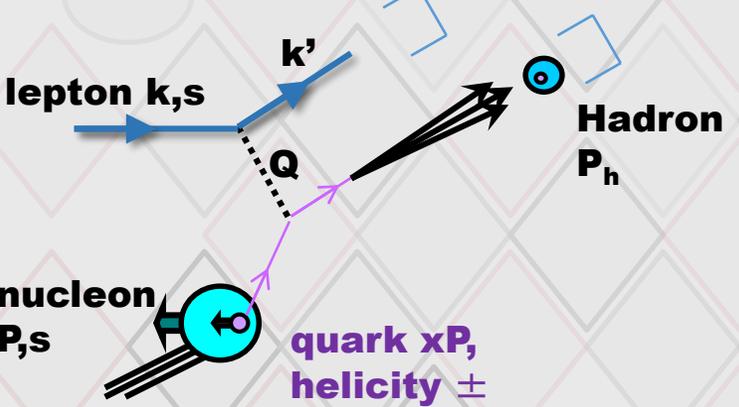


# The role of fragmentation measurements for TMD extractions

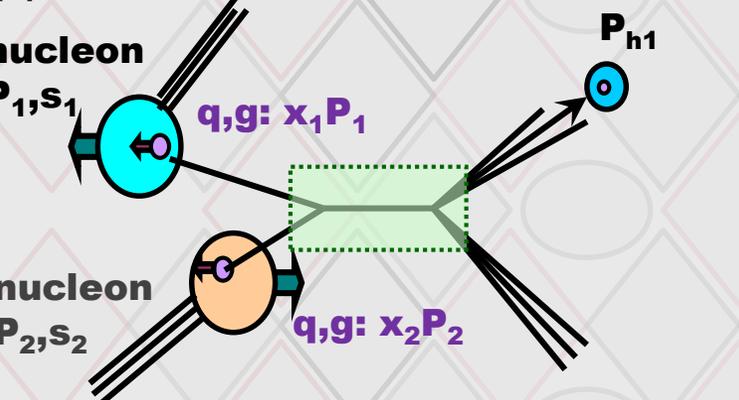
**TMD/Lattice workshop  
June 21**

**Ralf Seidl (RIKEN)**

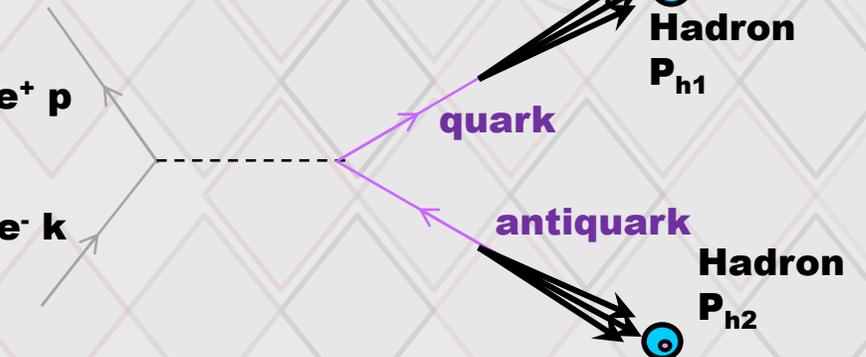
# SIDIS



# pp collisions



# e+e- Annihilation



# Access to Fragmentation functions

- SIDIS:
 
$$\sigma^h(x, z, Q^2, P_{h\perp}) \propto \sum_q e_q^2 q(x, p_t, Q^2) D_{1,q}^h(z, k_t, Q^2)$$
  - Relies on unpol PDFs
  - Parton momentum known at LO
  - Flavor structure directly accessible
  - Transverse momenta convoluted between FF and PDF

- pp:
 
$$\sigma^h(P_T) \propto \int_{x_1, x_2, z} \sum_{a, a' \in q, g} f_a(x_1) \otimes f_{a'}(x_2) \otimes \sigma_{aa'} \otimes D_{1,q}^h(z)$$
  - Relies on unpol PDFs
  - leading access to gluon FF
  - Parton momenta not directly known

- e+e-:
 
$$\sigma^h(z, Q^2, k_t) \propto \sum_q e_q^2 (D_{1,q}^h(z, k_t, Q^2) + D_{1,\bar{q}}^h(z, k_t, Q^2))$$
  - No PDFs necessary
  - Clean initial state, parton momentum known at LO
  - Flavor structure not directly accessible\*

## Single hadron FF

Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
Single hadron cross sections: $e^+e^- \rightarrow hX$ $D_{1,q}^h(z, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRL111 (2013) 062002</a>  <a href="#">PRD101(2020) 092004</a> </div>	Azimuthal asymmetries: $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2)$ $H_{1,q}^{\perp(1)h}(z, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRL 96 (2006) 232002</a>  <a href="#">PRD 78 (2008) 032011</a> </div>	Unpol SIDIS, pp: $\frac{d\sigma}{dz}$ $e^+e^- \rightarrow (h)(h)X$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRD92 (2015) 092007</a>  <a href="#">PRD101(2020) 092004</a> </div> and scale dependence
Transverse momentum dependent FFs: $e^+e^- \rightarrow (h)X$ $D_{1,q}^h(z, k_T, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRD 99 (2019) 112006</a> </div>	Transverse momentum dependent asymmetries $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2), Q_t$ $H_{1,q}^{\perp h}(z, k_T, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRD100 (2019) 92008</a> </div>	Polarizing $\Lambda$ fragmentation <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRL 122 (2019), 042001</a> </div> $D_{1,q}^{\perp h}(z, k_T, Q^2)$



## Dihadron FF (IFF)

Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
Dihadron cross sections $e^+e^- \rightarrow (hh)X$ $D_{1,q}^{h_1 h_2}(z, m, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRD96 (2017) 032005</a> </div>	Azimuthal asymmetries: $e^+e^- \rightarrow (hh)(hh)X,$ $\cos(\phi_1 + \phi_2),$ $H_{1,q}^{h_1, h_2, \triangleleft}(z, Q^2, M_h)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <a href="#">PRL107 (2011) 072004</a> </div>	Unpol SIDIS, pp: $\frac{d^2\sigma}{dzdm}$

## Single hadron measurements

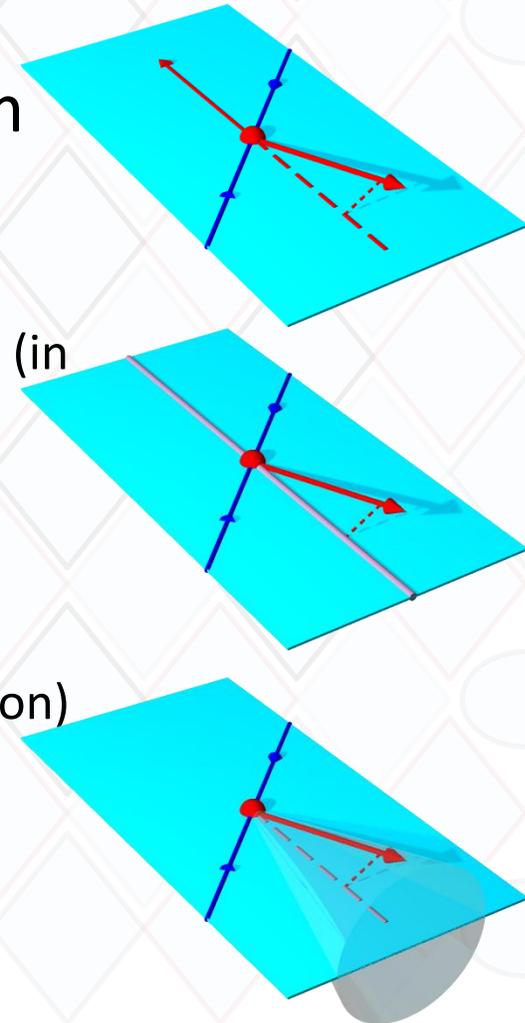
Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
<p>Single hadron cross sections:</p> $e^+e^- \rightarrow hX$ $D_{1,q}^h(z, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <p><a href="#">PRD 88 (2013) 032011</a> (Babar)</p> </div>	<p>Azimuthal asymmetries:</p> $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2)$ $H_{1,q}^{\perp(1)h}(z, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <p><a href="#">PRD 92 (2015) 111101</a> (Babar K) <a href="#">PRL 116 (2016) 042001</a> (BESIII)</p> </div>	<p>Unpol SIDIS, pp: <math>\frac{d\sigma}{dz}</math></p> $e^+e^- \rightarrow (h)(h)X$ <p style="text-align: center;">and scale dependence</p>
<p>Transverse momentum dependent FFs:</p> $e^+e^- \rightarrow (h)X$ $D_{1,q}^h(z, k_T, Q^2)$	<p>Transverse momentum dependent asymmetries</p> $e^+e^- \rightarrow (h)(h)X,$ $\cos(\phi_1 + \phi_2), Q_t$ $H_{1,q}^{\perp h}(z, k_T, Q^2)$ <div style="background-color: #d9ead3; padding: 5px; margin-top: 10px;"> <p><a href="#">PRD 90 (2014) 052003</a> (Babar)</p> </div>	

## Dihadron measurements

Unpolarized ingredients	Polarized ingredients	Flavor sensitivity
<p>Dihadron cross sections</p> $e^+e^- \rightarrow (hh)X$ $D_{1,q}^{h_1 h_2}(z, m, Q^2)$	<p>Azimuthal asymmetries:</p> $e^+e^- \rightarrow (hh)(hh)X,$ $\cos(\phi_1 + \phi_2),$ $H_{1,q}^{h_1, h_2, \triangleleft}(z, Q^2, M_h)$ <p style="font-size: small; text-align: center;">R.Seidl: FFs from e+e-</p>	<p>Unpol SIDIS, pp:</p> $\frac{d^2\sigma}{dzdm}$

# $K_T$ Dependence of FFs in $e^+e^-$

- Gain also sensitivity into transverse momentum generated in fragmentation
- Two ways to obtain transverse momentum dependence
  - Traditional 2-hadron FF
    - use transverse momentum between two hadrons (in opposite hemispheres)
    - Usual convolution of two transverse momenta
  - Single-hadron FF wrt to **Thrust** or jet axis
    - No convolution
    - Need correction for  $q\bar{q}$  axis (similar to a Jet function)



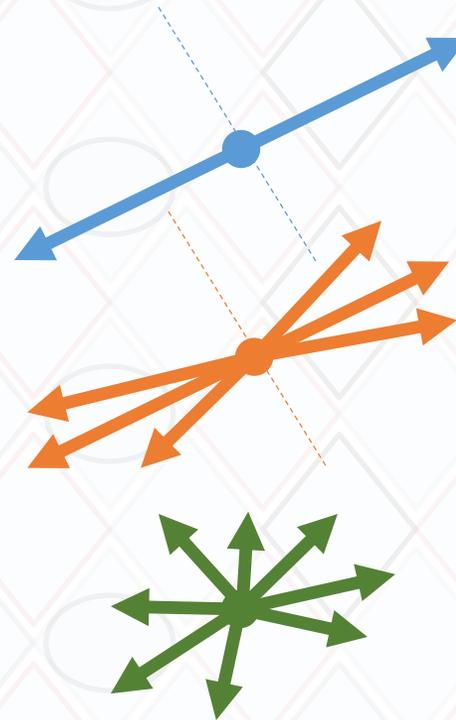
# Thrust definition

- Event shape variable thrust is defined as:

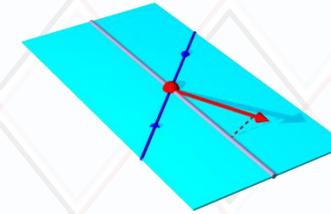
$$T \stackrel{max}{=} \frac{\sum_h |\mathbf{P}_h \cdot \hat{\mathbf{n}}|}{\sum_h |\mathbf{P}_h|}$$

- All final-state particles are included in the sum
- A **two-jet-like** event has a high thrust value
- A completely **spherical** event has a thrust value of 0.5

- Thrust axis  $\mathbf{n}$  also defines the hemispheres



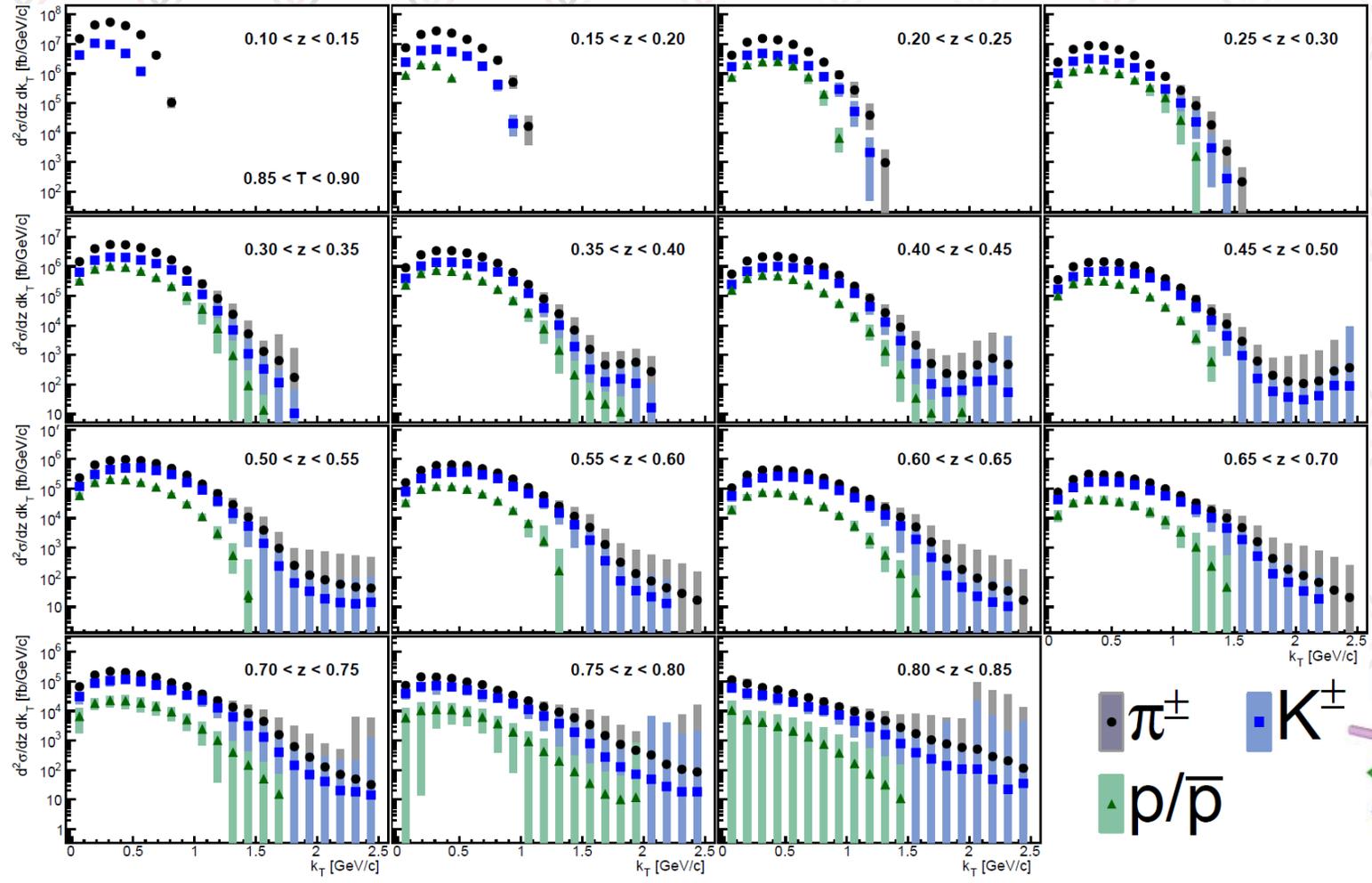
# Transverse momentum dependent cross sections for pions, kaons and protons



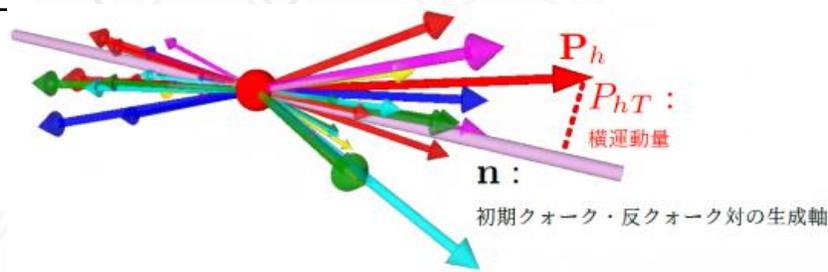
Important baseline for most transverse momentum/spin dependent measurements at RHIC and EIC

RIKEN Press release:  
[https://www.riken.jp/press/2019/20190615\\_1/](https://www.riken.jp/press/2019/20190615_1/)

$$\frac{d^2\sigma}{dzdP_{hT}}$$



$\bullet \pi^\pm$      $\blacksquare K^\pm$   
 $\blacktriangle p/\bar{p}$

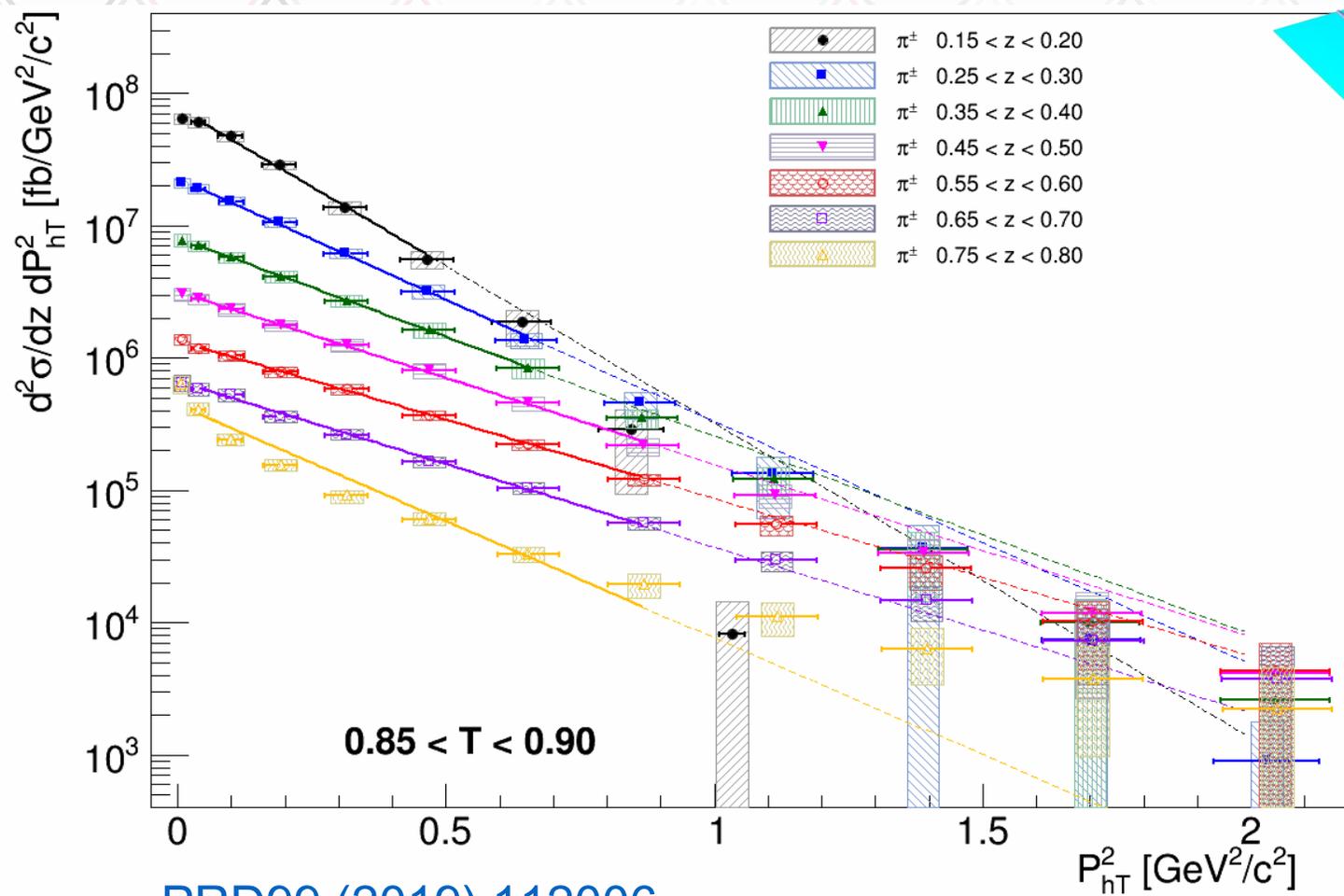


RS. et. al. [PRD99 \(2019\) 112006](https://arxiv.org/abs/1906.01120)

R.Seidl: FFs from e+e-

# Fits vs $P_{hT}^2$

Fit exponential to smaller transverse momenta for  
Gaussian  $P_{hT}$  dependence and power law at higher  $P_{hT}$



[PRD99 \(2019\) 112006](#)

# Gaussian widths comparison to MC

first direct (no convolutions) measurement of  $z$  dependence of Gaussian widths

MSTP(21):

0.28

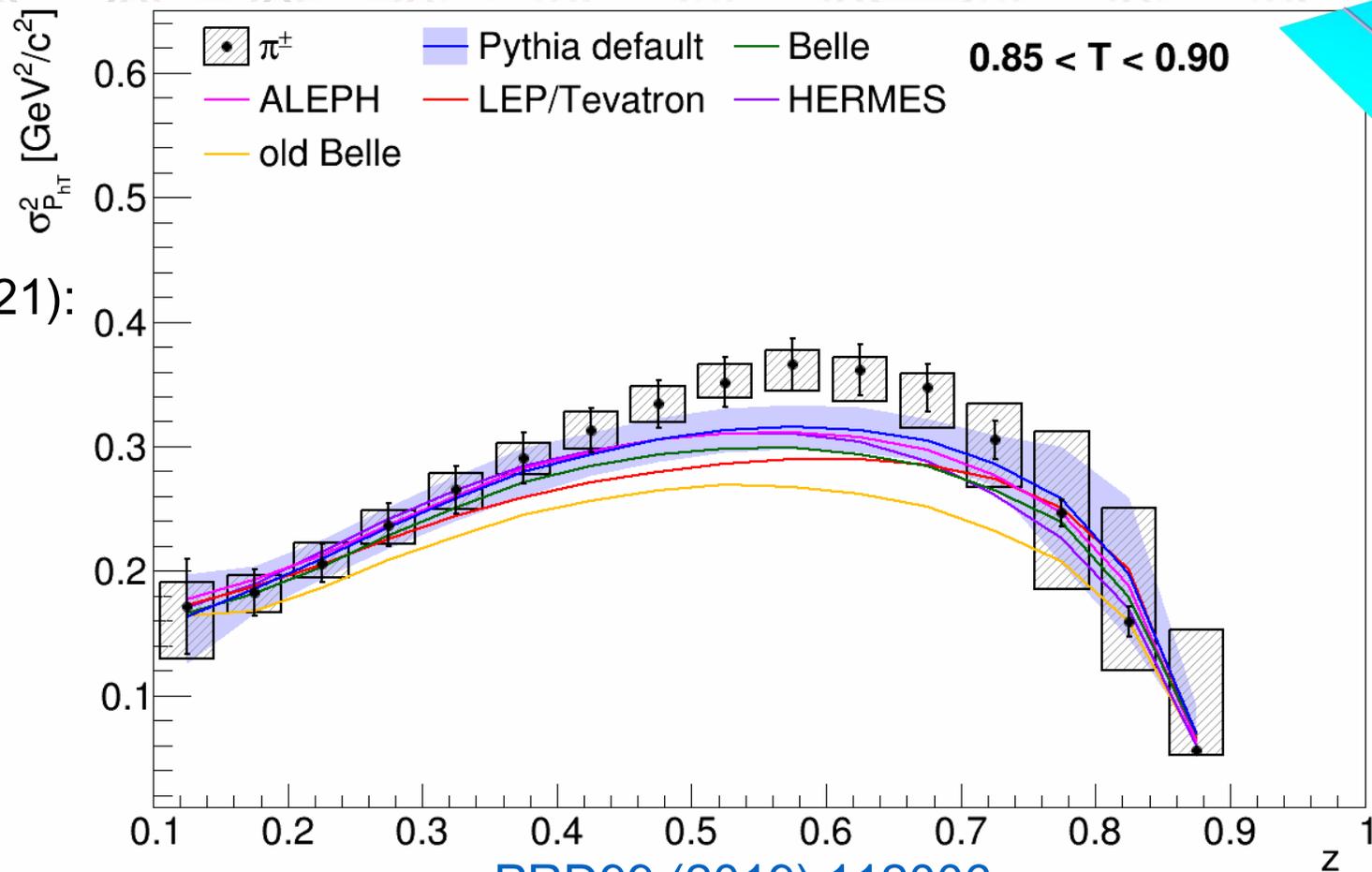
0.325

0.36

0.36

0.37

0.40

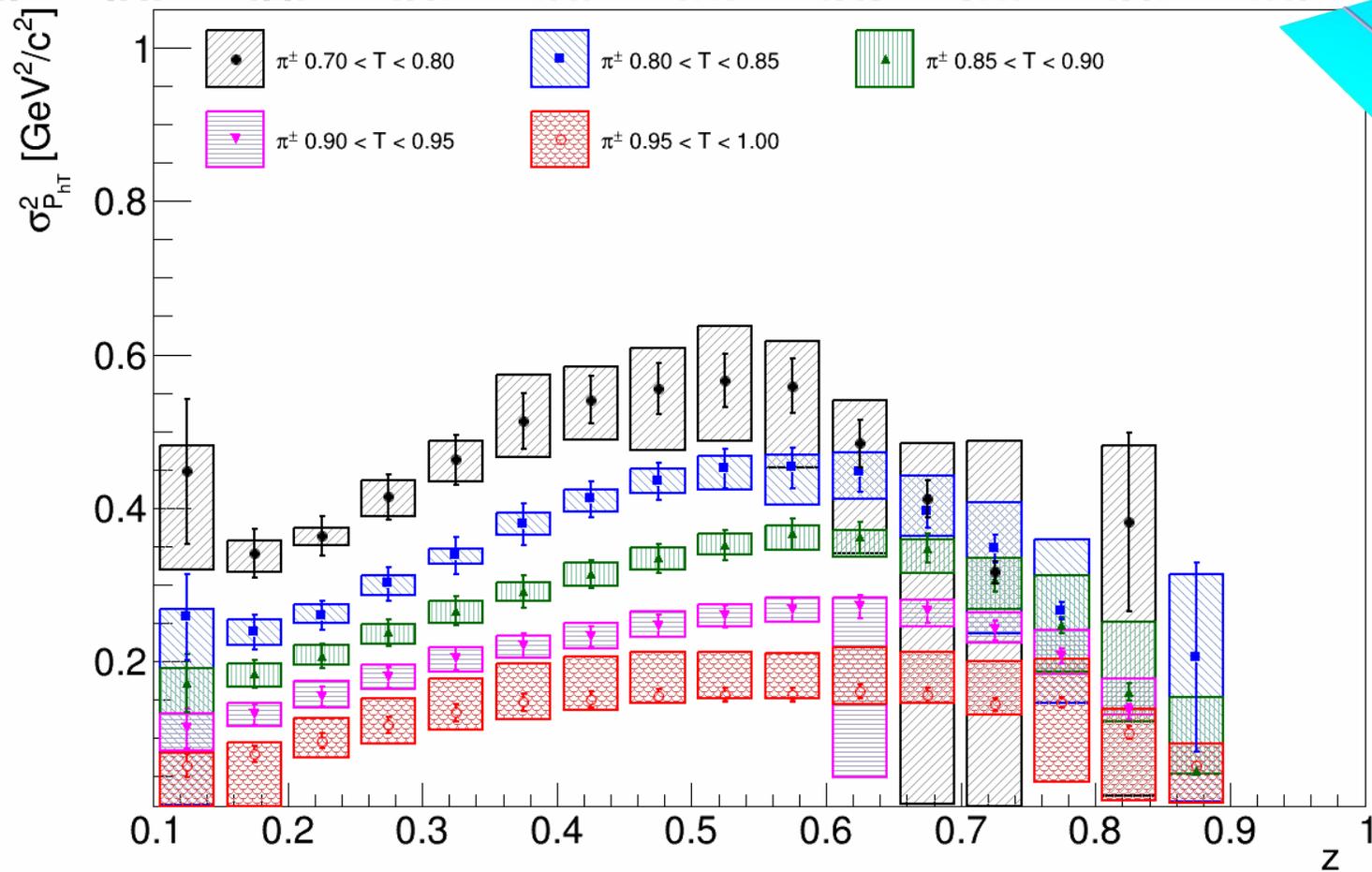


[PRD99 \(2019\) 112006](#)

R.Seidl: FFs from e+e-

# Gaussian widths, thrust dependence

Gaussian widths get narrower with higher Thrust



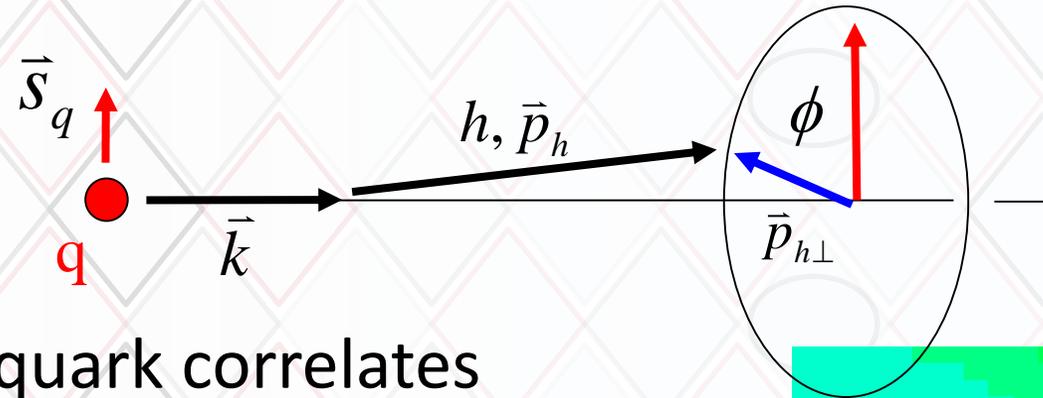
[PRD99 \(2019\) 112006](#)

R.Seidl: FFs from e+e-

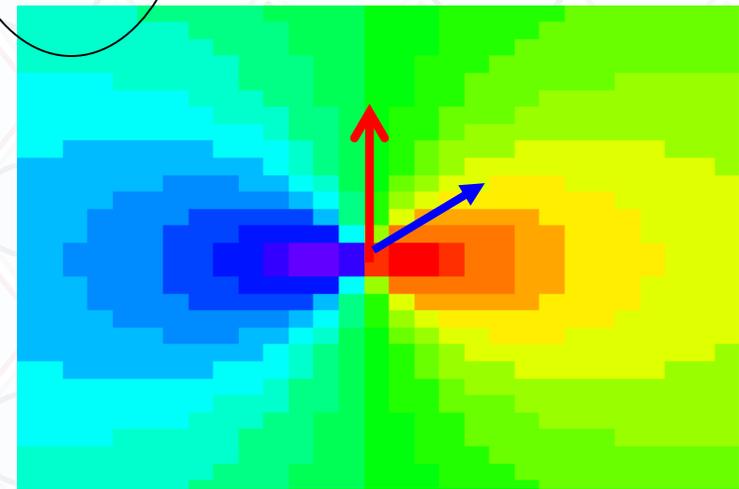
# Collins fragmentation function

J. Collins, Nucl. Phys. B396, (1993) 161

$$D_{q\uparrow}^h(z, P_{h\perp}) = D_{1,q}^h(z, P_{h\perp}^2) + H_{1,q}^{\perp h}(z, P_{h\perp}^2) \frac{(\hat{\mathbf{k}} \times \mathbf{P}_{h\perp}) \cdot \mathbf{S}_q}{zM_h}$$



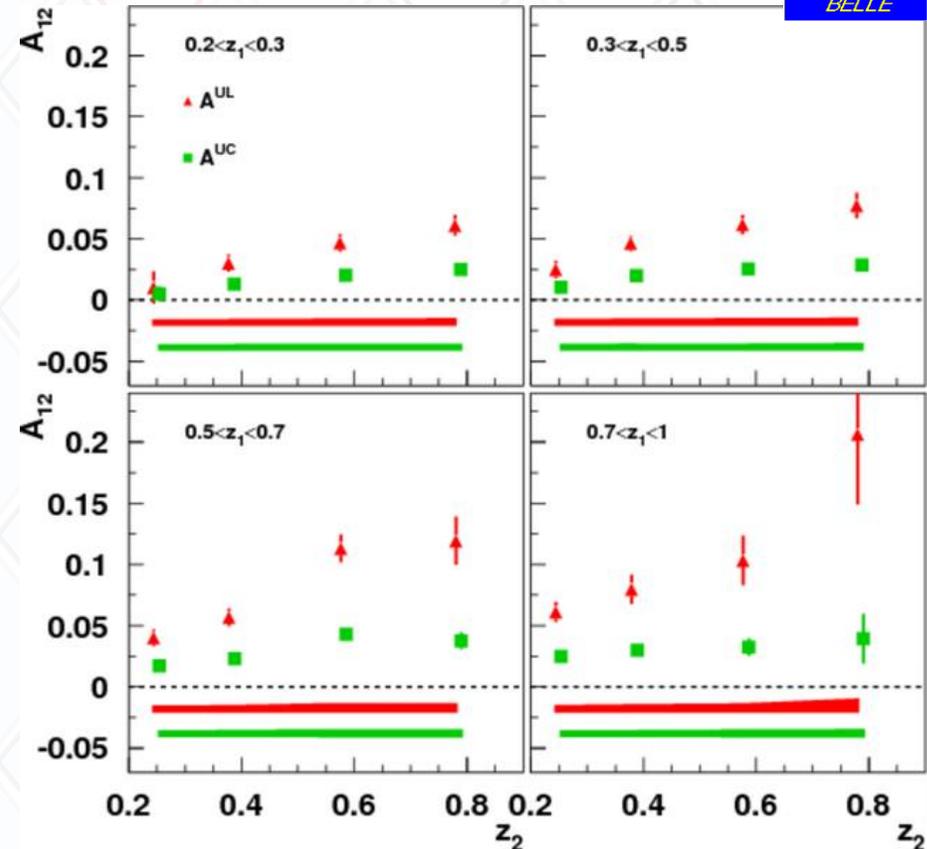
- Spin of quark correlates with hadron transverse momentum
- ➔ translates into azimuthal anisotropy of final state hadrons



# Belle Collins asymmetries



- **Red points** :  $\cos(\phi_1 + \phi_2)$  moment of **Unlike** sign pion pairs over **like** sign pion pair ratio :  $A^{UL}$
- **Green points** :  $\cos(\phi_1 + \phi_2)$  moment of **Unlike** sign pion pairs over **any charged** pion pair ratio :  $A^{UC}$
- Collins fragmentation is large effect
- Consistent with SIDIS indication of sign change between favored and disfavored Collins FF

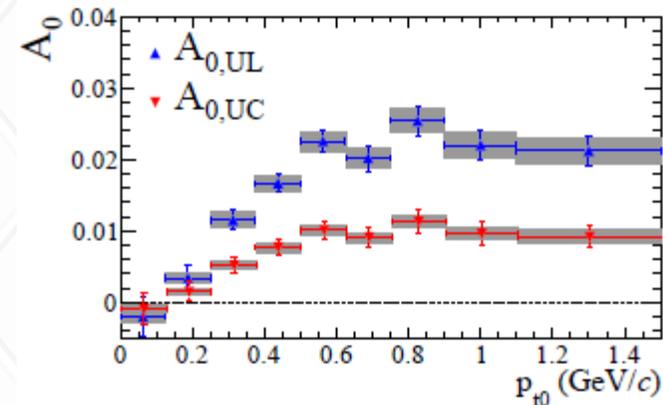
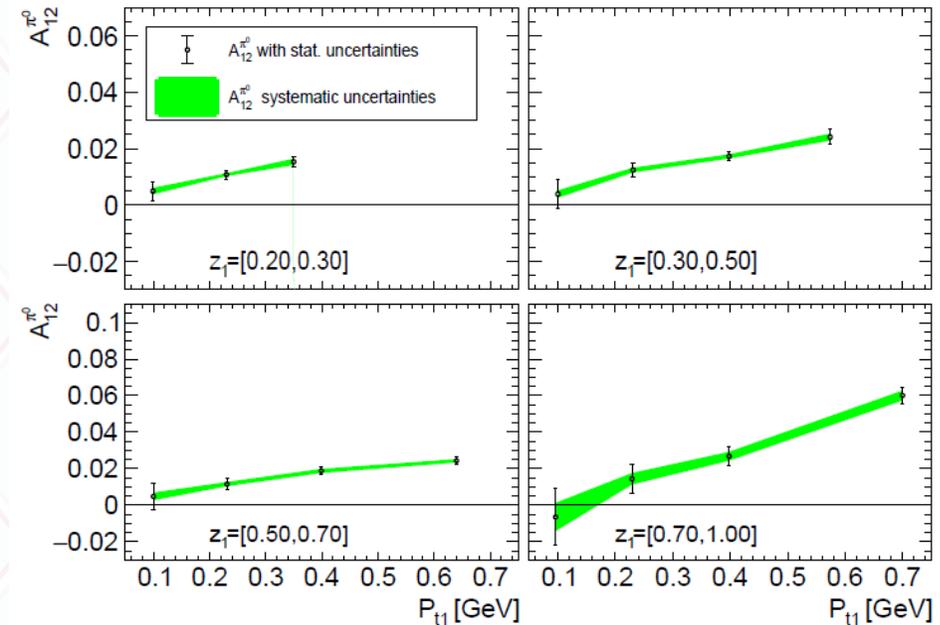


RS et al (Belle), PRL96: 232002  
PRD 78:032011, Erratum D86:039905

# Transverse momentum

- Add transverse momentum to Collins asymmetries'  $z$  dependence
- Currently only 1 or 2-dimensional extractions available ( $q_t, z_1 \times z_2, p_{t1} \times p_{t2}, z_1 \times p_{t1}$ )
- Increasing asymmetries with both  $z$  and  $p_t$ , but  $p_t$  reach limited
- Multidimensional extractions needed

PRD100 (2019) 92008

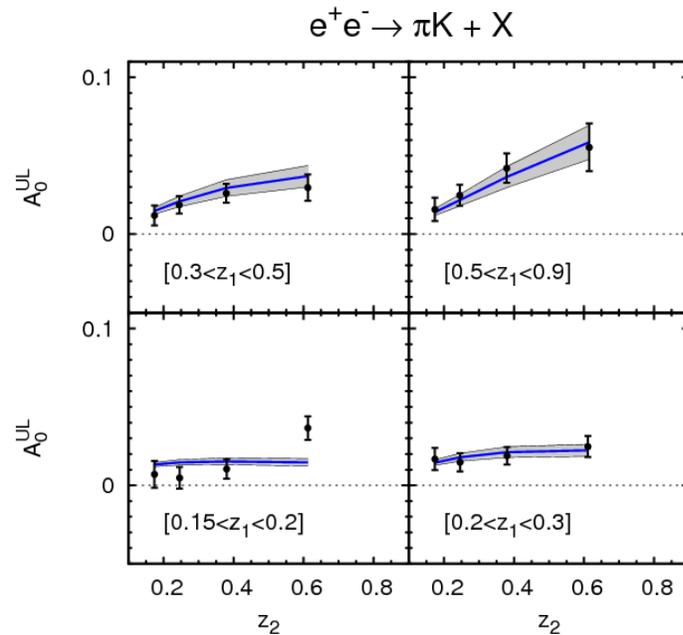


PRD 90 (2014) 052003 (Babar)

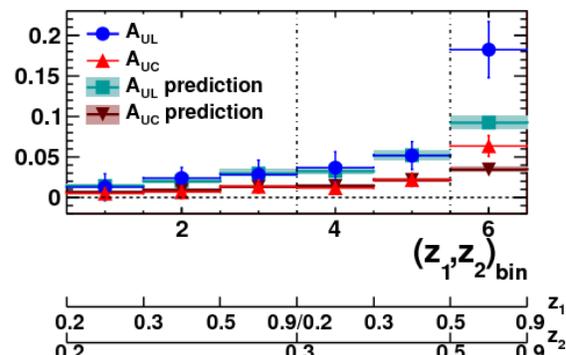
# Quark transversity via Collins: Kaons

BABAR: [PRD 92 \(2015\) 111101](#)

Anselmino et al: [PRD 93 \(2016\) 034025](#)



BESIII: [PRL 116 \(2016\) 042001](#)



- Addition of kaon Collins fragmentation strongly needed for flavor decomposition of quark transversity
- Large amount of potentially participating FFs well described by light and “heavy” favored and disfavored FFs
- Allows inclusion of HERMES and COMPASS kaon asymmetries (+eventually EIC) in fits
- Also: pion Collins at lower scale (BESIII) consistent with TMD evolution

# Ongoing work: Collins multidimensional analysis and Kaon combinations

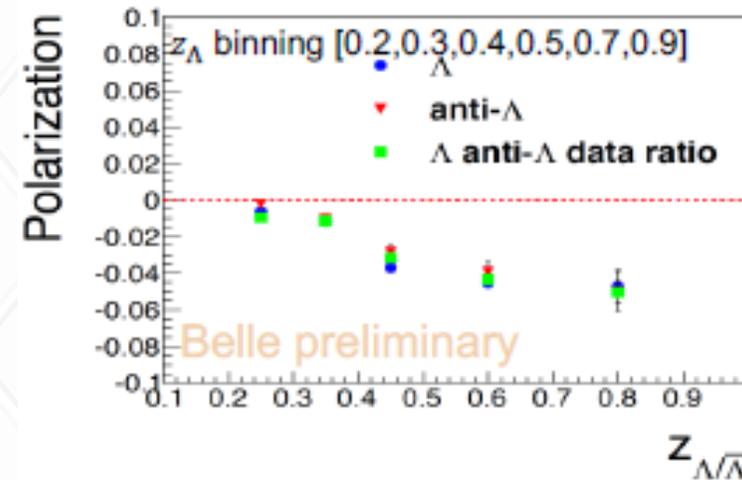
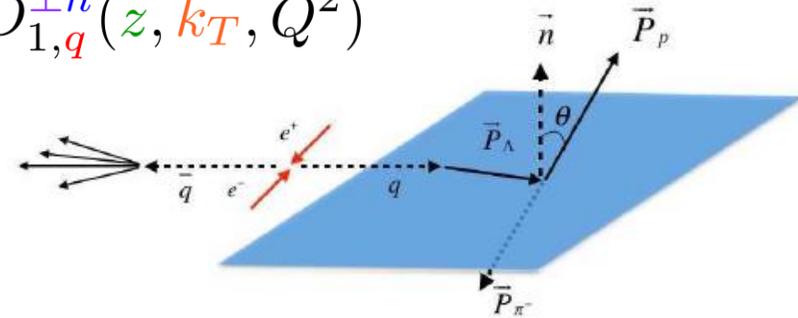
- Currently revisiting kaon combinations of the Collins asymmetries
- While doing so, try to perform a full multi-dimensional analysis:
  - Consider :
    - $6(z_1) \times 6(z_2) \times 5(k_{t1}) \times 5(k_{t2}) \times 1(\text{costheta}) \times 8(\text{phi})$  for  $A_{12}$  method
    - $6(z_1) \times 6(z_2) \times 10(q_t) \times 1(\text{costheta}) \times 8(\text{phi})$  for  $A_0$  method
- Perform most correction steps similar to recent analyses (PID, smearing)
  - Possibly simplified smearing unfolding as each  $z_1$ - $z_2$  bin separately (z smearing almost nonexistent in such a binning)
  - non-qqbar removal, charm removal, ISR correction and acceptance might require introduction of nonzero MC asymmetries

# Single $\Lambda$ polarization measurements

- Related to open question about  $\Lambda$  polarization in hadron collisions from 40 years ago!
- Fragmentation counterpart to the Sivers Function:
  - unpolarized parton fragments into transversely polarized baryon with transverse momentum wrt to parton direction
- Reconstruct  $\Lambda$ , its transverse momentum and polarization

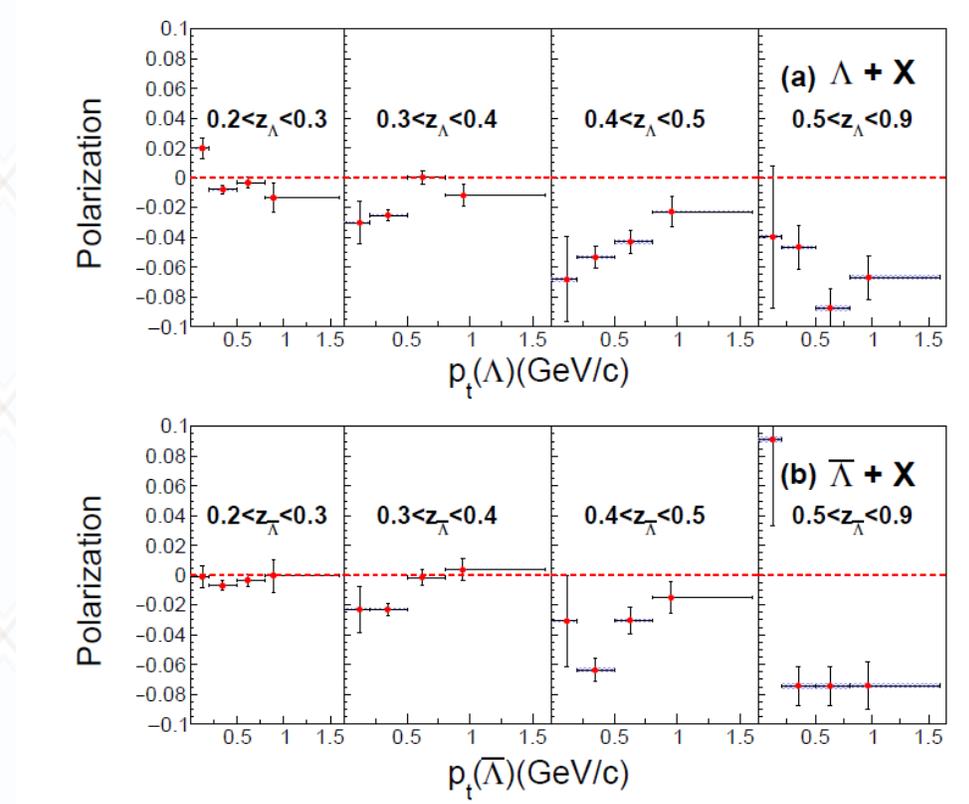
YingHui Guan (Indiana/KEK):  
[PRL 122 \(2019\), 042001](https://arxiv.org/abs/1808.07501)

$$D_{1,q}^{\perp h}(z, k_T, Q^2)$$



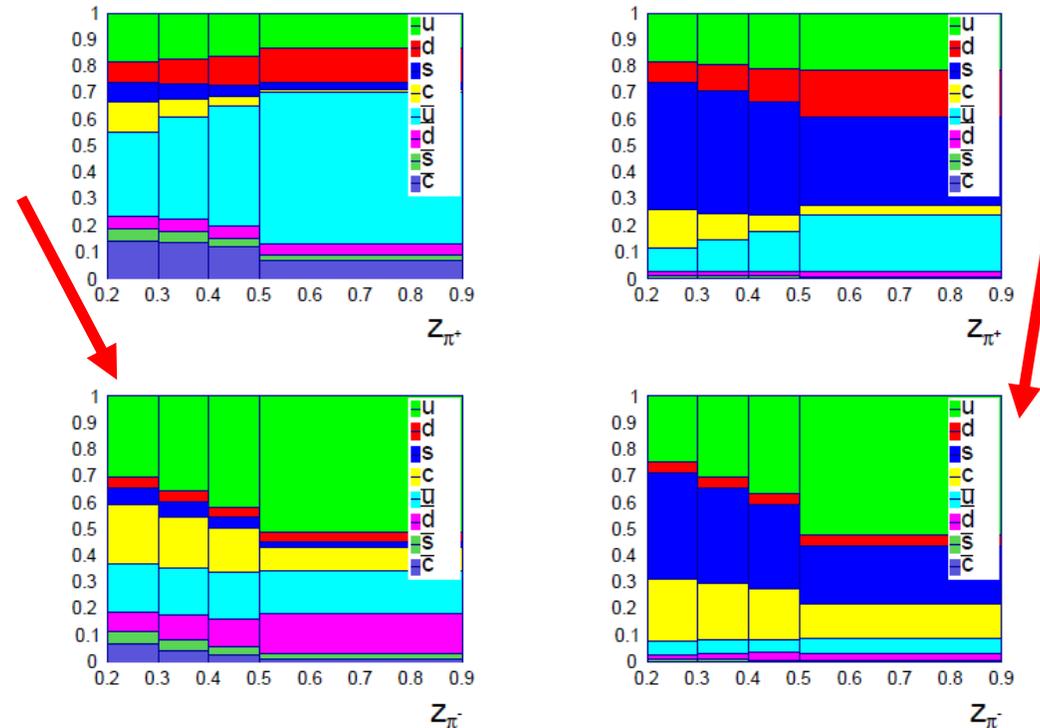
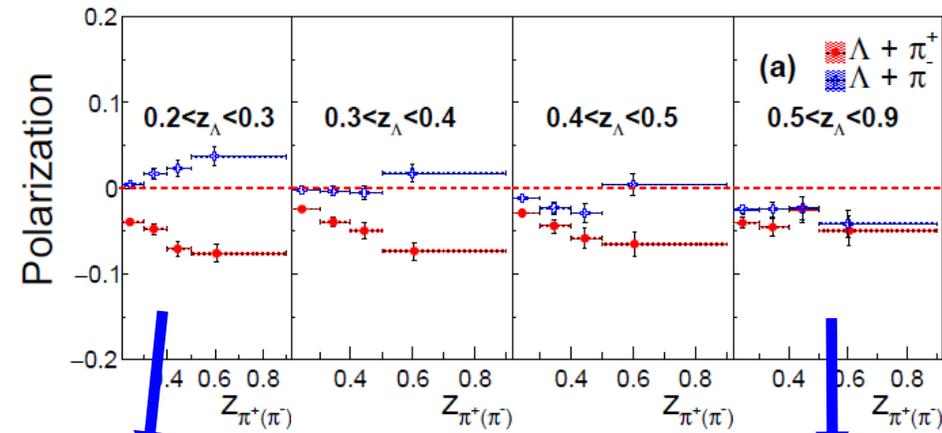
# Transverse momentum dependence

- Different behavior for low and high- $z$  :
- At low  $z$  small
- At intermediate  $z$  falling Polarization with  $P_t$
- At high  $z$  increasing polarization with  $P_t$



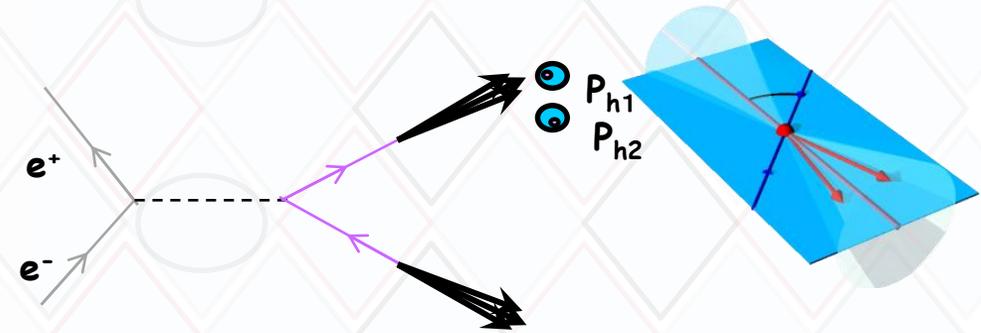
# Opposite hemisphere pion correlation

- Interesting  $z_\pi$  and  $z_\Lambda$  dependence :
- At low  $z_\Lambda$  light quark fragmentation dominant, some charm in  $\pi^- \rightarrow$  different signs
- At high  $z_\Lambda$  strange + charm fragmentation more relevant  $\rightarrow$  same signs
- Several fits to data with slightly different results

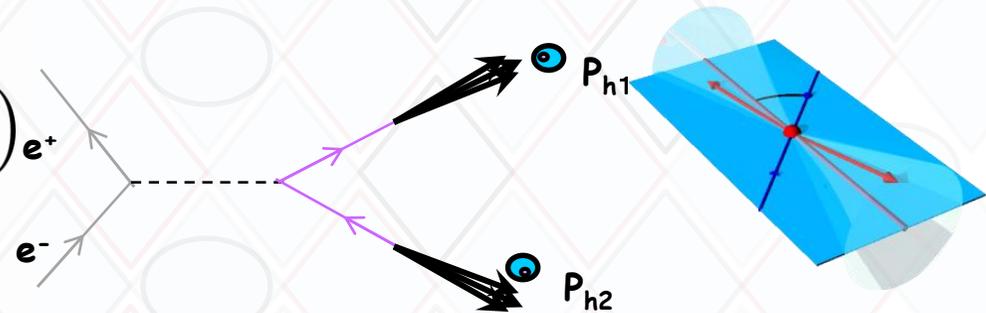


# Di-hadron fragmentation functions

$$D_{1,q}^{h_1 h_2}(z, m, Q^2)$$



$$D_{1,q}^h(z_1, Q^2) D_{1,q}^h(z_2, Q^2)$$



# Additional fractional energy definitions

- Traditional definition as:
- “New” definition based on 4-momentum dot products (Altarelli et al NPB160 (1979) , 301): automatic stress on nearly back-to-back pairs, better applicability for NLO analysis
- Also new definition interested in transverse momentum in pair (Mulders/vanHulse: [PRD 20100 \(2019\), 034011](#)):

$$z_{1,2} = \frac{2E_{1,2}}{q^2}$$

$$z_1 = \frac{2P_1 \cdot q}{q^2}$$

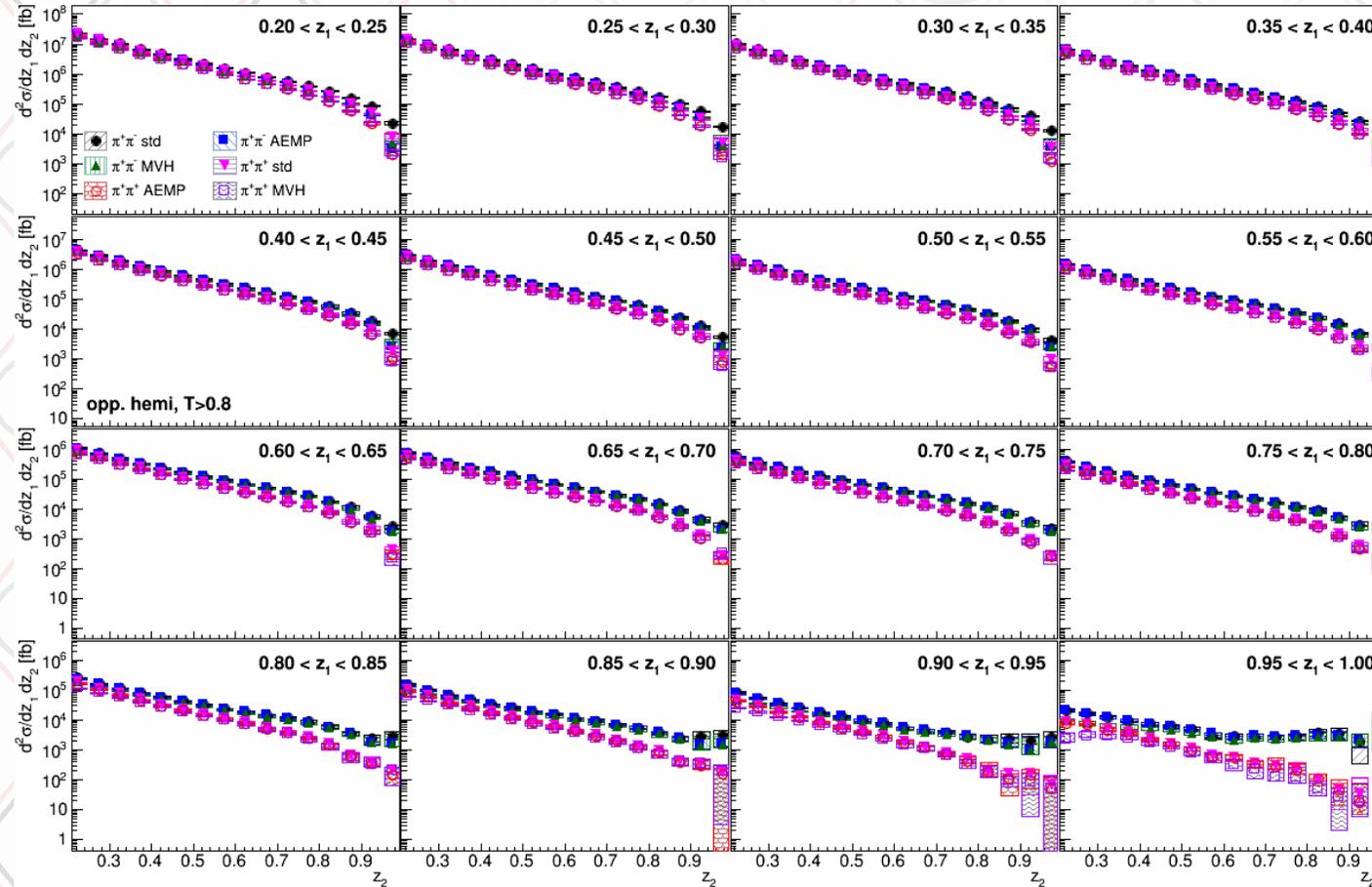
$$z_2 = u = \frac{P_1 \cdot P_2}{P_1 \cdot q}$$

$$z_1 = \left( P_1 \cdot P_2 - \frac{M_{h1}^2 M_{h2}^2}{P_1 \cdot P_2} \right) \frac{1}{P_2 \cdot q - M_{h2}^2 \frac{P_1 \cdot q}{P_1 \cdot P_2}}$$

$$1 \leftrightarrow 2$$

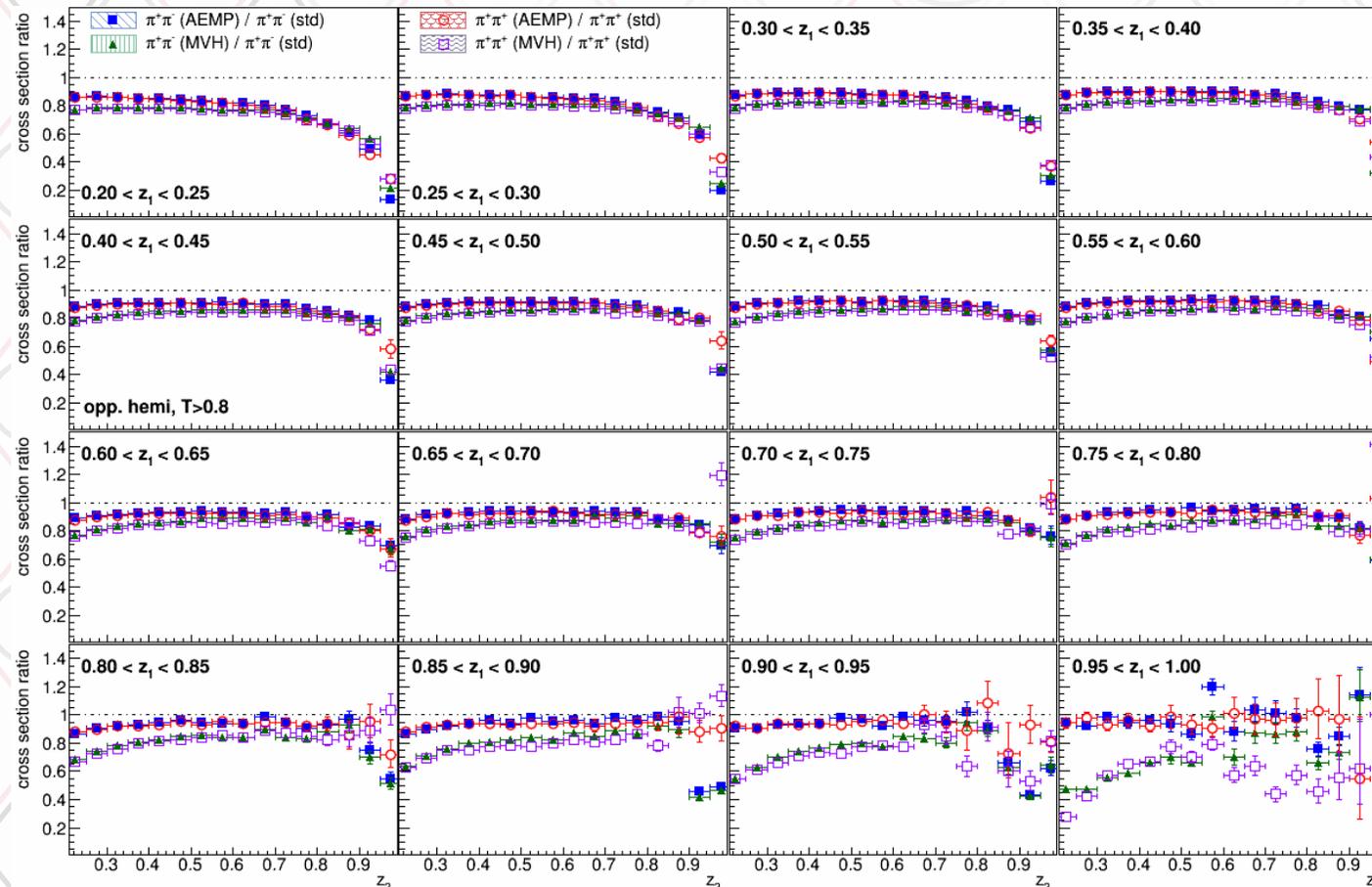
# Comparison of definitions: pion pairs in opposite hemispheres

[RS et al, PRD 101 \(2020\) 092004](#)



# Ratios to standard definitions: pion pairs in opposite hemisphere

[RS et al, PRD 101 \(2020\) 092004](#)



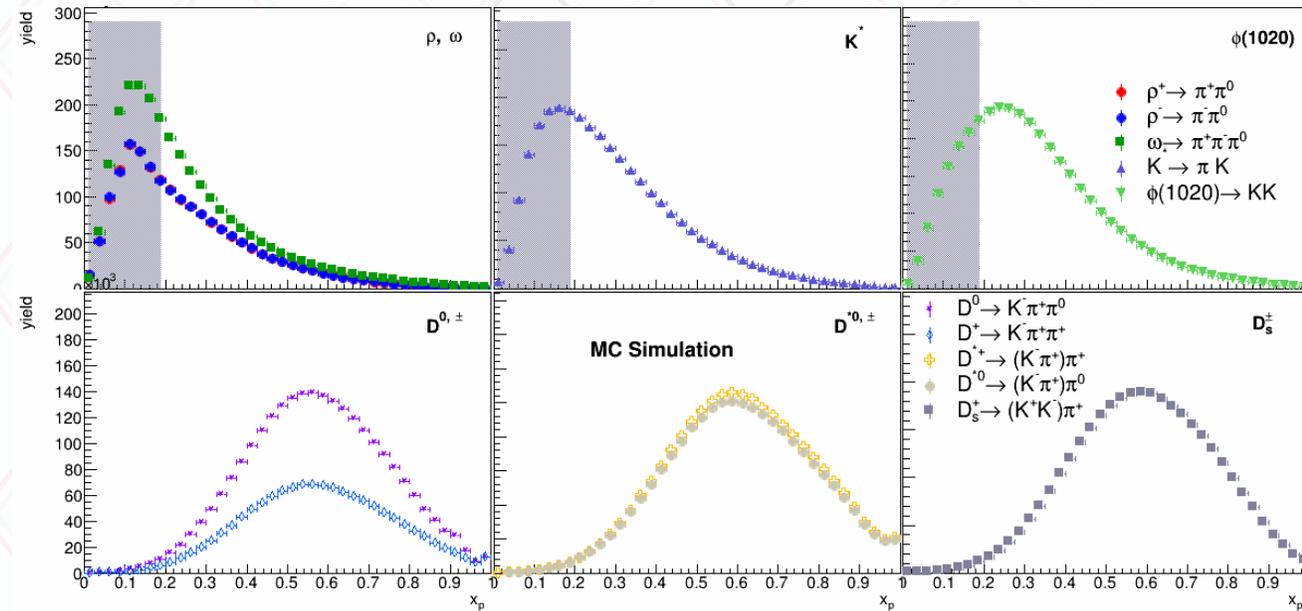
AEMP : suppressed at small fractional energies, at high  $z$  close to unity  
 MvH : further suppressed due to transverse momentum

R.Seidl: FFs from  $e^+e^-$

# Ongoing: Decaying particle FFs

- Study the explicit differential cross sections for VMs, D mesons as a function of  $x_p$
- Mostly mass distributions and fits well-behaved, except for  $\rho$ – $\omega$  (interference) and more exotic resonances
- Also of interest for ultra high-energetic cosmic ray air shower research (muon problem)

- Example from MC at Belle energies (for  $4\pi$  acceptance):



Important resource for EIC, RHIC and HI physics

# Summary

- $e^+e^-$  is an excellent source to study fragmentation
- Crucial information for any SIDIS/pp measurements to access PDFs
- Sensitivity not only to collinear FFs but also TMD FFs:
  - Explicitly  $kt$  dependent unpol FFs
  - Collins asymmetries
  - Polarizing L fragmentation
  - Nearly back-to-back di-hadron measurements
  - Role of D and Vector Meson fragmentation and its impact on light hadrons
- More to come

# Belle Detector and KEKB

- Asymmetric collider
- $8\text{GeV } e^- + 3.5\text{GeV } e^+$
- $\sqrt{s} = 10.58\text{GeV } (Y(4S))$
- $e^+e^- \rightarrow Y(4S) \rightarrow B \bar{B}$
- Continuum production:  $10.52\text{ GeV}$
- $e^+e^- \rightarrow q \bar{q} \text{ (u,d,s,c)}$
- Integrated Luminosity:  $>1000\text{ fb}^{-1}$
- $>70\text{fb}^{-1} \Rightarrow$  continuum

