SIDIS_RC EvGen: A Monte-Carlo generator for SIDIS e-p with QED radiative corrections

Shuo Jia

Medium Energy Physics Group
Department of Physics
Duke University

Vladimir Khachatryan

Hadronic Physics Group
Department of Physics
Indiana University

Workshop: Precision QCD Predictions for ep Physics at the EIC (II)
CFNS, Department of Physics and Astronomy
Stony Brook University, Stony Brook, NY 11794

September 18-22 2023

Outline

- ➢ Introduction
  - transverse-momentum-dependent parton distributions (TMDs) in semi-inclusive deep inelastic scattering (SIDIS), then radiative corrections (RCs)
- ➢ Giving more details on SIDIS process and structure functions (SFs)
- ➢ Discussing QED radiative effects in SIDIS with conventional approach
- ➢ **SIDIS-RC EvGen:** MC event generator
  Monte-Carlo (MC) frameworks for evaluation of RC effects in SIDIS
- ➢ Present numerical results and compare two RC (conventional and factorized) approaches
- ➢ Summary and next steps

SIDIS-TMD

Semi-Inclusive Deep Inelastic Scattering to access TMDs

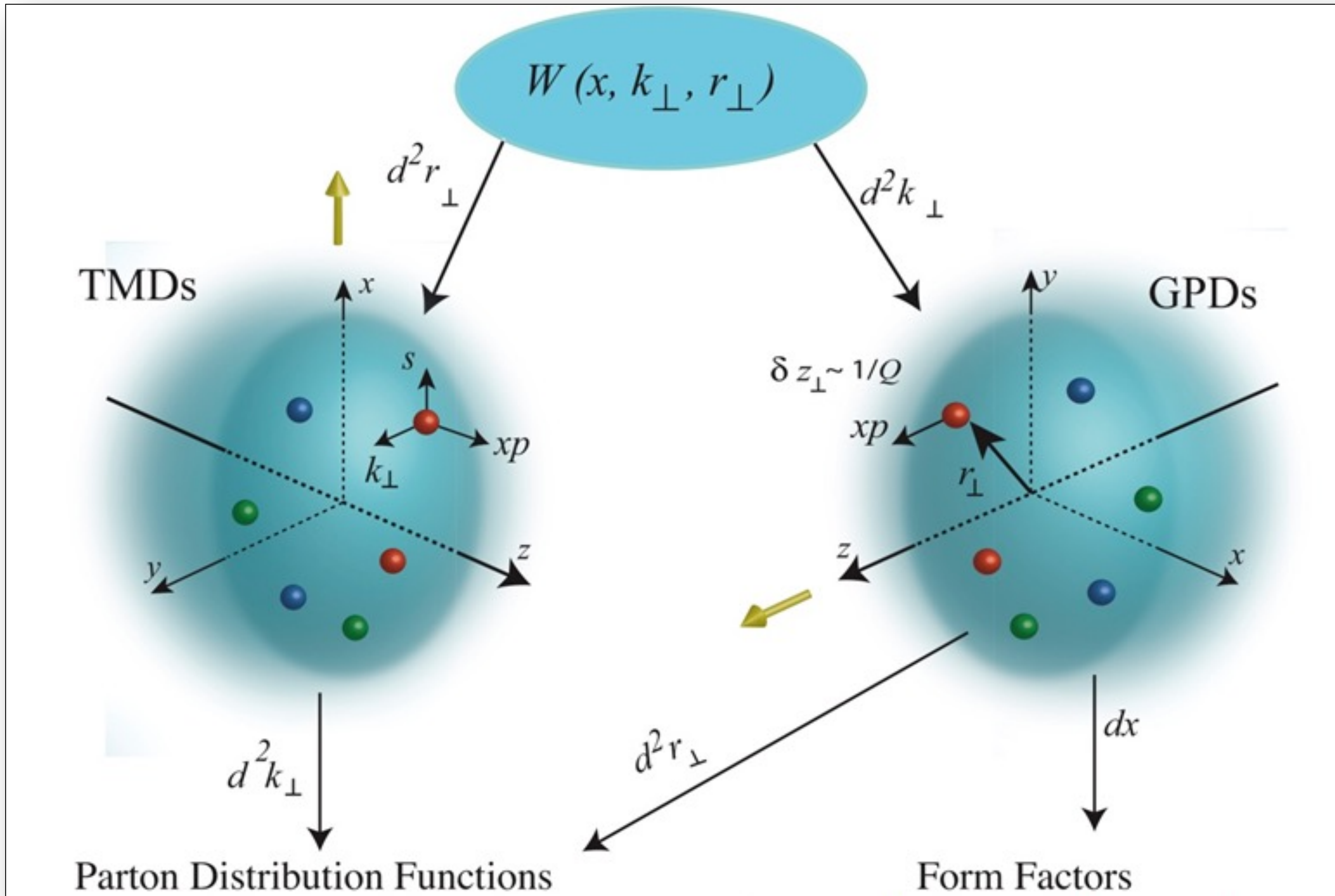


Image from Dudek et al., EPJA 48,187 (2012)

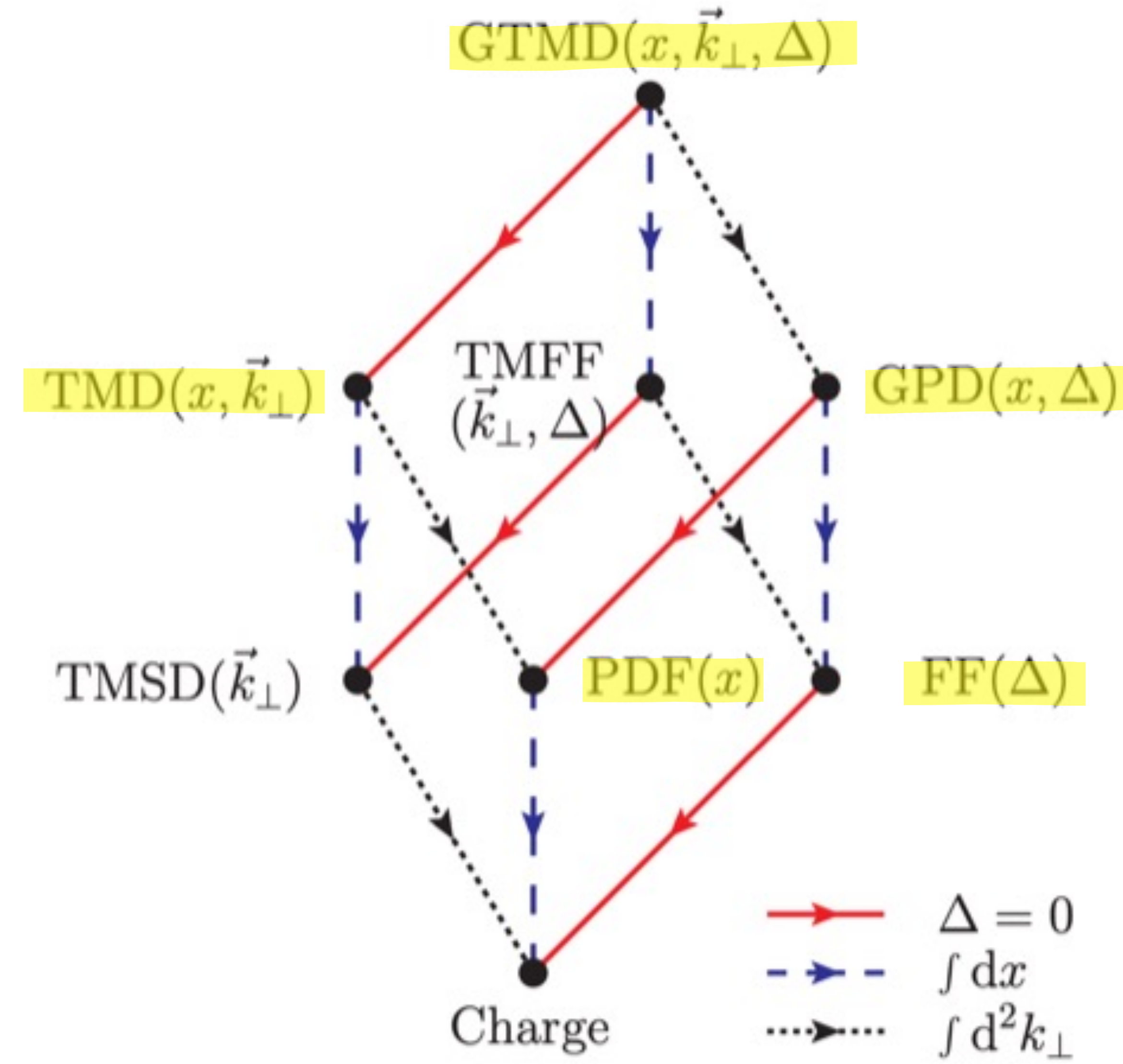


Image from Lorce et al., JHEP 05, 041 (2011)

- Transverse momentum dependent parton distribution (TMD)
- Generalized parton distribution (GPD)

Ji, PRL91, 062001 (2003)
Belitsky, Ji, Yuan, PRD69,074014 (2004)

- Radiative Correction is one of dominant sources of systematic uncertainties, due to radiation of photons
  - taking place with and without polarization of lepton beam and nucleon target

- First compute lowest-order model-independent part of QED RCs to SIDIS cross section in conventional method
  - alternative method of accounting for QED RCs in SIDIS based on factorization approach

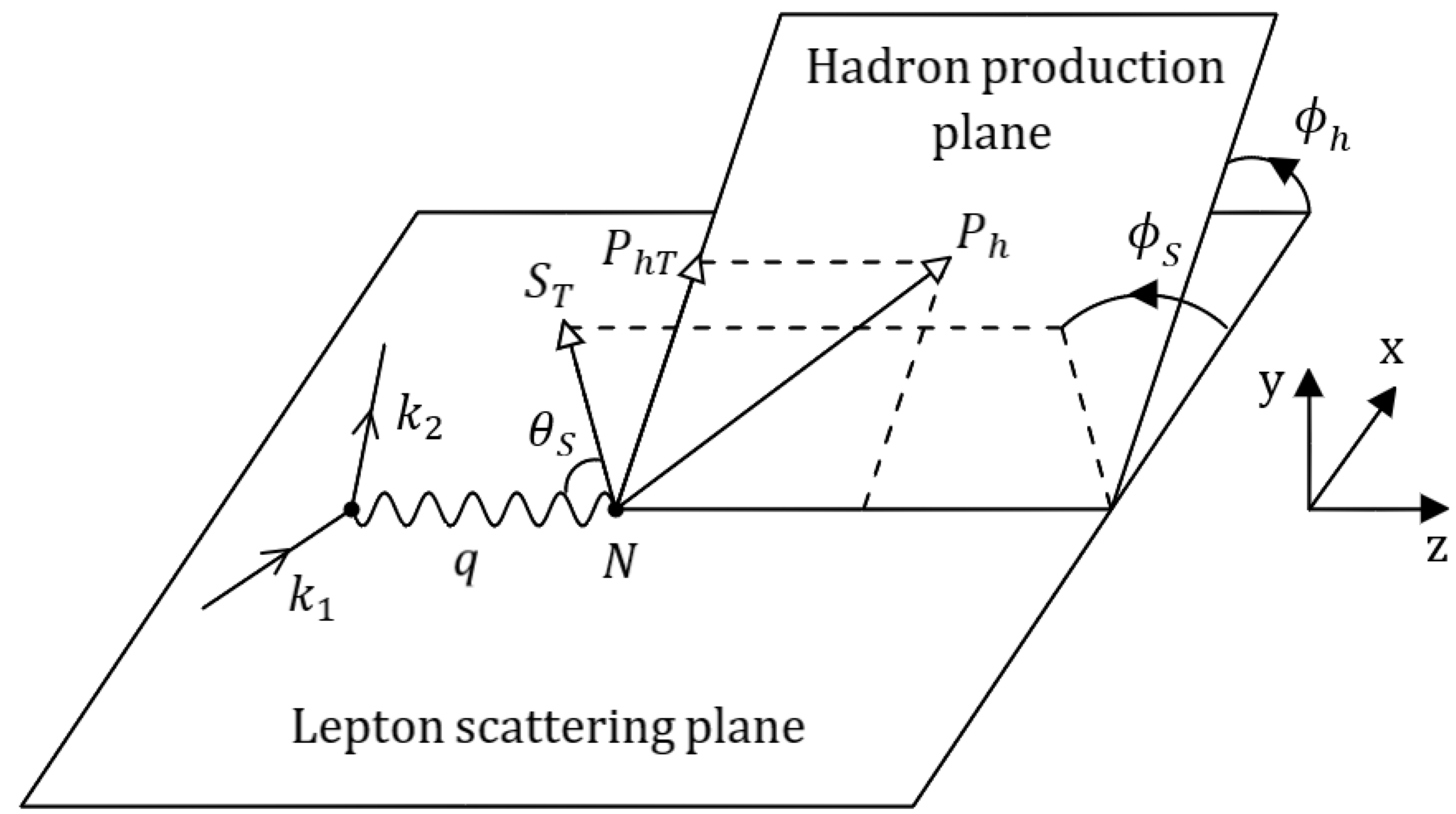
- **SIDIS-RC EvGen:** MC event generator for SIDIS processes including RCs
  - to generate radiative and non-radiative channels of scattering
  - to generate scattered lepton kinematics (i.e., Q^2 and x_{Bj}) and final-state hadron kinematics (i.e., z_h, P_{hT}, and ϕ_h)
  - to generate real photon radiation kinematics
  - to calculate full SIDIS cross section in any generated phase-space point with RCs included

- **SIDIS-RC EvGen** aiming to aid in multifaceted efforts for studying
  - TMD evolution effects
  - nucleon internal spin structure, spin-orbit and quark-gluon correlations
  - and nucleon 3D momentum structure in general

- Final-state hadron *h* detected in coincidence with lepton ℓ' scattered off target *N*

SIDIS process $\ell(k_1) + N(P) \rightarrow \ell'(k_2) + h(P_h) + X(P_X)$ **SIDIS process**

- k_1 and P: to be four-momenta of polarized/unpolarized incident lepton & nucleon target
- k_2 and P_h: to be four-momenta of scattered lepton & detected hadron
- P_X to be four-momentum of unobserved state of all undetected hadrons



SIDIS process kinematics in one-photon exchange approximation

Drawing made based on Trento conventions

- SIDIS kinematic variables

$$x_{Bj} = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot k_1}, \quad z_h = \frac{P\cdot P_h}{P\cdot q}, \quad \gamma = \frac{2M_N x_{Bj}}{Q}$$

- Six-fold differential cross section

$$\frac{d\sigma_{\text{SIDIS}}}{dx_{Bj}dydz_h dP_{hT}^2 d\phi_h d\phi_S}$$

- ϕ_S : azimuthal angle related to target-spin direction (spin-vector), if transversely polarized targets are applied

- SIDIS differential cross section by set of eighteen SFs at leading and subleading order in 1/Q expansion

→ Nucleon Spin
→ Quark Spin

See also Backups

Leading Twist TMDs

Nucleon Polarization	Quark polarization: Un-Polarized	Quark polarization: Longitudinally Polarized	Quark polarization: Transversely Polarized
U	$f_1 =$		$h_1^\perp =$ − Boer-Mulder
L		$g_1 =$ − Helicity	$h_{1L}^\perp =$ −
T	$f_{1T}^\perp =$ − Sivers	$g_{1T}^\perp =$ −	$h_{1T} =$ − Transversity $h_{1T}^\perp =$ − Pretzelosity

The Solenoidal Large Intensity Device (SoLID) for JLab 12 GeV arXiv:2209.13357 [nucl-ex]

Twist 3 TMDs

	U	L	T$_{J=0}$	T$_{J=1}$	T$_{J=2}$
U	$f_\bullet^\perp$	$g_\bullet^\perp$		$h_\bullet$	$h_\bullet^\perp$
L	$f_{\bullet L}^\perp$	$g_{\bullet L}^\perp$	$h_{\bullet L}$		$h_{\bullet L}^\perp$
T	$f_{\bullet T}$, $f_{\bullet T}^\perp$	$g_{\bullet T}$, $g_{\bullet T}^\perp$	$h_{\bullet T}^{D\perp}$	$h_{\bullet T}^{A\perp}$	$h_{\bullet T}^{S\perp}$, $h_{\bullet T}^{T\perp}$

Rodini and Vladimirov, JHEP 08, 031 (2022)

See also Backups

- SIDIS differential cross section expressed by set of eighteen SFs at leading and subleading order in 1/Q expansion
  - within TMD factorization framework
  - λ_e denoting lepton beam helicity
- In *XY* subscripts of most SFs
  - *X = U / L:* unpolarized or longitudinally polarized beam
  - *Y = U / L:* unpolarized or longitudinally polarized target with respect to *q*
  - *Y = U / T:* unpolarized or transversely polarized target with respect to *q*
- In *XY,Z* subscripts of remaining SFs
  - *Z = T / L* giving virtual photon polarizations
- Superscript in SF like $F_{UU}^{\cos(\phi_h)}$ showing azimuthal dependence

$$\frac{d\sigma_{\rm SIDIS}}{dx_{Bj}\,dy\,dz_h\,dP_{hT}^2\,d\phi_h\,d\phi_S} = \frac{\alpha^2}{x_{Bj}\,y\,Q^2}\left(1+\frac{\gamma^2}{2x_{Bj}}\right)\times$$
$$\times\Bigg\{\Big[c_1\,F_{UU,T} + c_2\,F_{UU,L} + c_3\cos(\phi_h)\,F_{UU}^{\cos(\phi_h)} +$$
$$+c_2\cos(2\phi_h)\,F_{UU}^{\cos(2\phi_h)} + \lambda_e c_4\sin(\phi_h)\,F_{LU}^{\sin(\phi_h)}\Big] +$$
$$+S_L\Big[c_3\sin(\phi_h)\,F_{UL}^{\sin(\phi_h)} + c_2\sin(2\phi_h)\,F_{UL}^{\sin(2\phi_h)}\Big] +$$
$$+S_L\lambda_e\Big[c_5\,F_{LL} + c_4\cos(\phi_h)\,F_{LL}^{\cos(\phi_h)}\Big] +$$
$$+S_T\Big[\sin(\phi_h-\phi_S)\left(c_1\,F_{UT,T}^{\sin(\phi_h-\phi_S)} + c_2\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) +$$
$$+c_2\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + c_2\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} +$$
$$+c_3\sin(\phi_S)\,F_{UT}^{\sin(\phi_S)} + c_3\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] +$$
$$+S_T\lambda_e\Big[c_5\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + c_4\cos(\phi_S)\,F_{LT}^{\cos(\phi_S)} +$$
$$+c_4\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Bigg\},$$

See also Backups

- Factors c_1, c_2, c_3, c_4, c_5, given by

$$c_1 = \frac{y^2}{2(1-\varepsilon)}, \quad c_2 = \frac{y^2}{2(1-\varepsilon)}\varepsilon, \quad c_3 = \frac{y^2}{2(1-\varepsilon)}\sqrt{2\varepsilon(1+\varepsilon)},$$

$$c_4 = \frac{y^2}{2(1-\varepsilon)}\sqrt{2\varepsilon(1-\varepsilon)}, \quad c_5 = \frac{y^2}{2(1-\varepsilon)}\sqrt{1-\varepsilon^2},$$

- ε to be ratio of longitudinal and transverse photon fluxes

$$\varepsilon = \frac{1 - y - (\gamma^2 y^2/4)}{1 - y + (y^2/2) + (\gamma^2 y^2/4)}$$

- Hadron azimuthal angle defined in

$$\cos(\phi_h) = -\frac{k_{1\mu} P_{h\nu}\, g_\perp^{\mu\nu}}{\sqrt{k_{1T}^2 P_{hT}^2}}, \qquad \sin(\phi_h) = -\frac{k_{1\mu} P_{h\nu}\, \epsilon_\perp^{\mu\nu}}{\sqrt{k_{1T}^2 P_{hT}^2}}, \qquad \text{with } k_{1T}^\mu = g_\perp^{\mu\nu} k_{1\nu} \text{ and } P_{hT}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

- Tensors $g_\perp^{\mu\nu}$ and $\epsilon_\perp^{\mu\nu}$ expressed as

$$g_\perp^{\mu\nu} = g^{\mu\nu} - \frac{q^\mu P^\nu + P^\mu q^\nu}{P\cdot q\,(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M_N^2}\right) \qquad \epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{P_\rho q_\sigma}{P\cdot q\sqrt{1+\gamma^2}}$$

- For TMDs and FFs, Gaussian ansatz used well supported by phenomenological analyses
- For given quark flavor, unpolarized TMD and FF given by

$$f^a(x_{Bj}, k_\perp^2) = f_c^a(x_{Bj}) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle}}{\pi \langle k_\perp^2 \rangle}, \quad D^a(z_h, p_\perp^2) = D_c^a(z_h) \frac{e^{-p_\perp^2/\langle p_\perp^2 \rangle}}{\pi \langle p_\perp^2 \rangle}$$

- $f_c^a(x_{Bj})$ being collinear PDF, and $D_c^a(z_h)$ being collinear FF
- Grid files used by us for both collinear PDF and FF

from Prokudin-Tezgin WW-SIDIS library https://github.com/prokudin /WW-SIDIS

and

from Martin-Stirling-Thorne-Watt MSTWPDF library https://mstwpdf.hepforge.org/

- As examples, write down general analytical forms of three SFs in Gaussian approximation

$$\text{(i) Leading-twist } F_{UU}(x,z,P_{hT}) = \left\{F_{UU} \equiv F_{UU,T}\right\}$$

$$= x_{Bj} \sum_a e_a^2\, f_c^a(x_{Bj})\, D_c^a(z_h)\, \frac{e^{-P_{hT}^2/\langle P_{hT}^2\rangle}}{\pi \langle P_{hT}^2\rangle}, \qquad \langle P_{hT}^2\rangle = \langle p_\perp^2\rangle_D + z_h^2 \langle k_\perp^2\rangle_f$$

- Other two SFs represented as

  - Collins:

$$\text{(ii) Leading-twist } F_{UT}^{\sin(\phi_h+\phi_S)}(x,z,P_{hT}) =$$

$$= x_{Bj} \sum_a e_a^2\, h_1^a(x_{Bj})\, H_1^{\perp(1)a}(z_h)\, b_A^{(1)} \left[\frac{z_h P_{hT}}{\langle P_{hT}^2\rangle}\right] \frac{e^{-P_{hT}^2/\langle P_{hT}^2\rangle}}{\pi \langle P_{hT}^2\rangle}$$

  - Sivers:

$$\text{(iii) Leading-twist } F_{UT}^{\sin(\phi_h-\phi_S)}(x,z,P_{hT}) = \left\{F_{UT}^{\sin(\phi_h-\phi_S)} \equiv F_{UT,T}^{\sin(\phi_h-\phi_S)}\right\}$$

$$= -x_{Bj} \sum_a e_a^2\, f_{1T}^{\perp(1)a}(x_{Bj})\, D_c^a(z_h)\, b_B^{(1)} \left[\frac{z_h P_{hT}}{\langle P_{hT}^2\rangle}\right] \frac{e^{-P_{hT}^2/\langle P_{hT}^2\rangle}}{\pi \langle P_{hT}^2\rangle}$$

J. High Energy Phys. 06 (2019) 007

- SIDIS process with incident lepton ξ and target nucleon $\boldsymbol{\eta}$ polarization vectors is

SIDIS process polarized

$$\ell(k_1,\xi) + N(P,\boldsymbol{\eta}) \to \ell'(k_2) + h(P_h) + X(P_X)$$

SIDIS process polarized

- Cross section differential of lowest-order QED (Born) contribution to SIDIS to be given by convolution of hadronic ($W_{\mu\nu}$) and leptonic tensors ($L_B^{\mu\nu}$):

$$d\sigma^B_{\text{SIDIS}} = \frac{(4\pi\alpha)^2}{2\sqrt{\lambda_S}Q^4} W_{\mu\nu} L_B^{\mu\nu} d\Gamma_B \qquad \text{with} \qquad \lambda_S = (2P\cdot k_1)^2 - 4M_N^2 m_l^2$$

- Phase-space parametrized by

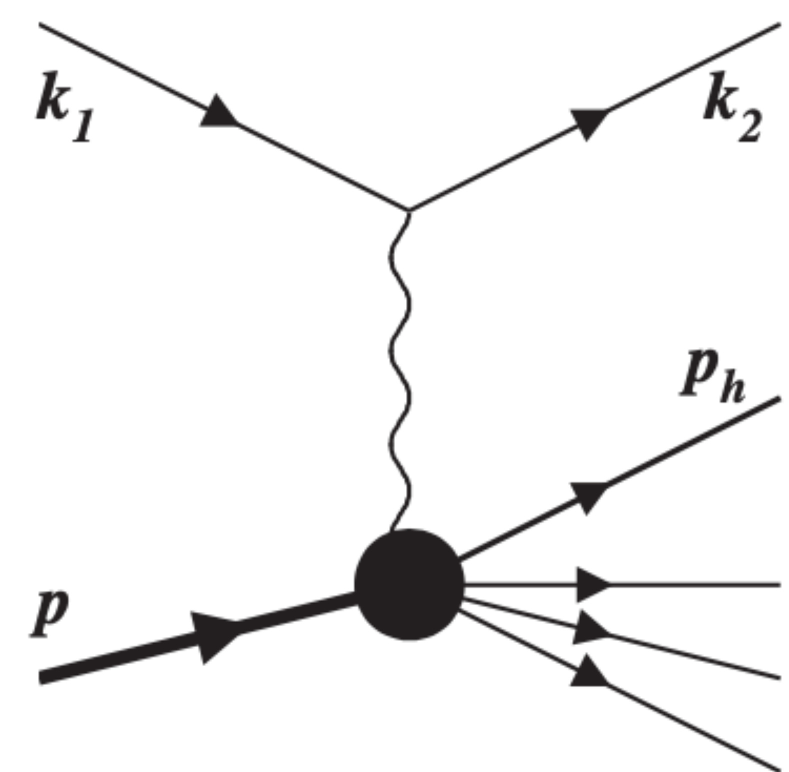
$$d\Gamma_B = (2\pi)^4 \frac{d^3k_2}{(2\pi)^3 2k_{20}} \frac{d^3P_h}{(2\pi)^3 2P_{h0}} = \frac{1}{4(2\pi)^2} \frac{SS_x\, dx_{Bj}\, dy\, d\phi_s}{2\sqrt{\lambda_S}} \frac{S_x\, dz_h\, dP_{hT}^2\, d\phi_h}{4M_N P_{hL}}$$

- with k_{20} - scattered lepton energy,
  P_{h0} (P_{hL}) - charged hadron energy (longitudinal momentum)

- Following set of variables used

$$S = 2P\cdot k_1, \quad S_x = 2P\cdot q, \quad P_{h0} = \frac{z_h S_x}{2M_N}, \quad P_{hT} = \sqrt{P_{h0}^2 - P_{hL}^2 - M_h^2}$$

$$P_{hL} = \frac{z_h S_x^2 + 2M_N^2\left(t + Q^2 - M_h^2\right)}{2M_N\sqrt{\lambda_Y}}, \quad \text{with } \lambda_Y = S_x^2 + 4M_N^2 Q^2.$$



Feynman diagram describing Born SIDIS process

See also Backups

- Leptonic tensor given by

$$L_B^{\mu\nu} = \frac{1}{2}\mathrm{Tr}\Big[\big(\hat{k}_2 + m_l\big)\gamma_\mu\big(\hat{k}_1 + m_l\big)\big(1 + \gamma_5\hat{\xi}\big)\gamma_\nu\Big] =$$

$$= 2\left[k_1^\mu k_2^\nu + k_2^\mu k_1^\nu - \frac{Q^2}{2}g^{\mu\nu} + \frac{i\lambda_e}{\sqrt{\lambda_S}}\epsilon^{\mu\nu\rho\sigma}\left(S\,k_{2\rho}k_{1\sigma} + 2m_l^2 q_\rho P_\sigma\right)\right]$$

- Incident lepton polarization vector reads as

$$\xi = \frac{\lambda_e S}{m_l\sqrt{\lambda_S}}k_1 - \frac{2\lambda_e m_l}{\sqrt{\lambda_S}}P = \xi_0 + \xi_1$$

- Hadronic tensor partitioned to spin-independent $H_{ab}^{(0)}$, and spin-dependent $H_{abi}^{(S)}$ scalar structure functions:

$$W_{\mu\nu} = \sum_{a,b=0}^{3} e_\mu^{\gamma(a)} e_\nu^{\gamma(b)}\left(H_{ab}^{(0)} + \sum_{\rho,i=0}^{3}\eta^\rho e_\rho^{h(i)} H_{abi}^{(S)}\right)$$

- Cross section of Born contribution to SIDIS to be following

$$\frac{d\sigma_{\mathrm{SIDIS}}^B}{dx_{Bj}\,dy\,dz_h\,dP_{hT}^2\,d\phi_h\,d\phi_s} = \frac{\alpha^2 S S_x^2}{8M_N Q^4 P_{hL}\lambda_S}\sum_{i=1}^{9}\theta_i^B\mathcal{H}_i$$

See Eq. (16) in Phys. Rev. D 100(3) (2019)

See also Backups

- Nucleon polarized three-vector $\boldsymbol{\eta} = (\eta_1, \eta_2, \eta_3)$ decomposition is

$$\eta_1 = \cos(\phi_s - \phi_h) S_T$$

$$\eta_2 = \sin(\phi_s - \phi_h) S_T$$

$$\eta_3 = S_L$$

- Examples of scalar functions expressed through structure functions given by

$$H_{00}^{(0)} = C_1 F_{UU,L},$$

$$H_{01}^{(0)} = -C_1 (F_{UU}^{\cos\phi_h} + i F_{LU}^{\sin\phi_h}),$$

$$H_{11}^{(0)} = C_1 (F_{UU}^{\cos 2\phi_h} + F_{UU,T}),$$

$$H_{22}^{(0)} = C_1 (F_{UU,T} - F_{UU}^{\cos 2\phi_h}),$$

$$H_{002}^{(S)} = C_1 F_{UT,L}^{\sin(\phi_h - \phi_s)},$$

$$H_{012}^{(S)} = C_1 (F_{UT}^{\sin\phi_s} - F_{UT}^{\sin(2\phi_h - \phi_s)} - i(F_{LT}^{\cos\phi_s} - F_{LT}^{\cos(2\phi_h - \phi_s)})),$$

$$H_{021}^{(S)} = C_1 (F_{UT}^{\sin(2\phi_h - \phi_s)} + F_{UT}^{\sin\phi_s} - i(F_{LT}^{\cos(2\phi_h - \phi_s)} + F_{LT}^{\cos\phi_s})),$$

$$H_{023}^{(S)} = C_1 (F_{UL}^{\sin\phi_h} - i F_{LL}^{\cos\phi_h}),$$

$$H_{121}^{(S)} = C_1 (-F_{UT}^{\sin(3\phi_h - \phi_s)} - F_{UT}^{\sin(\phi_h + \phi_s)} + i F_{LT}^{\cos(\phi_h - \phi_s)}),$$

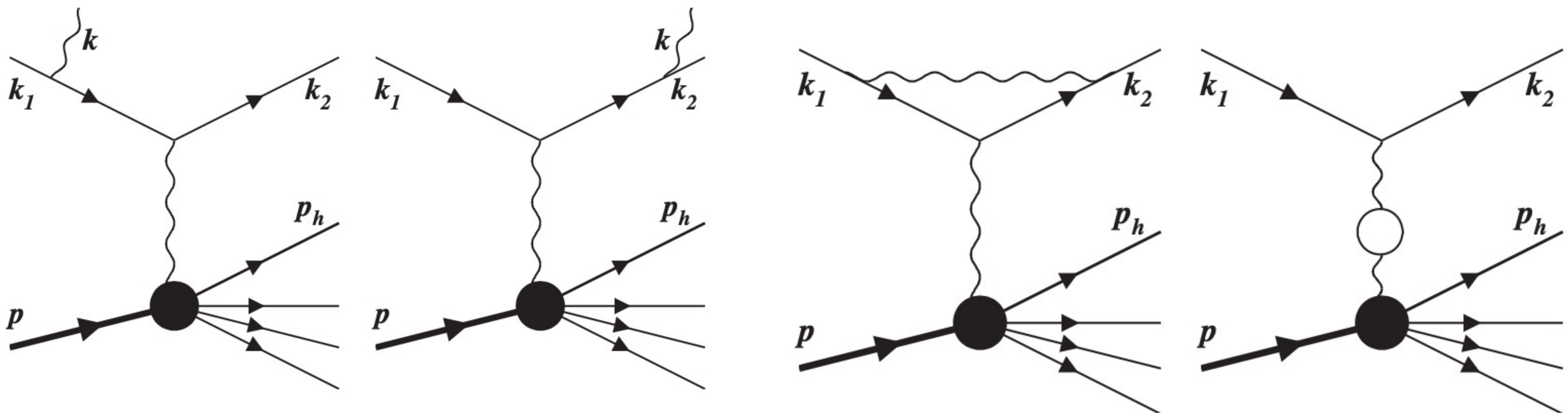
$$H_{123}^{(S)} = C_1 (-F_{UL}^{\sin 2\phi_h} + i F_{LL}),$$

$$H_{112}^{(S)} = C_1 (F_{UT}^{\sin(3\phi_h - \phi_s)} + F_{UT,T}^{\sin(\phi_h - \phi_s)} - F_{UT}^{\sin(\phi_h + \phi_s)}),$$

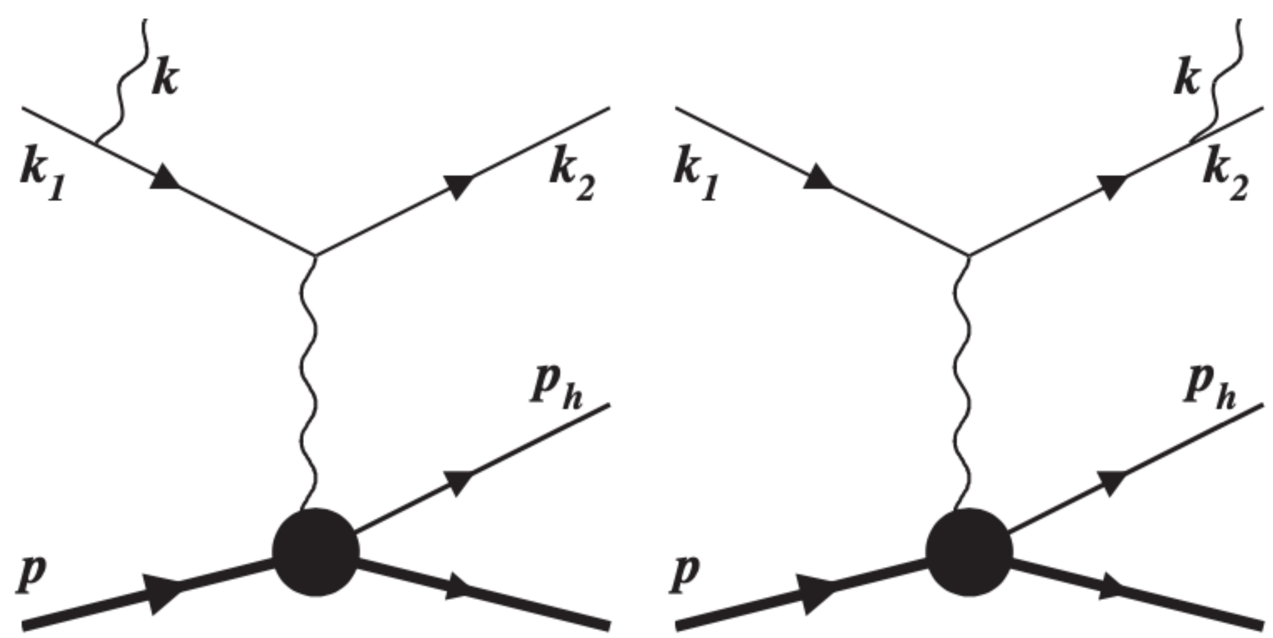
$$H_{222}^{(S)} = C_1 (F_{UT}^{\sin(\phi_h + \phi_s)} + F_{UT,T}^{\sin(\phi_h - \phi_s)} - F_{UT}^{\sin(3\phi_h - \phi_s)}),$$

$$C_1 = \frac{4 M p_l (Q^2 + 2 x M^2)}{Q^4}$$

- Radiative Correction being one of dominant sources of systematic uncertainties, due to radiation of photons off leptons, can be calculated by traditional (conventional) method



Next-to-leading (NLO) order RC contribution



Exclusive radiative tail contributions to NLO RC

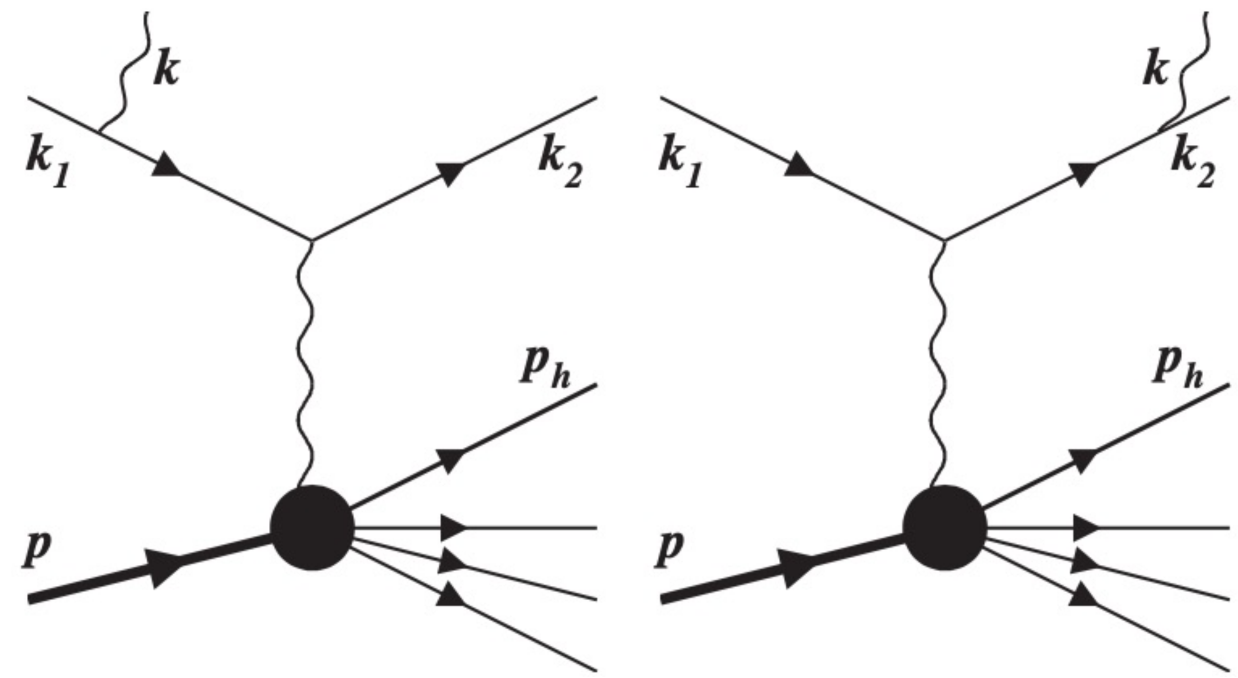
See also Backups

- Real photon emission in SIDIS given by

SIDIS process real γ emission

$$\ell(k_1,\xi) + N(P,\boldsymbol{\eta}) \to \ell'(k_2) + h(P_h) + X(\tilde{P}_X) + \gamma(k)$$

  - k to be four-momentum of radiated real photon γ



- Three additional photonic variables introduced
  - ϕ_k to be angle between $(\mathbf{k_1}, \mathbf{k_2})$ and $(\mathbf{k}, \mathbf{q})$ planes

$$R = 2k\cdot P, \qquad \tau = \frac{k\cdot q}{k\cdot P}, \qquad \phi_k$$

- Cross section differential of real photon radiation from leptonic leg is

$$d\sigma_R = \frac{(4\pi\alpha)^3}{2\sqrt{\lambda_S}\,\tilde{Q}^4}\tilde{W}_{\mu\nu}L_R^{\mu\nu}d\Gamma_R \qquad \tilde{Q}^2 = -(q-k)^2 = Q^2 + R\tau$$

$$d\Gamma_R = (2\pi)^4\frac{d^3k}{(2\pi)^3 2k_0}\frac{d^3k_2}{(2\pi)^3 2k_{20}}\frac{d^3P_h}{(2\pi)^3 2P_{h0}} \qquad \frac{d^3k}{k_0} = \frac{R\,dR\,d\tau\,d\phi_k}{2\sqrt{\lambda_Y}}$$

See also Backups

- For convenience, designate six-fold differential cross section as

$$\sigma \equiv \frac{d\sigma}{dx_{Bj}\, dy\, dz_h\, dP_{hT}^2\, d\phi_h\, d\phi_S}$$

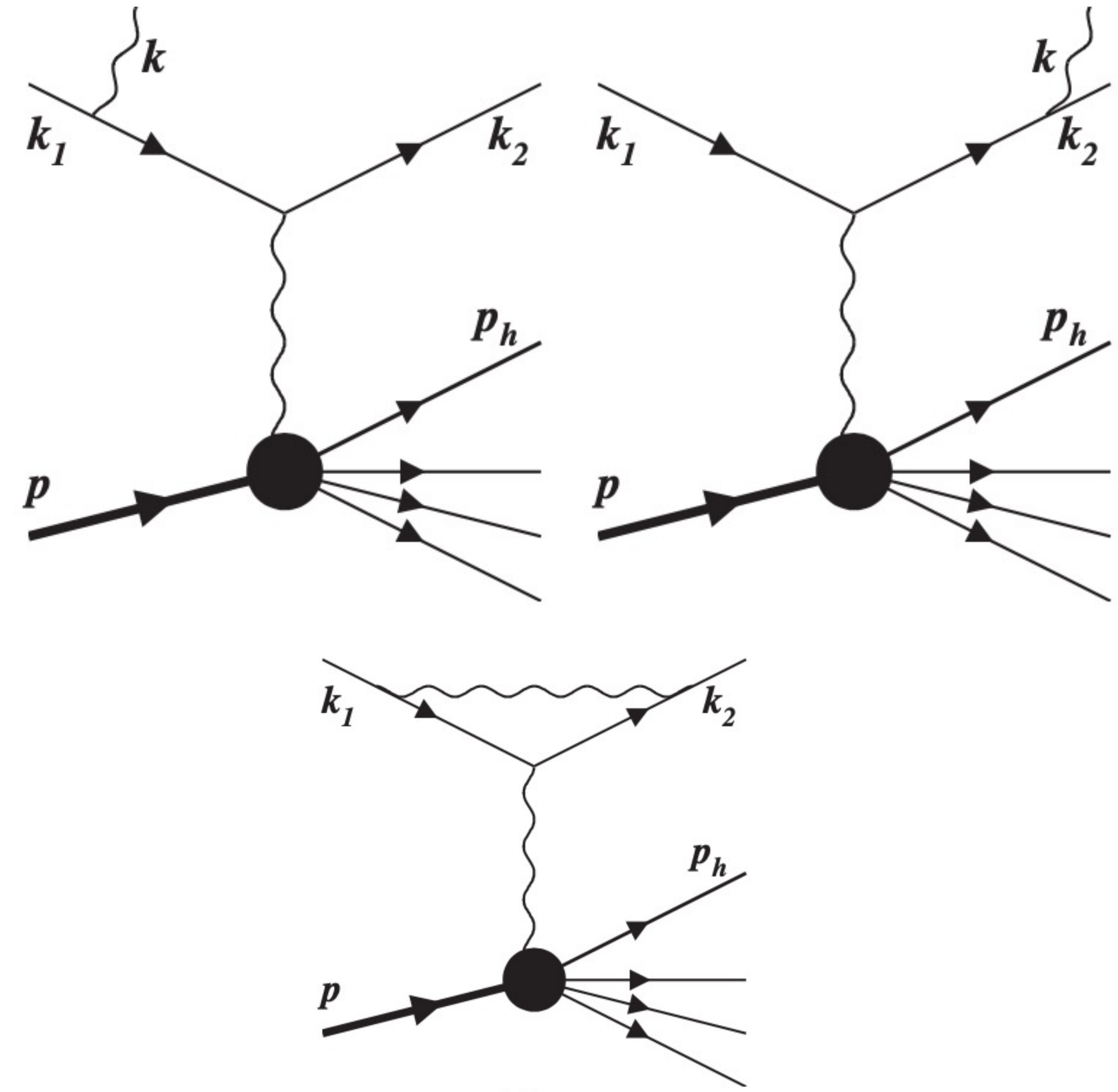
- **Inelastic tail of SIDIS total cross section** (with several RC components included) given by

$$\sigma_{\text{SIDIS}}^{in} = \frac{\alpha}{\pi}\left(\delta_{VR} + \delta_{\text{vac}}^{l} + \delta_{\text{vac}}^{h}\right)\sigma_{\text{SIDIS}}^{B} + \sigma_R^F + \sigma^{AMM}$$

- $\sigma_{\text{SIDIS}}^{B}$ - Born cross section in SIDIS
- δ_{VR} - sum of infrared divergent terms that is finite
- δ_{vac}^{l} - contribution of vacuum polarization by leptons
- δ_{vac}^{h} - contribution of vacuum polarization by hadrons
- σ_R^F - infrared free contribution to cross section, obtained after integration over three photonic variables
- σ^{AMM} - anomalous magnetic moment contribution to cross section

- δ_{VR} - sum of infrared divergent terms that is finite

$$\delta_{VR} = \delta_S + \delta_H + \delta_{\rm vert}$$
$$= 2(Q_m^2 L_m - 1)\log\frac{p_x^2 - M_{th}^2}{m\sqrt{p_x^2}} + \frac{1}{2}S' L_{S'}$$
$$+ \frac{1}{2}X' L_{X'} + S_\phi - 2 + \left(\frac{3}{2}Q^2 + 4m^2\right)L_m$$
$$- \frac{Q_m^2}{\sqrt{\lambda_m}}\left(\frac{1}{2}\lambda_m L_m^2 + 2{\rm Li}_2\left(\frac{2\sqrt{\lambda_m}}{Q^2 + \sqrt{\lambda_m}}\right) - \frac{\pi^2}{2}\right),$$



Bardin-Shumeiko approach
D. Yu. Bardin and N. M. Shumeiko, Nucl. Phys. B127, 242 (1977)

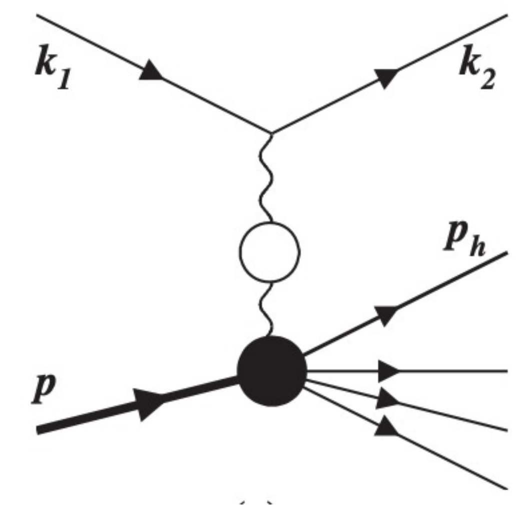
- σ_R^F - infrared free contribution to cross section, obtained after integration over three photonic variables

$$\sigma_R^F = -\frac{\alpha^3 S S_x^2}{64\pi^2 M_N P_{hL} \lambda_S \sqrt{\lambda_Y}} \int_{\tau_{\rm min}}^{\tau_{\rm max}} d\tau \int_0^{2\pi} d\phi_k \int_0^{R_{\rm max}} dR \times \sum_{i=1}^{9} \left(\frac{\theta_{i1}}{R} \left(\frac{\tilde{\mathcal{H}}_i}{\tilde{Q}^4} - \frac{\mathcal{H}_i}{Q^4} \right) + \sum_{j=2}^{k_i} \tilde{\mathcal{H}}_i \theta_{ij} \frac{R^{j-2}}{\tilde{Q}^4} \right)$$

$$R_{\rm max} = \frac{P_x^2 - M_{th}^2}{1 + \tau - \mu}, \qquad \tau_{\rm max/min} = \frac{S_x \pm \sqrt{\lambda_Y}}{2M_N^2}$$

- δ^l_{vac} - contribution of vacuum polarization by leptons

$$\delta^l_{\text{vac}} = \sum_{i=e,\mu,\tau} \delta^{l,i}_{\text{vac}} = \sum_{i=e,\mu,\tau} \left(\frac{2}{3}(Q^2 + 2m_i^2)L^i_m - \frac{10}{9} + \frac{8m_i^2}{3Q^2}(1 - 2m_i^2 L^i_m) \right)$$



- δ^h_{vac} - contribution of vacuum polarization by hadrons

$$\delta^h_{\text{vac}} = -\frac{2\pi}{\alpha}\left[A + B\log(1 + C|t_h|)\right]$$

- σ^{AMM} - anomalous magnetic moment contribution to cross section

$$\sigma^{AMM} = \frac{\alpha^3 m_l^2 S S_x^2}{16\pi M_N Q^2 P_{hL} \lambda_S} L_m \sum_{i=1}^{9} \theta_i^{AMM} \mathcal{H}_i$$

- **Inelastic tail of SIDIS total cross section** (with several RC components included) given by

$$\sigma^{in}_{\text{SIDIS}} = \frac{\alpha}{\pi}\left(\delta_{VR} + \delta^l_{\text{vac}} + \delta^h_{\text{vac}}\right)\sigma^B_{\text{SIDIS}} + \sigma^F_R + \sigma^{AMM}$$

based on the paper of Akushevich and Ilyichev Phys. Rev. D 100(3) (2019) 033005

➢ Our Generator:

- as a standalone C++ MC generator
- for generating SIDIS events and calculating cross sections
- to calculate SIDIS Born cross section
- to calculate SIDIS RCs at NLO level
- from medium to high beam energies
- with incident unpolarized or longitudinally polarized beam
- with unpolarized, longitudinally or transversely polarized target
- to compute azimuthal single-target asymmetries (SSA) and double-beam-target spin asymmetries (DSA)

Computer Physics Communications 287 (2023) 108702

Contents lists available at ScienceDirect

Computer Physics Communications

journal homepage: www.elsevier.com/locate/cpc

ELSEVIER

SIDIS-RC EvGen: A Monte-Carlo event generator of semi-inclusive deep inelastic scattering with the lowest-order QED radiative corrections ☆,☆☆

Check for updates

Duane Byer [a,b], Vladimir Khachatryan [a,b,*], Haiyan Gao [a,b], Igor Akushevich [a], Alexander Ilyichev [c,d], Chao Peng [e], Alexei Prokudin [f,g], Stan Srednyak [a,b], Zhiwen Zhao [a,b]

[a] Department of Physics, Duke University, Durham, NC 27708, USA
[b] Triangle Universities Nuclear Laboratory, Durham, NC 27708, USA
[c] Belarusian State University, Minsk, 220030, Belarus
[d] Institute for Nuclear Problems, Belarusian State University, Minsk, 220006, Belarus
[e] Physics Division, Argonne National Laboratory, Lemont, IL 60439, USA
[f] Division of Science, Penn State Berks, Reading, PA 19610, USA
[g] Theory Center, Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

ARTICLE INFO

Article history:
Received 12 October 2022
Received in revised form 4 February 2023
Accepted 14 February 2023
Available online 24 February 2023

Dataset link: https://github.com/duanebyer/sidis

Keywords:
Monte-Carlo event generators for radiative events
FOAM Monte-Carlo event generator
ROOT, GSL, VEGAS, cubature packages
Semi-inclusive deep inelastic scattering
Transverse momentum-dependent distribution, fragmentation functions
QED radiative corrections

ABSTRACT

SIDIS-RC EvGen is a C++ standalone Monte-Carlo event generator for studies of semi-inclusive deep inelastic scattering (SIDIS) processes at medium to high lepton beam energies. In particular, the generator contains binary and library components for generating SIDIS events and calculating cross sections for unpolarized or longitudinally polarized beam and unpolarized, longitudinally or transversely polarized target. The structure of the generator incorporates transverse momentum-dependent parton distribution and fragmentation functions, whereby we obtain multi-dimensional binned simulation results, which will facilitate the extraction of important information about the three-dimensional nucleon structure from SIDIS measurements. In order to build this software, we have used recent elaborate QED calculations of the lowest-order radiative effects, applied to the leading order Born cross section in SIDIS. In this paper, we provide details on the theoretical formalism as well as the construction and operation of SIDIS-RC EvGen, e.g., how we handle the event generation process and perform multi-dimensional integration. We also provide example programs, flowcharts, and numerical results on azimuthal transverse single-spin asymmetries.

Program summary
Program title: SIDIS-RC EvGen
CPC Library link to program files: https://doi.org/10.17632/thrkn96ydd.1
Licensing provisions: GNU General Public License Version 3
Developer's repository link: https://github.com/duanebyer/sidis
Programming language: C++, Python
External packages: FOAM, ROOT, GSL, VEGAS, Cubature, Cog, WW-SIDIS, MSTWPDF
Nature of problem: The task is to first create a code for calculations of the leading order Born cross section as well as radiative corrections (RCs) at the next-to-leading order (NLO) of the cross section of lepton-hadron semi-inclusive deep inelastic scattering (SIDIS) at medium to high beam energies with incident unpolarized or longitudinally polarized lepton beam and unpolarized, longitudinally or transversely polarized target, enabling to compute azimuthal single-target and double-beam-target spin asymmetries. Afterwards, a Monte-Carlo event generator based upon this code is developed, where in the coding and simulation processes multi-dimensional integrals need to be calculated precisely to obtain the exact NLO RCs to the SIDIS cross section with high precision beyond ultra-relativistic limit, which means that the lepton mass is taken into account.

See also Backups

- **SIDIS-RC EvGen** hosted at https://github.com/duanebyer/sidis

- Generator's core has *sidis package* divided into two components
  - C++ library (library component), called *libsidis*, for calculating SIDIS cross sections including NLO RCs
  - MC generator (binary component), called *sidisgen*, for producing random events

- For purpose of efficiently generating events and calculating cross sections, inelastic cross-section formula being used in slightly modified form

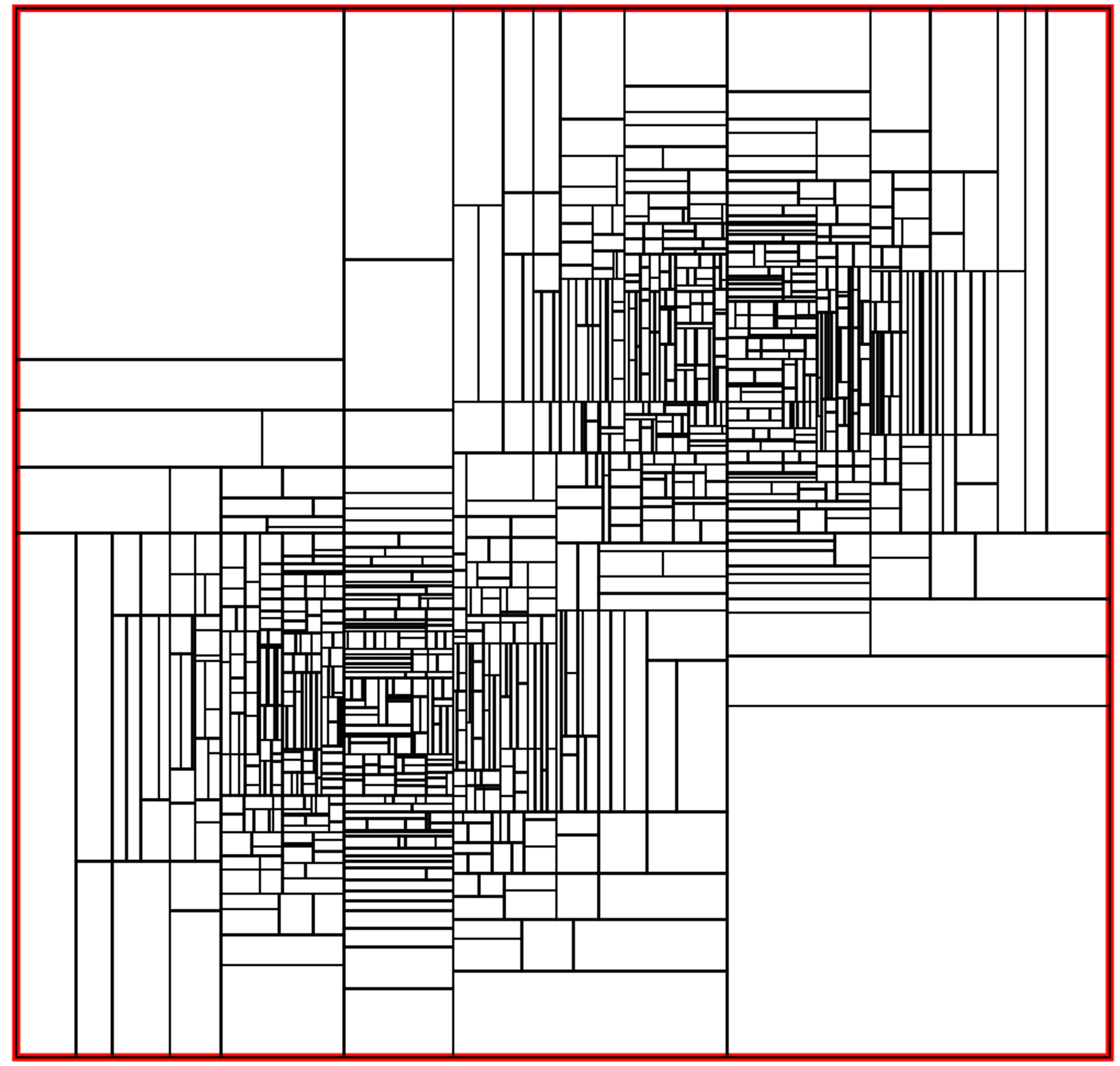
- Generated events randomly chosen to be either radiative or non-radiative, with chance proportional to total radiative/non-radiative cross-section
  - radiative cross section given as nine-fold differential cross section, $\sigma^{rad}_{\rm SIDIS}$ (six SIDIS degrees of freedom + three photon degrees of freedom)
  - non-radiative cross section given as six-fold differential cross section, $\sigma^{nrad}_{\rm SIDIS}$

- Set of functions provided by *libsidis* for computing $\sigma^{B}_{\rm SIDIS}$, $\sigma^{rad}_{\rm SIDIS}$, and $\sigma^{nrad}_{\rm SIDIS}$

- Auxiliary functions provided by *libsidis* for computing SFs, kinematic variables, and cross-section corrections

- Use FOAM library from ROOT for event generation

- FOAM library used by *sidisgen* as underlying MC engine for both non-radiative and radiative sub-generators
  - use spatial partitioning method with hyper-cubical "foam of cells" based on FOAM library
  - constructed through recursive process in which cross section is sampled randomly within each foam cell
  - cells being divided to minimize variance within each daughter cell
  - foam creates approximation of cross-section function using nested tree structure of hyper-cubes
  - foam is initialized before events can be generated
  - foam allows events to be generated with a weight close to 1 using Markov Chain Monte-Carlo method
  - foam is produced one time, and then used many times to provide events
  - during FOAM tree initialization, kinematic cuts can be provided
  - no out-of-bound events ever produced, and never need to be filtered out



Spatial indexing tree foam

- By default, use all eighteen leading-twist and subleading-twist SFs from Wandzura-Wilczek-type approximation J. High Energy Phys. 06 (2019) 007
  - MATHEMATICA implementation can be found at https://github.com/prokudin/WW-SIDIS

- Methods for specifying SFs directly as function of x_{Bj}, z_h, Q^2, P_{hT}, or on 4D grid
  - Specifying TMDs and FFs (either as functions or on grids), which are then convolved in 2D
  - Specifying TMDs and FFs, combined with Gaussian approximation, which can then be analytically convolved
    - ***Gaussian approximation***: Simplifies $k_\perp$ and $p_\perp$ dependence of TMDs and FFs, allowing for analytic evaluation of convolution integrals
  - TMDs and FFs are given in gaussian and WW approximations
    - ***Wandzura-Wilczek-type (WW) approximation***: use $\left|\frac{\langle \bar{q}gq\rangle}{\langle \bar{q}q\rangle}\right| \ll 1$ to express some TMDs and FFs in terms of others, given quark-gluon-quark, $\bar{q}gq$, correlations and quark-quark, $\bar{q}q$, correlations
    - $\langle \bar{q}gq\rangle$ and $\langle \bar{q}q\rangle$ denote matrix elements that enter definitions of TMDs or FFs
    - Reduce down to eight basis functions (six leading-twist TMDs + two leading-twist FFs) for WW approximation

- Input parameter file written by user with all parameters needed for generator
- Call sidisgen --initialize <params> to produce FOAM tree
- Call sidisgen --generate <params> to generate variable-weighted events
- Resulting events provided in ROOT file format but can be converted into other formats if needed
- Check what parameters previous set of events were generated with `sidisgen --inspect <ROOT file>`

```
num-events     3000000 # 300000000
num-init       10000
event-file     gen-2.root
# Random seed to be used for events.
#seed           0
#gen-rad        1
#gen-nrad       1
beam-energy    11.0
beam           e
target         p
hadron         pi+
mass-threshold 1.073249081
target-pol     0.0 1.0 0.0
beam-pol       0.0
# Use structure functions compiled into 'MyStructureFunctions.so'. This
# compilation process can be done easily with ROOT.
sf-set         prokudin
# Several radiative correction methods are available:
#  * none: use Born cross-section only.
#  * approx: use an approximation that neglects an expensive integral that is
#    small for small soft photon threshold.
#  * exact: calculate RC without neglecting any terms.
rc-method      none
# What energy threshold to use for dividing "soft" and "hard" events.
#soft-threshold 0.01
# Cuts.
# x-cut          0.0 0.1
# Q-sq-cut       1.0 10000.0
# theta-q-cut    0.0 0.1
k2-0-cut         1.0         7.0
theta-k2-cut     0.140070782 0.420212347
ph-0-cut         2.5         7.5
theta-h-cut      0.139626338 0.261799383
w-cut            5.29        1e10
mx-sq-cut        2.56        1e10
z-cut            0.3         0.7
```

Generator user interface

- Some important observables in electron scattering and hadronic physics to discuss
  - Transverse SSAs
  - In particular, extensively studied by HERMES, COMPASS and Jlab experiments

- SSA studies essential for cardinal understanding of nucleon 3D momentum structure

- Asymmetries generally defined as various ratios of polarized and unpolarized cross sections

$$A_{XY}^{\text{a.d.}} \equiv A_{XY}^{\text{a.d.}}(x_{Bj}, Q^2, z_h, P_{hT}) = \frac{F_{XY}^{\text{a.d.}}(x_{Bj}, Q^2, z_h, P_{hT})}{F_{UU}(x_{Bj}, Q^2, z_h, P_{hT})}$$

- Totally five SSAs with unpolarized beam and transversely polarized target existing

- Two of these SSAs given as twist-2 observables, due to Collins effect and Sivers effect

Collins transverse SSA

$$A_{UT}^{\text{Collins}} \equiv A_{UT}^{\sin(\phi_h+\phi_S)} \equiv 2\langle\sin(\phi_h+\phi_S)\rangle = 2\frac{\int_0^{2\pi} d\phi_S \int_0^{2\pi} d\phi_h \sin(\phi_h+\phi_S)\,\sigma^{\text{B}}}{\int_0^{2\pi} d\phi_S \int_0^{2\pi} d\phi_h\,\sigma^{\text{B}}} = \frac{c_2}{c_1}\frac{F_{UT}^{\sin(\phi_h+\phi_S)}}{F_{UU}}$$

Sivers transverse SSA

$$A_{UT}^{\text{Sivers}} \equiv A_{UT}^{\sin(\phi_h-\phi_S)} \equiv 2\langle\sin(\phi_h-\phi_S)\rangle = 2\frac{\int_0^{2\pi} d\phi_S \int_0^{2\pi} d\phi_h \sin(\phi_h-\phi_S)\,\sigma^{\text{B}}}{\int_0^{2\pi} d\phi_S \int_0^{2\pi} d\phi_h\,\sigma^{\text{B}}} = \frac{1}{c_1}\frac{F_{UT}^{\sin(\phi_h-\phi_S)}}{F_{UU}}$$

- Collins effect emerging from convolution of transversity TMD and Collins FF

- Sivers effect stemming from convolution of Sivers TMD and Unpolarized FF

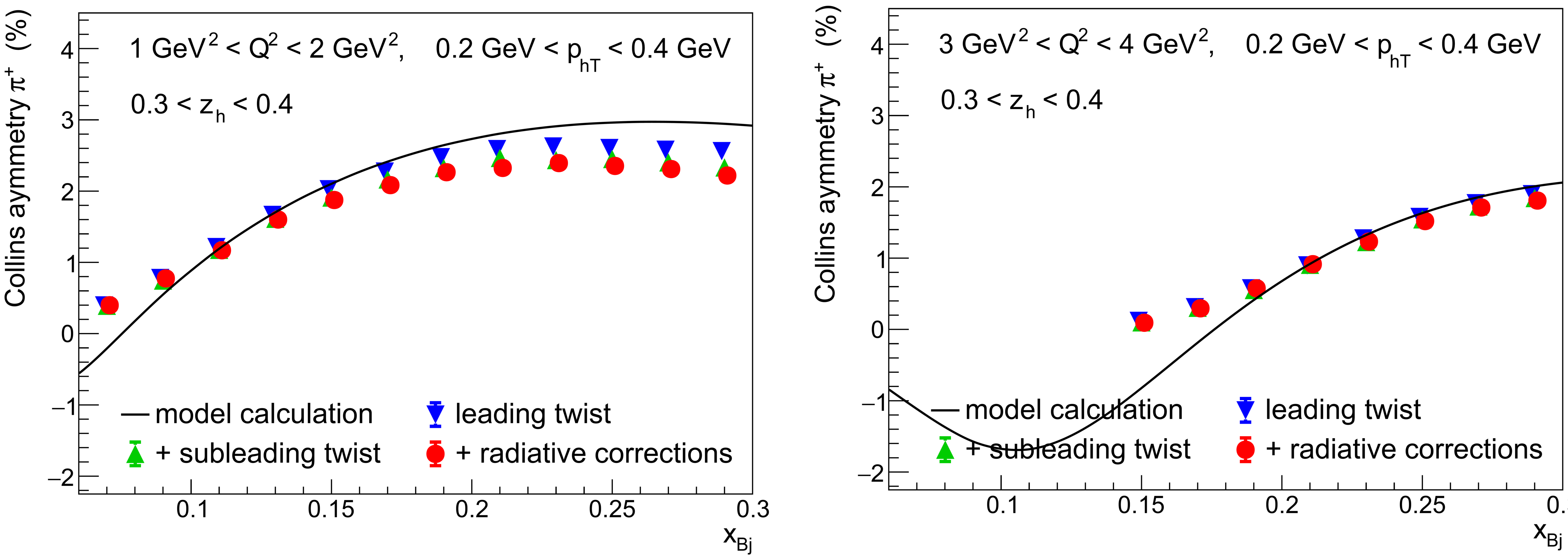
- General form of transverse non-separated SSA can be written with all **three twist-2 (or leading-twist)** and **two twist-3 (or subleading-twist)** terms

$$A_{UT} = A_{UT}^{\text{Collins}} \sin(\phi_h + \phi_s) + A_{UT}^{\text{Sivers}} \sin(\phi_h - \phi_s) + \\ + A_{UT}^{\text{Pretzelosity}} \sin(3\phi_h - \phi_s) + A_{UT}^{\text{sl-t1}} \sin(\phi_s) + A_{UT}^{\text{sl-t2}} \sin(2\phi_h - \phi_s)$$

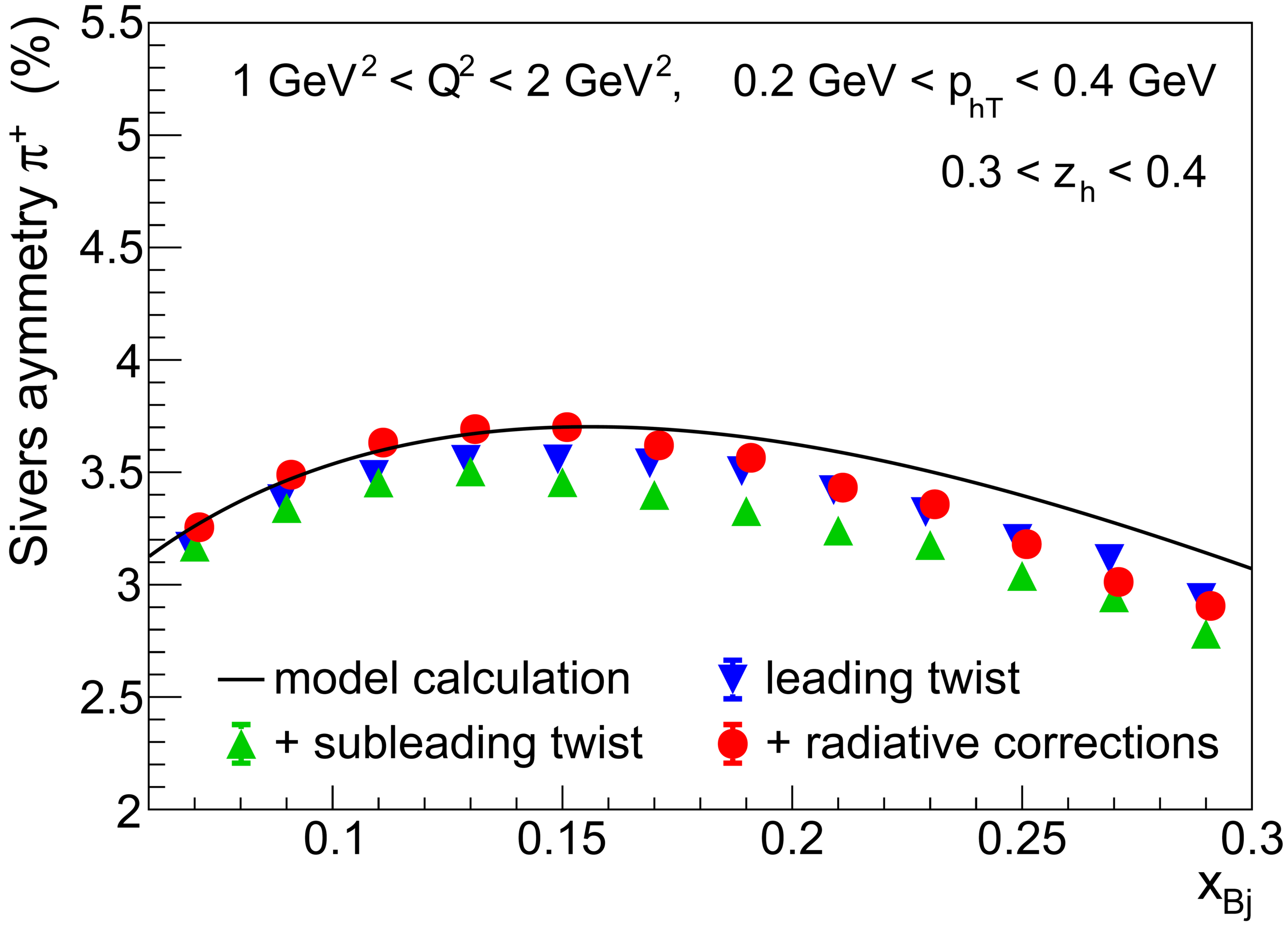
- Address following questions with all five transverse SSAs
  - whether one can provide high precision test of lattice QCD predictions via tensor charge
  - whether there are clear signatures of relativistic effects inside nucleon
  - how one can extract quantitative information about contribution of quark orbital angular momentum to proton spin
  - how to quantify quark transverse motion inside nucleon and observe spin-orbit correlations

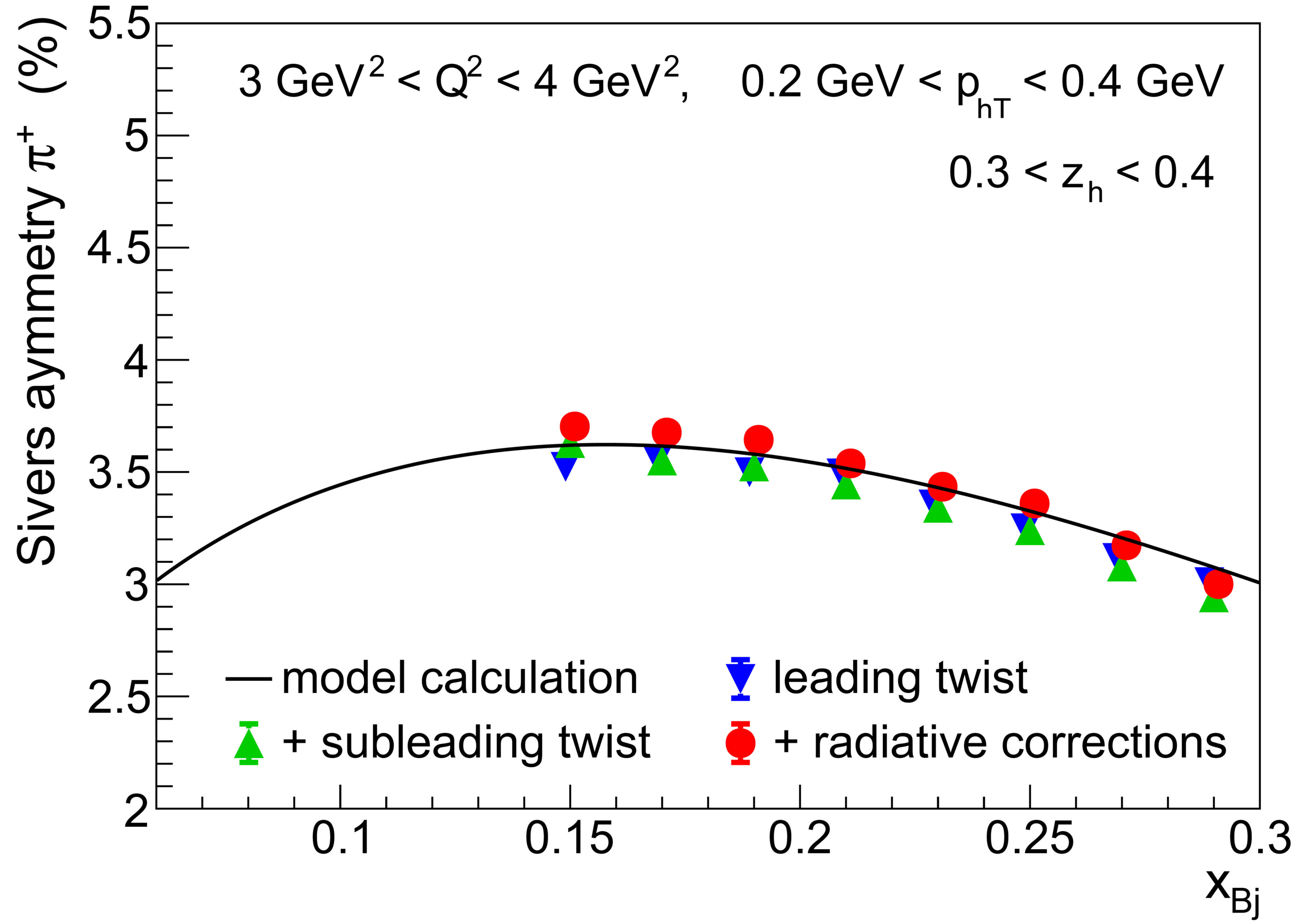
- Next, we can see RC effects on both Collins and Sivers asymmetries

- Figures showing **Collins SSA** for positively charged pions as function of x_{Bj} in given kinematic bins of Q^2, z_h, and P_{hT}



➢ Figures showing **Sivers SSA** for positively charged pions as function of x_{Bj} in given kinematic bins of Q^2, z_h, and P_{hT}





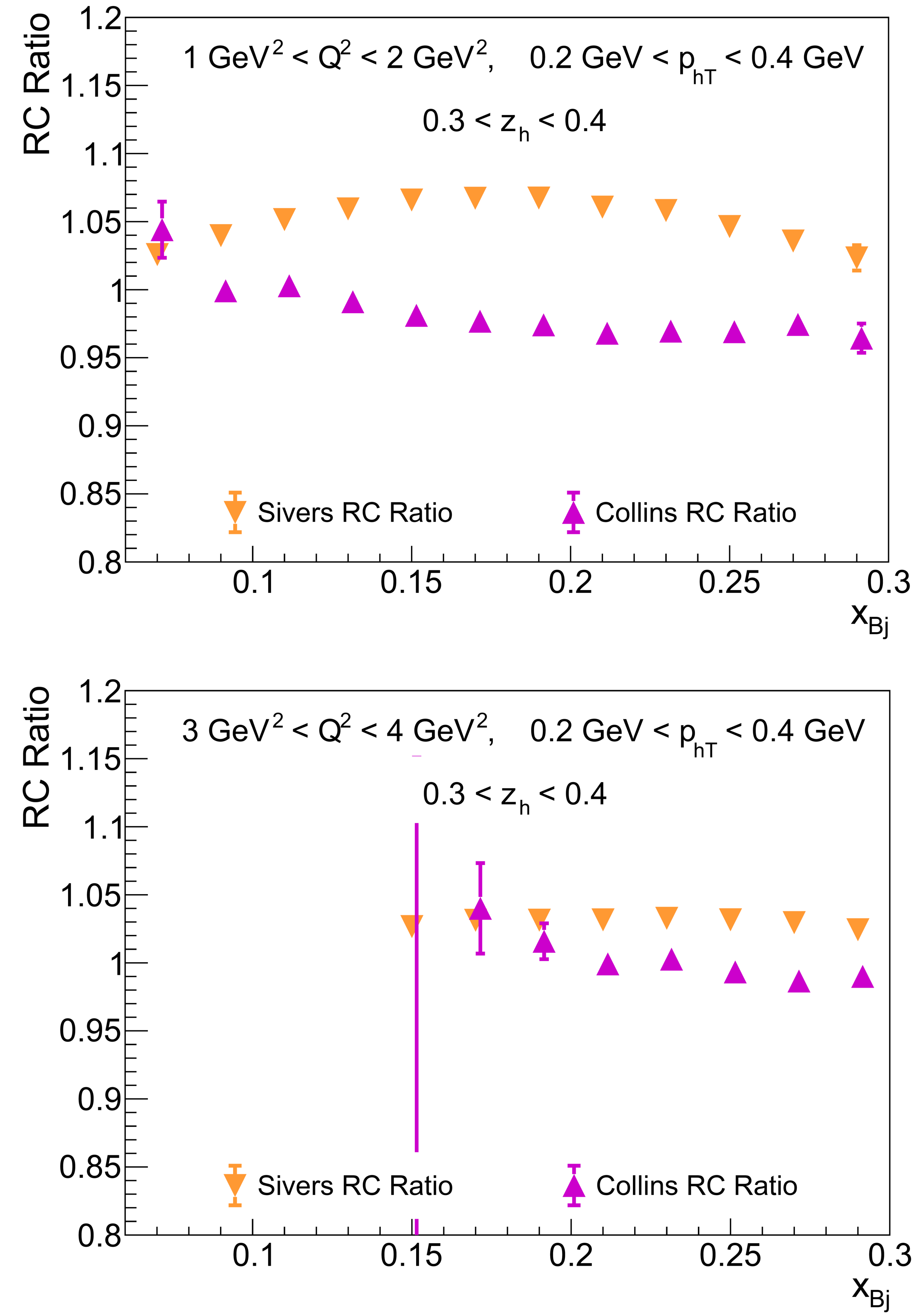
➢ Figures showing ratios describing RC effects on both **Collins SSA** and **Sivers SSA**

$$\text{RC Ratio} = \frac{A_{UT}^{\text{Collins}|\text{Sivers}}|_{RC}}{A_{UT}^{\text{Collins}|\text{Sivers}}}$$

Ratios of red circle pseudo-data to green upward triangle pseudo-data shown in previous two slides

Ratios demonstrating lowest-order RC effects on **Collins** and **Sivers SSAs** when subleading-twist effects are also considered

Large error bars of Collins RC ratio in the lowest bin of $x_{Bj} \sim 0.15$ appearing because Collins asymmetry drops to near zero at this specific point



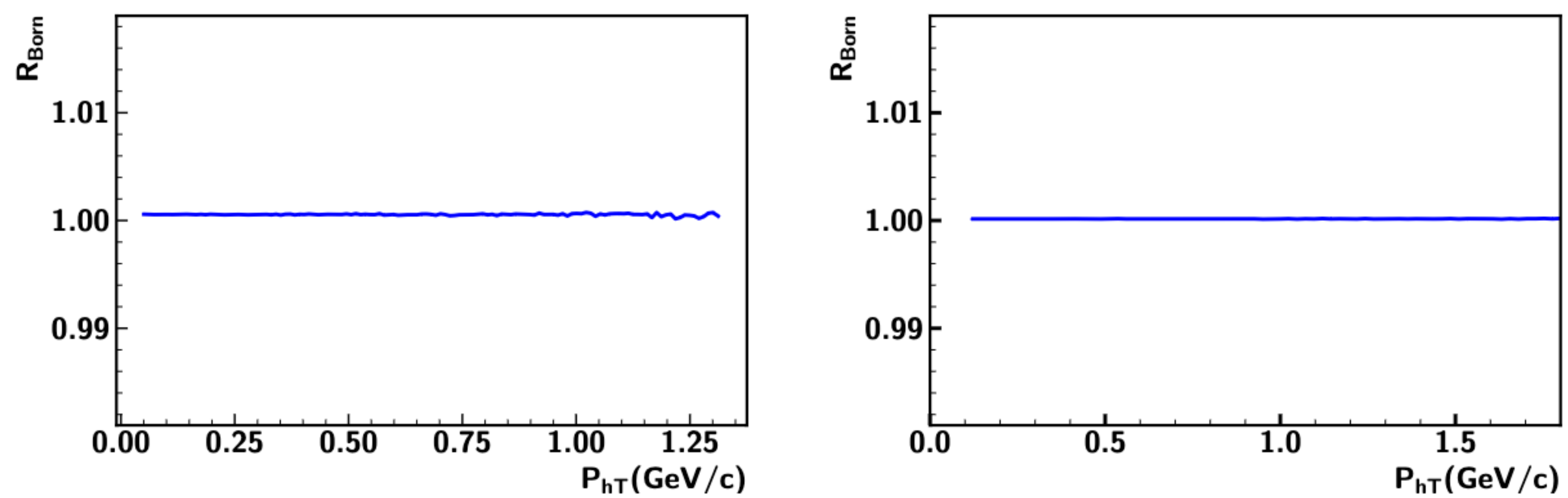
- Devoted to comparisons of some RC results between
  conventional approach
  and
  factorized approach

- QCD-like factorized approach to inclusive DIS and SIDIS developed in paper of
  Liu et al., J. High Energy Phys. 11 (2021) 157
  - treating QED and QCD radiation on equal footing
  - giving good approximation for QED radiative contributions by collinear factorization
  - providing improved approximation to extraction of TMDs

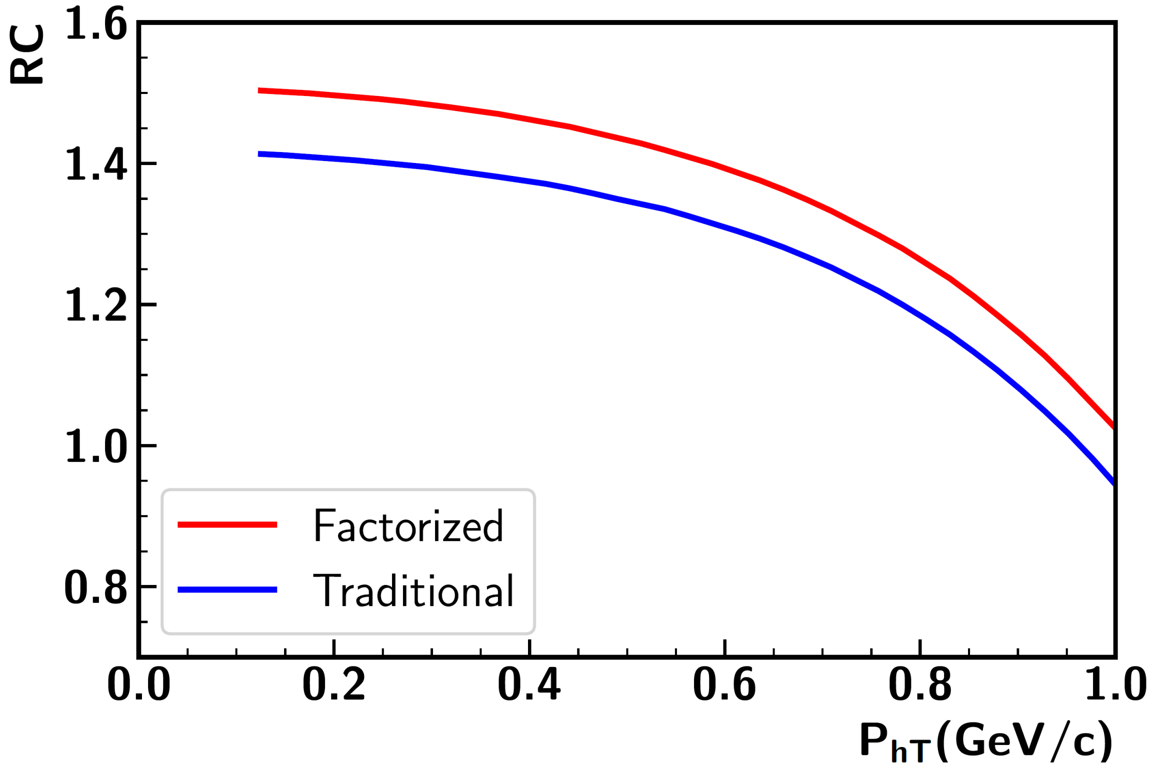
Radiative Correction Factor for Semi-Inclusive Deep Inelastic Scattering

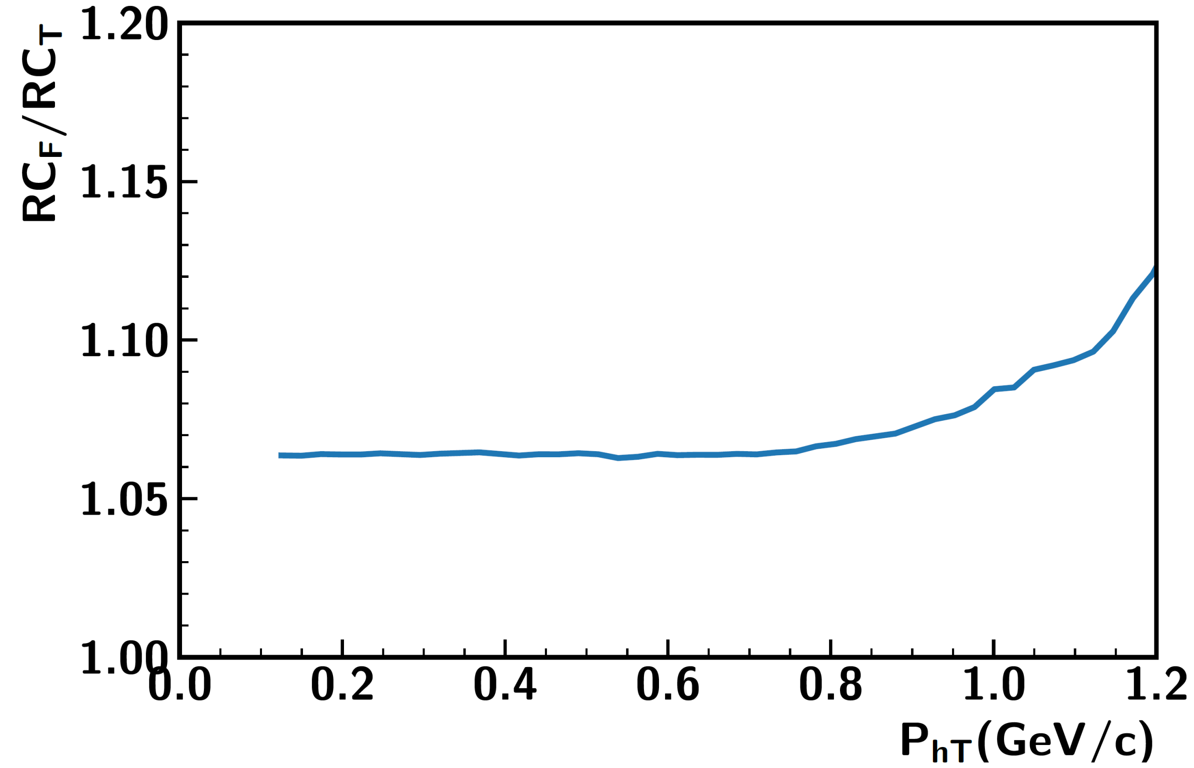
Bishnu Karki, Duane Byer, Shuo Jia, and Haiyan Gao
Duke University
(Dated: September 16, 2023)

Semi-inclusive deep inelastic scattering (SIDIS) is proven to be a powerful prob in accessing the partonic structure TMDs. The Radiative Correction (RC) for the SIDIS process becomes significant in future Electron-Ion Collider (EIC) and Jefferson Lab kinematics with a wide phase space. This work compares the RC using two methods, the traditional method and the factorized method, on several typical JLab and EIC kinematics for SIDIS $H(e, e'\pi^+)X$ reaction and Sivers and Collins asymmetries. Using the same structure functions, the Born level cross-section and angular modulations are nearly identical. The RC difference from the two methods changes from 3 % to 6 % depending on different kinematics.



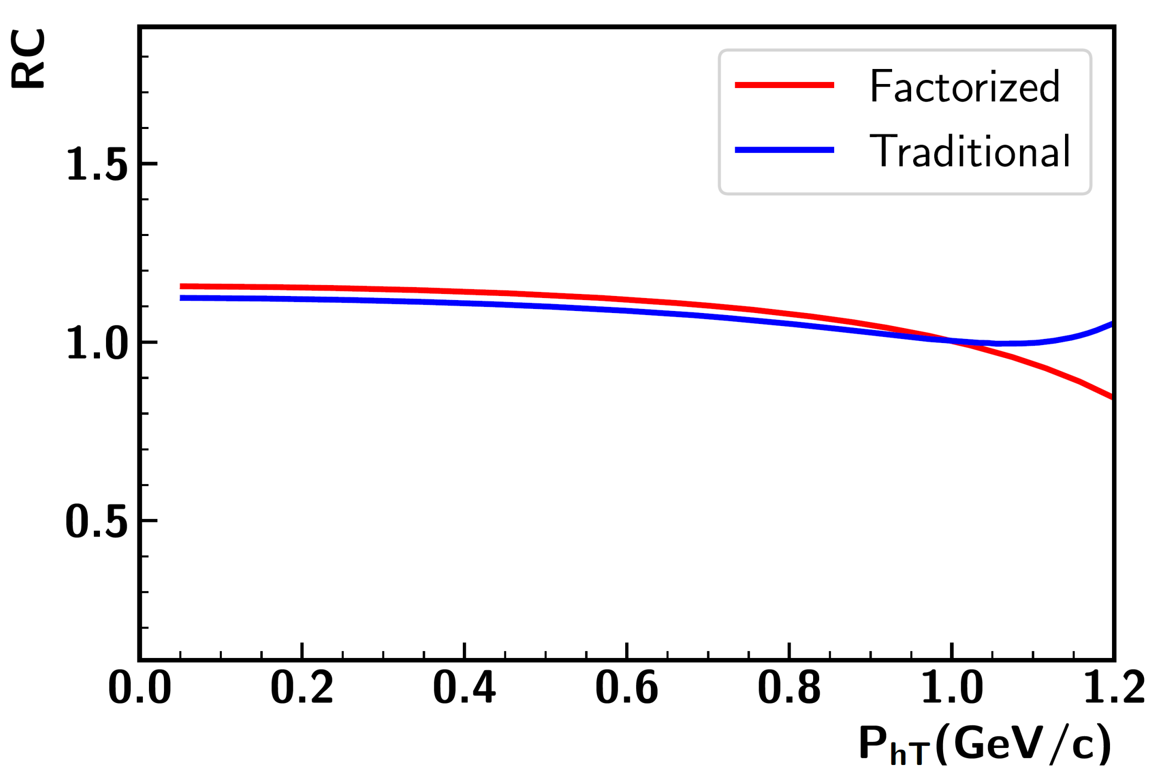
Left: JLab kinematic $\sqrt{s} = 4.9$ GeV, $Q^2 = 8\ (\mathrm{GeV/c})^2$, $z_h = 0.375$, $x_{Bj} = 0.48$
Right: EIC kinematic $\sqrt{s} = 140$ GeV, $Q^2 = 25\ (\mathrm{GeV/c})^2$, $z_h = 0.5$, $x_{Bj} = 0.01$
Using JAM3D20 Structure function, Born level cross section from two methods are nearly identical.

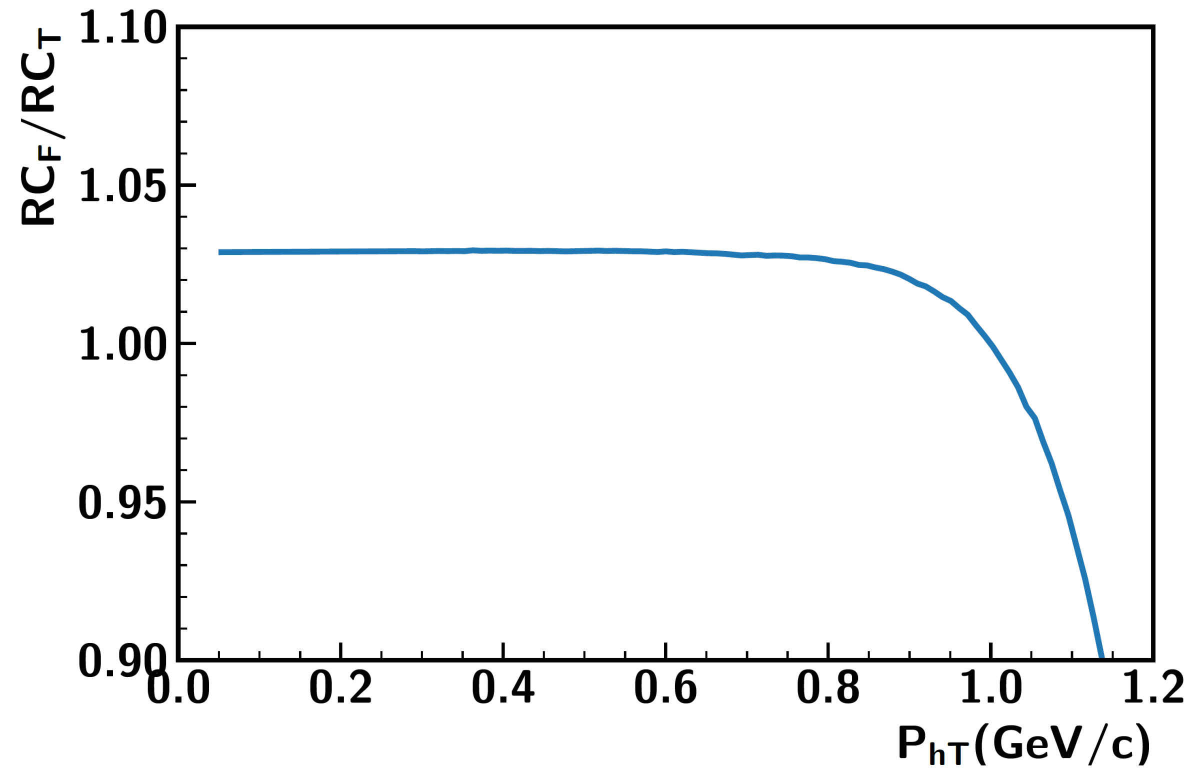




Left panel: RC factor computed using factorized approach (RC_F) to RC factor computed using traditional / conventional approach (RC_T) at $\sqrt{s} = 140$ GeV, $Q^2 = 25\ (\mathrm{GeV/c})^2$, $z_h = 0.5$, $x_{Bj} = 0.01$, $\phi_h = 0$

Right panel: RC factor ratios from two methods at given **EIC** kinematics



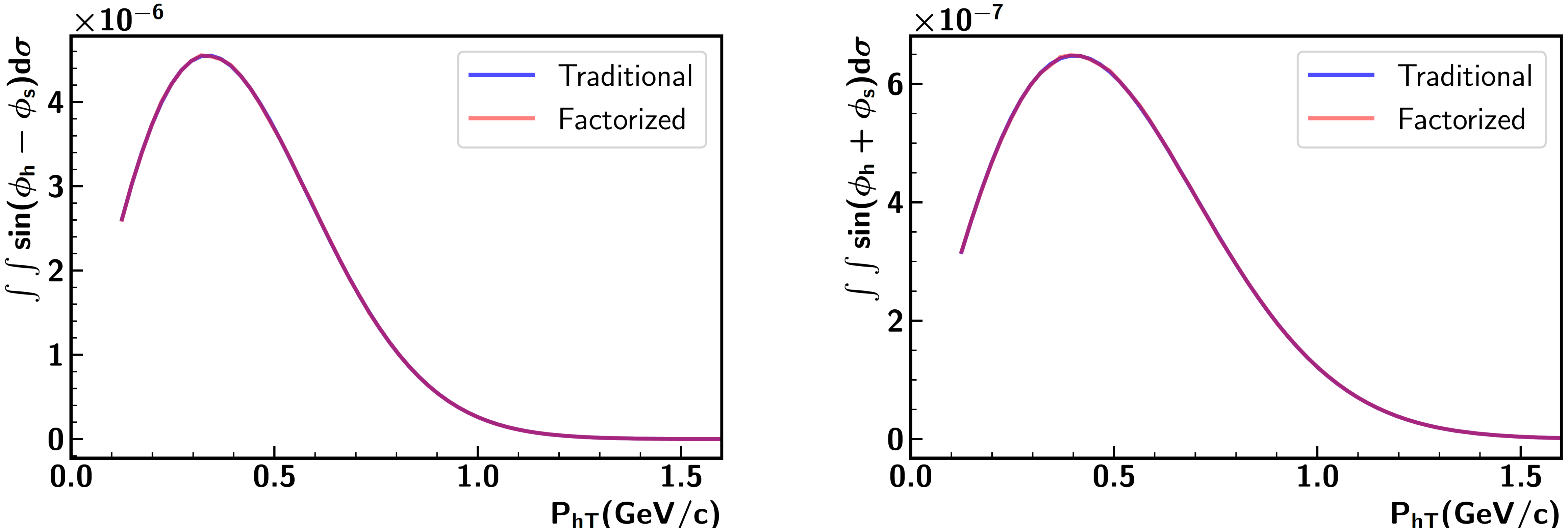


Left panel: RC factor computed using factorized approach (RC_F) to RC factor computed using traditional / conventional approach (RC_T) at $\sqrt{s} = 4.9$ GeV, $Q^2 = 8\ (\mathrm{GeV/c})^2$, $z_h = 0.375$, $x_{Bj} = 0.48$, $\phi_h = 0$

Right panel: RC factor ratios from two methods at given **JLab** kinematics

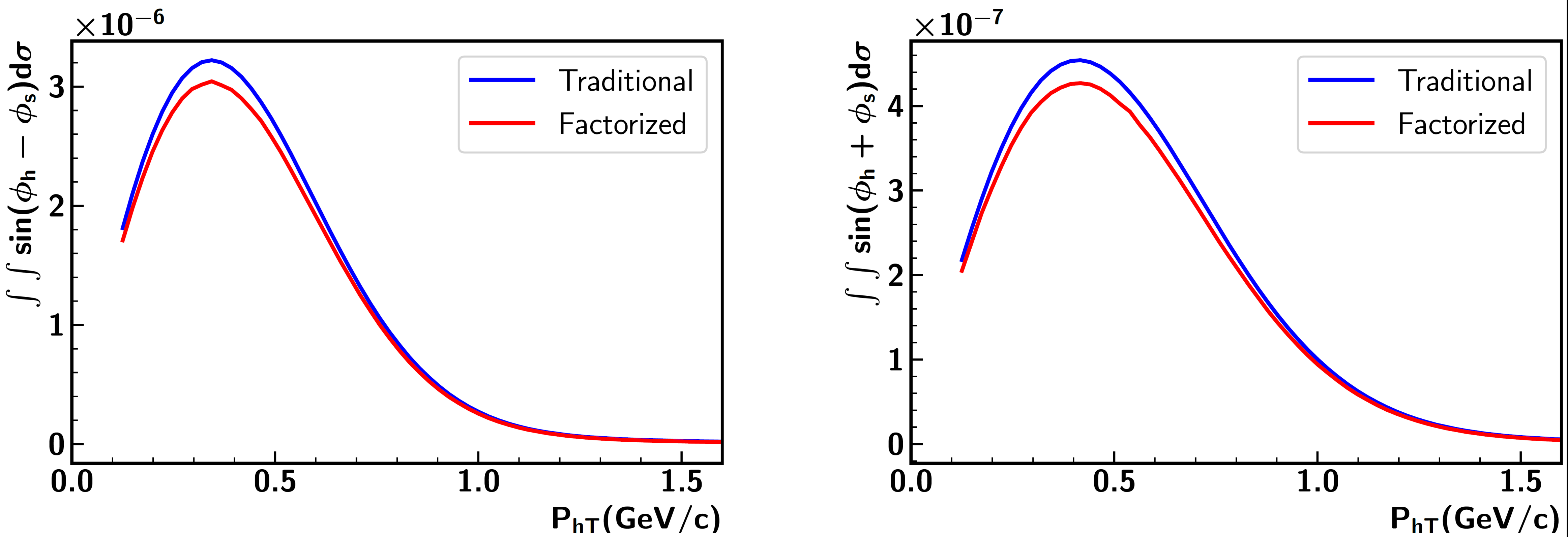
$\sqrt{s}$ (GeV)	x_B	Q^2 $(\mathrm{GeV}/c)^2$	z_h	RC ratio
Jefferson Lab Kinematics				
3.2	0.32	2.3	0.55	1.025
4.9	0.48	8	0.375	1.025
6.7	0.48	15	0.375	1.025
EIC Kinematics				
140	0.01	9	0.5	1.042
140	0.01	25	0.5	1.038
140	0.01	100	0.5	1.06

RC ratios from factorized approach is up to 6% larger compared to those from traditional approach in EIC kinematics



Comparison of angular modulations as function of P_{hT} between two different approaches **in absence of QED radiation**. Plots are for **EIC** kinematics at $\sqrt{s} = 140$ GeV, $Q^2 = 25\ (\mathrm{GeV/c})^2$, $z_h = 0.5$, $x_{Bj} = 0.01$, $\phi_h = 0$

Left: angular modulation $\sin(\phi_h - \phi_S)$. Right: angular modulation $\sin(\phi_h + \phi_S)$

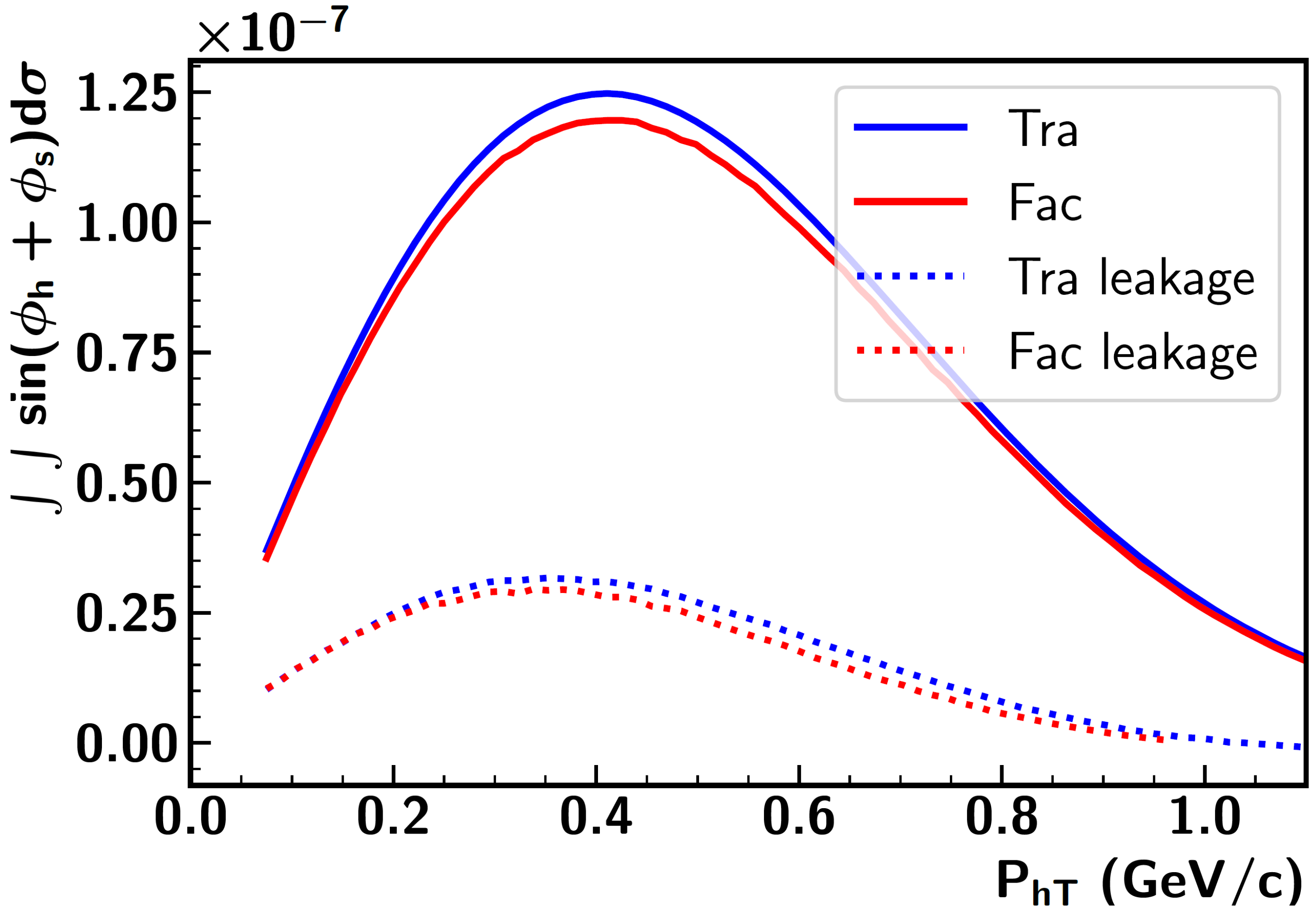


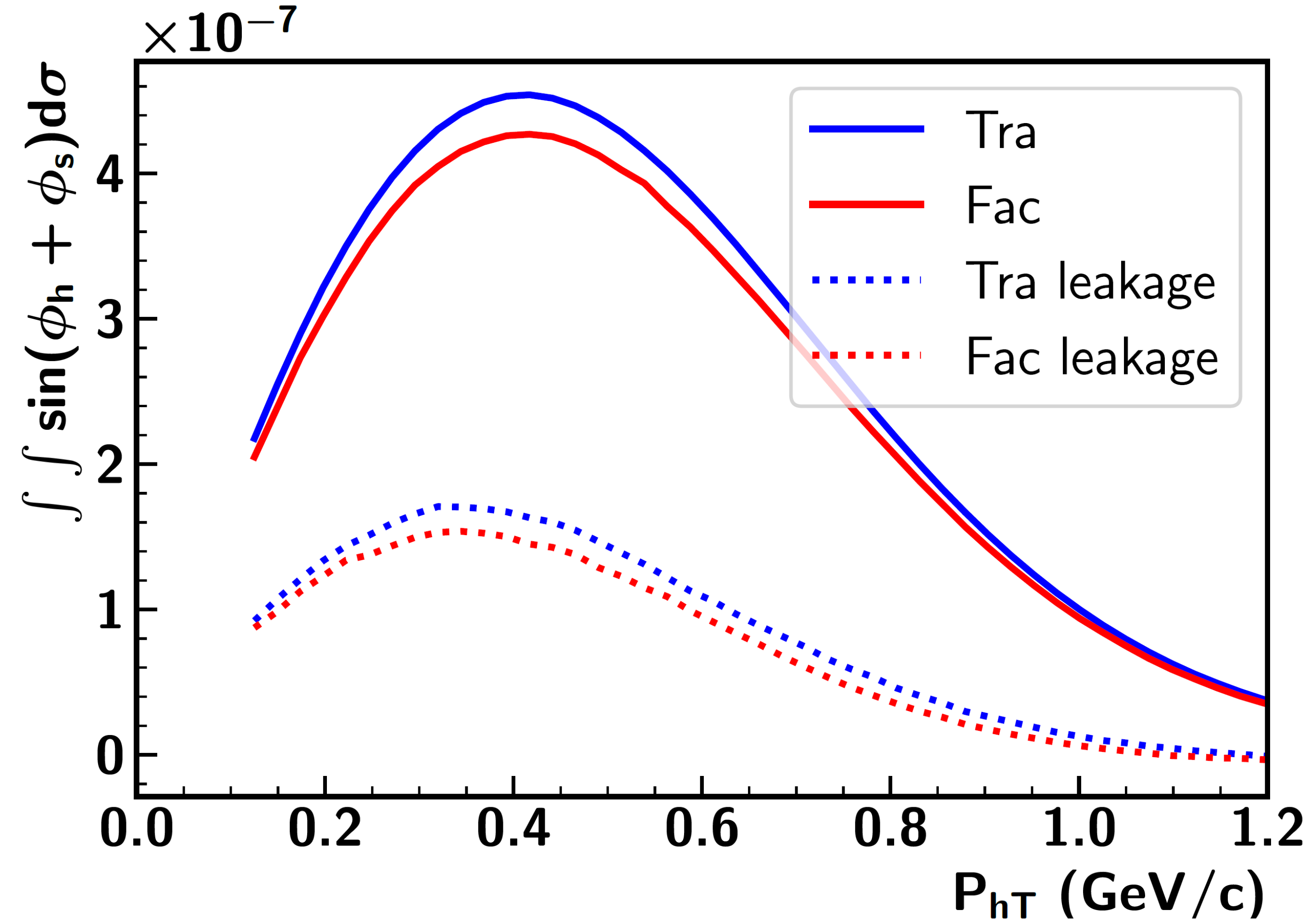
Comparison of angular modulations as function of P_{hT} between two different approaches **in presence of QED radiation**. Plots are for **EIC** kinematics at $\sqrt{s} = 140$ GeV, $Q^2 = 25\ (\mathrm{GeV/c})^2$, $z_h = 0.5$, $x_{Bj} = 0.01$, $\phi_h = 0$

Left: angular modulation $\sin(\phi_h - \phi_S)$. Right: angular modulation $\sin(\phi_h + \phi_S)$

$\sqrt{s}$ (GeV)	x_B	Q^2 $(GeV/c)^2$	z_h	Unpol ratio	Siv ratio	Col ratio
Jefferson Lab Kinematics						
3.2	0.32	2.3	0.55	1.025	1.025	1.025
4.9	0.48	8	0.375	1.025	1.03	1.025
6.7	0.48	15	0.375	1.025	1.03	1.03
EIC Kinematics						
140	0.01	9	0.5	1.042	1.042	1.05
140	0.01	25	0.5	1.038	1.043	1.047
140	0.01	100	0.5	1.065	1.06	1.06

Comparison of RC ratios between factorized and traditional approaches at different **JLab** and **EIC** kinematics





Comparison of Collins $\sin(\phi_h + \phi_S)$ angular modulation "leakage" as function of P_{hT} between two different approaches **in presence of QED radiation**.

Left: For **EIC** kinematics at $\sqrt{s} = 140$ GeV, $Q^2 = 9\ (\mathrm{GeV/c})^2$, $z_h = 0.5$, $x_{Bj} = 0.01$, $\phi_h = 0$

Right: For **EIC** kinematics at $\sqrt{s} = 140$ GeV, $Q^2 = 25\ (\mathrm{GeV/c})^2$, $z_h = 0.5$, $x_{Bj} = 0.01$, $\phi_h = 0$

$\sqrt{s}$ (GeV)	x_B	Q^2 $(GeV/c)^2$	z_h	Unpol ratio	Siv ratio	Col ratio	Col ratio including leakage
EIC Kinematics							
140	0.01	9	0.5	1.042	1.042	1.05	1.05
140	0.01	25	0.5	1.038	1.043	1.047	1.04
140	0.01	100	0.5	1.065	1.06	1.06	1.06

Comparison of RC ratios between factorized and traditional approaches at different EIC kinematics including "leakage" effect

- C++ coded standalone MC event generator called **SIDIS-RC EvGen** presented in this talk
  - SIDIS twist-2 and twist-3 SFs (TMDs & FFs) used in Gaussian and WW-type approximations
  - Generator's structure and functionality focusing on its library component for computing cross sections and binary (generator) component for event generation
  - Inelastic tail to SIDIS six-fold differential cross section can be obtained, including RCs
  - Partonic structure of nucleon in 3D momentum space, using SIDIS with lowest-order RCs

- Extensively used for making predictions and preparations for designed experiments
  - Some physics models and frameworks used for underlying event generation to understand various systematic effects in studies of various observables
  - RCs in SIDIS giving one source of such systematic effects

- The traditional method used for RC for the generator is compared with factorized method
  - Using same structure function, the Born cross section from two methods are almost identical
  - RCs ranges from 3% to 6% depending on different kinematics.

➢ Future developments and potential prospects related to **SIDIS-RC EvGen**

- Increase generator's efficiency by improving foam efficiency, or even replace FOAM algorithm by VEGAS algorithm;
- Provide with other options for including state-of-the-art parameterizations of TMDs, used in most recent and upcoming phenomenological studies
- Incorporate exclusive SFs into generator's current framework
- Implement neutron SFs in addition to proton SFs
- Improve parameterization that describes contribution of vacuum polarization by hadrons
  (i) making use of most recent hadronic data for fitting
  (ii) advanced calculations
  (iii) and software package *alphaQED* of Fred Jegerlehner
- Include higher-order SIDIS RCs in generator's framework
- Compare generator's output with data
  (i) with HERMES, COMPASS and JLab SSA data
  (ii) with HERMES, COMPASS and JLab charged hadron multiplicity data
  (iii) and make predictions for EIC
- Incorporate **SIDIS-RC EvGen** into detector simulations, allowing precise predictions and verification for some key aspects of entire experimental setups, such as SoLID, CLAS12 at Jlab, and EIC

Thanks !

Backups

- 3D nucleon structure in momentum space encoded in transverse momentum dependent distribution and fragmentation functions, TMD PDFs and TMD FFs, or just TMDs and FFs

- TMDs and FFs being generalizations of collinear PDFs and fragmentation functions appearing in the standard collinear factorization

- TMDs and FFs depending on two independent variables: x_{Bj} and $k_\perp$ for TMDs, as well as z_h and $p_\perp$ for FFs
  - $k_\perp$ to be parton (quark) intrinsic transverse momentum
  - z_h to be fraction of quark momentum transferred to produced (final-state) hadron
  - $p_\perp$ to be transverse momentum of the same hadron with respect to direction of fragmenting quark

- QCD factorization theorems proven for processes with two distinct measured scales, $Q_1 \ll Q_2$
  - For SIDIS process, Drell-Yan process, and production of two hadrons in e^+e^--annihilation

- For SIDIS
  - transverse momentum P_{hT} of produced charged hadron with respect to virtual photon momentum playing role of small scale Q_1
  - virtual photon virtuality Q playing role of large scale Q_2

- Currently, exact and more general results existing on six-fold differential cross section

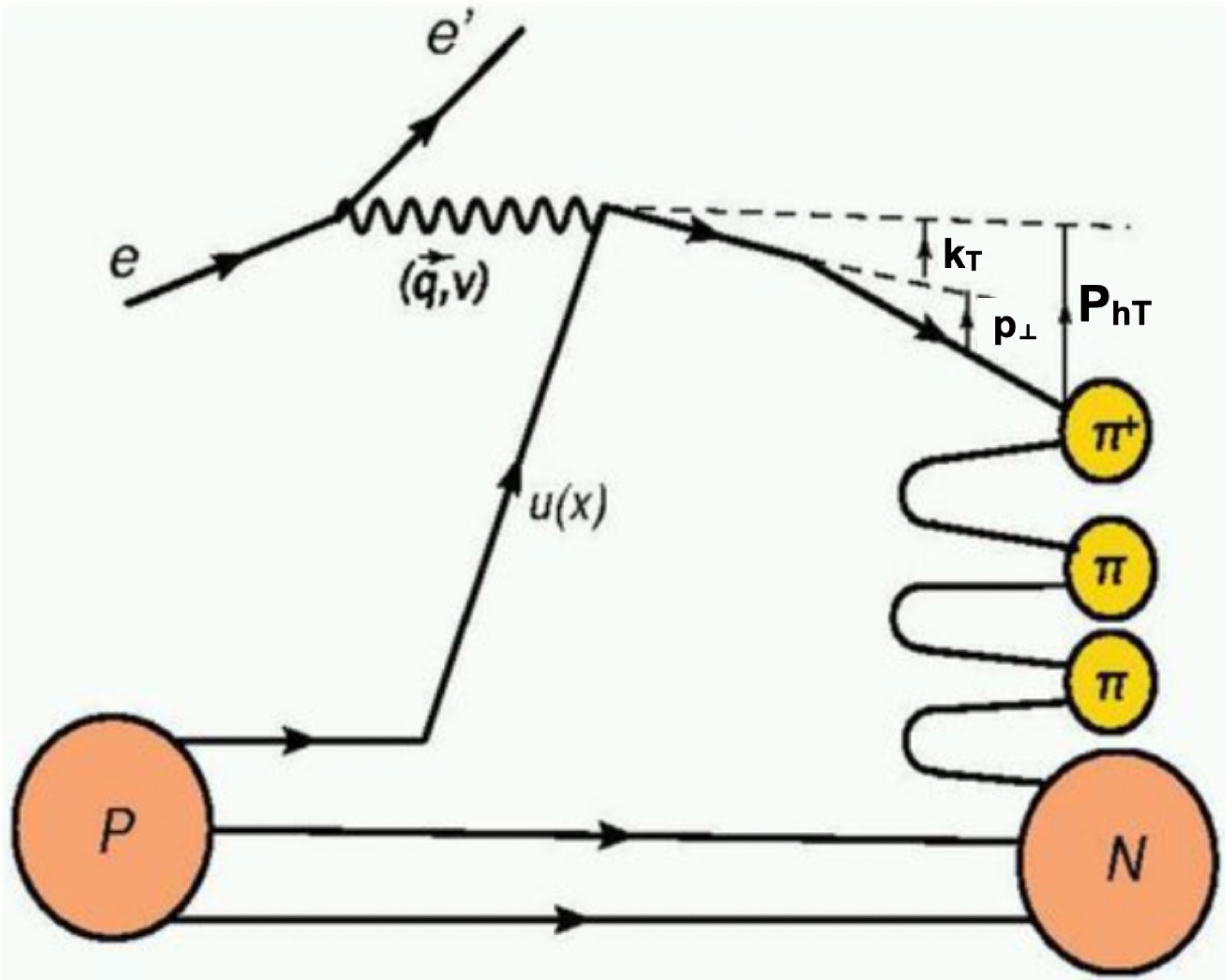
$$\frac{d\sigma_{\text{SIDIS}}}{dx_{Bj}dydz_h dP_{hT}^2 d\phi_h d\phi_S}$$

  - variable ϕ_S to be another azimuthal angle describing target-spin direction (spin-vector), if transversely polarized targets are applied

- MC event generator for SIDIS processes including RCs created (called **SIDIS-RC EvGen**)
  - to generate radiative and non-radiative channels of scattering;
    for generated events to be selected as either radiative or non-radiative, with probability of being proportional to radiative/non-radiative cross section
  - to generate scattered lepton kinematics (i.e., Q^2 and x_{Bj}) and final-state hadron kinematics (i.e., z_h, P_{hT}, and ϕ_h)
  - to generate real photon radiation kinematics
  - to calculate full SIDIS cross section in any generated phase-space point with RCs included

- **SIDIS-RC EvGen** aiming to aid in multifaceted efforts for studying
  - TMD evolution effects
  - nucleon internal spin structure, spin-orbit and quark-gluon correlations
  - and nucleon 3D momentum structure in general

Illustration of internal structure of ep scattering process



- MC and computation frameworks for evaluation of RCs in SIDIS much needed
  - to evaluate RC effects, for example, by having information obtained during event generation

- Examples below from RADIATIVE CORRECTION HELPDESK https://ily.hep.by/rc.html

- RADGEN 1.0: MC generator of polarized/unpolarized DIS radiative events
  - applicable for RC generation in inclusive, semi-inclusive and exclusive DIS processes

- ELRADGEN 2.0: MC generator for simulation of radiative events in elastic *ep* scattering of polarized particles

- POLRAD 2.0: FORTRAN code for treating experimental data with implemented RC procedure, in polarized inclusive and semi-inclusive DIS

- HAPRAD 2.0: FORTRAN code for calculation of RCs to semi-inclusive hadron leptoproduction including radiative tail from exclusive hadron production
  - performs RC calculations to five-fold differential cross section of unpolarized particles

$$\frac{d\sigma_{\mathrm{SIDIS}}}{dx_{Bj}\,dy\,dz_h\,dP_{hT}^2\,d\phi_h}$$

  - variable y to be lepton beam energy fraction carried by virtual photon
  - variable ϕ_h to be hadron azimuthal angle measured with respect to lepton scattering plane

See slide 7

- SIDIS differential cross section expressed by set of eighteen SFs at leading and subleading order in 1/Q expansion,
  in terms of asymmetries A_{XY}^{weight}

$$A_{XY}^{\text{weight}} \equiv A_{XY}^{\text{weight}}(x,z,P_{hT}) = \frac{F_{XY}^{\text{weight}}(x,z,P_{hT})}{F_{UU}(x,z,P_{hT})}$$

$$\begin{aligned}
\sigma_{\text{SIDIS}}^{B} = & \frac{\alpha^2}{x_{Bj}\, y\, Q^2}\left(1+\frac{\gamma^2}{2x_{Bj}}\right) c_1\, F_{UU} \times \\
& \times \Bigg\{\Bigg[1+\left(\frac{c_3}{c_1}\right)\cos(\phi_h)\, A_{UU}^{\cos(\phi_h)} + \\
& + \left(\frac{c_2}{c_1}\right)\cos(2\phi_h)\, A_{UU}^{\cos(2\phi_h)} + \lambda_e\left(\frac{c_4}{c_1}\right)\sin(\phi_h)\, A_{LU}^{\sin(\phi_h)}\Bigg] + \\
& + S_L\left[\left(\frac{c_3}{c_1}\right)\sin(\phi_h)\, A_{UL}^{\sin(\phi_h)} + \left(\frac{c_2}{c_1}\right)\sin(2\phi_h)\, A_{UL}^{\sin(2\phi_h)}\right] + \\
& + S_L\lambda_e\left[\left(\frac{c_5}{c_1}\right) A_{LL} + \left(\frac{c_4}{c_1}\right)\cos(\phi_h)\, A_{LL}^{\cos(\phi_h)}\right] + \\
& + S_T\Bigg[\sin(\phi_h-\phi_S)A_{UT,T}^{\sin(\phi_h-\phi_S)} + \\
& + \left(\frac{c_2}{c_1}\right)\sin(\phi_h+\phi_S)\, A_{UT}^{\sin(\phi_h+\phi_S)} + \left(\frac{c_2}{c_1}\right)\sin(3\phi_h-\phi_S)\, A_{UT}^{\sin(3\phi_h-\phi_S)} + \\
& + \left(\frac{c_3}{c_1}\right)\sin(\phi_S)\, A_{UT}^{\sin(\phi_S)} + \left(\frac{c_3}{c_1}\right)\sin(2\phi_h-\phi_S)\, A_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] + \\
& + S_T\lambda_e\Bigg[\left(\frac{c_5}{c_1}\right)\cos(\phi_h-\phi_S)\, A_{LT}^{\cos(\phi_h-\phi_S)} + \left(\frac{c_4}{c_1}\right)\cos(\phi_S)\, A_{LT}^{\cos(\phi_S)} + \\
& + \left(\frac{c_4}{c_1}\right)\cos(2\phi_h-\phi_S)\, A_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},
\end{aligned}$$

- In *XY* subscripts of most SFs
  - *X* = *U* / *L* referring to unpolarized or longitudinally polarized beam
  - *Y* = *U* / *L* referring to unpolarized or longitudinally polarized target with respect to *q*
  - *Y* = *U* / *T* referring to unpolarized or transversely polarized target with respect to *q*

- In *XY*,*Z* subscripts of remaining SFs
  - *Z* = *T* / *L* giving virtual photon polarizations

See slide 7

Leading order SIDIS cross section

$$\frac{d^6\sigma_{\text{leading}}}{dx\,dy\,dz\,d\psi_l\,d\phi_h\,dP_{hT}^2} = \frac{\alpha_{em}^2}{x\,y\,Q^2}\left(1-y+\frac{1}{2}y^2\right) F_{UU}(x,z,P_{hT}^2)$$
$$\times\Big\{ 1+\cos(2\phi_h)\,p_1\,A_{UU}^{\cos(2\phi_h)} + S_L\sin(2\phi_h)\,p_1\,A_{UL}^{\sin(2\phi_h)} + \lambda\,S_L\,p_2\,A_{LL}$$
$$+\ S_T\sin(\phi_h-\phi_S)\,A_{UT}^{\sin(\phi_h-\phi_S)} +\ S_T\sin(\phi_h+\phi_S)\,p_1\,A_{UT}^{\sin(\phi_h+\phi_S)}$$
$$+\ S_T\sin(3\phi_h-\phi_S)\,p_1\,A_{UT}^{\sin(3\phi_h-\phi_S)} + \lambda\,S_T\cos(\phi_h-\phi_S)\,p_2\,A_{LT}^{\cos(\phi_h-\phi_S)}\Big\}.$$

Subleading order

$$\frac{d^6\sigma_{\text{subleading}}}{dx\,dy\,dz\,d\psi_l\,d\phi_h\,dP_{hT}^2} = \frac{\alpha_{em}^2}{x\,y\,Q^2}\left(1-y+\frac{1}{2}y^2\right) F_{UU}(x,z,P_{hT}^2)\Big\{\cos(\phi_h)\,p_3\,A_{UU}^{\cos(\phi_h)}$$
$$+\lambda\sin(\phi_h)\,p_4\,A_{LU}^{\sin(\phi_h)} + S_L\sin(\phi_h)\,p_3\,A_{UL}^{\sin(\phi_h)} + \lambda\,S_L\cos(\phi_h)\,p_4\,A_{LL}^{\cos(\phi_h)}$$
$$+S_T\sin(2\phi_h-\phi_S)\,p_3\,A_{UT}^{\sin(2\phi_h-\phi_S)} + S_T\sin(\phi_S)\,p_3\,A_{UT}^{\sin(\phi_S)}$$
$$+\lambda\,S_T\cos(\phi_S)\,p_4\,A_{LT}^{\cos(\phi_S)} + \lambda\,S_T\cos(2\phi_h-\phi_S)\,p_4\,A_{LT}^{\cos(2\phi_h-\phi_S)}\Big\}. \qquad (2.2b)$$

See slide 8

- Target covariant spin-vector $S\nu$ being decomposed as

$$S\nu^{\mu} = S_L \frac{P^{\mu} - \left[q^{\mu} M_N^2/(P\cdot q)\right]}{M_N\sqrt{1+\gamma^2}} + S_T^{\mu},$$

$$\text{with} \quad S_L = \frac{S\nu\cdot q}{P\cdot q}\frac{M_N}{\sqrt{1+\gamma^2}}, \quad \text{and} \quad S_T^{\mu} = g_{\perp}^{\mu\nu} S\nu_{\nu},$$

- For transversely polarized targets, angle ϕ_S defined as

$$\cos(\phi_S) = -\frac{k_{1\mu} S_\nu g_{\perp}^{\mu\nu}}{\sqrt{k_{1T}^2 S_T^2}}, \qquad \sin(\phi_S) = -\frac{k_{1\mu} S_\nu \epsilon_{\perp}^{\mu\nu}}{\sqrt{k_{1T}^2 S_T^2}}$$

See slides 12 and 13

$\mathcal{H}_i$ to be generalized structure function, expressed through spin-independent $H^{(0)}_{ab}$, and spin-dependent $H^{(S)}_{abi}$ scalar structure functions

$$
\begin{aligned}
\mathcal{H}_1 &= H^{(0)}_{22} - \eta_2 H^{(S)}_{222},\\
\mathcal{H}_2 &= \frac{4}{\lambda_Y^2 p_t^2}[\lambda_Y p_t^2 Q^2 (H^{(0)}_{00} - \eta_2 H^{(S)}_{002}) + \lambda_3^2 S_x^2 (H^{(0)}_{11} \\
&\quad -\eta_2 H^{(S)}_{112}) - \lambda_2 \lambda_Y (H^{(0)}_{22} - \eta_2 H^{(S)}_{222}) \\
&\quad -2 S_x \lambda_3 p_t Q \sqrt{\lambda_Y}(\mathrm{Re} H^{(0)}_{01} - \eta_2 \mathrm{Re} H^{(S)}_{012})],\\
\mathcal{H}_3 &= \frac{1}{p_t^2}(H^{(0)}_{11} - H^{(0)}_{22} + \eta_2 (H^{(S)}_{222} - H^{(S)}_{112})),\\
\mathcal{H}_4 &= \frac{2}{\lambda_Y p_t^2}[\lambda_3 S_x (H^{(0)}_{22} - H^{(0)}_{11} + \eta_2 (H^{(0)}_{112} - H^{(S)}_{222})) \\
&\quad + p_t Q \sqrt{\lambda_Y}(\mathrm{Re} H^{(0)}_{01} - \eta_2 \mathrm{Re} H^{(S)}_{012})],\\
\mathcal{H}_5 &= \frac{2Q}{p_t \sqrt{\lambda_Y}}(\mathrm{Im} H^{(0)}_{01} - \eta_2 \mathrm{Im} H^{(S)}_{012}),\\
\mathcal{H}_6 &= \frac{4M}{\lambda_Y^{3/2} p_t^2}[Q p_t \sqrt{\lambda_Y}(\eta_1 \mathrm{Re} H^{(S)}_{021} + \eta_3 \mathrm{Re} H^{(S)}_{023}) \\
&\quad -\lambda_3 S_x (\eta_1 \mathrm{Re} H^{(S)}_{121} + \eta_3 \mathrm{Re} H^{(S)}_{123})],\\
\mathcal{H}_7 &= \frac{4M}{\lambda_Y^{3/2} p_t^2}[Q p_t \sqrt{\lambda_Y}(\eta_1 \mathrm{Im} H^{(S)}_{021} + \eta_3 \mathrm{Im} H^{(S)}_{023}) \\
&\quad -\lambda_3 S_x (\eta_1 \mathrm{Im} H^{(S)}_{121} + \eta_3 \mathrm{Im} H^{(S)}_{123})],\\
\mathcal{H}_8 &= \frac{2M}{\sqrt{\lambda_Y} p_t^2}(\eta_1 \mathrm{Re} H^{(S)}_{121} + \eta_3 \mathrm{Re} H^{(S)}_{123}),\\
\mathcal{H}_9 &= \frac{2M}{\sqrt{\lambda_Y} p_t^2}(\eta_1 \mathrm{Im} H^{(S)}_{121} + \eta_3 \mathrm{Im} H^{(S)}_{123}).
\end{aligned}
$$

See slide 13

$$
\begin{aligned}
H^{(0)}_{00} &= C_1 F_{UU,L},\\
H^{(0)}_{01} &= -C_1 (F_{UU}^{\cos\phi_h} + i F_{LU}^{\sin\phi_h}),\\
H^{(0)}_{11} &= C_1 (F_{UU}^{\cos 2\phi_h} + F_{UU,T}),\\
H^{(0)}_{22} &= C_1 (F_{UU,T} - F_{UU}^{\cos 2\phi_h}),\\
H^{(S)}_{002} &= C_1 F_{UT,L}^{\sin(\phi_h - \phi_s)},\\
H^{(S)}_{012} &= C_1 (F_{UT}^{\sin\phi_s} - F_{UT}^{\sin(2\phi_h - \phi_s)} \\
&\quad - i(F_{LT}^{\cos\phi_s} - F_{LT}^{\cos(2\phi_h - \phi_s)})),\\
H^{(S)}_{021} &= C_1 (F_{UT}^{\sin(2\phi_h - \phi_s)} + F_{UT}^{\sin\phi_s} \\
&\quad - i(F_{LT}^{\cos(2\phi_h - \phi_s)} + F_{LT}^{\cos\phi_s})),\\
H^{(S)}_{023} &= C_1 (F_{UL}^{\sin\phi_h} - i F_{LL}^{\cos\phi_h}),\\
H^{(S)}_{121} &= C_1 (-F_{UT}^{\sin(3\phi_h - \phi_s)} - F_{UT}^{\sin(\phi_h + \phi_s)} \\
&\quad + i F_{LT}^{\cos(\phi_h - \phi_s)}),\\
H^{(S)}_{123} &= C_1 (-F_{UL}^{\sin 2\phi_h} + i F_{LL}),\\
H^{(S)}_{112} &= C_1 (F_{UT}^{\sin(3\phi_h - \phi_s)} + F_{UT,T}^{\sin(\phi_h - \phi_s)} \\
&\quad - F_{UT}^{\sin(\phi_h + \phi_s)}),\\
H^{(S)}_{222} &= C_1 (F_{UT}^{\sin(\phi_h + \phi_s)} + F_{UT,T}^{\sin(\phi_h - \phi_s)} \\
&\quad - F_{UT}^{\sin(3\phi_h - \phi_s)}),
\end{aligned}
$$

$$
C_1 = \frac{4 M p_l (Q^2 + 2xM^2)}{Q^4}
$$

See slide 15

- Separate leptonic tensor into two parts $L_R^{\mu\nu} = L_{R0}^{\mu\nu} + L_{R1}^{\mu\nu}$

- First term given by

$$L_{R0}^{\mu\nu} = -\frac{1}{2}\mathrm{Tr}\Big[\big(\hat{k}_2 + m_l\big)\Gamma_R^{\mu\alpha}\big(\hat{k}_1 + m_l\big)\big(1 + \gamma_5\hat{\xi}_0\big)\bar{\Gamma}_{R\alpha}^{\nu}\Big]$$

with

$$\Gamma_R^{\mu\alpha} = \left(\frac{k_1^\alpha}{k\cdot k_1} - \frac{k_2^\alpha}{k\cdot k_2}\right)\gamma^\mu - \frac{\gamma^\mu\hat{k}\gamma^\alpha}{2k\cdot k_1} - \frac{\gamma^\alpha\hat{k}\gamma^\mu}{2k\cdot k_2},$$

$$\bar{\Gamma}_{R\alpha}^{\nu} = \gamma_0\,\Gamma_{R\alpha}^{\nu\dagger}\,\gamma_0 = \left(\frac{k_{1\alpha}}{k\cdot k_1} - \frac{k_{2\alpha}}{k\cdot k_2}\right)\gamma^\nu - \frac{\gamma^\nu\hat{k}\gamma_\alpha}{2k\cdot k_2} - \frac{\gamma_\alpha\hat{k}\gamma^\nu}{2k\cdot k_1}$$

- Second term given by

$$L_{R1}^{\mu\nu} = -\frac{1}{2}\mathrm{Tr}\Big[\big(\hat{k}_2 + m_l\big)\Gamma_R^{\mu\alpha}\big(\hat{k}_1 + m_l\big)\,\gamma_5\hat{\xi}_1\bar{\Gamma}_{R\alpha}^{\nu}\Big]$$

- Convolutions of both separated leptonic tensors with shifted hadronic tensor are given by

$$\tilde{W}_{\mu\nu}L_{R0}^{\mu\nu} = -2\sum_{i=1}^{9}\sum_{j=1}^{k_i}\tilde{\mathcal{H}}_i\,\theta_{ij}^{0}\,R^{j-3},$$

$$\tilde{W}_{\mu\nu}L_{R1}^{\mu\nu} = -2\sum_{i=5,7,9}\sum_{j=1}^{k_i}\tilde{\mathcal{H}}_i\,\theta_{ij}^{1}\,R^{j-3}$$

See Appendix B in Phys. Rev. D 100(3) (2019)

See slide 16

$$\sigma^{in}_{\rm SIDIS} = \frac{\alpha}{\pi}\left(\delta_{vert} + \delta^{l}_{\rm vac} + \delta^{h}_{\rm vac}\right)\sigma^{B}_{\rm SIDIS} + \sigma^{AMM} + \int\limits_{0}^{\infty} \bar{\sigma}_R \, d^3\mathbf{k}$$

$$\int\limits_{0}^{\infty} \bar{\sigma}_R \, d^3\mathbf{k} = \int\limits_{0}^{\bar{k}_0} \bar{\sigma}^{IR}_R \, d^3\mathbf{k} + \int\limits_{0}^{\bar{k}_0} \bar{\sigma}^{F}_R \, d^3\mathbf{k} + \int\limits_{\bar{k}_0}^{\infty} \bar{\sigma}_R \, d^3\mathbf{k}$$

$$= \frac{\alpha}{\pi}\delta_S \sigma^B + \int\limits_{0}^{\bar{k}_0} \bar{\sigma}^{F}_R \, d^3\mathbf{k} + \int\limits_{\bar{k}_0}^{\infty} \bar{\sigma}_R \, d^3\mathbf{k}.$$

$$\sigma^{in}_{\rm SIDIS} = \left[\frac{\alpha}{\pi}\left(\delta_{VS} + \delta^{l}_{\rm vac} + \delta^{h}_{\rm vac}\right)\sigma^{B}_{\rm SIDIS} + \sigma^{AMM} + \int\limits_{0}^{\bar{k}_0} \bar{\sigma}^{F}_R \, d^3\mathbf{k}\right]_{\text{non-rad. part}} +$$

$$+ \left[\int\limits_{\bar{k}_0}^{\infty} \bar{\sigma}_R \, d^3\mathbf{k}\right]_{\text{rad. part}} \equiv$$

$$\equiv \sigma^{nrad}_{\rm SIDIS} + \sigma^{rad}_{\rm SIDIS},$$