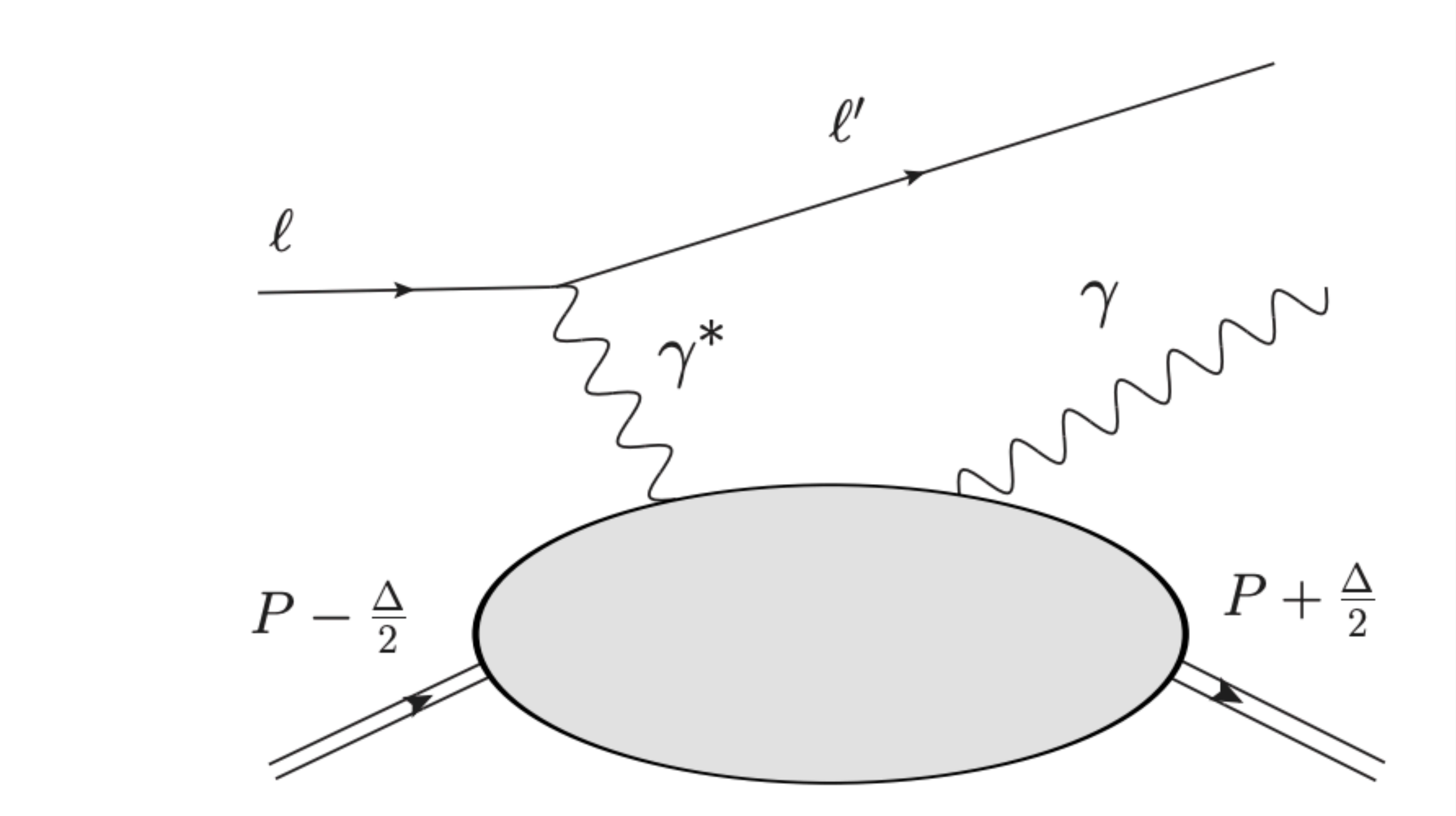
Outline

- Generalized Parton Distributions
- Covariant Extension via Radon Transform (RT) inversion
- RT inversion using Artificial Neural Networks
- Results on analytical models
- Conclusions and outlook

Generalized Parton Distribution (GPD)

Exclusive processes (DVCS)

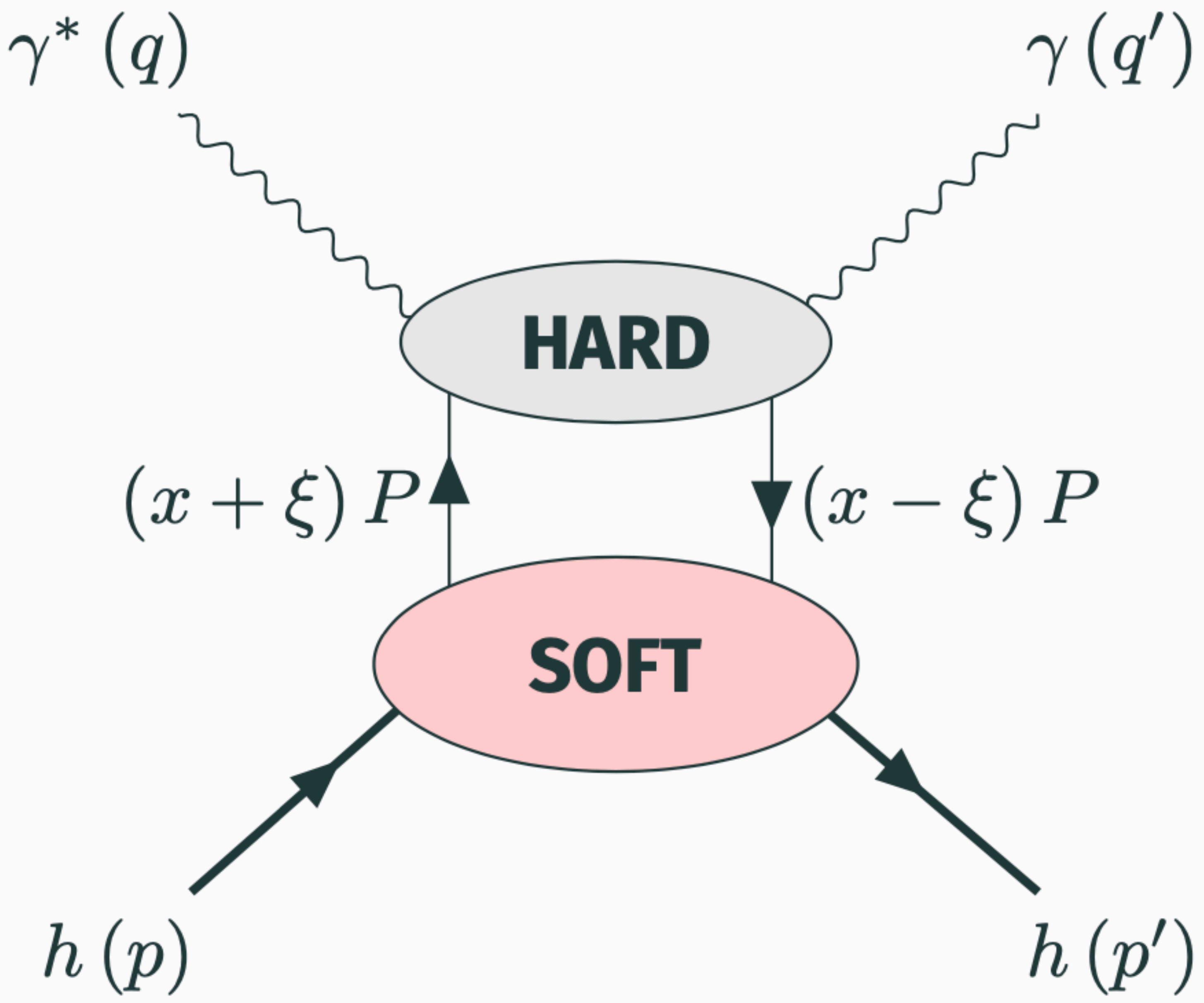


$Q^2 \to \infty$

$$P = \frac{p+p'}{2}, \quad \Delta = p' - p$$

$$\xi = \frac{(p'-p)^+}{2P^+} = -\frac{\Delta^+}{2P^+}$$

$$t = \Delta^2$$



Factorization of the amplitude

$$\mathscr{M}(\xi, t; Q^2) = \sum_{p=q,g} \int_{-1}^{1} \frac{dx}{\xi} K^p\left(\frac{x}{\xi}, \frac{Q^2}{\mu_F^2}, \alpha_s(\mu_F)\right) \mathscr{H}^p\left(x, \xi, t; \mu_F\right) + O(1/Q^2)$$

hard / perturbative

soft / non perturbative GPD

Ex: Chiral even twist-2 operators

Nucleon quark GPD

$$\mathcal{H}^q(x,\xi,t) = \frac{1}{2}\int dz^- \, e^{ixP^+z^-} \langle P+\Delta/2 \,|\, \bar{\psi}^q(-z/2)\gamma^+\psi^q(z/2) \,|\, P-\Delta/2\rangle \Big|_{z^+=z_\perp=0}$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\bar{u}(p')\gamma^+u(p) + E^q(x,\xi,t)\bar{u}(p')\frac{i\sigma^{+\mu}\Delta_\mu}{2M}u(p)\right]$$

$$\tilde{\mathcal{H}}^q(x,\xi,t) = \frac{1}{2}\int dz^- \, e^{ixP^+z^-} \langle P+\Delta/2 \,|\, \bar{\psi}^q(-z/2)\gamma^+\gamma^5\psi^q(z/2) \,|\, P-\Delta/2\rangle \Big|_{z^+=z_\perp=0}$$

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quark GPD of spin zero hadron

$$H^q_\pi(x,\xi,t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle \pi(P+\Delta/2) \,|\, \bar\psi^q(-z/2)\gamma^+\psi^q(z/2) \,|\, \pi(P-\Delta/2)\rangle \Big|_{z^+=0,\ z^\perp=0}$$

Ex: Chiral even twist-2 operators

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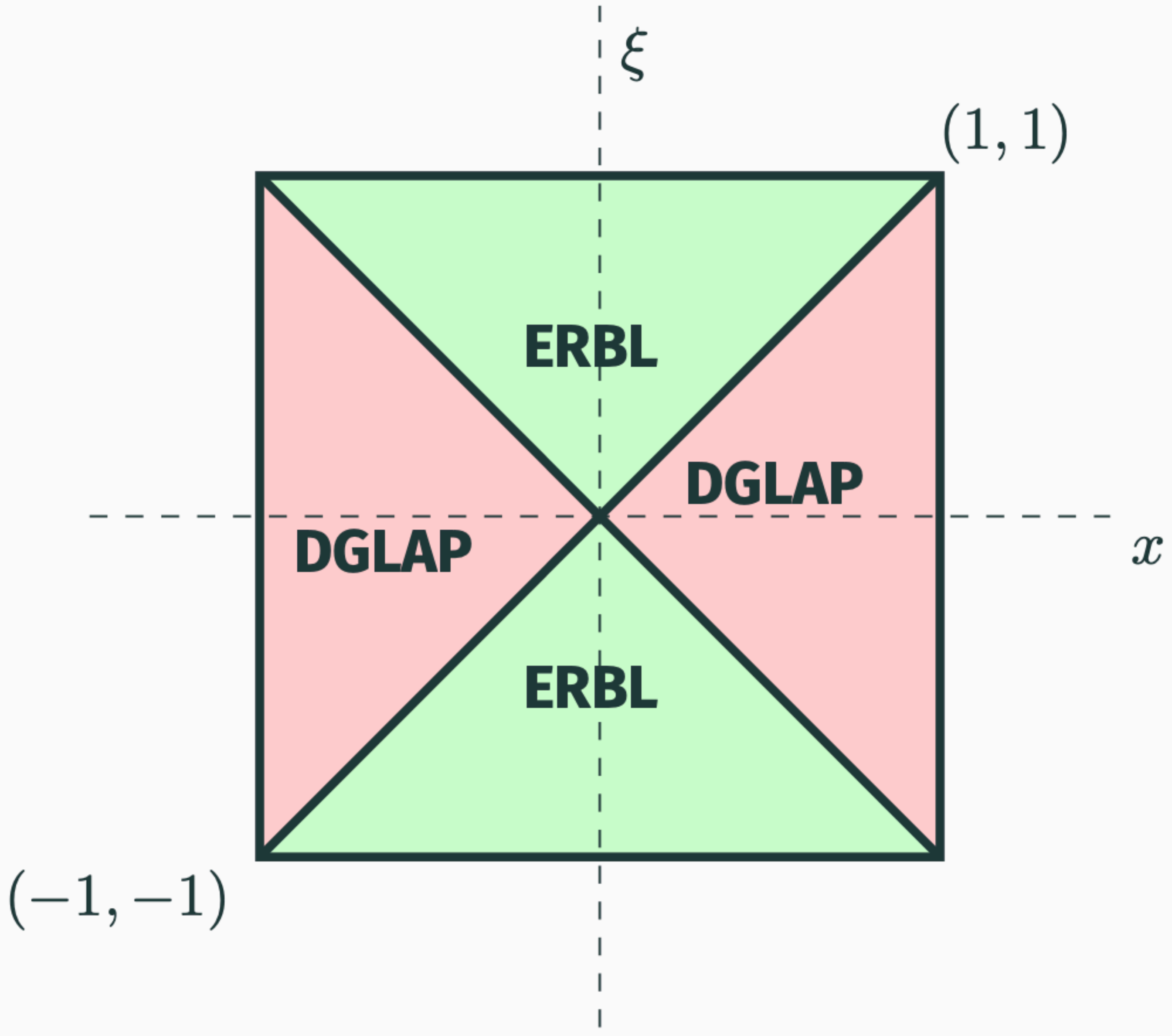
No Wilson line in light-cone gauge $A^+=0$

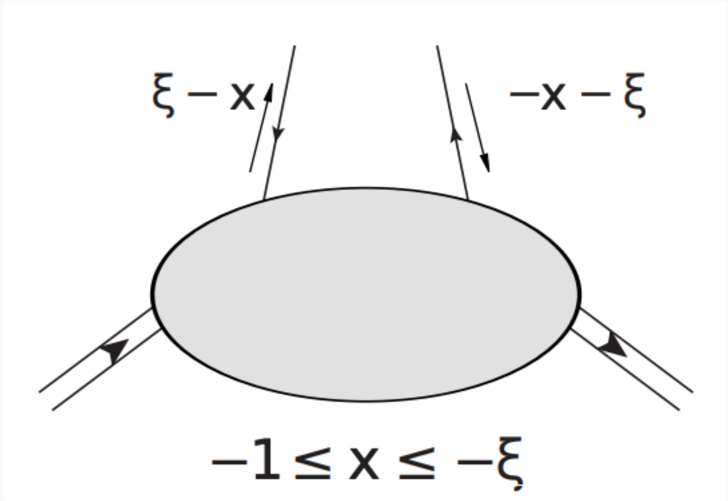
GPD properties

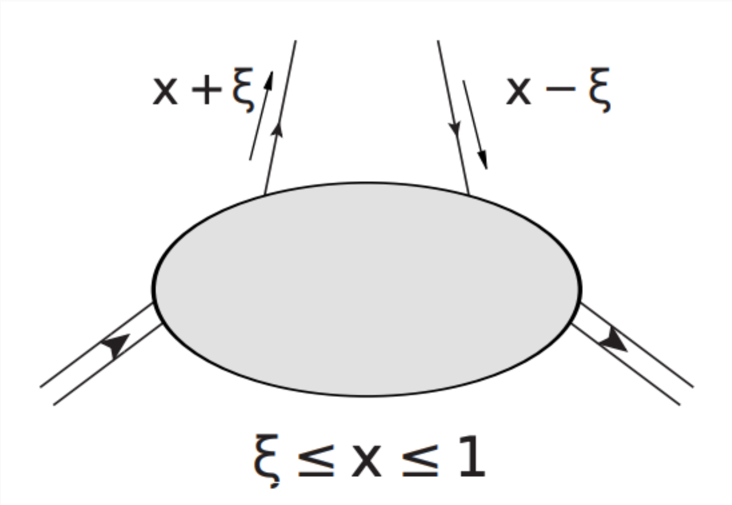
- **Support** $(x, \xi) \in [-1,1] \otimes [-1,1]$



DGLAP $|x| > |\xi|$

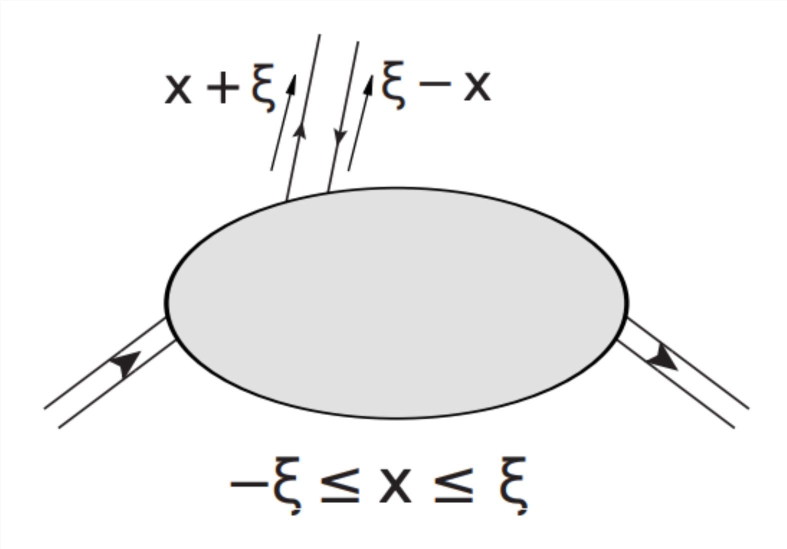
Emission/absorption quark $(x > 0)$ or antiquark $(x < 0)$





ERBL $|x| < |\xi|$

Emission of quark/antiquark pair



- ξ−parity

$$H^q(x,-\xi,t) = H^q(x,\xi,t)$$ Time inversion symmetry

- Polynomiality

$$\mathcal{A}_m(\xi,t) = \int_{-1}^{1} dx\, x^m H^q(x,\xi,t) = \sum_{\substack{k=0 \\ k\,even}}^{m+1} C_{k,m}(t)\,\xi^k$$ Lorentz symmetry

- Positivity

$$\left| H^q(x,\xi,t=0) \right| \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}, \quad |x| > |\xi|$$ Hilbert space norm

- Form factors

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) = F^q(t), \quad \int_{-1}^{1} dx\, x\, H^q(x,\xi,t) = A^q(t) + \xi^2 C^q(t)$$

- **Forward limit**

$$H^q(x,0,0) = q(x)\theta(x) - \bar{q}(-x)\theta(-x)$$

- **Hadron 3D tomography**

$$\rho^q(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-ib_\perp\cdot\Delta_\perp} H^q(x,0,-\Delta_\perp^2)$$

Probability density of finding a parton with longitudinal momentum fraction x and position $b_\perp$ in the transverse plane.

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Truncation in Overlap representation of LCWF is consistent only in DGLAP region

$$H^q(x,\xi,t)\Big|_{|x|>|\xi|} = \sum_{N,\beta} \sqrt{1-\xi^2}^{\,2-N} \int [d\bar{x}]_N [d^2\bar{\mathbf{k}}_\perp]_N \, \delta(x-\bar{x}_i)\, \psi^*_{N,\beta}(x_i^{out}, \mathbf{k}_{i\perp}^{out})\, \psi_{N,\beta}(x_i^{in}, \mathbf{k}_{i\perp}^{in})$$

Positivity is built in, but for polynomiality GPD must be extended to ERBL region

Covariant extension via Radon transform inversion

Is it possible to extend the GPD from DGLAP to ERBL preserving polynomiality?

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$$\langle P+\Delta/2\,|\,\bar{\psi}(-z/2)\gamma_\mu\psi(z/2)\,|\,P-\Delta/2\rangle\Big|_{z^2=0} = \int d\beta\, d\alpha e^{-i\beta(P\cdot z)+i\alpha\frac{\Delta\cdot z}{2}}\left[2P_\mu f(\beta,\alpha,t) - \Delta_\mu g(\beta,\alpha,t)\right]$$

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$$= \int_\Omega d\beta\, d\alpha\, \big(f(\beta,\alpha,t) + \xi\, g(\beta,\alpha,t)\big)\, \delta(x - \beta - \xi\alpha)$$

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Radon transform

- Integral over line parametrized by x, ξ
- It guarantees polynomiality

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Double Distributions (DD)

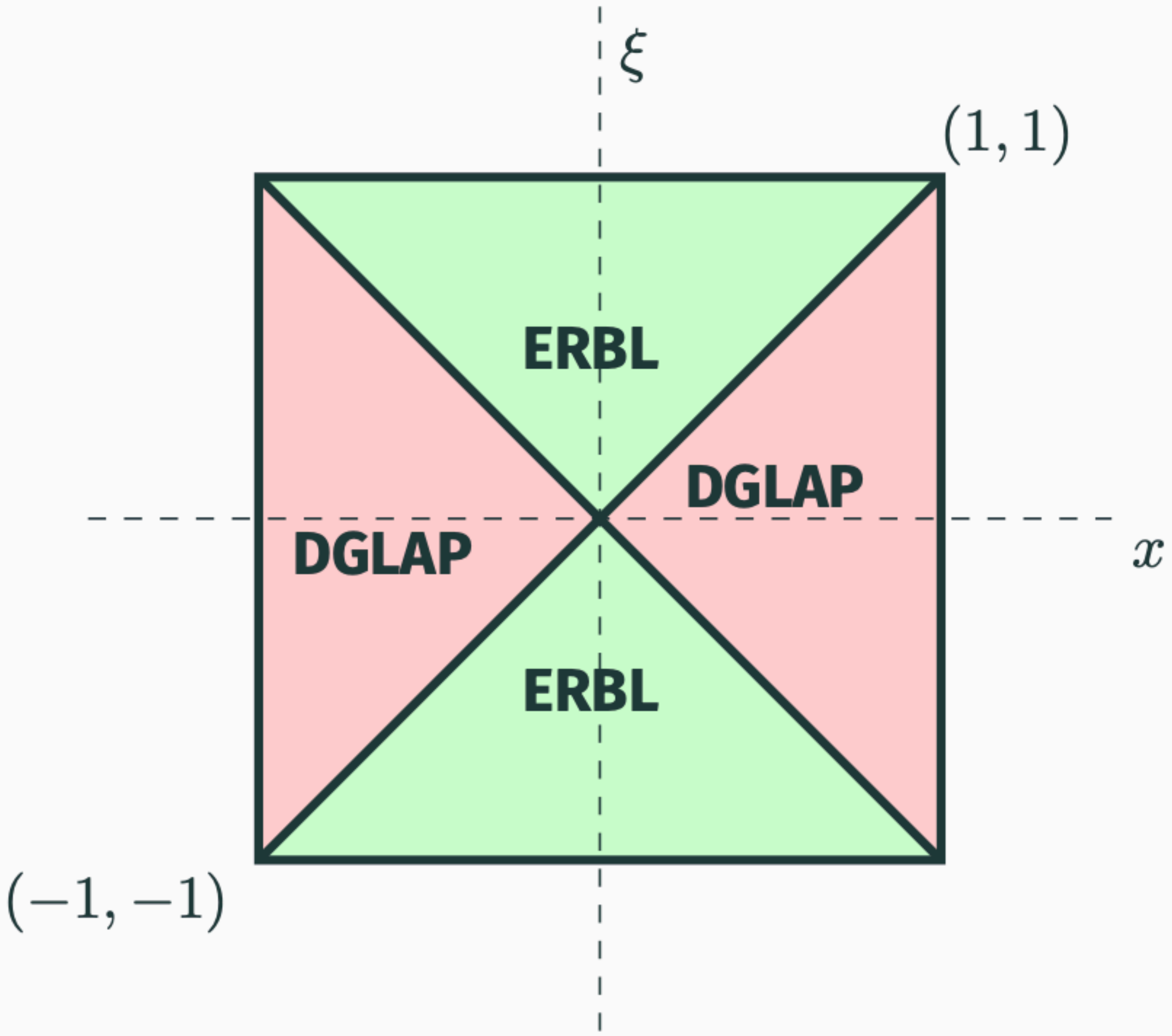
Not uniquely fixed

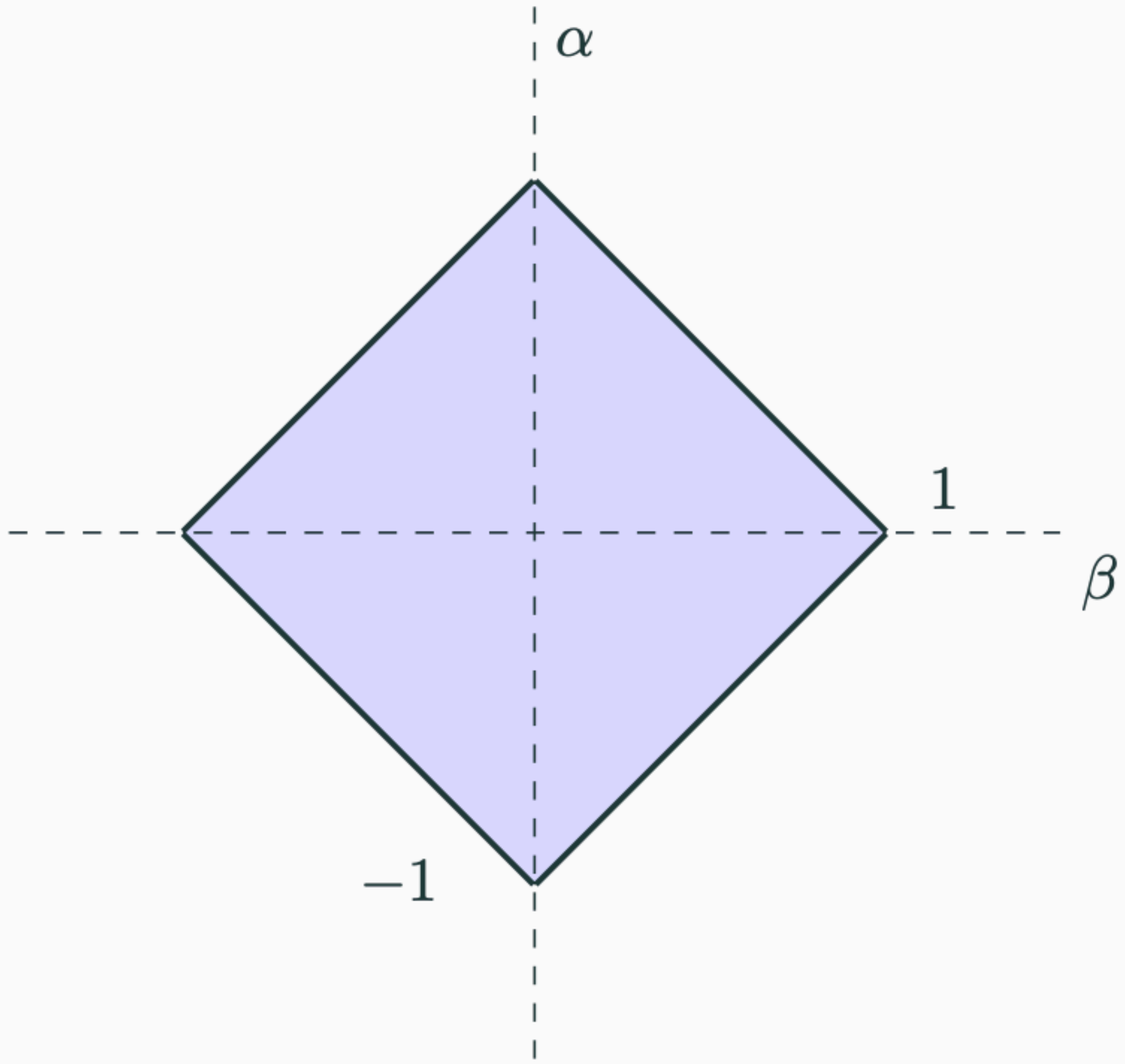
Radon transform

- Integral over line parametrized by x, ξ
- It guarantees polynomiality

$$H^q(x,\xi) = \mathscr{R}[h] = \int_\Omega d\beta d\alpha\, \delta\left(x - \beta - \alpha\xi\right) h(\beta,\alpha) \quad + \quad \theta(|\xi| - |x|) \text{ D-terms}$$

Support: $\Omega = \{(\beta,\alpha) \,|\, |\beta| + |\alpha| \leq 1\} = \Omega^+\theta(\beta) + \Omega^-\theta(-\beta)$





line $\alpha = -\dfrac{\beta}{\xi} + \dfrac{x}{\xi}$

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- Cannot be fixed by DGLAP data.
- Its DD has support only in $\beta = 0$
- Contribute only to $C_{m,m+1}(t)$

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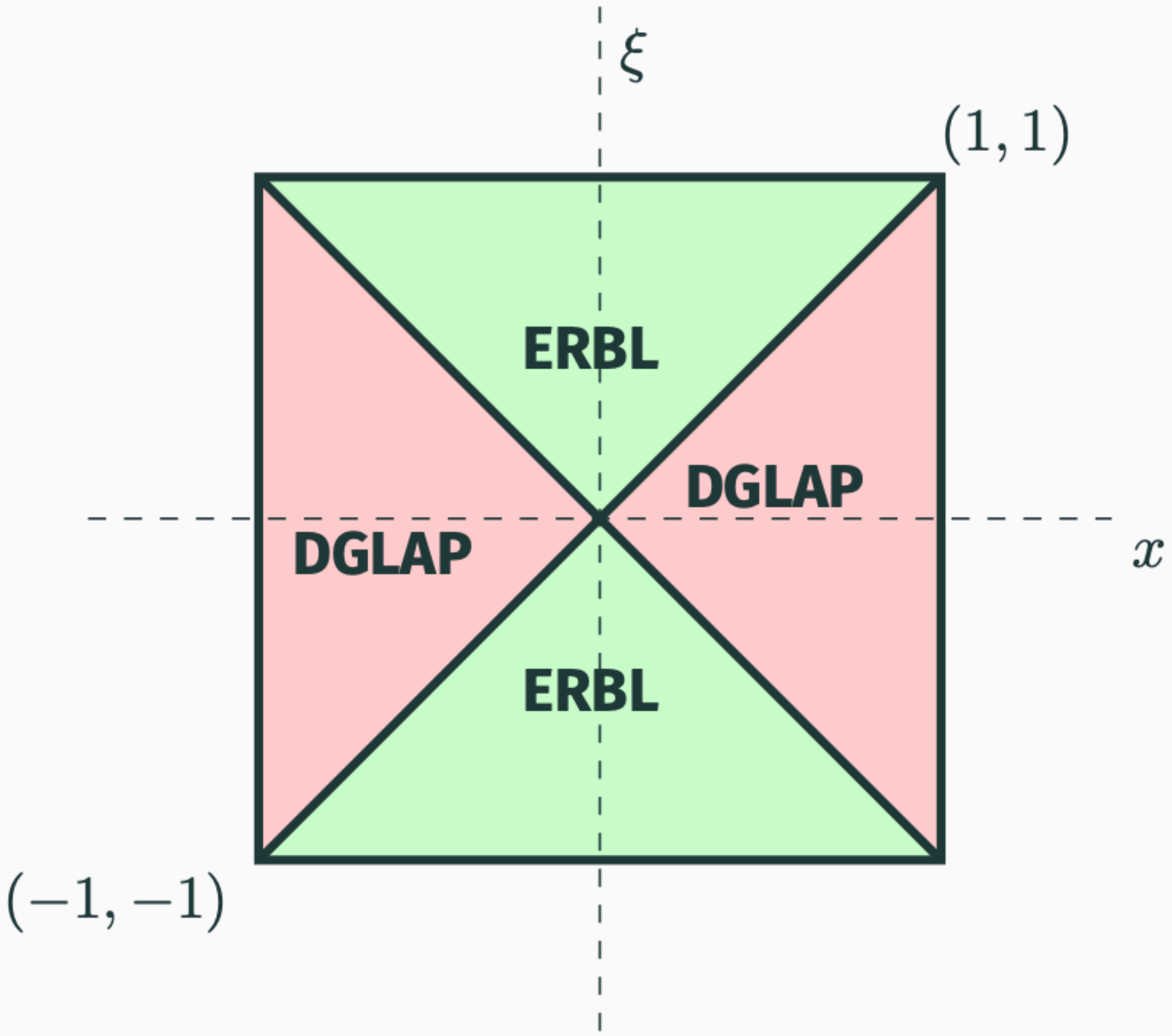

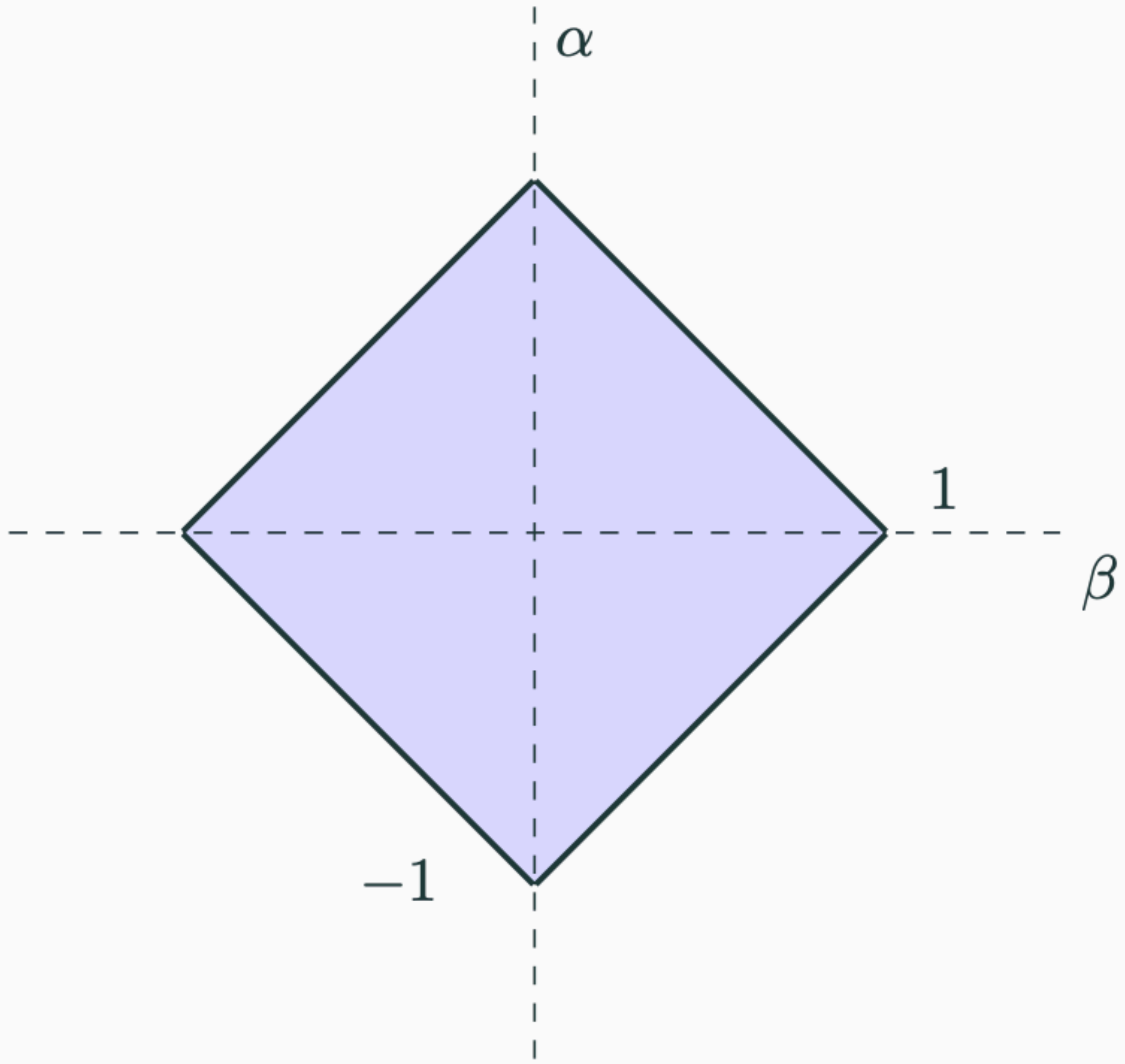

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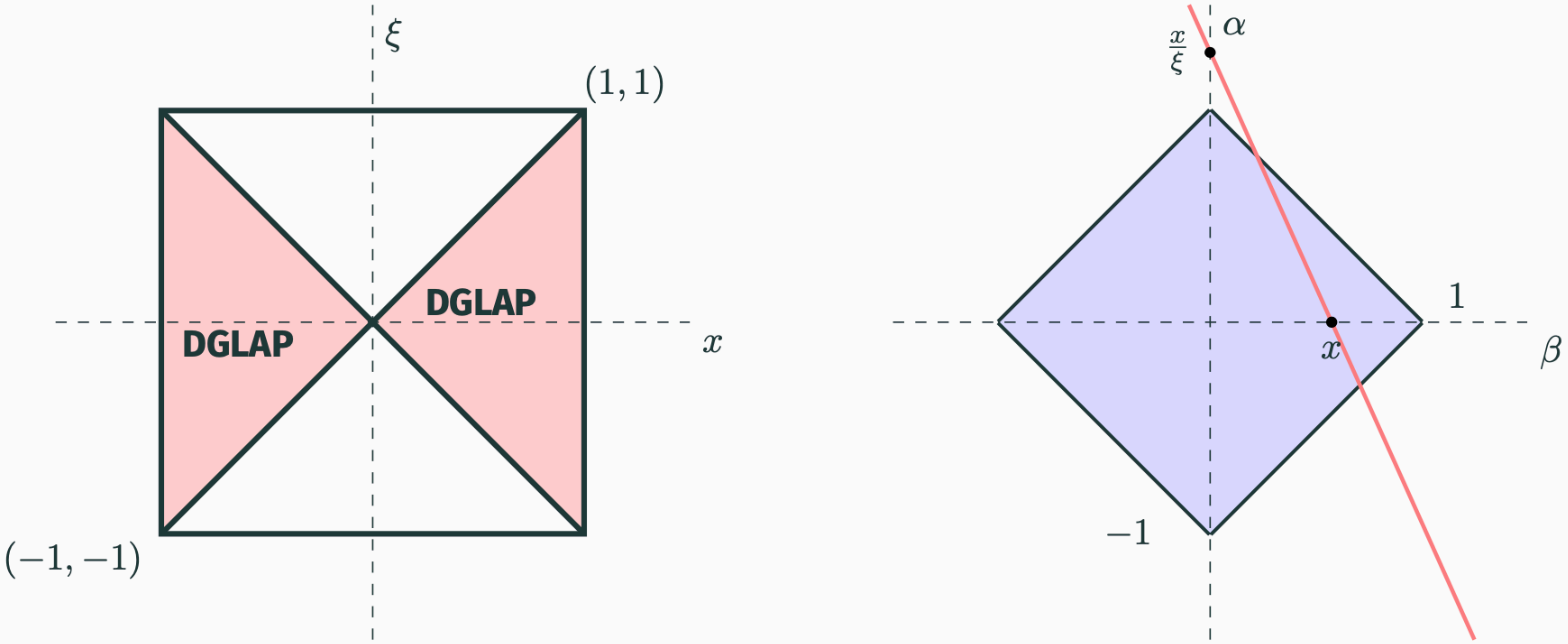

line $\alpha = -\dfrac{\beta}{\xi} + \dfrac{x}{\xi}$

$$H^q(x,\xi) = \mathscr{R}[h] = \int_\Omega d\beta d\alpha\, \delta\left(x - \beta - \alpha\xi\right) h(\beta,\alpha) \quad + \quad \theta(|\xi| - |x|)$$ **D-terms**

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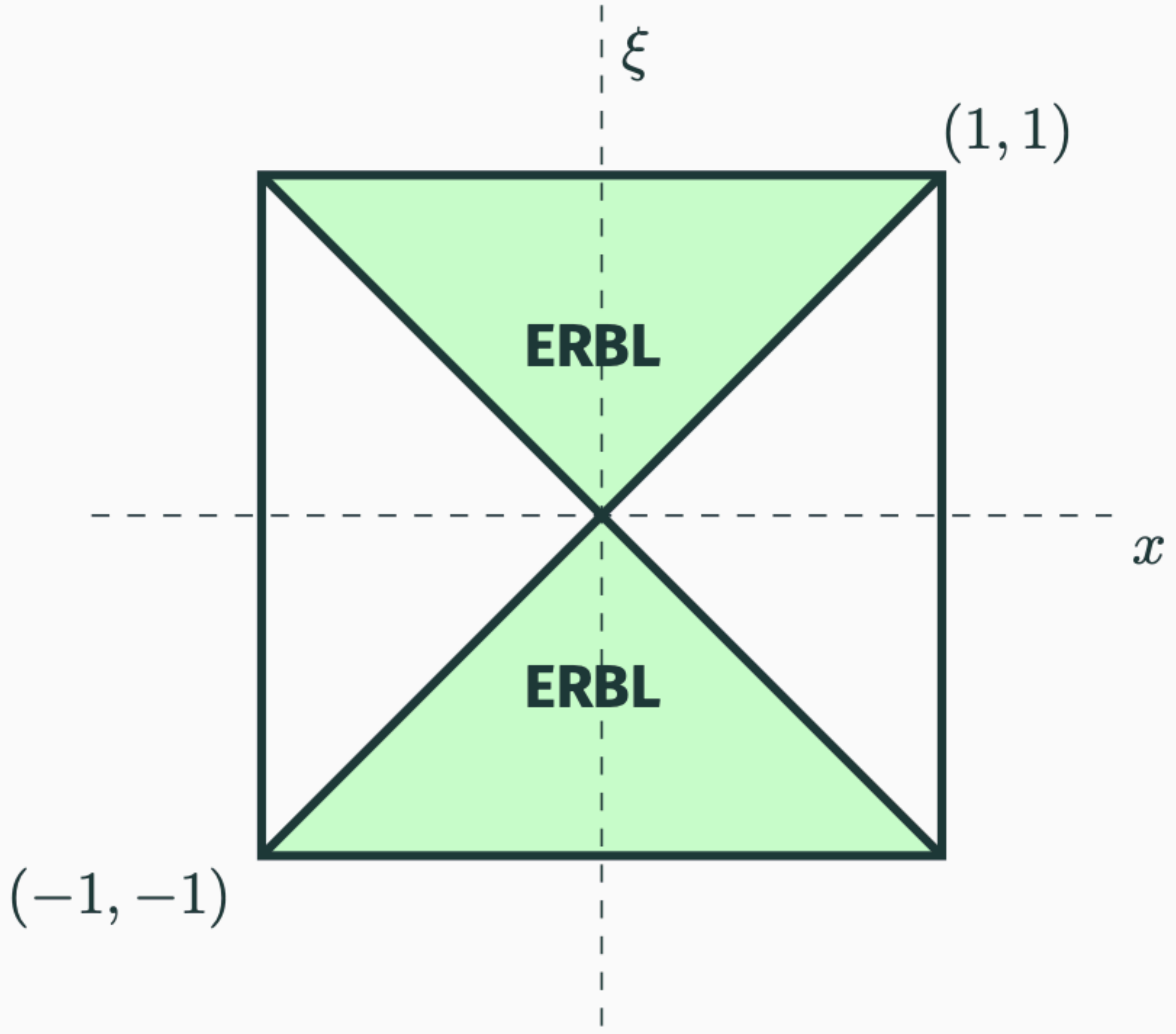

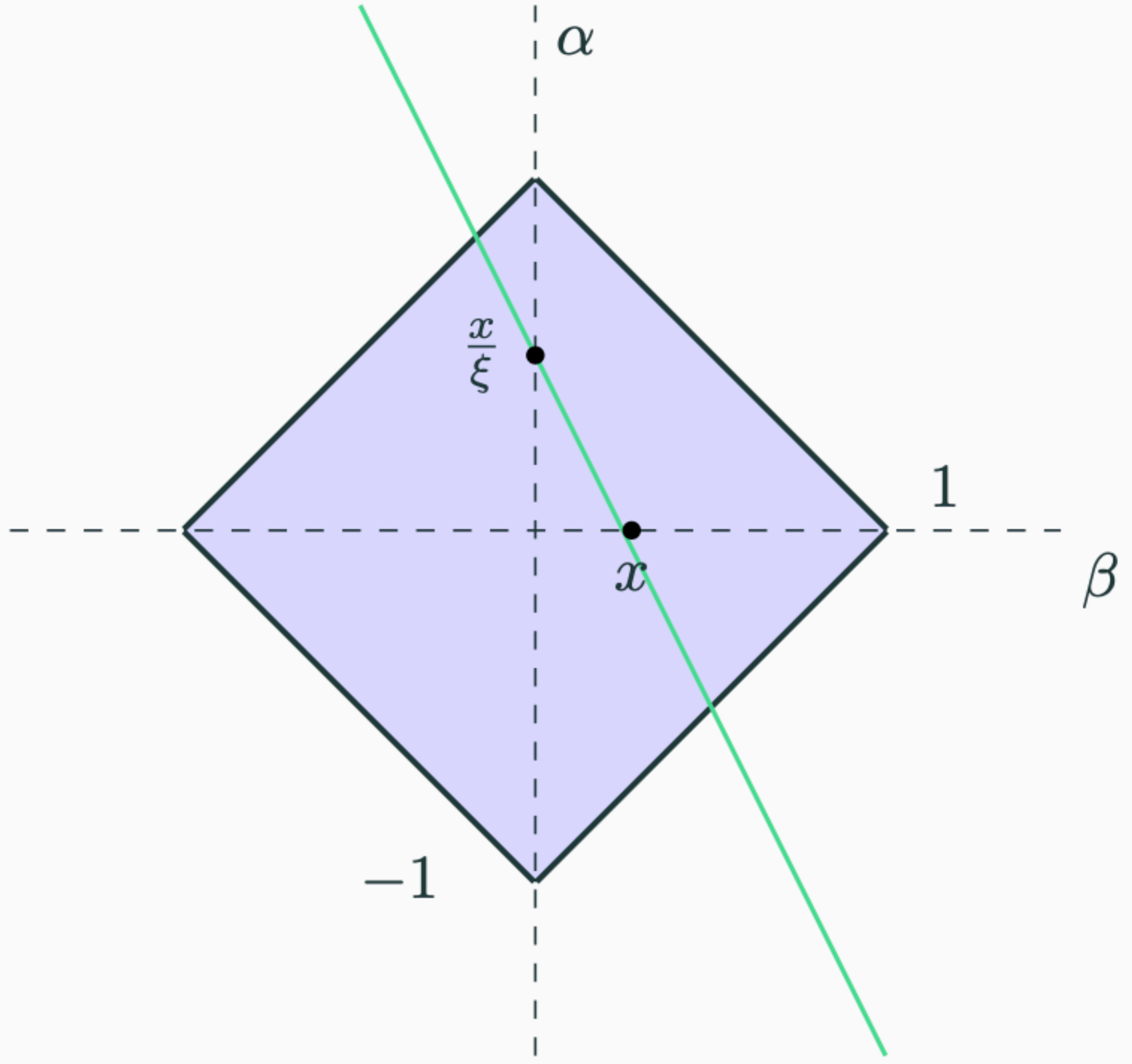

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Is it possible to invert the Radon transform knowing the GPD only in DGLAP?

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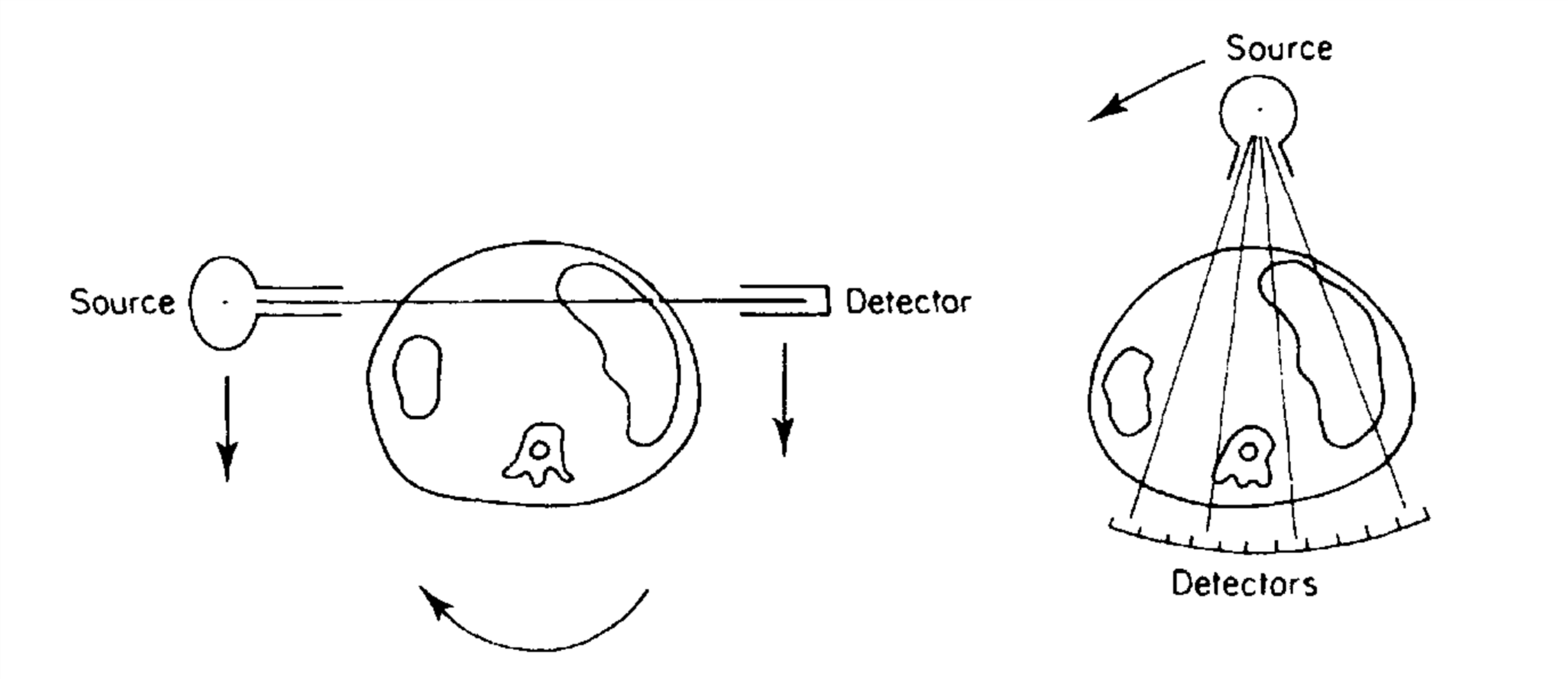
Answer from Computerized Tomography



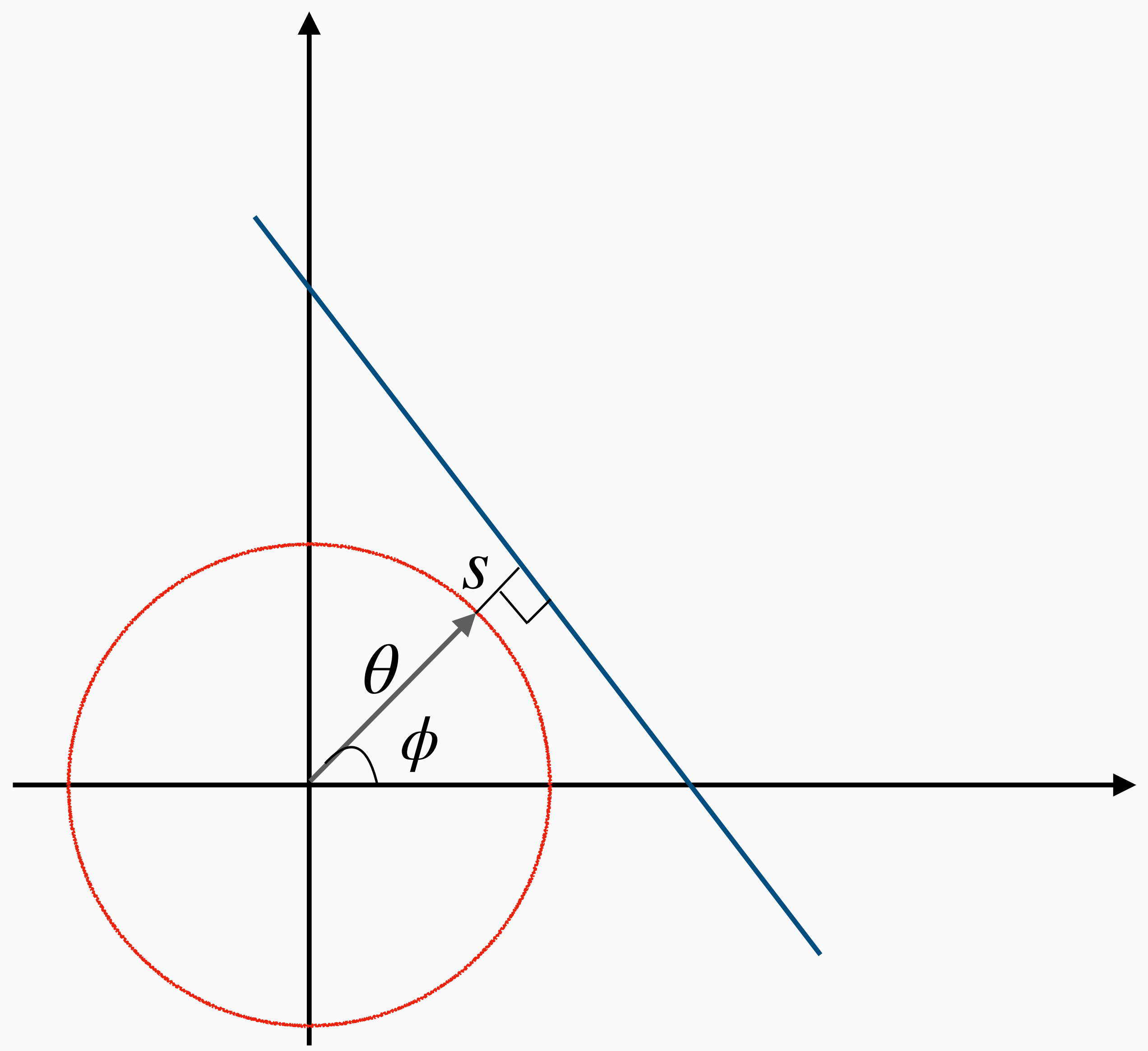
Parallel scanning

Fan-beam scanning

$$\frac{\Delta I}{I} = f(x)\,dx$$

$$\frac{I_{out}}{I_{in}}[L] = \exp\left\{-\int_L f(x)\,dx\right\}$$

$$\mathscr{R}f(\theta, s) \equiv \int_{z\cdot\theta=s} dz\, f(z) \qquad f \in \mathscr{S}(\mathbb{R}^n)\,, \quad \theta \in S^{n-1}\,, \quad s \in \mathbb{R}$$



GPD: $z \to (\beta, \alpha)\,, \quad \theta \to (\cos(\phi), \sin(\phi))\,, \quad x = \dfrac{s}{\cos(\phi)}\,, \quad \xi = \tan(\phi)$

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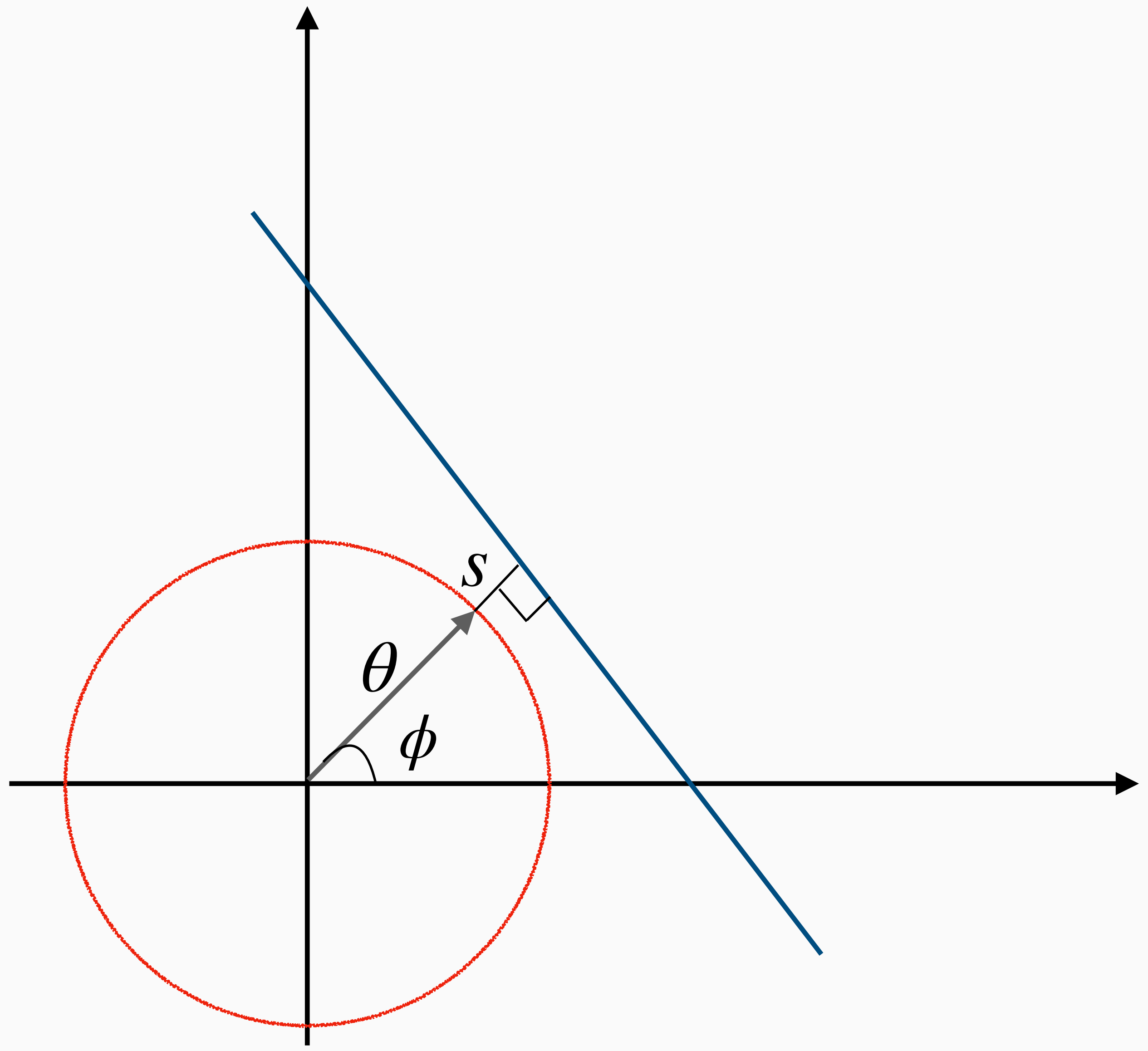

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Uniqueness theorems:

[F. Natterer, *The Mathematics of Computerized Tomography*]

Given f $\in \mathscr{S}(\mathbb{R}^n)$, if $\mathscr{R}f(\theta, s) = 0$ $\forall$ hyperplane $z \cdot \theta = s$ that does not intersect a compact convex set K $\subset \mathbb{R}^n \Rightarrow f(z) = 0$ outside K

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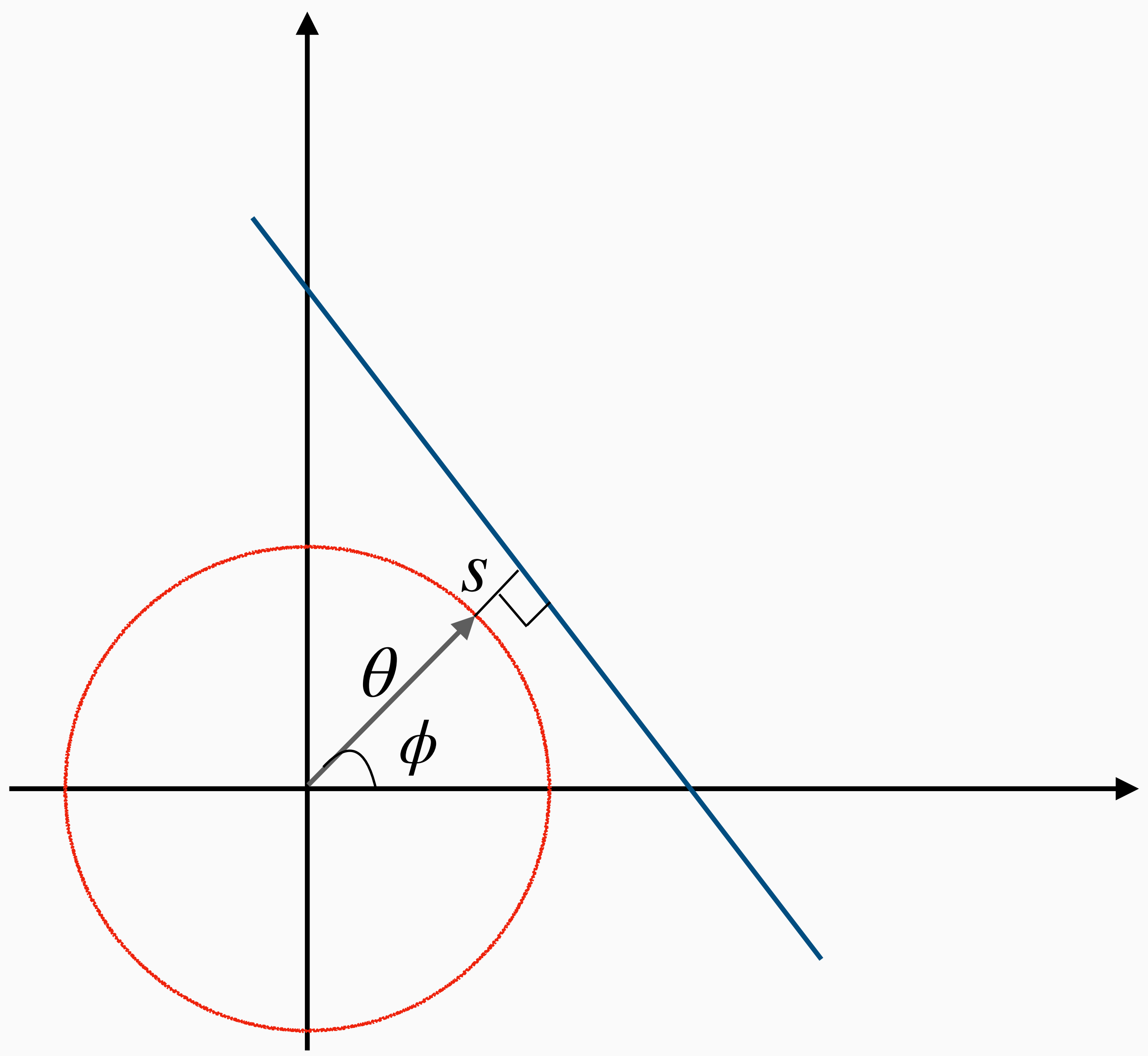

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→ DD is uniquely fixed by GPD in DGLAP (except for $\beta = 0$)

J. Boman and E.T. Quinto, Mathematical Journal 55 (1987)

Given f in $\mathbb{R}^2$ compactly-supported and locally summable, $(\theta_0, s_0) \in S^1 \times \mathbb{R}$, U_0 open neighborhood of θ_0 :

$$\mathscr{R}f(\theta, s) = 0 \;\; \forall s > s_0, \;\; \theta \in U_0 \quad \Rightarrow \quad f(z) = 0 \quad if\ z \cdot \theta_0 > s_0$$

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Given f in $\mathbb{R}^2$ compactly-supported and locally summable, $(\theta_0, s_0) \in S^1 \times \mathbb{R}$, U_0 open neighborhood of θ_0 :

$$\mathscr{R}f(\theta, s) = 0 \ \ \forall s > s_0, \ \ \theta \in U_0 \quad \Rightarrow \quad f(z) = 0 \quad \textit{if } z \cdot \theta_0 > s_0$$

→ DD is uniquely fixed (except for $\beta = 0$) by knowing GPD in:

$x \in [-1,1]\,, \quad \xi \in [0, \lambda x]\,, \ \ 0 < \lambda \leq 1$ (H. Moutarde)

In experiments $\xi \lesssim 0.2$

Numerical Inversion of Radon Transform

Artificial Neural Networks

H. Dutrieux et al. *Eur.Phys.J.C.* 82 (2022)

Finite Elements Methods

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J.M. Morgado et al. *Phys.Rev.D* 105 (2022)

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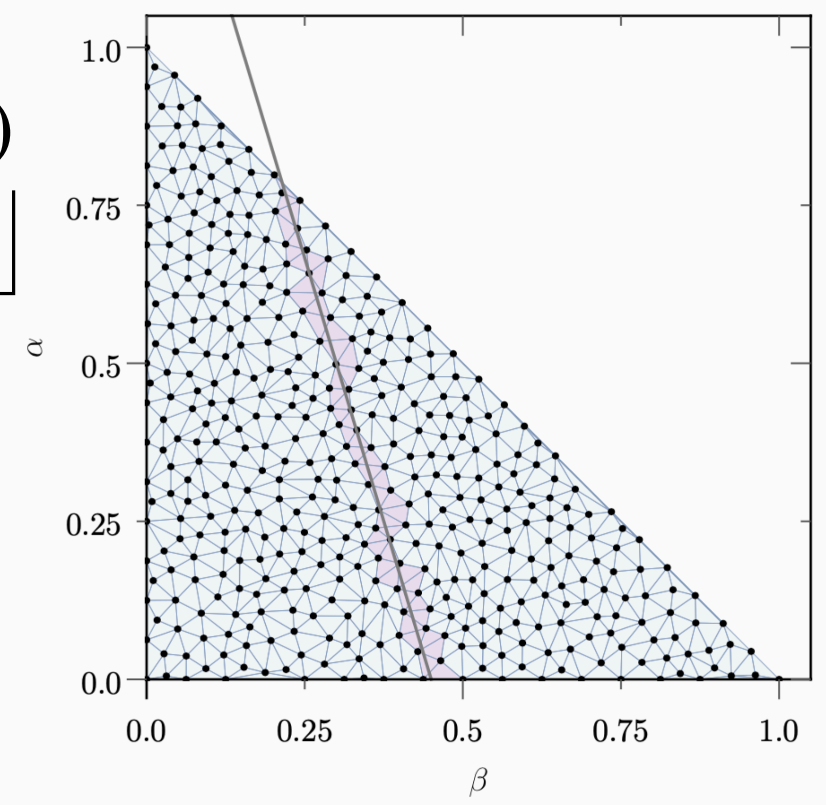
Lagrange Polynomial

$$H_i \equiv H(x_i, \xi_i) = \sum_j h_j \int_{\Omega^+} d\beta d\alpha \delta(x_i - \beta - \alpha\xi_i)\, v_j(\beta, \alpha)$$

R_{ij}

$$h = \left(R^T R\right)^{-1} R^T H$$



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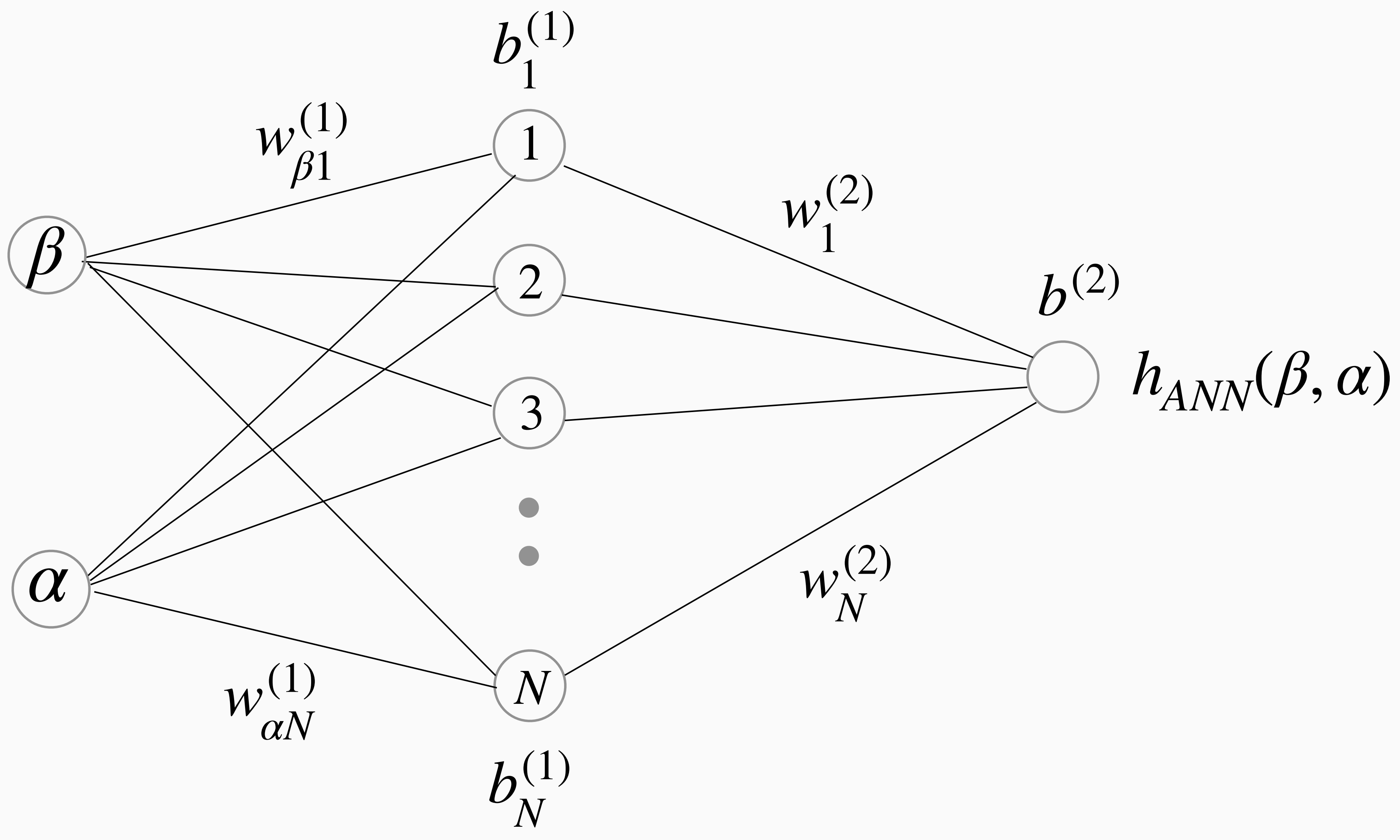

Unbiased parametrization of DD

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J.M. Morgado et al. *Phys.Rev.D* 105 (2022)

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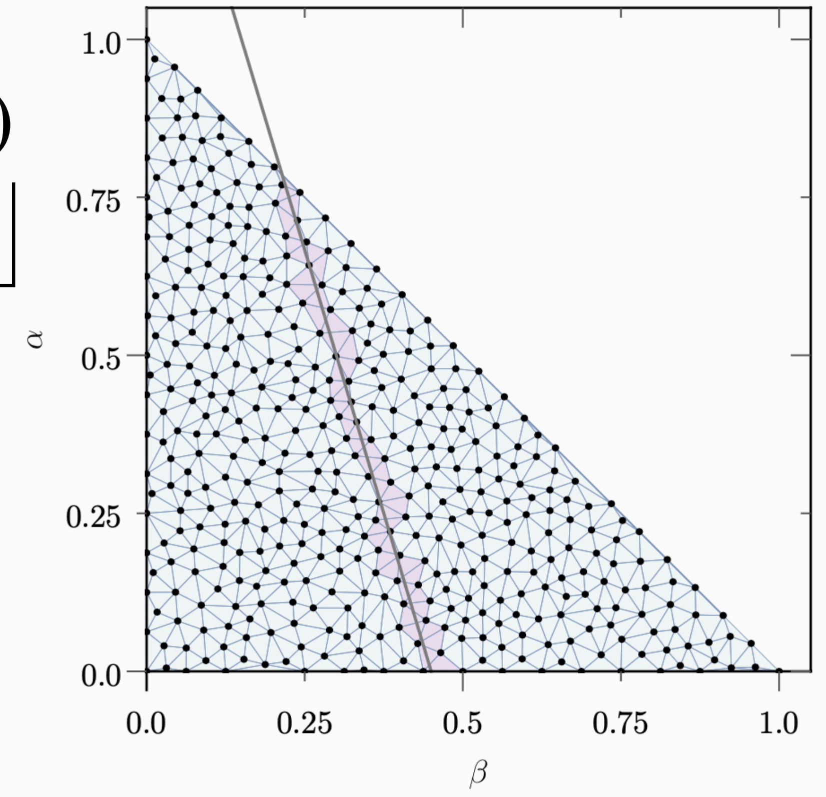
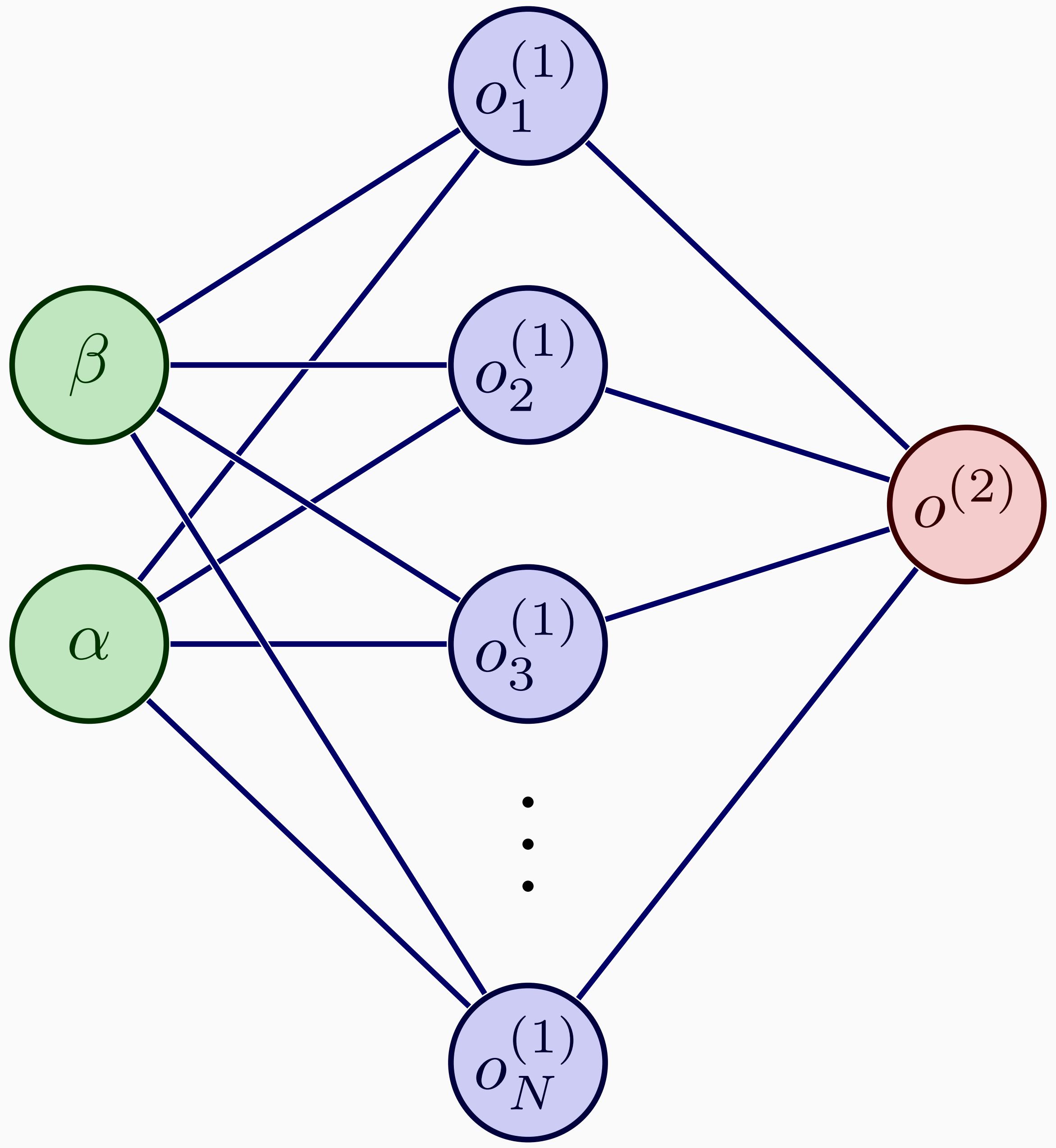


Parametrizing $h(\beta, \alpha)$ using **Artificial Neural Networks (ANN)**



$$= h_{ANN}(\beta, \alpha) = \sum_{i=1}^{N} w_i^{(2)} o_i^{(1)} + b^{(2)}$$

$$= \sum_{i=1}^{N} w_i^{(2)} \left[\sigma\left(w_{\beta i}^{(1)} \beta + w_{\alpha i}^{(1)} \alpha + b_i^{(1)} \right) + \sigma\left(w_{\beta i}^{(1)} \beta - w_{\alpha i}^{(1)} \alpha + b_i^{(1)} \right) \right] + b^{(2)}$$

- Adam optimizer (gradient descent)
- Dropout regularization

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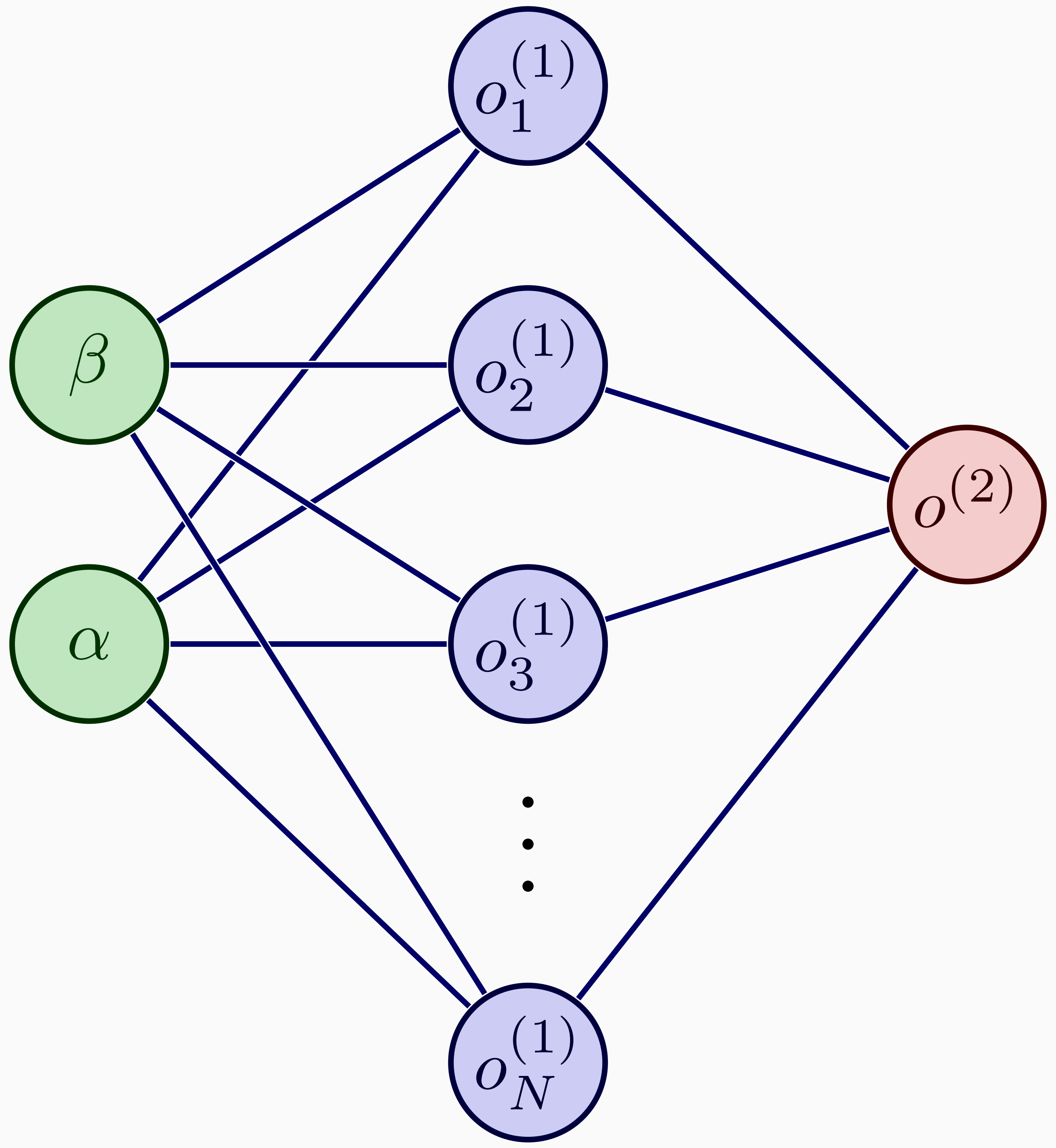

$$= h_{ANN}(\beta, \alpha) = \sum_{i=1}^{N} w_i^{(2)} o_i^{(1)} + b^{(2)}$$

$$h(\beta, -\alpha) = h(\beta, \alpha) \leftrightarrow H(x, -\xi) = H(x, \xi)$$

$$\uparrow$$

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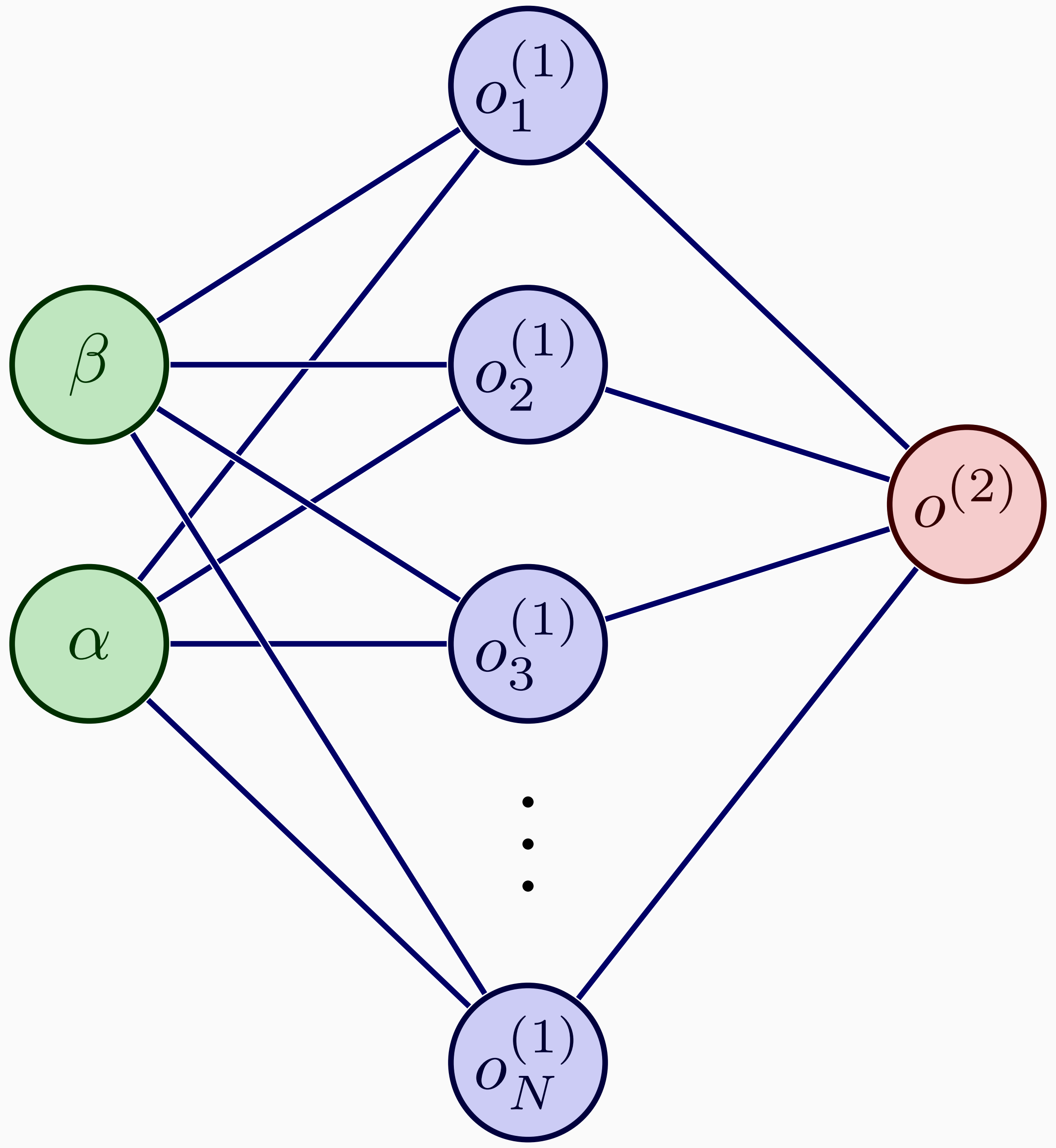

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$$h(\beta, -\alpha) = h(\beta, \alpha) \leftrightarrow H(x, -\xi) = H(x, \xi)$$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

- Adam optimizer (gradient descent)
- Dropout regularization

Algorithm

- Initialize the ANN parameters (randomly).
- Given a sampling set of GPD values $H_i(x_i, \xi_i)$ in the DGLAP region, numerically evaluate the RT along each line (x_i, ξ_i) using $h_{ANN}(\beta, \alpha)$ as DD. $\mathscr{R}h_{ANN}(x_i, \xi_i) = \hat{H}(x_i, \xi_i)$.
- Update the ANN parameters using Adam optimization algorithm in order to minimize

$$MSE = \frac{1}{N_{sample}} \sum_{i=1}^{N_{sample}} \left(H_i - \hat{H}_i\right)^2$$

- Iterate until convergence.

Testing with analytical models

Nakanishi based model for pion N.Chouika et al. *Phys.Lett.B*:780(2018)

$$H(x,\xi,t=0) = \begin{cases} 30\,\frac{(1-x)^2(x^2-\xi^2)}{(1-\xi^2)^2}, & |x| > \xi \\ 15\,\frac{(1-x)(\xi^2-x^2)(x+2x\xi+\xi^2)}{2\xi^3(1+\xi)^2}, & |x| < \xi \end{cases}$$

$$H(x,\xi,t=0) = (1-x)\int_{\Omega^+} d\beta d\alpha \delta(x-\beta-\alpha\xi)\, h(\beta,\alpha)$$

$$h(\beta,\alpha) = \frac{15}{2}\left(1-3(\alpha^2-\beta^2)-2\beta\right)$$

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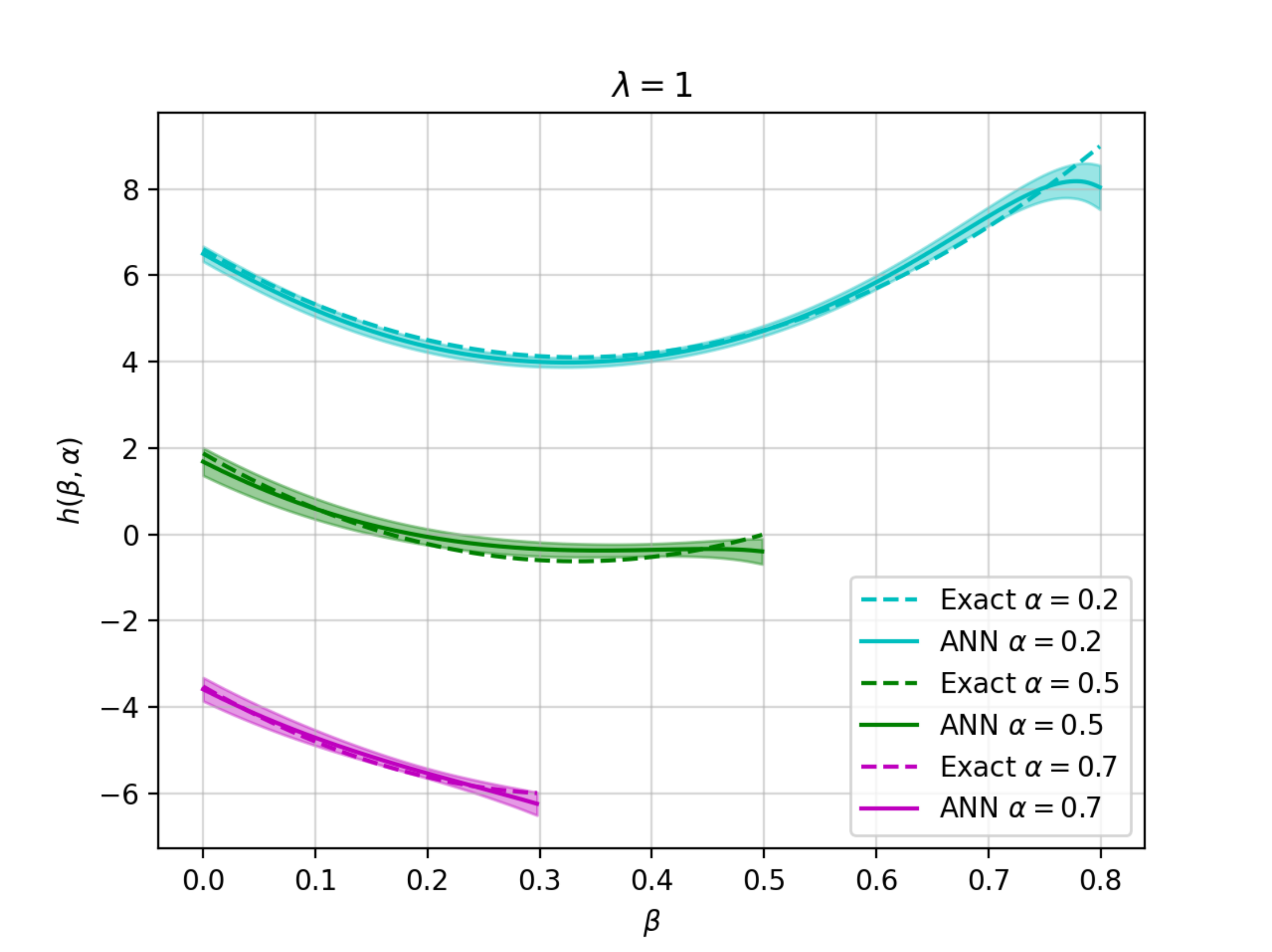
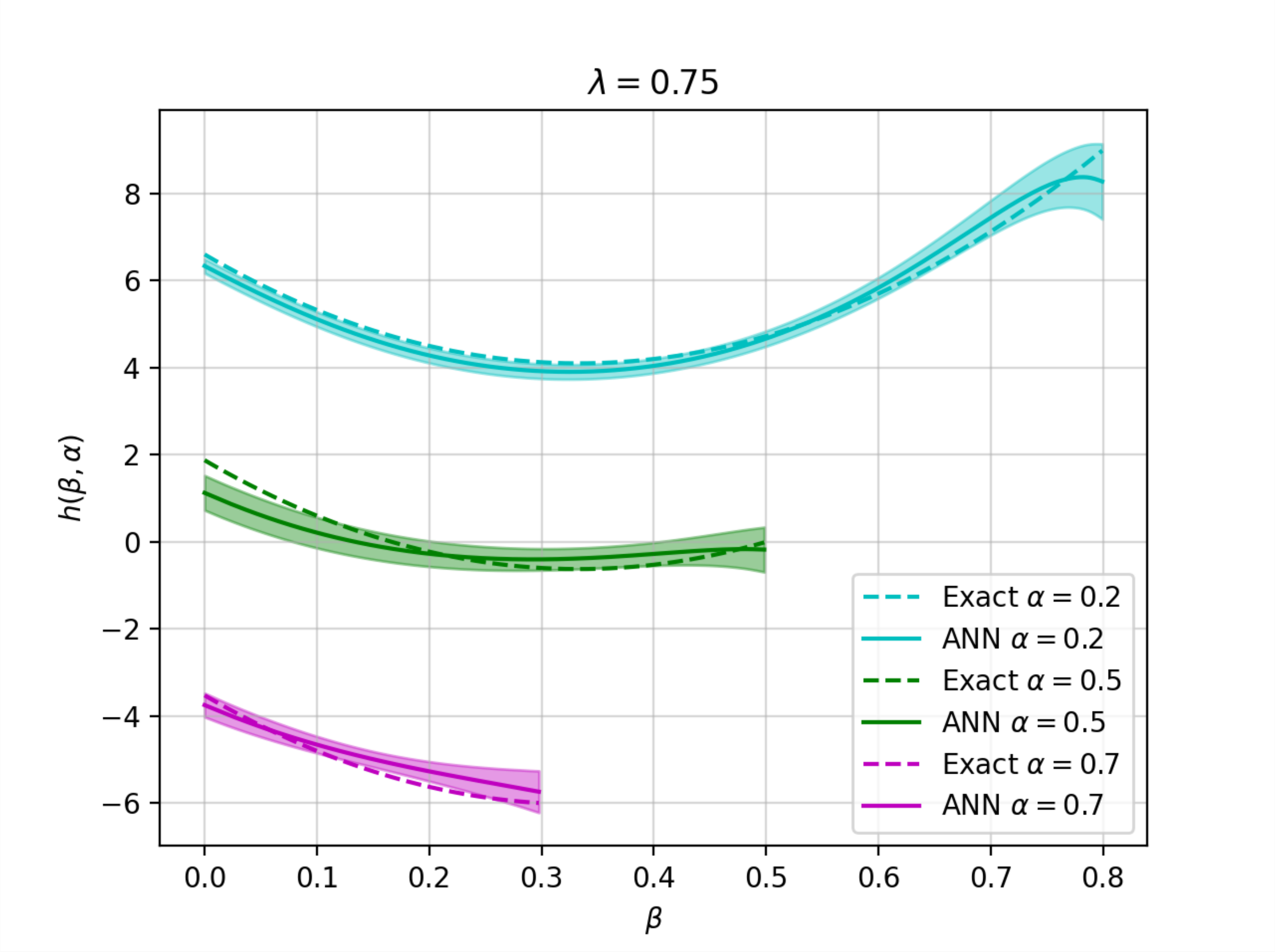
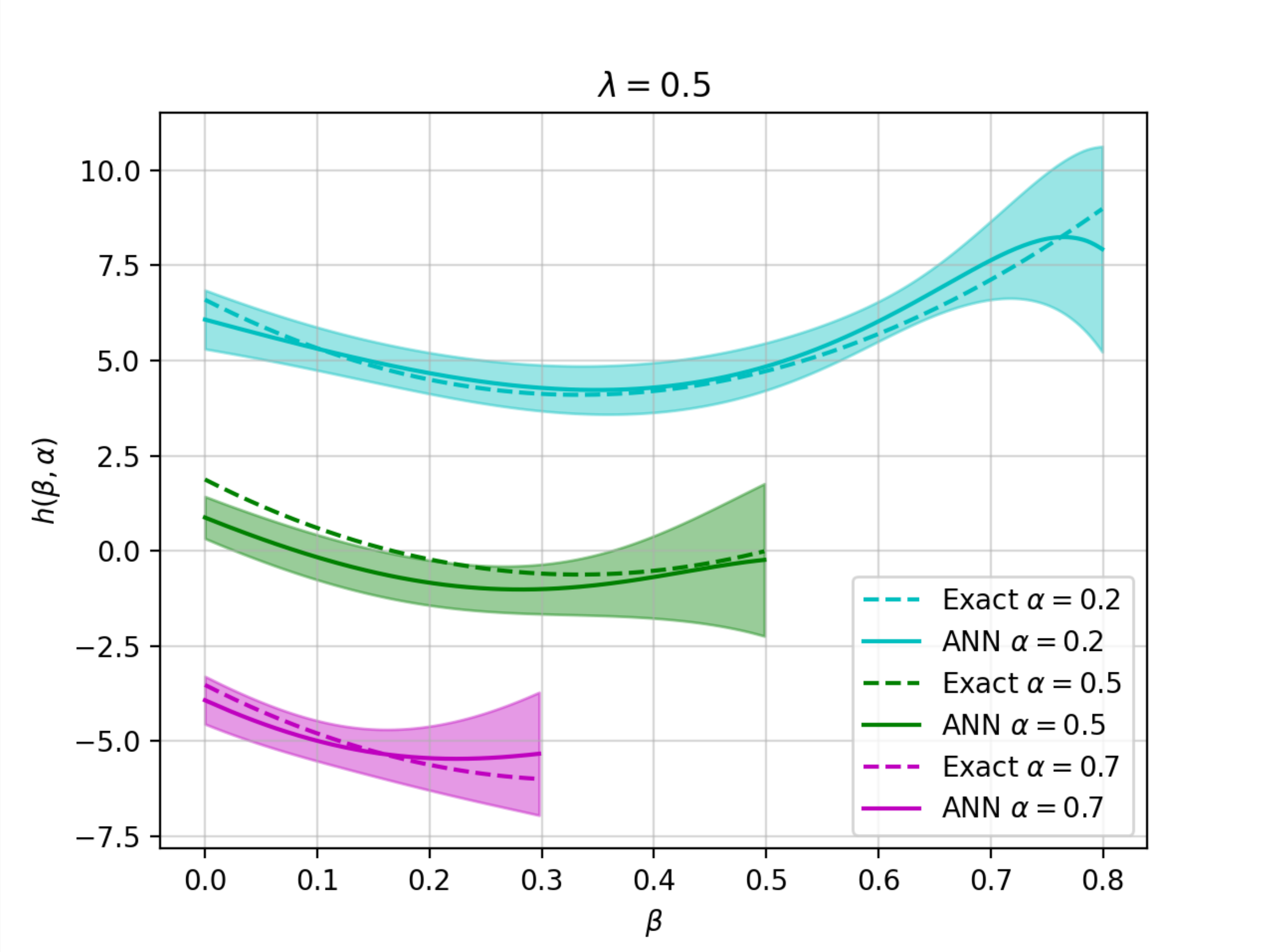
$N_{neurons} = 10^2, \quad N_{sample} = 10^4$

$\xi \in [0, \lambda x]$







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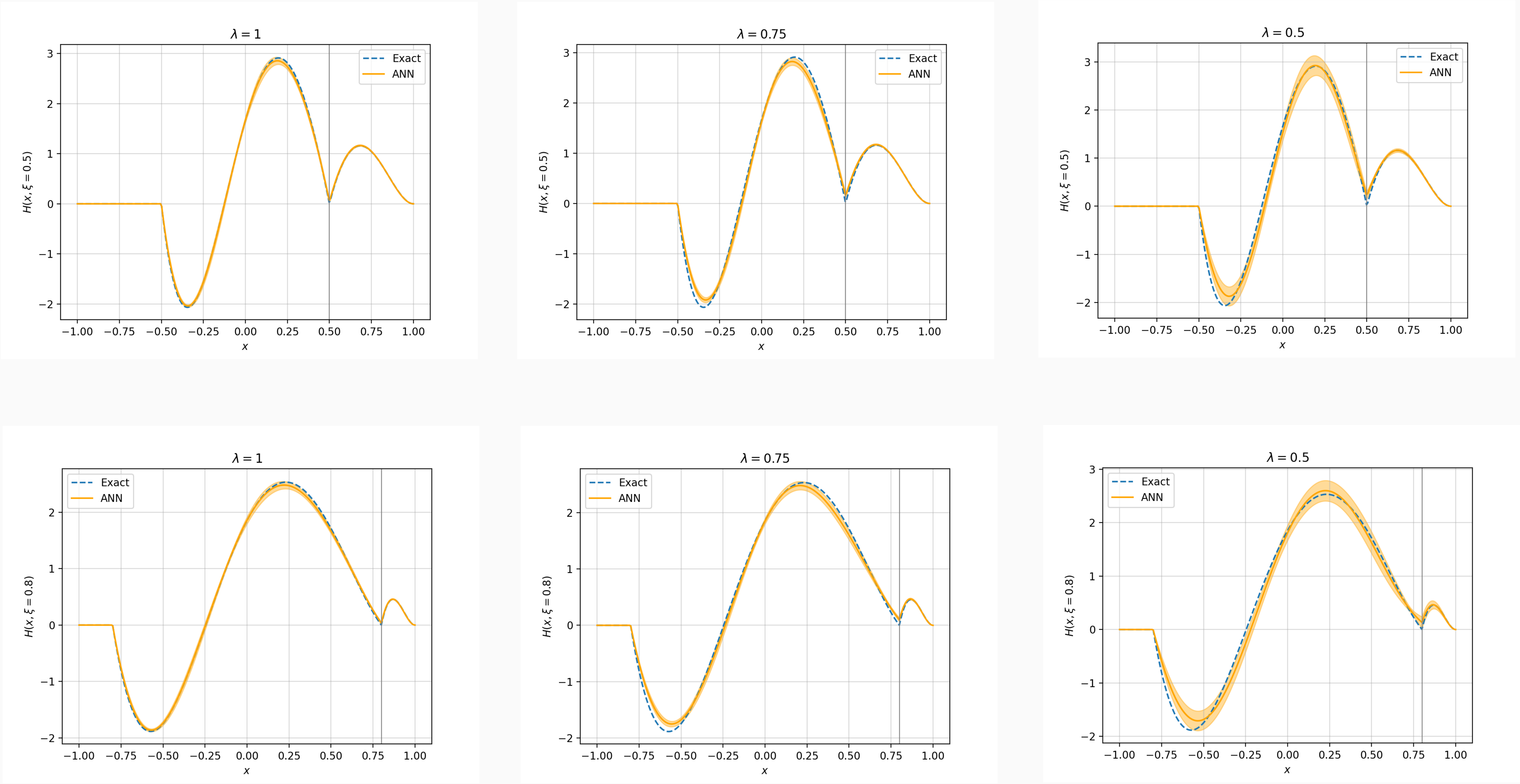

Goloskokov-Kroll model

S.V. Goloskokov, P. Kroll, Eur.Phys.J.C. 50 (2007)

$$H(x,\xi) = \int_\Omega d\beta d\alpha\, \delta\left(x - \beta - \alpha\xi\right) q(\beta)\, h_{GK}(\beta,\alpha)\,,$$

$$h_{GK}(\beta,\alpha) = \frac{\Gamma(n+2)}{2^{n+1}\Gamma^2(n+1)} \frac{\left[(1-\beta)^2 - \alpha^2\right]^n}{(1-\beta)^{2n+1}}$$

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Valence distribution $\quad q^u_{val}(\beta) = \beta^{-\delta}(1-\beta)^{2n+1} \sum_{j=0}^{2} c_j\, \beta^{j/2}\,, \quad n = 1,\ \ \delta = 0.48$

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Sea distribution $$q_{sea}(\beta) = \beta^{-\delta}(1-\beta)^{2n+1}\sum_{j=0}^{3} c_j\,\beta^{j/2}\,, \quad n=2,\ \ \delta=1.1$$

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$$H_{sea}(x,\xi) = \int_\Omega d\beta d\alpha\, \delta\left(x - \beta - \alpha\xi\right)\ sign(\beta)\, q_{sea}(|\beta|)\, h_{GK}(|\beta|,\alpha) \longrightarrow \ H_{sea}(-x,\xi) = -\,H_{sea}(x,\xi)$$

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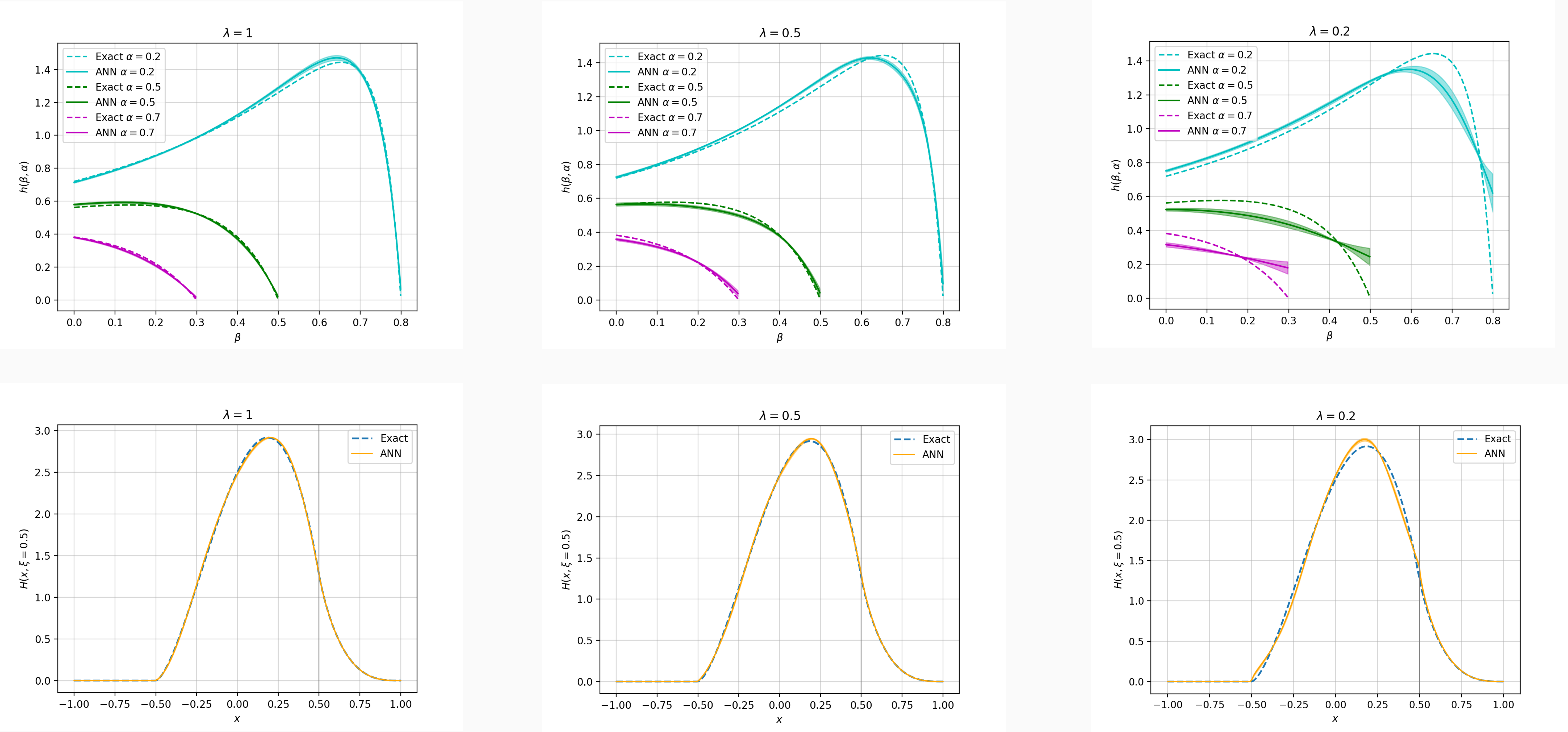
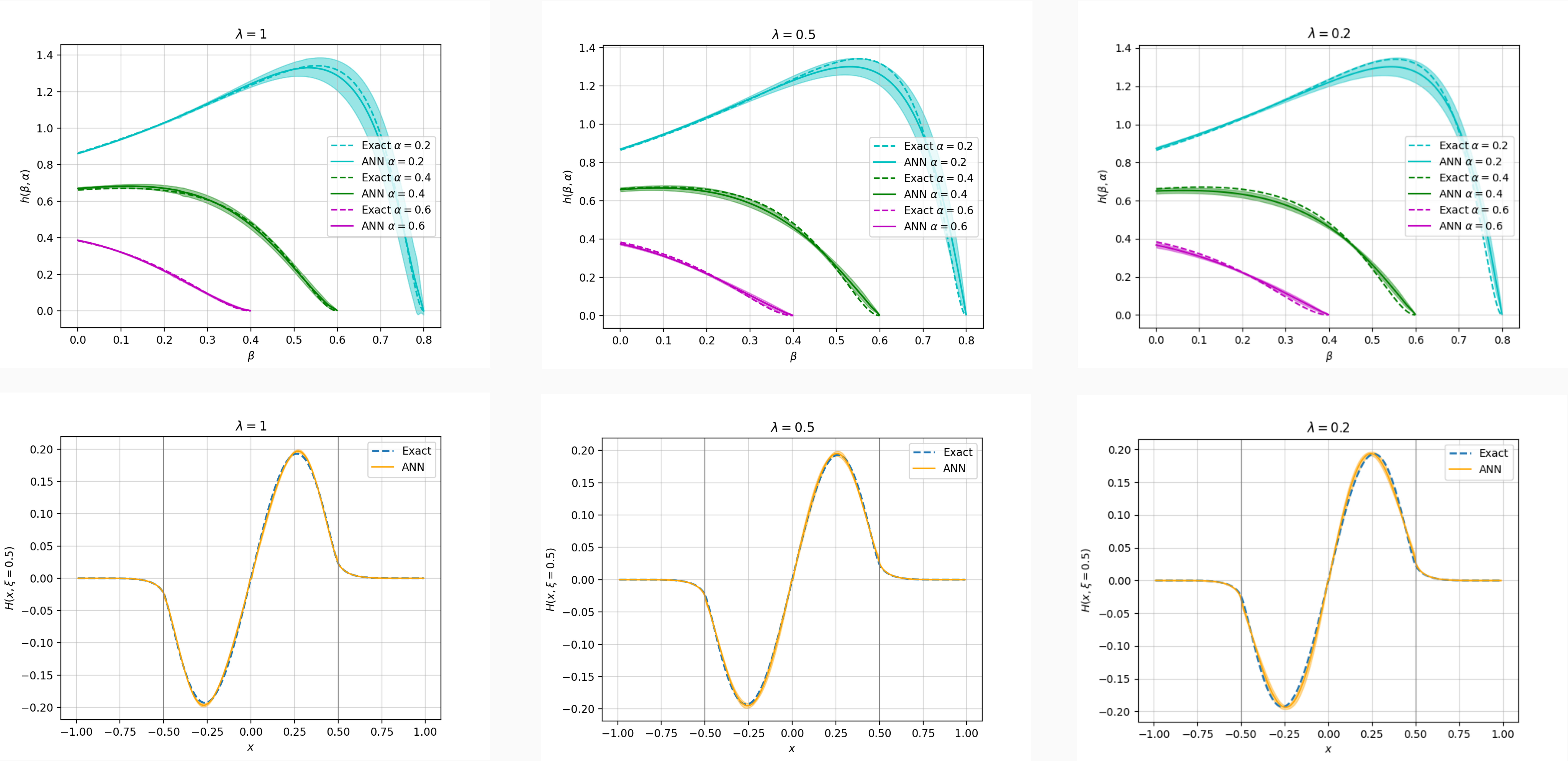
$$N_{neurons} = 25\,, \quad N_{sample} = 5 \times 10^3 \qquad h_{GK}(\beta,\alpha) \simeq \frac{h_{ANN}(\beta,\alpha)}{\int_{-1+|\beta|}^{1-|\beta|} d\alpha\, h_{ANN}(\beta,\alpha)}$$

Valence DD/GPD



Sea DD/GPD



$\lambda = 1$
$\lambda = 0.5$
$\lambda = 0.2$
$h(\beta, \alpha)$
β
Exact $\alpha = 0.2$
ANN $\alpha = 0.2$
Exact $\alpha = 0.4$
ANN $\alpha = 0.4$
Exact $\alpha = 0.6$
ANN $\alpha = 0.6$
$H(x, \xi = 0.5)$
x
Exact
ANN

Conclusions and Outlook

- GPD can be extended from DGLAP (and proper subsets) to ERBL inverting its RT
- ANNs are a good tool for inverting RT (Unbiased parametrization)
- Testing the method on experimental results (EIC)

- Applying ANNs to PDF reconstruction from a sequence of Mellin moments

  (with K. Raya and J. Quintero)

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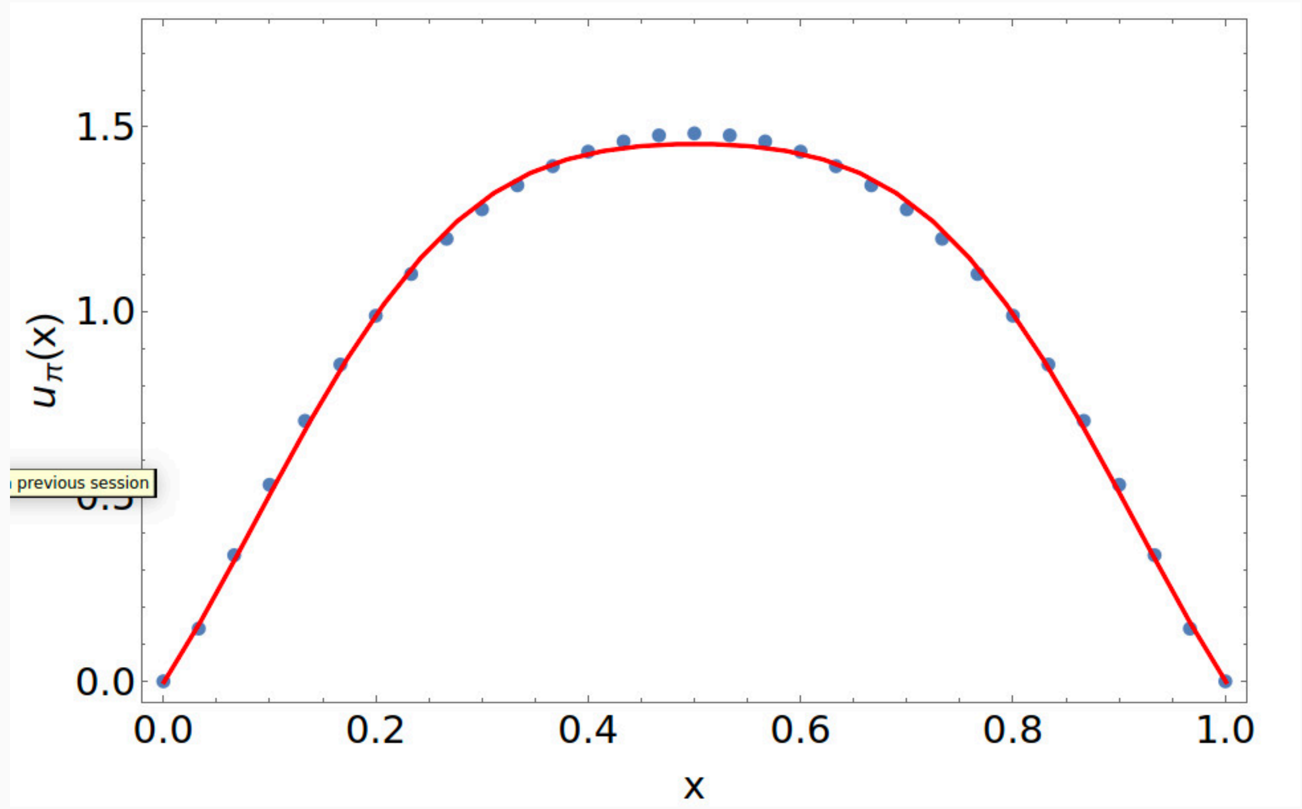

It works when PDF is symmetric, i.e. for the valence pion PDF (isospin symmetry)

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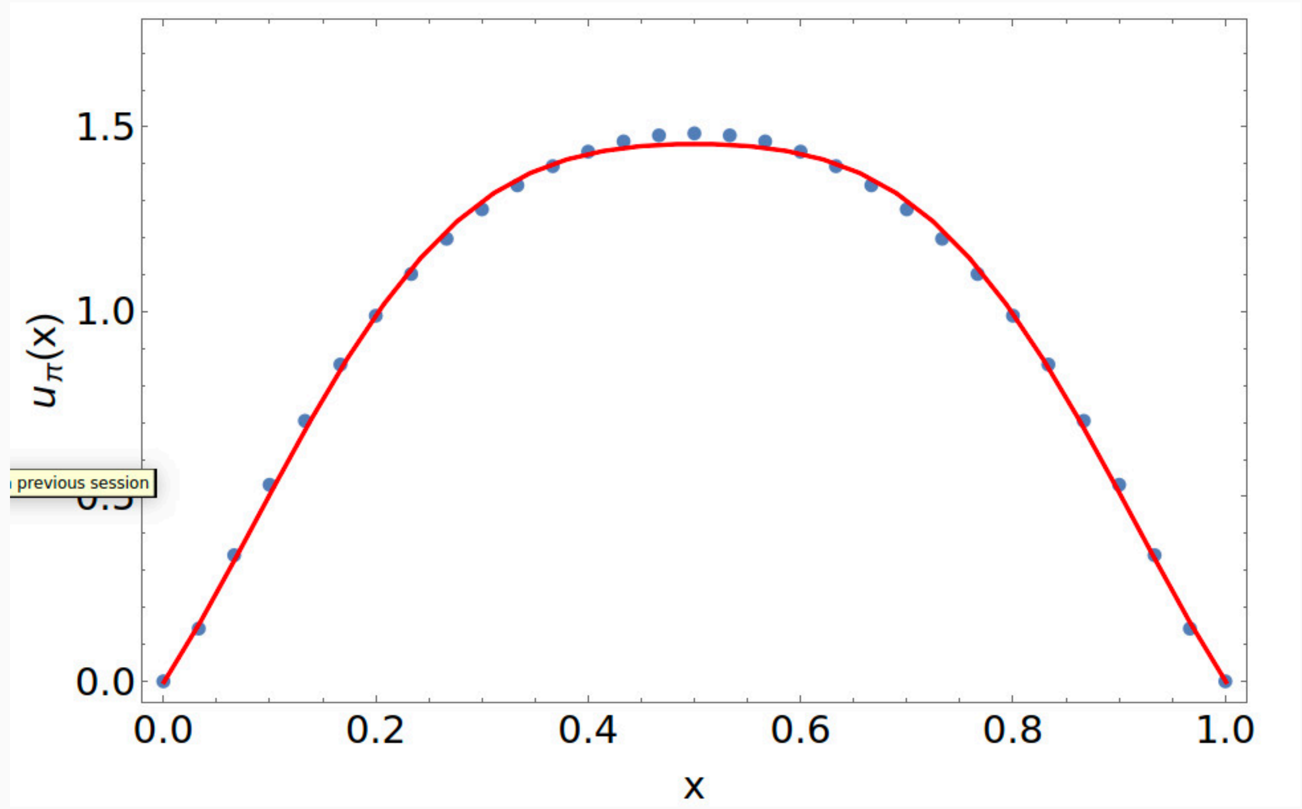

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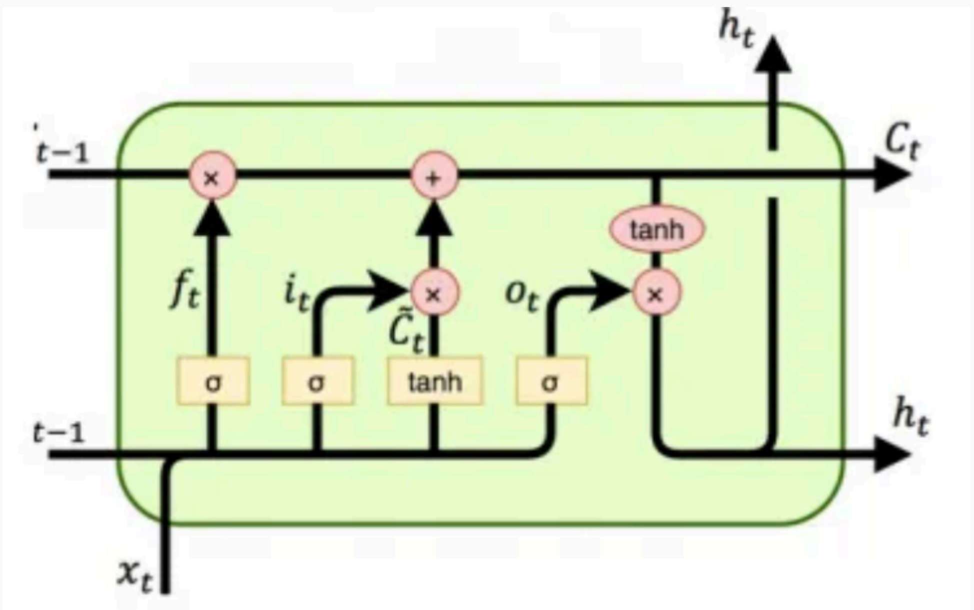
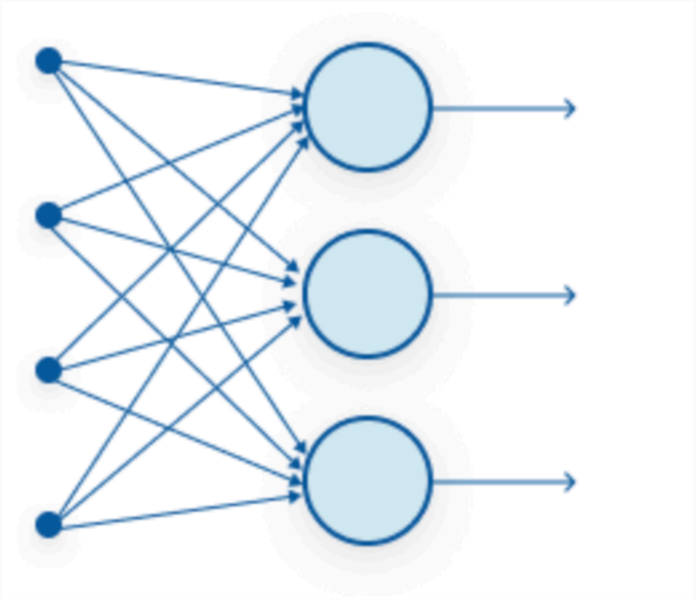
Alternative

LSTM

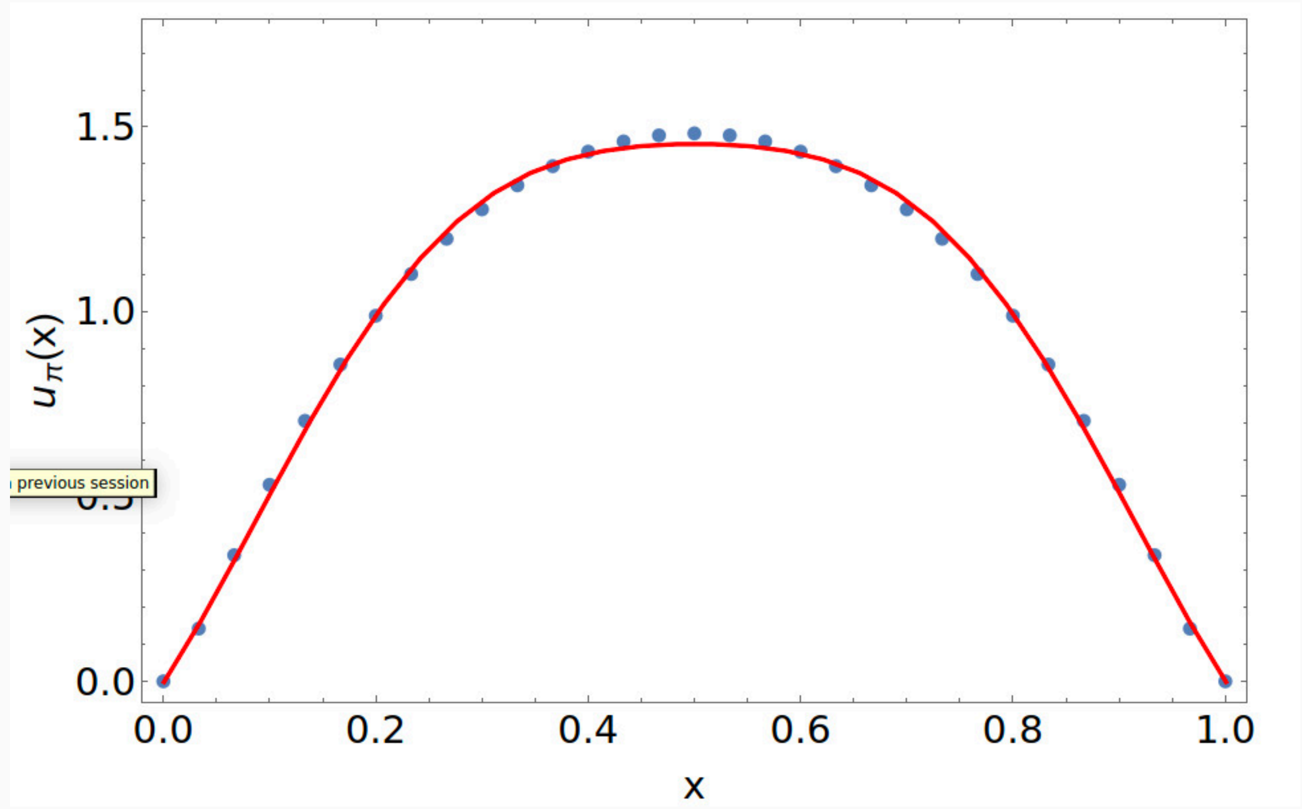
$[\mathscr{A}_1, \mathscr{A}_2, \ldots, \mathscr{A}_n]$





$[q(x_1), q(x_2), \ldots, q(x_m)]$

- Applying ANNs to PDF reconstruction from a sequence of Mellin moments (with K. Raya and J. Quintero)



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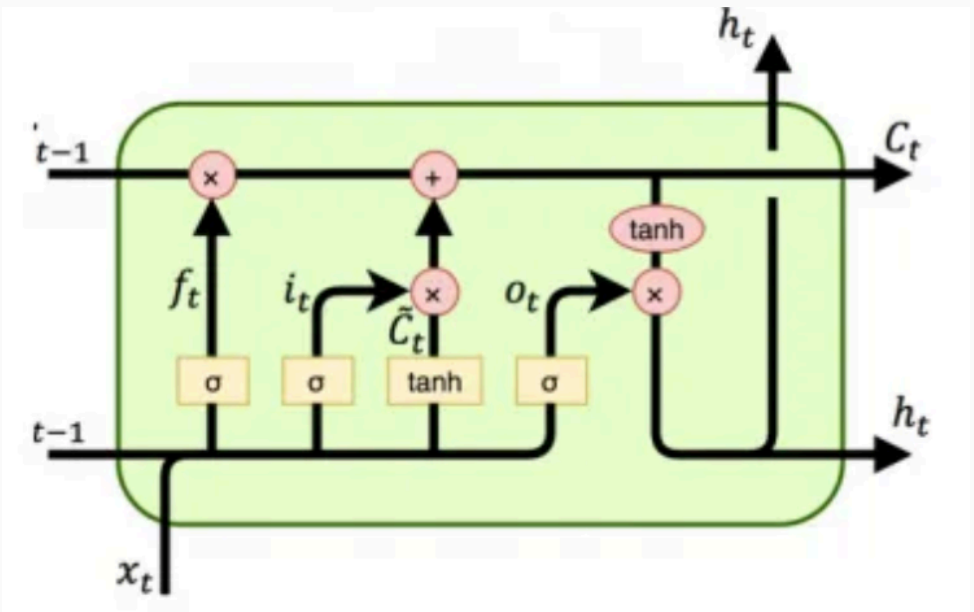

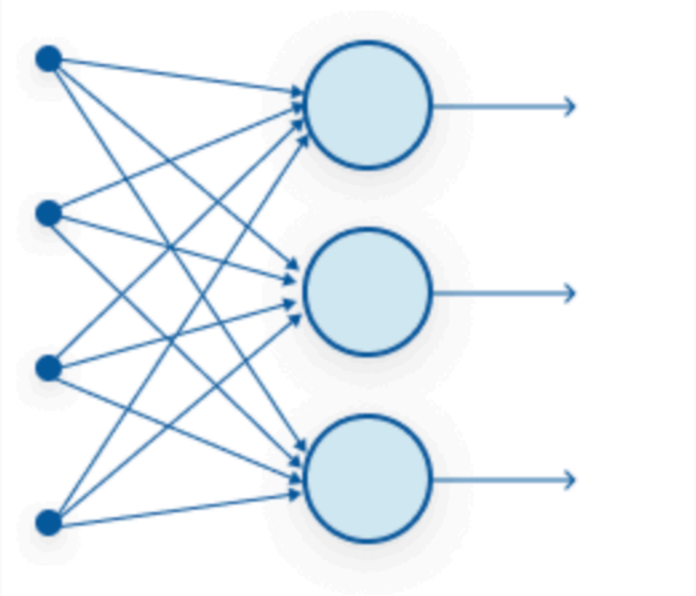

$$[q(x_1), q(x_2), \ldots, q(x_m)]$$

Memory loss problem → Trying with Transformer