Energy-Energy Correlators at the EIC

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References: Liu, Xiao-Hui Liu, Pan, Yuan, Hua-Xing Zhu, 2301.01788
Li, Xiao-Hui Liu, Yuan, Hua-Xing Zhu, 2308.10942

EIC Science: from quark/gluon to cosmo

- How do the nucleonic properties such as mass and spin emerge from partons and their underlying interactions?
- How are partons inside the nucleon distributed in both momentum and position space?
- What happens to the gluon density in nucleons and nuclei at small *x*? Does it saturate at high energy, giving rise to gluonic matter with universal properties in all nuclei (and perhaps even in nucleons)?
- How do color-charged quarks and gluons, and jets, interact with a nuclear medium? How do confined hadronic states emerge from these quarks and gluons? How do the quark-gluon interactions generate nuclear binding?
- Do signals from beyond-the-standard-model physics manifest in electron-proton/ion collisions? If so, what can we learn about the nature of these new particles and forces?

EIC Whitepaper for LRP
QCD Whitepaper, 2303.02579

Theory opportunities

- Precision computations and phenomenology developments toward a global analysis to extract nucleon/nucleus structure from various observables
- New observables, new structure, new dynamics

QGT Collaboration

SURGE COLLABORATION

Energy-Energy Correlations: a QCD playground since its birth

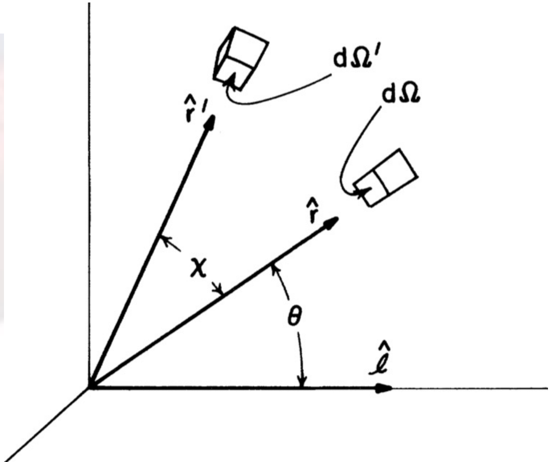
Energy Correlations in Electron-Positron Annihilation: Testing Quantum Chromodynamics

C. Louis Basham, Lowell S. Brown, Stephen D. Ellis, and Sherwin T. Love PRL 41, 1585 (1978)

This measure involves the asymmetry in the energy-weighted opening angles of the jets of hadrons produced in the process $e^+e^- \to$ hadrons at energy W. It is special for several reasons: It is reliably calculable in asymptotically free perturbation theory; it has rapidly vanishing (order $1/W^2$) corrections due to nonperturbative confinement effects; and it is straightforward to determine experimentally.

$$\frac{d\Sigma}{d\cos\chi} = \sum_{i,j} \int \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos\chi) d\sigma,$$

$$\frac{1}{\sigma_0}\frac{d\Sigma}{d\cos\chi} = \frac{\alpha_s(\mu)}{2\pi} C_F \frac{3-2z}{4(1-z)z^5} \times [3z(2-3z) + 2(2z^2 - 6z + 3)\log(1-z)]$$

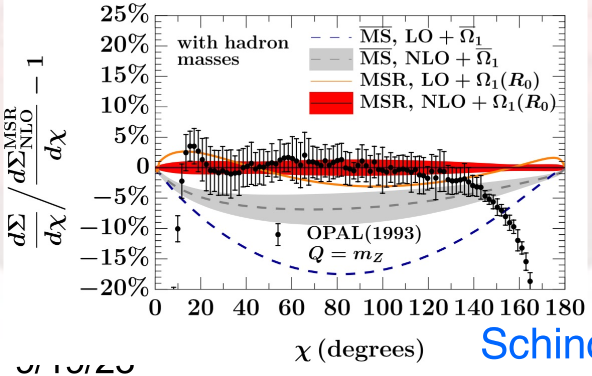


High order corrections

- NLO analytic result Dixon, Luo, Shtabovenko, Yang, Zhu, PRL 18

$$\frac{1}{\sigma_0}\frac{d\Sigma}{d\cos\chi} = \frac{\alpha_s(\mu)}{2\pi}A(z) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2\left(\beta_0 \log\frac{\mu}{Q}A(z) + B(z)\right)$$

  - B(z)~Log(z)/z when z→0, collinear splitting; B(z)~Log^3(1-z)/(1-z) when z→1, soft and collinear contributions
- NNLO numeric results Del Duca, Duhr, Kardos, Somogyi, Trócsányi, PRL 2016
- Renormalon resummation



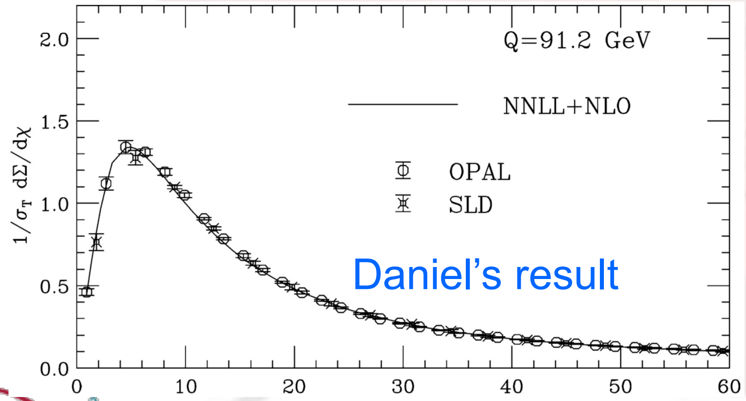
Schindler, Stewart, Sun, 2305.19311

Applying TMD in the back-to-back limit

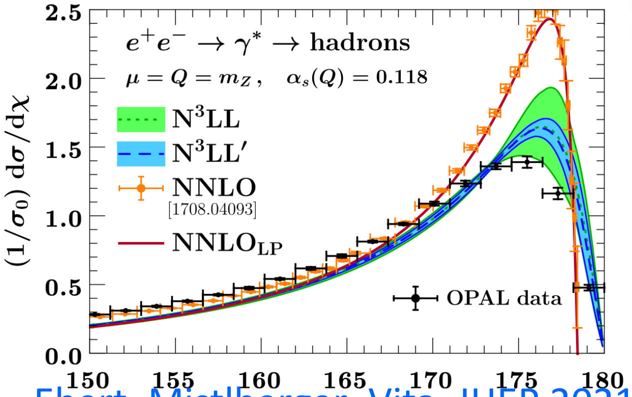
$$\frac{1}{\sigma_T}\frac{d\Sigma^{(\text{res.})}}{d\cos\chi} = \frac{Q^2}{8} H(\alpha_S(Q^2)) \int_0^\infty db\, b\, J_0(bq_T) S(Q,b)$$

$$S(Q,b) = \exp\left\{-\int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\left[A(\alpha_S(q^2))\ln\frac{Q^2}{q^2} + B(\alpha_S(q^2))\right]\right\}$$

Collins-Soper, 1981
de Florian-Grazzini, 2005
up to NLO+NNLL:
$A^{(1,2,3)}$, $B^{(1,2)}$, $H^{(1,2)}$



Daniel's result



Ebert, Mistlberger, Vita, JHEP 2021
See also: Kardos et al, 2018, 2017

Lost in translation: where are the TMDs

- Full TMD dependence

Collins-Soper, 1981

$$\frac{1}{\sigma_T}\frac{d\Sigma}{dk_T^2}=\frac{\pi}{2}\Big(\sum_i e_i^2\Big)^{-1}\sum_A\int_0^1 dx_A x_A\int d^3P_A^T\sum_B\int_0^1 dx_B x_B\int d^3P_B^T\sum_j e_j^2\mathcal{P}_{A/j}(x_A,P_A^T)\mathcal{P}_{B/j}(x_B,P_B^T)$$

- Restore

Moult-Zhou, 1801.02627

$$\frac{\hat{\sigma}_0}{8}H_{q\bar{q}}(Q,\mu)\int_0^\infty \mathrm{d}(b_TQ)^2\,J_0\big(b_TQ\sqrt{1-z}\big)J_q\Big(b_T,\mu,\frac{\nu}{Q}\Big)J_{\bar{q}}\Big(b_T,\mu,\frac{\nu}{Q}\Big)\tilde{S}_q(b_T,\mu,\nu)$$

- Fragmentation integral may matters

Nadolsky-Stump-Yuan, PRD99

$$e^{-S_z(b,x)}=\frac{1}{C_z^{out}(b_*,\mu)}\sum_B\int zdze^{-S_{BA}(b,x,z,Q)}$$

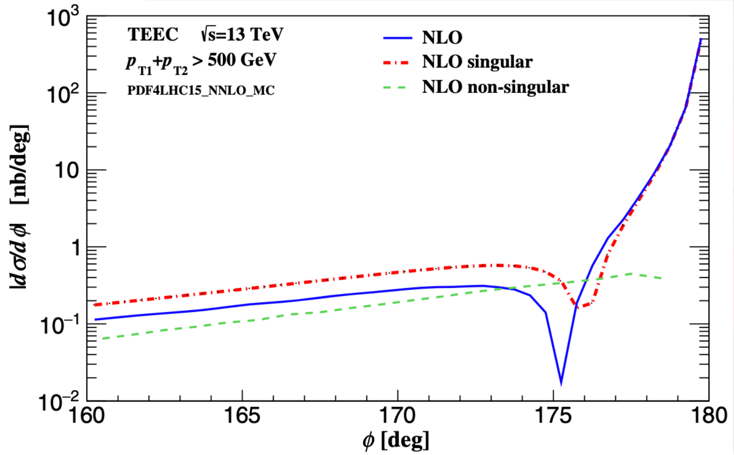
$$\times(D_{B/b}\otimes C_{b,j}^{out})(z,b_*,\mu).$$

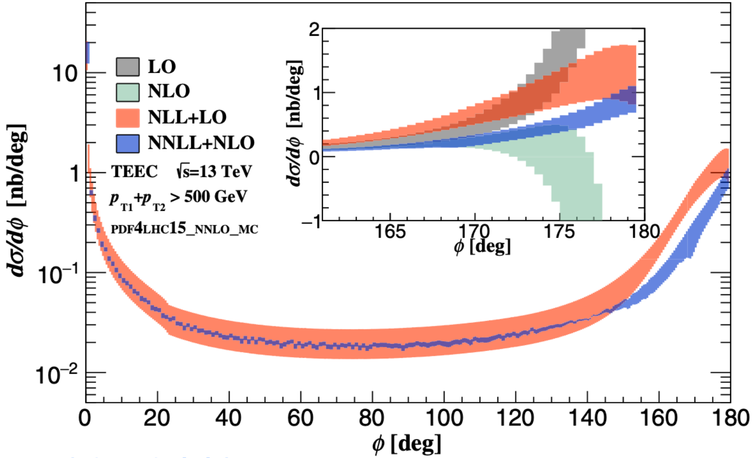
Transverse EEC at Colliders: Probing the TMDs

$$\frac{d\sigma}{d\cos\phi} = \sum_{a,b} \int d\sigma_{pp\to a+b+X} \frac{2E_{T,a}E_{T,b}}{|\sum_i E_{T,i}|^2} \delta(\cos\phi_{ab} - \cos\phi)$$

$$\to B_{f_1/N_1}(b,\xi_1,\mu,\nu) B_{f_2/N_2}(b,\xi_2,\mu,\nu) J_{f_3}(b,\mu,\nu) J_{f_4}(b,\mu,\nu)$$

Ali, Pietarinen, Stirling, PLB 1984
Ali, Barreiro, Llorente, W. Wang, PRD 2012;
Gao, Li, Moult, Zhu, PRL 2019





Gao, et al., 1901.04497

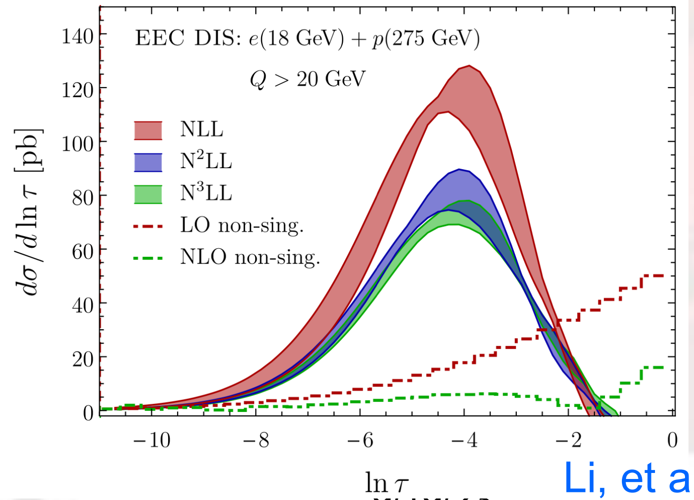
Transverse EEC at the EIC

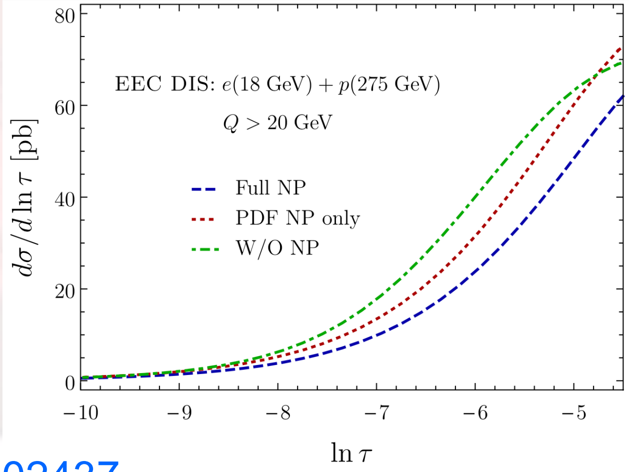
$$\mathrm{EEC_{DIS}} = \sum_a \int \frac{d\sigma_{ep\to e+a+X}}{\sigma} z_a \delta(\cos\theta_{ap} - \cos\theta)$$

$$\to B_{j/P}(x,b,\mu,\nu) S(b,\mu,\nu) J_i(b,\mu,\nu)$$

Nadolsky-Stump-Yuan, PRD99
Li-Makris-Vitev, PRD 2021

$$z_a \equiv \frac{P\cdot p_a}{P\cdot(\sum_i p_i)}$$

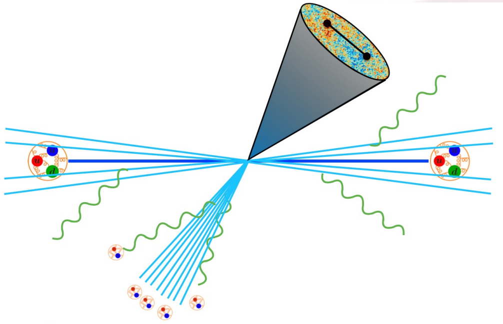


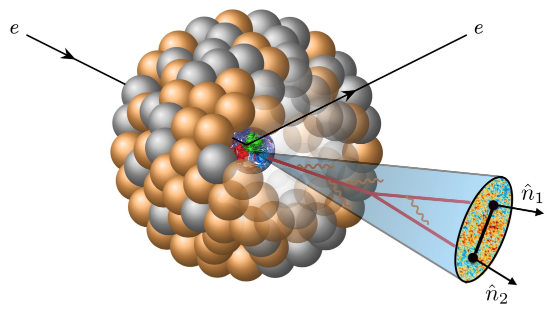


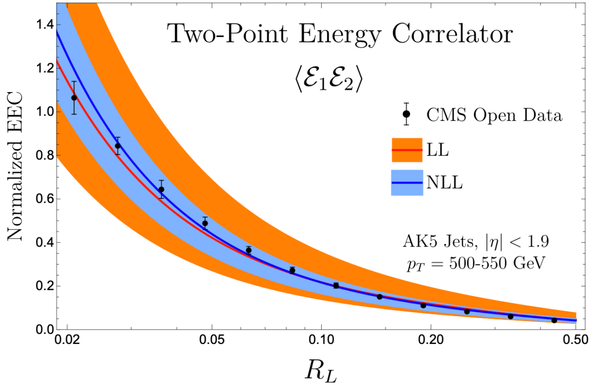
Li, et al, 2006.02437

Recent applications: Jet-substructure

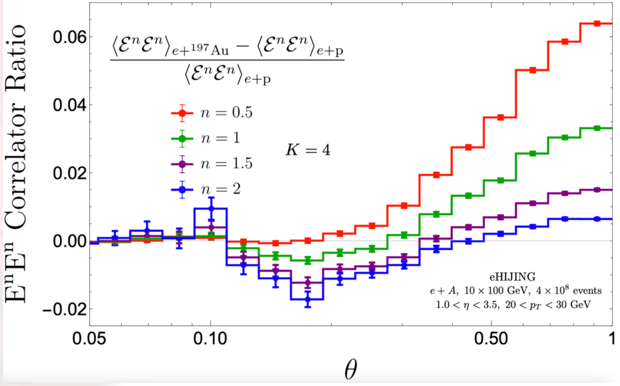
Moult, Zhu, Lee, et al, 20-23







Lee, Mecaj, Moult, 2205.03414



Devereaux, Fan, Ke,Lee,Moult,2303.08143

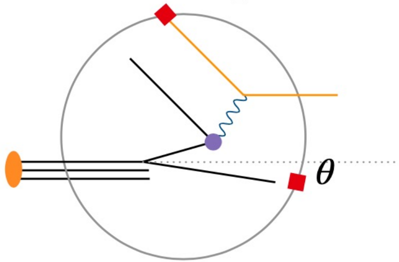
Hadron structure: nucleon EEC

$$f_{q,EEC}(x,\theta) = \int_{-\infty}^{\infty} \frac{dy^-}{2\pi E_P} e^{ixp^+y^-} \frac{\gamma^+}{2} \langle p | \bar{\psi}(0) \mathcal{E}(\theta) \mathcal{L} \psi(y^-) | p \rangle$$

$$= \sum_X \sum_{i\in X} \frac{E_i}{E_P} \delta(\theta_i^2 - \theta^2) \delta((1-x)p^+ - p_X^+) \frac{\gamma^+}{2} \langle p | \bar{\psi}(0) | X \rangle \langle X | \mathcal{L} \psi(0) | p \rangle$$

Liu, Zhu, 2209.02080
Cao, Liu, Zhu, 2303.01530

$$x_B = \frac{Q^2}{2P\cdot q}$$



$$\Lambda_{QCD} \ll Q\theta \ll Q$$

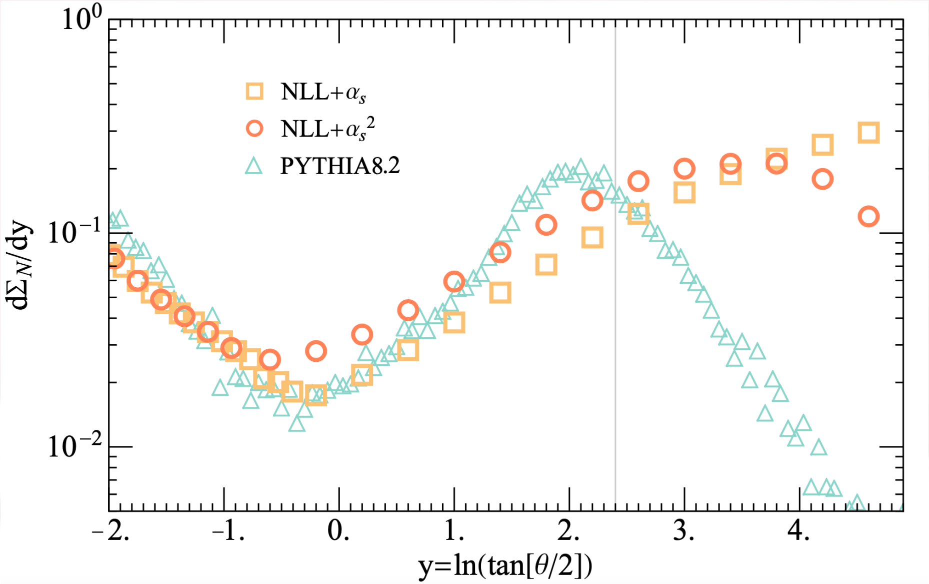
$$\Sigma(x_B, Q^2, \theta) = \int \frac{dz}{z} \hat{\sigma}\left(\frac{x_B}{z}, Q^2, \mu\right) f_{\rm EEC}(z, \theta, \mu)$$

$$\propto \int \frac{dz}{z} \hat{\sigma}\left(\frac{x_B}{z}\right) \frac{1}{\theta^2} \int \frac{d\xi}{\xi} (1 - \frac{z}{\xi}) P(\frac{z}{\xi}) [\xi f(\xi)]$$

- θ-distribution solely determined by $f_{\rm EEC}$
- In the collinear factorization:
  - $d\Sigma / d\ln\mu = P \otimes \Sigma$, solely determined by the vacuum splitting function
  - $\Sigma \sim \theta^{-2}$ at LO, $\Sigma \sim \theta^{-2+\gamma[\alpha_s]}$ to all orders

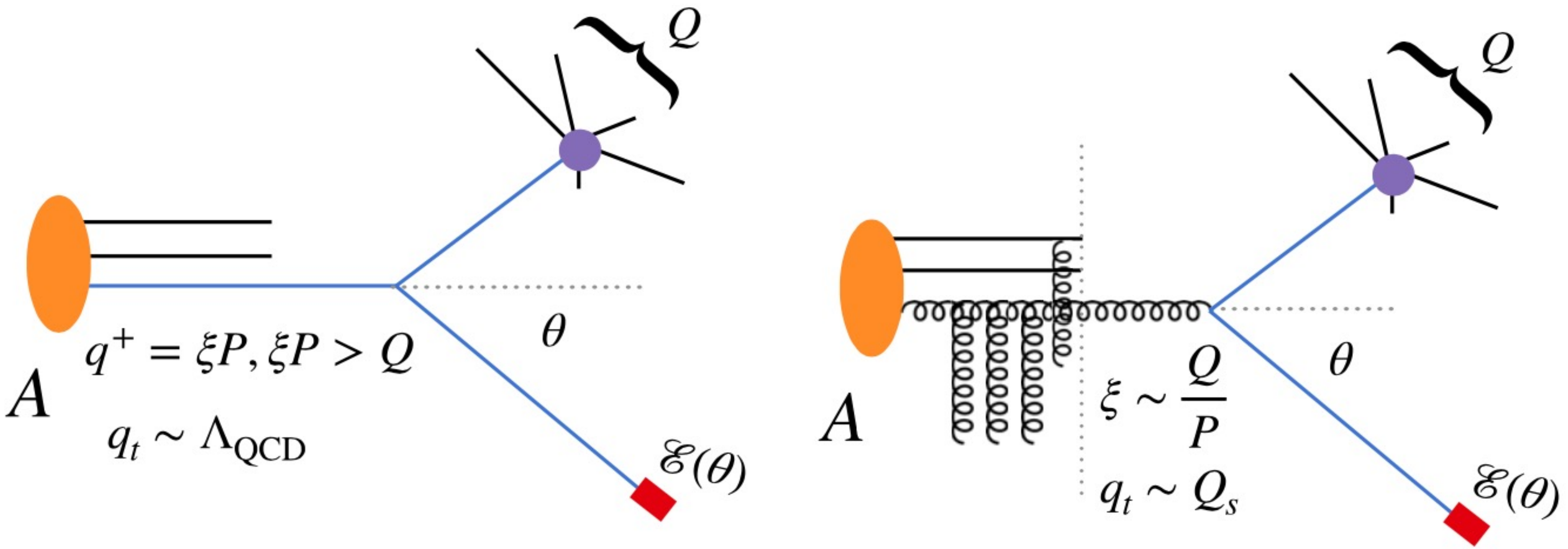
Resummation of collinear logs will modify the power behavior

$$\Sigma(x_B, Q^2, \theta) = \int \frac{dz}{z} \hat{\sigma}\left(\frac{x_B}{z}, Q^2, \mu\right) f_{\rm EEC}(z, \theta, \mu) \qquad \Sigma(x_B, Q^2, \theta) \sim \theta^{-2+\gamma}$$

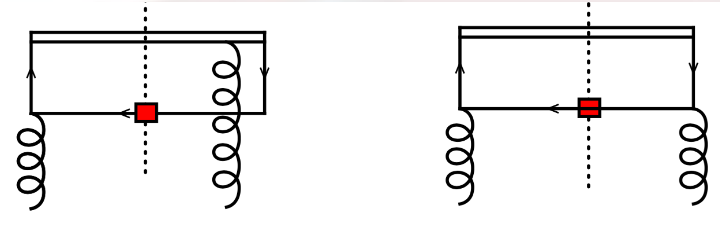


Cao, Liu, Zhu, 2303.01530

What happens at small-x



Collinear vs CGC



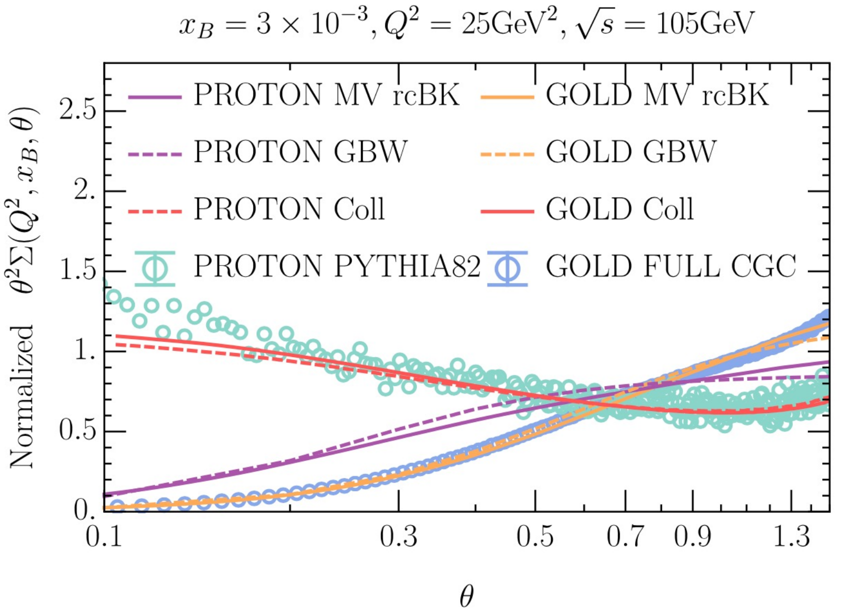
Collinear: $$f_{q,\mathrm{EEC}}(x,\theta)=\frac{\alpha_s T_R}{2\pi\theta^2}\int_x^1\frac{d\xi}{\xi}(1-\xi)(\xi^2+(1-\xi)^2)\left[\frac{x}{\xi}f_g\left(\frac{x}{\xi}\right)\right]$$

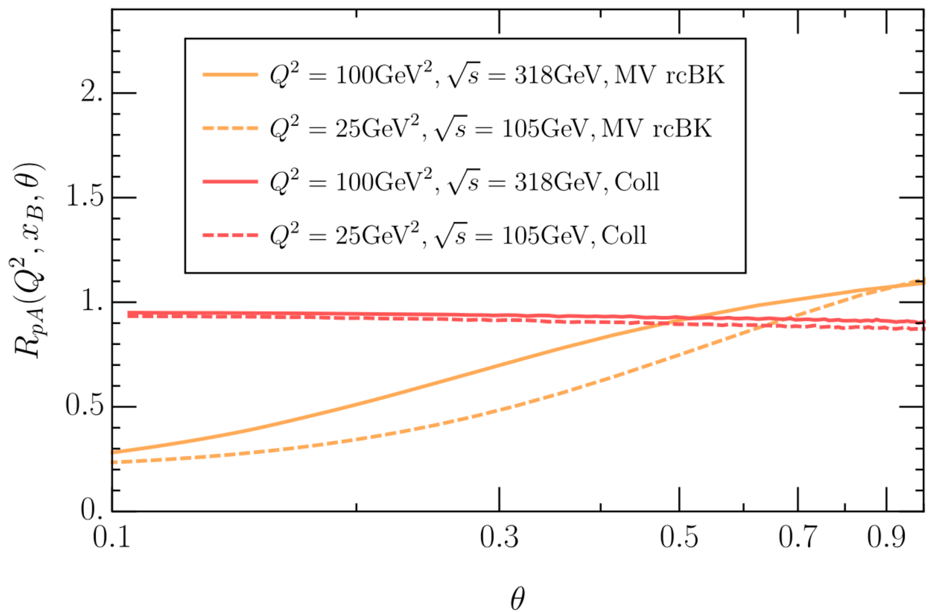
CGC: $$f_{q,\mathrm{EEC}}(x_B,\theta)=\frac{N_C S_\perp}{8\pi^4}\int d^2\vec{g}_t\int_{\xi_{\mathrm{cut}}}^1\frac{d\xi}{\xi}A_{qg}(\xi,\theta,\vec{g}_t)F_{g,x_B}(\vec{g}_t)$$

$$A_{qg}(\xi,\theta,\vec{g}_t)=\frac{1}{\theta^2}(1-\xi)\vec{k}_t^2(\vec{k}_t-\vec{g}_t)^2\left|\frac{\vec{k}_t}{\xi\vec{k}_t^2+(1-\xi)(\vec{k}_t-\vec{g}_t)^2}-\frac{\vec{k}_t-\vec{g}_t}{(\vec{k}_t-\vec{g}_t)^2}\right|^2$$

$$k_t=[(1-\xi)/2](Q/2)\theta,$$

Gluon saturation modify small- θ behavior





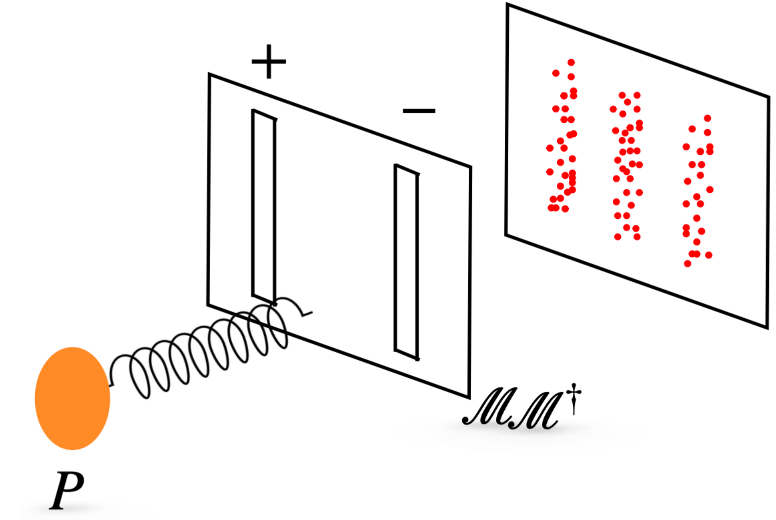
Liu, Liu, Pan, Yuan, Zhu, 2301.01788

NEEC with the linearly polarized gluons

$$f_{g,\mathrm{EEC}}^{\alpha\beta}(x,\vec{n}_a) = \int \frac{dy^-}{4\pi x P^+} e^{-ixP^+ \frac{y^-}{2}} \times \langle P|\mathcal{F}^{+\alpha}(y^-)\,\mathcal{L}^\dagger[\infty,y^-]\hat{\mathcal{E}}(\vec{n}_a)\mathcal{L}[\infty,0]\mathcal{F}^{+\beta}(0)|P\rangle$$

$$= -g_T^{\alpha\beta} f_{g,\mathrm{EEC}} + \left(\frac{n_{a,T}^\alpha n_{a,T}^\beta}{n_{a,T}^2} - \frac{g_T^{\alpha\beta}}{2}\right) d_{g,\mathrm{EEC}}\,, \qquad (2)$$

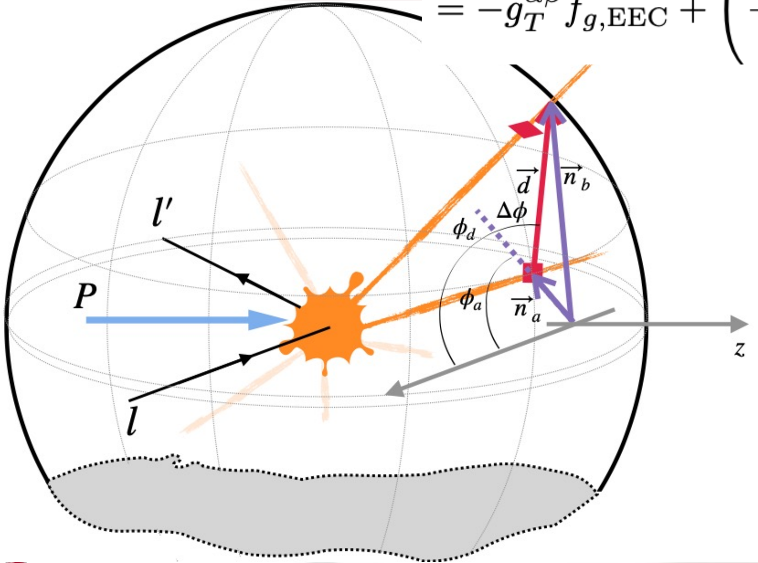
$$d_{g,\mathrm{EEC}}(x,\theta_a^2) = \frac{\alpha_s}{4\pi^2}\frac{2}{\theta_a^2}\int \frac{dz}{z}(1-z)\frac{1-z}{z} \times \frac{x}{z}\left[C_F f_q\left(\frac{x}{z}\right) + C_A f_g\left(\frac{x}{z}\right)\right],$$

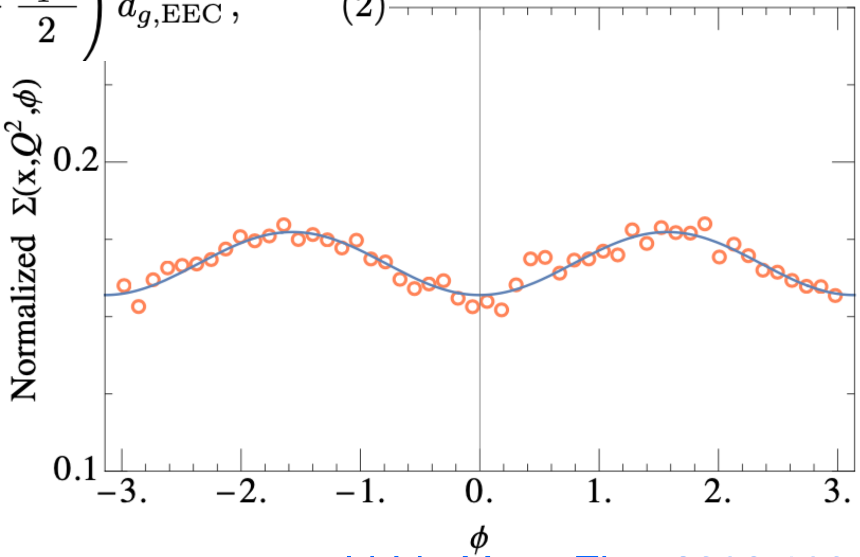


TMD counterparts:
Mulders et al, Vogelsang et al, Brodsky et al, Metz et al, 2000s-2020s
GPD counterparts:
Ji et al, Diehl et al, Belitsky et al, around 1990s

Cos(2ϕ) to probe the linearly polarized gluon

$$f_{g,\mathrm{EEC}}^{\alpha\beta}(x,\vec{n}_a) = \int \frac{dy^-}{4\pi x P^+} e^{-ixP^+\frac{y^-}{2}} \times \langle P|\mathcal{F}^{+\alpha}(y^-)\,\mathcal{L}^\dagger[\infty,y^-]\hat{\mathcal{E}}(\vec{n}_a)\mathcal{L}[\infty,0]\mathcal{F}^{+\beta}(0)|P\rangle = -g_T^{\alpha\beta} f_{g,\mathrm{EEC}} + \left(\frac{n_{a,T}^\alpha n_{a,T}^\beta}{n_{a,T}^2} - \frac{g_T^{\alpha\beta}}{2}\right) d_{g,\mathrm{EEC}}\,, \quad (2)$$



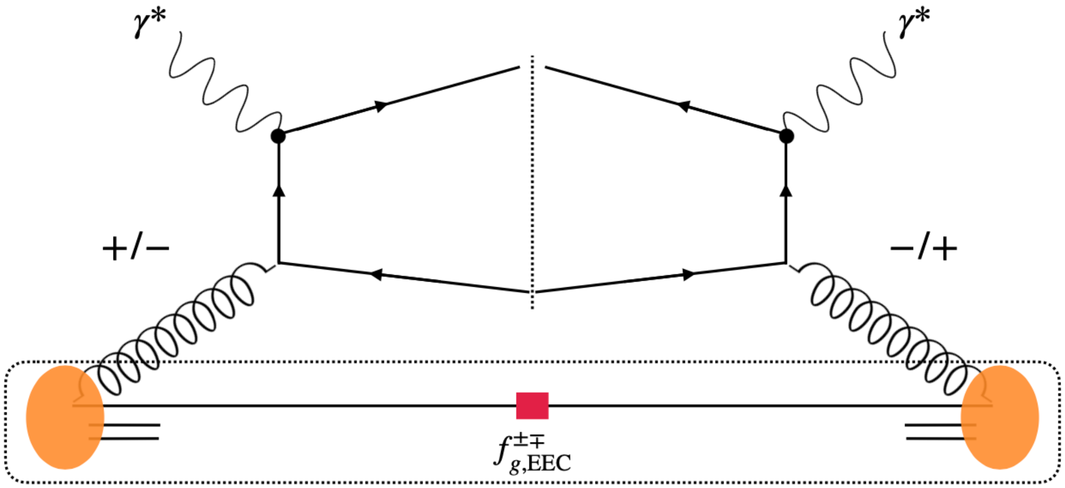


Li-Liu-Yuan-Zhu, 2308.10942

Looking forward…

- Nucleon energy-energy correlators show promising features to probe the internal structure of hadrons
  - Need more theoretical studies, such as high order calculations, extension to small-x domain, etc.
- EEC for final state jet production can probe the hadronization and multiple scattering of hard partons with the cold nuclear matter
  - Comparison with observables in heavy ion collisions will inform us the difference between hot and cold QCD matter

Back-up



$$l_{\mu\nu}\Sigma^{\mu\nu} = \int \frac{dz}{z}\Bigg[\sum_{i=q,g} \hat{H}_i(z,y,c_b)\frac{x_B}{z} f_{i,\mathrm{EEC}}\left(\frac{x_B}{z},\theta_a^2\right)$$

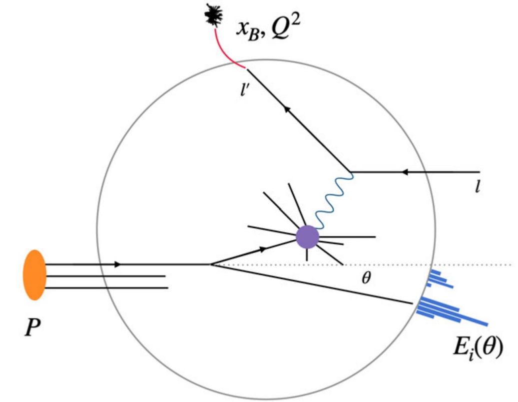
$$+ \frac{1}{2}\cos(2\phi)\Delta\hat{H}_g(z,y,c_b)\frac{x_B}{z} d_{g,\mathrm{EEC}}\left(\frac{x_B}{z},\theta_a^2\right)\Bigg], \qquad (7)$$

Factorization and evolution

$$\Sigma_N(Q^2,\theta) = \sum_i \int d\sigma(x_B, Q^2, p_i) x_B^{N-1} \frac{E_i}{E_P} \Theta(\theta - \theta_i)$$

$$\Sigma_N(Q^2,\theta) = \int dx_B x_B^{N-1} \int_{x_B}^1 \frac{dz}{z} \hat{\sigma}_i\left(\frac{x_B}{z}\right) f_{i,\mathrm{EEC}}(z,\theta)$$

$$\Sigma_N(Q^2,\theta) = \int dx_B x_B^{N-1} \int dz \big(H_q(z, x_B, Q^2) f_{q,\mathrm{EEC}}(z, P^+\theta) + H_g(z, x_B, Q^2) f_{g,\mathrm{EEC}}(z, P^+\theta)\big), \qquad (20)$$



$$f_{q,\mathrm{EEC}}(z, P^+\theta) = \int \frac{dy^-}{4\pi} e^{-izP^+\frac{y^-}{2}} \langle P|\bar{\chi}_n\left(\frac{y^-}{2} n^\mu\right) \frac{\gamma^+}{2} \hat{\mathcal{E}}(\theta) \chi_n(0)|P\rangle$$

$$f_{g,\mathrm{EEC}}(z, P^+\theta) = \int \frac{dy^-}{4\pi} e^{-izP^+\frac{y^-}{2}} P^+ \langle P|\mathcal{B}_\perp\left(\frac{y^-}{2} n^\mu\right) \hat{\mathcal{E}}(\theta) \mathcal{B}_\perp(0)|P\rangle$$

$$\Sigma_N(Q^2,\theta) = \sum_{i=q,g} \int du u^{N-1} \hat{\sigma}_i(u,Q^2) f_{i,\mathrm{EEC}}\left(N, \ln\frac{Q\theta}{u\mu}\right)$$

$$f_{i,\mathrm{EEC}}\left(N, \ln\frac{Q\theta}{u\mu}\right) = \int_0^1 dz z^{N-1} f_{i,\mathrm{EEC}}\left(z, \ln\frac{Q\theta}{zu\mu}\right)$$

$$\frac{d}{d\ln\mu^2} f_{i,\mathrm{EEC}}\left(N, \ln\frac{Q\theta}{u\mu}\right) = \sum_j \int d\xi \xi^{N-1} P_{ij}(\xi) f_{j,\mathrm{EEC}}\left(N, \ln\frac{Q\theta}{\xi u\mu}\right) = P_{ij} \odot f_{j,\mathrm{EEC}}(u),$$

$$\frac{d}{d\ln\mu^2} f_{i,\mathrm{EEC}}\left(z, \ln\frac{Q\theta}{zu\mu}\right) = \sum_j \int_z^1 \frac{d\xi}{\xi} P_{ij}\left(\frac{z}{\xi}\right) f_{j,\mathrm{EEC}}\left(\xi, \ln\frac{Q\theta}{zu\mu}\right)$$