Imprints of chiral & trace anomalies in GPDs

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Precision QCD Predictions for ep Physics at the EIC (II)

Center for Frontiers in Nuclear Science

Stony Brook University

In Collaboration with:

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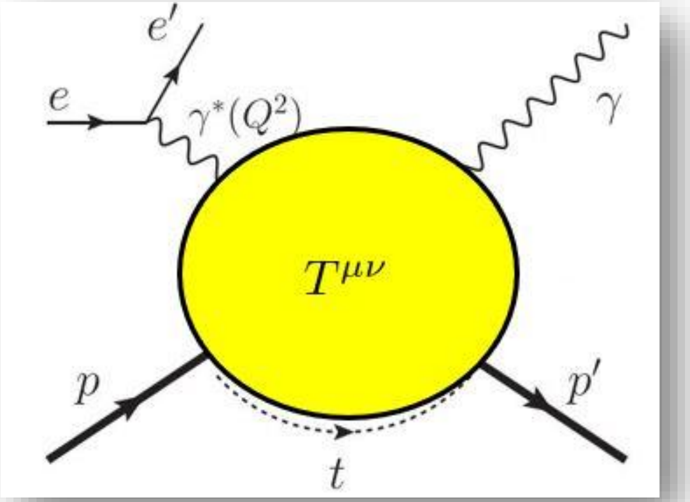
Werner Vogelsang (Tubingen U.)

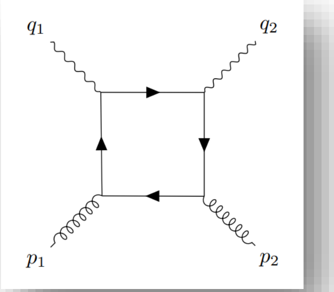
Based on:
arXiv:2210.13419, 2305.09431

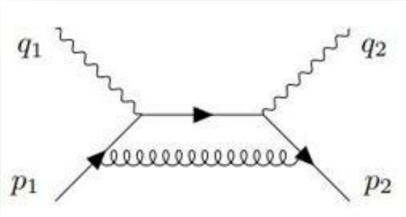
Outline

- Chiral & trace anomalies in QCD
- Anomaly in polarized DIS & proton's spin puzzle : History
- Connection between GPDs & anomalies:



Calculation of box diagrams relevant for Compton scattering:





- Polarized case
- Unpolarized case

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Calculation of box diagrams relevant for Compton scattering:

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Chiral anomaly

Recap on chiral anomaly in QCD:

- **Lagrangian invariant under global chiral rotation** $\psi \to e^{i\alpha\gamma_5}\psi$
- **Axial-vector current:** $J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$

Chiral anomaly

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- **Axial-vector current:** $J_5^\mu = \sum_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$
- **But measure of the path integral is not invariant, which breaks the conservation of the axial current**

K. Fujikawa, PRL 1979

Chiral anomaly

Anomaly equation:

$$\boxed{\partial_\mu J_5^\mu = -\frac{n_f \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}} \qquad \tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

A fundamental property of axial-vector current is the anomaly equation

Chiral anomaly

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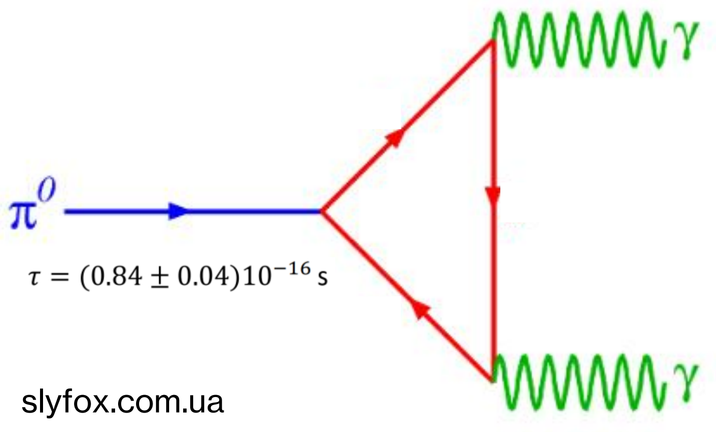
Adler – Bell - Jackiw chiral anomaly

Famous example: ABJ anomaly contribution to $\pi^0 \to 2\gamma$



In the chiral limit, without the anomaly,

π^0 **does not decay!**

Chiral anomaly

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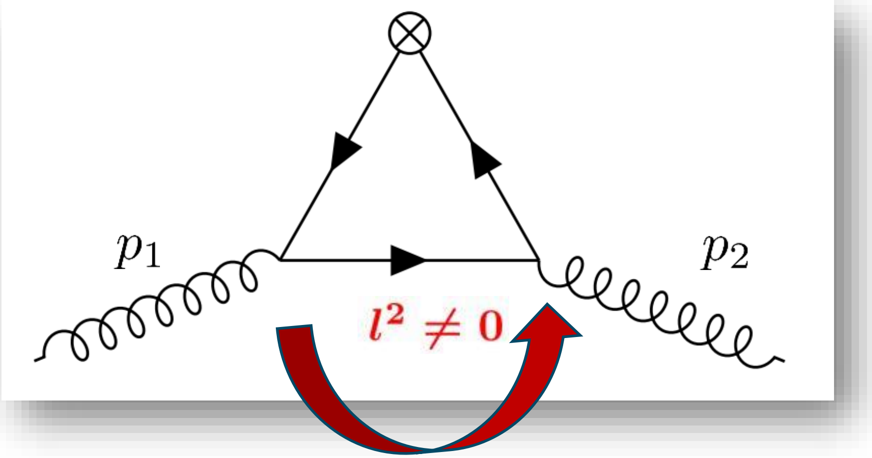
A perturbative solution to anomaly equation:



Calculation in off-forward kinematics $(l = p_2 - p_1)$**:**

$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{i l^\mu}{l^2} \langle p_2 | F_a^{\alpha\beta} \tilde{F}^a_{\alpha\beta} | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole

Chiral anomaly

Axial Form Factors:

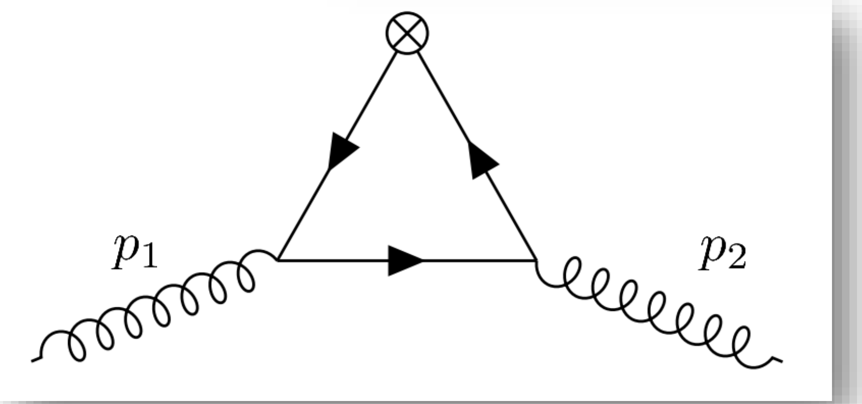
$$\boxed{\langle P_2|J_5^\mu|P_1\rangle = \bar{u}(P_2)\left[\gamma^\mu\gamma_5 g_A(l^2) + \frac{l^\mu\gamma_5}{2M}g_P(l^2)\right]u(P_1)}$$

A fundamental property of axial-vector current is the anomaly equation

Massless pole in pseudo scalar Form Factor? $g_P(l^2) \sim \frac{1}{l^2}$

A perturbative solutio

$g_A(0) = \Delta\Sigma$ **: Fraction of proton spin carried by quarks**



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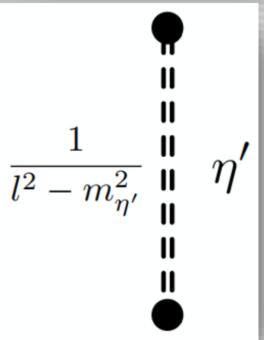
$F_{\rho\sigma}$

A fundamental property of axial-vector current is the anomaly equation

Massless pole in pseudo scalar Form Factor? $g_P(l^2) \sim \cancel{\frac{1}{l^2}}$

In QCD, we expect: $g_P(l^2) \sim \frac{1}{l^2 - m_{\eta'}^2}$

eta meson mass generation



A perturbative solutio

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Calculation in off-forward kinematics $(l$

Deeply tied to the UA(1) problem: Why is the η' so massive (957 MeV!)?

Triangle diagram is dominated by infra-red pole

Chiral anomaly

Taking divergence of axial-vector matrix element:

$$\boxed{2Mg_A(l^2) + \frac{l^2}{2M}g_P(l^2) = \frac{i\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}(l^2)|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}}$$

Massless pole in pseudo scalar Form Factor? $g_P(l^2) \sim \cancel{\frac{1}{l^2}}$

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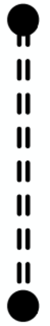

η'

In the presence of chiral anomaly:

: Fraction of proton spin carried by quarks

$$\boxed{\frac{g_P(l^2)}{2M}} = -\frac{2Mg_A(l^2)}{l^2} + \frac{i\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}(l^2)|P_1\rangle}{l^2\bar{u}(P_2)\gamma_5 u(P_1)}$$

$$\boxed{\approx -\frac{i}{l^2}\left(\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\bigg|_{l^2=0} - \frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\right)}$$

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$$\rangle = \frac{n_f\alpha_s}{4\pi}\frac{il^\mu}{l^2}\langle p_2|F^{\alpha\beta}_a\tilde{F}^a_{\alpha\beta}|p_1$$

Pole cancellation at Form Factor level

Same pole as what one naively gets from perturbation theory

Chiral anomaly

Axial Form Factors:

$\langle P_2 | J_5^\mu | P_1 \rangle = \bar{u}($

A fundamental property ... equation

Twist-2 GPDs

Pol. \ Γ	γ^+	$\gamma^+\gamma_5$	$\sigma^{+j}\gamma_5$
U	H		E_T
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Ma... or? $g_P(l^2) \sim \cancel{\frac{1}{l^2}}$

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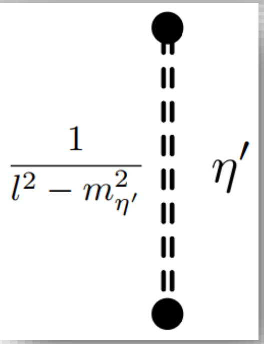

A perturbative solution to anomaly equation:

Any implications for the corresponding GPD?

$$g_P(l^2) = \int_{-1}^{1} dx\, \tilde{E}(x, \xi, l^2)$$

Triangle diagram is dominated by infra-red pole

Trace anomaly

Recap on trace anomaly in QCD:

- **Lagrangian invariant under scale transformation** $x^\mu \to e^\sigma x^\mu \quad \phi \to e^{-D\sigma}\phi$

- **Dilatation current:** $D^\mu = \Theta^{\mu\nu} x_\nu$ $\qquad \Theta^{\mu\nu}$ **: Energy Momentum Tensor (EMT)**

Energy Momentum Tensor (EMT)

$$T^{\mu\nu} = -F^{\mu\lambda}F^{\nu}{}_{\lambda} + \frac{\eta^{\mu\nu}}{4}F^2 + i\bar{\psi}\gamma^{(\mu}\overleftrightarrow{D}^{\nu)}\psi$$

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- **Conformal symmetry explicitly broken by quantum effects**

$$\partial_\mu D^\mu = \Theta^\mu_\mu \neq 0$$

Trace anomaly

Recap on trace anomaly in QCD:

- A quantum anomaly in the trace of its energy momentum tensor (conformal anomaly) breaks conformal invariance

Trace anomaly:

$$\boxed{\Theta^{\mu}_{\mu} = \frac{\beta(g)}{2g} F^{\mu\nu} F_{\mu\nu}}$$

$\Theta^{\mu\nu}$ **: Energy Momentum Tensor (EMT)**

Fundamentally important in QCD: Trace anomaly is the origin of hadron masses

$$\langle P | \Theta^{\mu}_{\mu} | P \rangle = 2M^2$$

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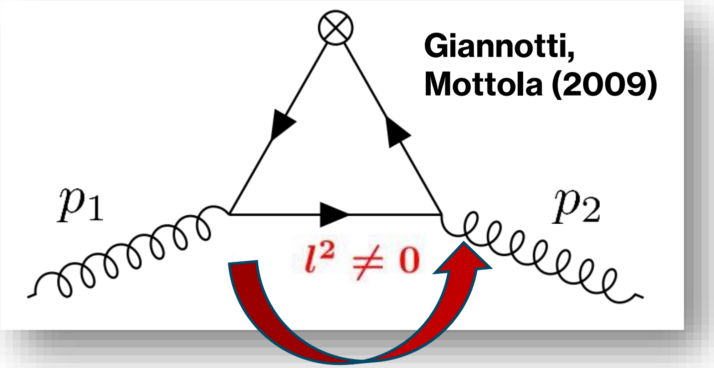


Calculation in off-forward kinematics $(l = p_2 - p_1)$**:**

$$\langle p_2 | \Theta^{\mu\nu}_{\mathrm{QED}} | p_1 \rangle = -\frac{e^2}{24\pi^2 l^2} \left(p^\mu p^\nu + \frac{l^\mu l^\nu - l^2 g^{\mu\nu}}{4} \right) \langle p_2 | F^{\alpha\beta} F_{\alpha\beta} | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole

Trace anomaly

Re

...maly)

...Tensor (EMT)

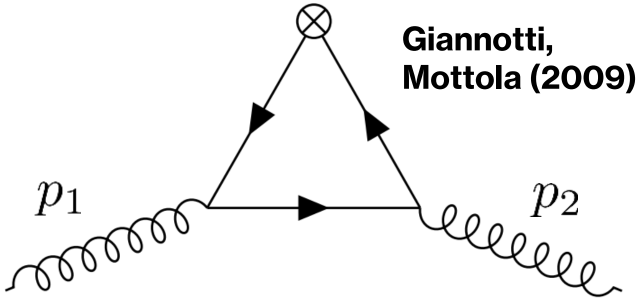
Gravitational Form Factors:

$$\boxed{\langle P_2|\Theta_f^{\mu\nu}|P_1\rangle = \frac{1}{M}\bar{u}(P_2)\left[P^\mu P^\nu A_f + (A_f+B_f)\frac{P^{(\mu}i\sigma^{\nu)\rho}l_\rho}{2} + \frac{D_f}{4}(l^\mu l^\nu - g^{\mu\nu}l^2) + M^2\bar{C}_f g^{\mu\nu}\right]u(P_1)}$$

Massless poles in Gravitational Form Factors?

$$A_f(l^2), B_f(l^2), D_f(l^2) \sim \frac{1}{l^2}$$

A perturbative solution to anomaly equation:



Giannotti, Mottola (2009)

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$$\frac{1}{l^2} \rightarrow \frac{1}{l^2 - m_G^2}$$

glueball mass generations

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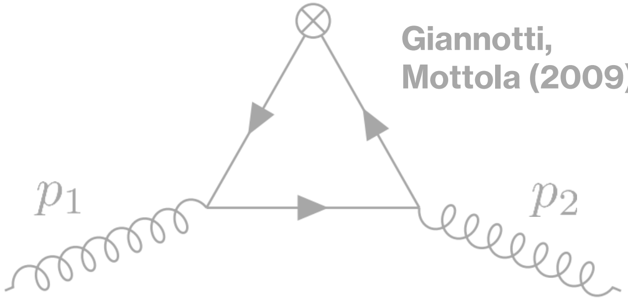

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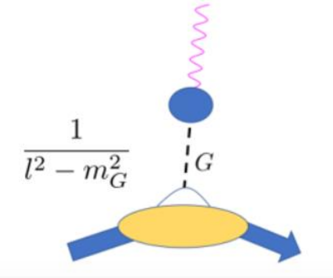

$$\langle p_2|\Theta^{\mu\nu}_{\mathrm{QED}}|p_1\rangle = -\frac{e^2}{24\pi^2 l^2}\Big(p^\mu p^\nu + \ldots \; {}_2|F^{\alpha\beta}F_{\alpha\beta}|p_1\rangle$$

Triangle diagram is domin

Fujita, Hatta, Sugimoto, Ueda (2022)
Mamo, Zahed (2019)

Trace anomaly

Gravitational Form Factors:

Trace of EMT:

$$\langle P_2|(\Theta)^\alpha_\alpha|P_1\rangle = M\left(A(t) + \frac{B(t)}{4M^2}t - \frac{3D(t)}{4M^2}t\right)\bar{u}(P_2)u(P_1) = \langle P_2|\frac{\beta(g)}{2g}F^{\mu\nu}F_{\mu\nu}|P_1\rangle$$

Massless poles in Gravitational Form Factors?

In the presence of trace anomaly: $A_f(l^2), B_f(l^2), D_f(l^2) \sim \cancel{\frac{1}{l^2}}$

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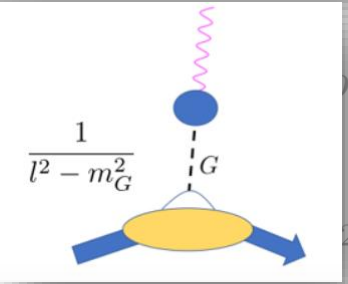
$$\frac{1}{l^2} \to \frac{1}{l^2 - m_G^2}$$

glueball mass generations



D term

$$\boxed{\frac{3D(t)}{4M^2}} \approx \frac{1}{t}\left(A(t) - \frac{\langle P_2|\frac{\beta(g)}{2g}F^2|P_1\rangle}{M\bar{u}(P_2)u(P_1)}\right)$$

$$= \boxed{\frac{A(t)-A(0)}{t} - \frac{1}{t}\left(\frac{\langle P_2|\frac{\beta(g)}{2g}F^2|P_1\rangle}{M\bar{u}(P_2)u(P_1)} - \left.\frac{\langle P_2|\frac{\beta(g)}{2g}F^2|P_1\rangle}{M\bar{u}(P_2)u(P_1)}\right|_{t=0}\right)}$$

Pole cancellation at Form Factor level

Same pole as what one naively gets from perturbation theory

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Trace anomaly

Twist-2 GPDs

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Gravitational

$$\langle P_2|\Theta_f^{\mu\nu}|P \ldots \frac{{}^{\mu}i\sigma^{\nu)\rho}l_\rho}{2} + \frac{D_f}{4}(l^\mu l^\nu - g^{\mu\nu}l^2) + M^2\bar{C}_f g^{\mu\nu}\Big] u(P_1)$$

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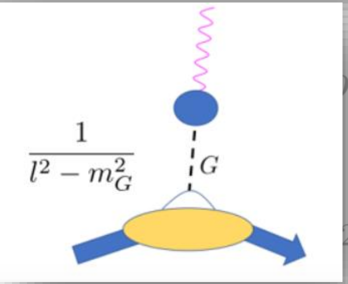
glueball mass generations



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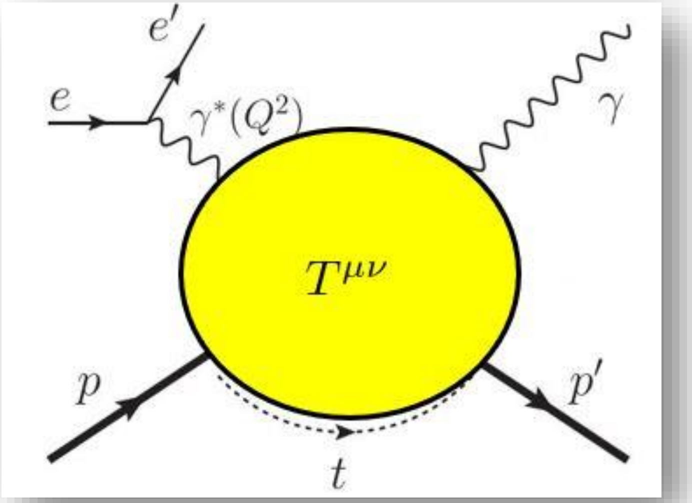

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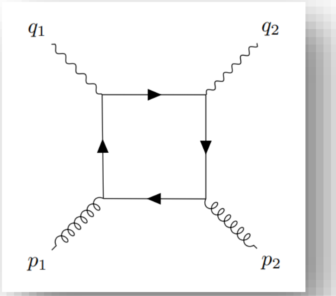

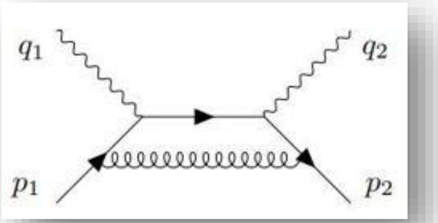

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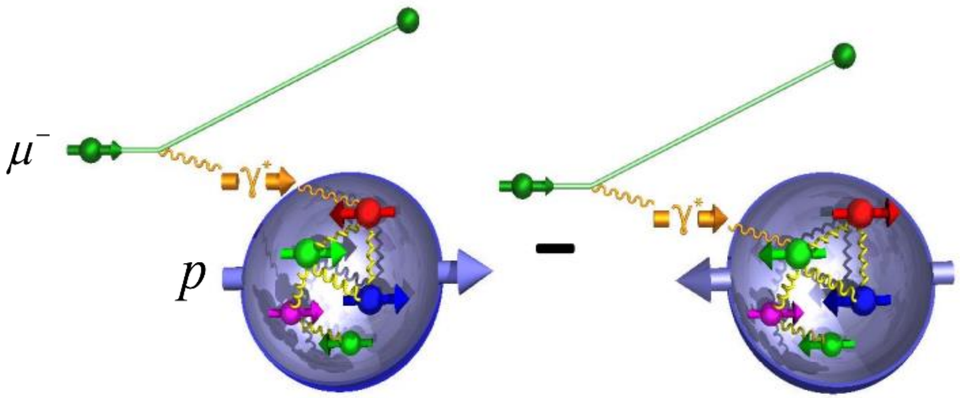
- **Unpolarized case**

Anomaly in polarized DIS: History

The role of chiral anomaly in polarized DIS is a well-known old story

Anomaly in polarized DIS: History

Polarized DIS & proton's spin puzzle:



g_1 **can be extracted from longitudinal double spin asymmetry:**

$$A_{LL} = \frac{\mu^\uparrow p^\Downarrow - \mu^\uparrow p^\Uparrow}{\mu^\uparrow p^\Uparrow + \mu^\uparrow p^\Downarrow} \sim \left(1 + \frac{\sigma_L}{\sigma_T}\right)\frac{2x g_1}{F_2}$$

First moment of g_1**:**

$\Delta\Sigma$

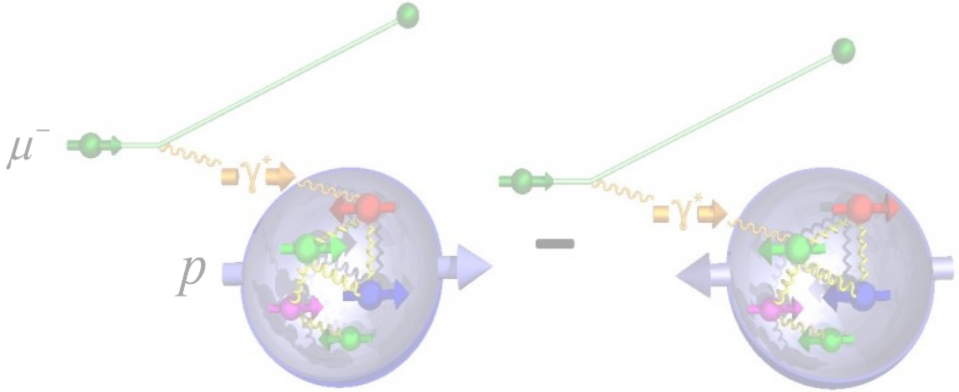
$$\int_0^1 dx g_1(x) = \frac{1}{9}(\Delta u + \Delta d + \Delta s) + \frac{1}{12}(\Delta u - \Delta d) + \frac{1}{36}(\Delta u + \Delta d - 2\Delta s) + \mathcal{O}(\alpha_s)$$

$g_A(0) = \Delta\Sigma$ **: Fraction of proton spin carried by quarks**

Deep inelastic scattering (DIS) experiments showed that quarks carry only about 30% of the proton's spin: $\Delta\Sigma \approx 0.32$, which is much smaller than predicted by the quark model $\Delta\Sigma \sim 1$ - spin puzzle

Anomaly in polarized DIS: History

Polarized DIS & proton's spin puzzle:



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$$+ \frac{1}{12}(\Delta u \ldots$$

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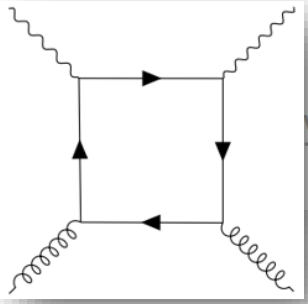
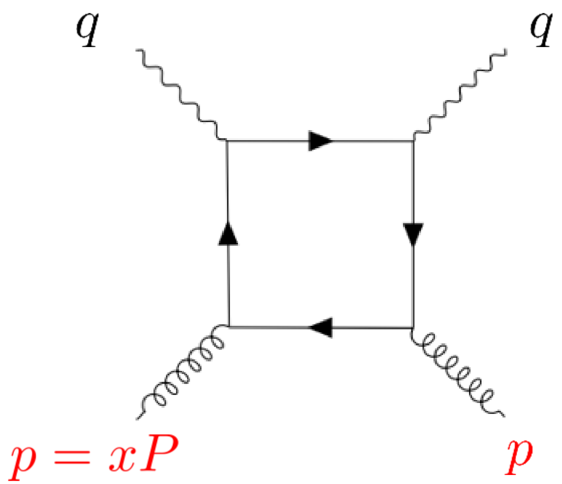
Deep inelastic scattering (DIS) expe... that quarks carry only about 30% of the ... y the

Calculate "box diagram", which is a controversial diagram



Anomaly in polarized DIS: History



Box diagram

One-loop correction to g_1 (gluon channel):

$$g_1(x) \sim \frac{\alpha_s}{2\pi}\left(\ln\frac{Q^2}{m_q^2}\Delta P_{qg}(x) + \delta C_{qg}(x)\right) \otimes \Delta G(x)$$

Polarized DGLAP splitting function: $\Delta P_{qg}(x) = 2x - 1$

Hard coefficient function (mass regularization):

A lot of controversy over this term in the past

$$\delta C_{qg}(x) = (2x-1)\left(\ln\frac{1-x}{x} - 1\right) + 2(1-x)$$

(same result in DR)

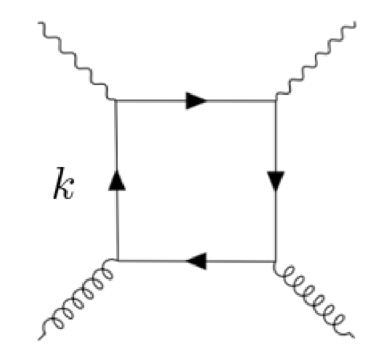
Anomaly in polarized DIS: History

A solution of spin problem?

One-loop correction to g_1 (gluon channel):

The "$1 - x$" comes from the infrared region of the box diagram:

$$(1-x)\int_0^{Q^2} dk_\perp^2 \frac{m_q^2}{(k_\perp^2 + m_q^2)^2} = \text{finite!}$$



$p = xP$ $\quad p$

But the coefficient function is supposed to be dominated by UV physics ...

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Anomaly in polarized DIS: History

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THE ANOMALOUS GLUON CONTRIBUTION TO POLARIZED LEPTOPRODUCTION

G. ALTARELLI and G.G. ROSS [1]
CERN, CH-1211 Geneva 23, Switzerland

Received 29 June 1988

THE ROLE OF THE AXIAL ANOMALY IN MEASURING SPIN-DEPENDENT PARTON DISTRIBUTIONS

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Received 22 August 1988

But the coefficient function is supposed to be dominated by UV physics ...

Consider this "anomalous" contribution as a part of "intrinsic spin":

$$\Delta\tilde{\Sigma} = \Delta\Sigma + \frac{n_f \alpha_s}{2\pi}\Delta G$$

Expect $\Delta\tilde{\Sigma} \sim 1$

If ΔG is large & positive, this can explain the smallness of $\Delta\Sigma$!

Reminder:

One-loop correction to g_1 (gluon channel):

$$g_1(x) \sim \frac{\alpha_s}{2\pi}\left(\ln\frac{Q^2}{m_q^2}\Delta P_{qg}(x) + \delta C_{qg}(x)\right) \otimes \Delta G(x)$$

First moment of g_1:

$$\int_0^1 dx g_1(x) = \Delta\Sigma + \mathcal{O}(\alpha_s)$$

Anomaly in polarized DIS: History

Critique 1

A solution of spin problem?

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THE ANOMALOUS GLUON CONTRIBUTION TO POLARIZED LEPTOPRODUCTION

Gluonic contribution to g_1 and its relationship to the spin-dependent parton distributions

Geoffrey T. Bodwin and Jianwei Qiu*

usual forms of the quark sum rules. We conclude that the size of the gluonic contribution to the first moment of g_1 is entirely a matter of the convention used in defining the quark distributions.

Consider this "anomalous" contribution as a part of "intrinsic spin":

$$\Delta\tilde{\Sigma} = \Delta\Sigma + \frac{n_f\alpha_s}{2\pi}\Delta G$$

A lot of controversy over this term in the past

Expect $\Delta\tilde{\Sigma} \sim 1$

If ΔG is large & positive, this can explain the smallness of $\Delta\Sigma$!

Anomaly in polarized DIS: History

Critique 2

spin problem?

Calculation in forward kinematics:

THE ANOMALOUS GLUON CONTRIBUT

No infrared pole!

G. A ... SS
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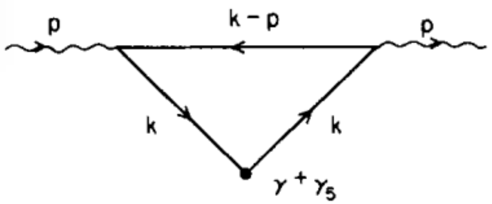

$$\frac{1}{2p^+}\langle p,\pm|j_5^+|p,\pm\rangle = \mp\frac{\alpha_s N_f}{2\pi}$$

(Carlitz, Collins, Mueller)

THE ROLE OF THE AXIAL ANOMALY
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But the coefficient function is supposed to be

THE g_1 PROBLEM: DEEP INELASTIC ELECTRON SCATTERING AND THE SPIN OF THE PROTON*

R.L. JAFFE and Aneesh MANOHAR**

Calculation in off-forward kinematics $(l = p_2 - p_1)$**:**

$$\langle p_2|J_5^\mu|p_1\rangle = \frac{n_f\alpha_s}{4\pi}\frac{il^\mu}{l^2}\langle p_2|F_a^{\alpha\beta}\tilde{F}^a_{\alpha\beta}|p_1\rangle$$

Triangle diagram is dominated by infra-red pole

direction? The answer lies in the triangle diagram. For massless quarks and on-shell gluons, the off-forward matrix element of the triangle diagram (see fig. 3) coincides with the matrix element of $-i(l^\mu/l^2)(\alpha_s/2\pi)\mathrm{Tr}\,F\tilde{F}$ [54]. This result is regularization-independent. In QCD, the pole at $l^2=0$ is unphysical and is cancelled by non-triangle contributions to the matrix element of A_μ^0. With the aid

Jaffe,Manohar (1990)

If ΔG is large & positive, this can explain

Anomaly in polarized DIS: History

Critique 2

spin problem?

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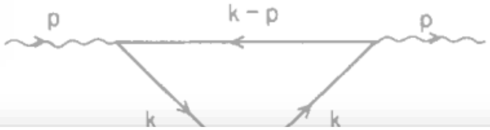
Calculation in forward kinematics:

THE ANOMALOUS GLUON CONTRIBUT

No infrared pole!



$$\frac{1}{2p^+}\langle p,\pm|j_5^+|p,\pm\rangle = \mp\frac{\alpha_s N_f}{2\pi}$$

Box diagram can be viewed as a non-local generalization of triangle diagram

s, Mueller)

A.H. MUELLER

If triangle is dominated by anomaly pole, trace of that should be visible in box diagram

TTERING
AND THE SPIN OF THE PROTON*

R.L. JAFFE and Aneesh MANOHAR**

But the coefficient function is supposed to be

Calculation in off-forward kinematics $(l = p_2 - p_1)$:

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Imprint of Anomalies in DIS

First calculation of box diagram with $l^2 \neq 0$:

Anomaly equation:

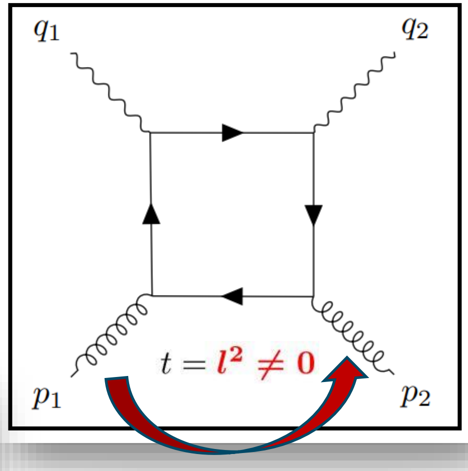
The role of the chiral anomaly in polarized deeply inelastic scattering I: Finding the triangle graph inside the box diagram in Bjorken and Regge asymptotics

Andrey Tarasov[1,2] and Raju Venugopalan[3]

A fundamental prop

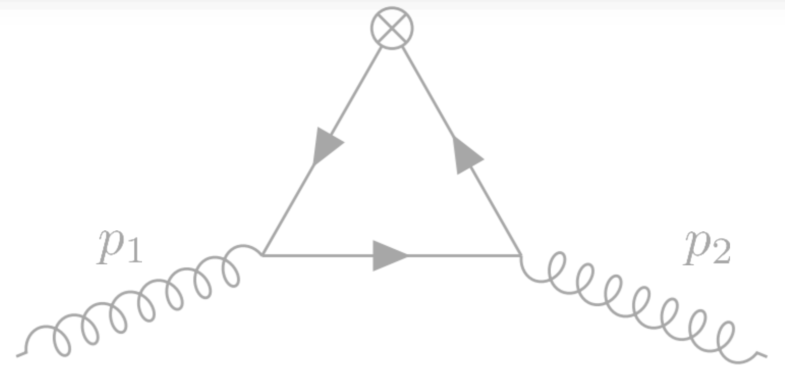
The role of the chiral anomaly in polarized deeply inelastic scattering II: Topological screening and transitions from emergent axion-like dynamics

Andrey Tarasov[1,2] and Raju Venugopalan[3]



Andrey & Raju demonstrated within world-line formalism that to capture the physics of anomaly we need to calculate everything in off-forward kinematics for polarized DIS

Box diagram



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$$\langle p_2 | J_5^\mu | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{i l^\mu}{l^2} \langle p_2 | F_a^{\alpha\beta} \tilde{F}^a_{\alpha\beta} | p_1 \rangle$$

Triangle diagram is dominated by infra-red pole

Imprint of Anomalies in QCD Compton scattering

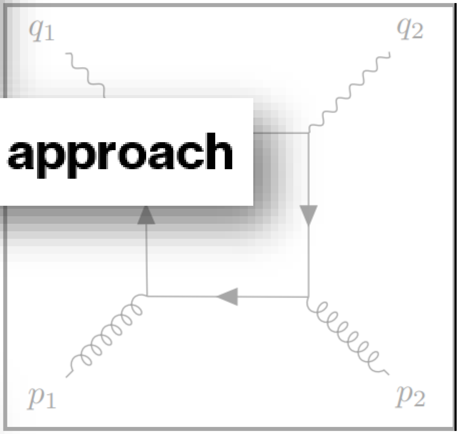
First calculation of b

The role of the

triangle gr

Chiral and trace anomalies in Deeply Virtual Compton Scattering : QCD factorization and beyond

Shohini Bhattacharya,[1,*] Yoshitaka Hatta,[2,1,†] and Werner Vogelsang[3,‡]



Box diagram

We explored the physics of anomaly in DVCS using Feynman-diagram approach

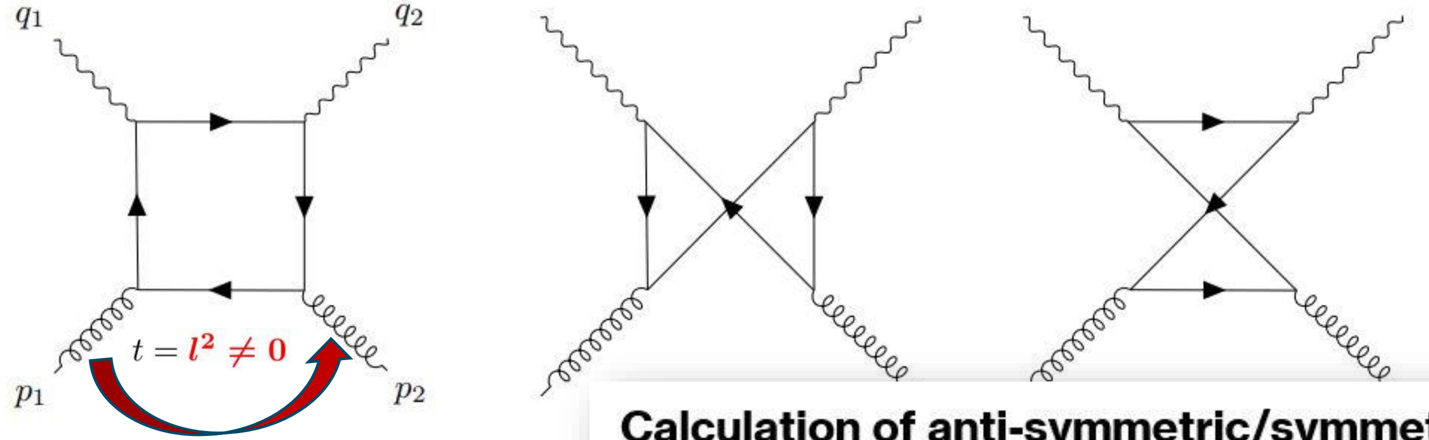


FIG. 1: Diagrams for the su

Calculation of anti-symmetric/symmetric (μ, ν) of Compton amplitude with $t = l^2 \neq 0$ $(\Lambda^2_{\rm QCD} \ll t \ll Q^2)$

Different than $t \sim \Lambda^2_{\rm QCD}$ regime

naly

d by infra-red pole

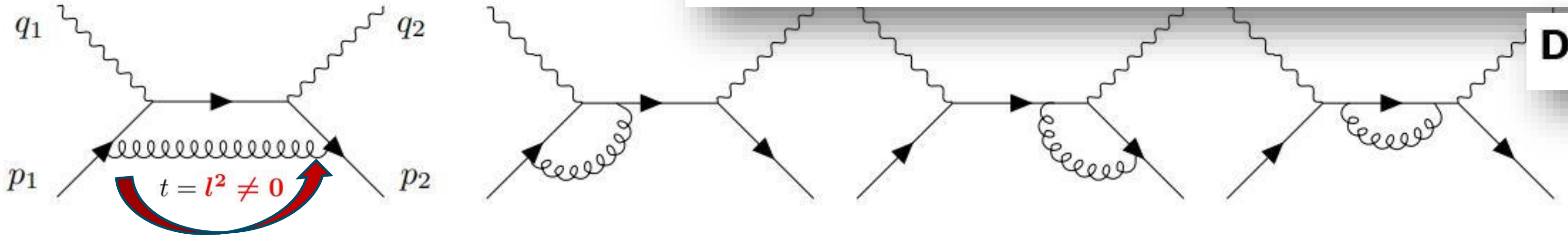
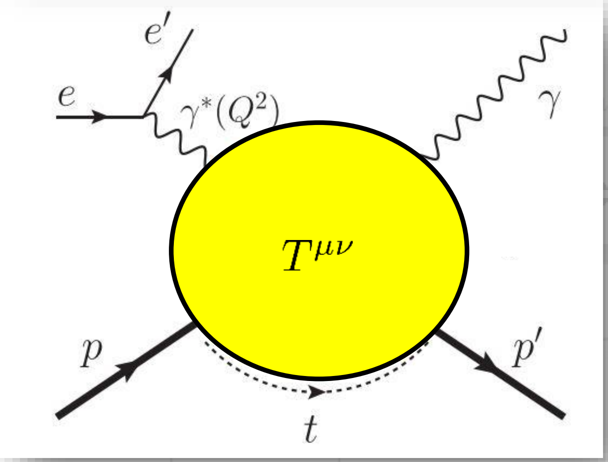


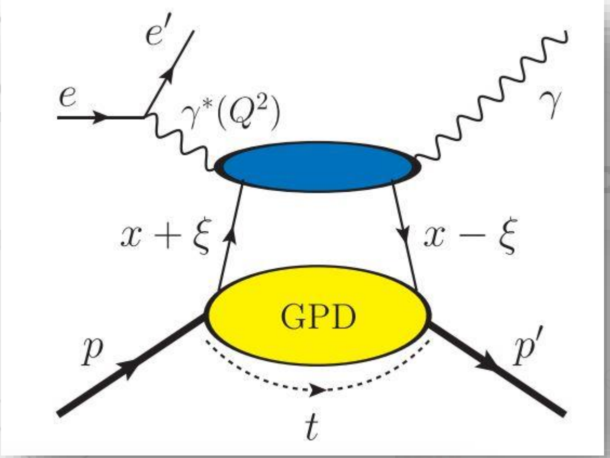
FIG. 2: Diagrams for the subprocess $\gamma^* q \to \gamma^* q$ in Compton scattering. Diagrams with photon lines crossed are not shown.

Remarks on DVCS:



Bjorken limit $\longrightarrow$

$t, \Lambda^2_{\mathrm{QCD}} \ll Q^2$



The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

$$T^{\mu\nu} = \sum_{a=q,g} \int \frac{dx}{x} C^{\mu\nu}_a\left(\frac{x_B}{x}, \frac{\xi}{x}\right) f_a(x,\xi,t) + \mathcal{O}(1/Q^2)$$

Original proof of factorization is for $t \sim \Lambda^2_{\mathrm{QCD}}$ **regime**

Different than $t \sim \Lambda^2_{\mathrm{QCD}}$ **regime**

We <u>extended factorization</u> for the first time within $\Lambda^2_{\mathrm{QCD}} \ll t \ll Q^2$ **regime**

Imprint of Anomalies in QCD Compton scattering

First calculation of b

Chiral and trace anomalies in Deeply Virtual Compton Scattering : QCD factorization and beyond

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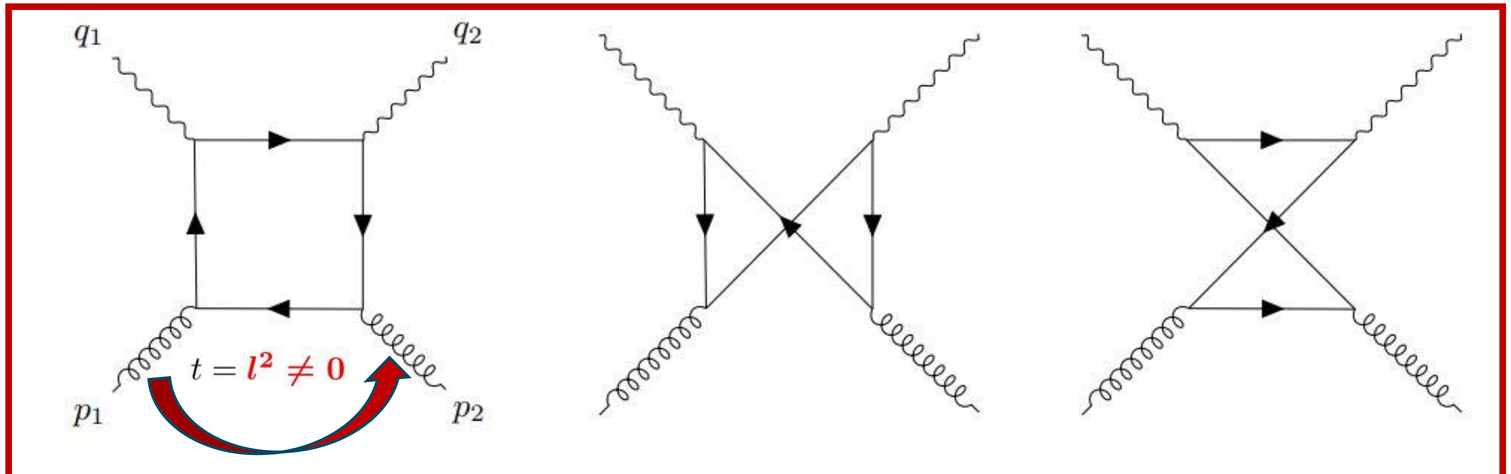
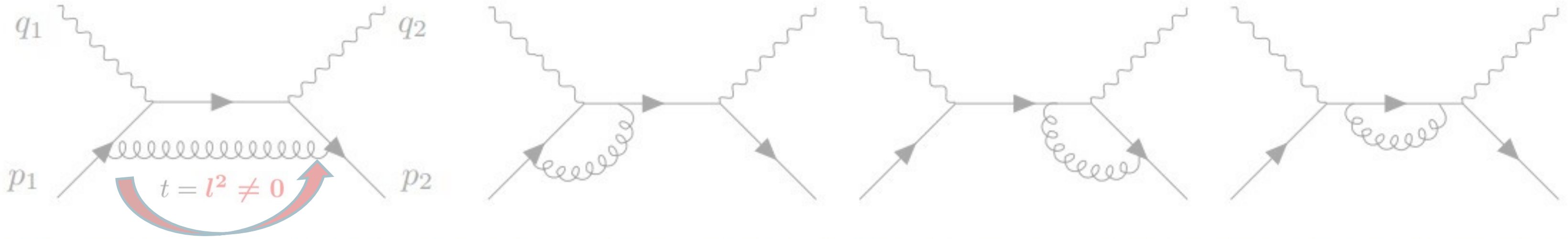


FIG. 1: Diagrams for the subprocess $\gamma^* g \to \gamma^* g$ in Compton scattering.



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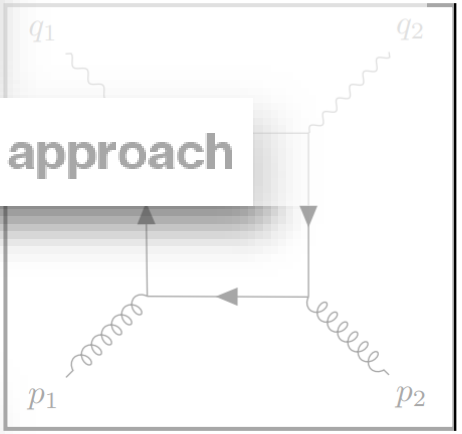
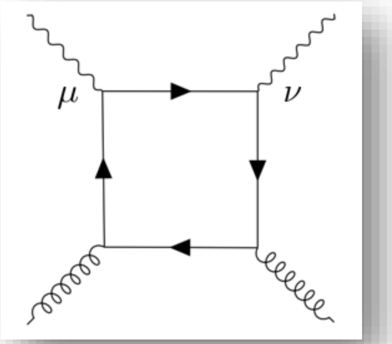


Box diagram

ematics $(l = p_2 - p_1)$:

d by infra-red pole

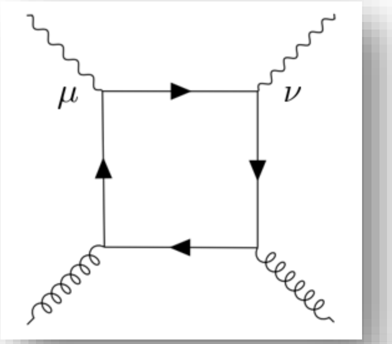
Imprint of Anomalies in QCD Compton scattering



Antisymmetric part of Compton amplitude

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \mathrm{Im} T^{\mathrm{asym}}_{\mu\nu}$$

Imprint of Anomalies in QCD Compton scattering



Antisymmetric part of Compton amplitude

Collinear singularity regularized by l^2

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \mathrm{Im}T^{\mathrm{asym}}_{\mu\nu} \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\bar{u}(P_2)\left[\left(\Delta P_{qg}\ln\frac{Q^2}{-l^2}+\delta C_g^{\mathrm{off}}\right)\otimes \Delta G(x_B)\gamma^\alpha\gamma_5 + \frac{l^\alpha}{l^2}\delta C_g^{\mathrm{anom}}\otimes \tilde{\mathcal{F}}(x_B)\gamma_5\right]u(P_1)$$

Polarized gluon distribution

Expected terms:

Splitting function $\Delta P_{qg}(\hat{x}) = 2T_R(2\hat{x}-1)$

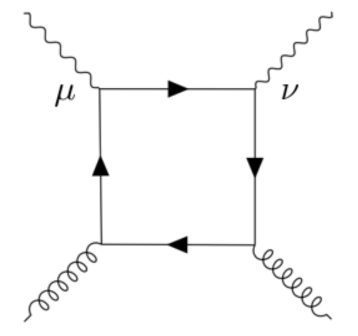
Coefficient function $\delta C_g^{\mathrm{off}}(\hat{x}) = 2T_R(2\hat{x}-1)\left(\ln\frac{1}{\hat{x}(1-\hat{x})}-1\right)$

Recall: In DR, one obtains

$$\Delta P_{qg}\frac{-1}{\epsilon}+\delta C_g^{\overline{\mathrm{MS}}}$$

$$\delta C_g^{\overline{\mathrm{MS}}}(\hat{x}) = 2T_R(2\hat{x}-1)\left(\ln\frac{1-\hat{x}}{\hat{x}}-1\right)+4T_R(1-\hat{x})$$

Imprint of Anomalies in QCD Compton scattering



Antisymmetric part of Compton amplitude

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \mathrm{Im} T^{\mathrm{asym}}_{\mu\nu} \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right) \bar{u}(P_2)\left[\left(\Delta P_{qg} \ln\frac{Q^2}{-l^2} + \delta C_g^{\mathrm{off}}\right) \otimes \Delta G(x_B)\gamma^\alpha\gamma_5 + \frac{l^\alpha}{l^2}\delta C_g^{\mathrm{anom}} \otimes \tilde{\mathcal{F}}(x_B)\gamma_5\right] u(P_1)$$

Pole term
In agreement with Tarasov, Venugopalan

Coefficient function $\delta C_g^{\mathrm{anom}}(\hat{x}) = 4T_R(1-\hat{x})$

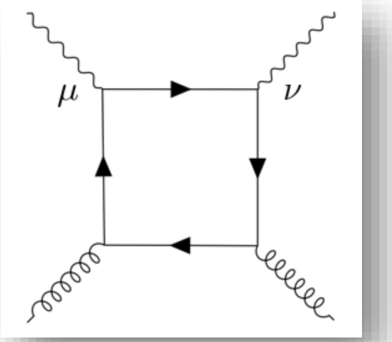
Same controversial "1-x" term as in previous calculations! (see earlier slide)

Twist-4 GPD:

$$\tilde{\mathcal{F}}(x, l^2) = \frac{iP^+}{\bar{u}(P_2)\gamma_5 u(P_1)} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P_2 | F_a^{\mu\nu}(-z^-/2)\tilde{F}^a_{\mu\nu}(z^-/2)|P_1\rangle$$

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude

Imprint of Anomalies in QCD Compton scattering



Antisymmetric part of Compton amplitude

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Twist-4 GPD

But no suppression in $1/Q^2$!

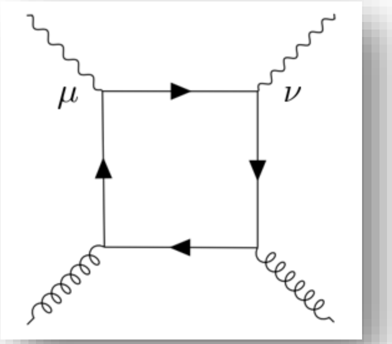
The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \mathrm{Im} T^{\mathrm{asym}}_{\mu\nu} = \frac{1}{2}\sum_f e_f^2 \bar{u}(P_2)\left[\gamma^\alpha\gamma_5\left(\tilde{H}_f(x_B,\xi,l^2)+\tilde{H}_f(-x_B,\xi,l^2)\right)+\frac{l^\alpha\gamma_5}{2M}\left(\tilde{E}_f^{\mathrm{bare}}(x_B,\xi,l^2)+\tilde{E}_f^{\mathrm{bare}}(-x_B,\xi,l^2)\right)\right]u(P_1)$$
$$+\mathcal{O}(\alpha_s)+\mathcal{O}(1/Q^2)\,,$$

Twist-2 GPDs to all orders

(Non-local) chiral anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering



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Anomalous contribution to GPD $\tilde{E}$ at one loop

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \mathrm{Im}T^{\mathrm{asym}}_{\mu\nu} = \frac{1}{2}\sum_f e_f^2\bar{u}(P_2)\left[\gamma^\alpha\gamma_5(\tilde{H}_f(x_B,\xi,l^2)+\tilde{H}_f(-x_B,\xi,l^2))+\frac{l^\alpha\gamma_5}{2M}(\tilde{E}_f^{\mathrm{bare}}(x_B,\xi,l^2)+\tilde{E}_f^{\mathrm{bare}}(-x_B,\xi,l^2))\right]u(P_1)$$
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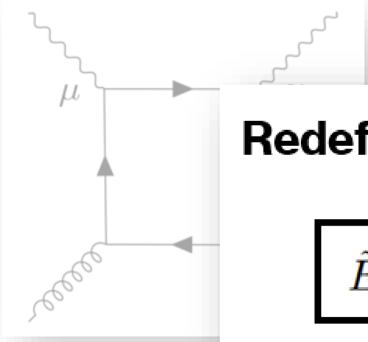


Antisymmetric p

Justifying factorization

Redefine

$$\boxed{\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2)} = \boxed{\tilde{E}_f^{\rm bare}(x_B,l^2)+\tilde{E}_f^{\rm bare}(-x_B,l^2)} + \frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B,l^2)$$

"Bare GPD" (tree level)

Perturbative pole (one loop)

$\otimes \tilde{\mathcal{F}}(x_B)\gamma_5\Big]\, u(P_1)$

one loop

The QCD factorization theorem: Collins, Freund; Ji, Osborne (1998)

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta \mathrm{Im}T^{\rm asym}_{\mu\nu} = \frac{1}{2}\sum_f e_f^2\bar{u}(P_2)\left[\gamma^\alpha\gamma_5(\tilde{H}_f(x_B,\xi,l^2)+\tilde{H}_f(-x_B,\xi,l^2)) + \frac{l^\alpha\gamma_5}{2M}(\tilde{E}_f^{\rm bare}(x_B,\xi,l^2)+\tilde{E}_f^{\rm bare}(-x_B,\xi,l^2))\right]u(P_1)$$
$$+\mathcal{O}(\alpha_s)+\mathcal{O}(1/Q^2)\,,$$

Perhaps not an ad hoc argument?

to all orders

chiral anomaly manifests itself in high energy scattering amplitude &

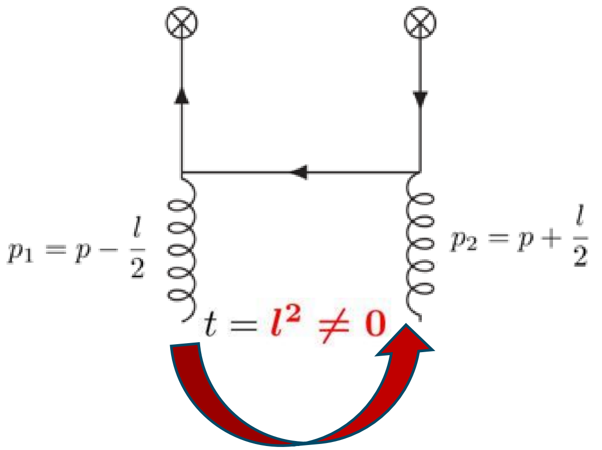
apparently breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering

Antisymmetric p

Justifying factorization

Redefine



Perturbative pole in GPD

$$\int \frac{dz^-}{4\pi} e^{i\hat{x}p^+z^-} \langle p_2|\bar{\psi}(-z^-/2)\gamma^+\gamma_5\psi(z^-/2)|p_1\rangle\Big|_{\text{pole}} \sim \frac{\alpha_s}{2\pi} T_R \frac{2il^+}{l^2}(1-\hat{x})\ \epsilon^{\epsilon_1\epsilon_2^* lp}$$

Same pole in one-loop calculation!

Perhaps not an ad hoc argument !

to all orders

chiral anomaly manife

The pole "belongs" to GPD

plitude &

apparently breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering

Antisymmetric part o

Elusive pole

ONE-LOOP QCD CORRECTIONS TO DEEPLY-VIRTUAL COMPTON SCATTERING: THE PARTON HELICITY-INDEPENDENT CASE

Xiangdong Ji and Jonathan Osborne

$$\sum_f e_f^2 \Big) \bar{u}(P_2) \Big[\Big(\Delta P_{qg} \ln \frac{Q^2}{-l^2} + \delta C_g^{\rm off} \Big) \otimes \Delta G(x_B) \gamma^\alpha \gamma_5 + \frac{l^\alpha}{l^2} \delta C_g^{\rm anom} \otimes \tilde{\mathcal{F}}(x_B) \gamma_5 \Big] u(P_1)$$

Anomalous contribution to GPD $\tilde{E}$ at one loop

Predictions from conformal algebra for the deeply virtual Compton scattering.

A.V. Belitsky D. Müller

, Osborne (1998)

$$\ldots \tilde{H}_f(x_B,\xi,l^2) + \tilde{H}_f(-x_B,\xi,l^2)) + \frac{l^\alpha \gamma_5}{2M} (\tilde{E}_f^{\rm bare}(x_B,\xi,l^2) + \tilde{E}_f^{\rm bare}(-x_B,\xi,l^2)) \Big] u(P_1)$$

NLO Corrections to Deeply-Virtual Compton Scattering *

L. Mankiewicz[†a], G. Piller[a], E. Stein[b], M. Vänttinen[a] and T. Weigl[a]

Twist-2 GPDs

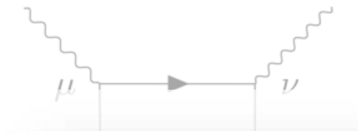
chiral anomaly manifests itself in high energy scattering amplitude &

ation

NLO corrections to timelike, spacelike and double deeply virtual Compton scattering.

B. Pire[1] and L. Szymanowski[2] and J. Wagner[2]

Imprint of Anomalies in QCD Compton scattering



Antisymmetric part

Elusive pole

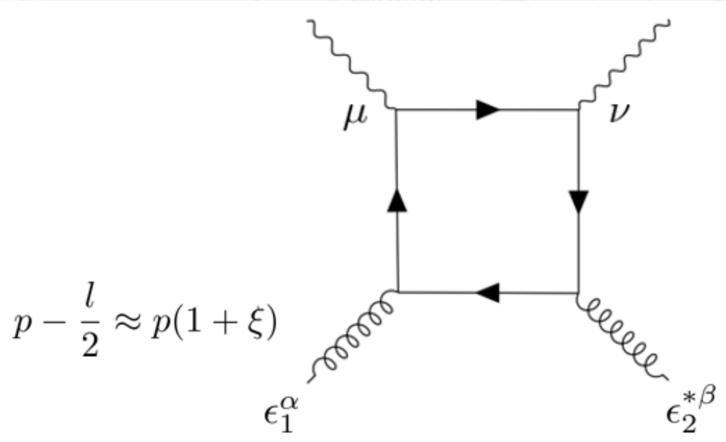
ONE-LOOP QCD CORRECTIONS TO

Pole was unnoticed in the GPD literature because one typically assumes

$$l^\mu = -2\xi p^\mu \;\to\; t = l^2 = 0$$

before loop integration

Usual rationale: Corrections supposedly higher twist $\dfrac{t}{Q^2}$



NL

However, box diagram is power-divergent in the IR!

Scattering

L. Mankiewicz[†a], G. Piller[a], E. Stein[b], M. Vänttinen[a] and T. Weigl[a]

$$\frac{l^\alpha}{l^2}\delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B)\gamma_5$$

Still, pole was never seen before because:

$$\langle p_2|F^{\mu\nu}\tilde{F}_{\mu\nu}|p_1\rangle \propto \epsilon^{\mu\nu\alpha\beta} l_\mu p_\nu \epsilon_{1\alpha}\epsilon^*_{2\beta}$$

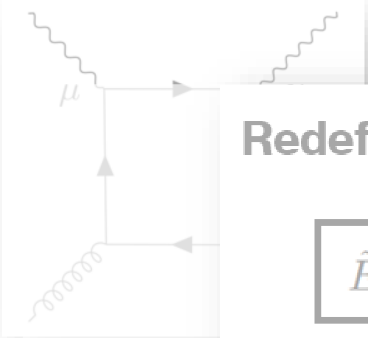
$$\to 0 \quad \textbf{when} \quad l^\mu \propto p^\mu$$

gy scattering amplitude &

NLO correction ... e and do

B. Pire[1] and L. Szymanow

Imprint of Anomalies in QCD Compton scattering



Antis

Beyond factorization: Fate of anomaly pole

Redefine

$$\boxed{\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2)} = \boxed{\tilde{E}_f^{\rm bare}(x_B,l^2)+\tilde{E}_f^{\rm bare}(-x_B,l^2)} + \frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B,l^2)$$

"Bare GPD" (tree level)

Perturbative pole (one loop)

$\otimes \tilde{\mathcal{F}}(x_B)\gamma_5\Big] u(P_1)$

one loop

The QCD

Perturbative calculations suggest that massless poles are induced in GPD $\tilde{E}$

$(-x_B,\xi,l^2))\Big] u(P_1)$

However, we know there are no massless poles in axial form factor (moment of GPD $\tilde{E}$)

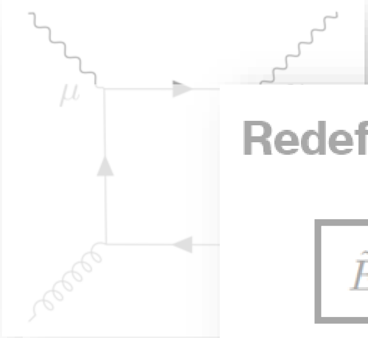
Twist-2 GPDs
to all orders

$$g_P(l^2) = \int dx\,\tilde{E}(x) \sim \cancel{\frac{1}{l^2}}$$

ests itself in high energy scattering amplitude &

apparently breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering



Beyond factorization: Fate of anomaly pole

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$$\boxed{\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2)} = \boxed{\tilde{E}_f^{\mathrm{bare}}(x_B,l^2)+\tilde{E}_f^{\mathrm{bare}}(-x_B,l^2)} + \frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\mathrm{anom}} \otimes \tilde{\mathcal{F}}(x_B,l^2)$$

"Bare GPD" (tree level)

Perturbative pole (one loop)

Postulate that the perturbative pole cancels the pre-existing pole in "bare" GPD:

$$\boxed{\tilde{E}_f^{\mathrm{bare}}(x_B,l^2)+\tilde{E}_f^{\mathrm{bare}}(-x_B,l^2)} \approx -\frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\mathrm{anom}} \otimes \tilde{\mathcal{F}}(x_B,l^2=0)$$

Postulate that the "renormalized" GPD integrates to $g_P(l^2)$:

$$g_P(l^2) = \sum_f \int_{-1}^{1} dx \tilde{E}_f(x,\xi,l^2) = \sum_f \int_0^1 \boxed{dx(\tilde{E}_f(x,\xi,l^2)+\tilde{E}_f(-x,\xi,l^2))}$$

Impr… Anomalies in QCD Compton scattering

Connection between twist 2 & twist 4 GPDs due to anomaly

Beyond factorization: Fate of anomaly pole

Redefine

$$\boxed{\tilde{E}_f(x_B,l^2)+\tilde{E}_f(-x_B,l^2)} = \boxed{\tilde{E}_f^{\rm bare}(x_B,l^2)+\tilde{E}_f^{\rm bare}(-x_B,l^2)} + \frac{\alpha_s}{2\pi}\frac{2M}{l^2}\delta C_g^{\rm anom}\otimes\tilde{\mathcal{F}}(x_B,l^2)$$

"Bare GPD" (tree level)

Perturbative pole (one loop)

Pole cancellation at $\int dx$

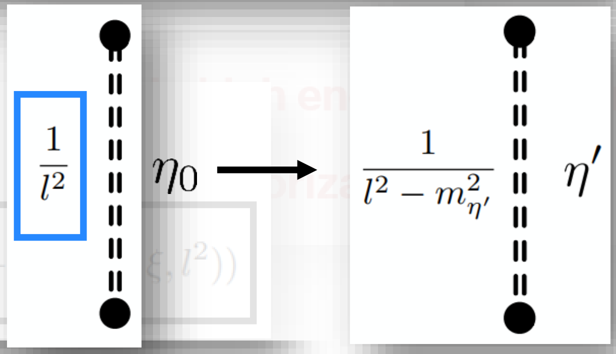
We find:

$$\frac{g_P(l^2)}{2M} = -\left[\boxed{\frac{2M\Delta\Sigma}{l^2}} - \frac{2M\Delta\Sigma}{l^2} + \frac{2M\Delta\Sigma}{l^2-m_{\eta'}^2}\right]$$

$$\sim \frac{1}{l^2-m_{\eta'}^2}$$

"We demonstrate that the dynamical interplay between the physics of the anomaly, and that of the isosinglet pseudoscalar $U_A(1)$ sector of QCD resolves both problems simultaneously: the lifting of the $\bar{\eta}$ pole by topological mass generation of the η' and the cancellation of the anomaly pole"

- Tarasov, Venugopalan



See also Jaffe Manohar, 1990

Story of a pole in GPD: a look at iso vector case

Connection between twist 2 ... due to anomaly

Antis Beyond fa

Example: Iso vector axial current

Axial Form Factors:

$$\langle P_2|J^{\alpha}_{5a}|P_1\rangle = \bar{u}(P_2)\left[\gamma^{\alpha}\gamma_5 F_A(t) + \frac{l^{\alpha}\gamma_5}{2M}F_P(t)\right]\frac{\tau^a}{2}u(P_1)$$

Current conservation (chiral symmetry) leads to:

$$F_P(t) \approx \frac{-2M^2 g_A^{(3)}}{t}, \qquad (t \to 0)$$

where $g_A^{(3)} = F_A(0) \approx 1.3$ is the isovector axial coupling constant. The pole is generated by the exchange of the massless pion which is the Nambu-Goldstone boson of spontaneously broken chiral symmetry.

Pole cancellation at $\int dx$

We find: $\frac{g_P(l^2)}{2M} = -\left[\boxed{\frac{2M\Delta\Sigma}{l^2}} - \frac{2M\Delta\Sigma}{l^2} + \frac{2M\Delta\Sigma}{l^2 - m^2_{\eta'}}\right] \sim \frac{1}{l^2 - m^2_{\eta'}}$

"We demonstrate that the dynamical interplay between the physics of the anomaly, and that of the isosinglet pseudoscalar $U_A(1)$ sector of QCD resolves both problems simultaneously: the lifting of the $\bar{\eta}$ pole by topological mass generation of the η' and the cancellation of the anomaly pole"

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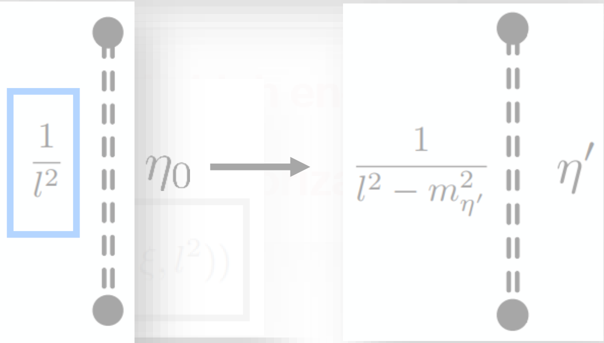


See also Jaffe Manohar, 1990

Story of a pole in GPD: a look at iso vector case

Iso vector GPD:

$$F_P(t) = \int_{-1}^{1} dx \left(\tilde{E}_u(x,\xi,t) - \tilde{E}_d(x,\xi,t) \right)$$

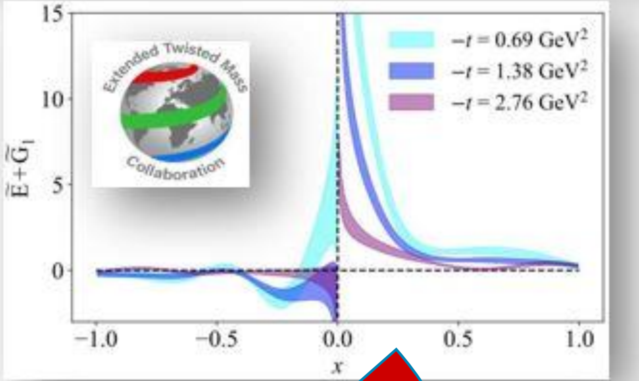
$$\tilde{E}_u(x,\xi,t) - \tilde{E}_d(x,\xi,t) \sim \theta(\xi - |x|) \frac{g_A^{(3)}}{t}$$

(Penttinen, Polyakov, Goeke)

First indication of pion pole from Lattice QCD:

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

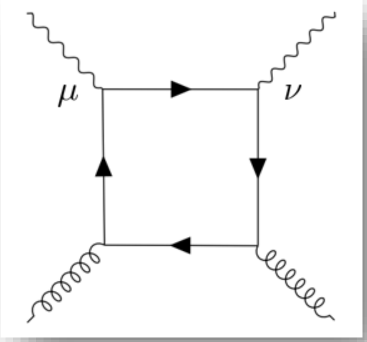
Shohini Bhattacharya[1,2], Krzysztof Cichy[3], Martha Constantinou[1], Jack Dodson[1], Andreas Metz[1], Aurora Scapellato[1], Fernanda Steffens[4]



In QCD, we expect:

$$\tilde{E}_u - \tilde{E}_d \sim \frac{1}{l^2 - m_\pi^2}$$

Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude

Pole! (New result)

$$\boxed{(H_f(x_B,\xi,l^2)-H_f(-x_B,\xi,l^2))}=\boxed{(H_f^{\rm bare}(x_B,\xi,l^2)-H_f^{\rm bare}(-x_B,\xi,l^2))}+\frac{\alpha_s}{2\pi}\frac{1}{l^2}C^{\rm anom}\otimes'\mathcal{F}(x_B,\xi,l^2)$$

$$\boxed{(E_f(x_B,\xi,l^2)-E_f(-x_B,\xi,l^2))}=\boxed{(E_f^{\rm bare}(x_B,\xi,l^2)-E_f^{\rm bare}(-x_B,\xi,l^2))}-\frac{\alpha_s}{2\pi}\frac{1}{l^2}C^{\rm anom}\otimes'\mathcal{F}(x_B,\xi,l^2)$$

Hatta, Zhao (2020); Radyushkin, Zhao (2021)

Twist-4 GPD:

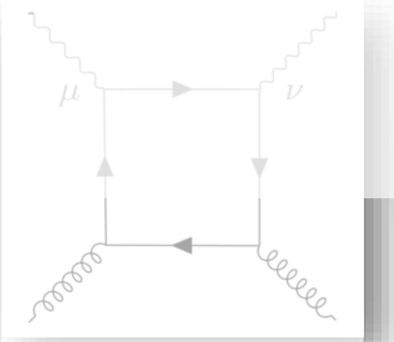
$$\mathcal{F}(x,\xi,l^2) = -4xP^+M\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\frac{\langle P_2|F^{\mu\nu}(-z^-/2)F_{\mu\nu}(z^-/2)|P_1\rangle}{\bar u(P_2)u(P_1)}$$

"Bare GPD" (tree level)

Perturbative pole (one loop)

(Non-local) trace anomaly manifests itself in high energy scattering amplitude & apparently breaks QCD factorization

Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude

Pole! (New result)

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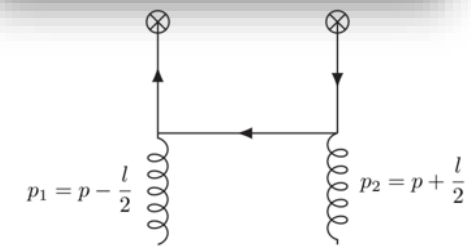
$$\boxed{(E_f(x_B,\xi,l^2) - E_f(-x_B,\xi,l^2))} = \boxed{(E_f^{\rm bare}(x_B,\xi,l^2) - E_f^{\rm bare}(-x_B,\xi,l^2))} - \frac{\alpha_s}{2\pi}\frac{1}{l^2} C^{\rm anom} \otimes' \mathcal{F}(x_B,\xi,l^2)$$

tive pol (one loop)

n energy scattering amplitude &

torization

The pole "belongs" to GPD

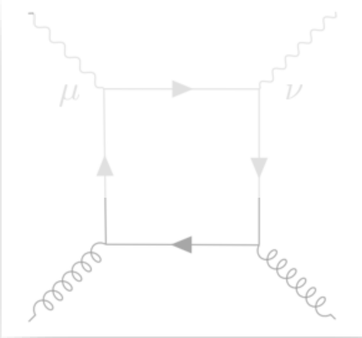


Perturbative pole in GPD

$$\int \frac{dk^- d^{2-2\epsilon}k_\perp}{(2\pi)^{3-2\epsilon}} \frac{\mathrm{Tr}[\gamma_\alpha(\not{k}+\not{l}/2)\gamma^+(\not{k}-\not{l}/2)\gamma^\beta(\not{k}-\not{p})]}{(p-k)^2(k-l/2)^2(k+l/2)^2}\epsilon^*_\alpha(p+l/2)\epsilon_\beta(p-l/2)\Bigg|_{\rm pole} \sim \frac{1}{l^2}\frac{x(1-x)}{1-\xi^2}\langle F\tilde{F}\rangle$$

Same pole in one-loop calculation!

Imprint of Anomalies in QCD Compton scattering



Symmetric part of Compton amplitude

Pole! (New result)

Glueball pole

Pole cancellation

$$\boxed{(H_f(x_B,\xi,l^2) - H_f(-x_B,\xi,l^2))} = \boxed{(H_f^{\rm bare}(x_B,\xi,l^2) - H_f^{\rm bare}(-x_B,\xi,l^2))} + \frac{\alpha_s}{2\pi}\frac{1}{l^2}C^{\rm anom}\otimes' \mathcal{F}$$

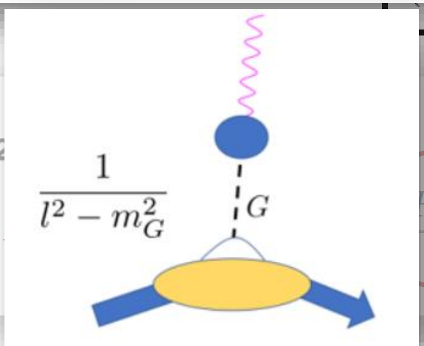
We proposed a possible scenario of pole cancellation

glueball mass generations

$$\boxed{(E_f(x_B,\xi,l^2) - E_f(-x_B,\xi,l^2))} = \boxed{(E_f^{\rm bare}(x_B,\xi,l^2) - E_f^{\rm bare}(-x_B,\xi,l^2))} - \frac{\alpha_s}{2\pi}\frac{1}{l^2}C^{\rm anom}\otimes' \mathcal{F}(x_B,\xi,l^2)$$

Hatta, Zhao (2020);
Radyushkin, Zhao (202

$$\mathcal{F}(x,\xi,l^2) = -4x \ldots \frac{\ldots F^{\mu\nu}(-z^-/2)F_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)u(P_1)}$$



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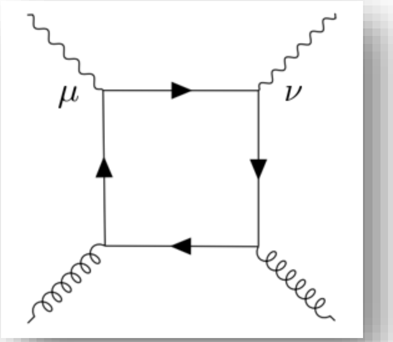
Imprint of Anomalies in QCD Compton scattering

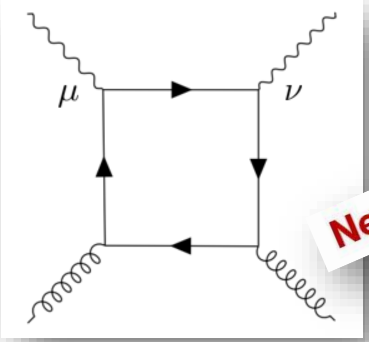
Real part of Compton amplitude:



Example: Antisymmetric case

Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude:



Example: Antisymmetric case

New result!

Pole in real part!

$$\sim \frac{16\langle F\tilde{F}\rangle}{l^2(-1+\xi^2)}\left(-(-1+x)\ln\frac{x-1}{x}+(x-\xi)\ln\frac{x-\xi}{x}\right)-(x\to -x)$$

$$+\tilde{\kappa}_{qg}(\hat{x},\hat{\xi})\ln\frac{Q^2}{-l^2}$$

$$+\tilde{C}_1^g(\hat{x},\hat{\xi})$$

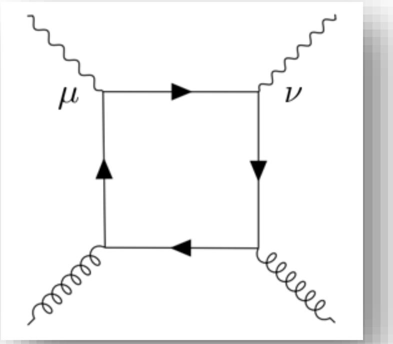
$$2T_R\int_0^1\frac{dx}{x}\frac{(1-\hat{x})\ln\frac{\hat{x}-1}{\hat{x}}+(\hat{x}-\hat{\xi})\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}-(\hat{x}\to-\hat{x})}{(1-\hat{\xi}^2)}\tilde{\mathcal{F}}(x,\xi,t)$$

$$=\int_0^1 dy\tilde{C}_0(y,x_B)\left[\underbrace{\int_y^1\frac{dx}{x}\tilde{K}\left(\frac{y}{x},\frac{\xi}{x}\right)\tilde{\mathcal{F}}(x,\xi,t)-\theta(\xi-y)\int_0^1\frac{dx}{x}\tilde{L}\left(\frac{y}{x},\frac{\xi}{x}\right)\tilde{\mathcal{F}}(x,\xi,t)}\right]$$

Same structure for convolution!
Even in real part, same mechanism should cancel pole as in imaginary part

Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude:



Example: Antisymmetric case

$$\sim \frac{16\langle F\tilde{F}\rangle}{l^2(-1+\xi^2)}\left(-(-1+x)\ln\frac{x-1}{x}+(x-\xi)\ln\frac{x-\xi}{x}\right)-(x\to -x)$$

$$+\tilde{\kappa}_{qg}(\hat{x},\hat{\xi})\ln\frac{Q^2}{-l^2}$$

$$+\tilde{C}_1^g(\hat{x},\hat{\xi})$$

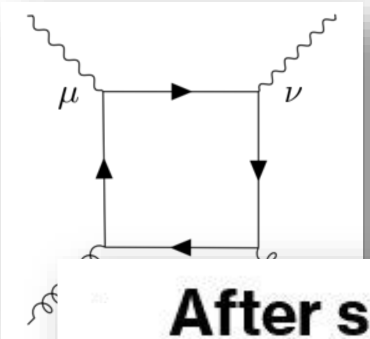
Reproduced the known logarithms from literature

$$\tilde{\kappa}_{qg}(\hat{x},\hat{\xi}) = \frac{2\hat{x}-1-\hat{\xi}^2}{2(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-1}{\hat{x}}-\frac{\hat{x}-\hat{\xi}}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}-(\hat{x}\to-\hat{x})$$

Ji, Osborne; Belitsky, Mueller

Imprint of Anomalies in QCD Compton scattering

Real part of Compton amplitude:



Example: Antisymmetric case

$$\sim \frac{16\langle F\tilde{F}\rangle}{l^2(-1+\xi^2)}\left(-(-1+x)\ln\frac{x-1}{x}+(x-\xi)\ln\frac{x-\xi}{x}\right)-(x\to -x)$$

After subtracting IR singularities and finite terms, $t\neq 0$ regularization is equivalent to $\overline{\mathrm{MS}}$ scheme

$$+\tilde{C}_1^g(\hat{x},\hat{\xi})$$

Coefficient function

$$\delta\tilde{C}_1^g(\hat{x},\hat{\xi}) = -\frac{2\hat{x}-1-\hat{\xi}^2}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-1}{\hat{x}}+2\frac{\hat{x}-\hat{\xi}}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}$$
$$+\frac{2\hat{x}-1-\hat{\xi}^2}{2(1-\hat{\xi}^2)^2}\ln^2\frac{\hat{x}-1}{\hat{x}}+\frac{\hat{\xi}}{1-\hat{\xi}^2}\ln^2\frac{\hat{x}-\hat{\xi}}{\hat{x}}-\frac{\hat{x}}{(1-\hat{\xi}^2)^2}\ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}\ln\frac{\hat{x}+\hat{\xi}}{\hat{x}}$$
$$+\frac{2\hat{\xi}}{(1-\hat{\xi}^2)^2}\mathrm{Li}_2\frac{-2\hat{\xi}}{\hat{x}-\hat{\xi}}+\frac{2\hat{x}-1-\hat{\xi}^2}{(1-\hat{\xi}^2)^2}\left(\mathrm{Li}_2\frac{1-\hat{\xi}}{1-\hat{x}}+\mathrm{Li}_2\frac{1+\hat{\xi}}{1-\hat{x}}\right)-(\hat{x}\to-\hat{x})$$

Imprint of Anomalies in QCD Compton scattering

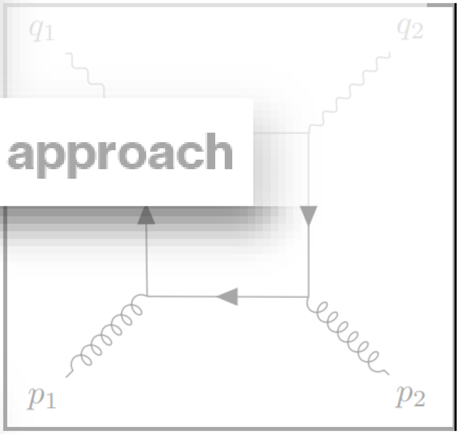
First calculation of b

Chiral and trace anomalies in Deeply Virtual Compton Scattering : QCD factorization and beyond

Shohini Bhattacharya,[1,*] Yoshitaka Hatta,[2,1,†] and Werner Vogelsang[3,‡]

The role of the
triangle gr

We explored the physics of anomaly in DVCS using Feynman-diagram approach



Box diagram

naly

ematics $(l = p_2 - p_1)$:

$\tilde{F}^a_{\alpha\beta}|p_1\rangle$

d by infra-red pole

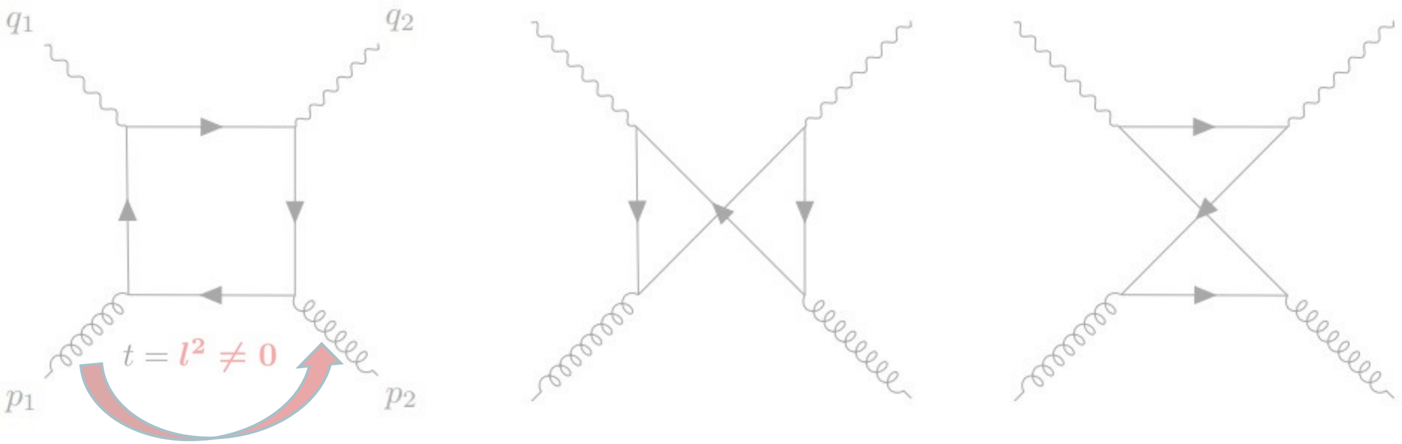


FIG. 1: Diagrams for the subprocess $\gamma^* g \to \gamma^* g$ in Compton scattering.

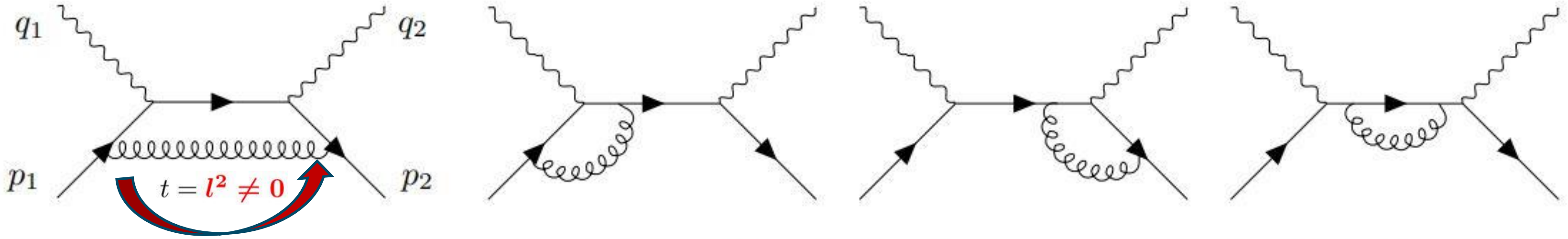
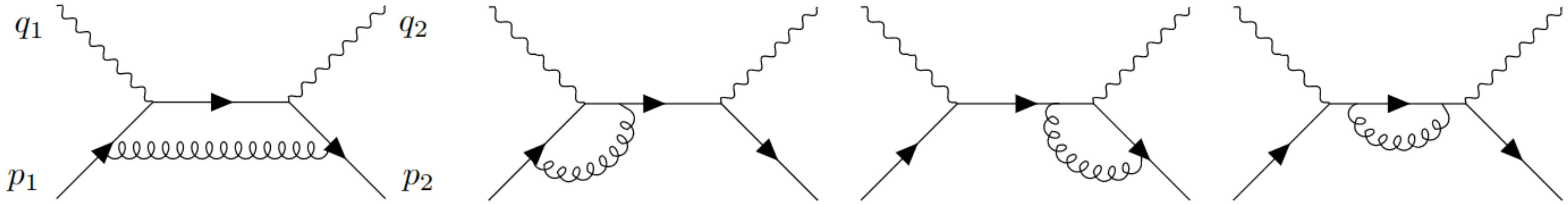


FIG. 2: Diagrams for the subprocess $\gamma^* q \to \gamma^* q$ in Compton scattering. Diagrams with photon lines crossed are not shown.

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS



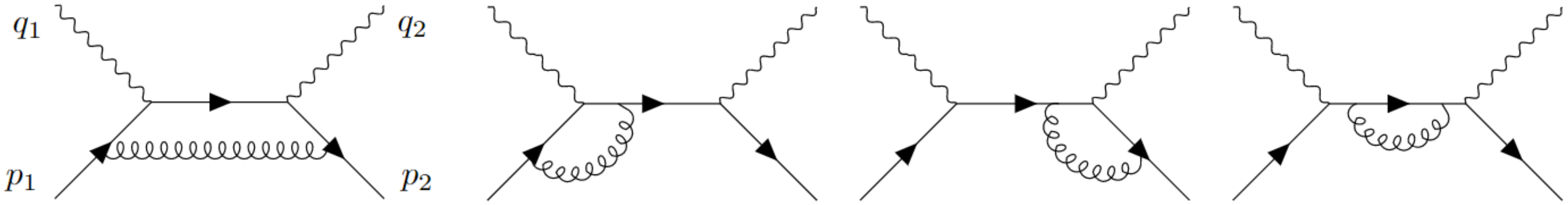
Example: Antisymmetric case

$$\sim \cancel{\frac{1}{l^2}} \quad + \tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) \ln \frac{Q^2}{-l^2} \quad + \delta \tilde{C}_1^q(\hat{x},\hat{\xi})$$

No pole!

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS



Example: Antisymmetric case

$$\sim \cancel{\frac{1}{l^2}} + \tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) \ln \frac{Q^2}{-l^2} + \delta \tilde{C}_1^q(\hat{x},\hat{\xi})$$

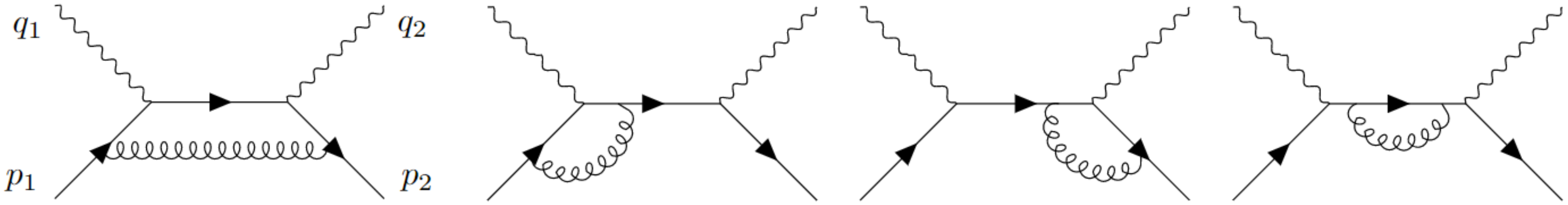
Reproduced the known logarithms from literature

$$\tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) = \frac{3}{2(1-\hat{x})} + \frac{\hat{x}^2+1-2\hat{\xi}^2}{(1-\hat{\xi}^2)(1-\hat{x})} \ln \frac{\hat{x}-1}{\hat{x}} - \frac{(\hat{x}-\hat{\xi})(1+\hat{x}^2+2\hat{x}\hat{\xi})}{(1-\hat{x}^2)(1-\hat{\xi}^2)} \ln \frac{\hat{x}-\hat{\xi}}{\hat{x}} - (\hat{x} \to -\hat{x})$$

Ji, Osborne; Belitsky, Mueller

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS



Example: Antisymmetric case

$$\sim \cancel{\frac{1}{l^2}} + \tilde{\kappa}_{qq}(\hat{x},\hat{\xi}) \ln \frac{Q^2}{-l^2} \quad + \delta\tilde{C}_1^q(\hat{x},\hat{\xi})$$

Coefficient function

$$\delta\tilde{C}_1^q(\hat{x},\hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{\epsilon_{IR}^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{2\epsilon_{IR}(1-\hat{x})} + \frac{-1+2\hat{x}-4\hat{x}^2+3\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)} \ln\frac{\hat{x}-1}{\hat{x}} + \frac{(\hat{x}-\hat{\xi})(1+2\hat{x}^2+3\hat{x}\hat{\xi})}{(1-\hat{x}^2)(1-\hat{\xi}^2)} \ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}$$

$$\ldots \ln^2\frac{\hat{x}-1}{\hat{x}} \ldots + \hat{x}^2 - 2\hat{\xi}^2) \ldots \ln\frac{\hat{x}-\hat{\xi}}{\hat{x}} \ln\frac{\hat{x}+\hat{\xi}}{\hat{x}} + \frac{\pi^2-54}{12(1-\hat{x})}$$

$$+\frac{\hat{\xi}}{1-\hat{\xi}^2}\mathrm{Li}_2 \ldots + \frac{1+\hat{x}^2}{(1-\hat{x})} \ldots$$

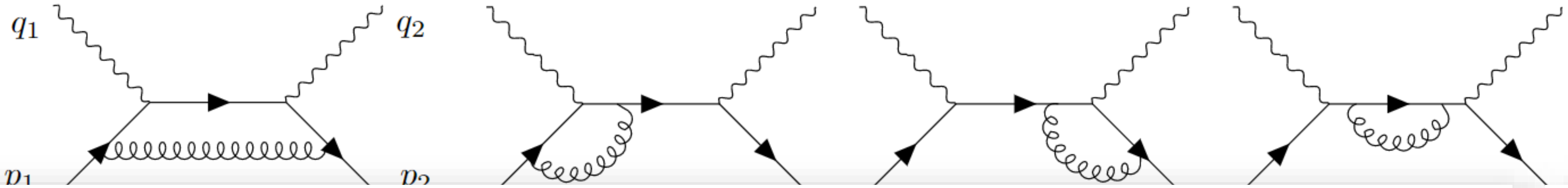
Unexpected double IR pole

Unexpected single IR pole

Sudakov logs! $\ln\left(\frac{Q^2}{-l^2}\right), \ln^2\left(\frac{Q^2}{-l^2}\right)$

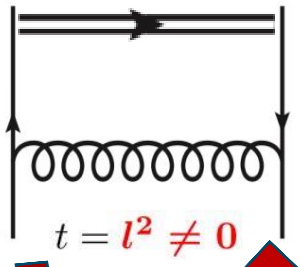
Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS



It looks like factorization is broken due to the unexpected double, single IR poles

But, when you compute GPD itself, you find the same double, single IR poles!
These poles can be systematically absorbed into GPD



Factorization restored

$$\delta\tilde{C}_1^q(\hat{x},\hat{\xi}) = -\frac{\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{\epsilon_{IR}^2(1-\hat{x})} - \frac{3\left(\frac{Q^2}{-l^2}\right)^{\epsilon_{IR}}}{2\epsilon_{IR}(1-\hat{x})} + \frac{-1+2\hat{x}-4\hat{x}^2+3\hat{\xi}^2}{2(1-\hat{x})(1-\hat{\xi}^2)}\ln\frac{\hat{x}-1}{\hat{x}} + \cdots$$

$$\cdots \ln\frac{\hat{x}-\hat{\xi}}{\hat{x}}\ln\frac{\hat{x}+\hat{\xi}}{\hat{x}} + \frac{\pi^2-54}{12(1-\hat{x})}$$

$$+\frac{\hat{\xi}}{1-\hat{\xi}^2}\mathrm{Li}_2\frac{-2\hat{\xi}}{\hat{x}-\hat{\xi}} + \frac{1+\hat{x}^2-2\hat{\xi}^2}{(1-\hat{x})(1-\hat{\xi}^2)}\left(\mathrm{Li}_2\frac{1-\hat{\xi}}{1-\hat{x}} + \mathrm{Li}_2\frac{1+\hat{\xi}}{1-\hat{x}}\right) - (\hat{x}\to-\hat{x})$$

Unexpected double IR pole

Unexpected single IR pole

Imprint of Anomalies in QCD Compton scattering

Quark-channel diagrams in DVCS

Remarks:

It look

- **Off-forwardness is an alternative factorization scheme that clarifies the physics of anomaly (More physical than other schemes.)**

But, when you compute GPD itself, you find the same double, single IR poles!

Th

- **After subtracting IR singularities and finite terms, $t \neq 0$ regularization is equivalent to $\overline{\mathrm{MS}}$ scheme**

This means that the result can be smoothly connected to the regime $t \sim \Lambda^2_{\mathrm{QCD}}$ as considered in the works by Collins, Freund; Ji, Obsborne

Unexpected double IR pole

Unexpected single IR pole

Summary

Factorization

- **Clarified QCD factorization for the first time within $\Lambda_{\mathrm{QCD}}^2 \ll t \ll Q^2$ regime: Crucial topic for ongoing & future experiments including at EIC**

- **Off-forwardness is an alternative factorization scheme that clarifies the physics of anomaly (More physical than other schemes)**

Summary

Factorization

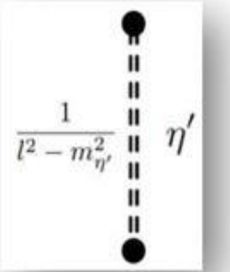
- **Clarified QCD factorization for the first time within $\Lambda^2_{\rm QCD} \ll t \ll Q^2$ regime: Crucial topic for ongoing & future experiments including at EIC**

- **Off-forwardness is an alternative factorization scheme that clarifies the physics of anomaly (More physical than other schemes)**

Fate of anomaly poles

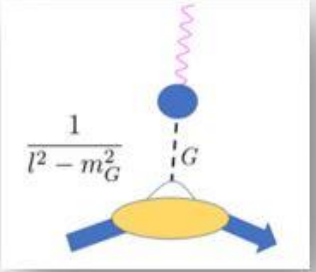
- **Novel connection between twist 2 & twist 4 sectors at the density level due to anomaly**

Imprints of chiral & trace anomalies in GPDs:



Eta-meson mass generation

$$\tilde{E}(x) \sim \frac{1}{l^2 - m^2_{\eta'}}$$



Glueball mass generation

$$H(x), E(x) \sim \frac{1}{l^2 - m^2_G}$$

Novel avenue of GPD research

Profound physical implication of anomaly poles:
Touches questions on mass generations, Chiral symmetry breaking, ...

Outlook

Novel connections between DVCS & chiral/trace anomalies:
This could be a new & potentially rich avenue for GPD research

Understand interplay between quark mass & $t = l^2 \neq 0$
Explore other structure functions

Sudakov resummation for GPDs

Novel avenue of GPD research

Profound physical implication of anomaly poles:
Touches questions on mass generations, Chiral symmetry breaking, ...

Backup slides

Imprint of Anomalies in QCD Compton scattering

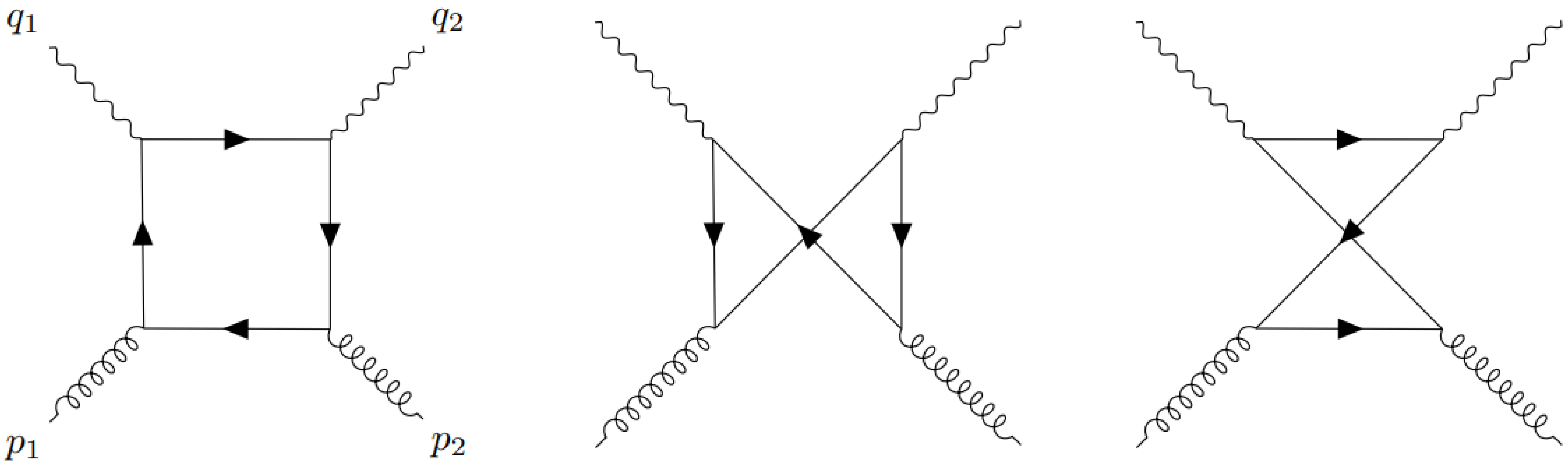
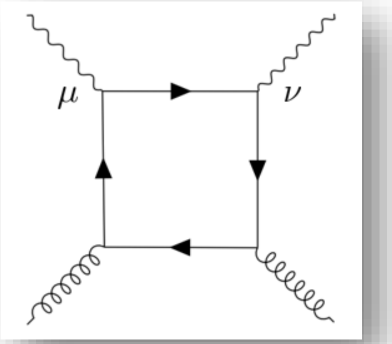


FIG. 1: Box diagrams for the Compton amplitude in off-forward kinematics.

Imprint of Anomalies in QCD Compton scattering

$$\frac{g_P(l^2)}{2M} \approx \frac{-2M\Delta\Sigma}{l^2 - m_{\eta'}^2}, \qquad i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)} \approx -2M\Delta\Sigma\frac{m_{\eta'}^2}{l^2 - m_{\eta'}^2}.$$

Imprint of Anomalies in QCD Compton scattering



Symmetric case:

Example: Antisymmetric part of Compton amplitude

$$\bar{F}_1^{\rm off}(x_B,l) \approx \frac{1}{2}\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\left[\left(P_{qg}\ln\frac{Q^2}{-l^2}+C_{1g}^{\rm off}\right)\otimes g(x_B)+\frac{1}{l^2}C^{\rm anom}\otimes' \mathcal{F}(x_B,\xi,l^2)\frac{\bar{u}(P_2)u(P_1)}{2M}\right],$$
$$\bar{F}_2^{\rm off}(x_B,l) \approx x_B\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\left[\left(P_{qg}\ln\frac{Q^2}{-l^2}+C_{2g}^{\rm off}\right)\otimes g(x_B)+\frac{1}{l^2}C^{\rm anom}\otimes' \mathcal{F}(x_B,\xi,l^2)\frac{\bar{u}(P_2)u(P_1)}{2M}\right]. \quad (31)$$

We recognize the expected structure of the one-loop corrections associated with the unpolarized gluon PDF $g(x)$, with the splitting function $P_{qg}(\hat{x}) = 2T_R((1-\hat{x})^2+\hat{x}^2)$. The coefficient functions are given by

$$C_{1g}^{\rm off}(\hat{x}) = 2T_R((1-\hat{x})^2+\hat{x}^2)\left(\ln\frac{1}{\hat{x}(1-\hat{x})}-1\right),$$
$$C_{2g}^{\rm off}(\hat{x}) = 2T_R((1-\hat{x})^2+\hat{x}^2)\left(\ln\frac{1}{\hat{x}(1-\hat{x})}-1\right)+8T_R\hat{x}(1-\hat{x}). \quad (32)$$

In addition, we find a pole $1/l^2$ in both $\bar{F}_1^{\rm off}$ and $\bar{F}_2^{\rm off}$ (but not in the difference $\bar{F}_2^{\rm off}-2x_B\bar{F}_1^{\rm off}$ relevant to the longitudinal structure function), with the following convolution formula

$$\boxed{C^{\rm anom}\otimes'\mathcal{F}(x_B,\xi,l^2) \equiv \int_{x_B}^1\frac{dx}{x}K(\hat{x},\hat{\xi})\mathcal{F}(x,\xi,l^2)-\frac{\theta(\xi-x_B)}{2}\int_{-1}^1\frac{dx}{x}L(\hat{x},\hat{\xi})\mathcal{F}(x,\xi,l^2),} \quad (33)$$

where

$$K(\hat{x},\hat{\xi}) = 2T_R\frac{\hat{x}(1-\hat{x})}{1-\hat{\xi}^2}, \qquad L(\hat{x},\hat{\xi}) = 2T_R\frac{\hat{x}(\hat{\xi}-\hat{x})}{1-\hat{\xi}^2}. \quad (34)$$

Antisymmetric case:

$$A\otimes B(x_B) \equiv \int_{x_B}^1\frac{dx}{x}A\left(\frac{x_B}{x}\right)B(x)$$

$-\epsilon^{\alpha\beta\mu\nu}P_\beta$

$-\epsilon^{\alpha\beta\mu\nu}P_\beta {\rm Im}T^{\rm asym}_{\mu\nu} =$

Twist-2 G

Imprint of Anomalies in QCD Compton scattering

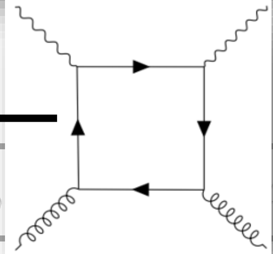
Polynomiality:

Symmetric part of Compton amplitude $(\xi \neq 0)$

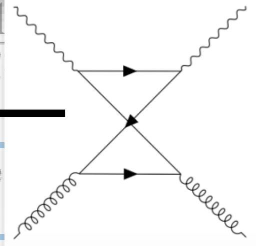
Structure of convolution:

$$C^{\text{anom}} \otimes' \mathcal{F}(x_B,\xi,l^2) \equiv \int_{x_B}^{1} \frac{dx}{x} K(\hat{x},\hat{\xi})\mathcal{F}(x,\xi,l^2) - \frac{\theta(\xi - x_B)}{2}\int_{-1}^{1}\frac{dx}{x} L(\hat{x},\hat{\xi})\mathcal{F}(x,\xi,l^2)$$

$$K(\hat{x},\hat{\xi}) = 2T_R \frac{\hat{x}(1-\hat{x})}{1-\hat{\xi}^2}$$



$$L(\hat{x},\hat{\xi}) = 2T_R \frac{\hat{x}(\hat{\xi}-\hat{x})}{1-\hat{\xi}^2}$$



Nonzero for nonzero skewness

Hatta, Zhao (2020);
Radyushkin, Zhao (2021)

Twist-4 GPD:

Example

$$\sum_f e_f^2 x_B (H_f^{\text{bare}}(x_B,\xi,l^2) - H_f^{\text{bare}}(-x_B,\xi,l^2)) \approx -\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\frac{x_B}{l^2} C^{\text{anom}} \otimes' \mathcal{F}(x_B,\xi,l^2=0)$$

Polynomiality

$$\sum_f e_f^2 (A_f^{\text{bare}}(l^2) + \xi^2 D_f^{\text{bare}}(l^2)) \approx \frac{T_R \alpha_s}{12\pi l^2}\left(\sum_f e_f^2\right)\left(\frac{\langle P|F^{\alpha\beta}(i\overleftrightarrow{D}^+)^2 F_{\alpha\beta}|P\rangle}{(P^+)^2} + \xi^2 \langle P|F^2|P\rangle\right)$$