Small-x TMD factorization at NLO

Farid Salazar (LBL/UCLA)

TMDs: Towards a Synergy between Lattice QCD
and Global Analysis (CFNS)

June 23rd, 2023

Based on

(1) 2108.06347 [JHEP 11 (2021) 222]
(2) 2208.13872 [JHEP 11 (2022) 169]
(3) 2304.03304 [Submitted to JHEP]
(4) 2307.XXXX [work in progress]

In collaboration with

Paul Caucal (Nantes)
Björn Schenke (BNL)
Tomasz Stebel (Jagiellonian)
Raju Venugopalan (BNL)

Outline

- Color Glass Condensate in a nutshell

- Dijet production in DIS in the CGC at NLO

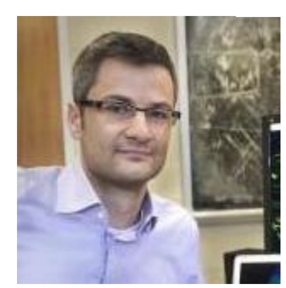
  P. Caucal, FS, R. Venugopalan. *2108.06347* *[JHEP 11 (2021) 222]*

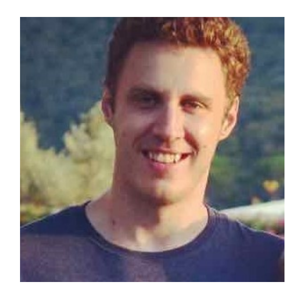
- The back-to-back limit: small-x TMD factorization at NLO

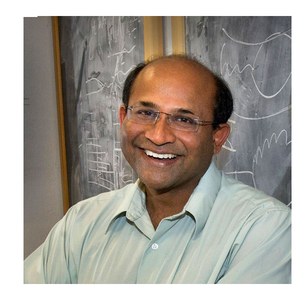
  PC, FS, B. Schenke ,RV. *2208.13872* *[JHEP 11 (2022) 169]*
  PC, FS, BS, T. Stebel, RV. *2304.03304* *[Submitted to JHEP]*
  2307.XXXX [work in progress]

- Outlook


Paul Caucal


Björn Schenke


Tomasz Stebel

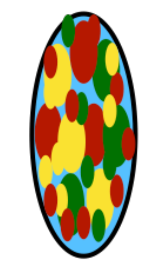

Raju Venugopalan

Color Glass Condensate

L. McLerran, R. Venugopalan (1993)

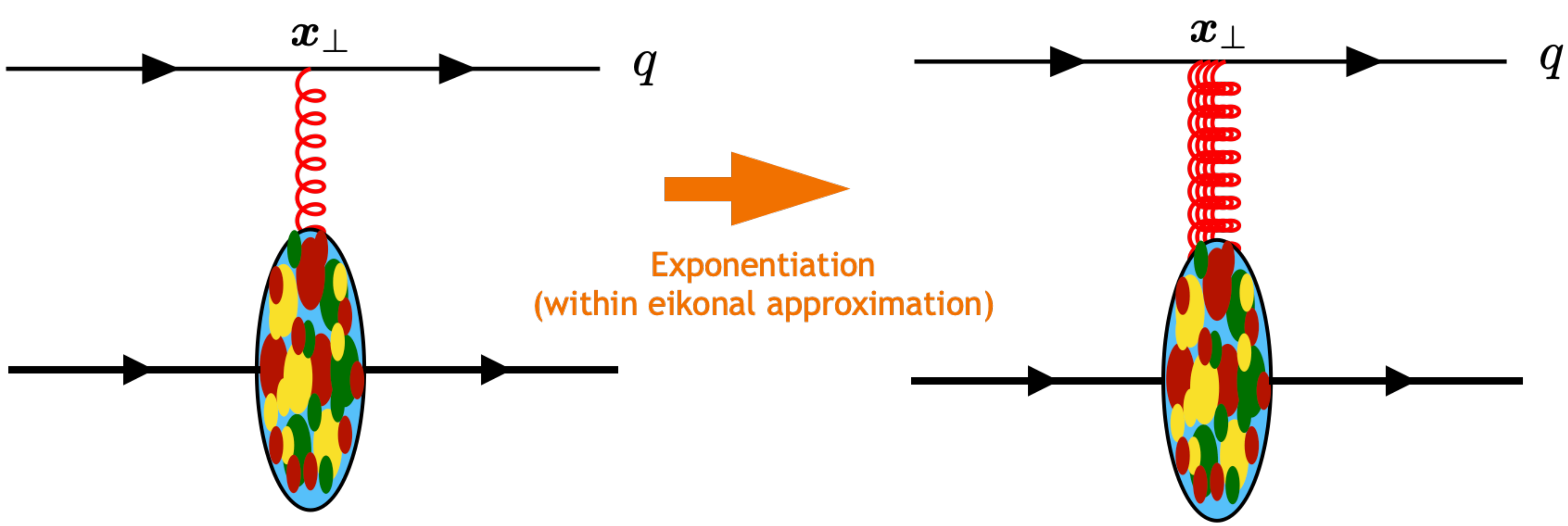
- **Effective field theory** for small-x gluons sourced by large-x partons

Color (QCD)
Glass (separation between slow and fast degrees of freedom)
Condensate (highly occupied system)



Large-x partons act as a classical color source which generates a background field A_{cl}

- **Multiple scattering** of partons with background field



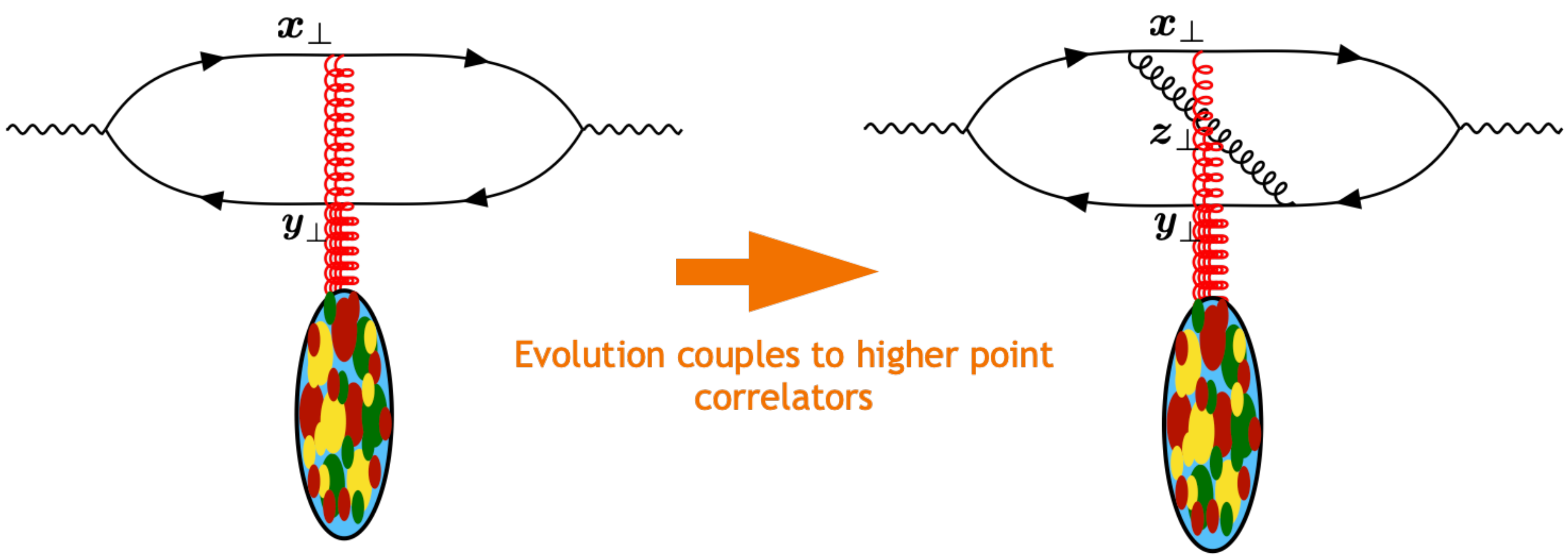
Ayala, Jalilian-Marian, McLerran, Venugopalan (PRD 1995) Balistky (NPB 1996)

High-energy scattering dofs = light-like Wilson line:

$$V(\boldsymbol{x}_\perp) = P\exp\left(ig\int \mathrm{d}x^- A^+_{\mathrm{cl}}(\boldsymbol{x}_\perp, x^-)\right)$$

Color Glass Condensate

- **Non-linear** renormalization group **evolution (BK-JIMWLK)**



$$\mathrm{Tr}\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp)\right] \qquad \mathrm{Tr}\left[V(\boldsymbol{x}_\perp)t^a V^\dagger(\boldsymbol{y}_\perp)t^b\right] U_{ab}(\boldsymbol{z}_\perp)$$

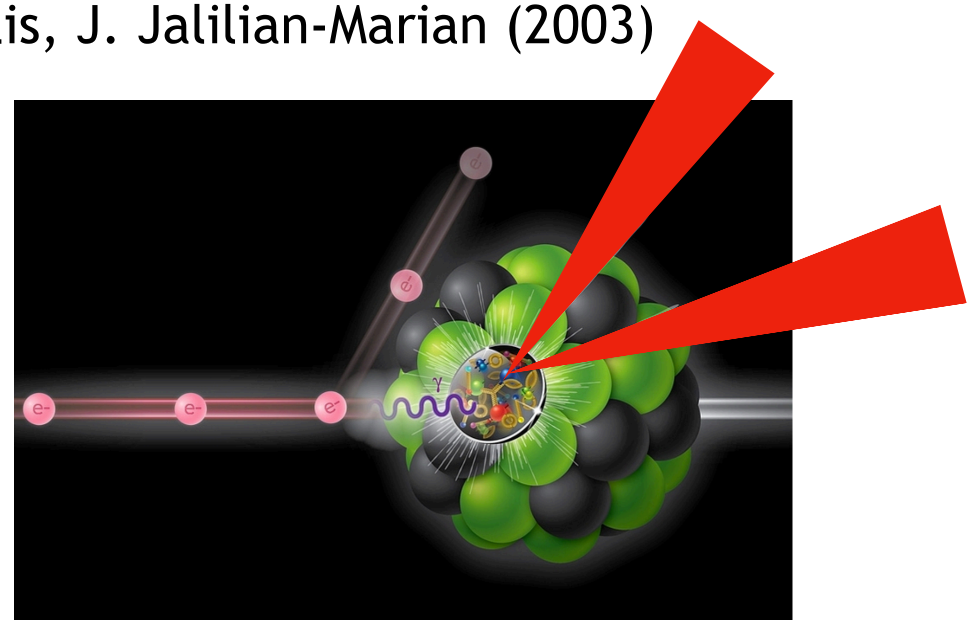
I. Balistky (1995), Y. Kovchegov (1999)
J. Jalilian-Marian, E. Iancu, L. McLerran,
H. Weigert, A. Leonidov, A. Kovner (1996-2002)

- Fast fields are non-perturbative, slow fields evolve perturbatively

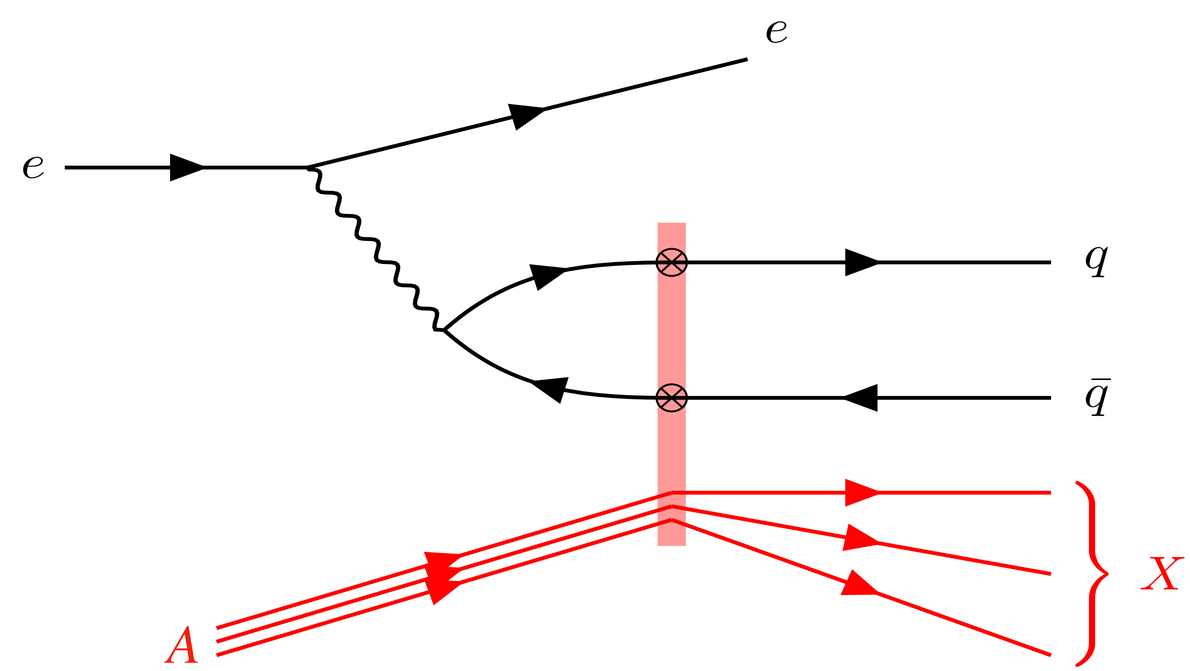
- **Probing CGC with dilute projectile**
  = pQCD embedded in strong gluon (non-perturbative) background field

Dijet production in DIS in the CGC at LO

F. Gelis, J. Jalilian-Marian (2003)



No need for hybrid factorization. Pin down photon kinematics from electron. Only one channel.



Dijet differential cross-section:

$$\frac{\mathrm{d}\sigma^{\gamma^*_\lambda + A \to q\bar{q} + X}}{\mathrm{d}^2\boldsymbol{k}_{\mathbf{1}\perp}\mathrm{d}^2\boldsymbol{k}_{\mathbf{2}\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} \propto \int \mathrm{d}^8\boldsymbol{X}_\perp\; e^{-i\boldsymbol{k}_{\mathbf{1}\perp}\cdot(\boldsymbol{x}_\perp - \boldsymbol{x}'_\perp)} e^{-i\boldsymbol{k}_{\mathbf{2}\perp}\cdot(\boldsymbol{y}_\perp - \boldsymbol{y}'_\perp)}$$

$$\times \langle \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp \boldsymbol{x}'_\perp)\rangle_Y \mathcal{R}^\lambda(\boldsymbol{x}_\perp - \boldsymbol{y}_\perp, \boldsymbol{x}'_\perp - \boldsymbol{y}'_\perp)$$

$$\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp \boldsymbol{x}'_\perp) = 1 - S^{(2)}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp) - S^{(2)}(\boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp) + S^{(4)}(\boldsymbol{x}_\perp, \boldsymbol{y}_\perp; \boldsymbol{y}'_\perp, \boldsymbol{x}'_\perp)$$

dipoles

quadrupole

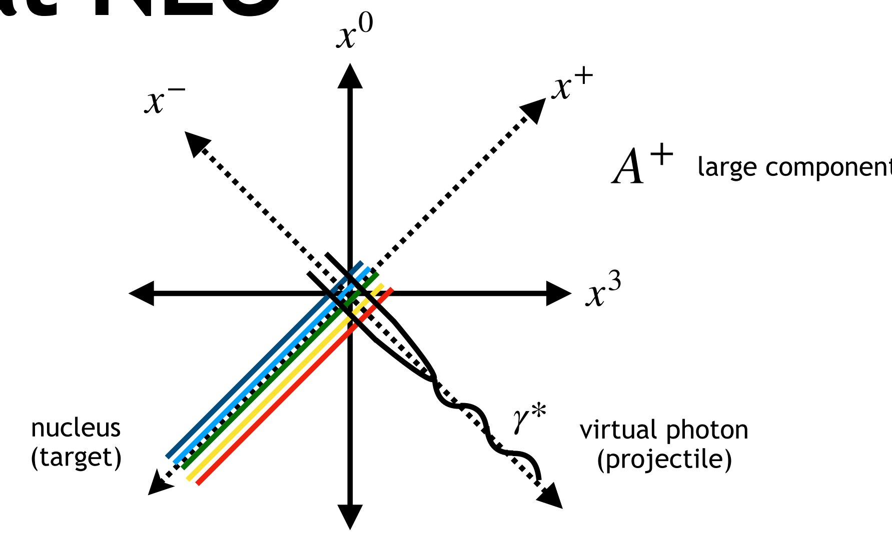
Implicitly contain saturation scale Q_s

First numerical evaluation
H. Mäntysaari, N. Mueller, FS, B. Schenke (PRL 2019)

Dijet production in DIS in the CGC at NLO

Our calculation

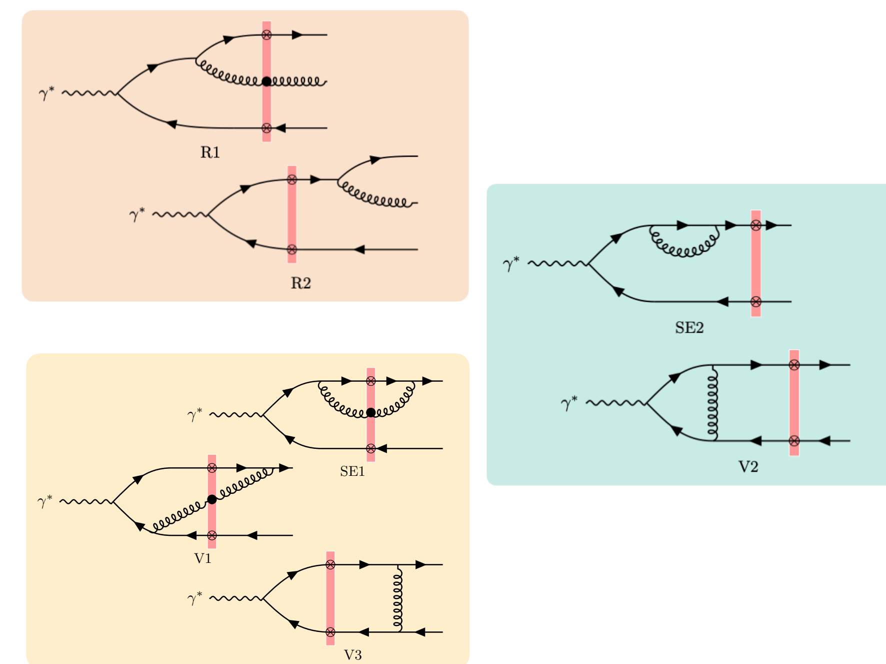
P. Caucal, FS, and R. Venugopalan. *JHEP 11 (2021) 222*



- Covariant perturbation theory Feynman rules in momentum space

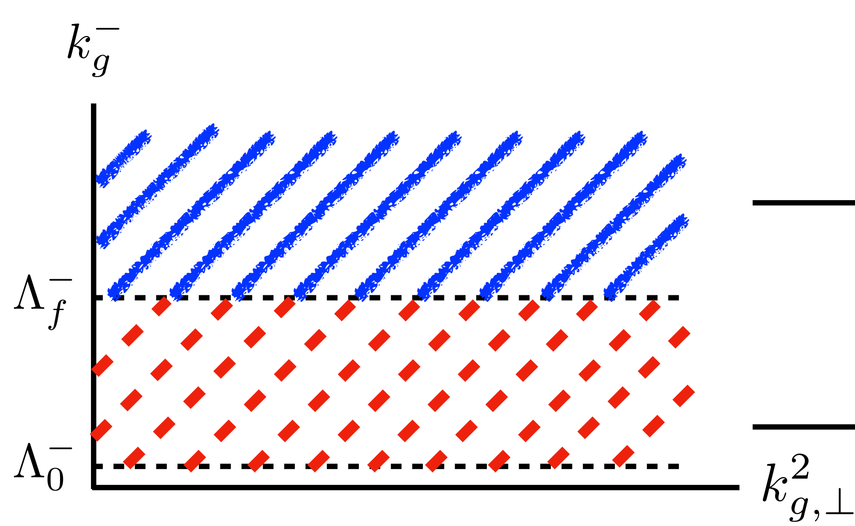
Dimensional regularization + longitudinal momentum cut-off + small-R cone algorithm

$$\int_{\Lambda_0^-} \frac{\mathrm{d}k_g^-}{k_g^-}\mu^{\varepsilon} \int \frac{\mathrm{d}^{2-\varepsilon}\boldsymbol{k_{g\perp}}}{(2\pi)^{2-\varepsilon}} f_{\Lambda^-}(k_g^-, \boldsymbol{k_{g\perp}})$$



- We showed cancellation of UV, soft and collinear divergences
- Absorbed large energy/rapidity logs into JIMWLK resummation
- Isolated genuine $\mathcal{O}(\alpha_s)$ contributions (aka NLO impact factor)

Gluon phase space (evolution vs impact factor)

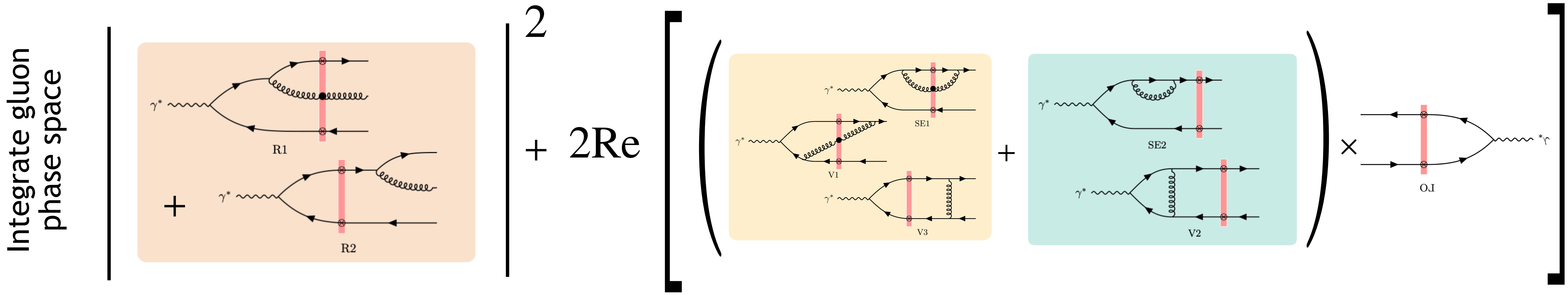


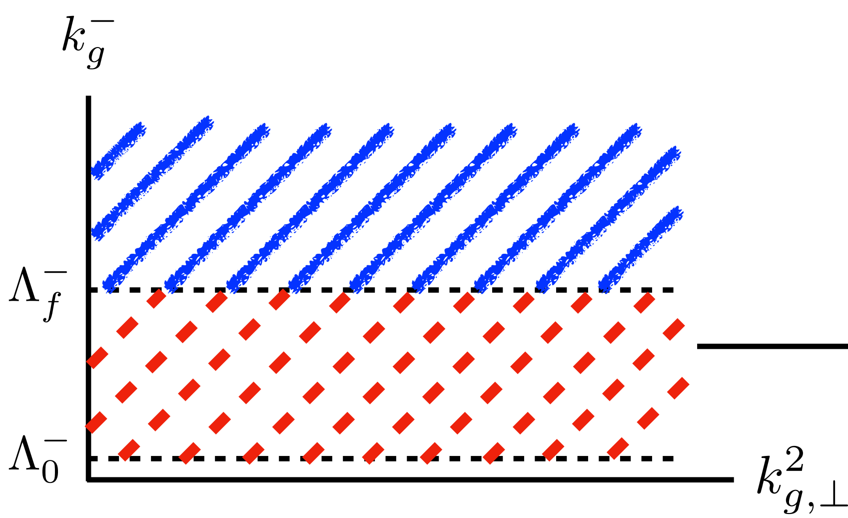
- Impact factor → **Finite piece (free of large rapidity logs)**
- Large rapidity (high-energy) logs → **Resummed via JIMWLK renormalization**

Dijet production in DIS in the CGC at NLO

Small-x evolution: JIMWLK factorization

P. Caucal, FS, and R. Venugopalan. *JHEP 11 (2021) 222*





Small-x evolution of dipole and quadrupole!

$$\left.\frac{\mathrm{d}\sigma^{\lambda}_{\mathrm{NLO}}}{\mathrm{d}^2\boldsymbol{k_{1\perp}}\mathrm{d}\eta_1\mathrm{d}^2\boldsymbol{k_{2\perp}}\mathrm{d}\eta_2}\right|_{\mathrm{LLx}} = \frac{\alpha_{\mathrm{em}}e_f^2N_c}{(2\pi)^6}\delta(1-z_q-z_{\bar{q}})\int \mathrm{d}\boldsymbol{X}_\perp \mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\ln\left(\Lambda_f^-/\Lambda^-\right)$$

$\mathcal{H}_{\mathrm{LL}}\left\langle \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp\boldsymbol{y}'_\perp)\right\rangle_Y$

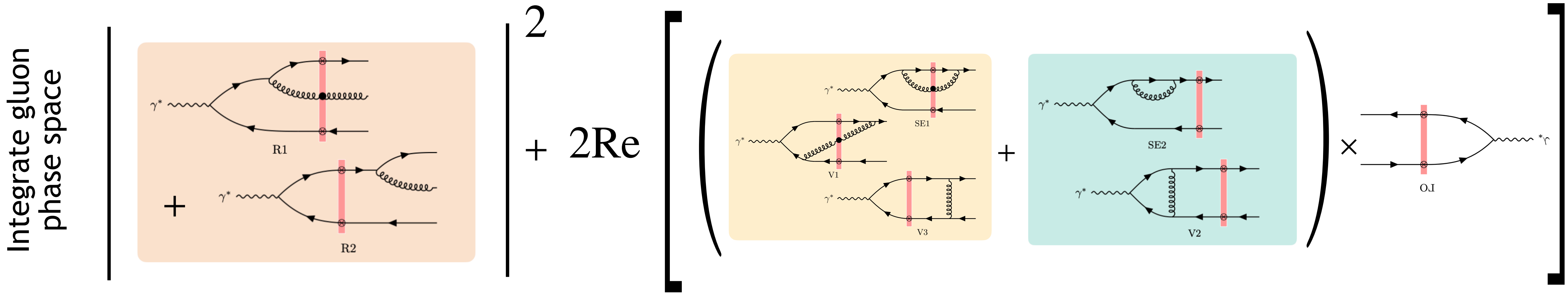
$$\begin{aligned}\times\frac{\alpha_sN_c}{4\pi^2}\Bigg\langle\int \mathrm{d}^2\boldsymbol{z}_\perp\Bigg\{&\frac{\boldsymbol{r}^2_{xy}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zy}}(2D_{xy}-2D_{xz}D_{zy}+D_{zy}Q_{y'x',xz}+D_{xz}Q_{y'x',zy}-Q_{xy,y'x'}-D_{xy}D_{y'x'})\\&+\frac{\boldsymbol{r}^2_{x'y'}}{\boldsymbol{r}^2_{zx'}\boldsymbol{r}^2_{zy'}}(2D_{y'x'}-2D_{y'z}D_{zx'}+D_{zx'}Q_{xy,y'z}+D_{y'z}Q_{xy,zx'}-Q_{xy,y'x'}-D_{xy}D_{y'x'})\\&+\frac{\boldsymbol{r}^2_{xx'}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zx'}}(D_{zx'}Q_{xy,y'z}+D_{xz}Q_{y'x',zy}-Q_{xy,y'x'}-D_{xx'}D_{y'y})\\&+\frac{\boldsymbol{r}^2_{yy'}}{\boldsymbol{r}^2_{zy}\boldsymbol{r}^2_{zy'}}(D_{y'z}Q_{xy,zx'}+D_{zy}Q_{y'x',xz}-Q_{xy,y'x'}-D_{xx'}D_{y'y})\\&+\frac{\boldsymbol{r}^2_{xy'}}{\boldsymbol{r}^2_{zx}\boldsymbol{r}^2_{zy'}}(D_{xx'}D_{y'y}+D_{xy}D_{y'x'}-D_{zx'}Q_{xy,y'z}-D_{zy}Q_{y'x',xz})\\&+\frac{\boldsymbol{r}^2_{x'y}}{\boldsymbol{r}^2_{zx'}\boldsymbol{r}^2_{zy}}(D_{xx'}D_{y'y}+D_{xy}D_{y'x'}-D_{y'z}Q_{xy,zx'}-D_{xz}Q_{y'x',zy})\Bigg\}\Bigg\rangle_Y\end{aligned}$$

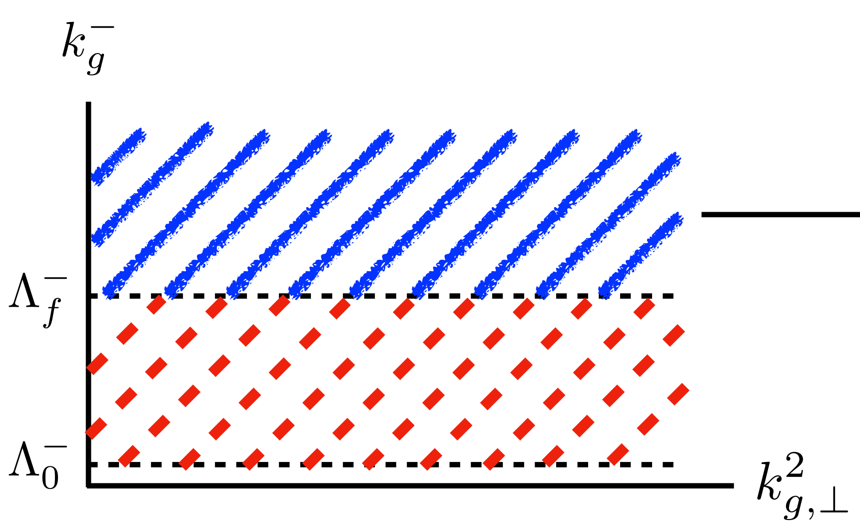
JIMWLK LL Hamiltonian acting on LO cross-section → Renormalization of Wilson line operators

Dijet production in DIS in the CGC at NLO

Impact factor

P. Caucal, FS, and R. Venugopalan. *JHEP 11 (2021) 222*





Only longitudinally polarized photon shown, lengthier expressions for transversely polarized photon

$$\mathrm{d}\sigma_{\mathrm{R}_2\times\mathrm{R}_2,\mathrm{sud2}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}} \mathcal{R}_{\mathrm{LO}}^{\lambda}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) \times C_F \Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times \frac{\alpha_s}{\pi}\int_0^1 \frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}}\right]\ln\left(\frac{\boldsymbol{k_{1\perp}}^2\boldsymbol{r}_{xx'}^2R^2\xi^2}{c_0^2}\right)$$

$$\mathrm{d}\sigma_{\mathrm{R}_2\times\mathrm{R}'_2,\mathrm{sud2}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}} \mathcal{R}_{\mathrm{LO}}^{\lambda}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'}) \times \Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times \frac{(-\alpha_s)}{\pi}\int_0^1 \frac{\mathrm{d}\xi}{\xi}\left[1-e^{-i\xi\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy'}}\right]\ln\left(\frac{\boldsymbol{P}_\perp^2\boldsymbol{r}_{xy'}^2\xi^2}{z_2^2c_0^2}\right)$$

$$\mathrm{d}\sigma_{\mathrm{R,no-sud,LO}}^{\gamma_{\mathrm{L}}^\star+A\to q\bar{q}g+X} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^8} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}} (4\alpha_s C_F)\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times \frac{e^{-i\boldsymbol{k_{g\perp}}\cdot\boldsymbol{r}_{xx'}}}{(\boldsymbol{k_{g\perp}}-\frac{z_g}{z_1}\boldsymbol{k_{1\perp}})^2}\left\{8z_1z_2^3(1-z_2)^2Q^2\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)K_0(\bar{Q}_{\mathrm{R2}}r_{xy})K_0(\bar{Q}_{\mathrm{R2}}r_{x'y'})\delta_z^{(3)} -\mathcal{R}_{\mathrm{LO}}^{\mathrm{L}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta(z_1-z_g)\delta_z^{(2)}\right\}+(1\leftrightarrow 2)$$

$$\mathrm{d}\sigma_{\mathrm{R,no-sud,NLO_3}}^{\gamma_{\mathrm{L}}^\star+A\to q\bar{q}g+X} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c}{(2\pi)^8} \int \mathrm{d}^2\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}} (-4\alpha_s)\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times \frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy'}}}{\boldsymbol{l}_\perp^2}\left\{8z_1^2z_2^2(1-z_2)(1-z_1)Q^2K_0(\bar{Q}_{\mathrm{R2}}r_{xy})K_0(\bar{Q}_{\mathrm{R2'}}r_{x'y'})\left[1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right] \times e^{-i\boldsymbol{l}_\perp\cdot\boldsymbol{r}_{xy'}}\frac{\boldsymbol{l}_\perp\cdot(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)}{(\boldsymbol{l}_\perp+\boldsymbol{K}_\perp)^2}\delta_z^{(3)} - \mathcal{R}_{\mathrm{LO}}^{\mathrm{L}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Theta\left(\frac{c_0^2}{\boldsymbol{r}_{xy'}^2}\geq\boldsymbol{l}_\perp^2\geq\boldsymbol{K}_\perp^2\right)\Theta(z_1-z_g)\delta_z^{(2)}\right\} + (1\leftrightarrow 2)$$

$$\mathrm{d}\sigma_{\mathrm{R,no-sud,other}}^{\gamma_{\mathrm{L}}^\star+A\to q\bar{q}g+X} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(3)}}{(2\pi)^8} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}} 8z_1^3z_2^3Q^2\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\frac{\mathrm{d}^2\boldsymbol{z}'_\perp}{\pi}e^{-i\boldsymbol{k_{g\perp}}\cdot\boldsymbol{r}_{zz'}}$$
$$\alpha_s\left\{-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2}}r_{w'y'})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO},1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp) + \frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(\bar{Q}_{\mathrm{R2'}}r_{w'y'})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO},1}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{w}'_\perp,\boldsymbol{y}'_\perp)\right.$$
$$+\frac{1}{2}\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)\Xi_{\mathrm{NLO},4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp) - \frac{1}{2}\frac{\boldsymbol{r}_{zy}\cdot\boldsymbol{r}_{z'x'}}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{z'x'}^2}K_0(QX_{\mathrm{R}})K_0(QX'_{\mathrm{R}})\left(1+\frac{z_g}{2z_1}+\frac{z_g}{2z_2}\right)\Xi_{\mathrm{NLO},4}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp,\boldsymbol{z}'_\perp)$$
$$\left.+(1\leftrightarrow 2)+c.c.\right\} - \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^8}\alpha_s\Theta(z_f-z_g)\times\text{“slow”}$$

$$\mathrm{d}\sigma_{\mathrm{V,no-sud,LO}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}} \mathcal{R}_{\mathrm{LO}}^{\lambda}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp) \times\frac{\alpha_sC_F}{\pi}\left\{-\frac{3}{4}\ln\left(\frac{\boldsymbol{k_{1\perp}}^2\boldsymbol{k_{2\perp}}^2\boldsymbol{r}_{xy}^2\boldsymbol{r}_{x'y'}^2}{c_0^4}\right)-3\ln(R)+\frac{1}{2}\ln^2\left(\frac{z_1}{z_2}\right)+\frac{11}{2}+3\ln(2)-\frac{\pi^2}{2}\right\}$$

$$\mathrm{d}\sigma_{\mathrm{V,no-sud,NLO_3}}^{\lambda=\mathrm{L}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int \mathrm{d}^3\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\boldsymbol{r}_{yy'}} 8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'}) \times\frac{\alpha_s}{\pi}\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}\left\{K_0(\bar{Q}_{\mathrm{V3}}r_{xy})\left[\left(1-\frac{z_g}{z_1}\right)^2\left(1+\frac{z_g}{z_2}\right)(1+z_g)e^{i(\boldsymbol{P}_\perp+z_g\boldsymbol{q}_\perp)\cdot\boldsymbol{r}_{xy}}K_0(-i\Delta_{\mathrm{V3}}r_{xy}) -\left(1-\frac{z_g}{2z_1}+\frac{z_g}{2z_2}-\frac{z_g^2}{2z_1z_2}\right)e^{i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xy}}\mathcal{J}_\odot\left(\boldsymbol{r}_{xy},\left(1-\frac{z_g}{z_1}\right)\boldsymbol{P}_\perp,\Delta_{\mathrm{V3}}\right)\right] +K_0(\bar{Q}r_{xy})\ln\left(\frac{z_gP_\perp r_{xy}}{c_0z_1z_2}\right)+(1\leftrightarrow 2)\right\}\Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)+c.c.$$

$$\mathrm{d}\sigma_{\mathrm{V,no-sud,other}}^{\lambda=\mathrm{L}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\boldsymbol{r}_{yy'}} 8z_1^3z_2^3Q^2K_0(\bar{Q}r_{x'y'})\int_0^{z_1}\frac{\mathrm{d}z_g}{z_g}$$
$$\times\frac{\alpha_s}{\pi}\int\frac{\mathrm{d}^2\boldsymbol{z}_\perp}{\pi}\left\{\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}K_0(QX_V)-\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO},1} -\frac{1}{\boldsymbol{r}_{zx}^2}\left[\left(1-\frac{z_g}{z_1}+\frac{z_g^2}{2z_1^2}\right)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})-\Theta(z_f-z_g)e^{-\frac{\boldsymbol{r}_{zx}^2}{\boldsymbol{r}_{xy}^2e^{\gamma_E}}}K_0(\bar{Q}r_{xy})\right]C_F\Xi_{\mathrm{LO}}\right.$$
$$\left.-\frac{\boldsymbol{r}_{zx}\cdot\boldsymbol{r}_{zy}}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zy}^2}\left[\left(1-\frac{z_g}{z_1}\right)\left(1+\frac{z_g}{z_2}\right)\left(1-\frac{z_g}{2z_1}-\frac{z_g}{2(z_2+z_g)}\right)e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}K_0(QX_V) -\Theta(z_f-z_g)K_0(\bar{Q}r_{xy})\right]\Xi_{\mathrm{NLO},1}+(1\leftrightarrow 2)\right\}+c.c..$$

$$\mathrm{d}\sigma_{\mathrm{sud1}} = \frac{\alpha_{\mathrm{em}} e_f^2 N_c \delta_z^{(2)}}{(2\pi)^6} \int \mathrm{d}^8\boldsymbol{X}_\perp e^{-i\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{xx'}-i\boldsymbol{k_{2\perp}}\cdot\boldsymbol{r}_{yy'}} \mathcal{R}_{\mathrm{LO}}^{\lambda}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\times\frac{\alpha_s}{\pi} \times\left\{C_F\Xi_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_f}{z_1}\right)\ln\left(\frac{\boldsymbol{r}_{xx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_f}{z_2}\right)\ln\left(\frac{\boldsymbol{r}_{yy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right] + \Xi_{\mathrm{NLO},3}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp;\boldsymbol{x}'_\perp,\boldsymbol{y}'_\perp)\left[\ln\left(\frac{z_1}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{xy'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)+\ln\left(\frac{z_2}{z_f}\right)\ln\left(\frac{\boldsymbol{r}_{yx'}^2}{|\boldsymbol{r}_{xy}||\boldsymbol{r}_{x'y'}|}\right)\right]\right\}$$

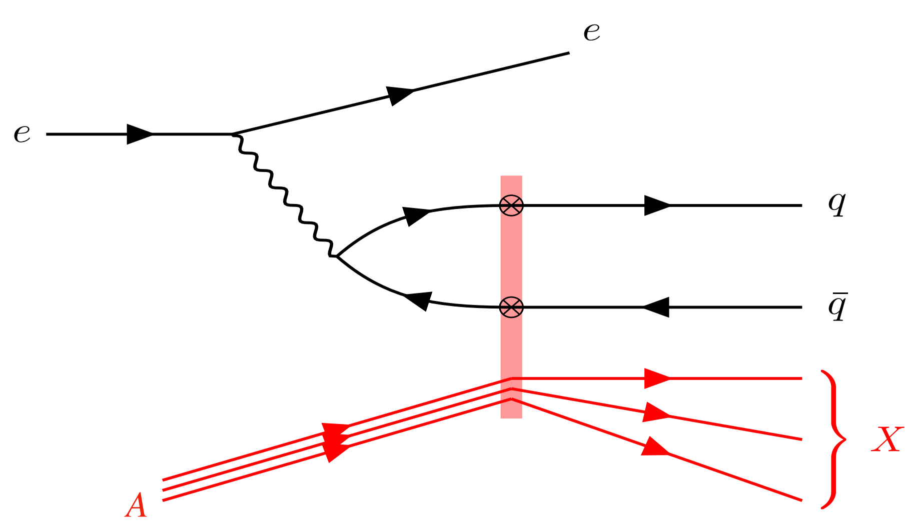
Back-to-back dijets at LO

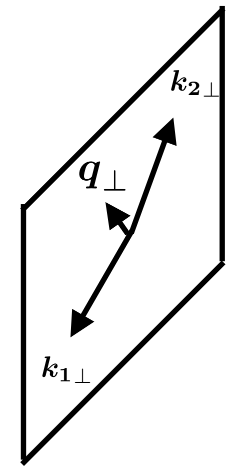
Small-x TMD factorization from CGC

F. Dominguez, C. Marquet, B-W. Xiao, F. Yuan (2011)

In the "correlation limit" $q_\perp, Q_s \ll P_\perp$ and high-energy limit $P_\perp \ll W$

→ Forward-jets (photon direction) but close to back-to-back in the transverse plane





$$\boldsymbol{q}_\perp = \boldsymbol{k_1}_\perp + \boldsymbol{k_2}_\perp$$
$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k_1}_\perp - z_1 \boldsymbol{k_2}_\perp$$

Perturbatively calculable

$$\mathrm{d}\sigma^{\gamma^*_\lambda + A \to q\bar{q} + X} \sim \mathcal{H}^{ij}(Q, \boldsymbol{P}_\perp)\, G^{ij}_Y(\boldsymbol{q}_\perp)$$

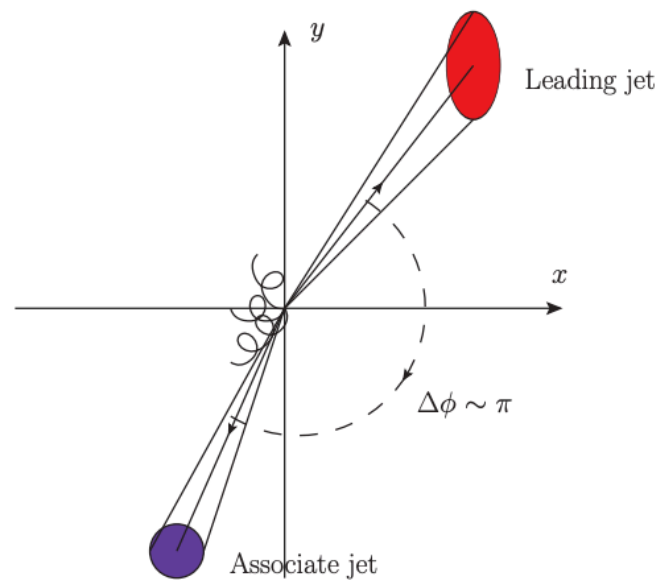
$$G^{ij}_Y(\boldsymbol{q}_\perp) = \frac{-2}{\alpha_s} \int \frac{\mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp}{(2\pi)^4} e^{-i\boldsymbol{q}_\perp \cdot (\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)} \left\langle \mathrm{Tr}\left[V(\boldsymbol{b}_\perp) \partial^i_\perp V^\dagger(\boldsymbol{b}_\perp) V(\boldsymbol{b}'_\perp) \partial^j_\perp V^\dagger(\boldsymbol{b}'_\perp) \right] \right\rangle_Y$$

"Bare" TMD built from Wilson lines correlators, saturation scale Q_s implicitly built in

Back-to-back dijets at NLO

Interplay of small-x & soft gluon resummation

A.H. Mueller, B-W. Xiao, F. Yuan (2013)



$$q_\perp^2 \ll P_\perp^2 \ll s \qquad \ln(s/P_\perp^2) \qquad \ln^2(P_\perp^2/q_\perp^2)$$

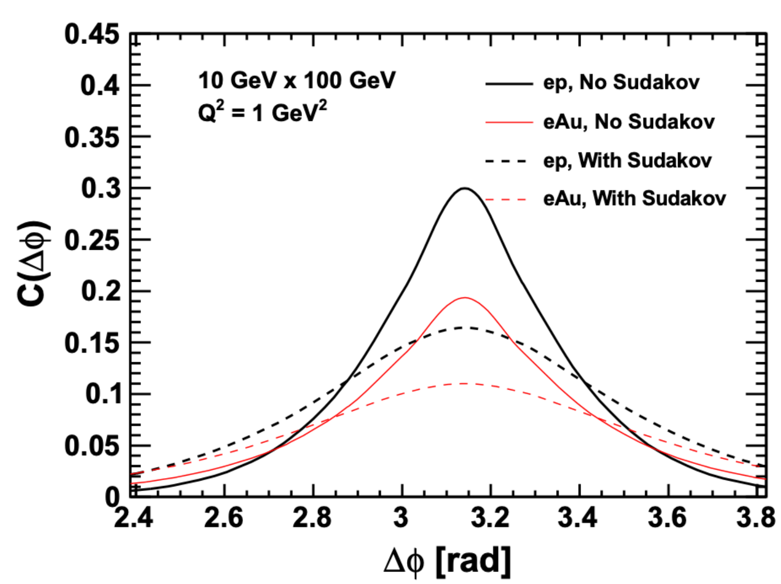
Joint small-x + Sudakov resummation

$$d\sigma^{\gamma_\lambda^* + A \to q\bar{q} + X} \propto \mathcal{H}(Q, \boldsymbol{P}_\perp) \int \frac{d^2 \mathbf{r}_{bb'}}{(2\pi)^2} e^{-i\boldsymbol{q}_\perp \cdot \mathbf{r}_{bb'}} \widetilde{G}_Y^0(\mathbf{r}_{bb'}) e^{-S_{\mathrm{Sud}}(\mathbf{r}_{bb'}, \boldsymbol{P}_\perp)}$$

Obeys JIMWLK equation (non-linear)
(see also Dominguez, Mueller, Munier, Xiao 2013)

Sudakov factor: $$S_{\mathrm{Sud}}(\mathbf{r}_{bb'}, P_\perp) = \frac{\alpha_s N_c}{\pi} \int_{c_0^2/\mathbf{r}_{bb'}^2}^{P_\perp^2} \frac{1}{2} \ln\left(\frac{P_\perp^2}{\mu^2}\right)$$

Soft gluon emissions change profile of azimutal correlations
Zheng, Aschenauer, Lee, Xiao 2013



Our goal:

Does the CGC/TMD correspondence hold at NLO?

$$\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X} \propto \mathcal{H}^{ij}_{\mathrm{LO+NLO}}(Q,\boldsymbol{P}_\perp)\int\frac{\mathrm{d}^2\boldsymbol{b}_\perp}{(2\pi)^2}e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}}\widetilde{G}^{ij}_Y(\mathbf{r}_{bb'})e^{-S_{\mathrm{Sud}}(\mathbf{r}_{bb'},\boldsymbol{P}_\perp)}$$

i) Is this evolution equation related to non-local RG equation known in small-x literature?

ii) Is it possible to pin down the Sudakov double and single logs?

iii) Is it possible to express the finite pieces in terms of a factorizable NLO coefficient function and the WW gluon TMD? Or do we expect factorization-breaking contributions?

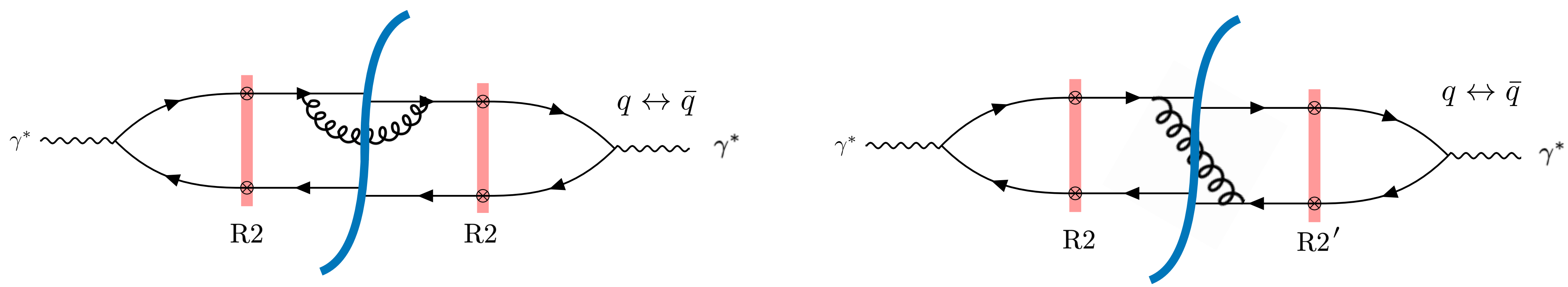
iv) Can we account both for unpolarized and linearly polarized contributions?

Back-to-back dijets at NLO

Small-x and Sudakov interplay

P. Caucal, FS, B. Schenke, and R. Venugopalan *JHEP 11 (2022) 169*

Real diagrams with soft double logarithmic enhancement:



Back-to-back limit gives Sudakov factor with positive (wrong) sign!

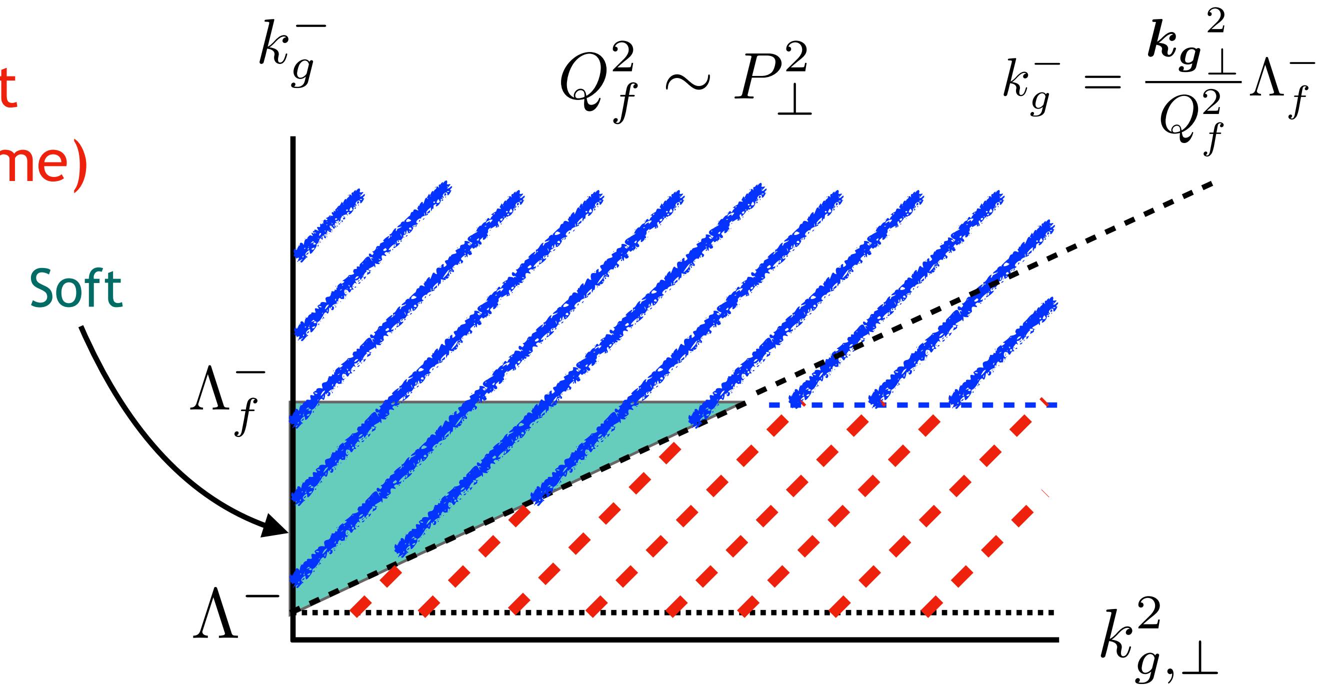
$$\mathrm{d}\sigma^{\gamma_\lambda^* + A \to q\bar{q} + X} \propto \mathcal{H}(Q, \boldsymbol{P}_\perp) \int \mathrm{d}^2 \mathbf{r}_{bb'} e^{-i\boldsymbol{q}_\perp \cdot \mathbf{r}_{bb'}} \left[1 + \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2 \mathbf{r}_{bb'}^2}{c_0^2}\right) + \cdots + \alpha_s \ln\left(\Lambda_f^-/\Lambda^-\right) \mathcal{K}_{LL} \otimes \right] \widetilde{G}_Y(\mathbf{r}_{bb'})$$

Problem: overlapping phase space between soft and small-x gluons

Back-to-back dijets at NLO

Small-x and Sudakov interplay

Solution: impose a kinematic constraint that enforces ordering in both k_g^- and k_g^+ (lifetime)



Proposed by P. Taels, T. Altinoluk, C. Marquet, G. Beuf (2022) for dijet photo-production

P. Caucal, FS, B. Schenke, and R. Venugopalan. *JHEP 11 (2022) 169*

$$\mathrm{d}\sigma^{\gamma_\lambda^*+A\to q\bar{q}+X} \propto \mathcal{H}(Q,\boldsymbol{P}_\perp)\int \mathrm{d}^2\mathbf{r}_{bb'} e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}}$$

$$\left[1 - \frac{\alpha_s N_c}{4\pi}\ln^2\left(\frac{\boldsymbol{P}_\perp^2\mathbf{r}_{bb'}^2}{c_0^2}\right) - \frac{\alpha_s}{\pi}s_L \ln\left(\frac{\boldsymbol{P}_\perp^2\mathbf{r}_{bb'}^2}{c_0^2}\right) + \alpha_s\mathcal{K}_{LL,\mathrm{coll}}\otimes\right]\widetilde{G}_Y(\mathbf{r}_{bb'}) + \mathcal{O}(\alpha_s)$$

Correct Sudakov double log

Kinematically improved small-x evolution

Single log coefficient: $s_L = C_F \ln\left(\frac{1}{z_1 z_2 R^2}\right) - N_c \ln\left(\frac{P_\perp^2}{z_1 z_2 c_0^2}\frac{z_f}{Q_f^2}\right)$

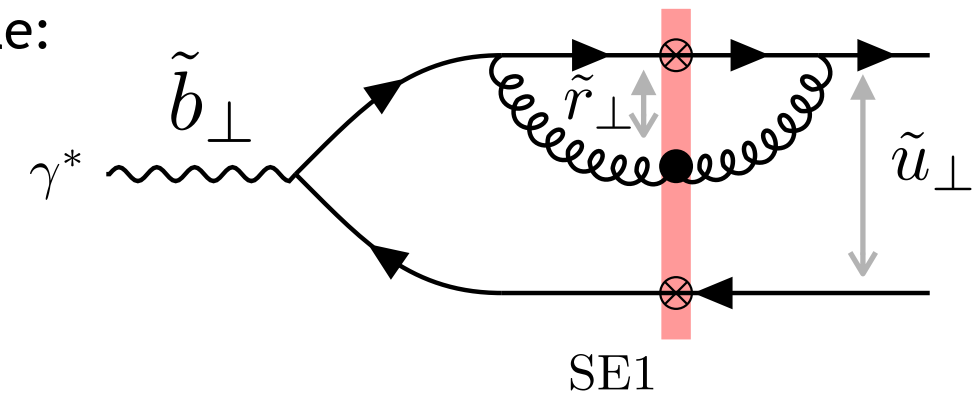
This term is seen by Hatta, Xiao, Yuan, Zhou (2021)

New contribution at small-x?

Back-to-back dijets at NLO

Finite pieces from diagrams where gluon crosses shock-wave

Example:



Correlation limit at NLO

$$q_\perp \ll P_\perp \longrightarrow \tilde{u}_\perp \ll \tilde{b}_\perp$$

Still involves operator beyond WW

Explicit expansion in $\tilde{r}_\perp$ comes with powers $1/P_\perp$, leading contribution only depends on WW distribution

Physically, the virtual loop is dominated by hard emissions $1/\tilde{r}_\perp \sim k_{g\perp} \sim P_\perp$

↳ All finite corrections @ leading power can be absorbed by defining a new NLO coefficient function

$$\mathcal{H}^{ij}_{\rm LO}(Q, \boldsymbol{P}_\perp) \rightarrow \mathcal{H}^{ij}_{\rm LO+NLO}(Q, \boldsymbol{P}_\perp)$$

By product: cancellation of singularities demands the kinematic constraint and

$$\frac{z_f}{Q_f^2} = \frac{e c_0^2 z_1 z_2}{P_\perp^2 + z_1 z_2 Q^2}$$

Back-to-back dijets at NLO

Analytic results for the NLO coefficient function

$$\mathcal{O}(\alpha_s) = \mathcal{H}^{ij}_{\rm LO} G^{ij}_Y(\boldsymbol{q}_\perp) \left[\frac{\alpha_s N_c}{2\pi} f_1 + \frac{\alpha_s}{2\pi N_c} f_2 \right]$$

$$\begin{aligned} f_1^{\lambda={\rm L}}(\chi, z_1, R; z_f) &= 7 - \frac{3\pi^2}{2} - \frac{3}{2}\ln\left(\frac{z_1 z_2 R^2}{\chi^2}\right) - \ln(z_1)\ln(z_2) + 2\ln\left(\frac{(1+\chi^2) z_f}{z_1 z_2}\right) \\ &- \ln(1+\chi^2)\ln\left(\frac{1+\chi^2}{z_1 z_2}\right) + \left\{ {\rm Li}_2\left(\frac{z_2 - z_1\chi^2}{z_2(1+\chi^2)}\right) - \frac{1}{4(z_2 - z_1\chi^2)} \right. \\ &\left. + \frac{(1+\chi^2)(z_2(2z_2 - z_1) + z_1(2z_1 - z_2)\chi^2)}{4(z_2 - z_1\chi^2)^2} \ln\left(\frac{z_2(1+\chi^2)}{\chi^2}\right) + (1 \leftrightarrow 2) \right\} \end{aligned}$$

$$\chi = \frac{Q}{M_{q\bar{q}}}$$

Remaining issues:

Similar expression for f_2

Residual $Y_f = \ln(z_f)$ dependence on the NLO coefficient function

The coefficient of single Sudakov log does not match the expected result from TMD (CSS)

$$s_L = C_F \ln\left(\frac{1}{z_1 z_2 R^2}\right) - N_c \ln\left[\left(1 + Q^2/M_{q\bar{q}}^2\right) - 1\right]$$

Back-to-back dijets at NLO

From projectile to target rapidity evolution

2307.XXXX [work in progress]

Assume Gaussian approximation, then enough to consider dipole evolution

For the BK equation, our kinematic constraint implies

$$\frac{\partial S_Y(\mathbf{r}_{bb'})}{\partial Y} = \frac{\alpha_s N_c}{\pi} \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta(-Y - \ln(r_<^2 \mu_\perp^2)) \frac{\mathbf{r}_{bb'}^2}{\mathbf{r}_{zb}^2 \mathbf{r}_{zb'}^2} \left[S_Y(\mathbf{r}_{zb}) S_Y(\mathbf{r}_{zb'}) - S_Y(\mathbf{r}_{bb'})\right]$$

Can be cast in terms of target evolution:

$$\frac{\partial \mathcal{S}_\eta(\mathbf{r}_{bb'})}{\partial \eta} = \frac{\alpha_s N_c}{\pi} \int \frac{\mathrm{d}^2 \boldsymbol{z}_\perp}{2\pi} \Theta(\eta - \delta_{bb'z}) \frac{\mathbf{r}_{bb'}^2}{\mathbf{r}_{zb}^2 \mathbf{r}_{zb'}^2} \left[\mathcal{S}_{\eta - \delta_{zb}}(\mathbf{r}_{zb}) \mathcal{S}_{\eta - \delta_{zb'}}(\mathbf{r}_{zb'}) - \mathcal{S}_\eta(\mathbf{r}_{bb'})\right]$$

Same as non-local equation found by Iancu, Mueller, Soyez, Triantafyllopolous (2019)

$$\mathcal{S}_{\eta_f} = S_{Y_f} \qquad \eta_f = Y_f + \ln(\mathbf{r}_{bb'} Q^2) - \ln(x_{\mathrm{Bj}}/x_0)$$

$$Y_f = -\ln(\mathrm{r}_{bb'}^2 \mu_\perp^2) \longrightarrow \eta_f = \ln(x_0/x_f) \qquad x_f = \mu_\perp^2/W^2$$

This choice leads to the cancellation of -1 in the coefficient of the Sudakov single log!

Leads to a natural choice of target rapidity scale

Back-to-back dijets at NLO

Final result for TMD factorized expression

2307.XXXX [work in progress]

Decomposed the differential cross-section in Fourier modes in the angle between $\mathbf{P}_\perp$ and $\mathbf{q}_\perp$ we only show the isotropic contribution. Other Fourier modes have similar expressions.

$$\mathrm{d}\sigma^{(0)}_{\mathrm{LO+NLO}} = \mathcal{H}_{\mathrm{LO}}(Q,\boldsymbol{P}_\perp)\int \mathrm{d}^2\mathbf{r}_{bb'}\, e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}}\, G^0_{\eta_f}(\mathbf{r}_{bb'})$$
$$\left[1 - \frac{\alpha_s N_c}{4\pi}\ln^2\left(\frac{\boldsymbol{P}^2_\perp\mathbf{r}^2_{bb'}}{c_0^2}\right) - \frac{\alpha_s}{\pi}s_L\ln\left(\frac{\boldsymbol{P}^2_\perp\mathbf{r}^2_{bb'}}{c_0^2}\right)\right.$$
$$\left. + \frac{\alpha_s N_c}{2\pi} f_1\left(Q/M_{q\bar{q}}, z_1, R\right) + \frac{\alpha_s}{2\pi N_c} f_2\left(Q/M_{q\bar{q}}, z_1, R\right)\right]$$
$$+\frac{\alpha_s}{2\pi}\mathcal{H}_{\mathrm{LO}}(Q,\boldsymbol{P}_\perp)\int \mathrm{d}^2\mathbf{r}_{bb'}\, e^{-i\boldsymbol{q}_\perp\cdot\mathbf{r}_{bb'}}\, h^0_{\eta_f}(\mathbf{r}_{bb'})\left[\frac{N_c}{2}(1+\ln(R^2)) - \frac{1}{2N_c}\ln(z_1 z_2 R^2)\right]$$

In practice, we will resum the Sudakov logs (ala CSS) via exponentiation

Last line: dependence on linearly polarized WW due to mixing of modes due to soft gluon radiation

Summary

- Performed a one-loop computation of dijet production in DIS at small-x in the CGC
- Isolated small-x and Sudakov logarithms, which required a kinematic constraint and reformulating the evolution in terms of “target” rapidity.
- Sudakov double and single logs agree with those expected from CSS evolution
- Showed that finite pieces can be factorized into analytically computable NLO coefficient functions multiplying the WW gluon TMD at small-x
- A full numerical study combining all these ingredients is in progress

A rigorous joint resummation/exponentiation of the Sudakov logarithms is beyond our current work. See e.g. Ian Balitsky’s talk