PDFs at approximate N3LO accuracy

Felix Hekhorn

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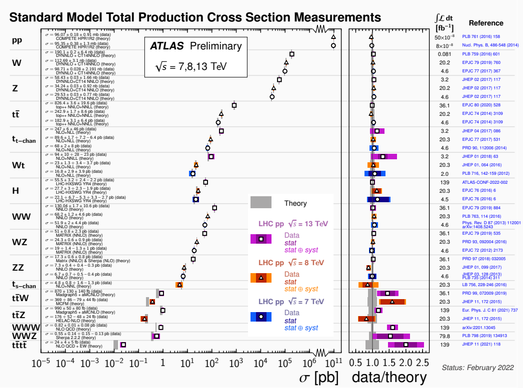
Outline

1. Motivation

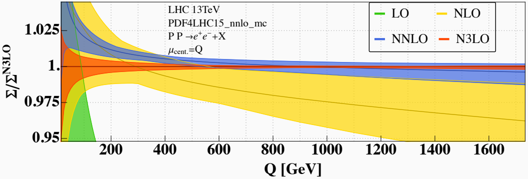
Call for Precision

Experiment:



taken from ATL-PHYS-PUB-2022-009

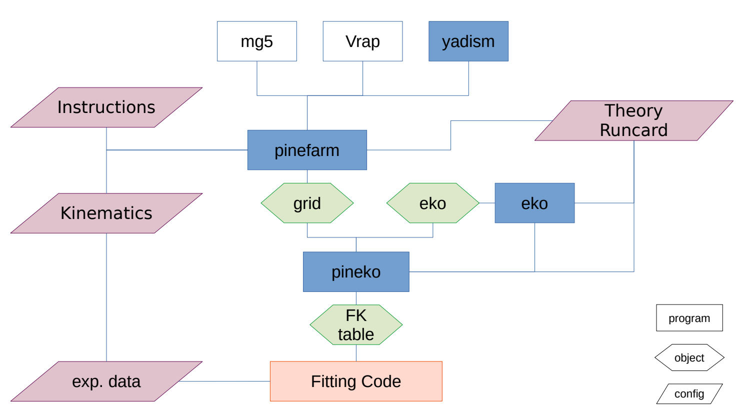
Theory:



taken from [JHEP03.116]

New Theory Prediction ~~Pipeline~~ Pineline

Produce FastKernel (FK) tables!



The workhorse in the background: PineAPPL

pineline: Industrialization of High-Energy Theory Predictions [2302.12124]

https://nnpdf.github.io/pineline

- **Industrialization**: collect diverse generators in an "assembly line"
  $\rightarrow$ NNPDF4.0[EPJC82.428]: $>$ 4.5k datapoints + $>$ 10 generators
- **I/O format**: single input $\rightarrow$ translation layer $\rightarrow$ single output
- **Reproducibility**: track data and metadata
- **Open Source**: crucial to the above

$\Rightarrow$ please provide new calculations in an "interfaceable" way

Ingredients for an aN3LO PDF Fit

cross-sections: matrix element providers

- DIS
- LHC (and everything else)

evolution: `eko`

- splitting functions/anomalous dimensions
- matching conditions/transition matrix elements

Account for uncertainties!

2. Theory Uncertainties

Covariance Matrix Formalism [EPJC79.931]

To maximize the (Bayesian) probability $P(T|D)$ for a theory prediction T to describe a data point D we minimize

$$\chi^2 = (T - D)^T C^{-1} (T - D) \tag{1}$$

and assuming the experimental cov. matrix C is independent of a theoretical cov. matrix S we can minimize:

$$\chi^2 = (T - D)^T (C + S)^{-1} (T - D) \tag{2}$$

Both can contain

- statistical uncertainties (e.g. QM vs. MC)
- systematic uncertainties (e.g. resolution vs. scale)

Decomposing the Theory Covariance Matrix

$$S^{tot} = S^{MHOU} + S^{IHOU} \tag{3}$$

missing higher order uncertainties (MHOU) via RGE invariance:

$$S_{ij}^{MHOU} = \langle (T[\mu] - T[\mu_0])_i (T[\mu] - T[\mu_0])_j \rangle_{\mu \in V_\mu} \quad i, j = 1, \ldots, N_{dat} \tag{4}$$

incomplete higher order uncertainties (IHOU) via parametrization freedom:

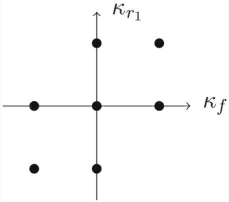
$$S_{ij}^{IHOU} = \langle (T[k] - T[k_0])_i (T[k] - T[k_0])_j \rangle_{k \in V_k} \quad i, j = 1, \ldots, N_{dat} \tag{5}$$

MHOU Using Scale Variations [EPJC79.931]

$$T[\mu_r, \mu_f] = C(Q, \mu_r) \otimes E(Q \leftarrow Q_0, \mu_f) \otimes f(Q_0) \tag{6}$$

Factorization scale variations:

- finite knowledge of anomalous dimensions
- correlated across datasets as PDFs are universal



Renormalization scale variations:

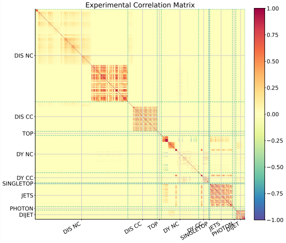
- finite knowledge of partonic cross-sections
- correlated for a given process (DIS NC, DIS CC, TOP, JETS, . . .)

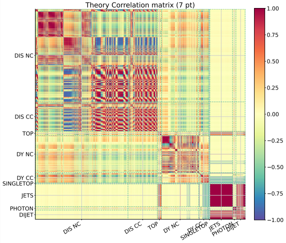
Choose:

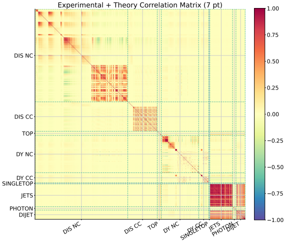
$$\mu = \kappa\mu_0 \qquad \kappa \in \{1/2, 1, 2\} \tag{7}$$

MHOU Covariance Matrices

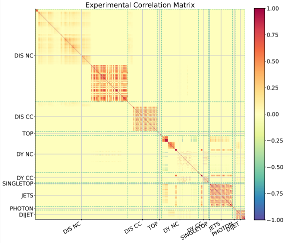
NLO

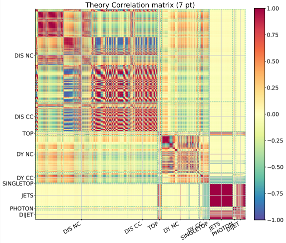


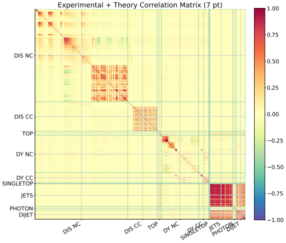




NNLO







3. Cross-Sections

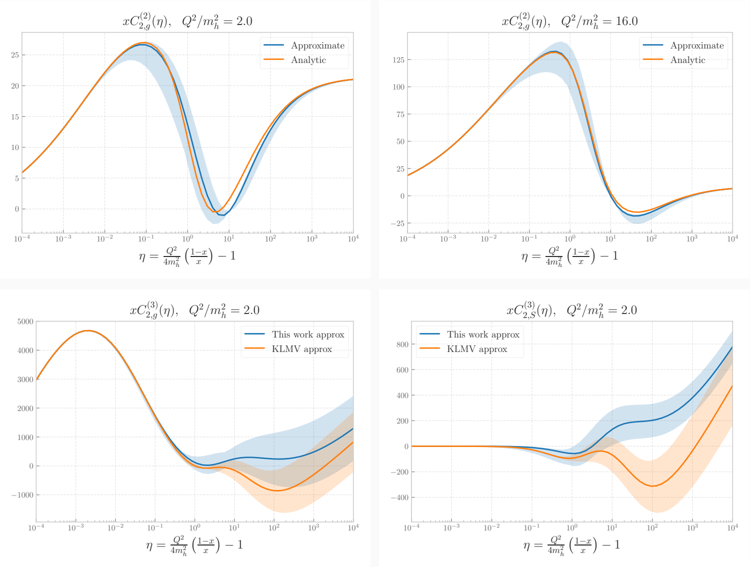
DIS with QCD@N3LO

- light coefficient functions [VVM05],[MVV05],[MV00],[MRV08],[MVV09] ✓
- massive coefficient functions $\to$ approximation in MSc thesis of N. Laurenti ✓
  i.e. combine in a suitable way:
  - threshold limit $s \to 4m^2$
  - high-energy limit $s \to \infty$
  - asymptotic limit $Q^2 \gg m^2$

DIS with QCD@N3LO

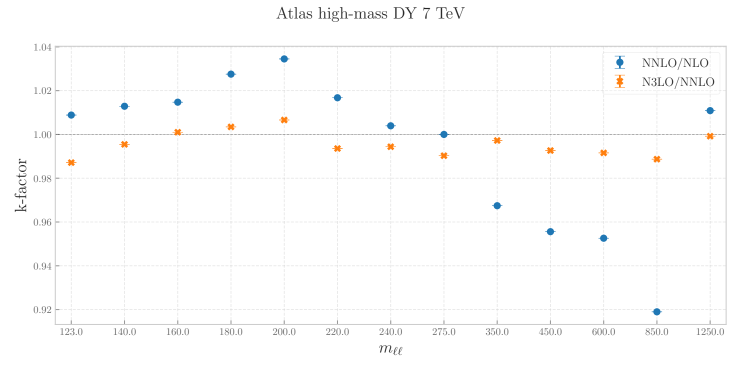
compare NNLO to exact [NPB392.162]

compare N3LO to [KLMV]



DY with QCD@N3LO

- even if available, most codes are private
- use `n3loxs` [JHEP12.066] to obtain k-factor for inclusive distributions, e.g. [PLB725.223]



4. Evolution

EKO Features

Evolution Kernel Operators

- unpolarized, space-like QCD@aN3LO ✓
- unpolarized, space-like QCD@N*LO×QED@NNLO ✓
- polarized, space-like QCD@NNLO ✓
- unpolarized, time-like QCD@NNLO ✓

Transition Matrix Elements

Use whatever is available in literature:

- I. Bierenbaum, J.Blümlein, and S. Klein. Mellin Moments of the $\mathcal{O}(\alpha_s^3)$ Heavy Flavor Contributions to unpolarized Deep-Inelastic Scattering at $Q^2 \gg m^2$ and Anomalous Dimensions. *Nucl. Phys. B*, 820:417–482, 2009. arXiv:0904.3563, doi:10.1016/j.nuclphysb.2009.06.005.
- J. Blümlein. Analytic continuation of mellin transforms up to two-loop order. *Computer Physics Communications*, 133(1):76–104, Dec 2000. URL: http://dx.doi.org/10.1016/S0010-4655(00)00156-9, doi:10.1016/s0010-4655(00)00156-9.
- J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, C. Schneider, and F. Wißbrock. The 3-Loop Non-Singlet Heavy Flavor Contributions and Anomalous Dimensions for the Structure Function $F_2(x,Q^2)$ and Transversity. *Nucl. Phys. B*, 886:733–823, 2014. arXiv:1406.4654, doi:10.1016/j.nuclphysb.2014.07.010.
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- J. Ablinger, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, and C. Schneider. The $\mathcal{O}(\alpha_s^3 T_f^2)$ contributions to the Gluonic Operator Matrix Element. *Nucl. Phys. B*, 885:280–317, 2014. arXiv:1405.4259, doi:10.1016/j.nuclphysb.2014.05.028.
- J. Ablinger, J. Blümlein, A. De Freitas, A. Hasselhuhn, A. von Manteuffel, M. Round, C. Schneider, and F. Wißbrock. The transition matrix element a_gq(n) of the variable flavor number scheme at o(α_s^3). *Nuclear Physics B*, 882:263–288, May 2014. URL: http://dx.doi.org/10.1016/j.nuclphysb.2014.02.007, doi:10.1016/j.nuclphysb.2014.02.007.
- A. Behring, I. Bierenbaum, J. Blümlein, A. De Freitas, S. Klein, and F. Wißbrock. The logarithmic contributions to the $\mathcal{O}(\alpha_s^3)$ asymptotic massive Wilson coefficients and operator matrix elements in deeply inelastic scattering. *Eur. Phys. J. C*, 74(9):3033, 2014. arXiv:1403.6356, doi:10.1140/epjc/s10052-014-3033-x.
- J. Ablinger, J. Blümlein, S. Klein, C. Schneider, and F. Wissbrock. The $\mathcal{O}(\alpha_s^3)$ Massive Operator Matrix Elements of $\mathcal{O}(n_f)$ for the Structure Function $F_2(x,Q^2)$ and Transversity. *Nucl. Phys. B*, 844:26–54, 2011. arXiv:1008.3347, doi:10.1016/j.nuclphysb.2010.10.021.
- J. Blümlein, J. Ablinger, A. Behring, A.De Freitas, A. von Manteuffel, and C. Schneider. Heavy Flavor Wilson Coefficients in Deep-Inelastic Scattering: Recent Results. *PoS*, QCDEV2017:031, 2017. arXiv:1711.07957, doi:10.22323/1.308.0031.
- J. Ablinger, A. Behring, J. Blümlein, A. De Freitas, A. Goedicke, A. von Manteuffel, C. Schneider and K. Schonwald. The Unpolarized and Polarized Single-Mass Three-Loop Heavy Flavor Operator Matrix Elements $A_{gg,Q}$ and $\Delta A_{gg,Q}$ arXiv:2211.0546

Known Information on Anomalous Dimensions

Splitting functions/anomalous dimensions are not known fully analytically, but some partial information:

- large n_f contributions γ_{ij,n_f} [NPB915.335], [JHEP10.041]
- small N limit (from BFKL) $\gamma_{ij,N\to 0}$ [JHEP06.145], [JHEP08.135]
- large N limit (from soft) $\gamma_{ij,N\to\infty}$ [NPB832.152], [JHEP04.018], [JHEP09.155]
- some (low) moments [JHEP10.041], [PLB825.136853], [PLB842.137944], [2307.04158]

Strategy:
combine known limits and add sub-leading functions to ensure moments $\Rightarrow$ IHOU

Constructing Anomalous Dimensions

$$\gamma_{ij}(N) = \gamma_{ij,n_f}(N) + \gamma_{ij,N\to 0}(N) + \gamma_{ij,N\to\infty}(N) + \tilde{\gamma}_{ij}(N) \tag{8}$$

Make ansatz:

$$\tilde{\gamma}_{ij}(N) = \sum_{l=0}^{n_{ij}} a_l^{ij} G_l^{ij}(N) \tag{9}$$

Choose:

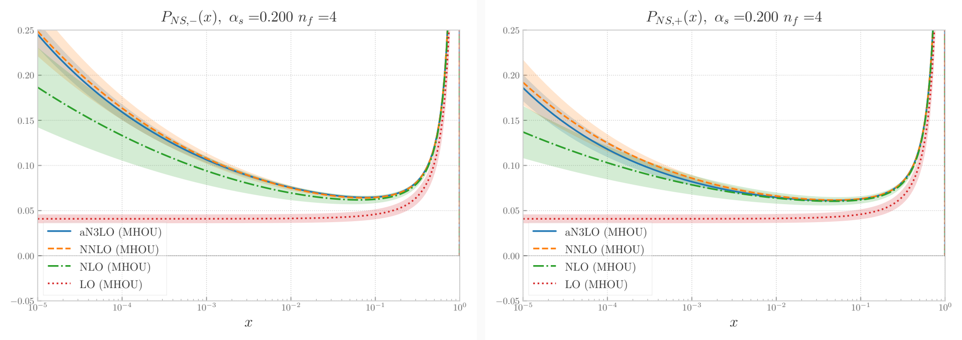
- G_1 the first unknown large N contribution - e.g. $G_1^{gg}(N) = S_1(N)/N$
- G_2 the first unknown small N contribution - e.g. $G_2^{gg}(N) = S_2(N-2)/N \overset{N\to 1}{\approx} \frac{1}{(N-1)^2}$
- vary others with sub-leading contributions - e.g. $G_3^{gg} \in \{\frac{1}{N-1}, \frac{1}{N}\}$, $G_4^{gg} \in \{\frac{1}{N-1}, \frac{1}{N^4}, \frac{1}{N^3}, \frac{1}{N^2}, \frac{1}{N}, \frac{1}{(N+1)^3}, \frac{1}{(N+1)^2}, \frac{1}{N+1}, \frac{1}{N+2}, \frac{S_1(N-2)}{N}, \mathcal{M}[\ln^3(1-x)], \mathcal{M}[\ln^2(1-x)], \frac{S_1(N)}{N}, \frac{S_1^2(N)}{N}\}$

$\Rightarrow$ fix a_l^{ij} $\Rightarrow$ $O(70)$ independent approximations $\Rightarrow$ $O(20)$ representative approximations

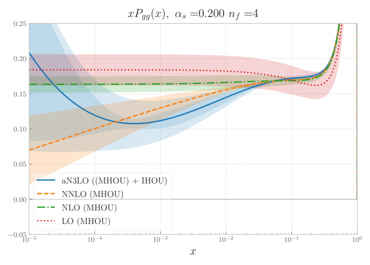
Non-Singlet Splitting Functions

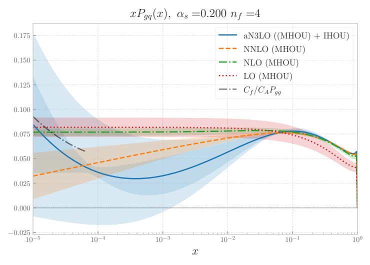
8 known moments $\to$ use a fixed set of G_l:

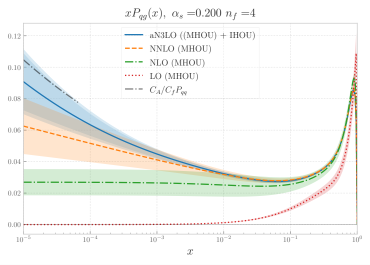
$$G_l \supset 1/(N+1)^2, S_1(N)/N^2, 1/(N+2), \dots \tag{10}$$

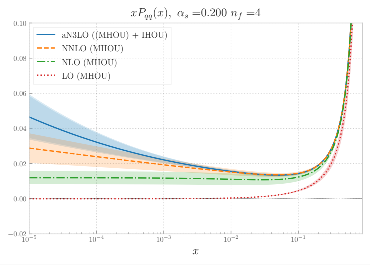


Singlet Splitting Functions

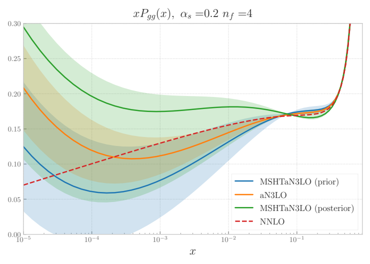


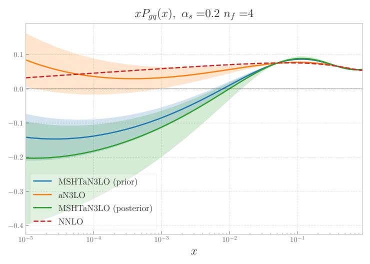


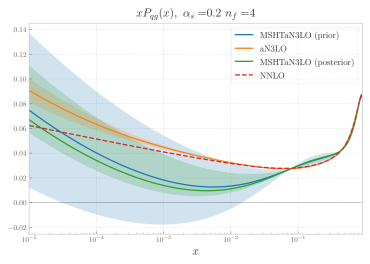


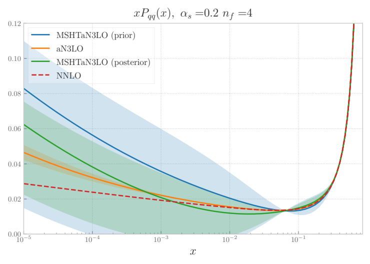


Comparison with MSHT [EPJC83.185]



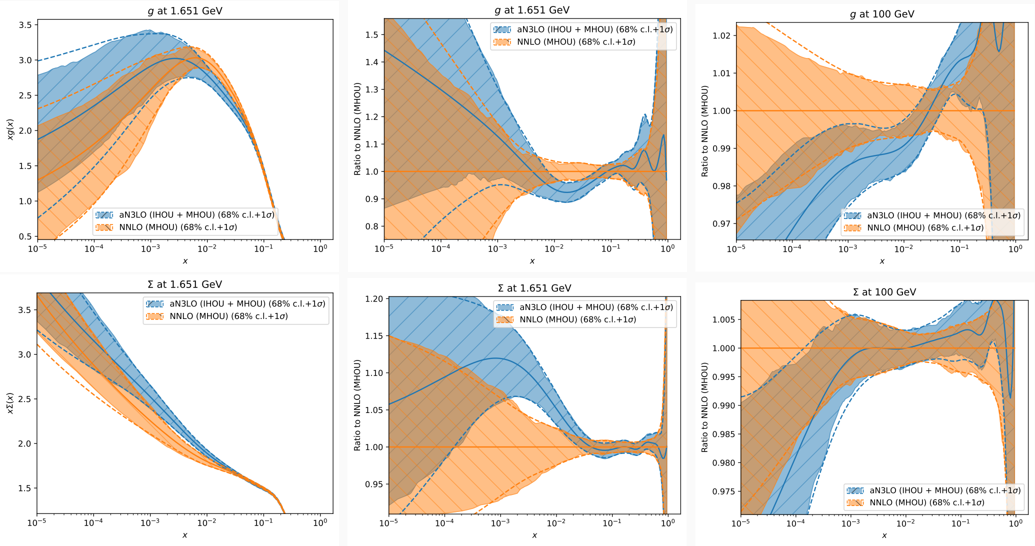






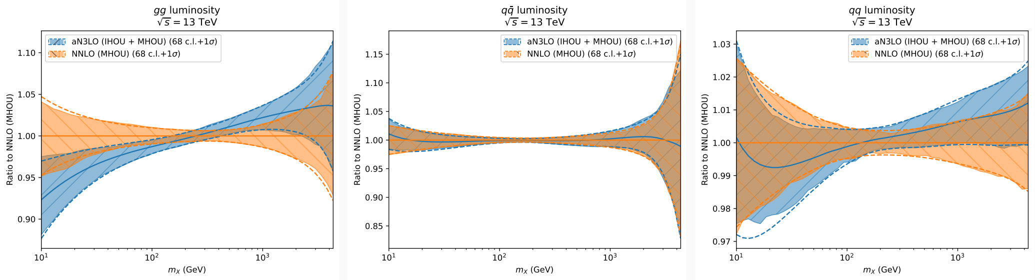
5. NNPDF4.0 with QCD@aN3LO

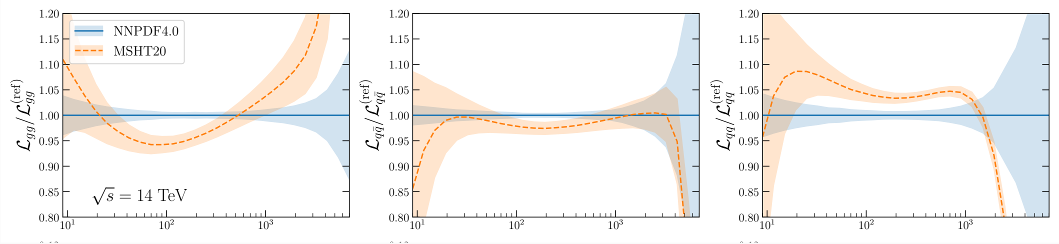
PDFs



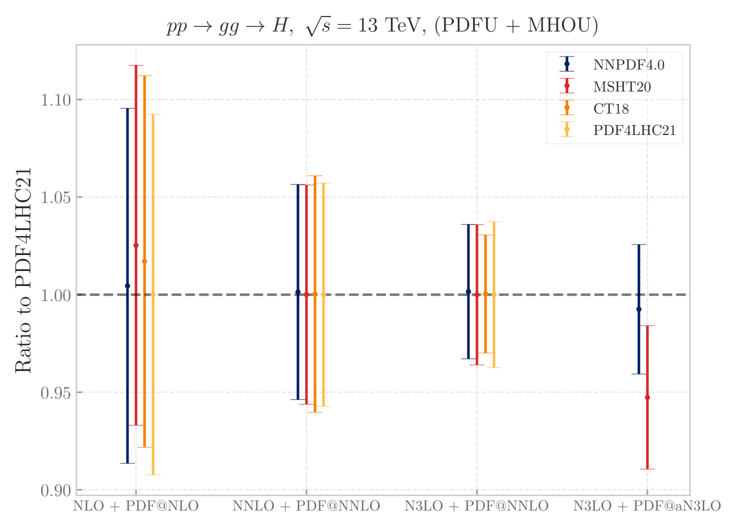
g at 1.651 GeV
xg(x)
x
aN3LO (IHOU + MHOU) (68% c.l.+1σ)
NNLO (MHOU) (68% c.l.+1σ)
g at 1.651 GeV
Ratio to NNLO (MHOU)
x
aN3LO (IHOU + MHOU) (68% c.l.+1σ)
NNLO (MHOU) (68% c.l.+1σ)
g at 100 GeV
Ratio to NNLO (MHOU)
x
aN3LO (IHOU + MHOU) (68% c.l.+1σ)
NNLO (MHOU) (68% c.l.+1σ)
Σ at 1.651 GeV
xΣ(x)
x
aN3LO (IHOU + MHOU) (68% c.l.+1σ)
NNLO (MHOU) (68% c.l.+1σ)
Σ at 1.651 GeV
Ratio to NNLO (MHOU)
x
aN3LO (IHOU + MHOU) (68% c.l.+1σ)
NNLO (MHOU) (68% c.l.+1σ)
Σ at 100 GeV
Ratio to NNLO (MHOU)
x
aN3LO (IHOU + MHOU) (68% c.l.+1σ)
NNLO (MHOU) (68% c.l.+1σ)

Luminosities





Some First Pheno . . .



6. Outlook

Usage of PDFs with Theory Uncertainties [EPJC79.931]

- PDFs are universal
- we can assume uncertainties for a given process independent of the PDF

In the end we can just do:

$$(\delta\sigma^{tot})^2 = (\delta\sigma^{MHOU})^2 + (\delta\sigma^{PDF})^2 \tag{11}$$

with (as usual):

$$(\delta\sigma^{MHOU})^2 = \langle (T_\sigma[\mu] - T_\sigma[\mu_0])^2 \rangle_{\mu \in V_\mu} \tag{12}$$

$$(\delta\sigma^{PDF})^2 = \langle (T_\sigma[f^{(k)}] - T_\sigma[f^{(0)}])^2 \rangle_{k=1\ldots N_{rep}} \tag{13}$$

Summary

For PDFs at % accurarcy we need:

- include QED and EW effects $\rightarrow$ NNPDF4.0QED
- account for theory uncertainties $\rightarrow$ NNPDF4.0MHOU
- use N3LO precision $\rightarrow$ NNPDF4.0aN3LO

For PDFs with QCD@aN3LO we need

- approximate splitting functions
- upgrade as many processes as possible
- account for theory uncertainties

Thank you!