# TENSOR CHARGE OF THE NUCLEON

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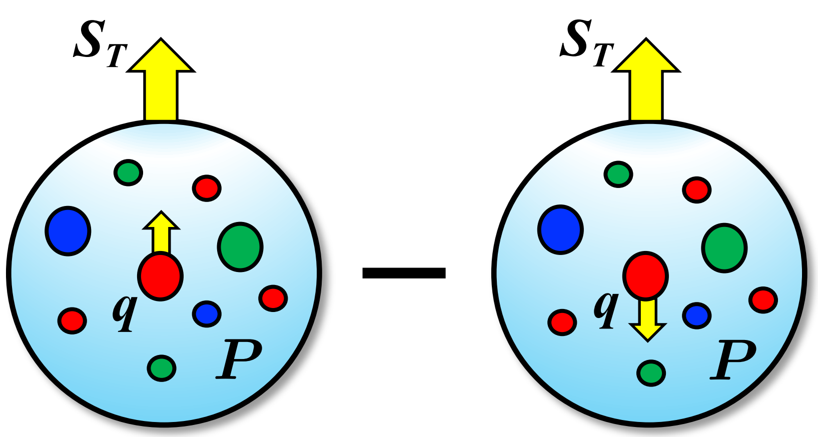
# MOTIVATION

# TRANSVERSITY

$$S_T^i\, \boldsymbol{h_1^q(x)} = \frac{1}{2}\int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-}\,\mathrm{Tr}[\langle P,S|\bar{\psi}_q(0)\mathcal{W}(0,\xi^-)\psi_q(\xi^-)i\sigma^{i+}\gamma_5|P,S\rangle]$$

transversity PDF - universal parton density encoding the difference between the number of quarks with their spin aligned versus anti-aligned to the proton's spin when it's in a transverse direction




$$\boldsymbol{\delta q} \equiv \int_0^1 dx\,[h_1^q(x) - h_1^{\bar{q}}(x)]$$

Tensor charge for an individual flavor

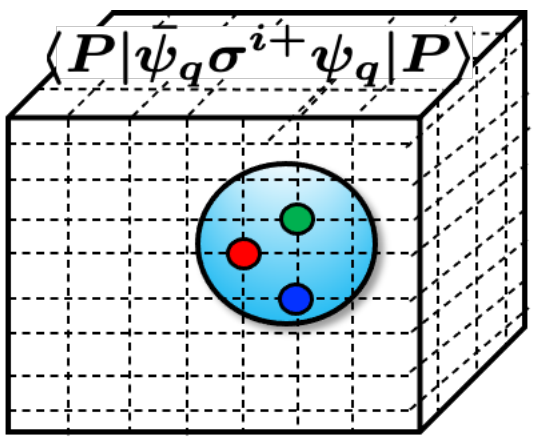
$$\boldsymbol{g_T} \equiv \delta u - \delta d$$

Isovector combination

# TENSOR CHARGE

$$\langle P|\bar{\psi}_q\sigma^{i+}\psi_q|P\rangle = \boldsymbol{\delta q}\,[\bar{u}_P\sigma^{i+}u_P]$$

local matrix element - can be computed in lattice QCD as well as other approaches like Dyson-Schwinger equations



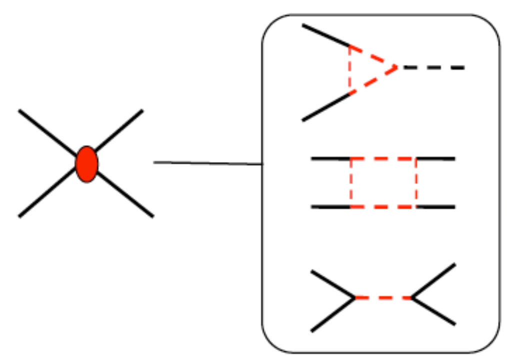

Gupta, et al. (2018);
Yamanaka, et al. (2018);
Hasan, et al. (2019);
Alexandrou, et al. (2019, 2023);

Yamanaka, et al. (2013);
Pitschmann, et al. (2015);
Xu, et al. (2015);
Wang, et al. (2018)

# TENSOR CHARGE

- Like the scalar, vector, and axial charges, it is a fundamental charge of the nucleon (although scale dependent)
- Since helicity PDF ≠ transversity PDF in relativistic quantum mechanics, it can be considered a measure of relativistic effects in the nucleon
- Key point of comparison between QCD phenomenology/experiment and ab initio approaches like lattice QCD and DSE
- Tensor couplings, not present in the SM Lagrangian, could be the footprints of new physics at higher scales

$\varepsilon_T\, g_T \approx M_W^2 / M_{BSM}^2$




Bhattacharya et al, PRD 85 (12)
Pattie et al., P.R. C88 (13)
Courtoy et al,  PRL 115 (2015)

Lagrangian for neutron beta decay

$$\mathcal{L}_{n\to pe\bar{\nu}_e} \sim \ \ldots + 4\sqrt{2}G_F V_{ud}\, \boldsymbol{g_T} \epsilon_T\, \bar{p}\,\sigma^{\mu\nu} n\, \bar{e}\,\sigma_{\mu\nu}\nu_e + \ldots$$

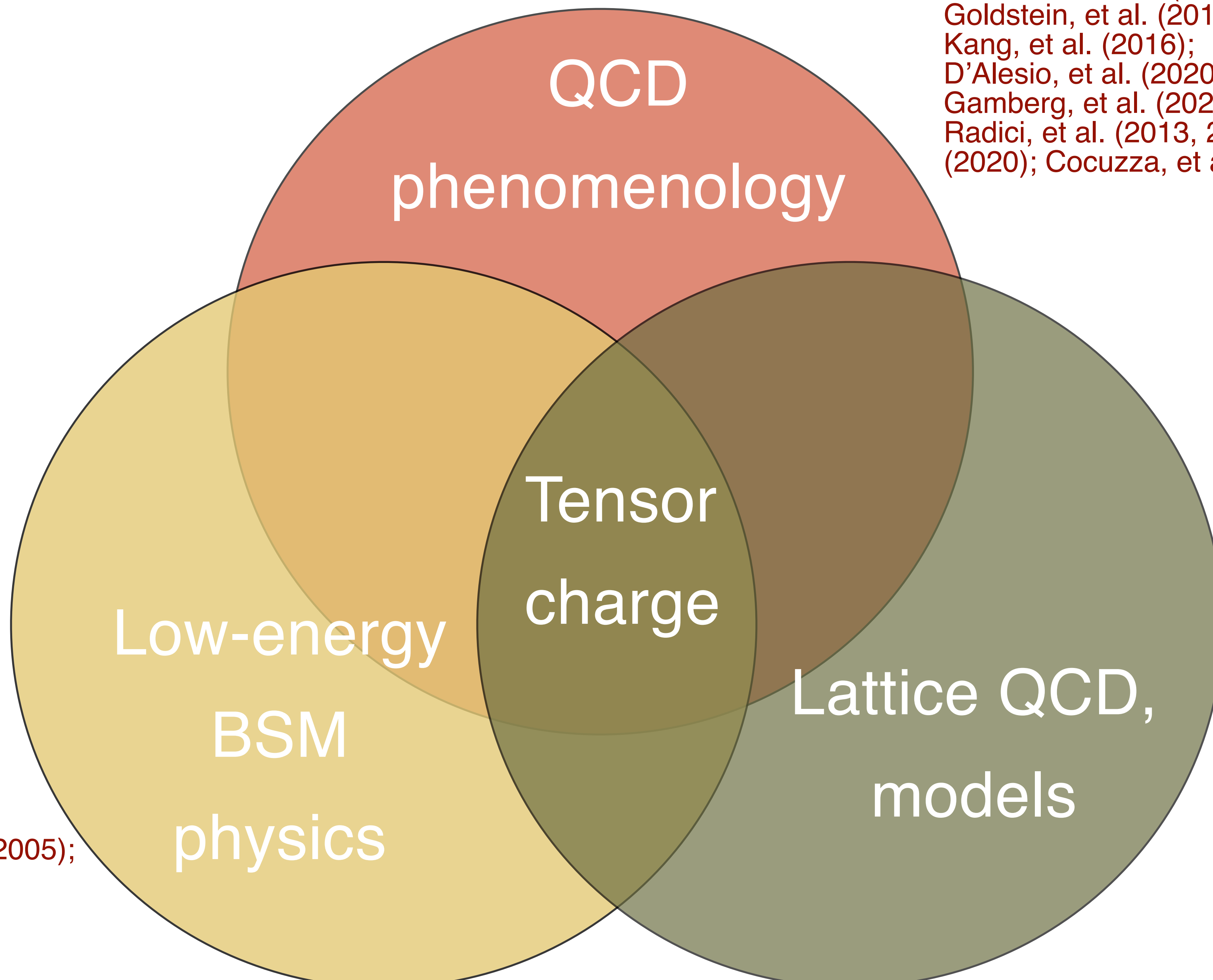
EDM of the proton

$$\tilde{d}_p = \tilde{d}_u\, \boldsymbol{\delta u} + \tilde{d}_d\, \boldsymbol{\delta d}$$

EDM of quarks

# TENSOR CHARGE

Anselmino, et al. (2007, 2009, 2013, 2015);
Goldstein, et al. (2014);
Kang, et al. (2016);
D'Alesio, et al. (2020); Cammarota, et al. (2020);
Gamberg, et al. (2022)
Radici, et al. (2013, 2015, 2018); Benel, et al. (2020); Cocuzza, et al. (2023)




He, Ji (1995); Barone, et al. (1997); Schweitzer, et al. (2001); Gamberg, Goldstein (2001); Pasquini, et al. (2005); Wakamatsu (2007); Lorce (2009); Gupta, et al. (2018); Yamanaka, et al. (2018); Hasan, et al. (2019); Alexandrou, et al. (2019, 2023) Yamanaka, et al. (2013);
Pitschmann, et al. (2015);
Xu, et al. (2015); Wang, et al. (2018); Liu, et al. (2019)

Herczeg (2001);
Erler, Ramsey-Musolf (2005);
Pospelov, Ritz (2005);
Severijns, et al. (2006);
Cirigliano, et al. (2013);
Courtoy, et al. (2015);
Yamanaka, et al. (2017); Liu, et al. (2018); Gonzalez-Alonso, et al. (2019)

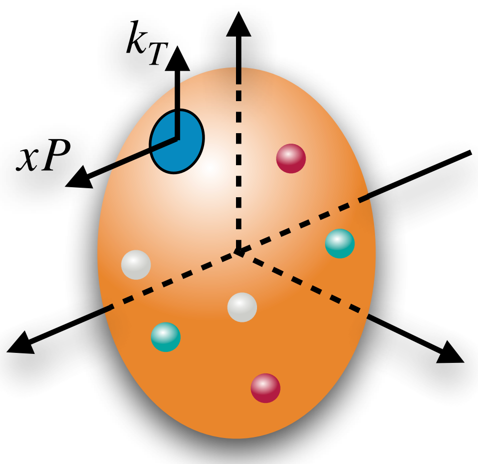
*Slide courtesy of Daniel Pitonyak and Chris Cocuzza*

# PHENOMENOLOGY

# TRANSVERSITY

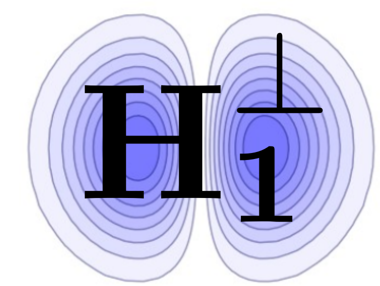
- Transversity is a chiral odd quantity, it must couple to another chair odd function to be measured
- Another transversity (or another chiral odd function) in double polarized Drell-Yan
- Chiral odd fragmentation function, Collins function or interference dihadron FF, in Semi Inclusive Deep Inelastic Scattering ✓
- Modulations measured in pion in jet, or left-right asymmetry in proton-proton scattering ✓
- Exclusive processes where transversity GPDs are accessible

# TRANSVERSE MOMENTUM DEPENDENT DISTRIBUTIONS




- TMDs depend both on collinear and transverse momenta

**Collins function**




Collins 1992

| | | Quark Polarization | | |
|---|---|---|---|---|
| | | Unpolarized (U) | Longitudinally Polarized (L) | Transversely Polarized (T) |
| Nucleon Polarization | U | $f_1(x,k_T^2)$ | | $h_1^\perp(x,k_T^2)$ - Boer-Mulders |
| | L | | $g_1(x,k_T^2)$ - Helicity | $h_{1L}^\perp(x,k_T^2)$ - |
| | T | $f_{1T}^\perp(x,k_T^2)$ - Sivers | $g_{1T}(x,k_T^2)$ - | $h_1(x,k_T^2)$ - Transversity<br>$h_{1T}^\perp(x,k_T^2)$ - Pretzelosity |

- Describes fragmentation of transversely polarized quark into an unpolarized nucleon
- Generates asymmetries in SIDIS and $e^+e^-$

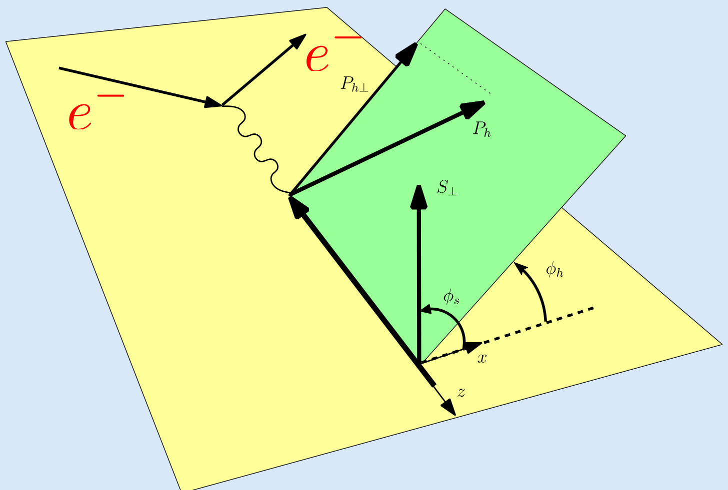
Kotzinian (1995)
Mulders, Tangerman (1995)
Boer, Jakob, Mulders (1997)

- Universal in SIDIS, $e^+e^-$, and PP

Metz, Collins (2004)
Yuan (2008)
Boer, Kang, Vogelsang, Yuan (2010)

- Transversity TMD should couple to another chiral odd function

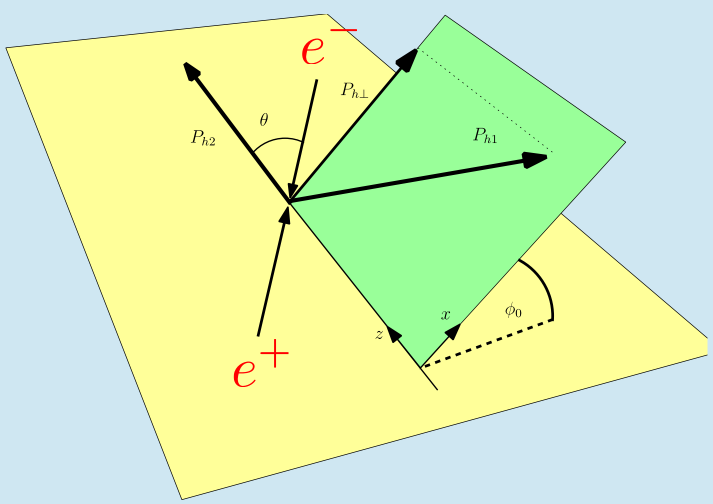
# TRANSVERSE SPIN ASYMMETRIES IN SIDIS AND E+E-




$$F_{UT}^{\sin(\phi_h+\phi_s)} \sim h_1(x_B, k_\perp) H_1^\perp(z_h, p_\perp)$$

transversity Collins function

$$\frac{d\sigma(S_\perp)}{dx_B dy dz_h d^2 P_{h\perp}} = \sigma_0(x_B, y, Q^2)\left[F_{UU} + \sin(\phi_h+\phi_s)\,\frac{2(1-y)}{1+(1-y)^2}\,F_{UT}^{\sin(\phi_h+\phi_s)} + \ldots\right]$$



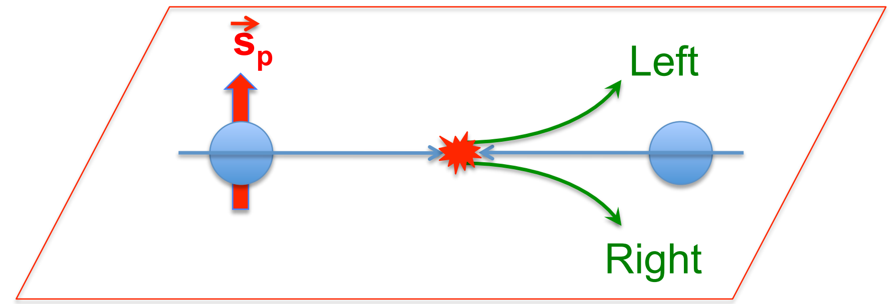

$$Z_{\text{collins}}^{h_1 h_2} \sim H_1^\perp(z_1, p_{1\perp}) H_1^\perp(z_2, p_{2\perp})$$

Collins function Collins function

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 + X}}{dz_{h1} dz_{h2} d^2 P_{h\perp}\, d\cos\theta} = \frac{N_c \pi \alpha_{\text{em}}^2}{2Q^2}\left[\left(1+\cos^2\theta\right) Z_{uu}^{h_1 h_2} + \sin^2\theta\cos(2\phi_0) Z_{\text{collins}}^{h_1 h_2}\right]$$

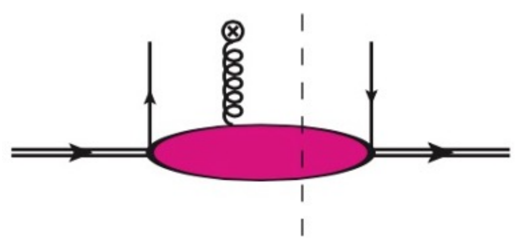
# TRANSVERSE SPIN ASYMMETRY IN PP SCATTERING

$A_N$ in pp scattering is related to collinear twist-3 (CT3) factorization




$$d\Delta\sigma(S_T) \sim H_{QS} \otimes f_1 \otimes \boldsymbol{F_{FT}} \otimes D_1 \; + \; H_F \otimes f_1 \otimes \boldsymbol{h_1} \otimes \left(\boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}}\right)$$

Qiu-Sterman term

$\boldsymbol{F_{FT}} \sim$ 

quark-gluon-quark correlator

Qiu, Sterman (91), Kouvaris, et al (06)

$$\pi \boldsymbol{F_{FT}(x,x)} = \int d^2\vec{k}_T \, \frac{k_T^2}{2M^2} \boldsymbol{f_{1T}^{\perp}(x, k_T^2)} \equiv f_{1T}^{\perp(1)}(x)$$ the first moment of Sivers function

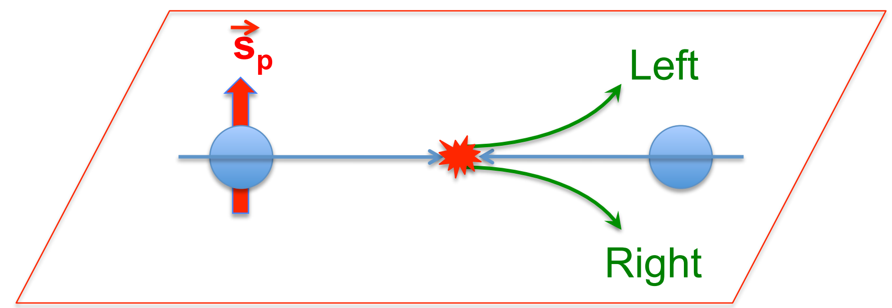
Boer, et al (03)

TMD and CT3 factorizations agree in their overlapping region of applicability

Ji, et al (06); Koike, et a. (08); Zhou, et al (08, 10); Yuan and Zhou (09)

# TRANSVERSE SPIN ASYMMETRIES

$A_N$ in pp scattering is related to collinear twist-3 (CT3) factorization



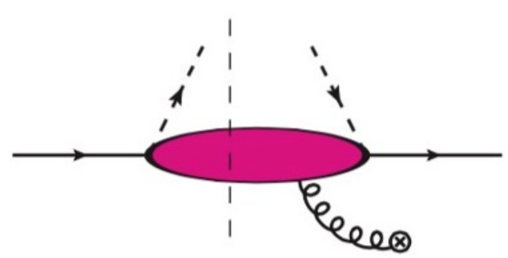

$$d\Delta\sigma(S_T) \sim H_{QS} \otimes f_1 \otimes \boldsymbol{F_{FT}} \otimes D_1 \; + \; H_F \otimes f_1 \otimes \boldsymbol{h_1} \otimes \left( \boldsymbol{H_1^{\perp(1)}}, \boldsymbol{\tilde{H}} \right)$$

Fragmentation term

$\boldsymbol{h_1}$ collinear transversity

Kanazawa, Koike, Metz, Pitonyak, Schlegel, (16)

$\boldsymbol{H_1^{\perp(1)}}$ $\boldsymbol{\tilde{H}}$ $\sim$



quark-gluon-quark fragmentation functions

$$\boldsymbol{H_1^{\perp(1)}(z)} \equiv z^2 \int d^2\vec{p}_\perp \frac{p_\perp^2}{2M_h^2} \boldsymbol{H_1^{\perp}(z, z^2 p_\perp^2)}$$ the first moment of Collins FF

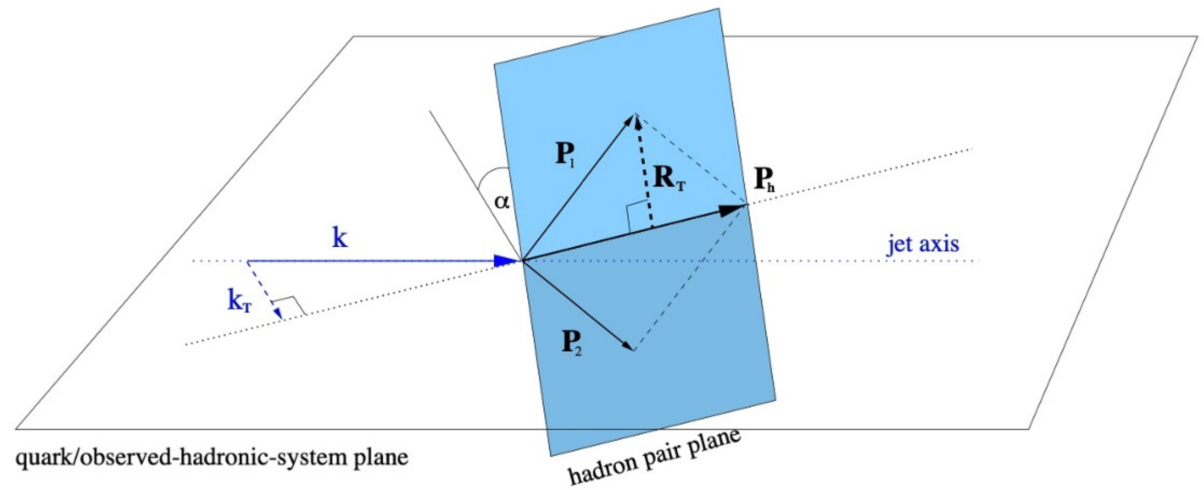
$$F_{UT}^{\sin\phi_S} \sim \sum_a e_a^2 \frac{2M_h}{Q} \boldsymbol{h_1^a(x)} \frac{\boldsymbol{\tilde{H}(z)}}{z}$$

Mulders, Tangerman (96); Bacchetta, et al (07)

# DIHADRON FRAGMENTATION APPROACH

collinear PDFs (x)

| N \ q | U | L | T |
|---|---|---|---|
| U | $f_1$ | | |
| L | | $g_1$ | |
| T | | | $h_1$ |




extDiFFs (z, $M_h$)

| N \ q | U | L | T |
|---|---|---|---|
| U | $D_1$ | | $H_1^{\sphericalangle}$ |

Collins, et al. (1994); Bianconi, et al. (1999), etc *

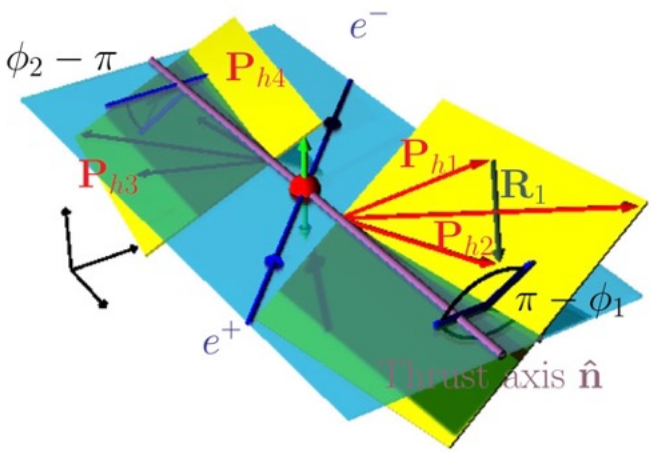
$z = z_1 + z_2$ , $M_h$ = invariant mass of dihadron. The "extended" DiFFs (extDiFFs) depend on z and $M_h$ (or equivalently $R_T$)

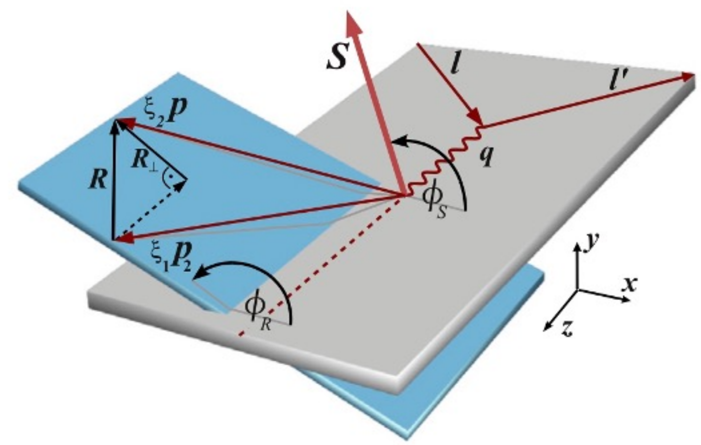
$H_1^{\sphericalangle}$ is chiral-odd "interference" FF (IFF)

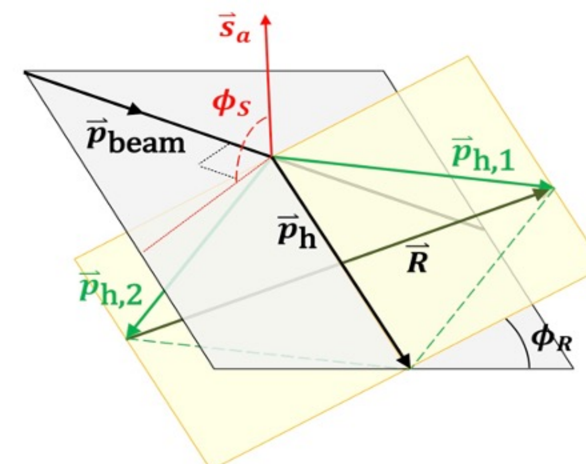
*New definition of DiFFs: Pitonyak et al, (2023) 2305.11995

# DIHADRON FRAGMENTATION APPROACH

$$e^+e^- \to (h_1h_2)(\bar{h}_1\bar{h}_2)\,X \qquad \ell\,N^\uparrow \to \ell\,(h_1h_2)\,X \qquad p^\uparrow p \to (h_1h_2)\,X$$




Collins, et al. (1994); Bianconi, et al. (1999); Bacchetta, Radici (2003, 2004); Courtoy, et al. (2012); Matevosyan, et al. (2018); Radici, et al. (2013, 2015, 2018); Benel, et al. (2020); Pitonyak et al (2023); Cocuzza et al (2023)

$$a_{12R} = \frac{\sin^2\theta \sum_q e_q^2\, H_1^{\sphericalangle,q}(z, M_h^2) H_1^{\sphericalangle,\bar{q}}(\bar{z}, \overline{M}_h^2)}{(1+\cos^2\theta)\sum_q e_q^2\, D_1^q(z, M_h^2) D_1^{\bar{q}}(\bar{z}, \overline{M}_h^2)}$$

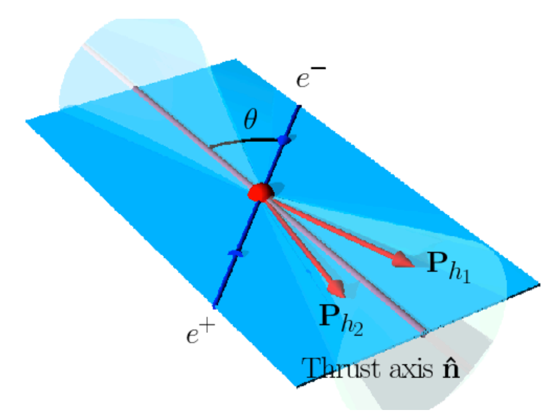
Artru-Collins asymmetry,

$D_1$ can be constrained using

measurements of $d\sigma/dzdM_h$ from BELLE (2017)

$$A_{UT}^{\sin(\phi_R+\phi_S)} = \frac{\sum_q e_q^2\, \boldsymbol{h_1^q(x)}\, H_1^{\sphericalangle,q}(z, M_h^2)}{\sum_q e_q^2\, f_1^q(x)\, D_1^q(z, M_h^2)}$$

$$A_{UT}^{\sin(\phi_R-\phi_S)} \sim \frac{\frac{d\Delta\hat{\sigma}_{a b^\uparrow \to c^\uparrow d}}{d\hat{t}} \otimes f_1^a(x_a) \otimes \boldsymbol{h_1^b(x_b)} \otimes H_1^{\sphericalangle,c}(z, M_h^2)}{\frac{d\hat{\sigma}_{ab\to cd}}{d\hat{t}} \otimes f_1^a(x_a) \otimes f_1^b(x_b) \otimes D_1^c(z, M_h^2)}$$




*Slide courtesy of Daniel Pitonyak*

# TMD/CT3 ANALYSES OF THE DATA

| | $e^+e^-$ Collins | SIDIS Collins | Hadron-in-jet Collins | Proton-proton $A_N$ | Lattice tensor charge(s) | Soffer bound | Framework |
|---|---|---|---|---|---|---|---|
| **Anselmino, et al. (2015)** | ✓ | ✓ | X | X | X | ✓ | Parton model |
| **Kang, et al. (2016)** | ✓ | ✓ | X | X | X | ✓ | CSS/TMD evolution |
| **Lin, et al. (2018)** | X | ✓ | X | X | ✓ $g_T$ | X | Parton model |
| **D'Alesio, et al. (2020)** | ✓ | ✓ | X | X | X | X† | Parton model |
| **Cammarota, et al. (2020) JAM3D-20*** | ✓ | ✓ | X | ✓ | X | X | Parton model |

*Also included Sivers effects in SIDIS and Drell-Yan

†Performed fit both with and without SB

Soffer bound (SB): $|h_1^q(x)| \leq \frac{1}{2}(f_1^q(x) + g_1^q(x))$

Note: Predictions exist for hadron-in-jet Collins effect (D'Alesio, et al. (2017); Kang, et al. (2017)) but no groups have included the STAR data in a fit. These are important measurements to use in future studies.

*Slide courtesy of Daniel Pitonyak*

# TMD/CT3 ANALYSES OF THE DATA

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| **Anselmino, et al. (2015)** | ✓ | ✓ | X | X | X | ✓ | Parton model |
| **Kang, et al. (2016)** | ✓ | ✓ | X | X | X | ✓ | CSS/TMD evolution |
| **Lin, et al. (2018)** | X | ✓ | X | X | ✓ $g_T$ | X | Parton model |
| **D'Alesio, et al. (2020)** | ✓ | ✓ | X | X | X | X$^\dagger$ | Parton model |
| **Cammarota, et al. (2020) JAM3D-20*** | ✓ | ✓ | X | ✓ | X | X | Parton model |
| **Gamberg, et al. (2022) JAM3D-22*** | ✓ | ✓ | X | ✓ | ✓ $g_T$ | ✓^ | Parton model |

*Also included Sivers effects in SIDIS and Drell-Yan

$^\dagger$Performed fit both with and without SB

^ Imposed the SB but allowed for violations given the uncertainties in $f_1(x)$ and $g_1(x)$

*Slide courtesy of Daniel Pitonyak*

# TMD/CT3 ANALYSES OF THE DATA

Single-transverse-spin asymmetries: From deep inelastic scattering to hadronic collisions

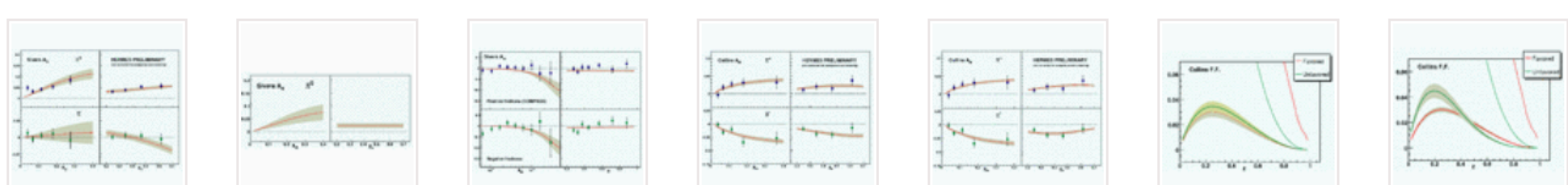
Werner Vogelsang and Feng Yuan

Phys. Rev. D **72**, 054028 – Published 30 September 2005

Article | References | Citing Articles (148) | PDF | HTML | Export Citation

ABSTRACT

We study single-spin asymmetries in semi-inclusive deep inelastic scattering with transversely polarized target. Based on the QCD factorization approach, we consider Sivers and Collins contributions to the asymmetries. We fit simple parametrizations for the Sivers and Collins functions to the recent HERMES data, and compare to results from COMPASS. Using the fitted parametrizations for the Sivers functions, we predict the single-transverse-spin asymmetries for various processes in $pp$ collisions at the Relativistic Heavy Ion Collider, including the Drell-Yan process and angular correlations in dijet and jet-plus-photon production. These asymmetries are found to be sizable at forward rapidities.



| | |
|---|---|
| Anse et al. | work |
| Kang (2( | model |
| Lin, (2( | MD tion |
| D'A et al. | model |
| Camr et al. JAM3 | model |
| Gam et al. JAM3 | model |
| | model |

*Also ... given the

†Performed fit both with and without SB ... uncertainties in $f_1(x)$ and $g_1(x)$

*Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)*

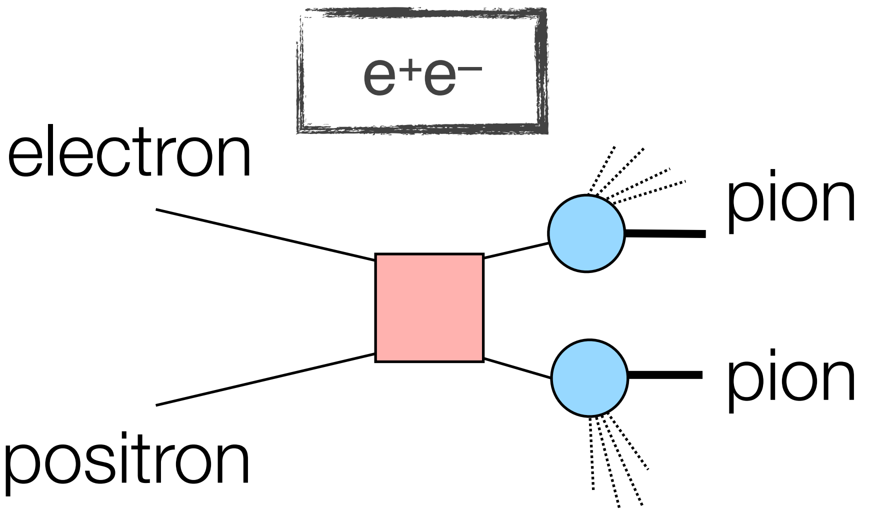
Jefferson Lab Angular Momentum Collaboration

# JAM20 ANALYSIS

# UNIVERSAL GLOBAL FIT 2020

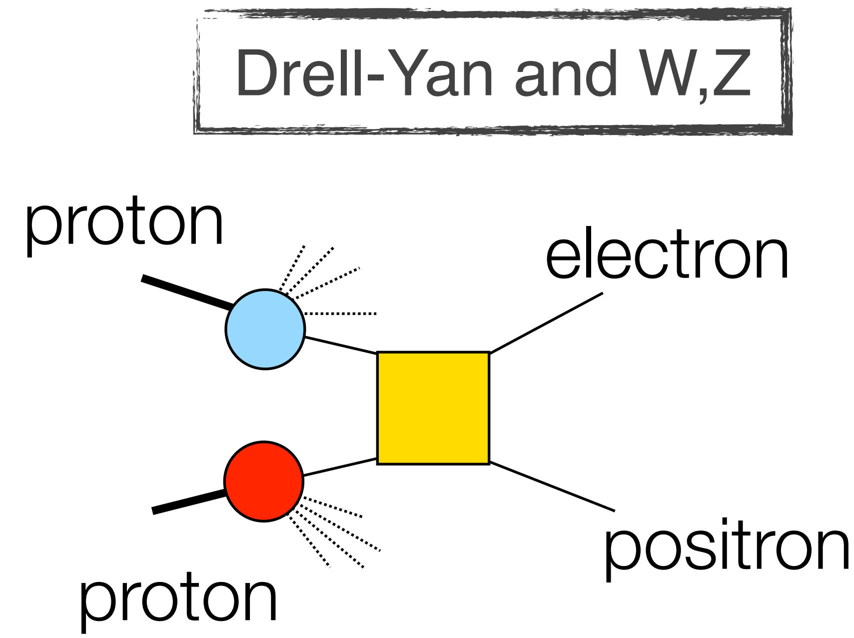
*JAM20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato Phys.Rev.D 102 (2020) 5, 05400 (2020)*

*JAM22: Gamberg, Malda, Miller, Pitonyak, Prokudin, Sato, Phys.Rev.D 106 (2022) 3, 034014*

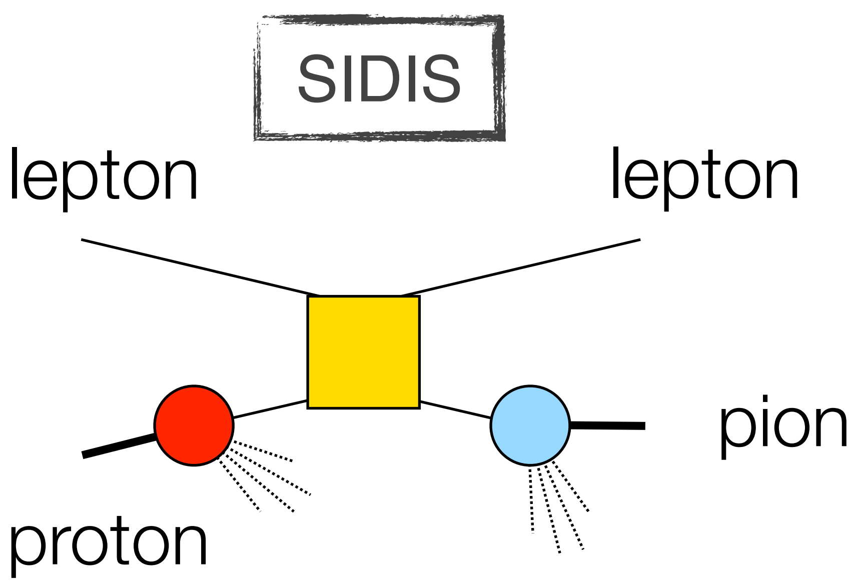



*Collins asymmetries*
*BELLE, BaBar, BESIII data*

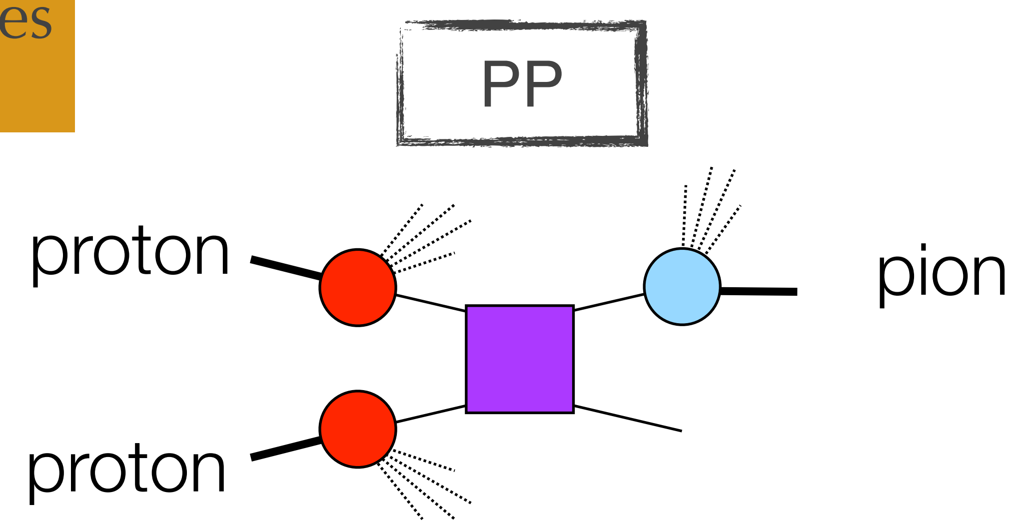
To demonstrate the common origin of SSAs in various processes, we will combine all available data and extract a universal set of non perturbative functions that describes all of them




*Sivers asymmetries*
*COMPASS, STAR data*




*Sivers, Collins asymmetries*
*COMPASS, HERMES, JLab data*




*$A_N$ asymmetry*
*STAR, PHENIX, BRAHMS data*

# UNIVERSAL GLOBAL FIT 2020

Jefferson Lab Angular Momentum Collaboration

https://www.jlab.org/theory/jam

| Observable | Reactions | Non-Perturbative Function(s) | $\chi^2/N_{\text{pts.}}$ |
|---|---|---|---|
| $A_{\text{SIDIS}}^{\text{Siv}}$ | $e+(p,d)^{\uparrow}\to e+(\pi^+,\pi^-,\pi^0)+X$ | $f_{1T}^{\perp}(x,k_T^2)$ | $150.0/126=1.19$ |
| $A_{\text{SIDIS}}^{\text{Col}}$ | $e+(p,d)^{\uparrow}\to e+(\pi^+,\pi^-,\pi^0)+X$ | $h_1(x,k_T^2),H_1^{\perp}(z,z^2p_{\perp}^2)$ | $111.3/126=0.88$ |
| $A_{\text{SIA}}^{\text{Col}}$ | $e^++e^-\to\pi^+\pi^-(UC,UL)+X$ | $H_1^{\perp}(z,z^2p_{\perp}^2)$ | $154.5/176=0.88$ |
| $A_{\text{DY}}^{\text{Siv}}$ | $\pi^-+p^{\uparrow}\to\mu^+\mu^-+X$ | $f_{1T}^{\perp}(x,k_T^2)$ | $5.96/12=0.50$ |
| $A_{\text{DY}}^{\text{Siv}}$ | $p^{\uparrow}+p\to(W^+,W^-,Z)+X$ | $f_{1T}^{\perp}(x,k_T^2)$ | $31.8/17=1.87$ |
| $A_N^h$ | $p^{\uparrow}+p\to(\pi^+,\pi^-,\pi^0)+X$ | $h_1(x),F_{FT}(x,x)=\frac{1}{\pi}f_{1T}^{\perp(1)}(x),H_1^{\perp(1)}(z)$ | $66.5/60=1.11$ |

*Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)*

- 18 observables and 6 non-perturbative functions (Sivers up/down; transversity up/down; Collins favored/unfavored)

$$h_1(x),\ F_{FT}(x,x),\ H_1^{\perp(1)}(z),\ \tilde{H}(z)$$

- Broad kinematical coverage to test universality
- The analysis is performed at parton level leading order, gaussian model is used for TMDs, and DGLAP-type evolution is implemented

# UNIVERSAL GLOBAL FIT 2020

*JAM20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato Phys.Rev.D 102 (2020) 5, 05400 (2020)*

The relevant set of collinear functions to extract

$h_1(x)$ transversity

$$\pi F_{FT}(x,x) = \int d^2\vec{k}_T \, \frac{k_T^2}{2M^2} f_{1T}^{\perp}(x,k_T^2) \equiv f_{1T}^{\perp(1)}(x)$$

$F_{FT}(x,x)$ Qiu-Sterman function (related to Sivers function)

$H_1^{\perp(1)}(z)$ the first $k_T$ moment of Collins FF

$$H_1^{\perp(1)}(z) \equiv z^2 \int d^2\vec{p}_\perp \frac{p_\perp^2}{2M_h^2} H_1^{\perp}(z, z^2 p_\perp^2)$$

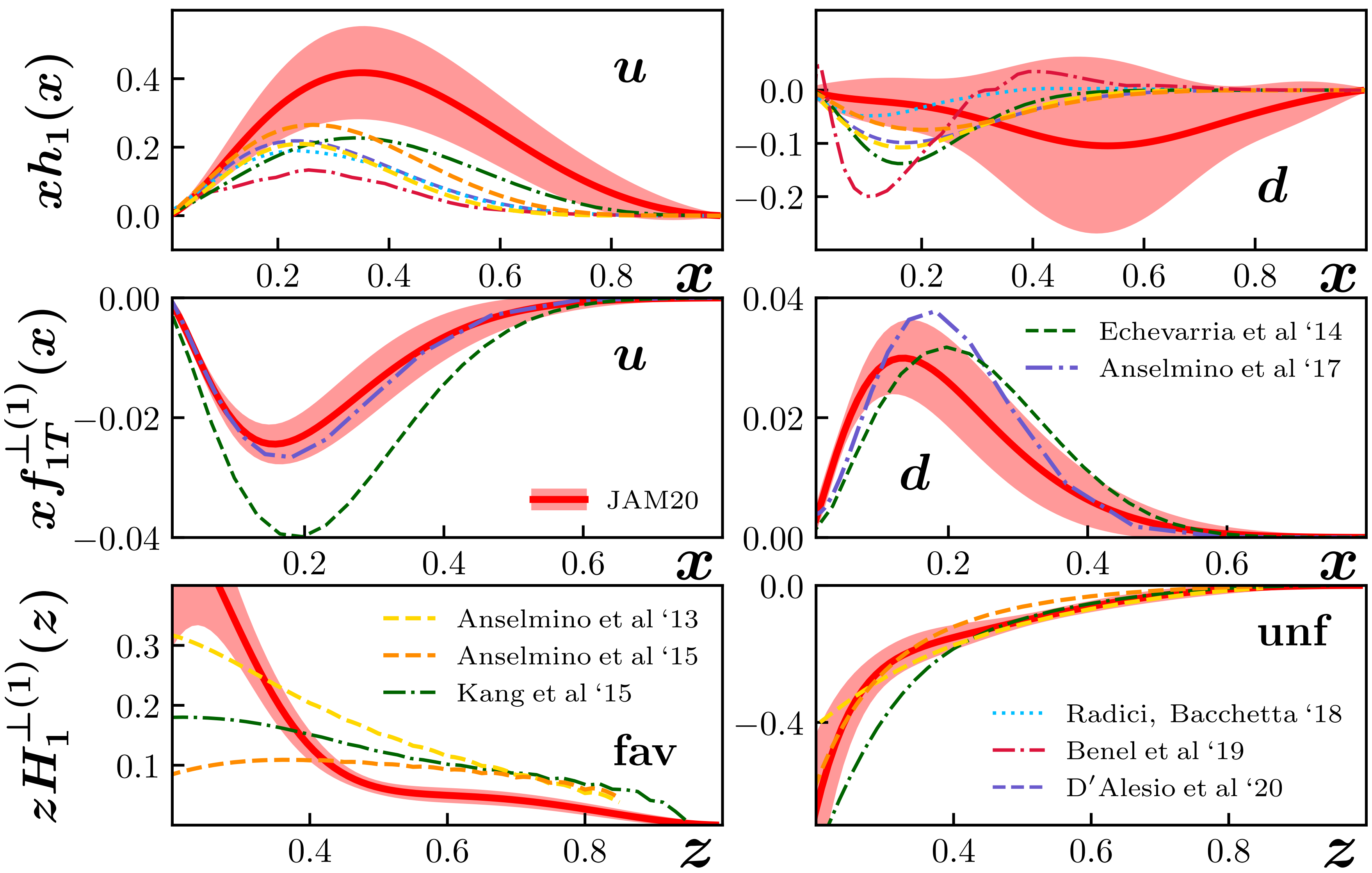
$\tilde{H}(z)$ fragmentation twist-3 function

Flexible parametrization

$$F^q(x) = \frac{N_q \, x^{a_q} (1-x)^{b_q} \, (1 + \gamma_q \, x^{\alpha_q} (1-x)^{\beta_q})}{\mathrm{B}[a_q+2, b_q+1] + \gamma_q \mathrm{B}[a_q+\alpha_q+2, b_q+\beta_q+1]}$$

# UNIVERSAL GLOBAL FIT 2020

*Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato (2020)*




Transversity

$h_1(x)$

Sivers

$f_{1T}^{\perp(1)}(x)$

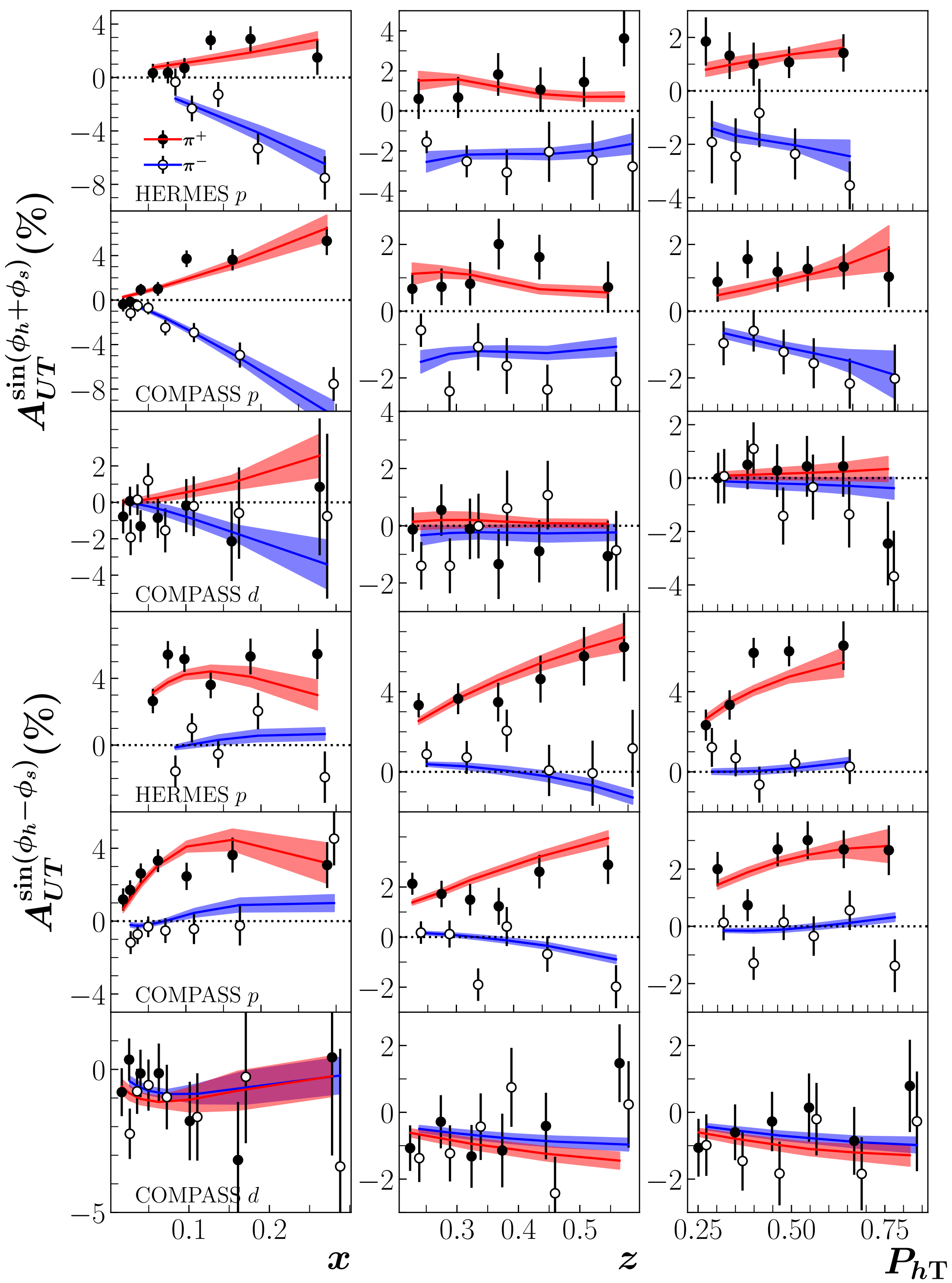
Collins FF

$H_1^{\perp(1)}(z)$

# UNIVERSAL GLOBAL FIT 2020

*JAM20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato Phys.Rev.D 102 (2020) 5, 05400 (2020)*

SIDIS




Collins asymmetry

$$\frac{\chi^2}{npoints} = \frac{107.1}{126} = 0.85$$

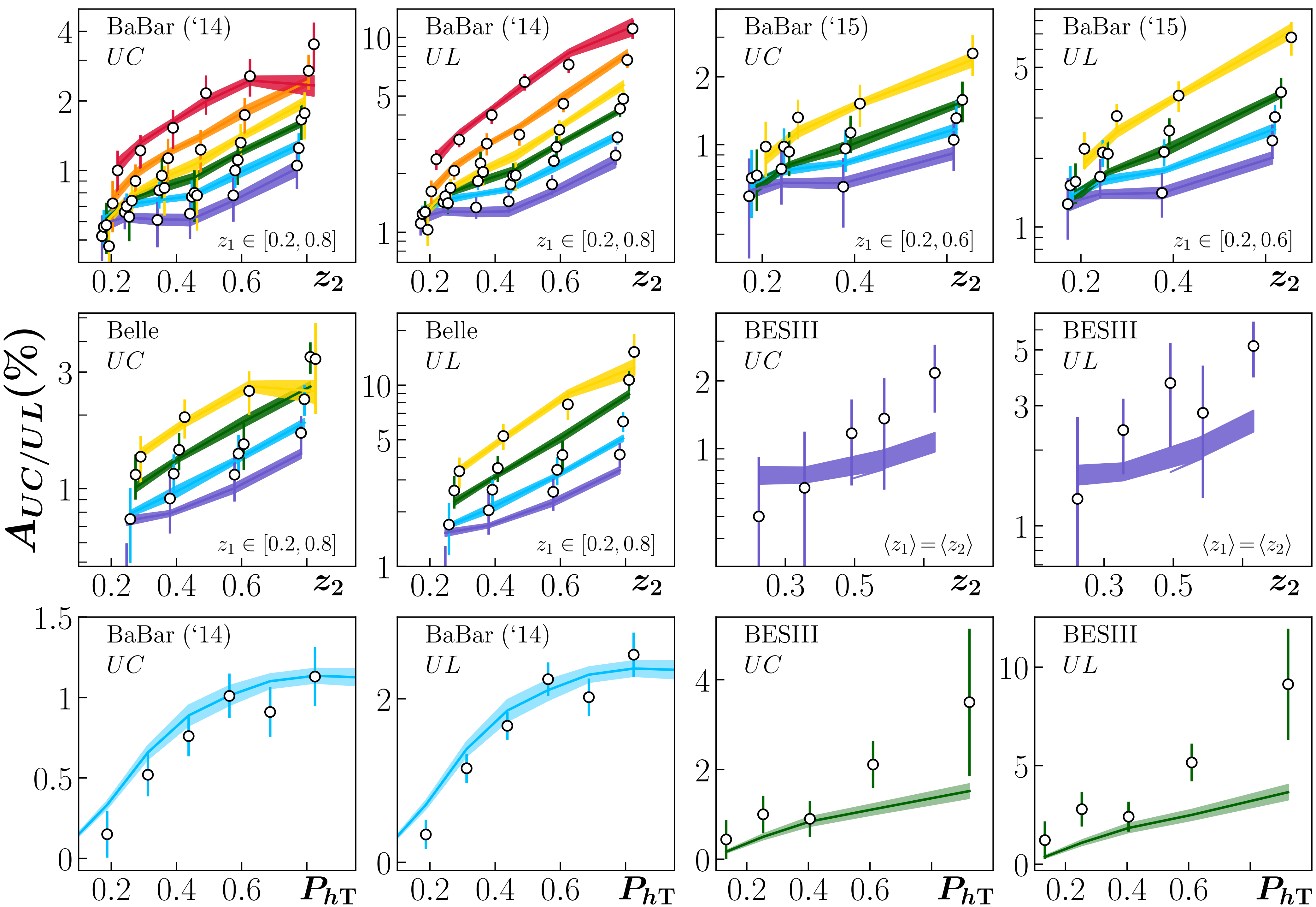
Sivers asymmetry

$$\frac{\chi^2}{npoints} = \frac{85.4}{88} = 0.97$$

# UNIVERSAL GLOBAL FIT 2020

*JAM20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato Phys.Rev.D 102 (2020) 5, 05400 (2020)*

e+e-

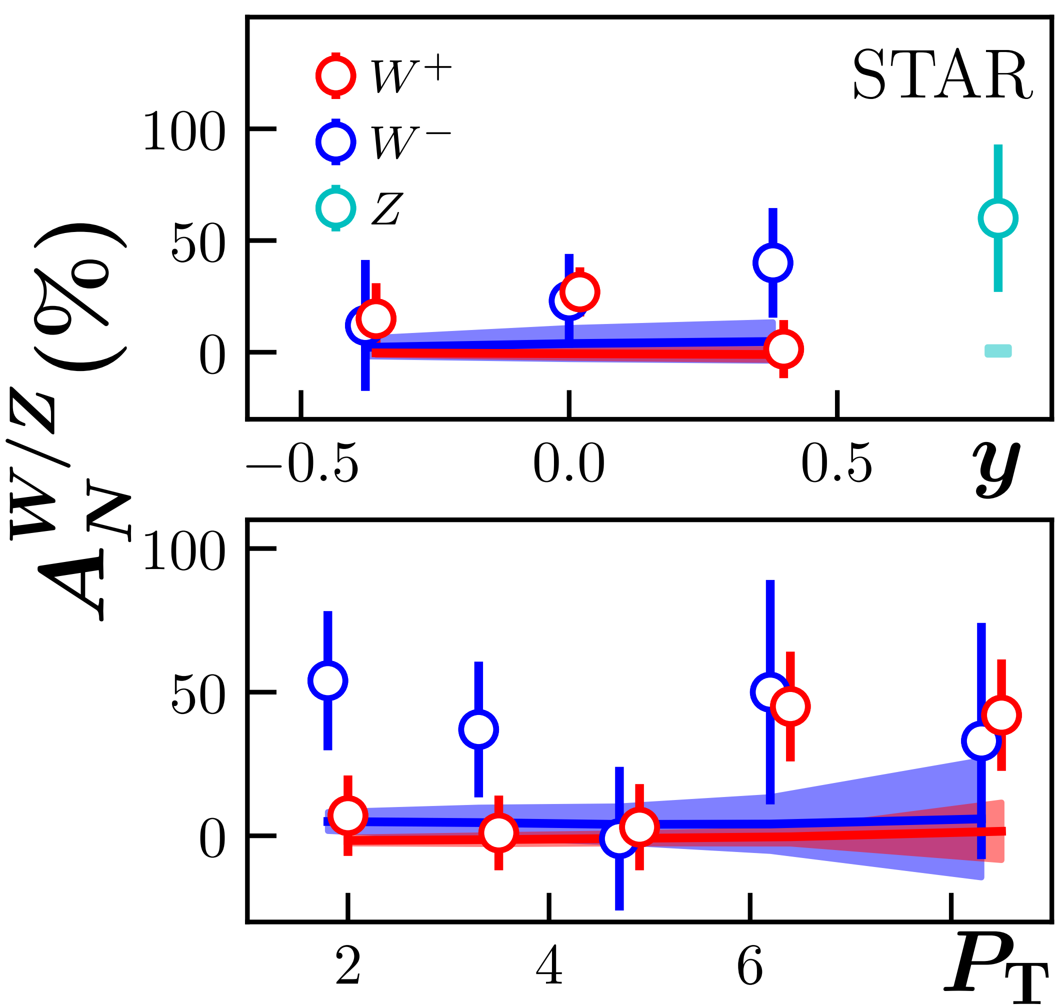



$$\frac{\chi^2}{npoints} = \frac{149.6}{176} = 0.85$$

# UNIVERSAL GLOBAL FIT 2020

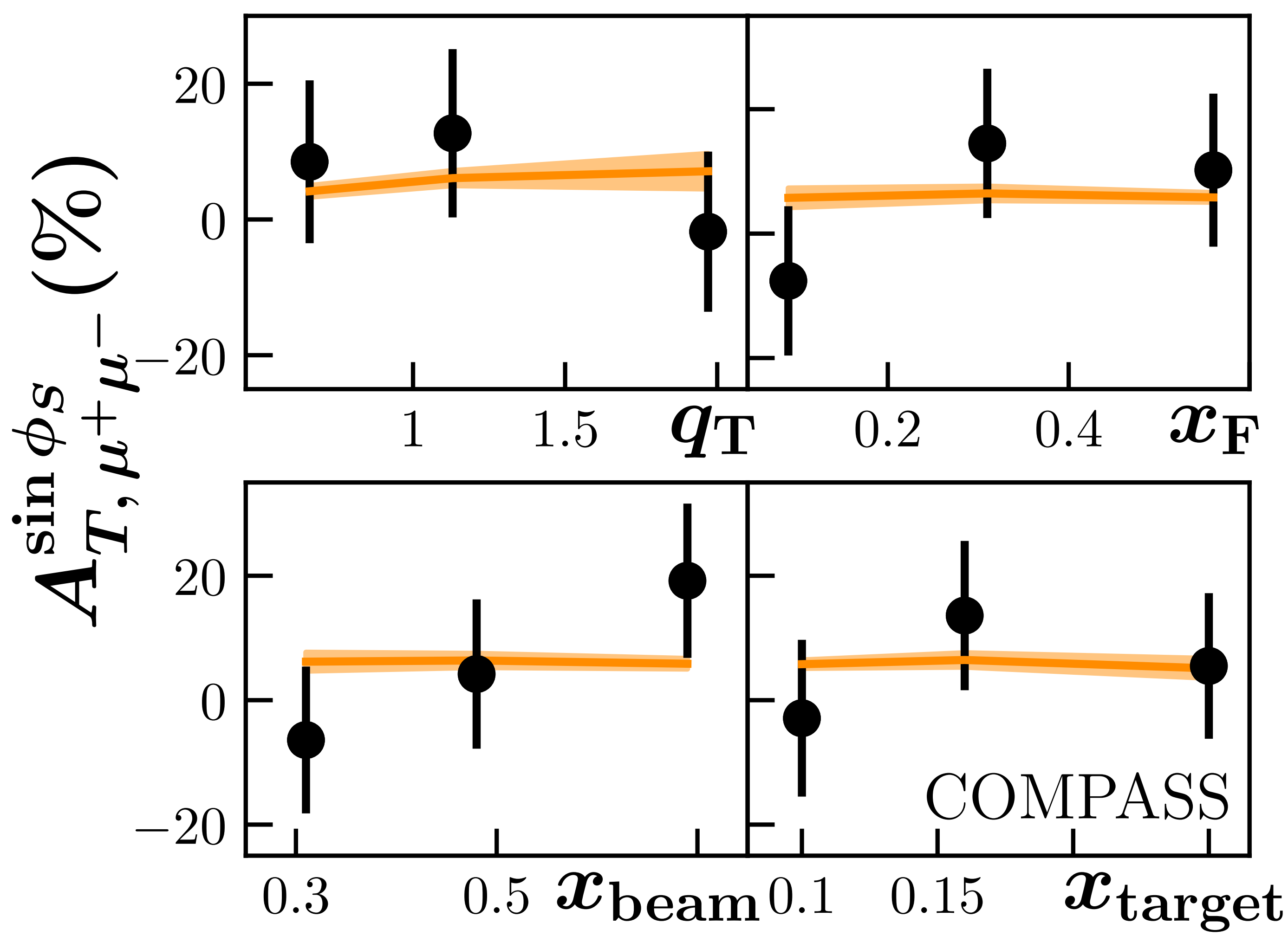
*JAM20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato Phys.Rev.D 102 (2020) 5, 05400 (2020)*

## Drell-Yan




$$\frac{\chi^2}{npoints} = \frac{29.8}{17} = 1.75$$

STAR



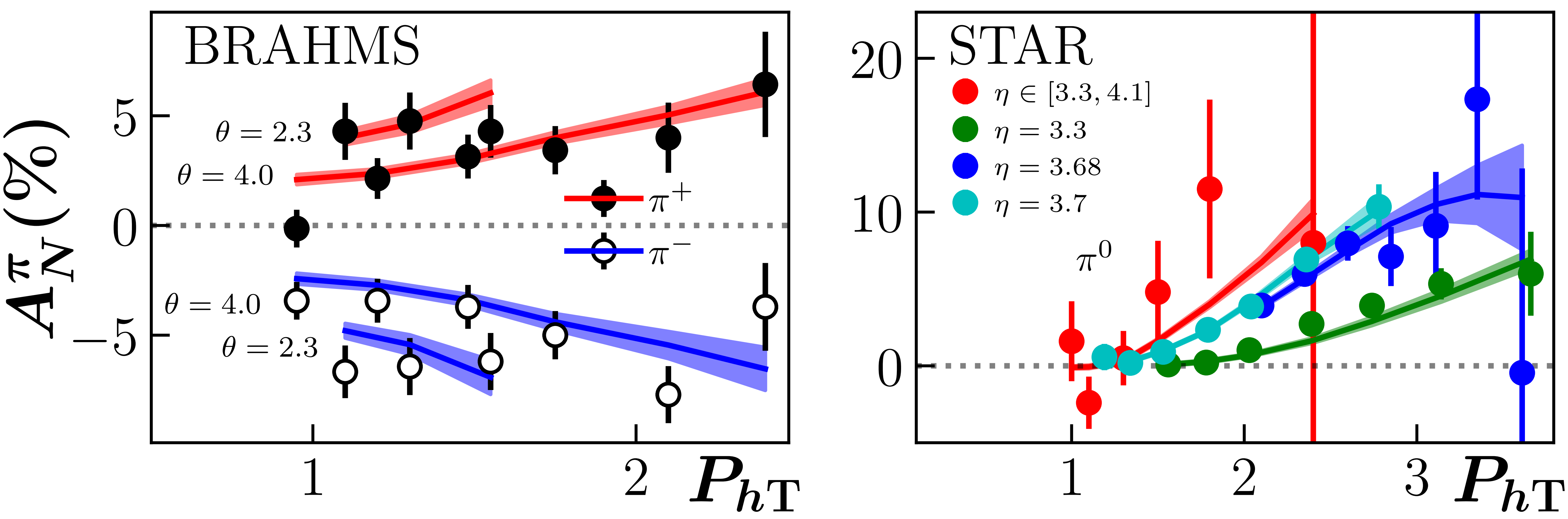

$$\frac{\chi^2}{npoints} = \frac{7.6}{12} = 0.63$$

COMPASS DY

# UNIVERSAL GLOBAL FIT 2020

*JAM20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato Phys.Rev.D 102 (2020) 5, 05400 (2020)*

proton-proton $A_N$




$$\frac{\chi^2}{npoints} = \frac{72.0}{60} = 1.2$$

# UNIVERSAL GLOBAL FIT 2020

*JAM20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato Phys.Rev.D 102 (2020) 5, 05400 (2020)*

$$\frac{E_h d\sigma^{Frag}(S_P)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\,\epsilon^{P'PP_hS_P}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\,\delta(\hat{s}+\hat{t}+\hat{u})$$

$$\times\frac{1}{\hat{s}\,(-x'\hat{t}-x\hat{u})}h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),\pi/c}(z)-z\frac{dH_1^{\perp(1),\pi/c}(z)}{dz}\right]S_{H_1^\perp}^i+\frac{1}{z}H^{\pi/c}(z)\,S_H^i\right.$$

$$\left.+\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat{H}_{FU}^{\pi/c,\Im}(z,z_1)\,S_{\hat{H}_{FU}}^i\right\},$$

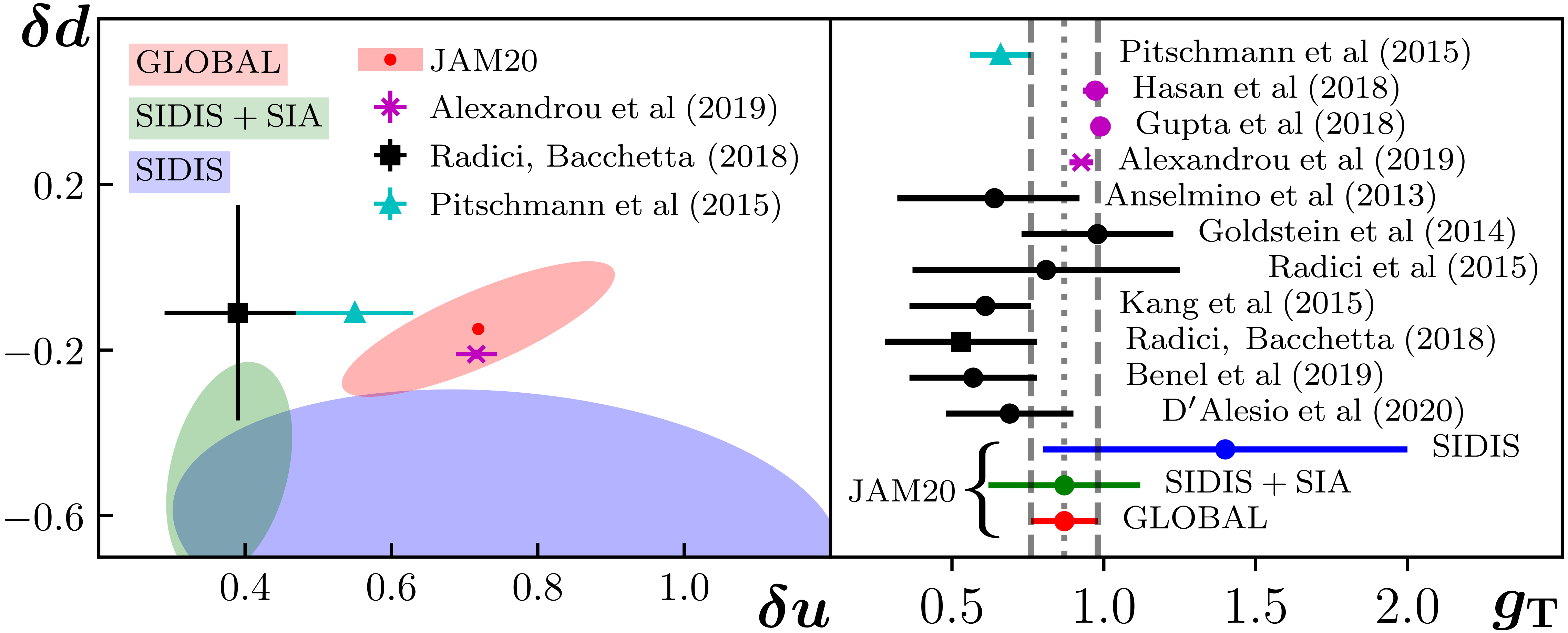
Integration over **x** for transversity, conservation of momenta in *ab→cd:*

$$\int_{x_{min}}^1\frac{dx}{x} \qquad\qquad x_{min} = -(U/z)/(T/z+S)$$

RHIC data is sensitive to high-x behavior of transversity quark-gluon channel is dominant contribution for large $x_F$

# UNIVERSAL GLOBAL FIT 2020

*JAM20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato Phys.Rev.D 102 (2020) 5, 05400 (2020)*




- Tensor charge from up and down quarks is constrained and compatible with lattice results
- Isovector tensor charge $g_T = \delta u - \delta d$

  $g_T = 0.89 \pm 0.12$ compatible with lattice results

**$\delta$u and $\delta$d $Q^2$=4 $GeV^2$**

**$\delta u = 0.65 \pm 0.22$**

**$\delta d = -0.24 \pm 0.2$**

*JAM22: Gamberg, Malda, Miller, Pitonyak, Prokudin, Sato, Phys.Rev.D 106 (2022) 3, 034014*

# JAM22 ANALYSIS

# JAM22: SET UP

*JAM22: Gamberg, Malda, Miller, Pitonyak, Prokudin, Sato, Phys.Rev.D 106 (2022) 3, 034014*

- Collins and Sivers (3D binned) SIDIS data from HERMES (2020)

  *HERMES Collaboration, A. Airapetian et al. JHEP 12 (2020) 010*

- $A_{UT}^{\sin\phi_S}$ (x and z projections only) from HERMES (2020)

- All other data sets are the same as in JAM20 (COMPASS, BELLE, RHIC), except for the new HERMES data that supersedes previous sets
- 19 observables and 8 non-perturbative functions (Sivers up/down; transversity up/down; Collins fav/unf, $\tilde{H}$ fav/unf)

$$\boldsymbol{h_1(x), F_{FT}(x,x), H_1^{\perp(1)}(z), \tilde{H}(z)}$$

- Lattice data on $g_T$ at the physical pion mass from Alexandrou, et al. (2020) (as a Bayesian prior)

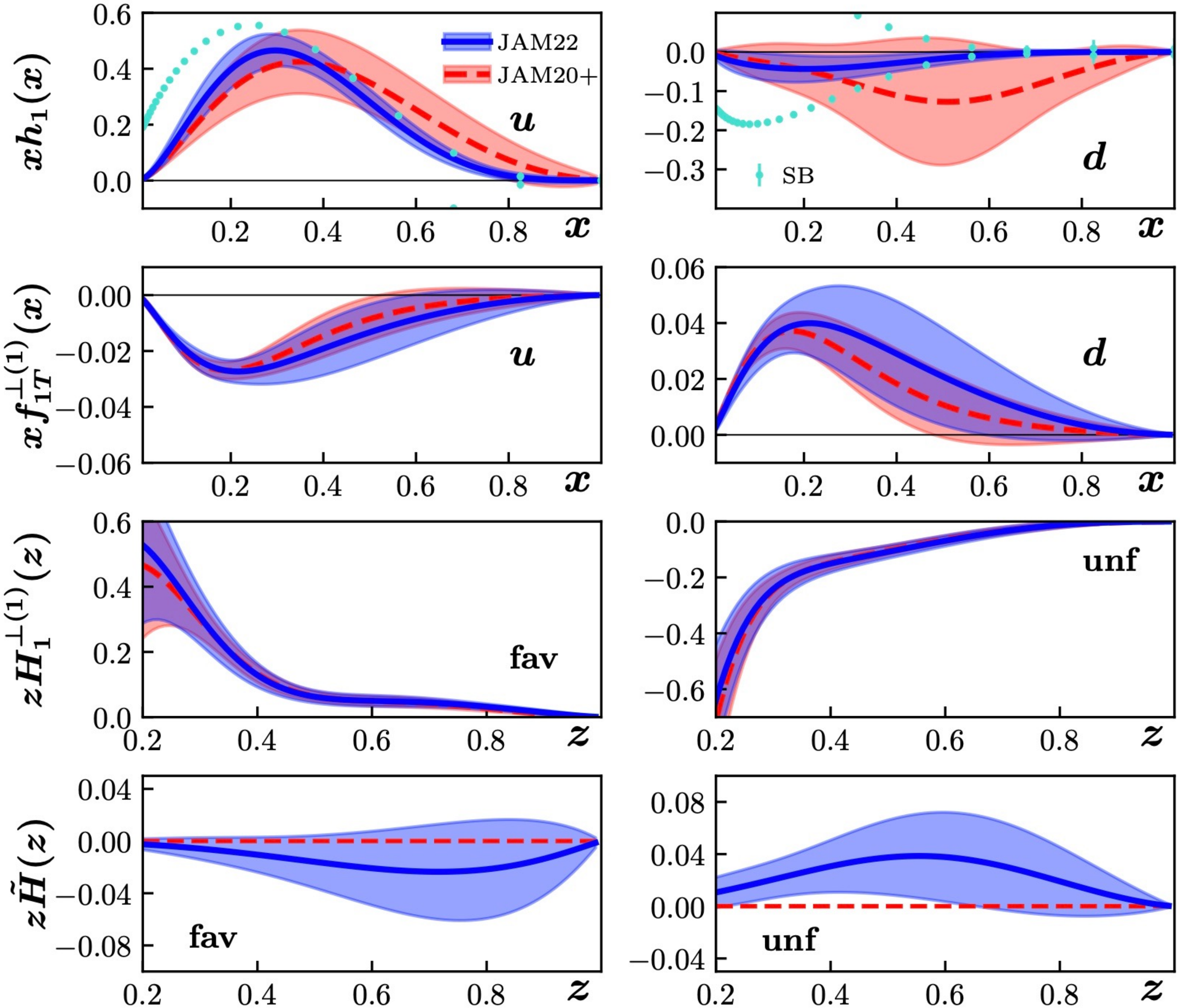
  *C. Alexandrou et al, Phys.Rev.D 102 (2020)*

- Imposing the Soffer bound on transversity (as a Bayesian prior) $|h_1^q(x)| \leq \frac{1}{2}(f_1^q(x) + g_1^q(x))$

  *J. Soffer, Phys.Rev.Lett. 74 (1995)*

# UNIVERSAL GLOBAL ANALYSIS 2022

*JAM22: Gamberg, Malda, Miller, Pitonyak, Prokudin, Sato, arXiv:2205.00999*




Transversity

$h_1(x)$

Sivers

$f_{1T}^{\perp(1)}(x)$

Collins FF

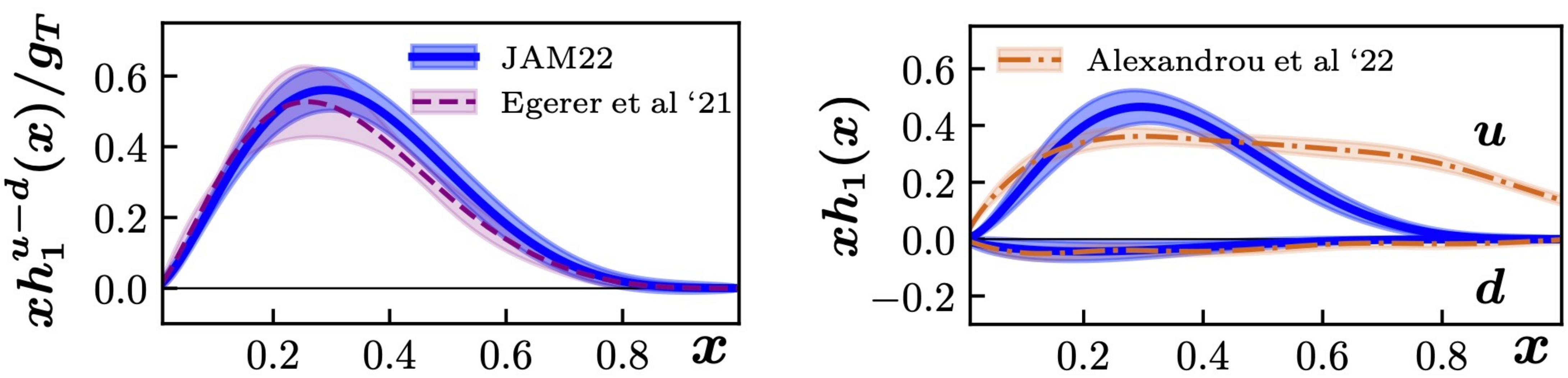
$H_1^{\perp(1)}(z)$

Twist-3 FF

$\tilde{H}(z)$

# JAM22: TRANSVERSITY AND LATTICE

*JAM22: Gamberg, Malda, Miller, Pitonyak, Prokudin, Sato, arXiv:2205.00999*




The raw lattice data for Egerer, et al. and Alexandrou, et al. are compatible, but the former uses pseudo-PDFs and the latter quasi-PDFs

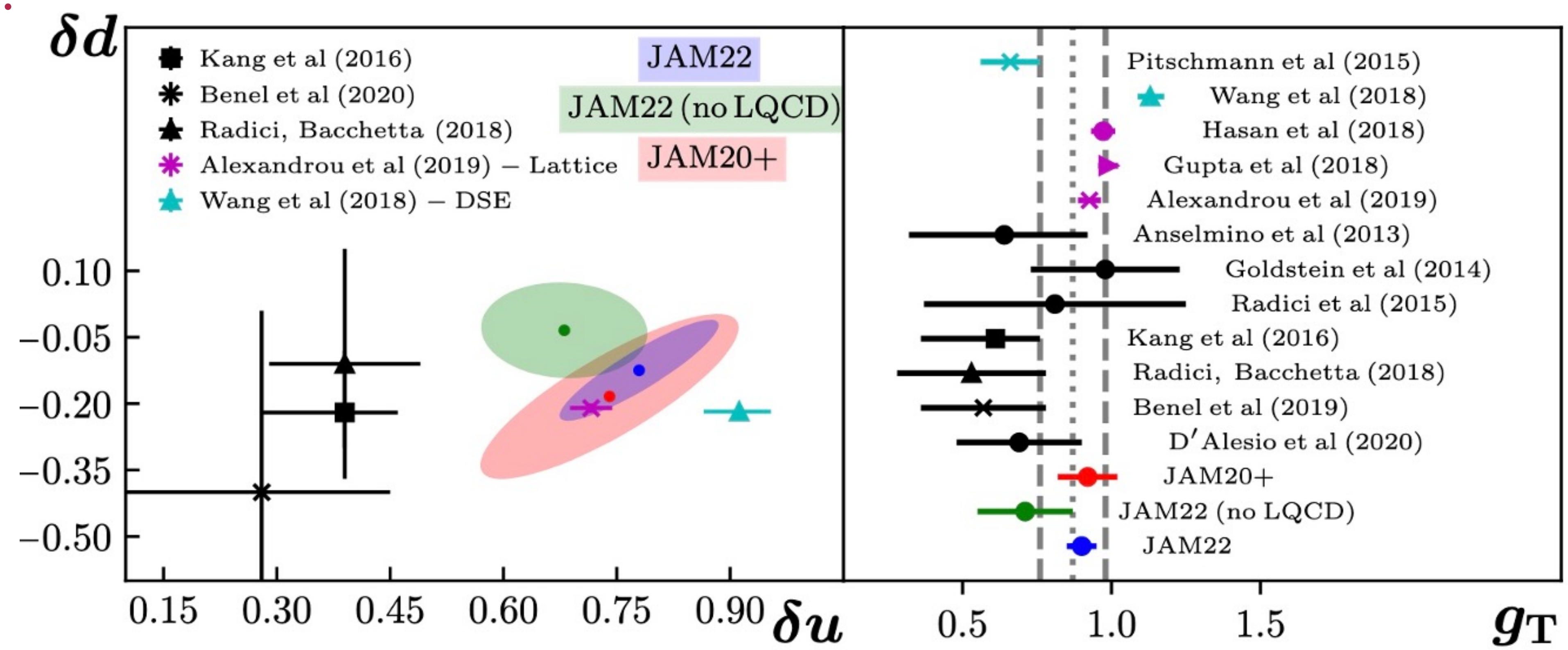
The behavior at large x for the up quark in Alexandrou, et al. is due to systematics in the reconstruction of the x dependence in the quasi-PDF approach

We find good agreement with lattice calculations of transversity

Now that the lattice $g_T$ data point is included in JAM3D-22, the uncertainties in the phenomenological extraction of transversity are compatible with lattice

# UNIVERSAL GLOBAL ANALYSIS 2022

*JAM22: Gamberg, Malda, Miller, Pitonyak, Prokudin, Sato, Phys.Rev.D 106 (2022) 3, 034014*




- Tensor charge from up and down quarks and $g_T = \delta u - \delta d$ are well constrained and compatible with both lattice results and the Soffer bound

**$\delta u$ and $\delta d$ $Q^2$=4 GeV$^2$**

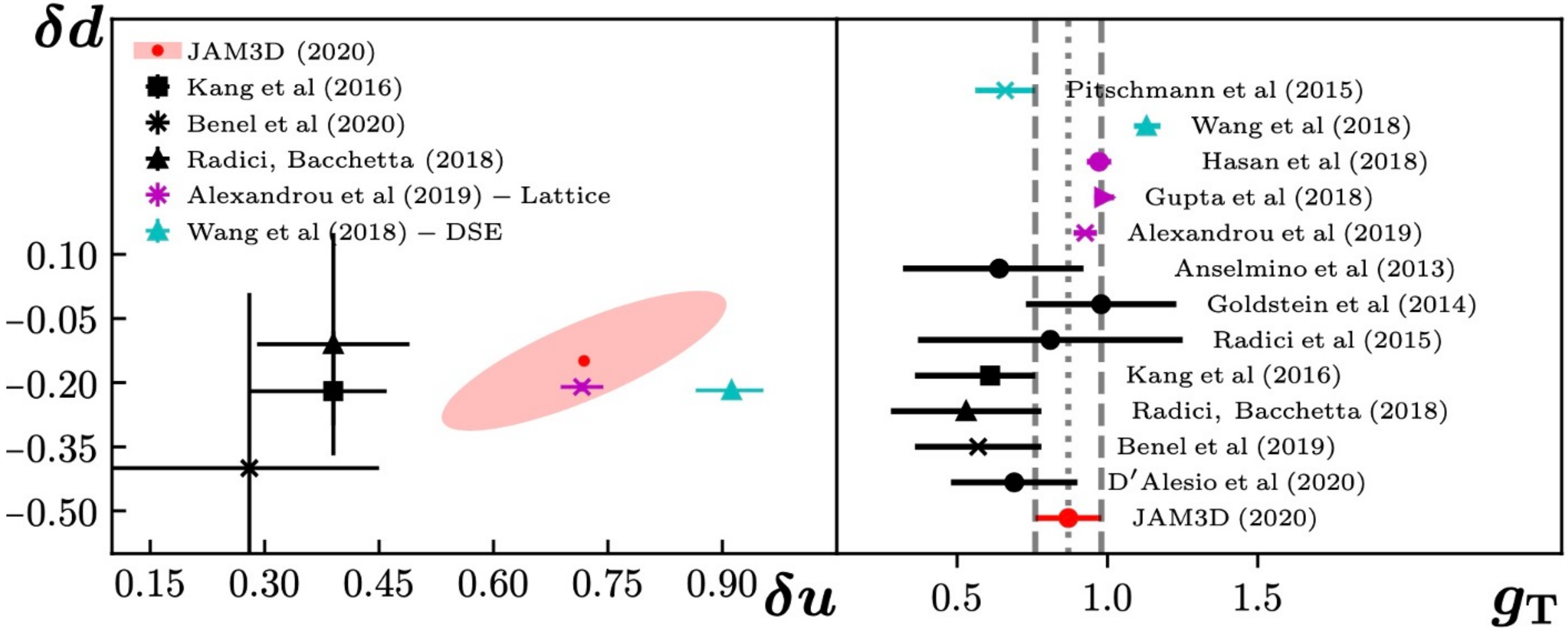
**$\delta u$= 0.74 $\pm$ 0.11**

**$\delta d$= -0.15 $\pm$ 0.12**

**$g_T$= 0.89 $\pm$ 0.06**

- Once the the lattice $g_T$ data point is included, we find the non-perturbative functions can accommodate it and still describe the experimental data well

# TRANSVERSE SPIN PUZZLE?




- Dihadron analyses (e.g., Benel, et al. (2020); Radici, Bacchetta (2018)), along with TMD fits that only include e+e- and SIDIS Collins effect data (e.g., Kang, et al. (2016)), are generally below the lattice values for $g_T$ and $\delta u$
- Data, theory, phenomenology?

*JAMDIFF23: C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2308.14857(2023)*
*C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2306.12998(2023)*

# JAMDIFF23 ANALYSIS

# DIHADRON STUDIES

| | $e^+e^-$ $d\sigma/dzdM_h$ | $e^+e^-$ Artru-Collins | SIDIS $\sin(\varphi_R+\varphi_S)$ | Proton-proton $\sin(\varphi_R-\varphi_S)$ | Lattice tensor charge(s) | Soffer bound |
|---|---|---|---|---|---|---|
| Radici, Bacchetta (2018) | ✓* PYTHIA | ✓* | ✓ | ✓ | X | ✓ |
| Benel, Courtoy, Ferro-Hernandez (2020) | ✓* PYTHIA | ✓* | ✓ | X | X | ✓^ |
| Cocuzza, et al. (2023) JAMDiFF-23 | ✓ | ✓ | ✓ | ✓ | ✓ $\delta u, \delta d$ | ✓^ |

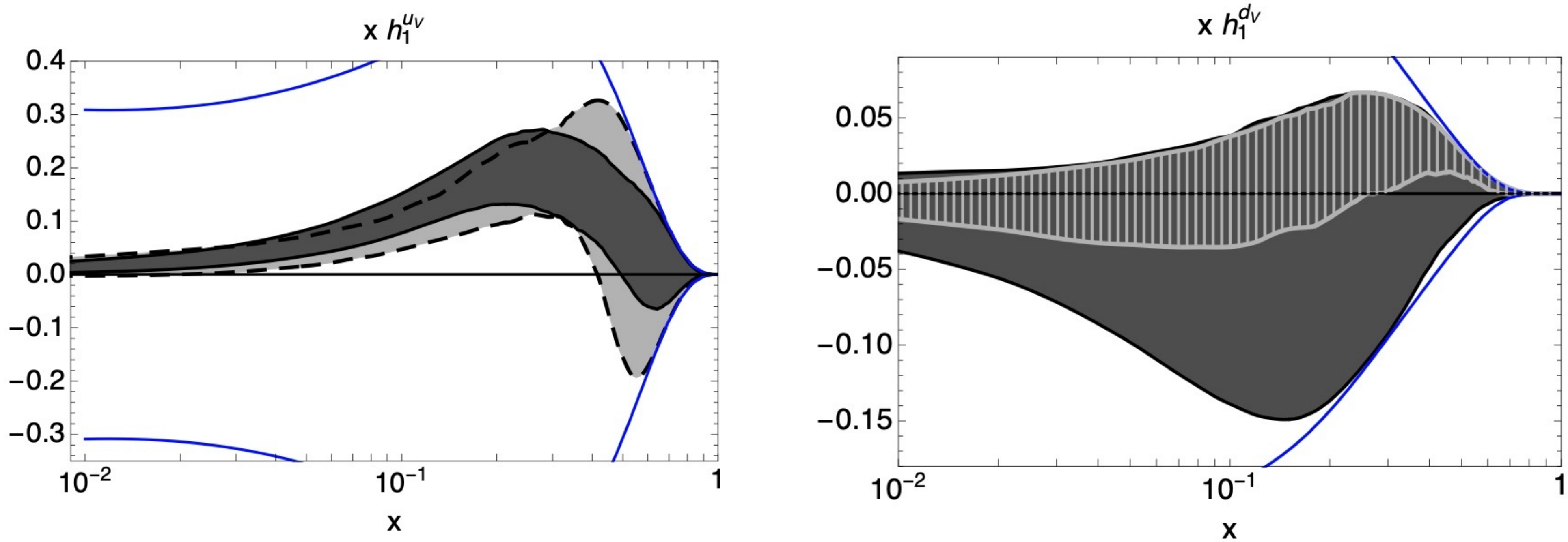
* $D_1(z,M_h)$ and $H_1^{\sphericalangle}(z,M_h)$ were fit in a separate analysis and then fixed when extracting $h_1(x)$

^ Imposed the SB but allowed for violations given the uncertainties in $f_1(x)$ and $g_1(x)$

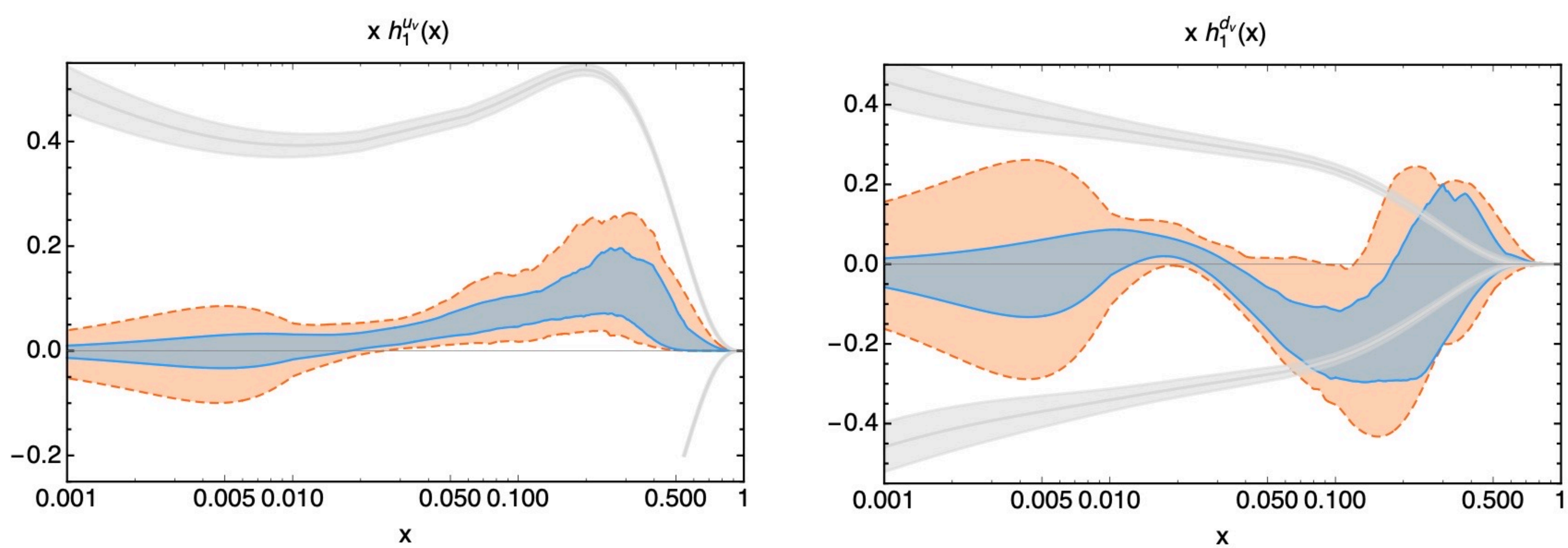
*Slide courtesy of Daniel Pitonyak*

# DIHADRON STUDIES

Radici, Bacchetta (2018)




Benel, et al. (2020)



JAMDIFF23: C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2308.14857(2023)
C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2306.12998(2023)

# JAMDIFF23 SETUP

- SIA cross section, Belle (2017), $\pi^+\pi^-$ 1121 points

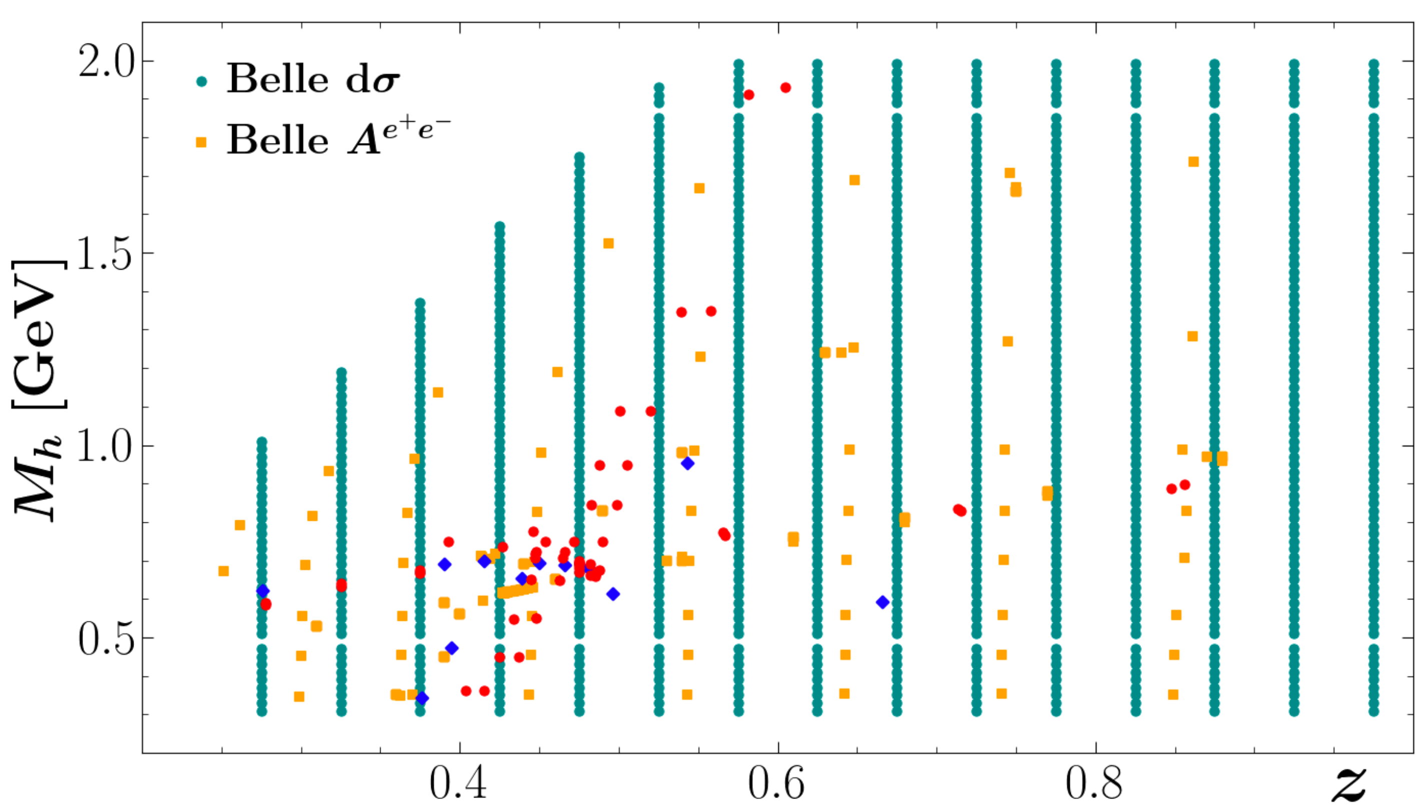
  *R. Seidl et al., Phys. Rev. D 96, no. 3, 032005 (2017)*

$$\frac{d\sigma}{dz\, dM_h} = \frac{4\pi\alpha_{em}^2}{s} \sum_q e_q^2 D_1^q(z, M_h)$$

- SIA Artru-Collins, Belle (2011), $\pi^+\pi^-$ 183 points

  *A. Vossen et al., Phys. Rev. Lett. 107, 072004 (2011)*

$$A^{e^+e^-}(z, M_h, \bar{z}, \overline{M}_h) = \frac{\sin^2\theta \sum_q e_q^2 H_1^{\sphericalangle,q}(z, M_h) H_1^{\sphericalangle,\bar{q}}(\bar{z}, \overline{M}_h)}{(1+\cos^2\theta)\sum_q e_q^2 D_1^q(z, M_h) D_1^{\bar{q}}(\bar{z}, \overline{M}_h)}$$




$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}},$$

$$D_1^s = D_1^{\bar{s}}, \quad D_1^c = D_1^{\bar{c}}, \quad D_1^b = D_1^{\bar{b}},$$

5 independent functions (w/ $D_1^g$)
[supplement with PYTHIA data]
for $\sigma^q/\sigma^{tot}$ for $q = s, c, b$

$$\sqrt{s} = [10.58, 30.73, 50.88, 71.04, 91.19]\ \text{GeV}$$

$$H_1^{\sphericalangle,u} = -H_1^{\sphericalangle,d} = -H_1^{\sphericalangle,\bar{u}} = H_1^{\sphericalangle,\bar{d}},$$

$$H_1^{\sphericalangle,s} = -H_1^{\sphericalangle,\bar{s}} = H_1^{\sphericalangle,c} = -H_1^{\sphericalangle,\bar{c}} = 0,$$

1 independent function

*A. Courtoy et al., Phys. Rev. D 85, 114023 (2012)*

Courtesy of Chris Cocuzza

# JAMDIFF23 SETUP

*JAMDIFF23: C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2308.14857(2023)*
*C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2306.12998(2023)*

- SIDIS (p,d) SOMPASS, HERMES, 64 points

*C. Adolph et al.,Phys. Lett. B 713, 10-16 (2012)*
*A. Airapetian et al., JHEP 06, 017 (2008)*

$$A_{UT}^{\mathrm{SIDIS}} = c(y) \frac{\sum_q e_q^2\, h_1^q(x)\, H_1^{\sphericalangle,q}(z, M_h)}{\sum_q e_q^2\, f_1^q(x)\, D_1^q(z, M_h)}$$

3 independent observables
3 independent functions

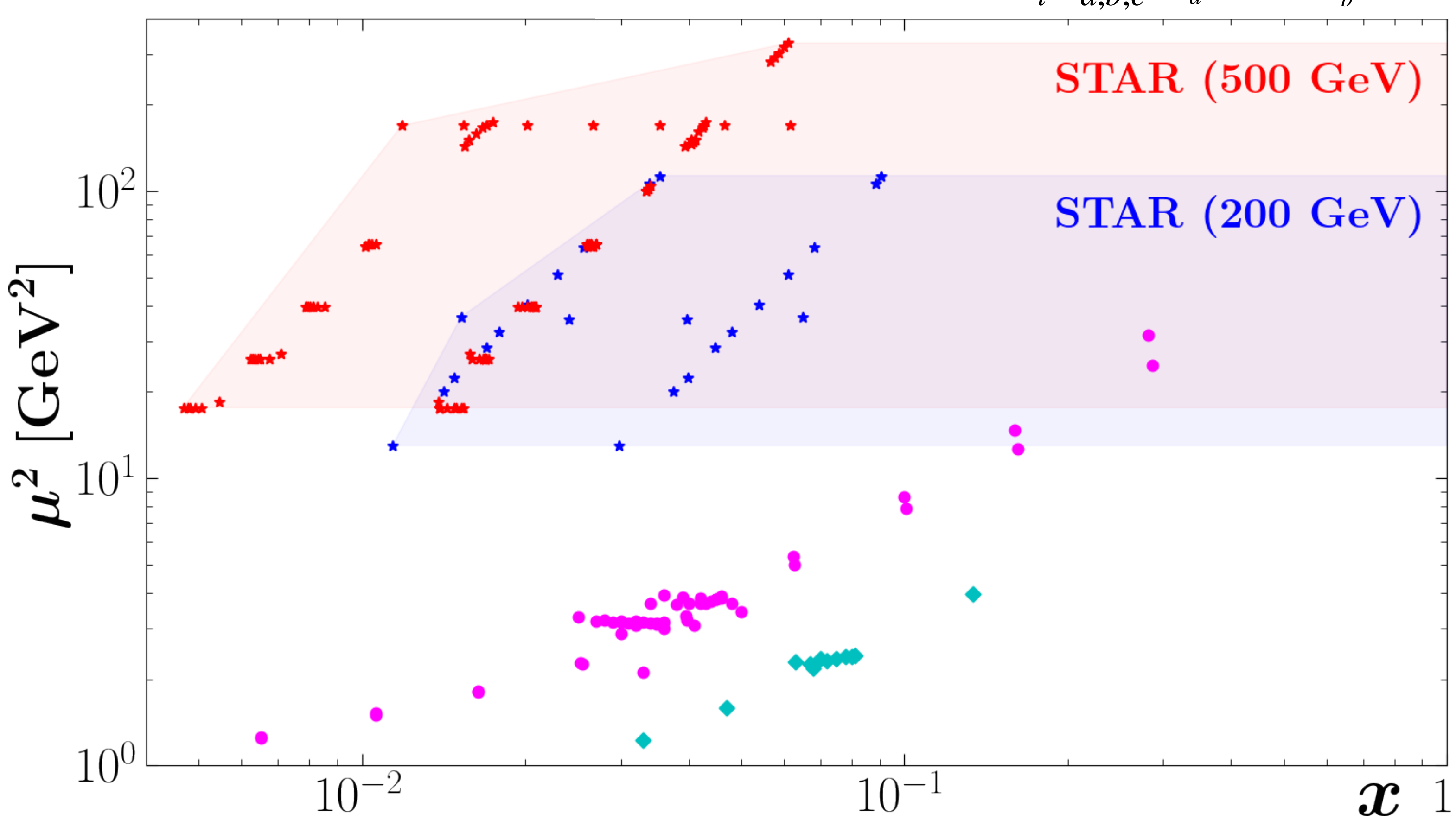
- Proton-proton, STAR, 269 points

*L. Adamczyk et al., Phys. Rev. Lett. 115, 242501 (2015)*

$$A_{UT}^{pp} = \frac{\mathscr{H}(M_h, P_{hT}, \eta)}{\mathscr{D}(M_h, P_{hT}, \eta)}$$

$$\mathscr{H}(M_h, P_{hT}, \eta) = 2P_{hT} \sum_i \sum_{a,b,c} \int_{x_a^{\min}}^1 \mathrm{d}x_a \int_{x_b^{\min}}^1 \frac{\mathrm{d}x_b}{z} f_1^a(x_a)\, h_1^b(x_b) \frac{\mathrm{d}\Delta\hat{\sigma}_{ab^\uparrow \to c^\uparrow d}}{\mathrm{d}\hat{t}} H_1^{\sphericalangle,c}(z, M_h)$$

$$\mathscr{D}(M_h, P_{hT}, \eta) = 2P_{hT} \sum_i \sum_{a,b,c} \int_{x_a^{\min}}^1 \mathrm{d}x_a \int_{x_b^{\min}}^1 \frac{\mathrm{d}x_b}{z} f_1^a(x_a)\, f_1^b(x_b) \frac{\mathrm{d}\hat{\sigma}_{ab \to cd}}{\mathrm{d}\hat{t}} D_1^c(z, M_h)$$




$h_1^{u_v}$

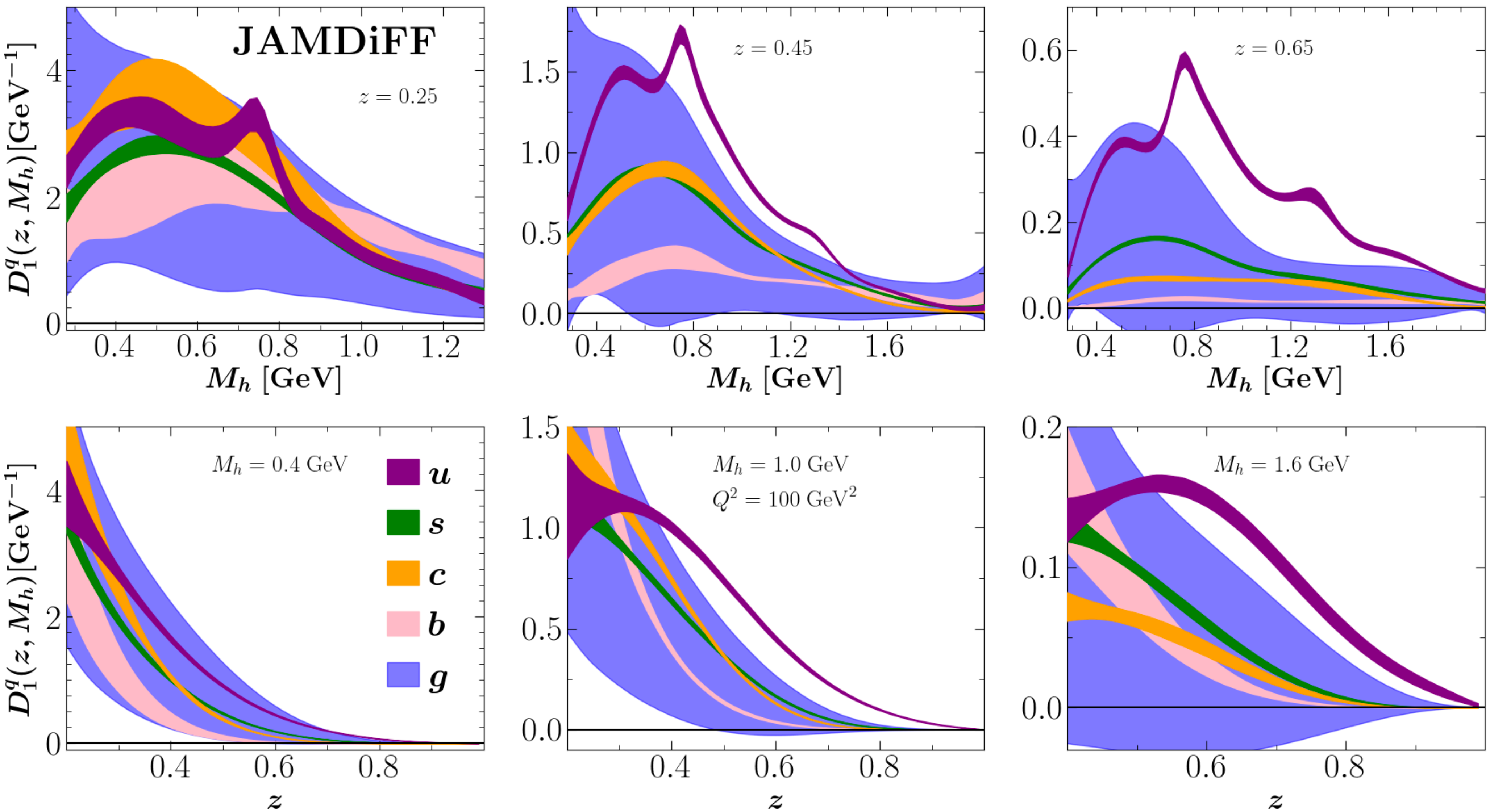
$h_1^{d_v}$

$h_1^{\bar{u}} = -\, h_1^{\bar{d}}$

*Courtesy of Chris Cocuzza*

*JAMDIFF23: C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2308.14857(2023)*
*C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2306.12998(2023)*

# EXTRACTED DIFFS




<u>Bound:</u> $D_1^q > 0$

*A. Bacchetta and M. Radici, Phys. Rev. D 67, 094002 (2003)*

- Due to the resonance structure we use a flexible spline parametrization on a grid $\mathbf{M}_h^u = [2m_\pi, 0.40, 0.50, 0.70, 0.75, 0.80, 0.90, 1.00, 1.20, 1.30, 1.40, 1.60, 1.80, 2.00]$ GeV
- Each point is interpolated

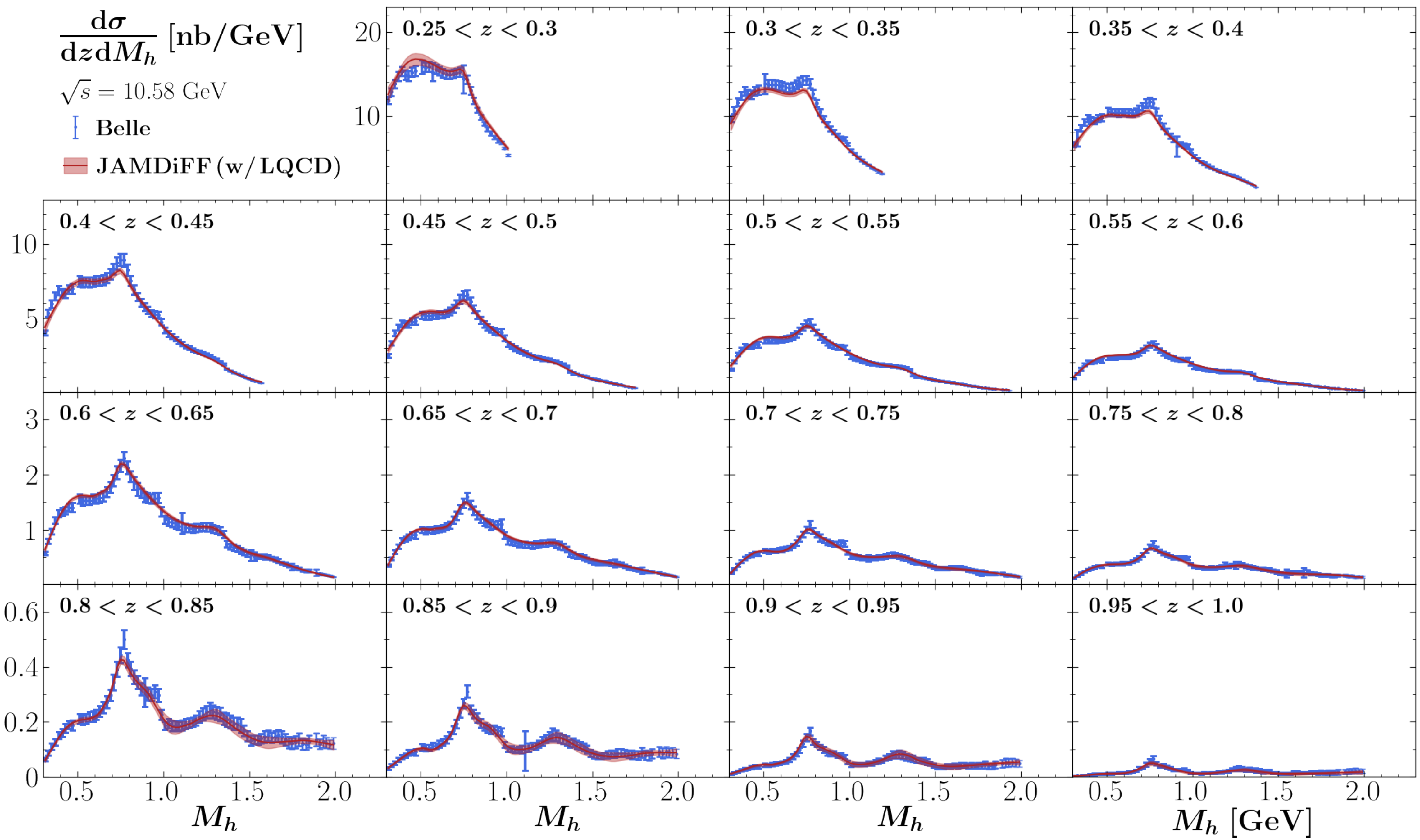
$$D_1^u(z, \mathbf{M}_h^{u,i}) = \sum_{j=1,2,3} \frac{N_{ij}^u z^{\alpha_{ij}^u}(1-z)^{\beta_{ij}^u}}{\mathrm{B}[\alpha_{ij}^u + 1, \beta_{ij}^u + 1]}$$

$\rightarrow$ 204 parameters for $D_1$ and 48 parameters for $H_1^{\sphericalangle}$

*JAMDIFF23: C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2308.14857(2023)*
*C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2306.12998(2023)*

# QUALITY OF THE FIT

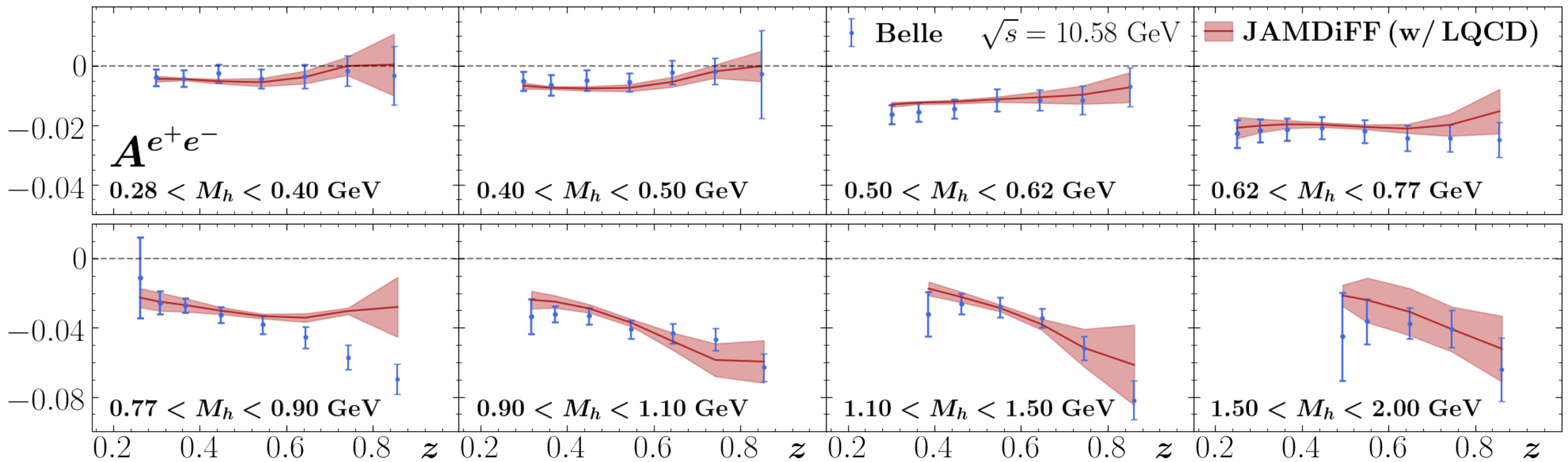
Data: *R. Seidl et al., Phys. Rev. D 96, no. 3, 032005 (2017)*



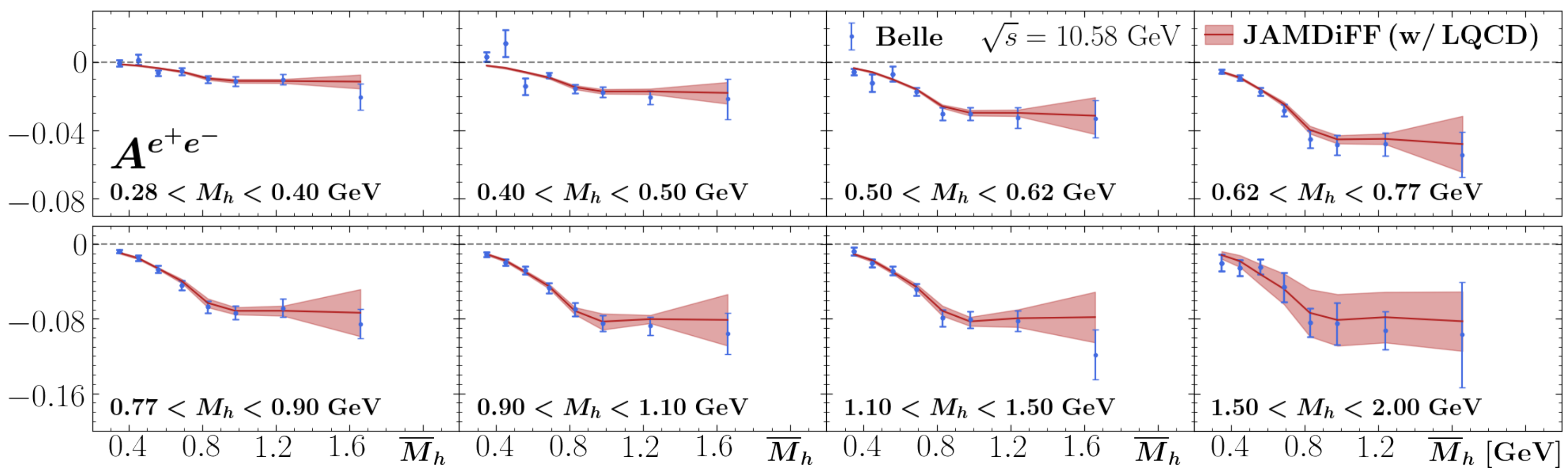
JAMDIFF23: C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2308.14857(2023)
C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2306.12998(2023)

# QUALITY OF THE FIT

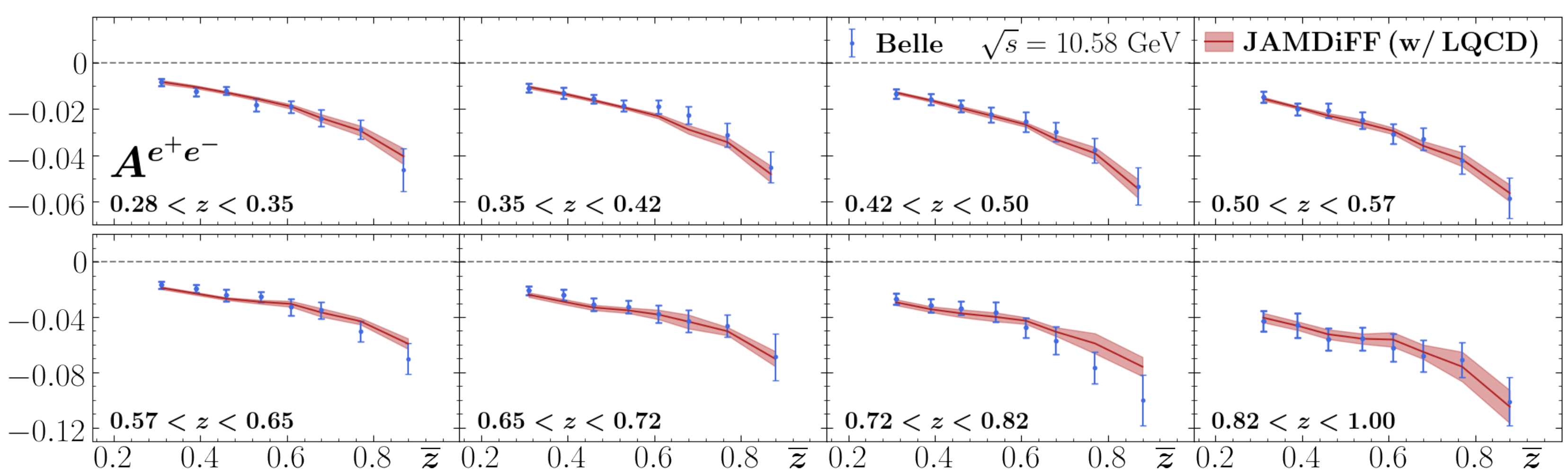
Data: *A. Vossen et al., Phys. Rev. Lett. 107, 072004 (2011)*




bins in $z$



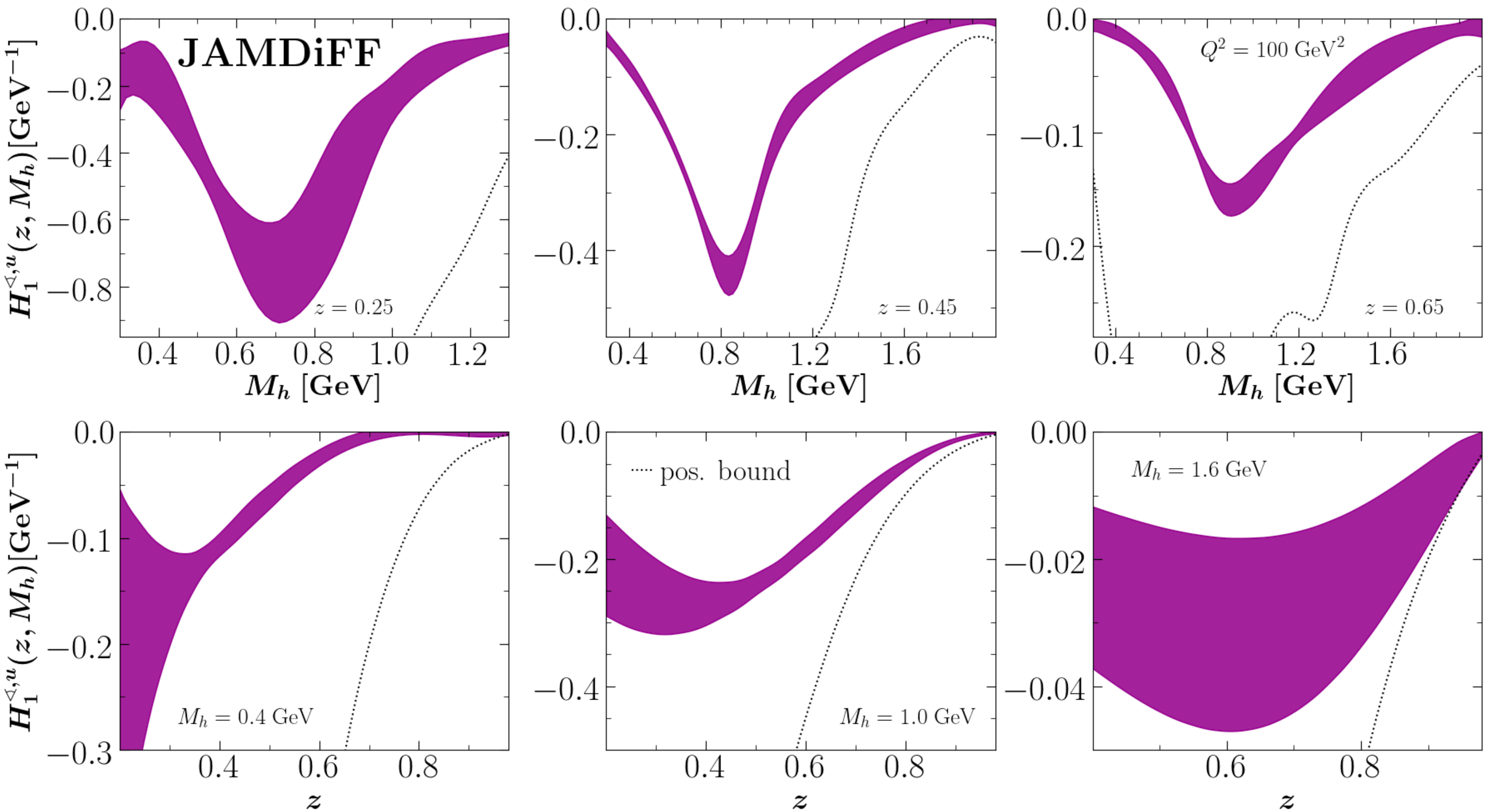

bins in $M_h, \bar{M}_h$




bins in $\bar{z}$

*JAMDIFF23: C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2308.14857(2023)*
*C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2306.12998(2023)*

# EXTRACTED DIFFS



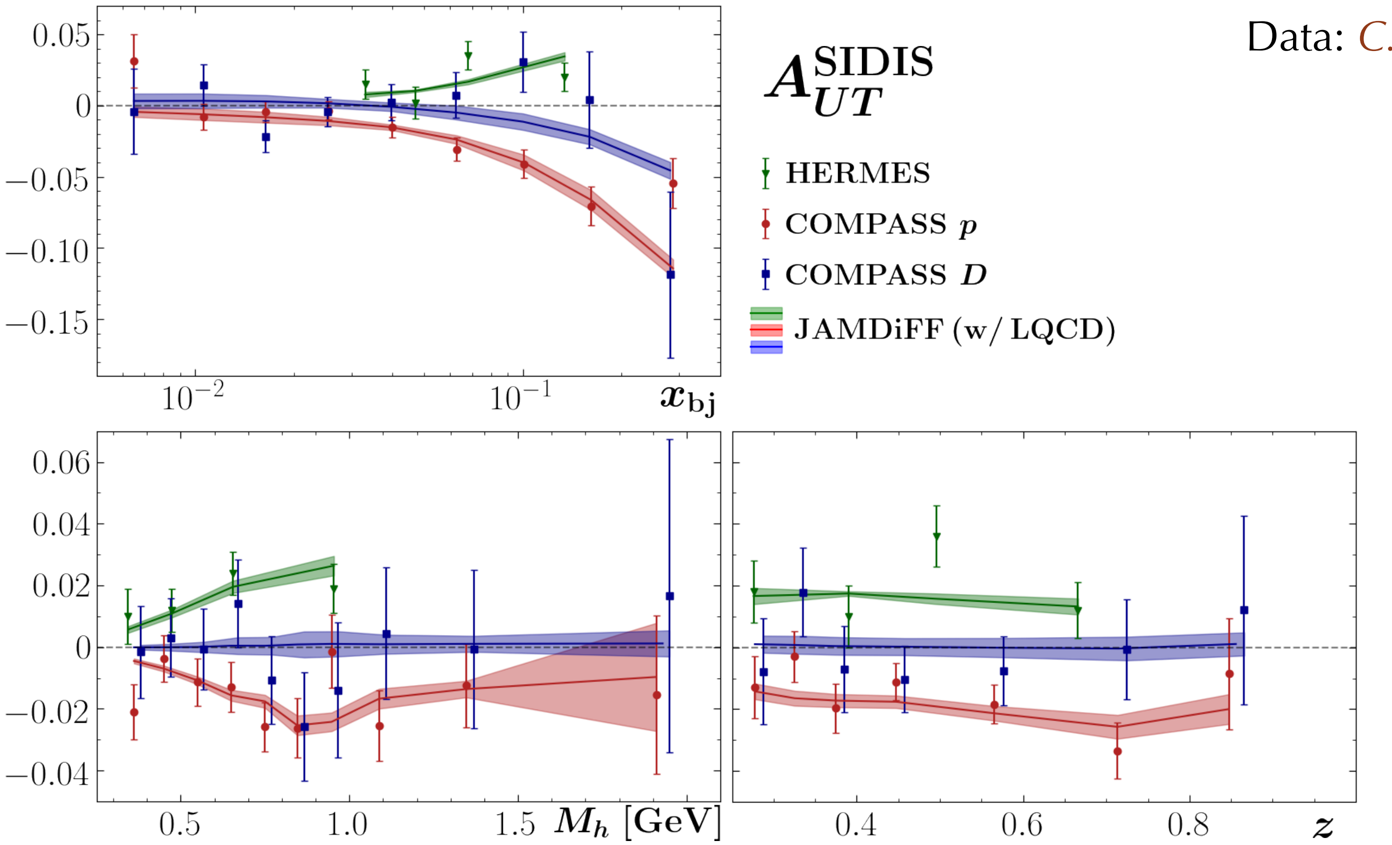

<u>Bound:</u>

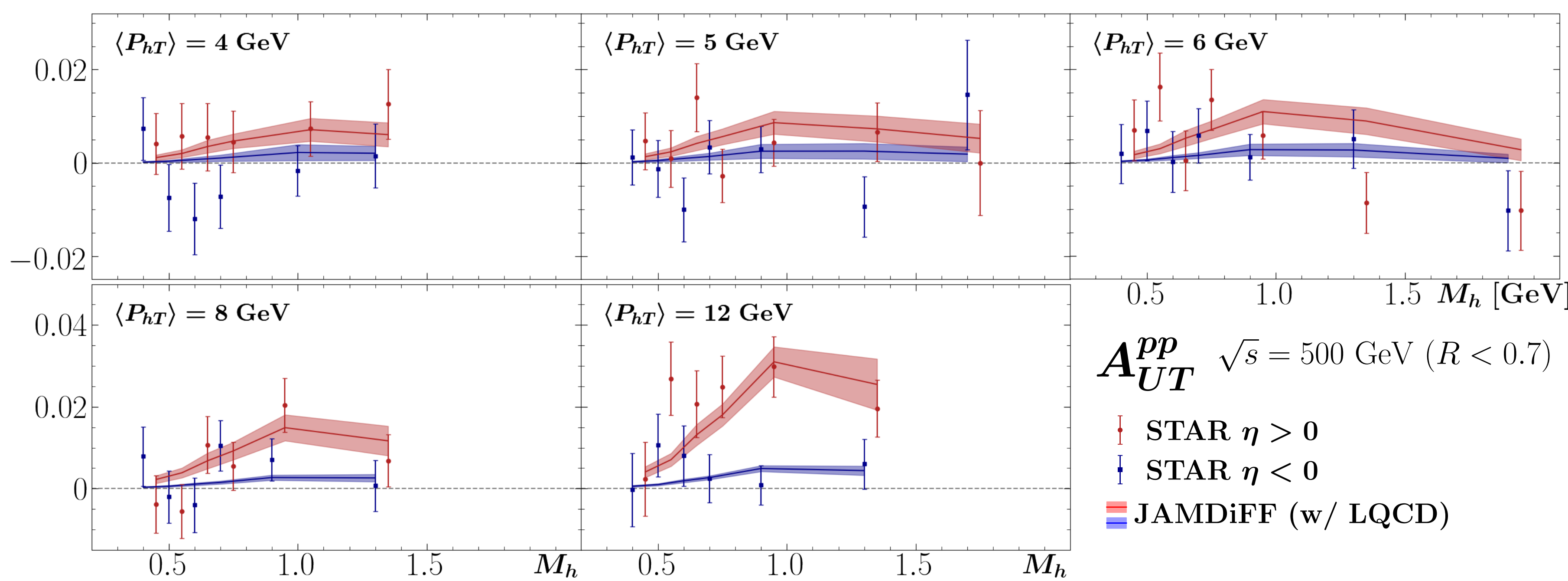
$$|H_1^{\sphericalangle,q}| < D_1^q$$

*A. Bacchetta and M. Radici, Phys. Rev. D 67, 094002 (2003)*

# QUALITY OF FIT SIDIS AND PP

Data: *C. Adolph et al., (COMPASS) Phys. Lett. B 713, 10-16 (2012)*
*A. Airapetian et al. (HERMES), JHEP 06, 017 (2008)*



# TRANSVERSITY

- We use the following parametrization for transversity PDFs $u_v$ , $d_v$ , and $\bar{u} = -\bar{d}$ (from large-$N_c$ limit (Pobylitsa (2003))) and impose the Soffer bound $|h_1^i(x;\mu)| \leq \frac{1}{2}\left[f_1^i(x;\mu) + g_1^i(x;\mu)\right]$

$$h_1^i(x) = \frac{N^i}{\mathcal{M}^i} x^{\alpha^i} (1-x)^{\beta^i} (1 + \gamma^i \sqrt{x} + \delta^i x)$$

$$\mathcal{M}^i = \mathrm{B}[\alpha^i + 1, \beta^i + 1] + \gamma^i \mathrm{B}[\alpha^i + \tfrac{3}{2}, \beta^i + 1] + \delta^i \mathrm{B}[\alpha^i + 2, \beta^i + 1]$$

$\rightarrow$ 15 parameters for $h_1$

- We include small-x constraint *Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D 99, 054033 (2019)*

$$\alpha^i \xrightarrow{x \to 0} 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}} \qquad \alpha = 0.170 \pm 0.085$$

- Perform the analysis with and without LQCD data for the tensor charges δu,δd from ETMC (Alexandrou, et al. (2019)) and PNDME (Gupta, et al. (2018)) (physical pion mass and 2+1+1 flavors)

JAMDIFF23: C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, R. Seidl e-Print: 2308.14857(2023)
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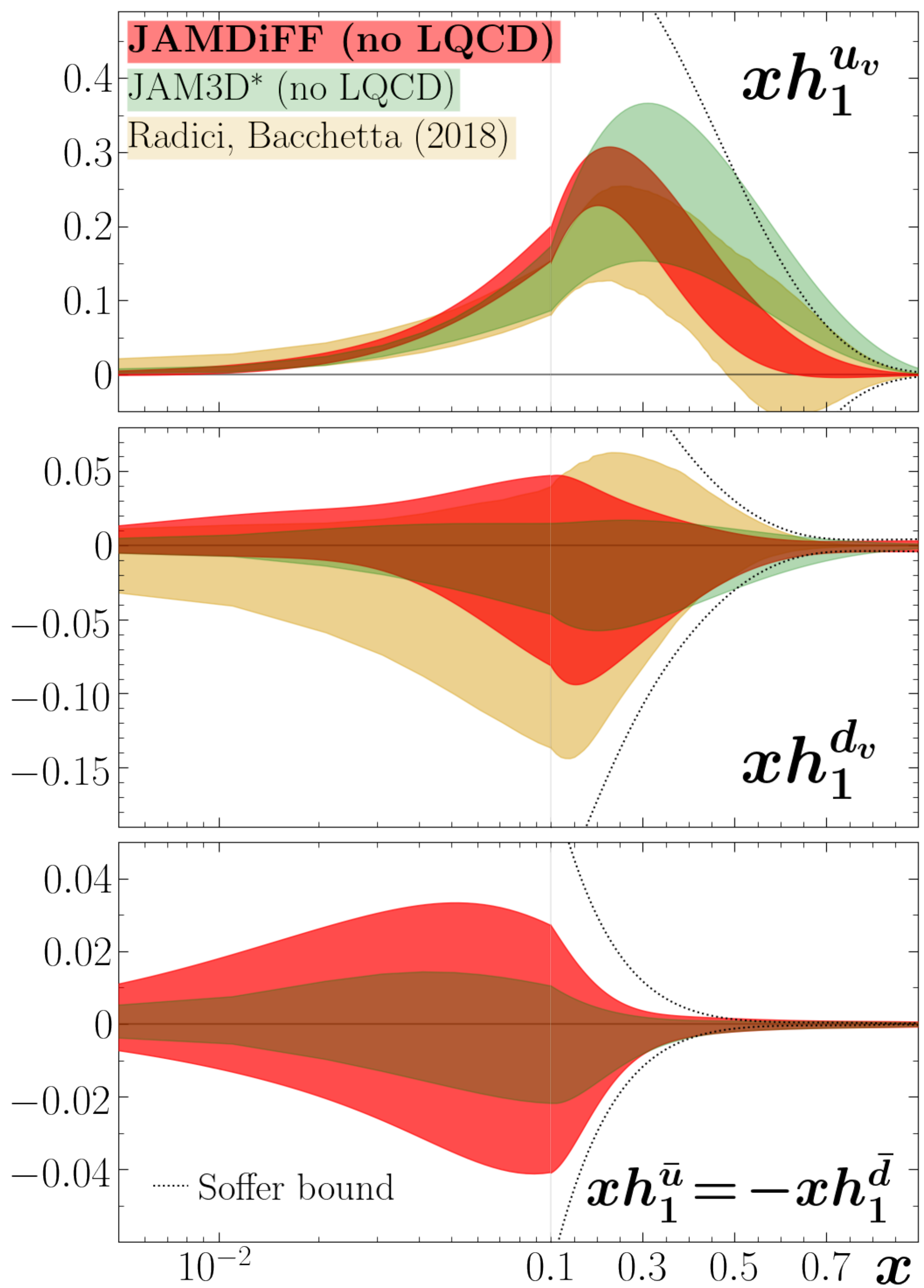
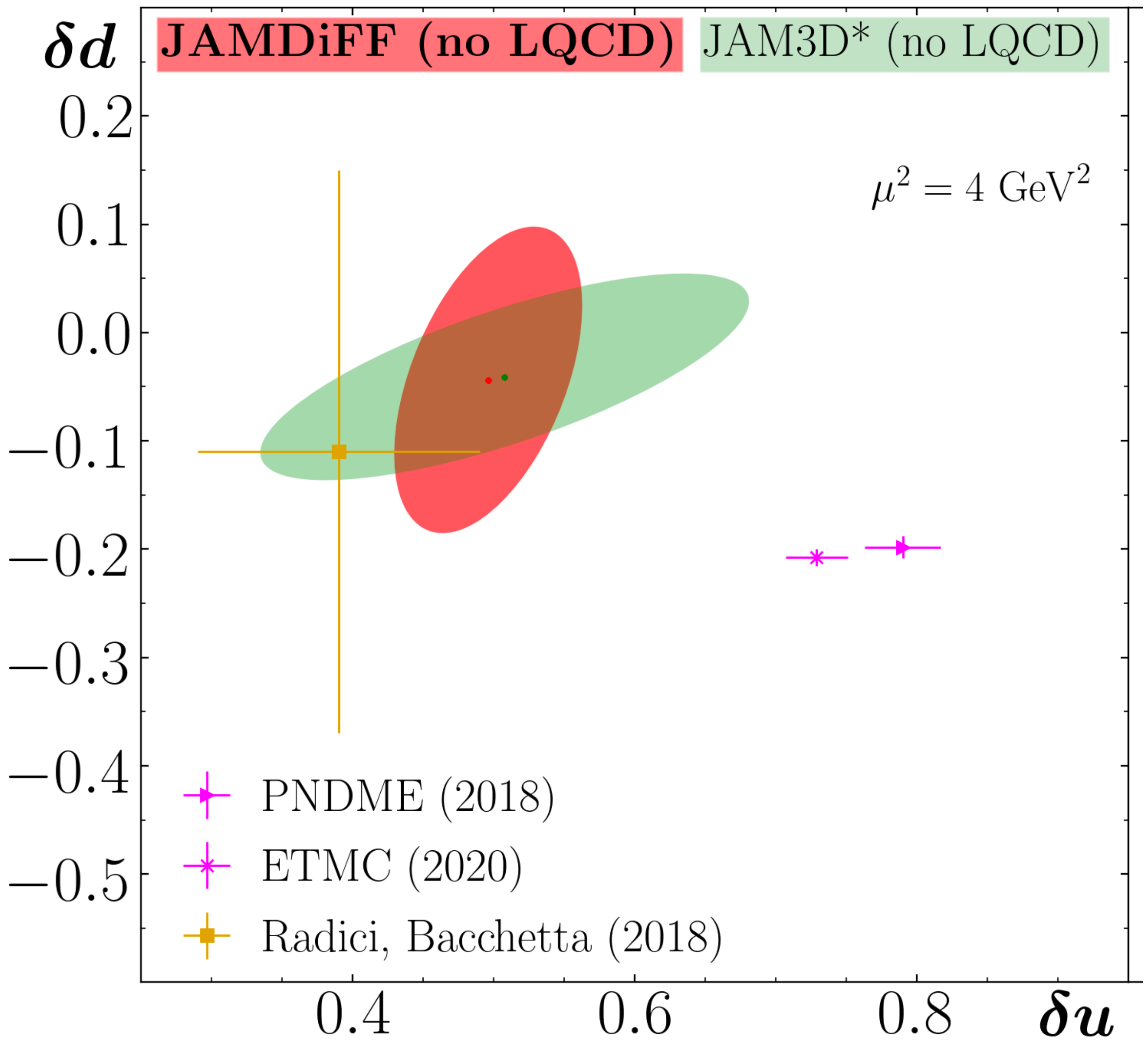
# TRANSVERSITY AND TENSOR CHARGE




- ➤ JAMDiFF (no LQCD) finds agreement with Radici, Bacchetta (2018) with a slightly larger $u_v$ function at larger x
- ➤ JAM3D* = JAM22 (no LQCD) + antiquarks w/ $\bar{u} = -\bar{d}$ + small-x constraint
- ➤ JAMDiFF agrees with JAM3D*
- ➤ Agreement between all three analyses within errors

# TRANSVERSITY AND TENSOR CHARGE



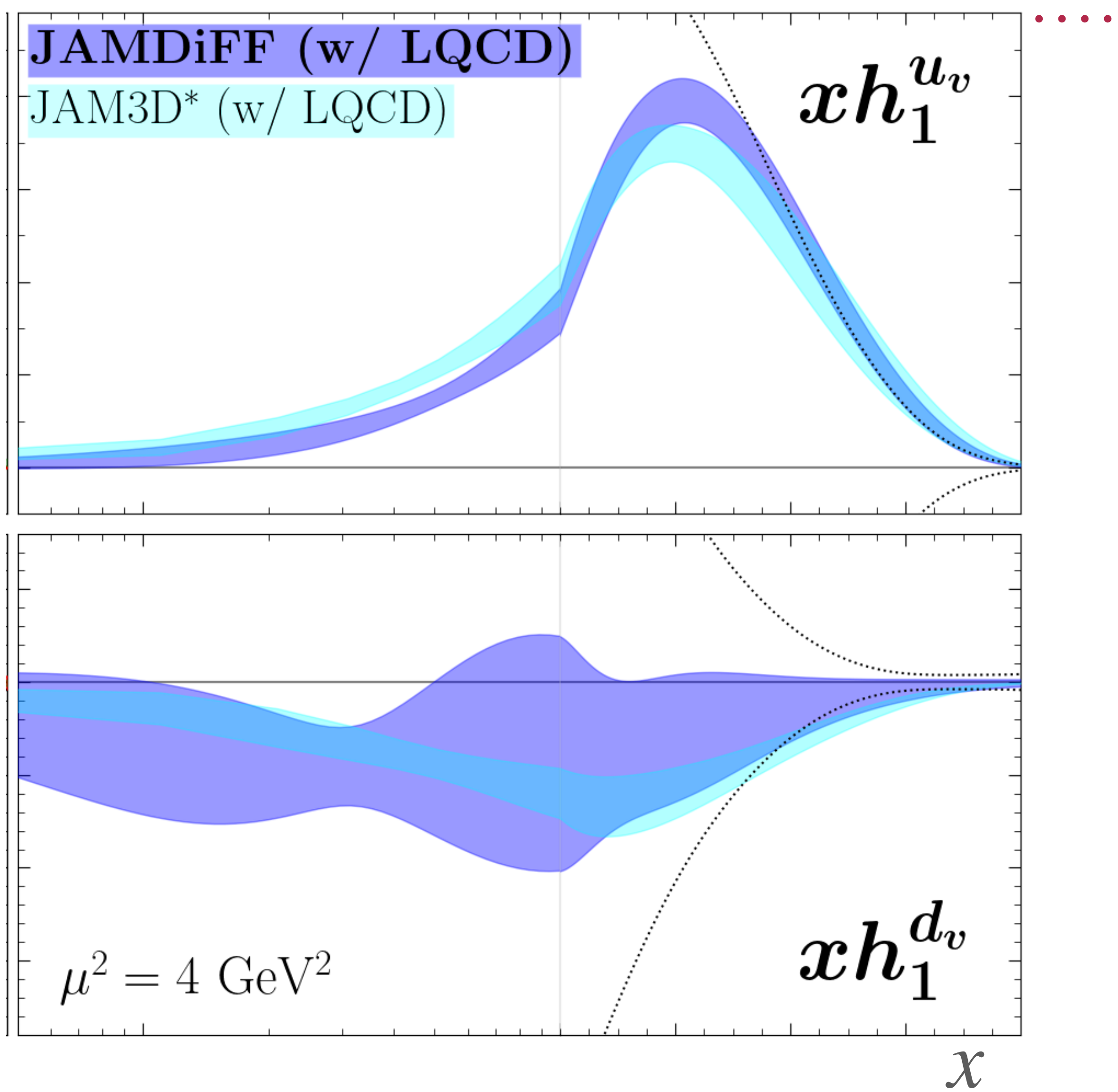

- Agreement between all three analyses within errors
- JAMDiFF and JAM3D*result in larger δu
- Before drawing a conclusion about the compatibility between LQCD tensor charges and experimental data, one needs first to include both in the analysis. One should only be concerned if the description of the lattice data remains poor even after its inclusion and/or if the description of the experimental data suffers significantly.
- NNPDF methodology was used to verify the compatibility of results

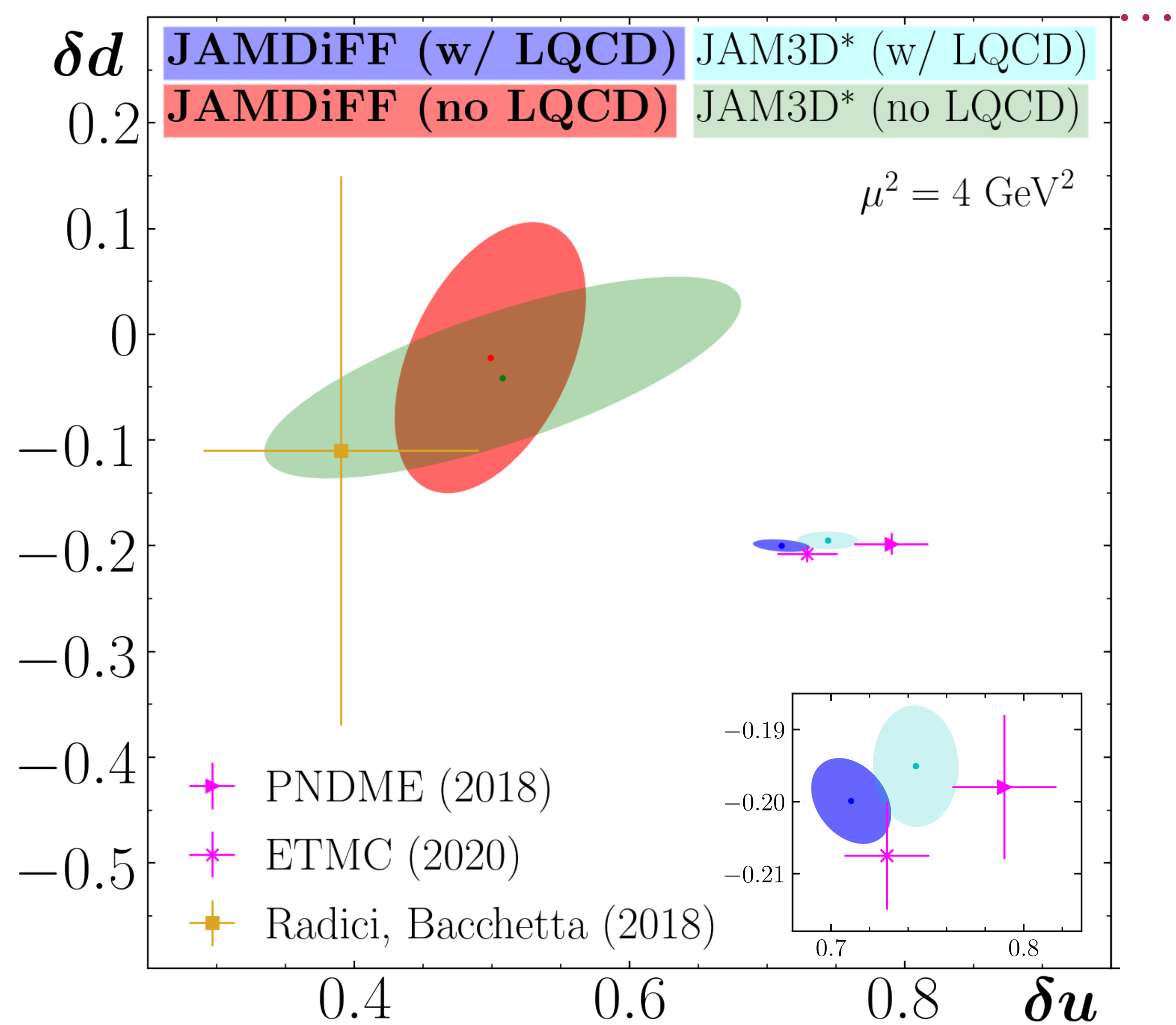
*R. D. Ball et al. (NNPDF), Eur. Phys. J. C 82, 428 (2022)*

# TRANSVERSITY AND TENSOR CHARGE

| Experiment | Binning | $N_{\text{dat}}$ | $\chi^2_{\text{red}}$ JAMDiFF (w/ LQCD) | (no LQCD) | (SIDIS only) |
|---|---|---|---|---|---|
| Belle (cross section)[64] | $z, M_h$ | 1094 | 1.01 | 1.01 | 1.01 |
| Belle (Artru-Collins) [112] | $z, M_h$ | 55 | 1.27 | 1.24 | 1.28 |
| | $M_h, \overline{M}_h$ | 64 | 0.60 | 0.60 | 0.60 |
| | $z, \bar{z}$ | 64 | 0.42 | 0.42 | 0.41 |
| HERMES [118] | $x_{\text{bj}}$ | 4 | 1.77 | 1.70 | 1.67 |
| | $M_h$ | 4 | 0.41 | 0.42 | 0.47 |
| | $z$ | 4 | 1.20 | 1.17 | 1.13 |
| COMPASS ($p$) [117] | $x_{\text{bj}}$ | 9 | 1.98 | 0.65 | 0.59 |
| | $M_h$ | 10 | 0.92 | 0.94 | 0.93 |
| | $z$ | 7 | 0.77 | 0.60 | 0.63 |
| COMPASS ($D$) [117] | $x_{\text{bj}}$ | 9 | 1.37 | 1.42 | 1.22 |
| | $M_h$ | 10 | 0.45 | 0.37 | 0.38 |
| | $z$ | 7 | 0.50 | 0.46 | 0.46 |
| STAR [121] $\sqrt{s} = 200$ GeV $R < 0.3$ | $M_h, \eta < 0$ | 5 | 2.57 | 2.56 | — |
| | $M_h, \eta > 0$ | 5 | 1.34 | 1.55 | — |
| | $P_{hT}, \eta < 0$ | 5 | 0.98 | 1.00 | — |
| | $P_{hT}, \eta > 0$ | 5 | 1.73 | 1.74 | — |
| | $\eta$ | 4 | 0.52 | 1.46 | — |
| STAR [97] $\sqrt{s} = 500$ GeV $R < 0.7$ | $M_h, \eta < 0$ | 32 | 1.30 | 1.10 | — |
| | $M_h, \eta > 0$ | 32 | 0.81 | 0.78 | — |
| | $P_{hT}, \eta > 0$ | 35 | 1.09 | 1.07 | — |
| | $\eta$ | 7 | 2.97 | 1.83 | — |
| ETMC $\delta u$ [77] | — | 1 | 0.71 | — | — |
| ETMC $\delta d$ [77] | — | 1 | 1.02 | — | — |
| PNDME $\delta u$ [71] | — | 1 | 8.68 | — | — |
| PNDME $\delta d$ [71] | — | 1 | 0.04 | — | — |
| **Total $\chi^2_{\text{red}}$** ($N_{\text{dat}}$) | | | **1.01** (1475) | **0.98** (1471) | 0.96 (1341) |

# TENSOR CHARGE




- The experimental measurements are sensitive to the x-dependence of the transversity PDFs, not the full moment like the lattice data (EIC and JLab are needed)
- JAM3D* and JAMDiFF agree on the x-dependence of transversity (nontrivial since the lattice data only constrains the full moment of the transversity PDFs )
- JAM3D* and JAMDiFF can successfully include lattice QCD data on the tensor charges in the analyses, thus showing for the first time the universal nature of all available information on transversity and the tensor charges of the nucleon

# IMPACT OF THE EIC

# GENERATED EIC PSEUDODATA

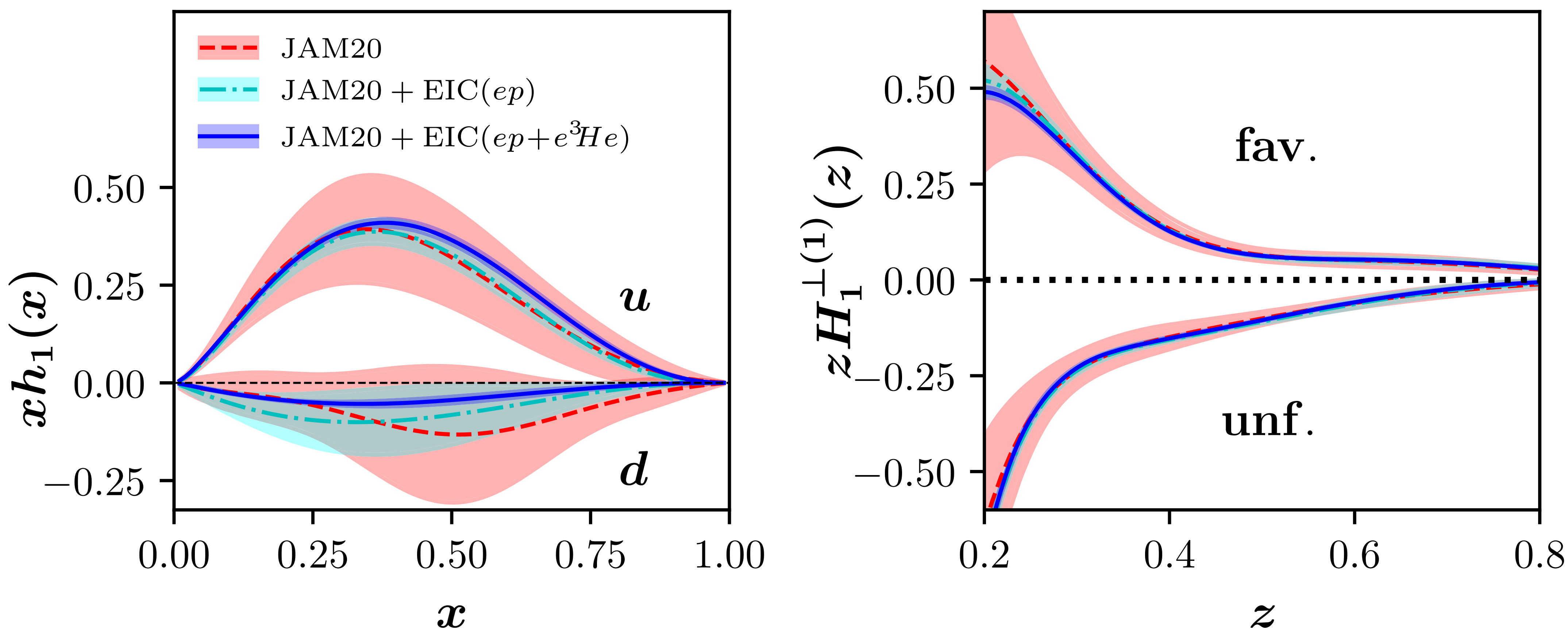
L. Gamberg, Z. Kang, D. Pitonyak, A. Prokudin, N. Sato, R.Seidl, Phys.Lett.B 816 (2021)

| EIC Pseudo-data | | | |
|---|---|---|---|
| **Observable** | **Reactions** | **CM Energy ($\sqrt{S}$)** | **$N_{\text{pts.}}$** |
| Collins (SIDIS) | $e + p^{\uparrow} \to e + \pi^{\pm} + X$ | 141 GeV | 756 ($\pi^+$)<br>744 ($\pi^-$) |
| | | 63 GeV | 634 ($\pi^+$)<br>619 ($\pi^-$) |
| | | 45 GeV | 537 ($\pi^+$)<br>556 ($\pi^-$) |
| | | 29 GeV | 464 ($\pi^+$)<br>453 ($\pi^-$) |
| | $e + {}^{3}He^{\uparrow} \to e + \pi^{\pm} + X$ | 85 GeV | 647 ($\pi^+$)<br>650 ($\pi^-$) |
| | | 63 GeV | 622 ($\pi^+$)<br>621 ($\pi^-$) |
| | | 29 GeV | 461 ($\pi^+$)<br>459 ($\pi^-$) |
| | | **Total EIC $N_{\text{pts.}}$** | **8223** |

Assumed accumulated luminosity 10 fb$^{-1}$, 70% polarization, conservatively accounted for detector smearing and acceptance effects

# EIC IMPACT

L. Gamberg, Z. Kang, D. Pitonyak, A. Prokudin, N. Sato, R.Seidl, Phys.Lett.B 816 (2021)

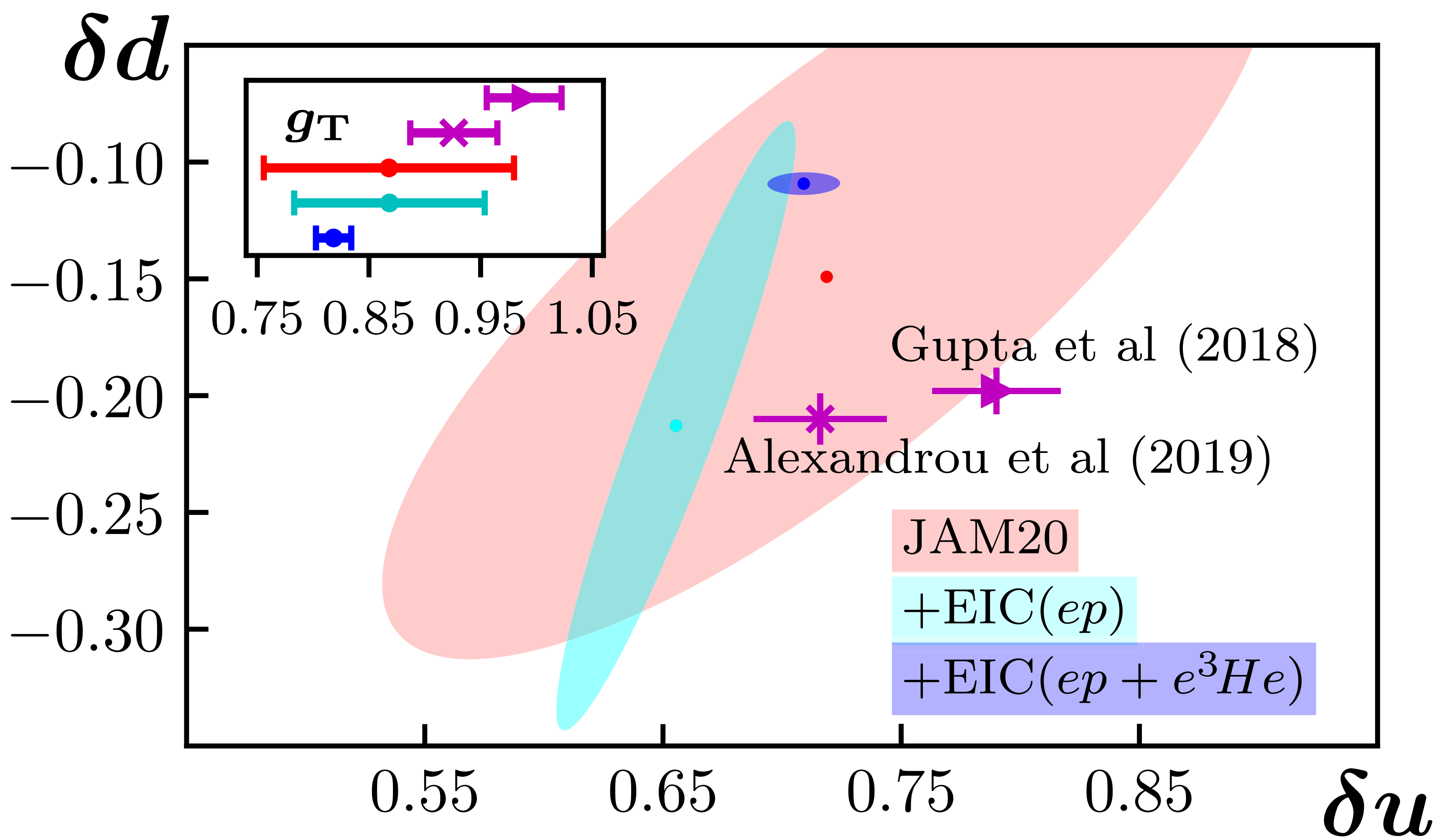



JAM20: Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato, Phys.Rev.D 102 (2020)

EIC data will significantly reduce uncertainties on transversity PDF (and Collins FF)

# EIC IMPACT

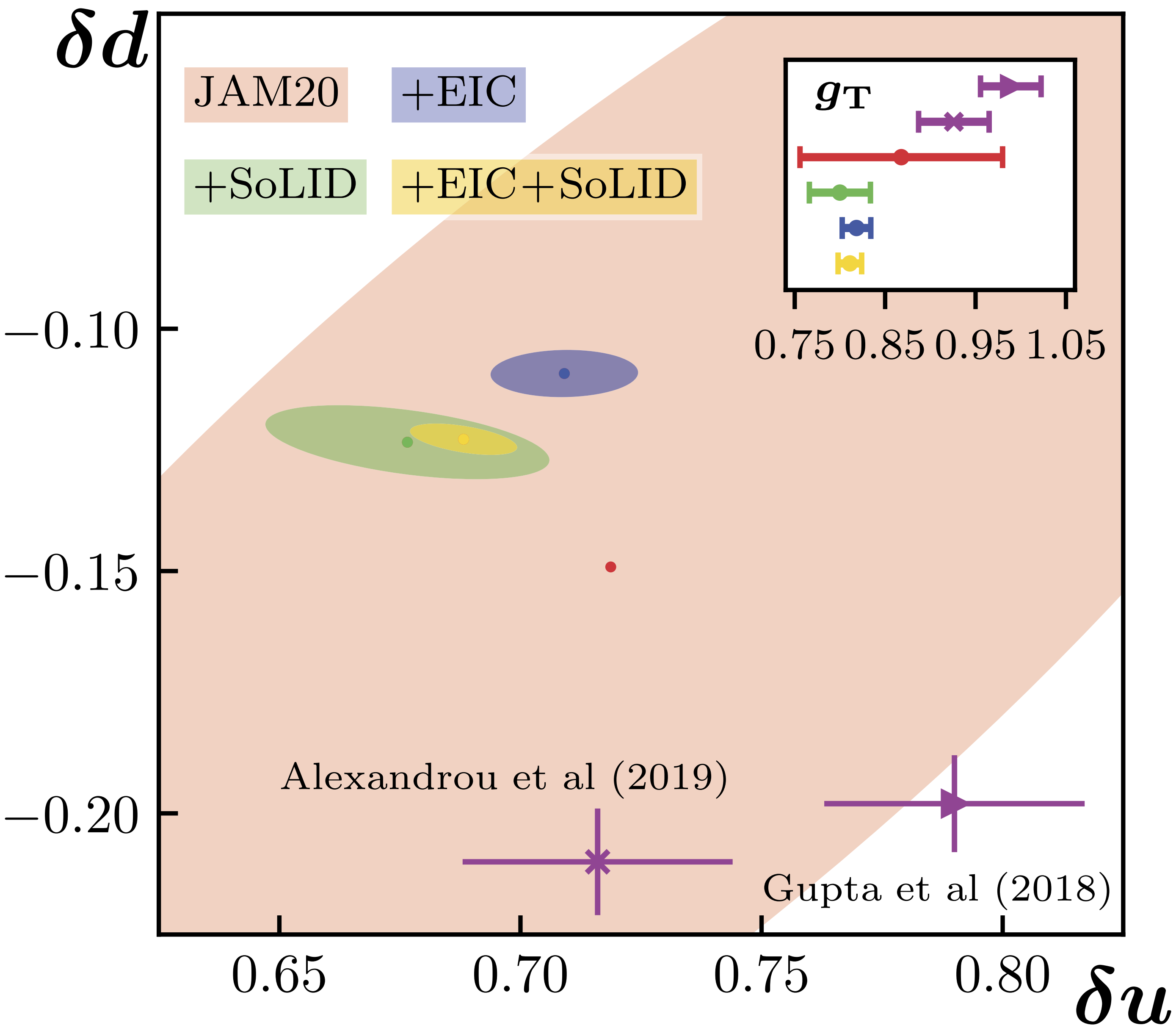
L. Gamberg, Z. Kang, D. Pitonyak, A. Prokudin, N. Sato, R.Seidl, Phys.Lett.B 816 (2021)




EIC data (combination of p and $^3$He) will allow extraction of the tensor charge at the level of precision of current lattice QCD calculations

# TENSOR CHARGE AT THE EIC AND JLAB

L. Gamberg, Z. Kang, D. Pitonyak, A. Prokudin, N. Sato, R.Seidl, Phys.Lett.B 816 (2021)




- EIC and JLab 12 data will allow to have complementary information on tensor charge to test the consistency of the extraction and expand the kinematical region

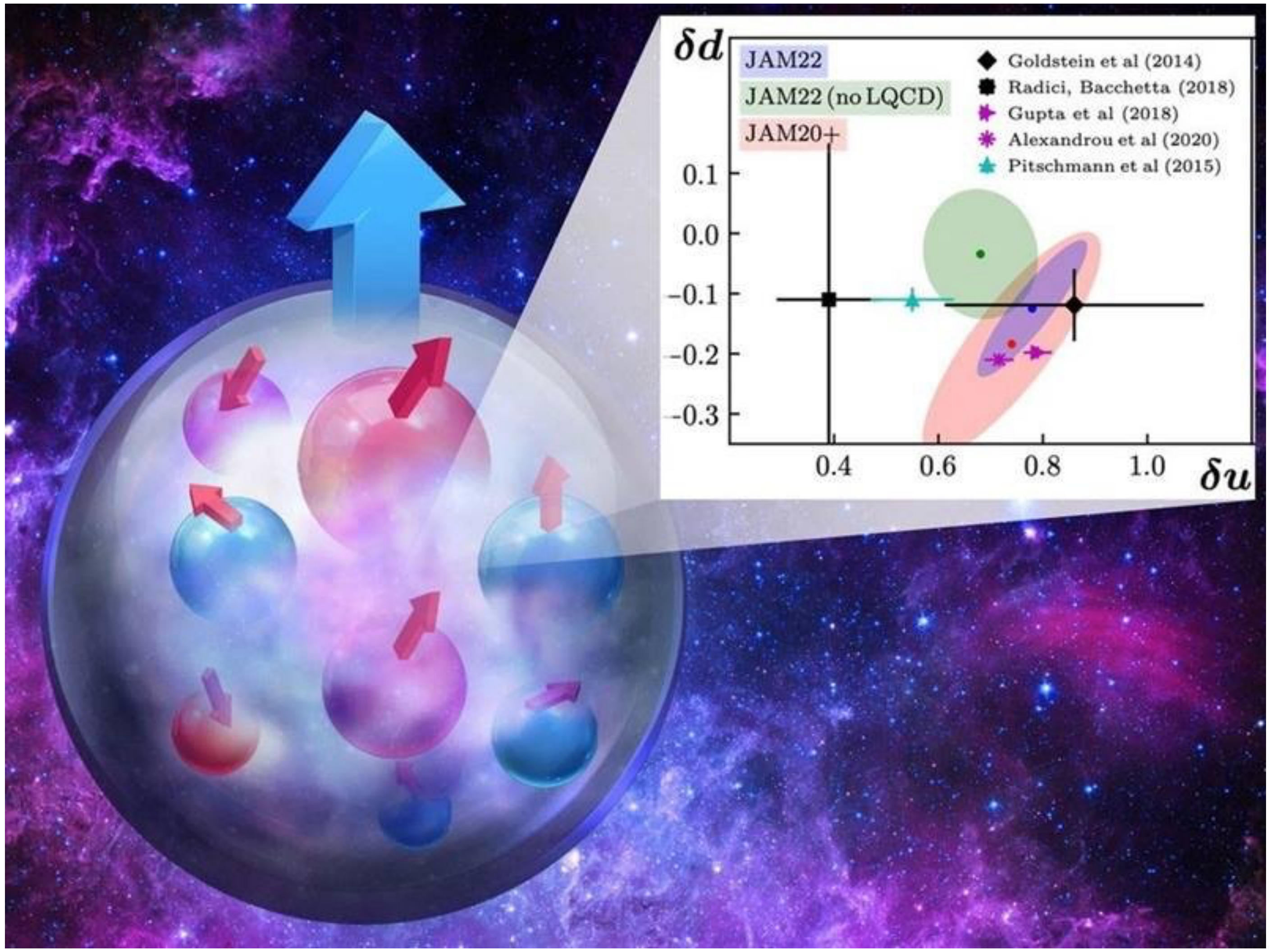
# CONCLUSIONS

Nuclear Physics

# Zeroing in on a Fundamental Property of the Proton's Internal Dynamics

APRIL 28, 2023






**A proton with transverse spin and quarks inside also with transverse spins. The tensor charge can be calculated for "up" and "down" quarks by various methods to quantify their total transverse spin in the proton (inset figure).**

Image courtesy of Thomas Jefferson National Accelerator Facility.

# COLLABORATION

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Ralf Seidl

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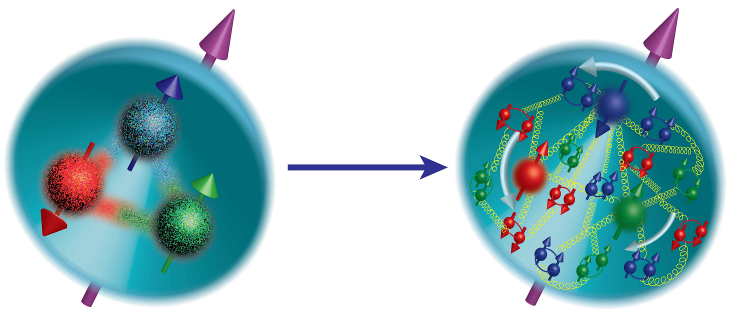
Zhongbo Kang

# CONCLUSIONS

- The tensor charges of the nucleon are quantities of particular interest - they are fundamental properties of the nucleon that have connections to QCD phenomenology, ab initio lattice QCD computations, model calculations, and low-energy beyond the Standard Model studies (e.g., beta decay, EDM)
- We have performed separate QCD global analyses of TSSAs in TMD/collinear twist-3 single-hadron observables and in dihadron fragmentation measurements, also studying the role of lattice QCD in our fits
- Recent analyses by the JAM Collaboration show agreement between single-hadron and dihadron approaches for extracting transversity as well as compatibility with lattice QCD tensor charges, thus showing for the first time the universal nature of all this information
- The EIC will play a transformative role in our understanding of the spin structure of the nucleon



# BACK UP

# FF

$$D_1^{h/q}(z, \vec{P}_\perp^2) = \frac{1}{N_c}\frac{\mathbf{1}}{\mathbf{4z}}\sum\!\!\!\!\!\!\int_X \int \frac{d\xi^+ d^2\vec{\xi}_\perp}{(2\pi)^3}\, e^{ik^-\xi^+}\,\mathrm{Tr}\Big[\langle 0|\mathcal{W}(\infty,\xi)\psi_q(\xi^+,0^-,\vec{\xi}_\perp)|P;X\rangle \times \langle P;X|\bar{\psi}_q(0^+,0^-,\vec{0}_\perp)\mathcal{W}(0,\infty)|0\rangle\,\gamma^-\Big]$$

This prefactor is key to the number density interpretation of single-hadron FFs → allows us to introduce the number operator when deriving the number sum rule

$$\hat{N} \equiv \sum_h \int \frac{dP^- d^2\vec{P}_\perp}{(2\pi)^3\, 2P^-}\, \hat{a}_h^\dagger \hat{a}_h = \sum_h \int \frac{dz\, d^2\vec{P}_\perp}{(2\pi)^3\, 2z}\, \hat{a}_h^\dagger \hat{a}_h$$

# DIFF

$$\Delta_{\alpha\beta}^{h_1 h_2/i}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) = \frac{1}{N_i} \sum\!\!\!\!\!\!\!\int_X \int \frac{d\xi^+ d^2\vec{\xi}_\perp}{(2\pi)^3}\, e^{ik\cdot\xi}\, \mathcal{O}_{\alpha\beta}^{h_1 h_2/i}(\xi)\Big|_{\xi^-=0}$$

<u>quark fragmentation</u> **($N_i$ = $N_c$)**

$$\begin{aligned}\mathcal{O}_{\alpha\beta}^{h_1 h_2/q}(\xi) =& \langle 0|\mathcal{W}(\infty,\xi)\psi_{q,\alpha}(\xi^+, 0^-, \vec{\xi}_\perp)|P_1, P_2; X\rangle \\ & \times \langle P_1, P_2; X|\bar{\psi}_{q,\beta}(0^+, 0^-, \vec{0}_\perp)\mathcal{W}(0,\infty)|0\rangle\end{aligned}$$

<u>gluon fragmentation</u> **($N_i$ = $N_c^2$ - 1)**

$$\begin{aligned}\mathcal{O}_{\alpha\beta}^{h_1 h_2/g}(\xi) =& \langle 0|\mathcal{W}^{ba}(\infty,\xi)F_{+\alpha}^{a}(\xi^+, 0^-, \vec{\xi}_\perp)|P_1, P_2; X\rangle \\ & \times \langle P_1, P_2; X|F_{+\beta}^{c}(0^+, 0^-, \vec{0}_\perp)\mathcal{W}^{cb}(0,\infty)|0\rangle\end{aligned}$$

# DIFF

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\left[\Delta^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})\gamma^-\right] = D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}^2, \vec{P}_{2\perp}^2, \vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

This ***prefactor is key to the number density interpretation*** of dihadron FFs (see also Majumder, Wang (2004)) because in order to prove a number sum rule we need to introduce the number operator separately for each hadron ( $j$ = 1, 2)

$$\hat{N}_{h_j} \equiv \int \frac{dP_j^- d^2\vec{P}_{j\perp}}{(2\pi)^3\, 2P_j^-}\, \hat{a}_{h_j}^\dagger \hat{a}_{h_j} = \int \frac{dz_j d^2\vec{P}_{j\perp}}{(2\pi)^3\, 2z_j}\, \hat{a}_{h_j}^\dagger \hat{a}_{h_j}$$

# DIFF

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\Big] = D_1^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})\gamma^-\gamma_5\Big] = -\frac{\epsilon_\perp^{ij}R_\perp^i P_{h\perp}^j}{zM_h^2}\,G_1^{\perp\,h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

$$\frac{1}{64\pi^3 z_1 z_2}\mathrm{Tr}\Big[\Delta^{h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp},\vec{P}_{2\perp})i\sigma^{i-}\gamma_5\Big] = -\frac{\epsilon_\perp^{ij}R_\perp^j}{M_h}H_1^{\sphericalangle'\,h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp}) + \frac{\epsilon_\perp^{ij}P_{h\perp}^j}{zM_h}H_1^{\perp'\,h_1h_2/q}(z_1,z_2,\vec{P}_{1\perp}^2,\vec{P}_{2\perp}^2,\vec{P}_{1\perp}\cdot\vec{P}_{2\perp})$$

***NB*: number density interpretation holds not only for unpolarized quarks ($\gamma^-$ projection) but also for longitudinally ($\gamma^-\gamma^5$ projection) and transversely ($i\sigma^{i-}\gamma^5$ projection) polarized quarks**

# DIFF

- Experimental measurements are sensitive to the so-called "extended" DiFFs where $k_T$ (and usually $\zeta$) is integrated out

$$D_1^{h_1 h_2/i}(z, M_h) \equiv \frac{\pi}{2} M_h \int_{-1}^{1} d\zeta\,(1-\zeta^2) D_1^{h_1 h_2/i}(z, \zeta, \vec{R}_T^2)$$

is a number density in $(z, M_h)$

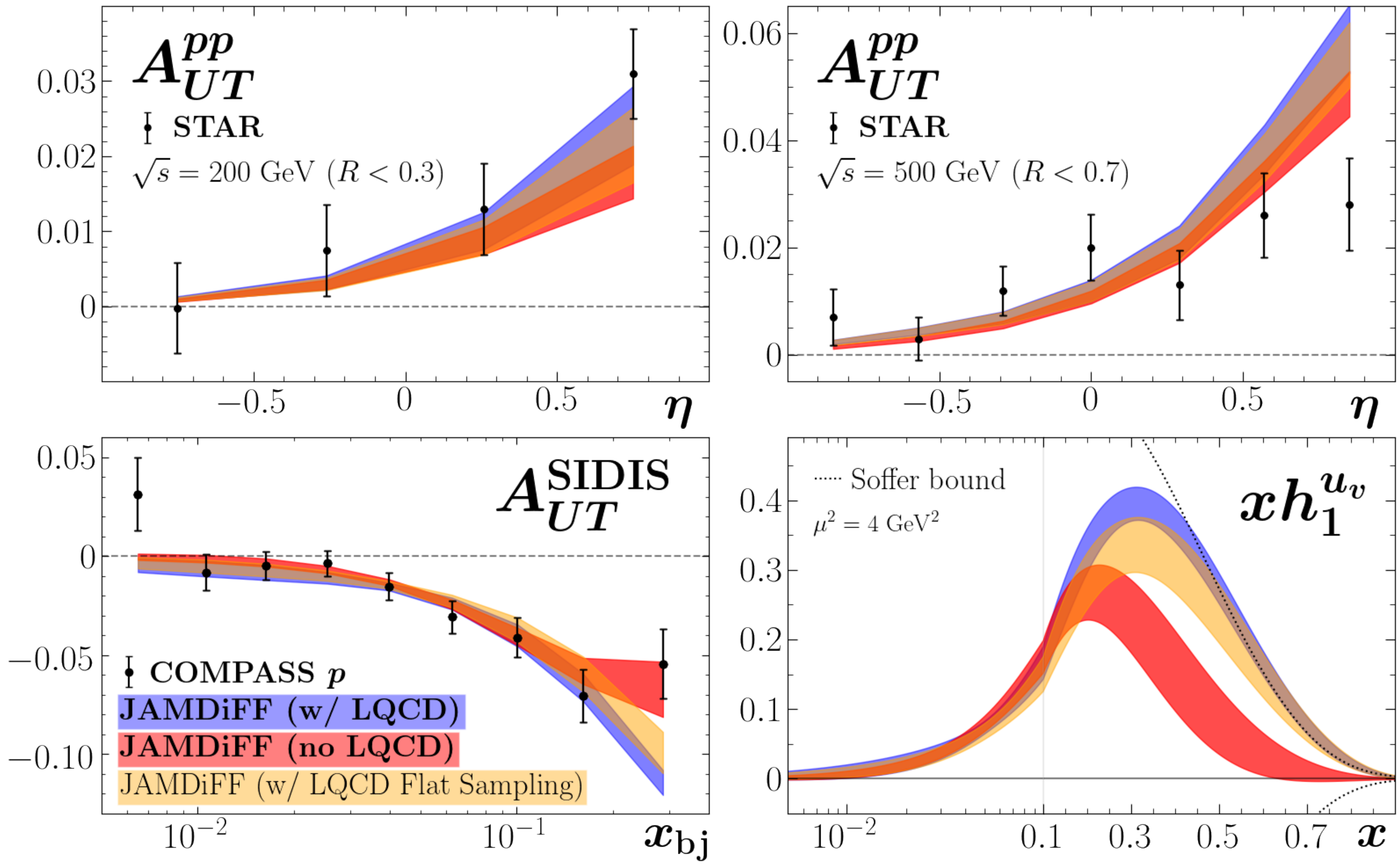
$$e^+e^- \to (h_1 h_2)\, X$$

$$\frac{d\sigma}{dz\, dM_h} = \boxed{\frac{4\pi N_c \alpha_{\rm em}^2}{3Q^2} \sum_q e_q^2}\, D_1^{h_1 h_2/q}(z, M_h)$$

partonic cross section for $e^+e^- \to \gamma \to q\bar{q}$

This is exactly the structure $d\sigma$ should have if $D_1$ has a number density interpretation

# LATTICE VS DATA




STAR 200 GeV data have a preference for a large $h_{uv}$ at large x, while the COMPASS proton data and STAR 500 GeV data prefer a smaller $h_{uv}$. In such a situation where there are competing preferences the choice of likelihood function and prior do not guarantee that the fits overlap within statistical uncertainties.

# JAM METHODOLOGY

$$\mathcal{L}\left(\boldsymbol{a}, \mathrm{data}\right) = \exp\left(-\frac{1}{2}\chi^2\left(\boldsymbol{a}, \mathrm{data}\right)\right)$$

$$\mathcal{P}(\boldsymbol{a}|\mathrm{data}) \sim \mathcal{L}(\boldsymbol{a}, \mathrm{data})\,\pi(\boldsymbol{a})$$