Transversity PDFs and GPDs from lattice QCD

Martha Constantinou

Temple University

CFNS Workshop

TMDs: Towards a Synergy between Lattice QCD & Global Analysis

June 22, 2023

Topics

★ Twist-2 PDFs & GPDs: "traditional calculations

PHYSICAL REVIEW D **98,** 091503(R) (2018)

Rapid Communications

Transversity parton distribution functions from lattice QCD

PHYSICAL REVIEW D **99,** 114504 (2019)

Systematic uncertainties in parton distribution functions from lattice QCD simulations at the physical point

Constantia Alexandrou,[1,2] Krzysztof Cichy,[3] Martha Constantinou,[4] Kyriakos Hadjiyiannakou,[2] Karl Jansen,[5] Aurora Scapellato,[1,6] and Fernanda Steffens[7]

YSICAL REVIEW D **105,** 034501 (2022)

Transversity GPDs of the proton from lattice QCD

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★ Twist-3 distribution functions

PHYSICAL REVIEW D **104,** 114510 (2021)

Parton distribution functions beyond leading twist from lattice QCD: The $h_L(x)$ case

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PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

+ Joshua Miller

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

★ Twist-2 GPDs: new approach

Distribution Functions

★ Key universal non-perturbative tools for study of hadron structure

★ Global analyses: main source of information for distribution functions

★ Global fits improved: theoretical advances & new data

★ ambiguities due to limited kinematic regions, limited measurements, etc

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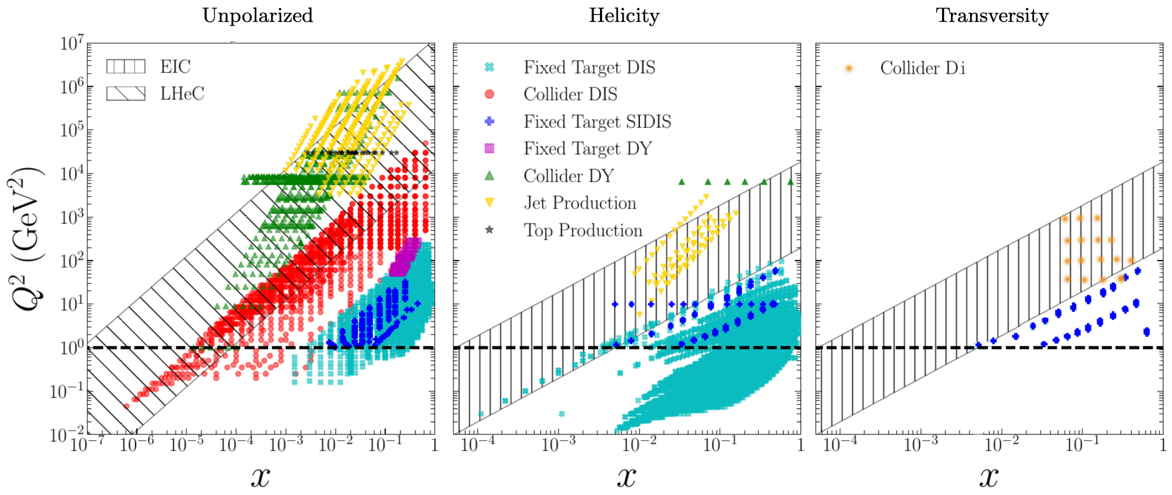

[Ethier & Nocera, *Ann. Rev. Nucl. Part. Sci.* 70 (2020) 1, arXiv:2001.07722]

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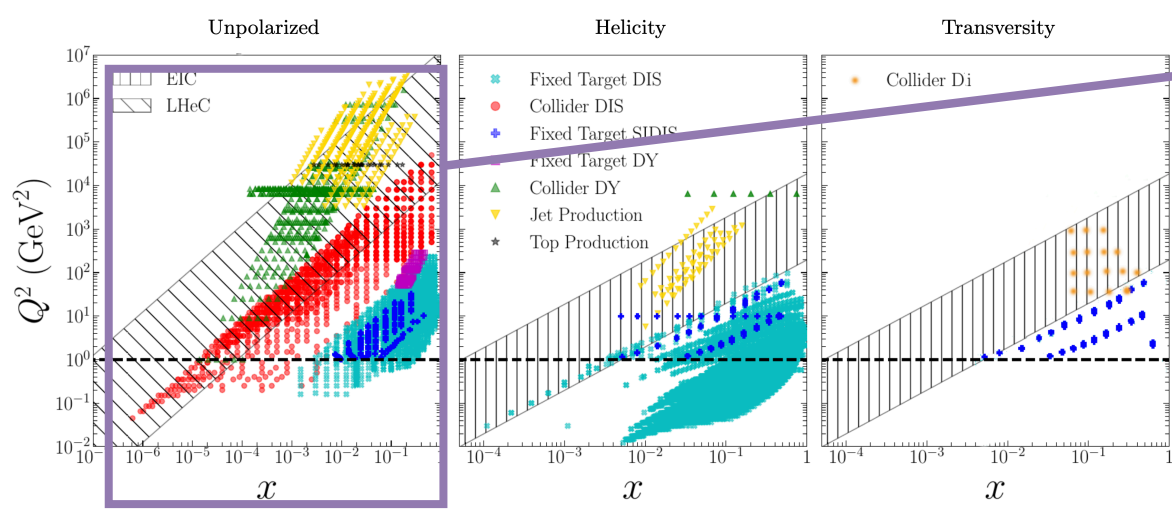

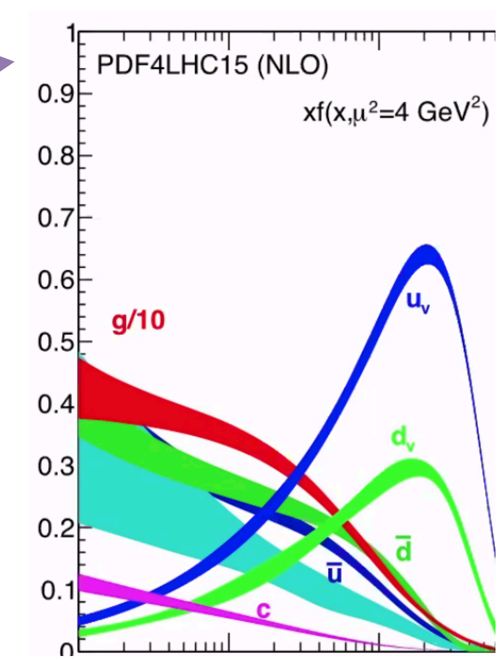

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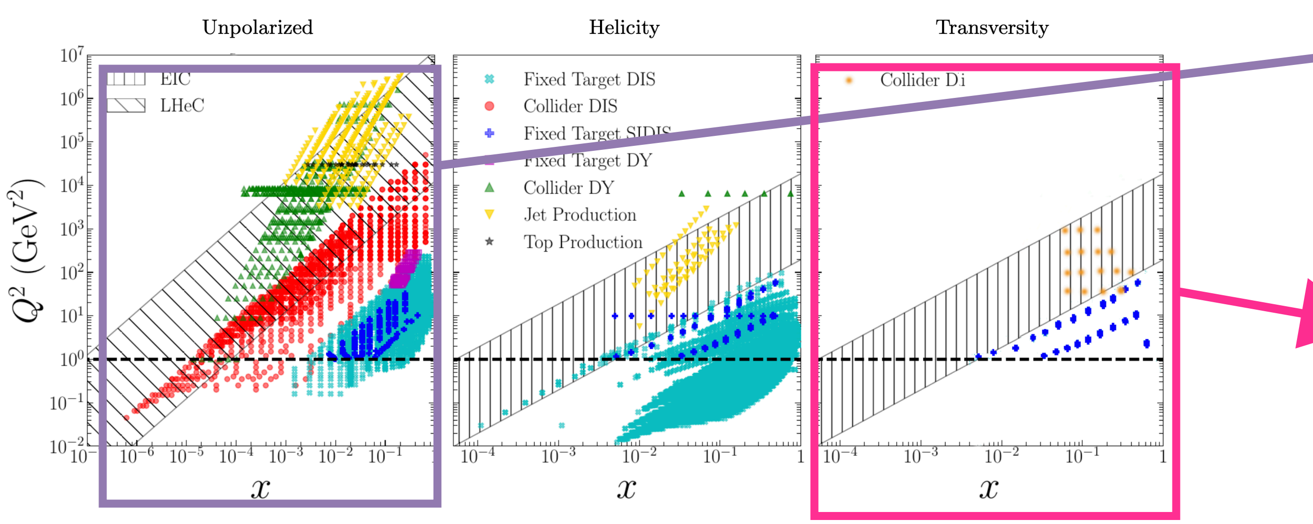

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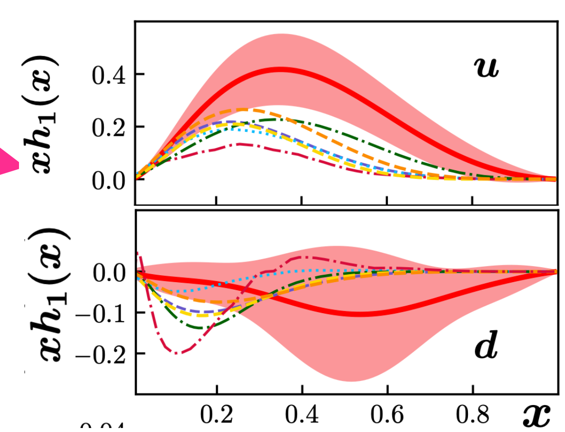

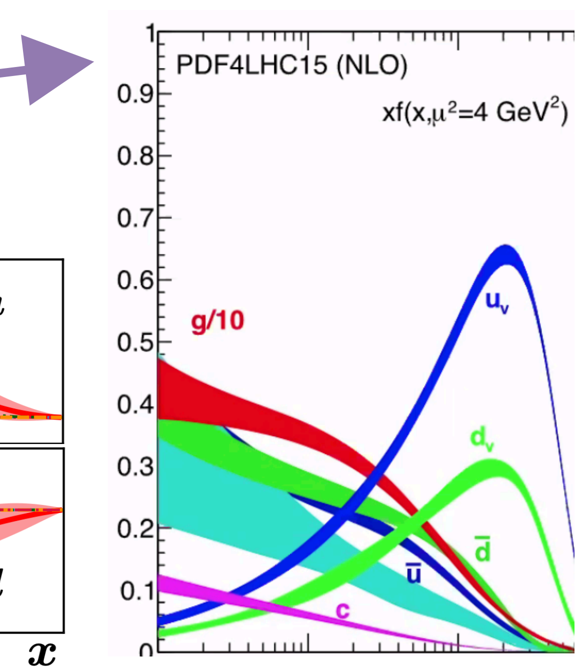

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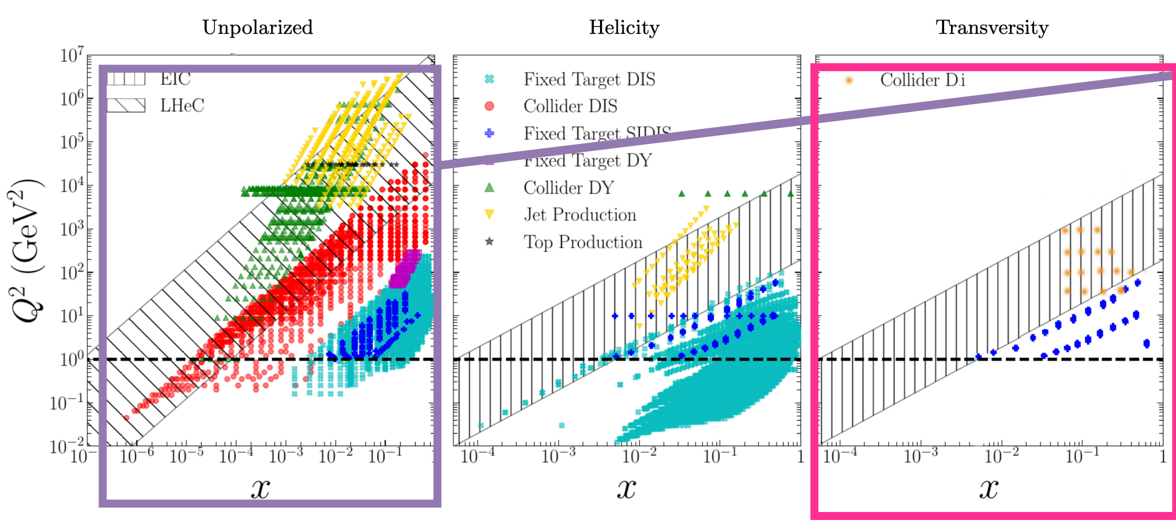

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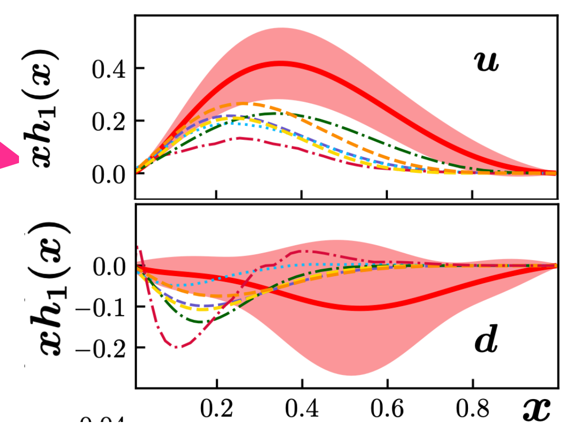

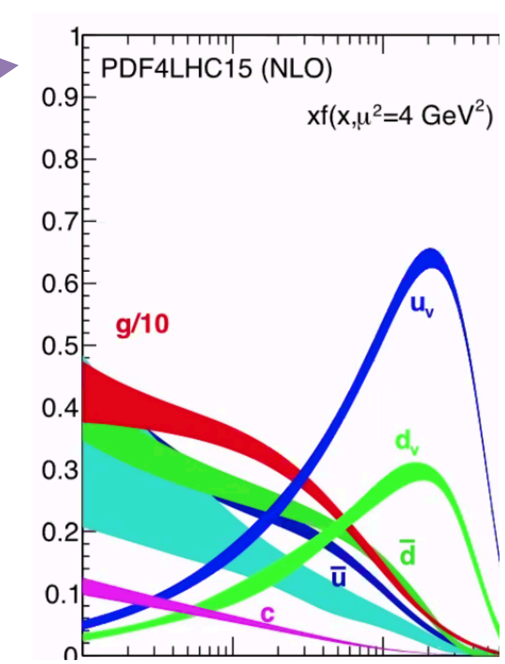

Calculation from first principle (lattice QCD) can help in the estimation of PDFs

Twist-classification of PDFs, GPDs, TMDs

- ★ Twist: specifies the order in $1/Q$ at which the function enters factorization formula for a given observable

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

- ★ Twist-2: probabilistic densities - a wealth of information exists

- ★ Twist-3: poorly known, but very important:
  - as sizeable as twist-2
  - contain information about quark-gluon correlations inside hadrons
  - appear in QCD factorization theorems for various observables (e.g. g_2)
  - certain twist-3 PDFs are related to the TMDs
  - physical interpretation (e.g. average force on partons inside hadron)

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While twist-3 $f_i^{(1)}$ share some similarities with twist-2 $f_i^{(0)}$ in their extraction, there are several challenges both experimentally and theoretically

Distribution functions

Through non-local matrix elements

of fast-moving hadrons

Access of PDFs/GPDs on a Euclidean Lattice

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]

Matrix elements of nonlocal (equal-time) operators with fast moving hadrons

$$\tilde{q}_{\Gamma}^{\mathrm{GPD}}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i x P_3 z} \quad \langle N(P_f) \,|\, \bar{\Psi}(z) \, \Gamma \, \mathscr{W}(z,0) \Psi(0) \,|\, N(P_i) \rangle_{\mu}$$

$$\Delta = P_f - P_i$$

$$t = \Delta^2 = -Q^2$$

$$\xi = \frac{Q_3}{2P_3}$$

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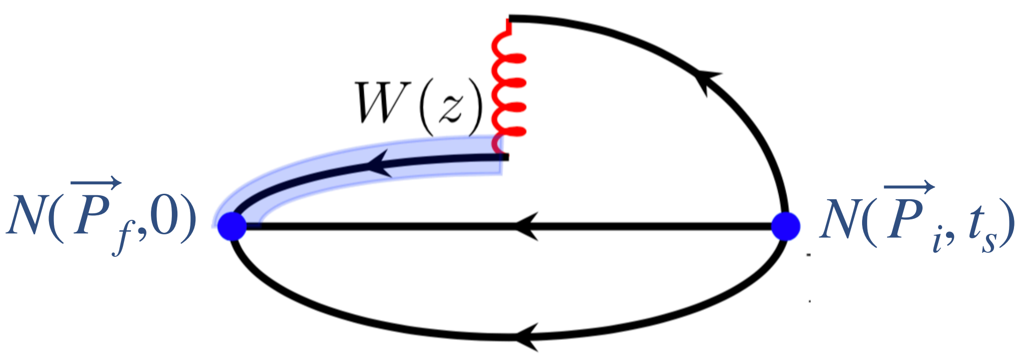

Computationally intensive

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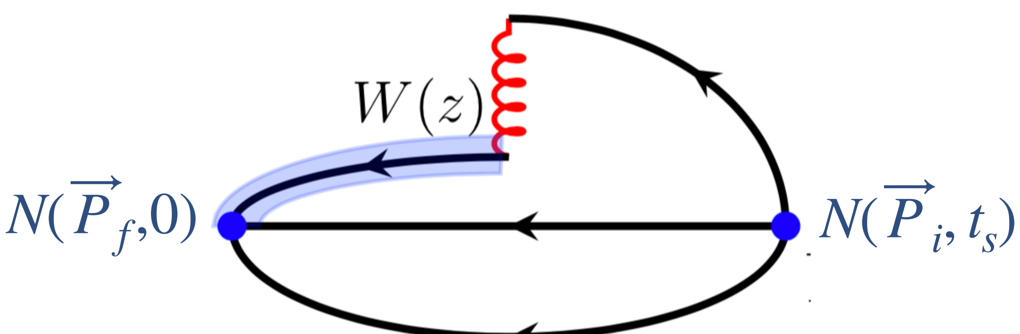

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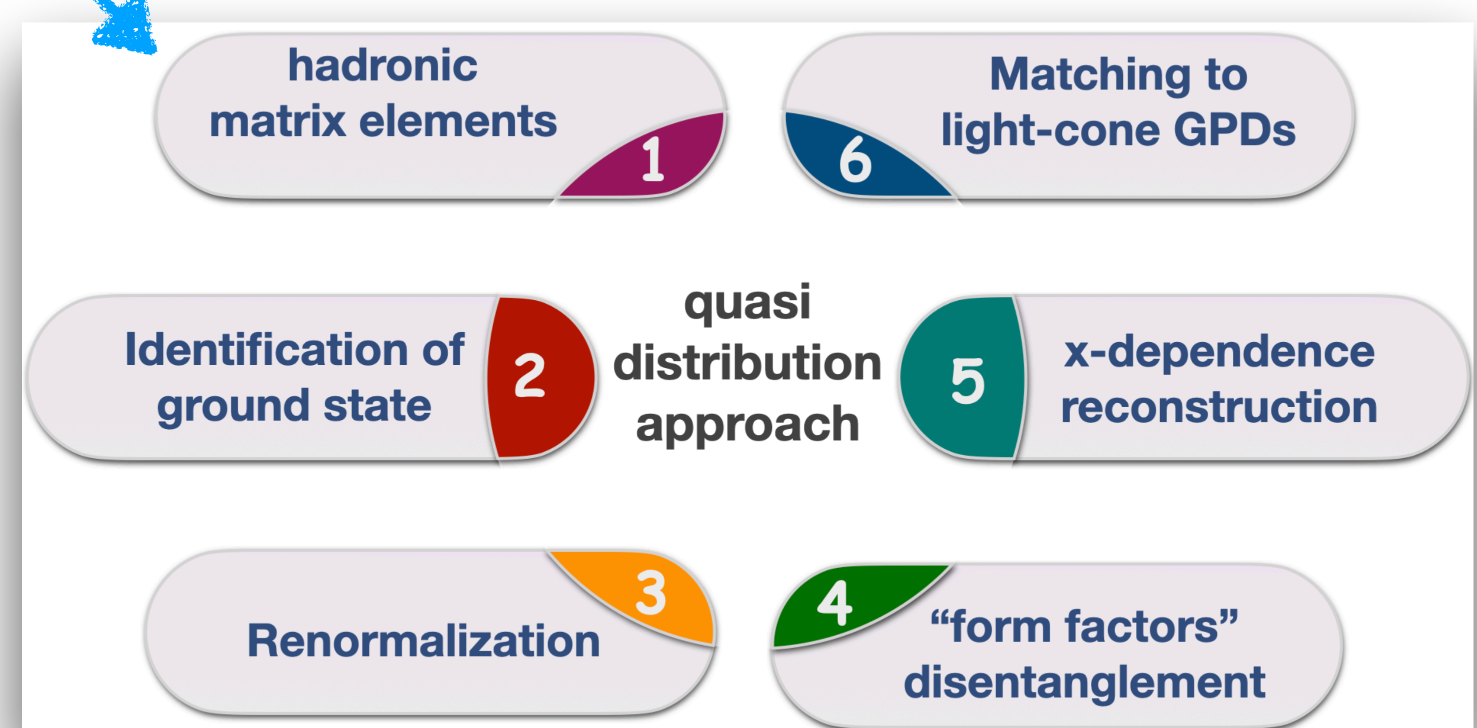

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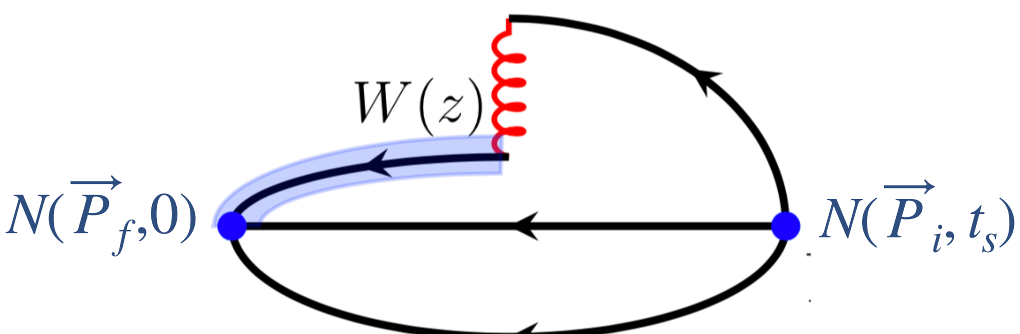

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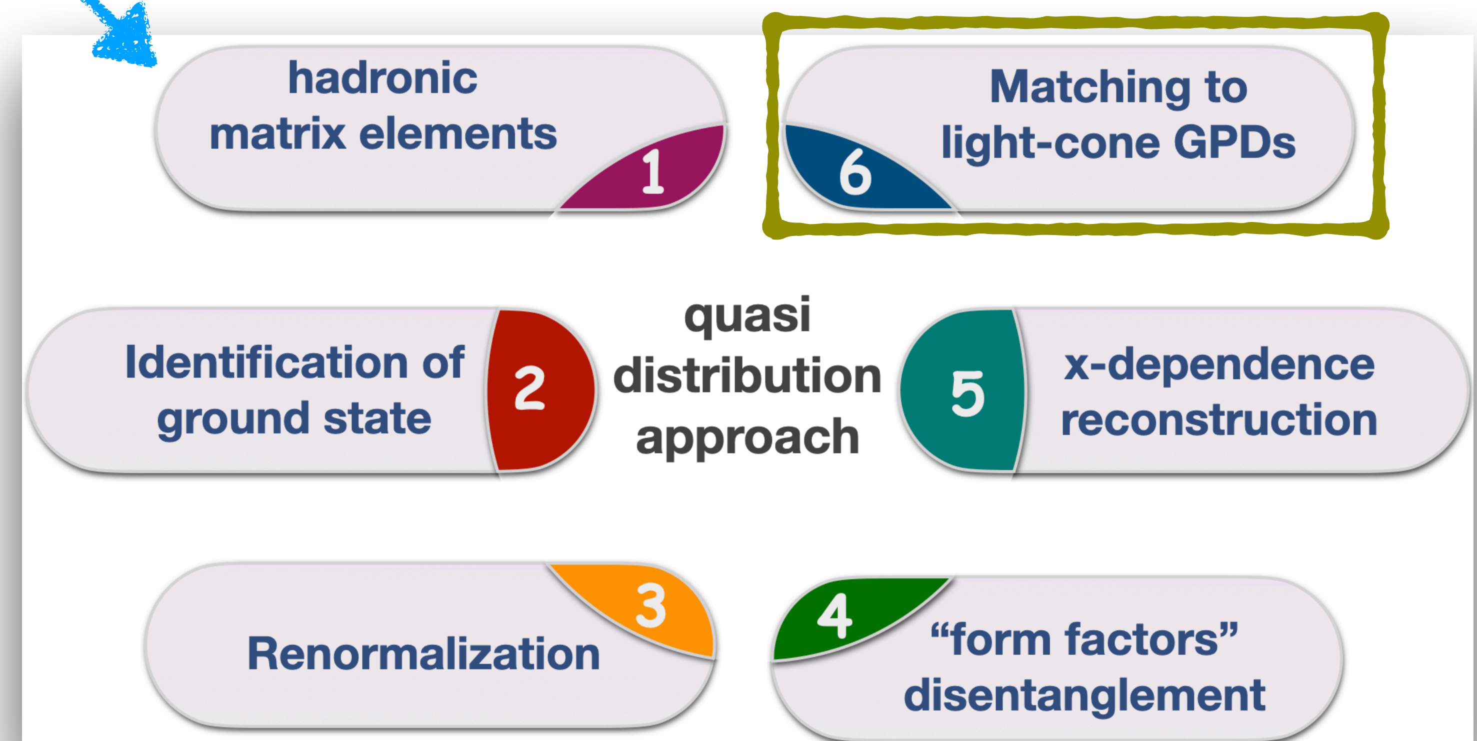

Transversity PDFs

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Parameters of calculations

★ Nf=2 twisted mass fermions with a clover term @ physical point

[Extended Twisted Mass Collaboration, Phys. Rev. D 95, 094515 (2017)]

$\beta=2.10,\ c_{SW}=1.57751,\ a=0.0938(3)(2)$ fm, $r_0/a=5.32(5)$	
$48^3\times 96,\ L\approx 4.5$ fm	$a\mu=0.0009$
	$m_\pi=0.1304(4)$ GeV
	$m_\pi L=2.98$
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Ins.	$N_{\rm conf}$	$N_{\rm HP}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm HP}$	$N_{\rm LP}$	$N_{\rm meas}$	Ins.	$N_{\rm conf}$	$N_{\rm HP}$	$N_{\rm LP}$	$N_{\rm meas}$
γ^3	100	16	9600	γ^3	425	1	16	38250	γ^3	811	1	16	72990
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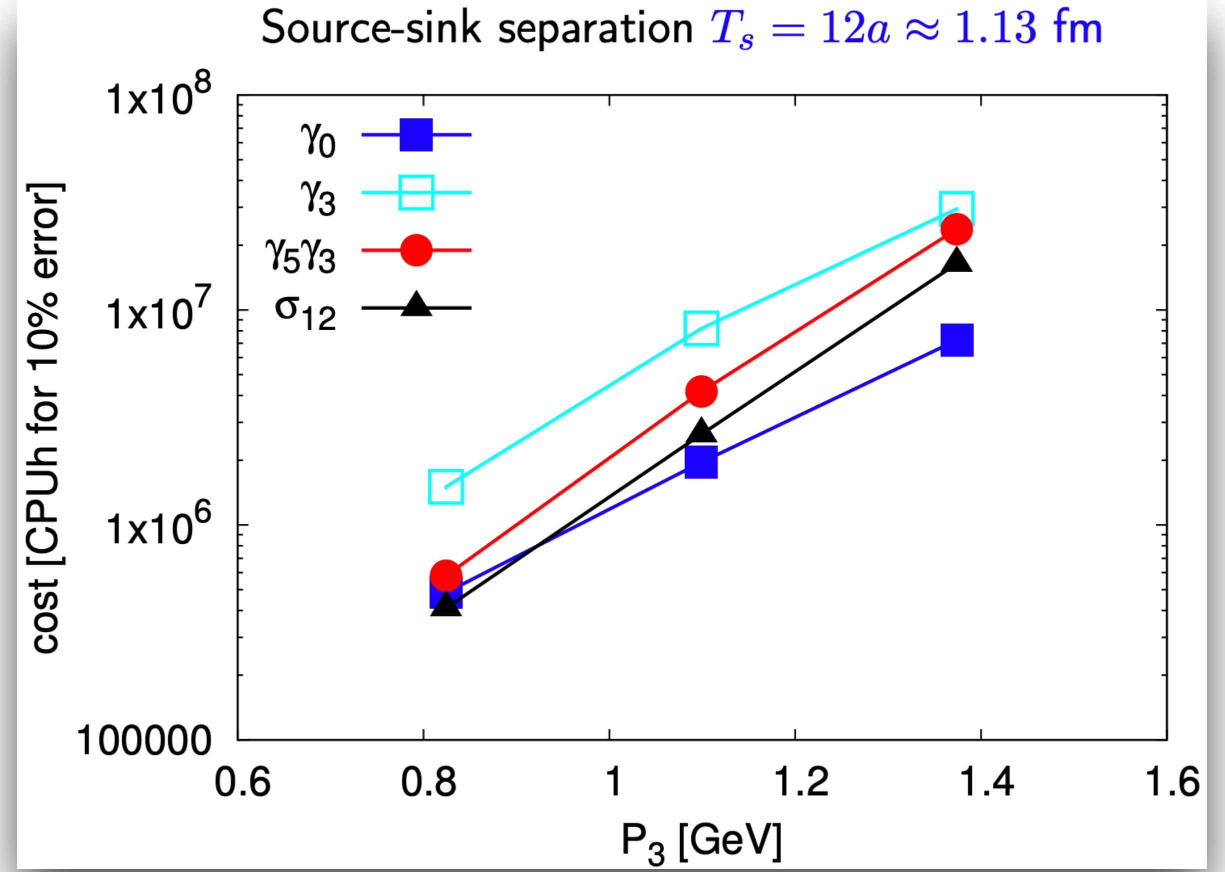

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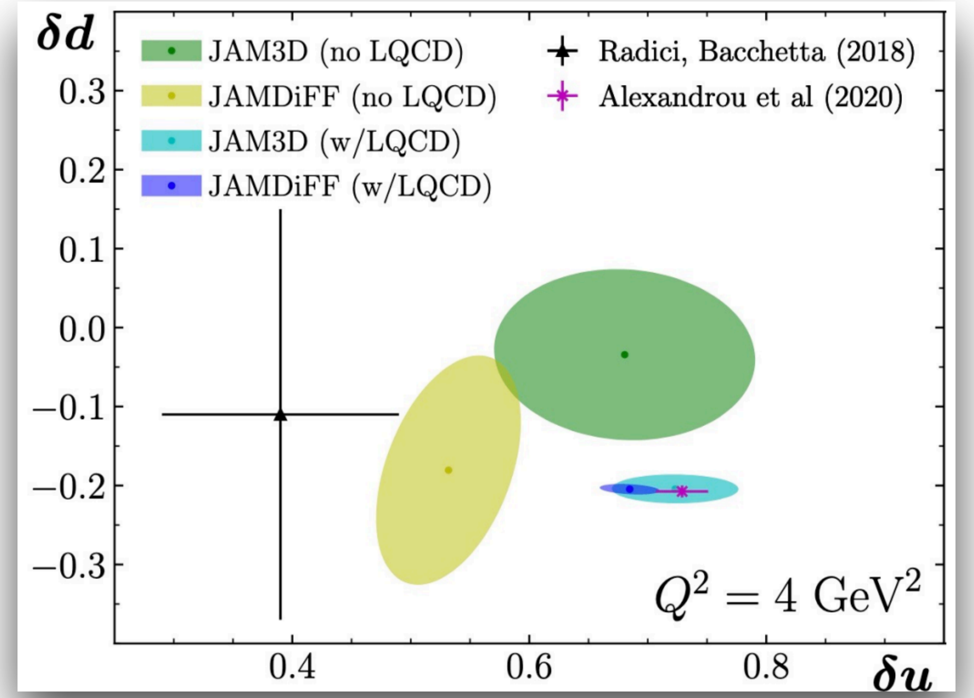
Transversity PDF



[Daniel Pitonyak, GHP 2023]

★ Opportunity for lattice QCD to complement global analysis

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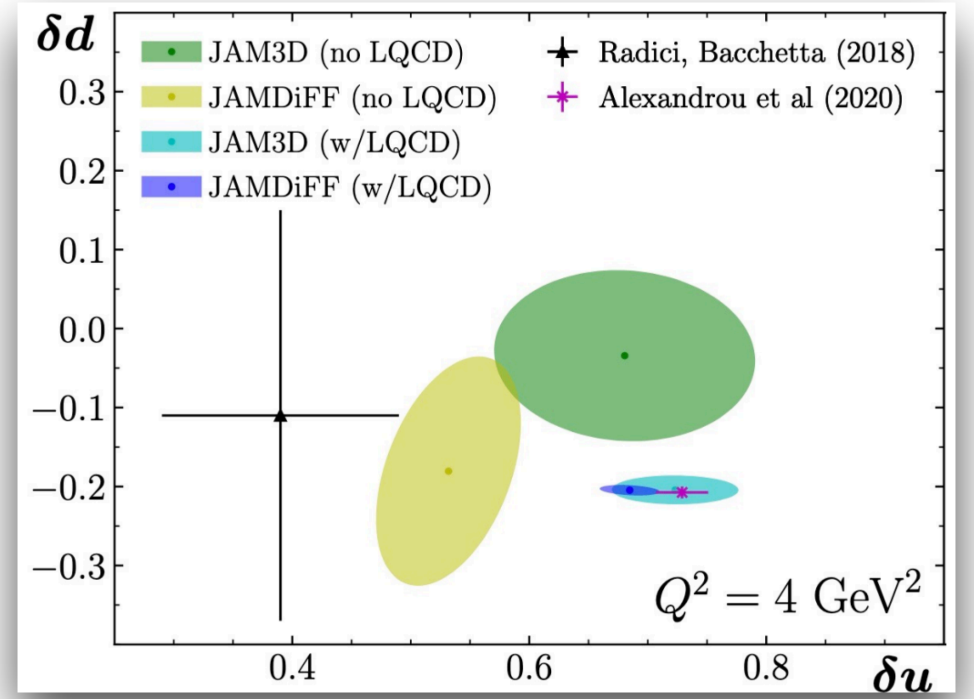

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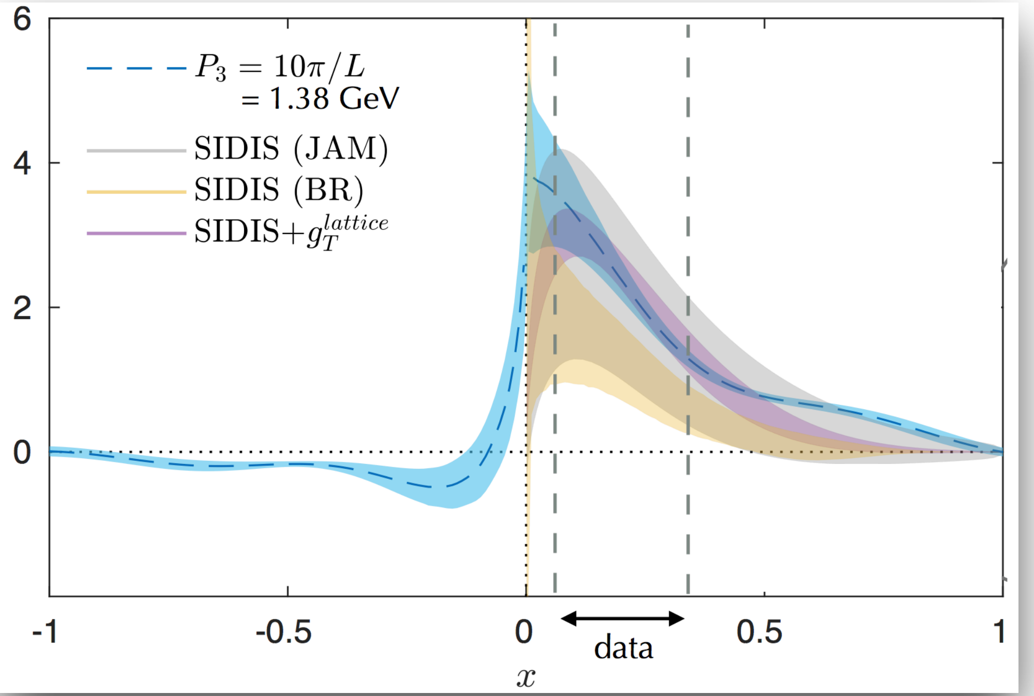

[C. Alexandrou et al. (ETMC), PRD 98 (2018) 091503 (R)]

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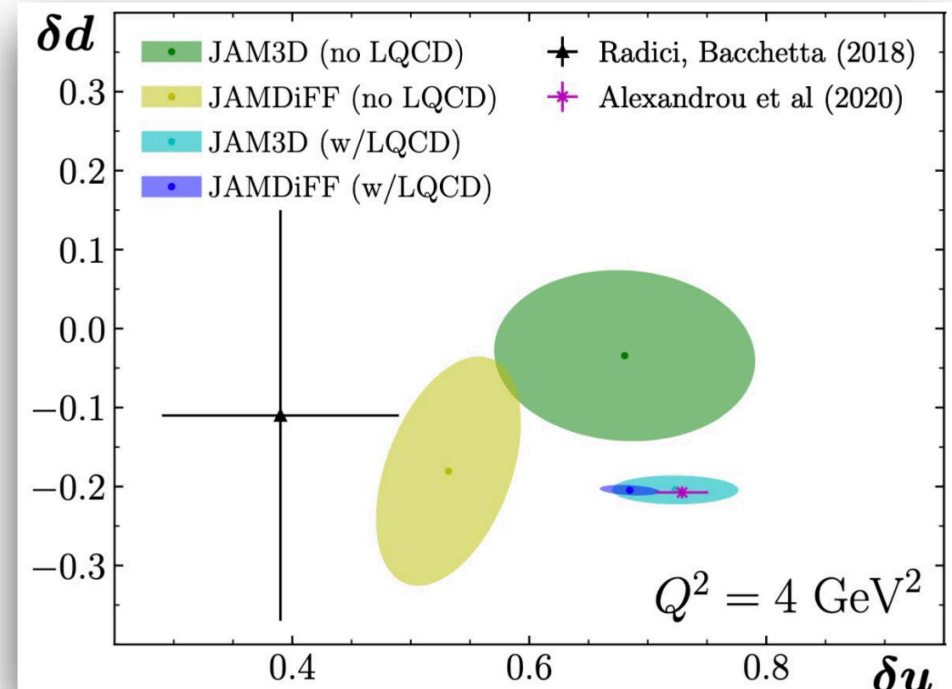

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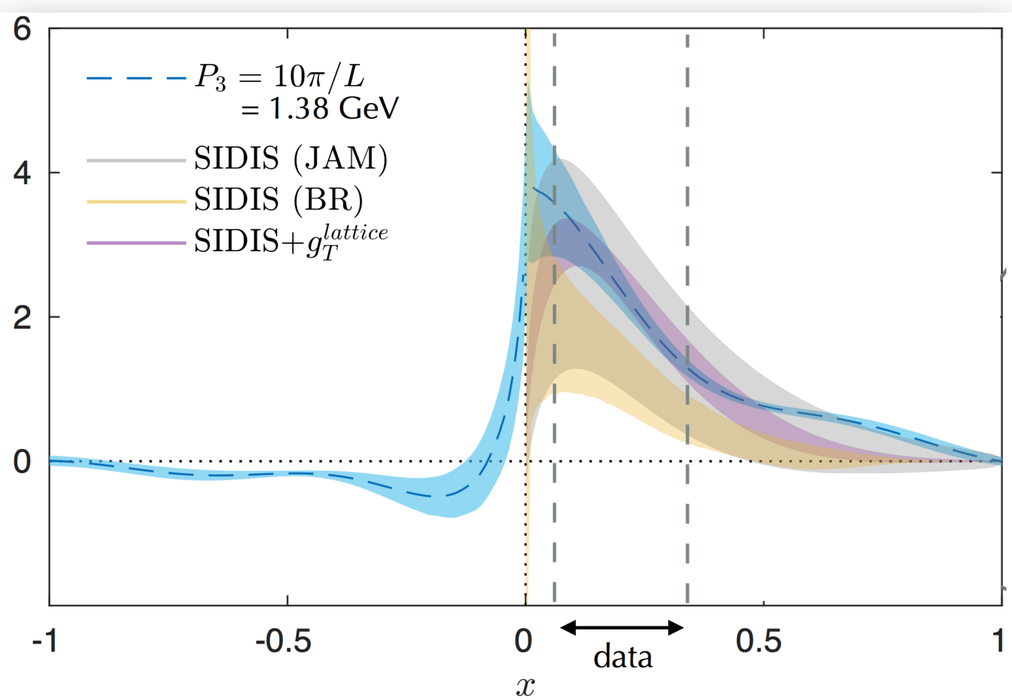
- ★ Field moves towards addressing systematic uncertainties
- ★ Disconnected contributions for flavor decomposition



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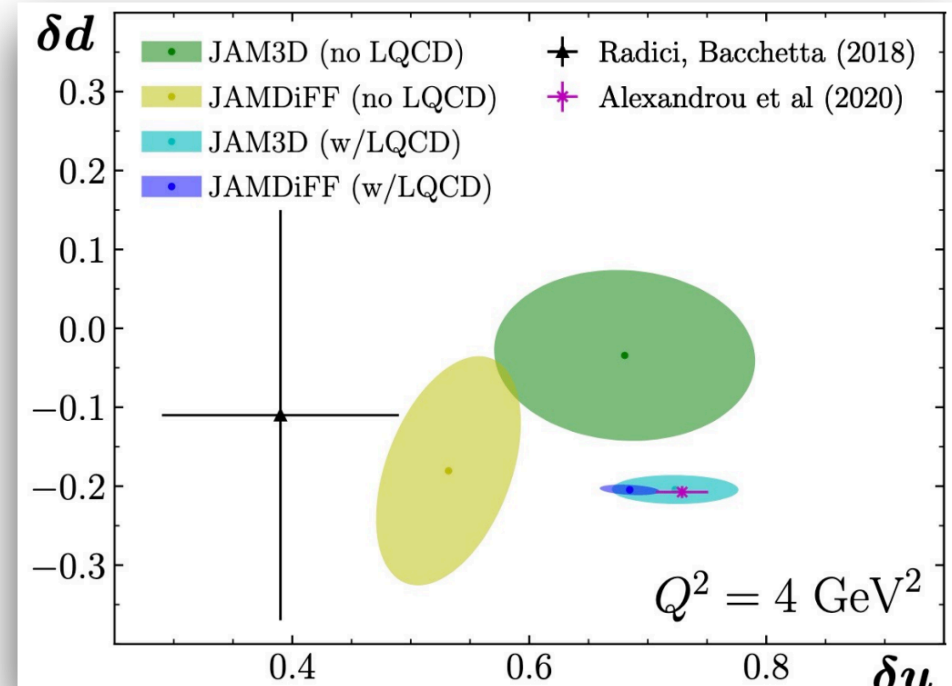

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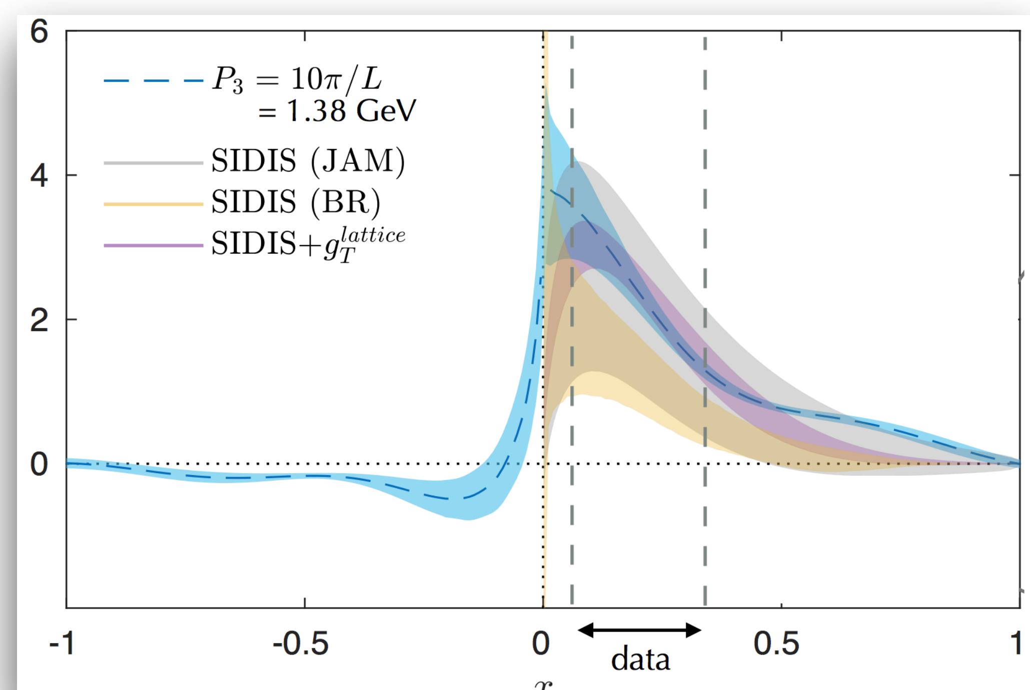

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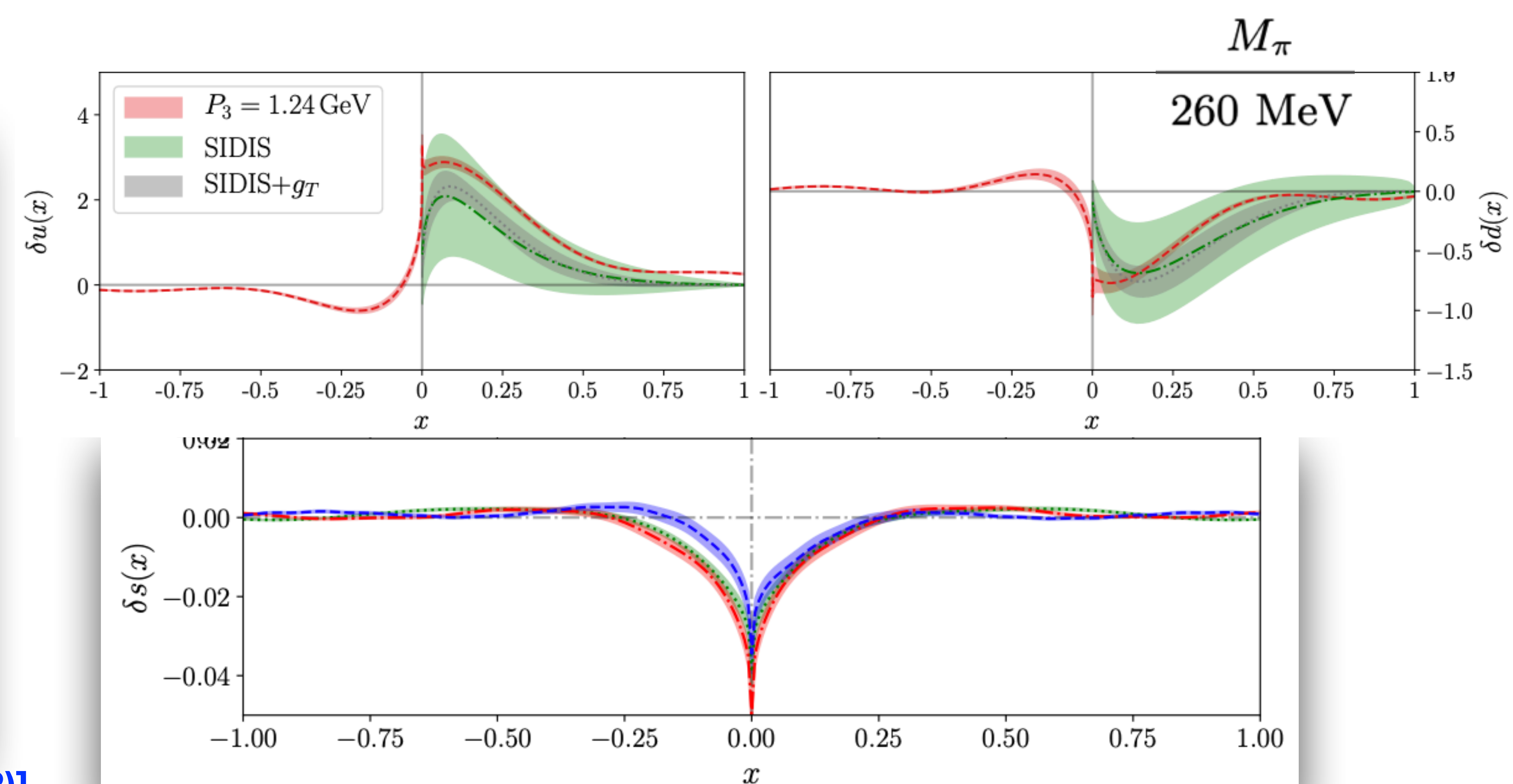

[C. Alexandrou et al., (ETMC), PRD 104 (2021) 5, 054503]

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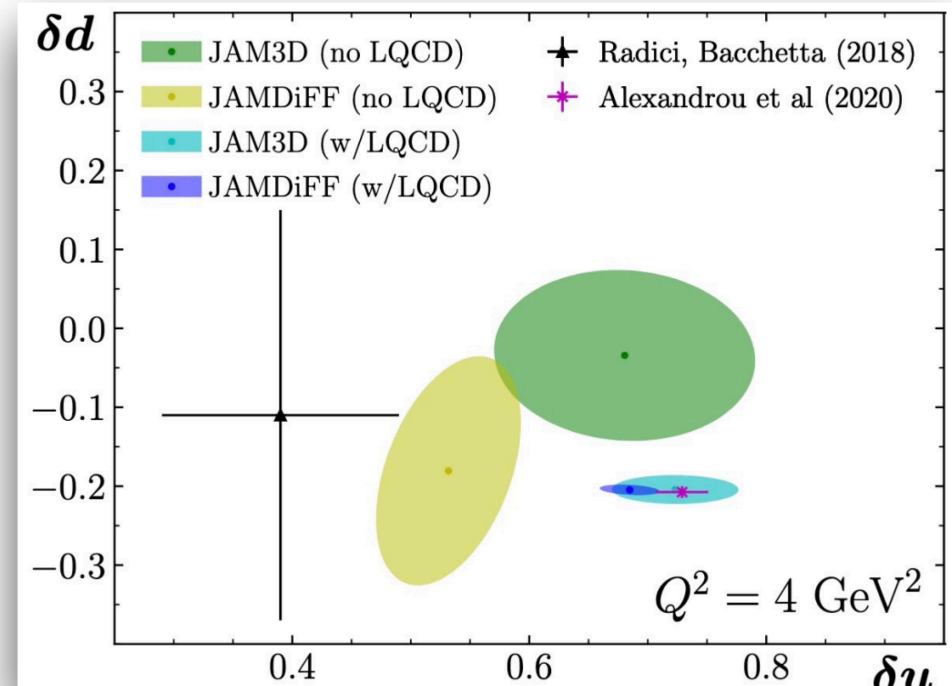

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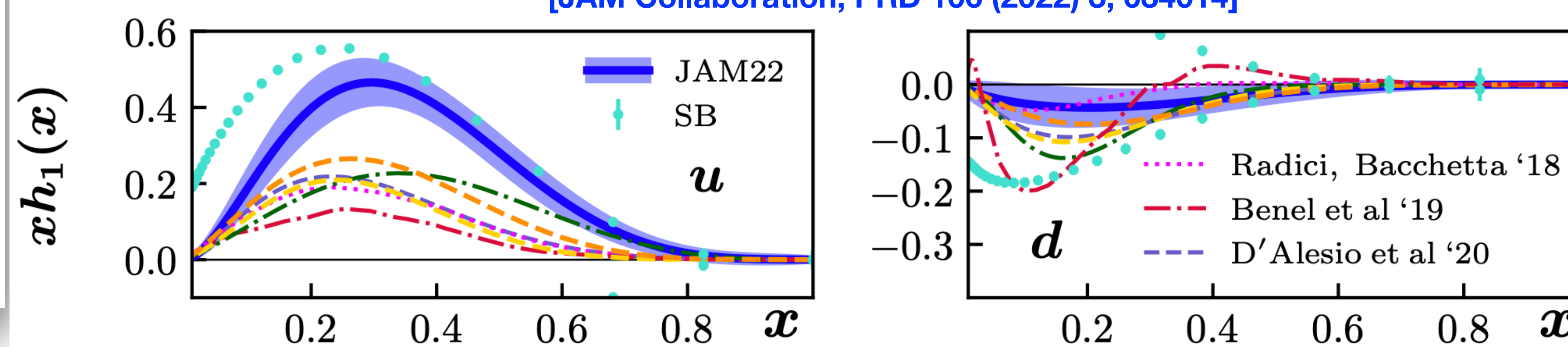
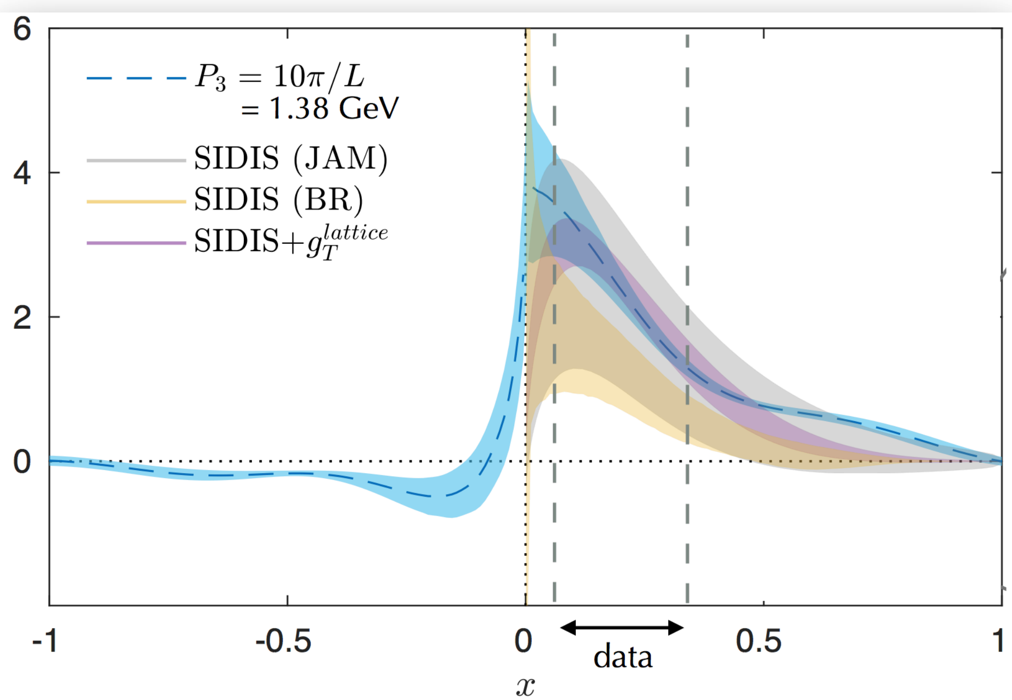
[JAM Collaboration, PRD 106 (2022) 3, 034014]





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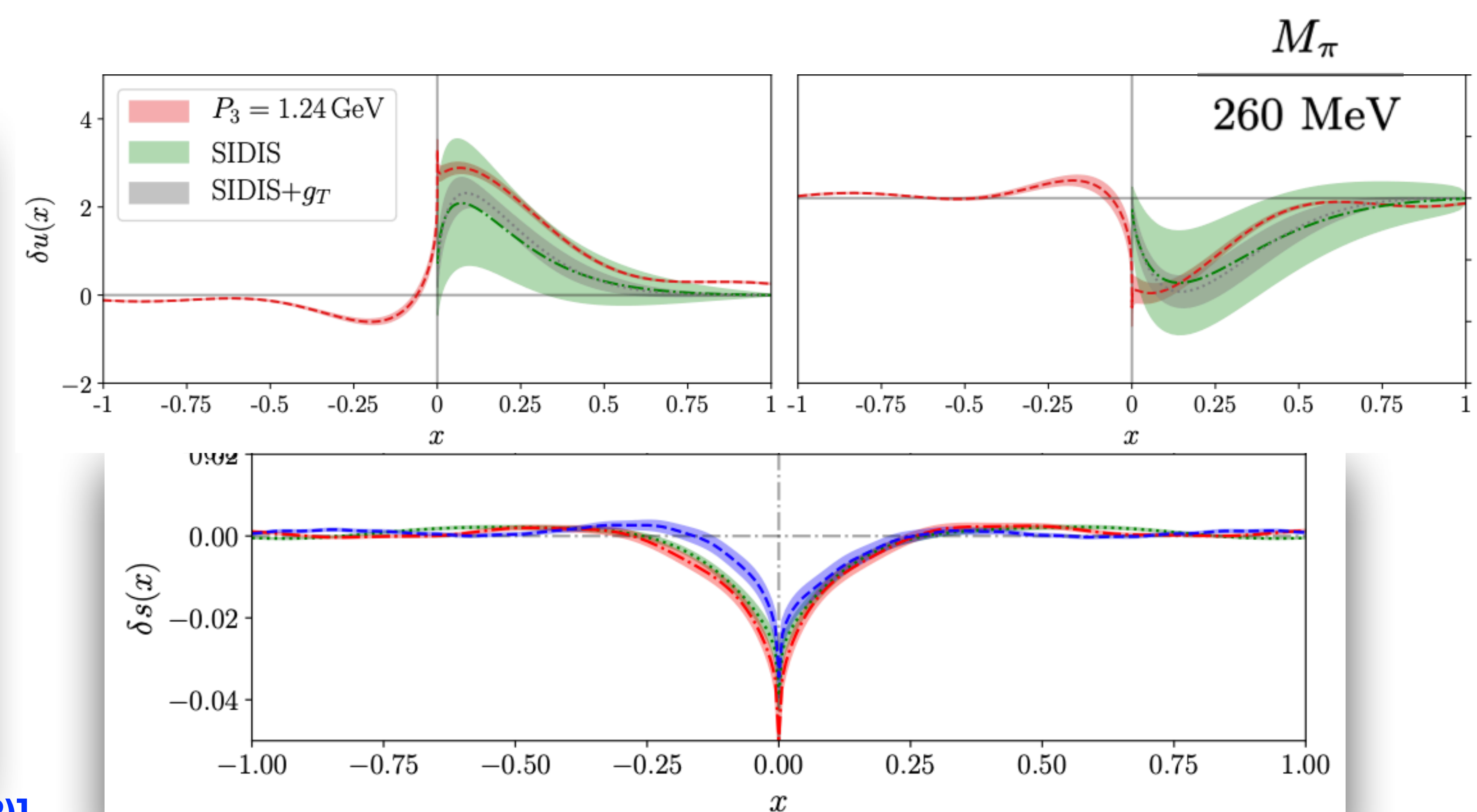

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Transversity GPDs

PHYSICAL REVIEW D **105,** 034501 (2022)

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★ Nf=2+1+1 twisted mass fermions with a clover term;

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cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm confs}$	$N_{\rm meas}$
0.83	(0,2,0)	0.69	0	519	4152
1.25	(0,2,0)	0.69	0	1315	42080
1.67	(0,2,0)	0.69	0	1753	112192
1.25	(0,2,2)	1.02	1/3	417	40032
1.25	(0,2,-2)	1.02	-1/3	417	40032

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P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm confs}$	$N_{\rm meas}$
0.83	(0,2,0)	0.69	0	519	4152
1.25	(0,2,0)	0.69	0	1315	42080
1.67	(0,2,0)	0.69	0	1753	112192
1.25	(0,2,2)	1.02	1/3	417	40032
1.25	(0,2,-2)	1.02	-1/3	417	40032

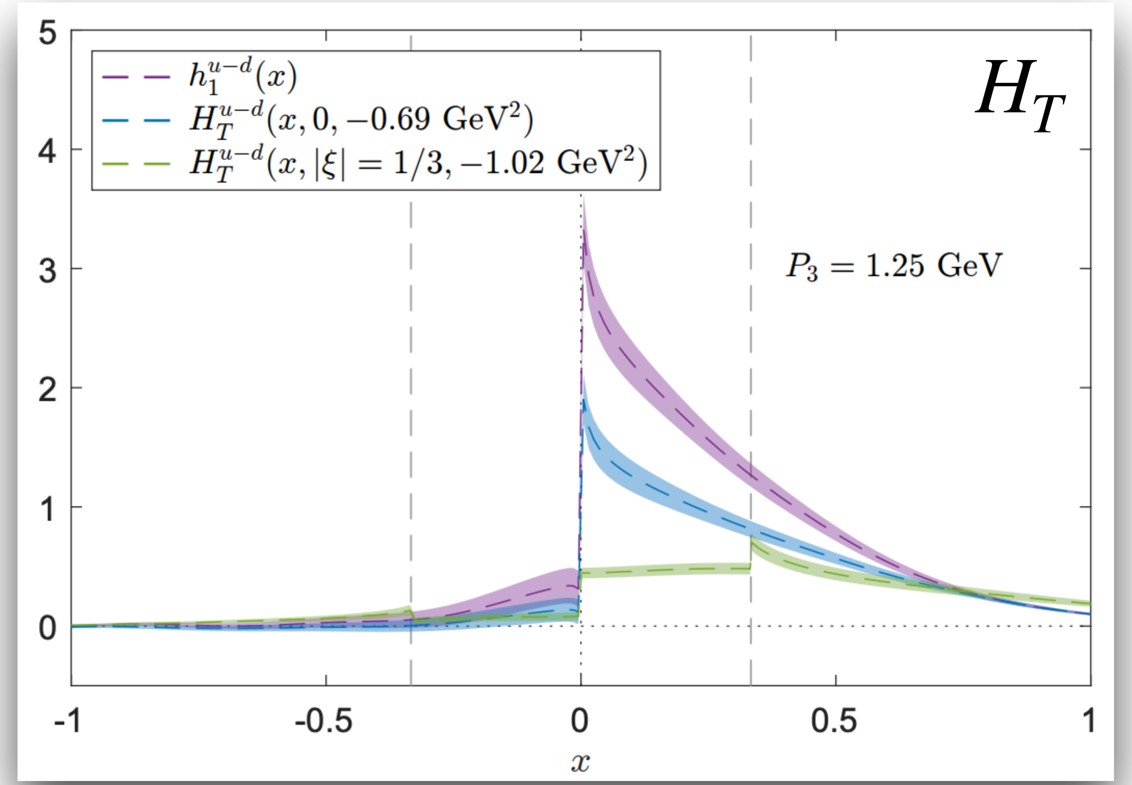
★ Parametrization [Meissner et al., *JHEP* 08 (2009) 056]

$$h_T^j(\Gamma_\nu, z, P_f, P_i) = \langle\langle \sigma^{3j} \rangle\rangle F_{H_T}(z,\xi,t,P_3) + \frac{\imath}{2m}\langle\langle \gamma^3\Delta_j - \gamma^j\Delta_3 \rangle\rangle F_{E_T}(z,\xi,t,P_3)$$

$$+ \frac{P_3\Delta_j - P_j\Delta_3}{m^2}\langle\langle \hat{1} \rangle\rangle F_{\widetilde{H}_T}(z,\xi,t,P_3) + \frac{1}{m}\langle\langle \gamma^3 P_j - \gamma^j P_3 \rangle\rangle F_{\widetilde{E}_T}(z,\xi,t,P_3)$$

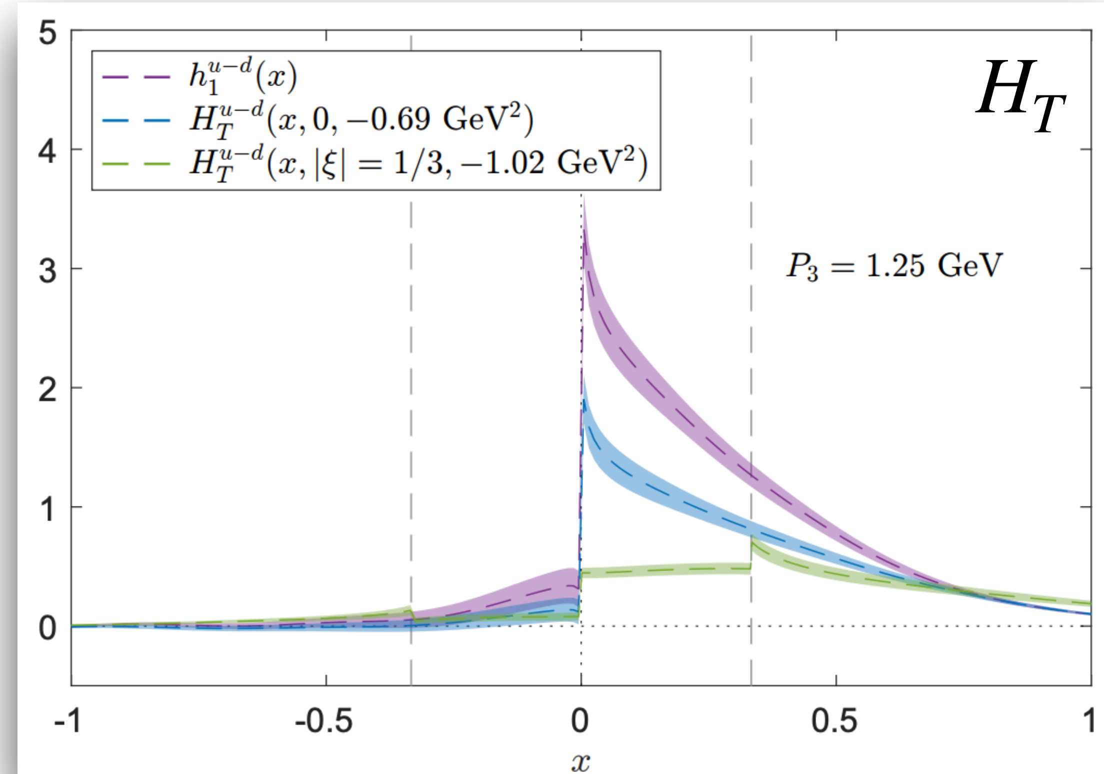
x-dependent transversity GPDs

x-dependent transversity GPDs



- ★ ERBL: decrease of H_T as $-t$ increases
- ★ DGLAP region shows less sensitivity in $-t$

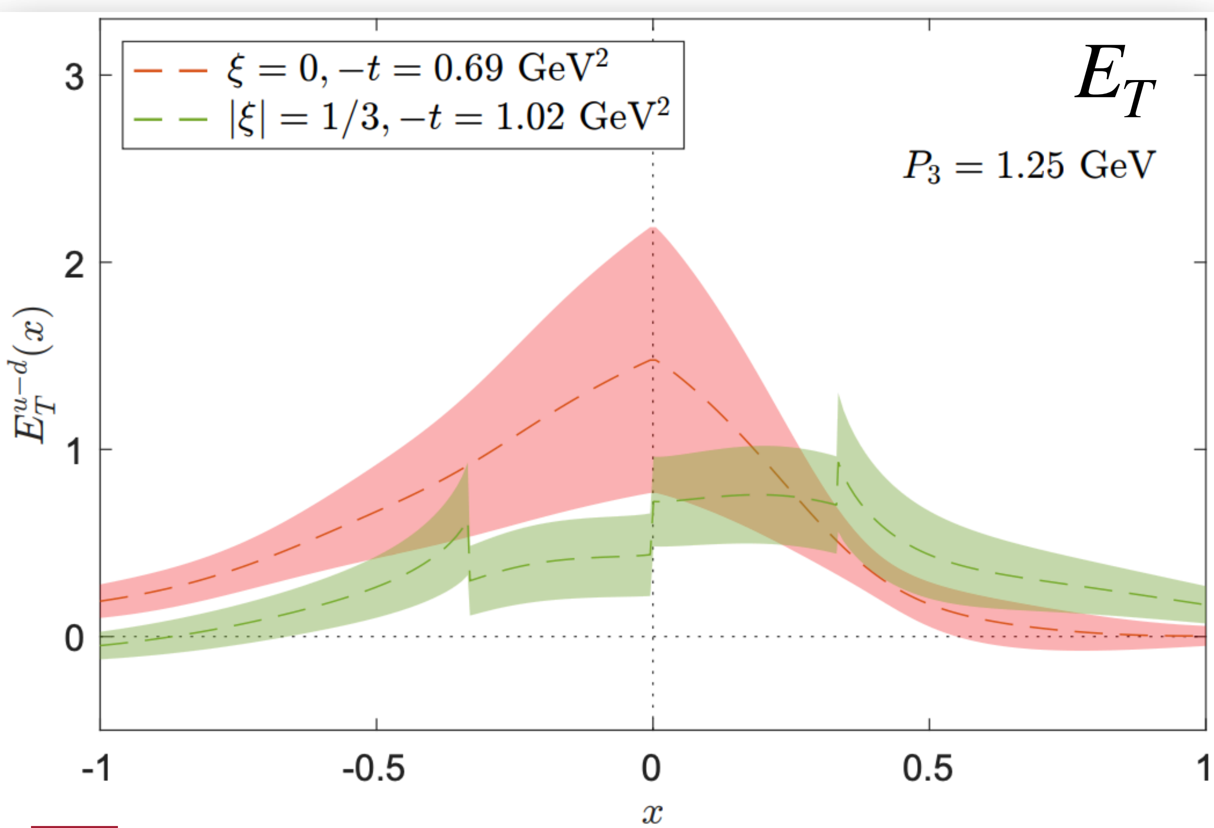
x-dependent transversity GPDs

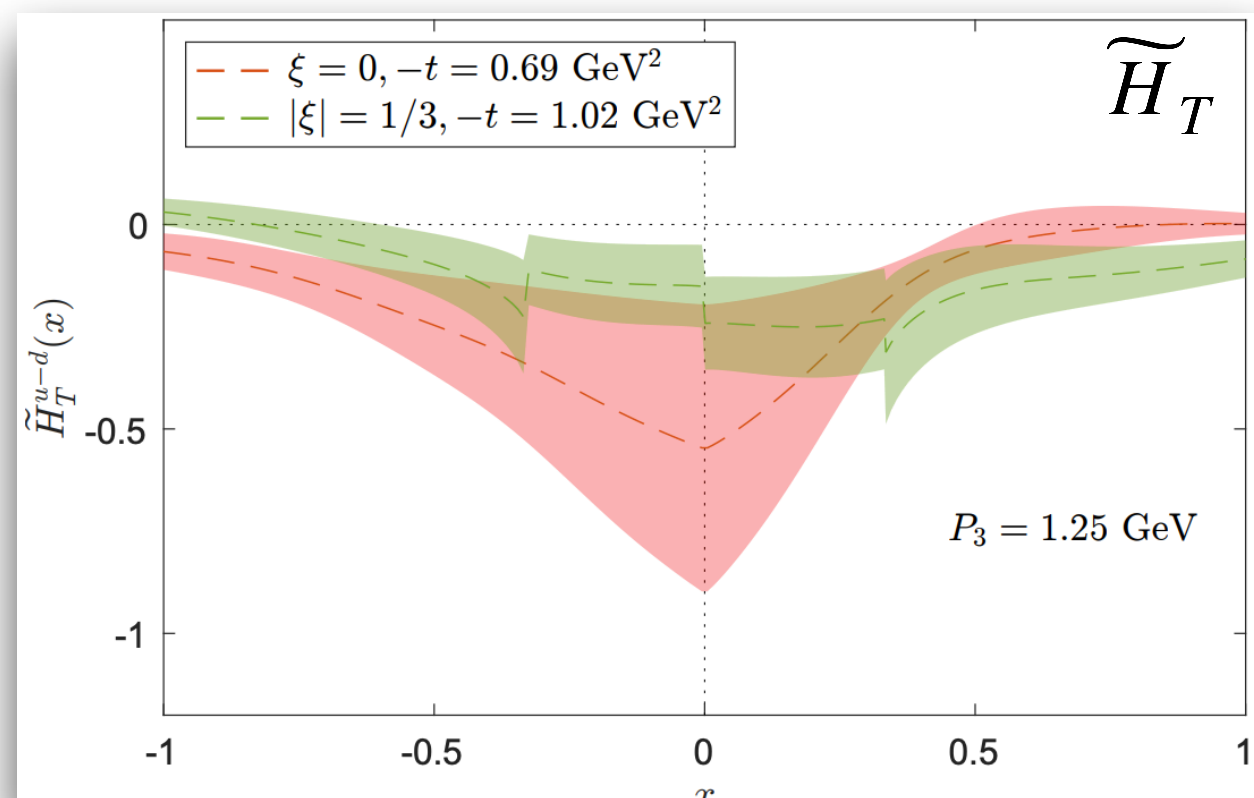


- ★ ERBL: decrease of H_T as $-t$ increases
- ★ DGLAP region shows less sensitivity in $-t$

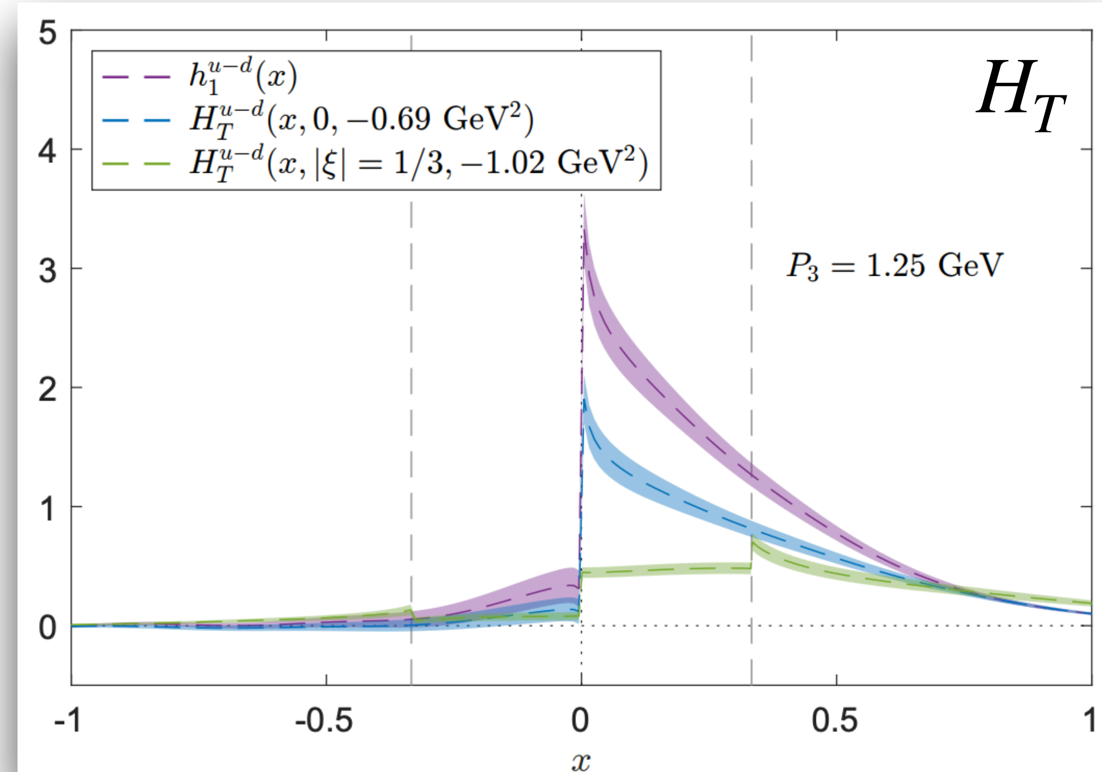
- ★ Qualitatively E_T, $\widetilde{H}_T$ show approximate symmetry of quark & antiquark regions
- ★ $\widetilde{H}_T < 0$ as in scalar diquark model

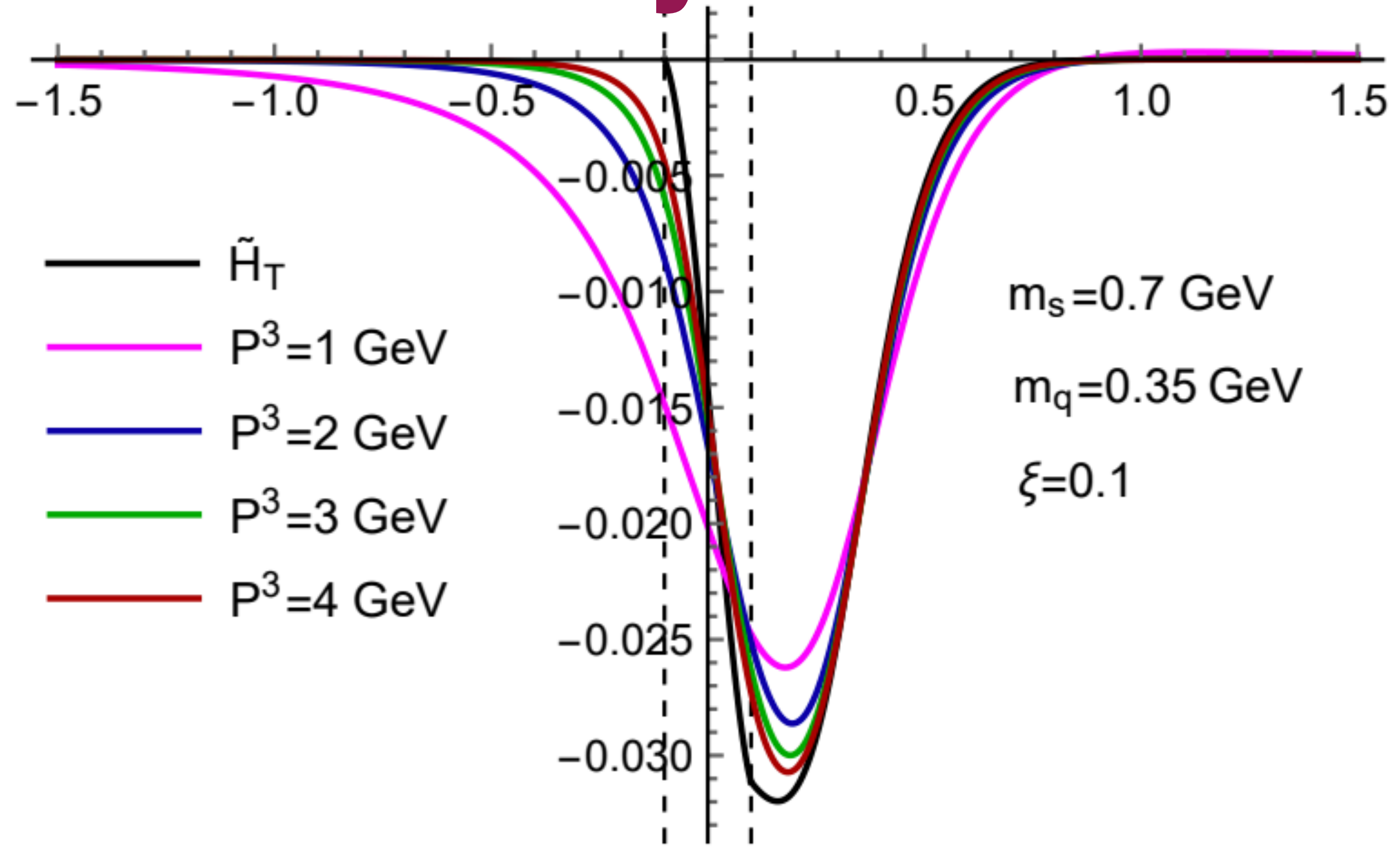
[S. Bhattacharya et al, Phys. Rev. D 102, 054021 (2020)]



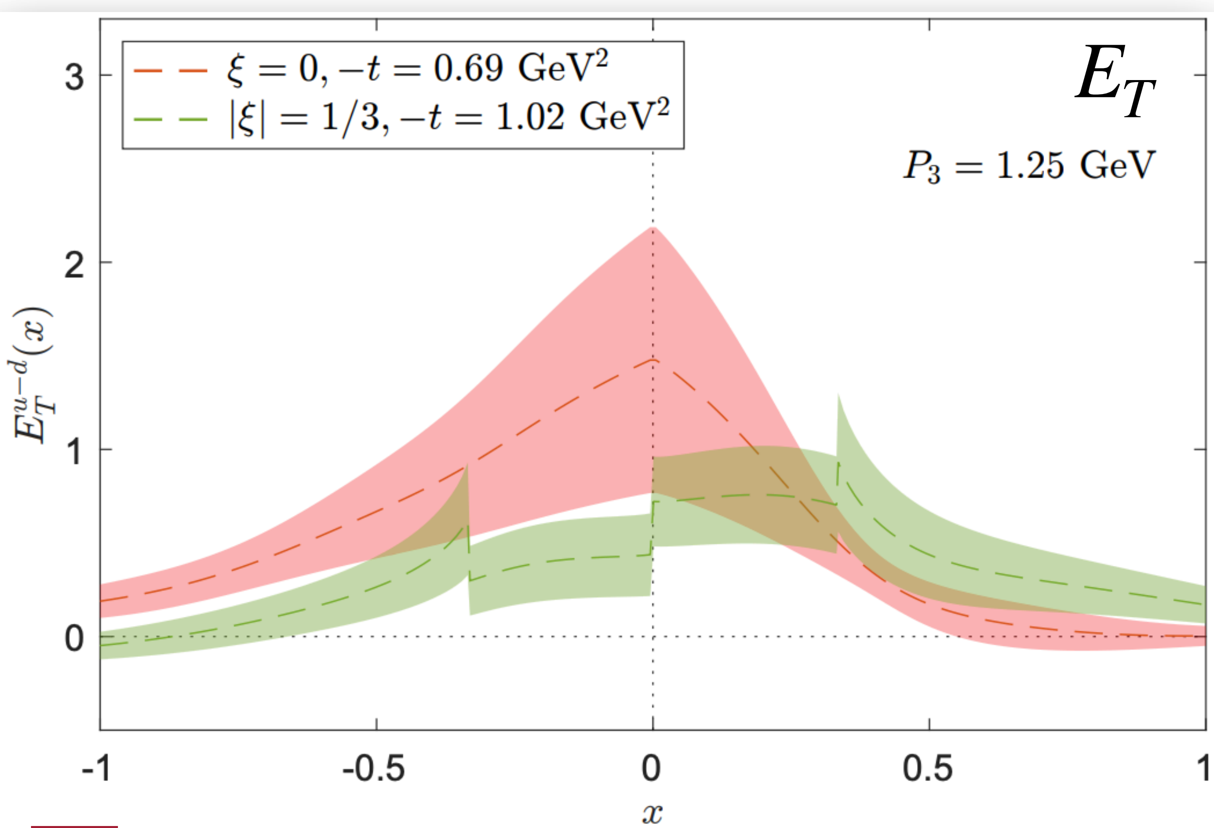


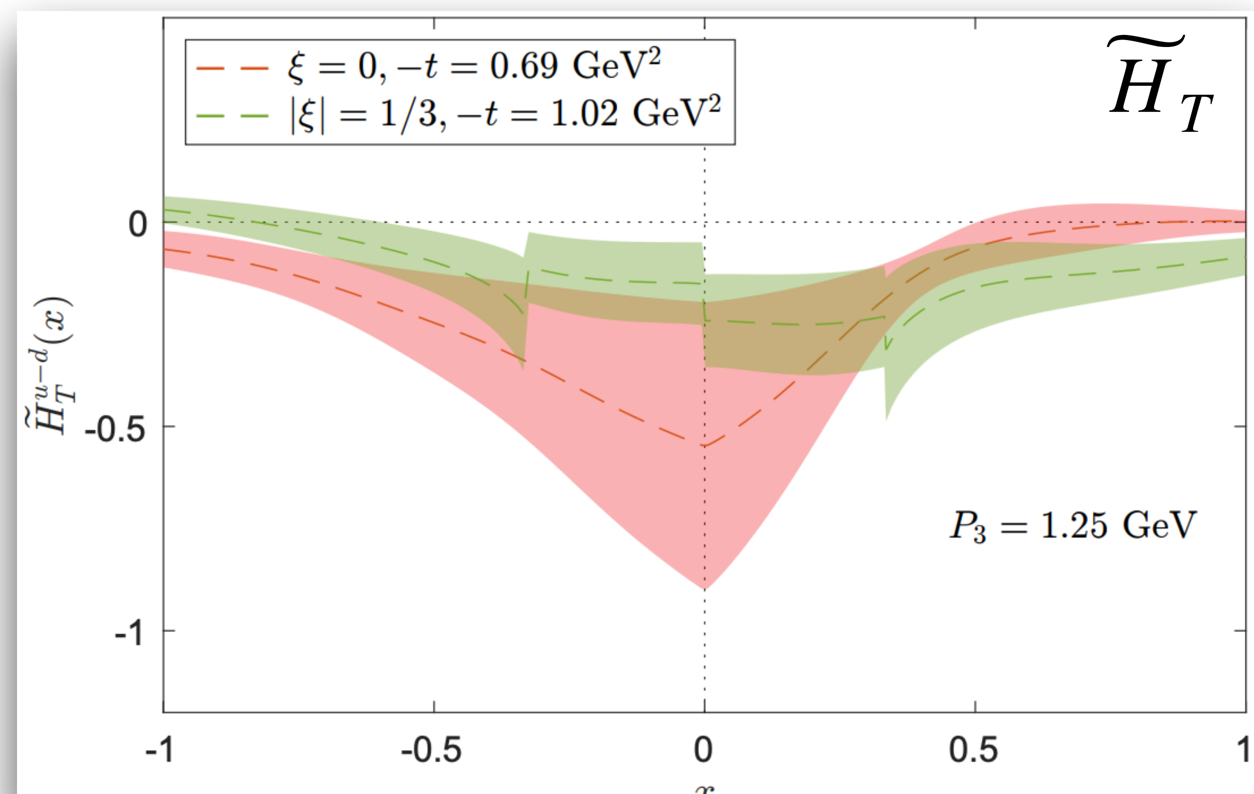
x-dependent transversity GPDs





[S. Bhattacharya et al, Phys. Rev. D 102, 054021 (2020)]





x-dependent transversity GPDs

- ★ $[2\widetilde{H}_T + E_T](0)$ lowest Mellin moment:
  transverse spin-flavor dipole moment in unpolarized target k_T

[M. Burkardt, Phys. Rev. D 72, 094020 (2005)]

- ★ $[2\widetilde{H}_T + E_T](0)$ second Mellin moment:
  related to transverse-spin quark angular momentum in unpolarized proton

x-dependent transversity GPDs

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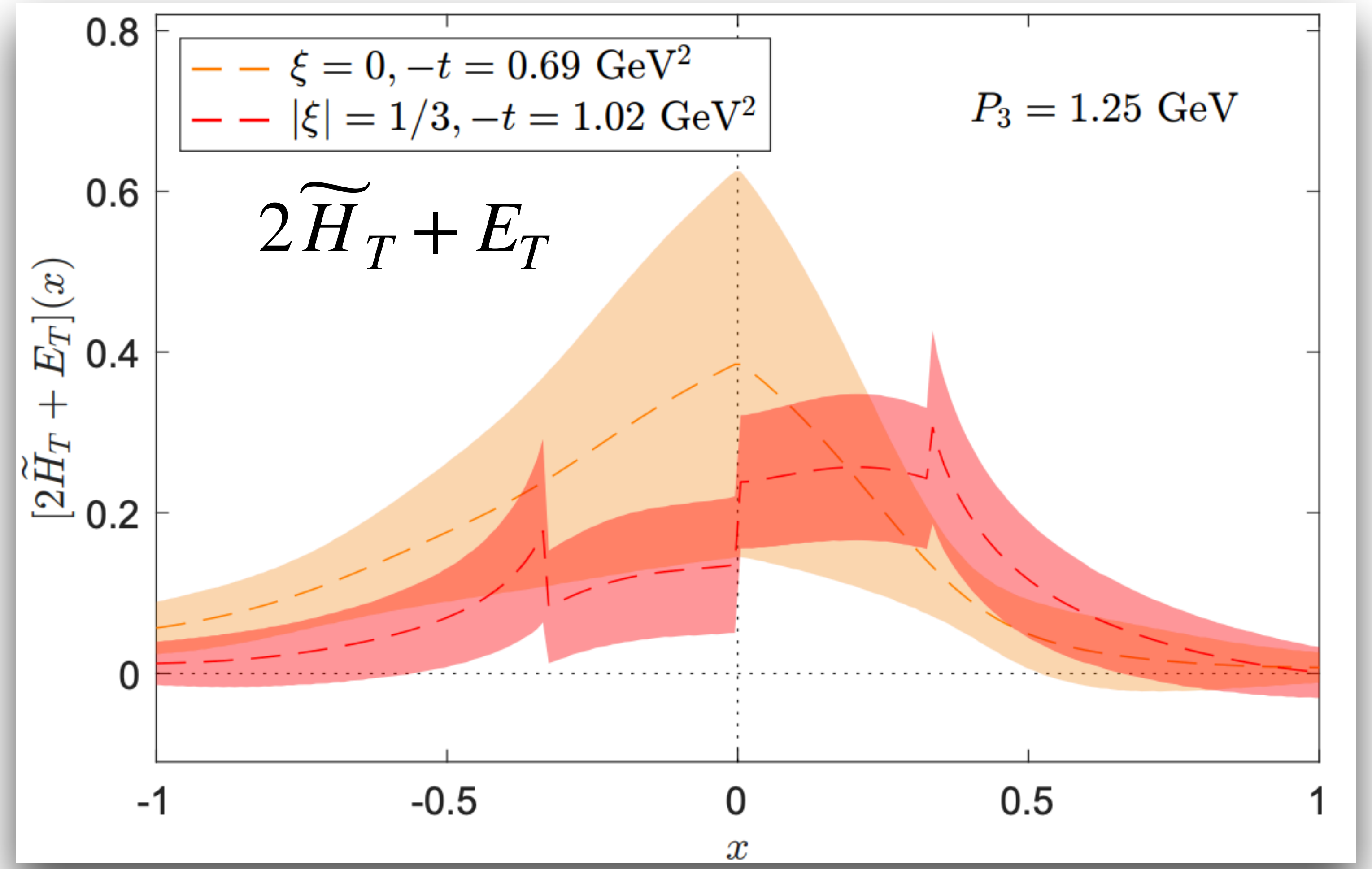
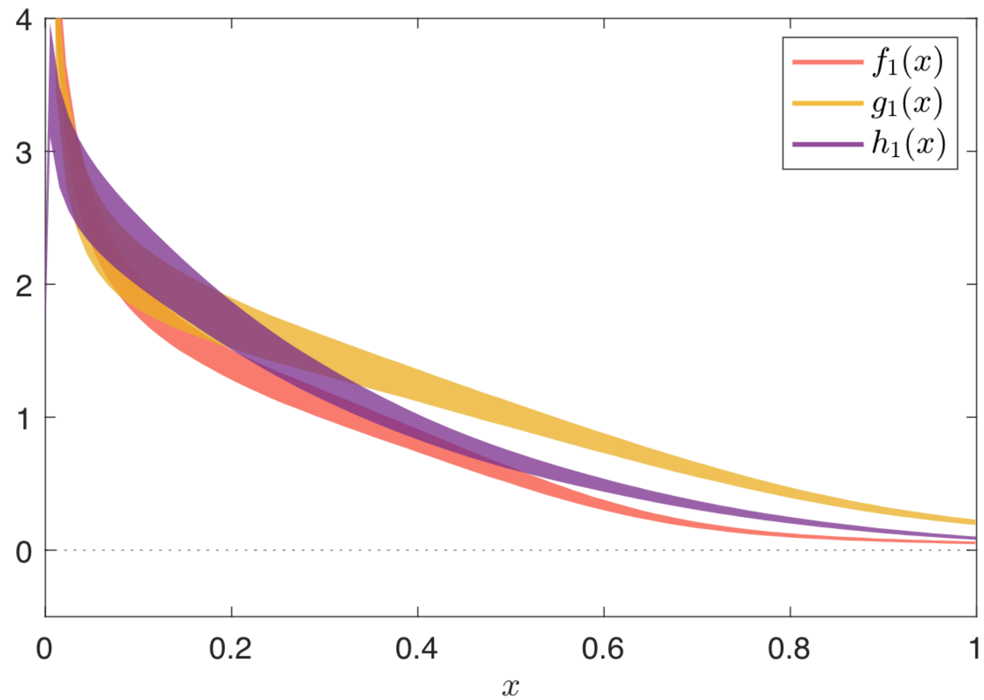


★ distribution for |ξ| = 1/3 tends to be systematically lower than the one at ξ = 0

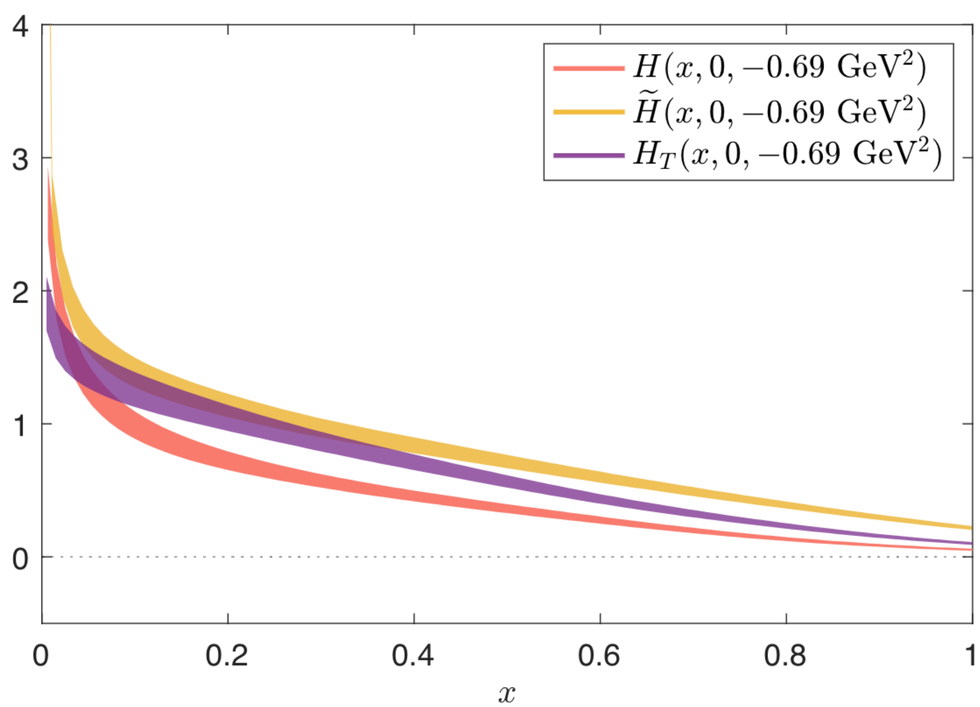
x-dependent transversity GPDs

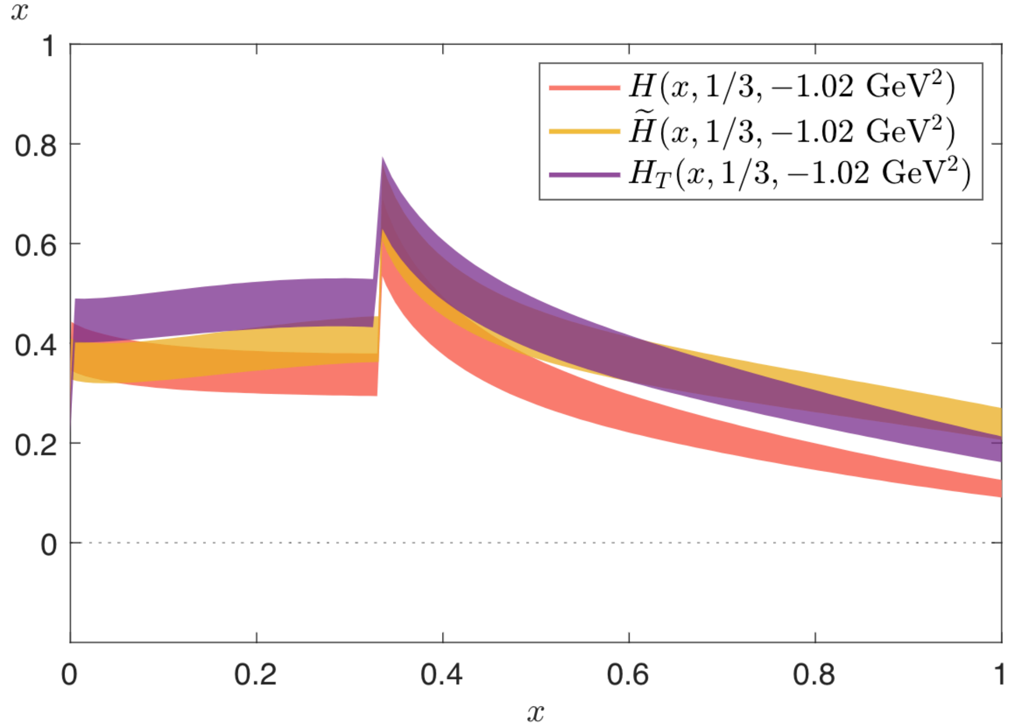
★ Qualitative understanding of GPDs and their relations

★ Qualitative understanding of ERBL and DGLAP regions



★ Relations can be identified for the t-dependence of GPDs





What can we currently check using lattice results?

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Adress system. effects through sum rules

[S. Bhattacharya et al., PRD 102, 054021 (2020)]

Sum rules not imposed in calculation

$$\int_{-1}^{1} dx\, H_T(x,\xi,t) = \int_{-\infty}^{\infty} dx\, H_{Tq}(x,\xi,t,P_3) = A_{T10}(t)\,, \qquad \int_{-1}^{1} dx\, x\, H_T(x,\xi,t) = A_{T20}(t)\,,$$

$$\int_{-1}^{1} dx\, E_T(x,\xi,t) = \int_{-\infty}^{\infty} dx\, E_{Tq}(x,\xi,t,P_3) = B_{T10}(t)\,, \qquad \int_{-1}^{1} dx\, x\, E_T(x,\xi,t) = B_{T20}(t)\,,$$

$$\int_{-1}^{1} dx\, \widetilde{H}_T(x,\xi,t) = \int_{-\infty}^{\infty} dx\, \widetilde{H}_{Tq}(x,\xi,t,P_3) = \widetilde{A}_{T10}(t)\,, \qquad \int_{-1}^{1} dx\, x\, \widetilde{H}_T(x,\xi,t) = \widetilde{A}_{T20}(t)\,,$$

$$\int_{-1}^{1} dx\, \widetilde{E}_T(x,\xi,t) = \int_{-\infty}^{\infty} dx\, \widetilde{E}_{Tq}(x,\xi,t,P_3) = 0\,. \qquad \int_{-1}^{1} dx\, x\, \widetilde{E}_T(x,\xi,t) = 2\xi \widetilde{B}_{T21}(t)\,.$$

What can we currently check using lattice results?

Adress system. effects through sum rules

[S. Bhattacharya et al., PRD 102, 054021 (2020)]

Sum rules not imposed in calculation

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★ Results

$$\int_{-2}^{2} dx H_{Tq}(x,0,-0.69\,\mathrm{GeV}^2,P_3) = \{0.65(4),\, 0.64(6),\, 0.81(10)\}\,, \qquad \int_{-2}^{2} dx H_{Tq}(x,\tfrac{1}{3},-1.02\,\mathrm{GeV}^2,1.25\,\mathrm{GeV}) = 0.49(5)\,,$$

$$\int_{-1}^{1} dx\, H_T(x,0,-0.69\,\mathrm{GeV}^2) = \{0.69(4),\, 0.67(6),\, 0.84(10)\}\,, \qquad \int_{-1}^{1} dx\, H_T(x,\tfrac{1}{3},-1.02\,\mathrm{GeV}^2) = 0.45(4)\,,$$

$$\int_{-1}^{1} dx\, x\, H_T(x,0,-0.69\,\mathrm{GeV}^2) = \{0.20(2),\, 0.21(2),\, 0.24(3)\}\,, \qquad \int_{-1}^{1} dx\, x\, H_T(x,\tfrac{1}{3},-1.02\,\mathrm{GeV}^2) = 0.15(2)\,.$$

$$A_{T10}(-0.69\,\mathrm{GeV}^2) = \{0.65(4)\,, 0.65(6)\,, 0.82(10)\}\,, \qquad A_{T10}(-1.02\,\mathrm{GeV}^2) = 0.49(5)$$

- lowest moments the same between quasi-GPDs and GPDs
- Values of moments decrease as t increases
- Higher moments suppressed compared to the lowest

New parametrization of GPDs

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

Parameters of calculations

★ Nf=2+1+1 twisted mass fermions with a clover term;

[Extended Twisted Mass Collaboration, Phys. Rev. D 104, 074515 (2021), arXiv:2104.13408]

Name	β	N_f	$L^3 \times T$	a [fm]	M_π	$m_\pi L$
cA211.32	1.726	u, d, s, c	$32^3 \times 64$	0.093	260 MeV	4

frame	P_3 [GeV]	$\mathbf{\Delta}$ $[\frac{2\pi}{L}]$	$-t$ [GeV^2]	ξ	$N_{\rm ME}$	$N_{\rm confs}$	$N_{\rm src}$	$N_{\rm tot}$
N/A	±1.25	(0,0,0)	0	0	2	731	16	23392
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
symm	±1.67	(±2,0,0), (0,±2,0)	0.69	0	8	294	32	75264
symm	±1.25	(±2, ±2, 0)	1.39	0	16	224	8	28672
symm	±1.25	(±4,0,0), (0,±4,0)	2.76	0	8	329	32	84224
asymm	±1.25	(±1,0,0), (0,±1,0)	0.17	0	8	429	8	27456
asymm	±1.25	(±1, ±1, 0)	0.33	0	16	194	8	12416
asymm	±1.25	(±2,0,0), (0,±2,0)	0.64	0	8	429	8	27456
asymm	±1.25	(±1,±2,0), (±2,±1,0)	0.80	0	16	194	8	12416
asymm	±1.25	(±2,±2,0)	1.16	0	16	194	8	24832
asymm	±1.25	(±3,0,0), (0,±3,0)	1.37	0	8	429	8	27456
asymm	±1.25	(±1,±3,0), (±3,±1,0)	1.50	0	16	194	8	12416
asymm	±1.25	(±4,0,0), (0,±4,0)	2.26	0	8	429	8	27456

Symmetric frame very expensive computationally

Theoretical setup

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu M A_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

Goals

(A) A_i are related to the standard H, E GPDs $\quad \mathcal{H}_0^s(A_i^s; z) = A_1 + \frac{z(\Delta_1^2 + \Delta_2^2)}{2P_3}A_6$

(B) Extraction of standard GPDs using A_i obtained from any frame

(C) quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone:

Theoretical setup

$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^\mu}{M}A_1 + z^\mu MA_2 + \frac{\Delta^\mu}{M}A_3 + i\sigma^{\mu z}MA_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{M}A_6 + \frac{z^\mu i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

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(B) Extraction of standard GPDs using A_i obtained from any frame

(C) quasi-GPDs may be redefined (Lorentz covariant) inspired by light-cone:

$$H(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = A_1 + \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3$$

$$E(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2) = -A_1 - \frac{\Delta_{s/a}\cdot z}{P_{avg,s/a}\cdot z}A_3 + 2A_5 + 2P_{avg,s/a}\cdot zA_6 + 2\Delta_{s/a}\cdot zA_8$$

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$$F^{\mu}_{\lambda,\lambda'} = \bar{u}(p',\lambda')\left[\frac{P^{\mu}}{M}A_1 + z^{\mu} M A_2 + \frac{\Delta^{\mu}}{M}A_3 + i\sigma^{\mu z} M A_4 + \frac{i\sigma^{\mu\Delta}}{M}A_5 + \frac{P^{\mu} i\sigma^{z\Delta}}{M}A_6 + \frac{z^{\mu} i\sigma^{z\Delta}}{M}A_7 + \frac{\Delta^{\mu} i\sigma^{z\Delta}}{M}A_8\right]u(p,\lambda)$$

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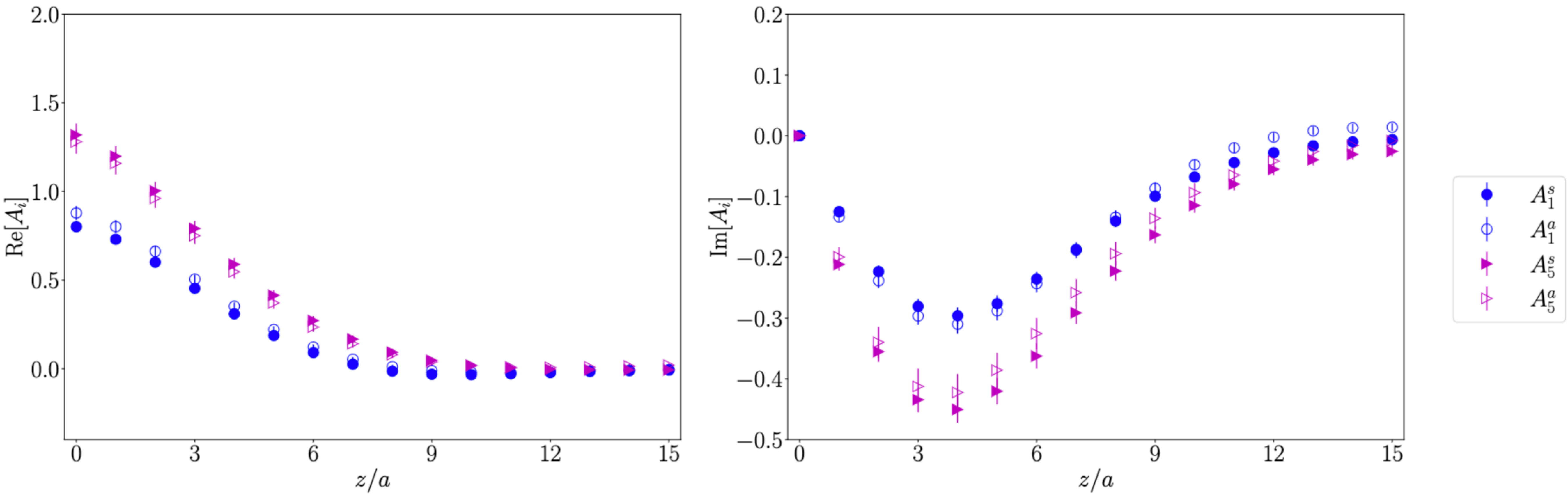
(A) Proof-of-concept calculation ($\xi = 0$):

- symmetric frame: $\vec{p}_f^{\,s} = \vec{P} + \frac{\vec{Q}}{2}, \quad \vec{p}_i^{\,s} = \vec{P} - \frac{\vec{Q}}{2} \qquad -t^s = \vec{Q}^2 = 0.69\, GeV^2$

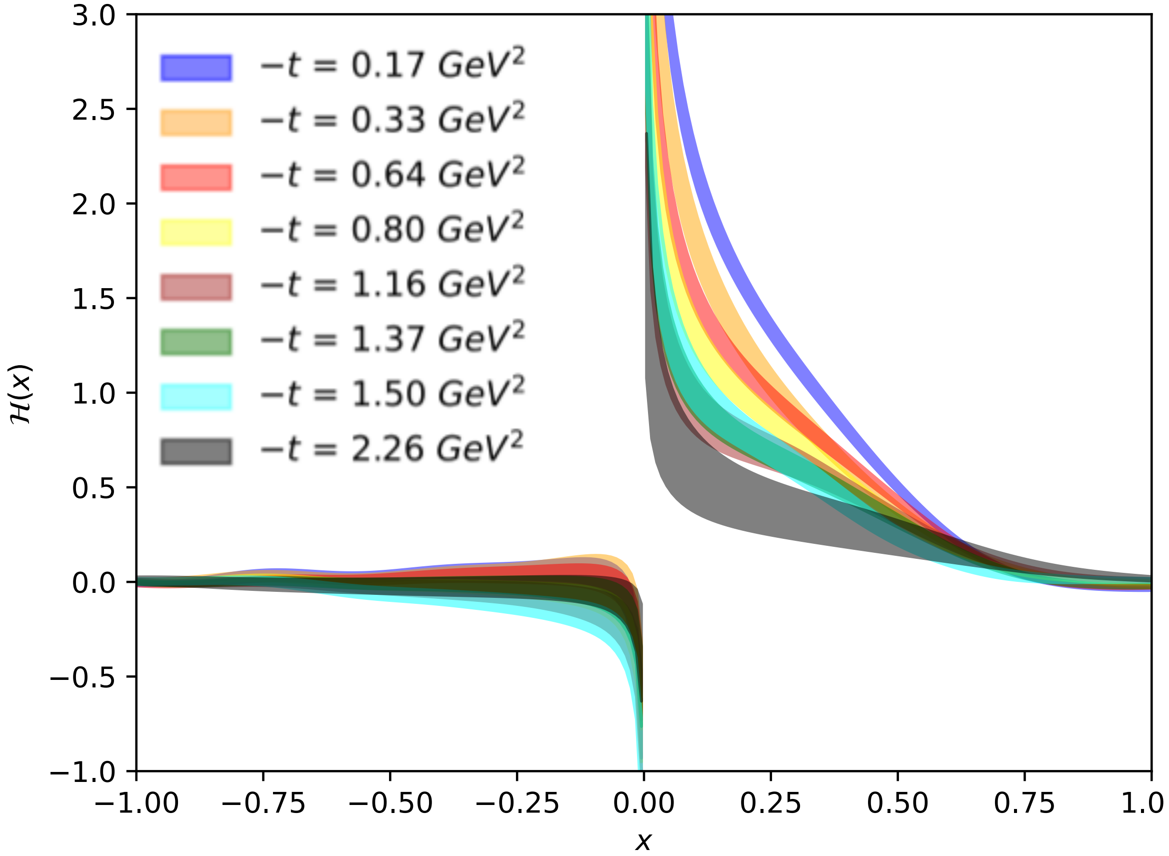
- asymmetric frame: $\vec{p}_f^{\,a} = \vec{P}, \qquad \vec{p}_i^{\,a} = \vec{P} - \vec{Q} \qquad t^a = -\vec{Q}^2 + (E_f - E_i)^2 = 0.65\, GeV^2$

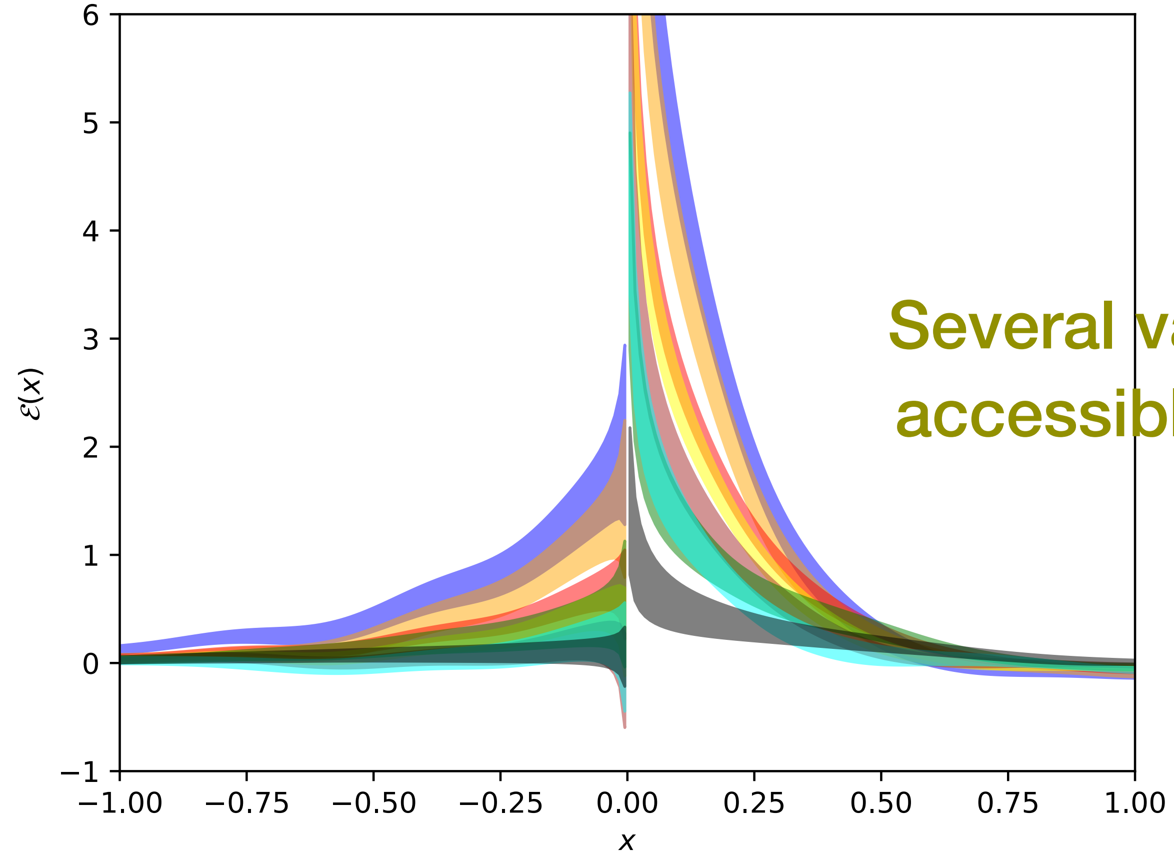
Comparison of A_i in two frames



- ★ A_1, A_5 dominant contributions
- ★ Full agreement in two frames for both Re and Im parts of A_1, A_5
- ★ A_3, A_4, A_8 zero at $\xi = 0$
- ★ A_2, A_6, A_7 suppressed (at least for this kinematic setup and $\xi = 0$)

H, E light-cone GPDs

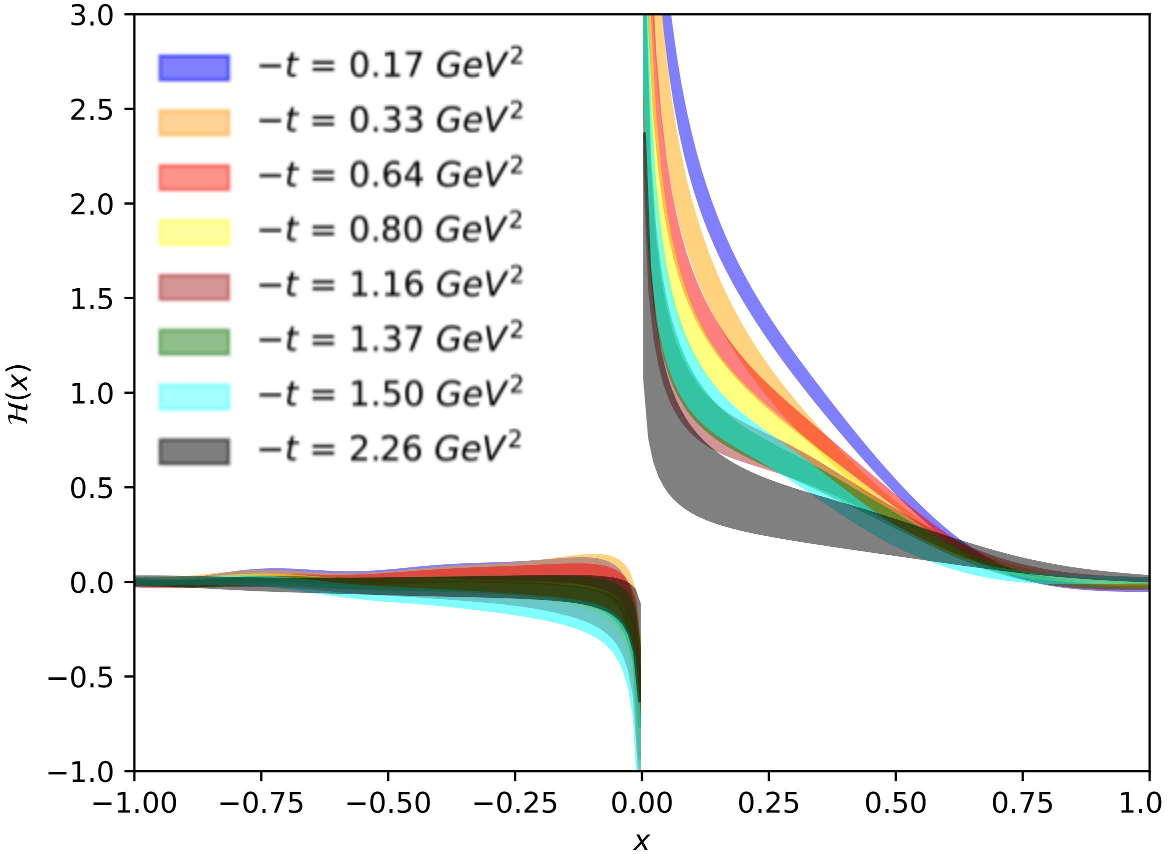


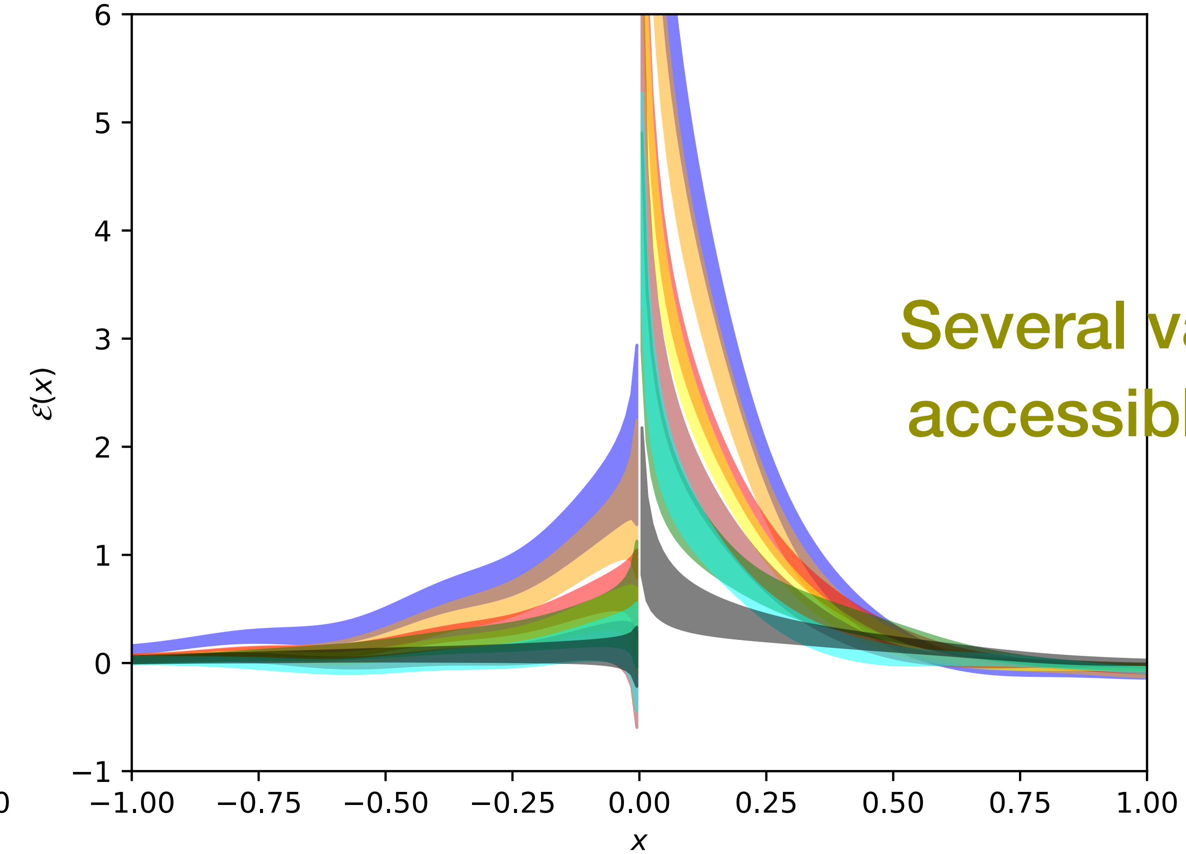


Several values of -t accessible at once

★ Anti-quark region susceptible to systematic uncertainties

H, E light-cone GPDs





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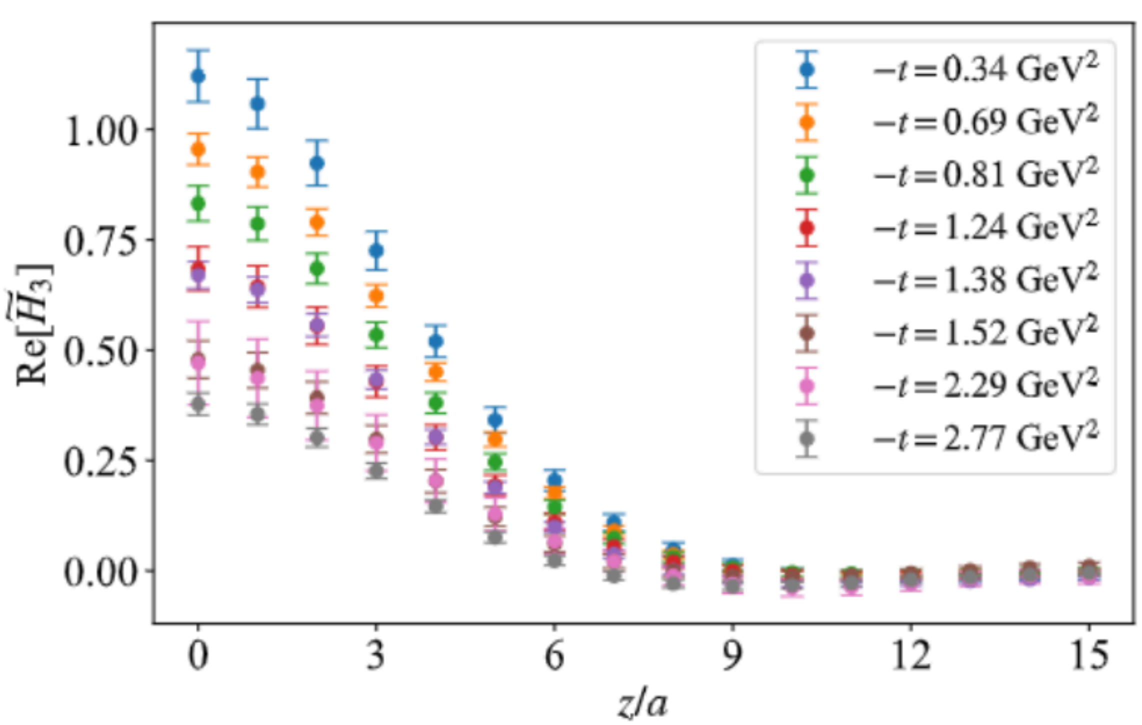
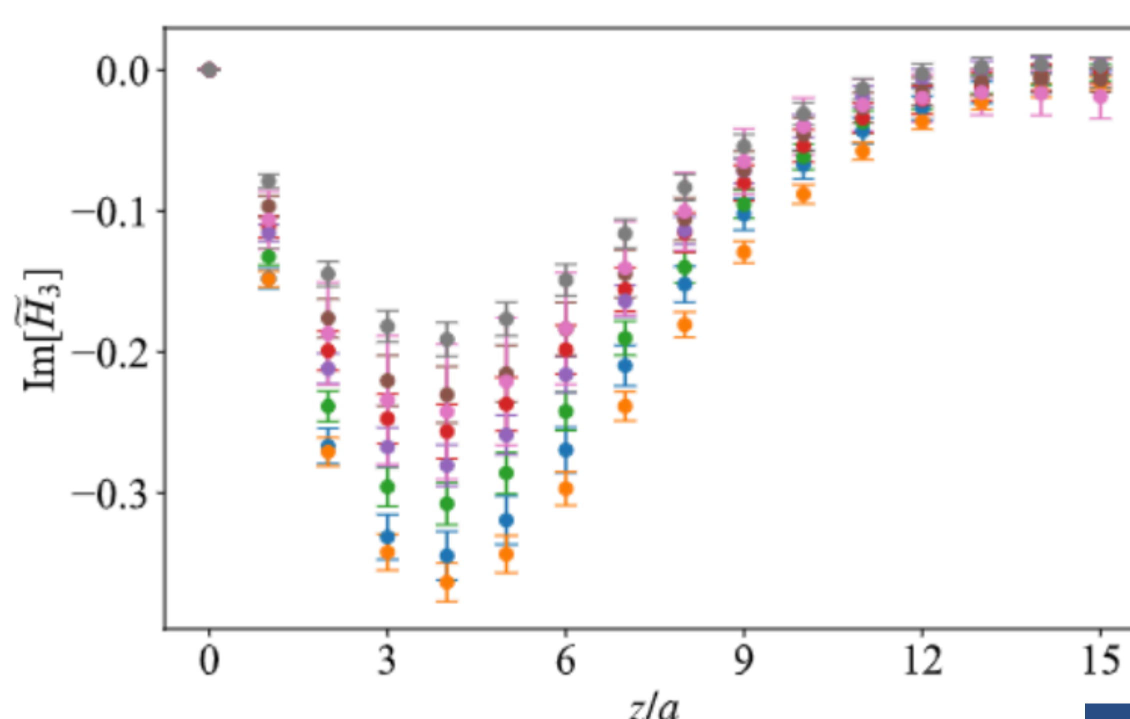
Similar analysis: helicity GPDs





Exploration of twist-3 PDFs & GPDs

PHYSICAL REVIEW D **102**, 111501(R) (2020)

Rapid Communications | Editors' Suggestion

Insights on proton structure from lattice QCD: The twist-3 parton distribution function $g_T(x)$

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[2] and Fernanda Steffens[3]

PHYSICAL REVIEW D **104**, 114510 (2021)

Parton distribution functions beyond leading twist from lattice QCD: The $h_L(x)$ case

Shohini Bhattacharya,[1] Krzysztof Cichy,[2] Martha Constantinou,[1] Andreas Metz,[1] Aurora Scapellato,[1] and Fernanda Steffens[3]

arXiv:2306.05533v1 [hep-lat] 8 Jun 2023

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya[1,2], Krzysztof Cichy[3], Martha Constantinou[1], Jack Dodson[1], Andreas Metz[1], Aurora Scapellato[1], Fernanda Steffens[4]

h_L twist-3 PDF

★ h_L decouples from usual DIS (chiral odd) - not trivial to extract

★ most elusive of the three twist-3 PDFs ($e(x)$, $g_T(x)$, $h_L(x)$)

★ h_L may be accessed via:

- double-polarized Drell-Yan process
[R. Jaffe, PRL 67 (1991) 552-555; Y. Koike et al., PLB 668 (2008) 286, arXiv:0805.2289]

- di-hadron single spin asymmetries (CLAS)
[Gliske et al., PRD 90 (2014) 11, 114027, arXiv:1408.5721; A. Vossen, CIPANP 2018, arXiv: 1810.02435]

- single-inclusive particle production in proton-proton collisions
[Y. Koike et al., PLB 759 (2016) 75, arXiv:1603.07908]

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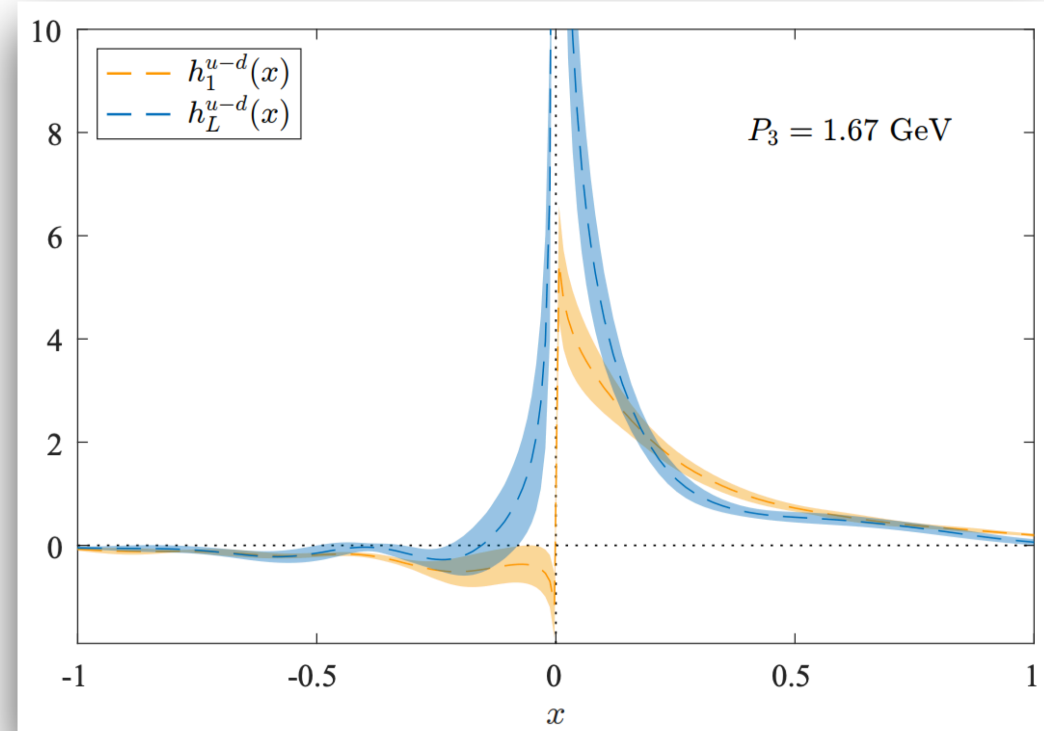
[Y. Koike et al., PLB 759 (2016) 75, arXiv:1603.07908]

★ Parametrization [Meissner et al., *JHEP* 08 (2009) 056]

$$F^{[\sigma^{+-}\gamma_5]} = \bar{u}(p') \left(\gamma^+ \gamma_5 \widetilde{H}_2' + \frac{P^+ \gamma_5}{M} \widetilde{E}_2' \right) u(p)$$

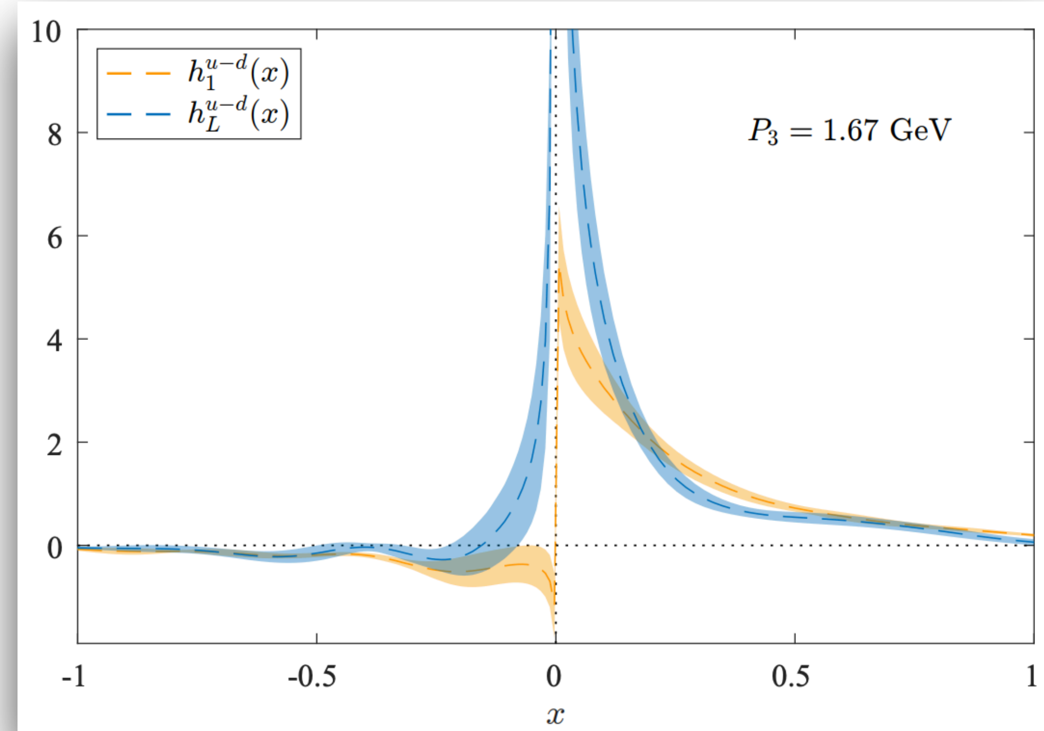
First lattice calculation of x-dependent GPDs



★ Twist-3 h_L as sizable as twist-2 h_1

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

First lattice calculation of x-dependent GPDs



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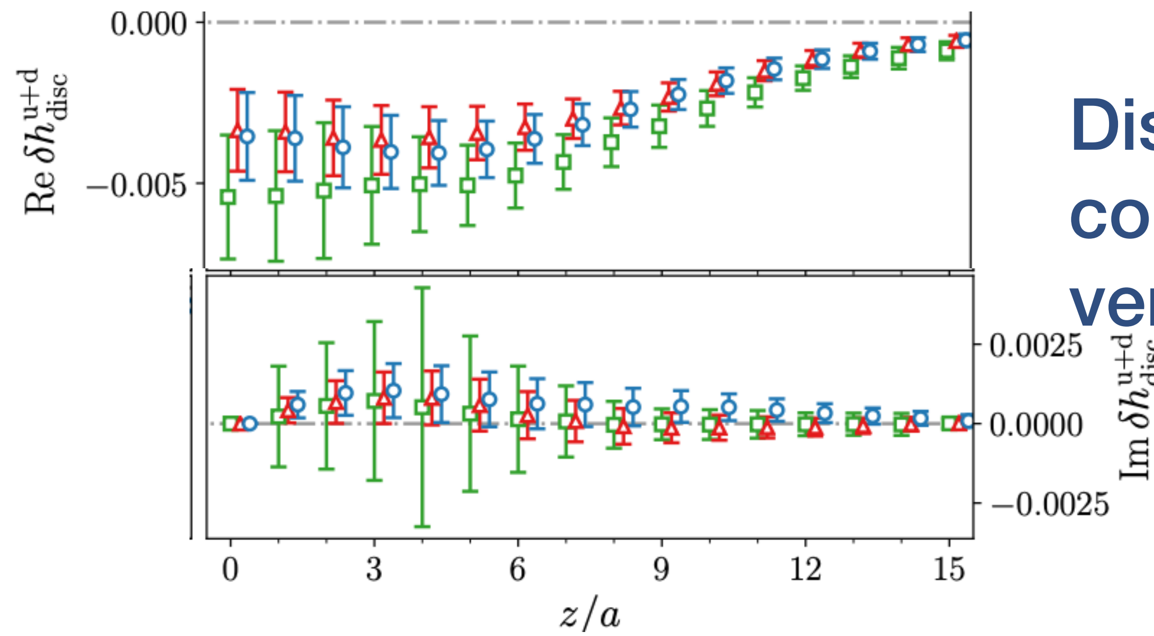
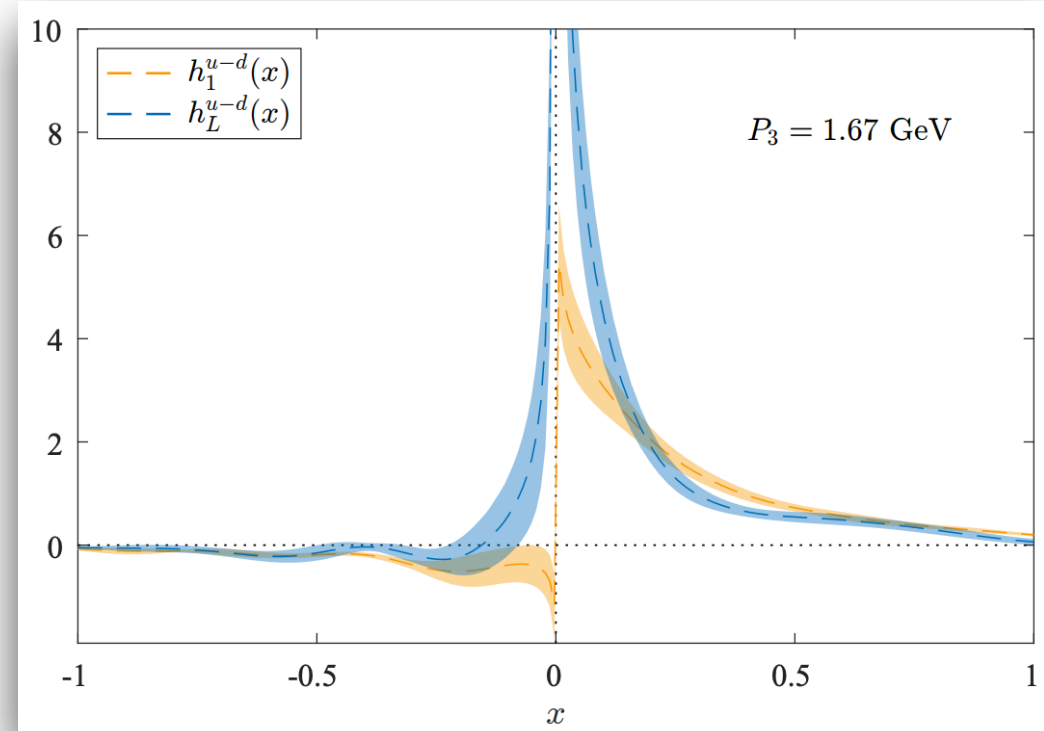


Disconnected contributions very small

First lattice calculation of x-dependent GPDs



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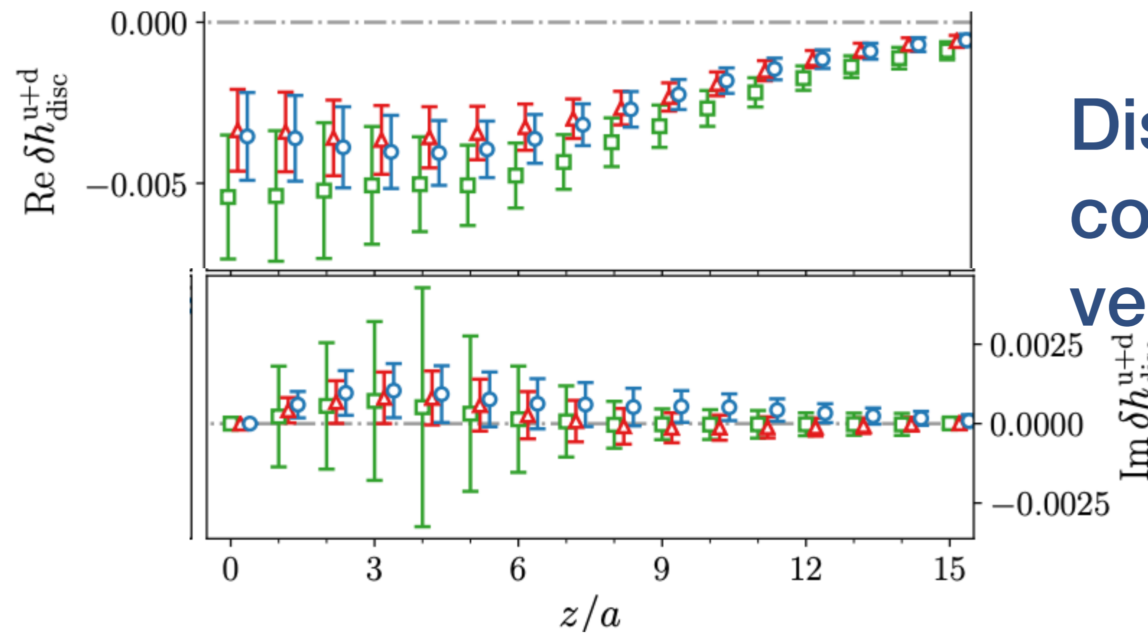
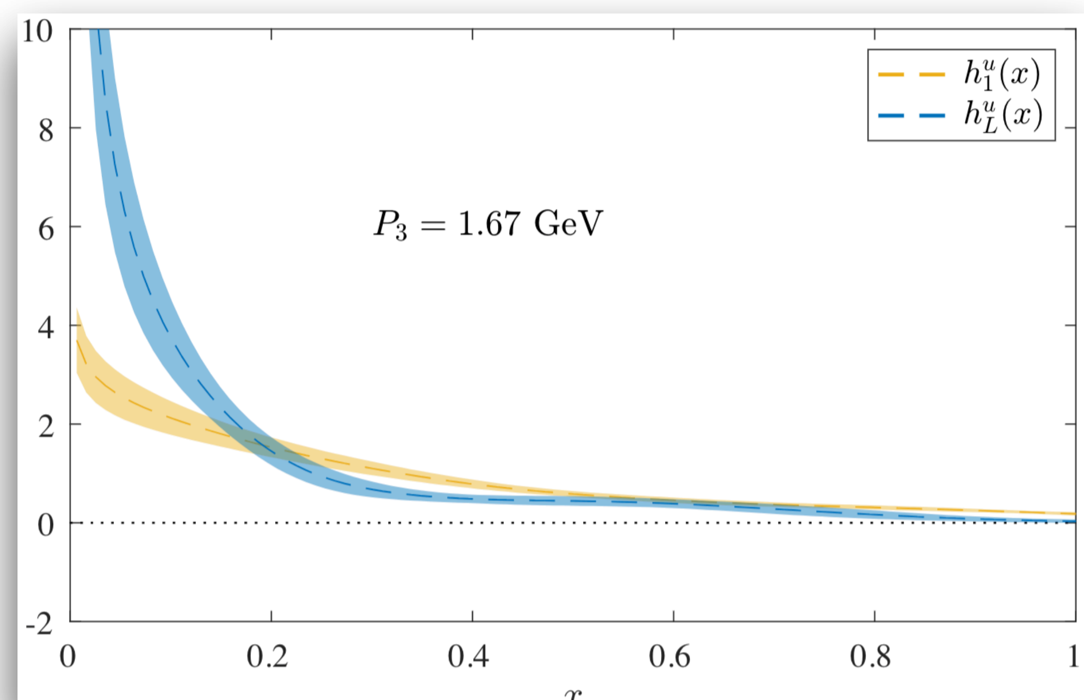
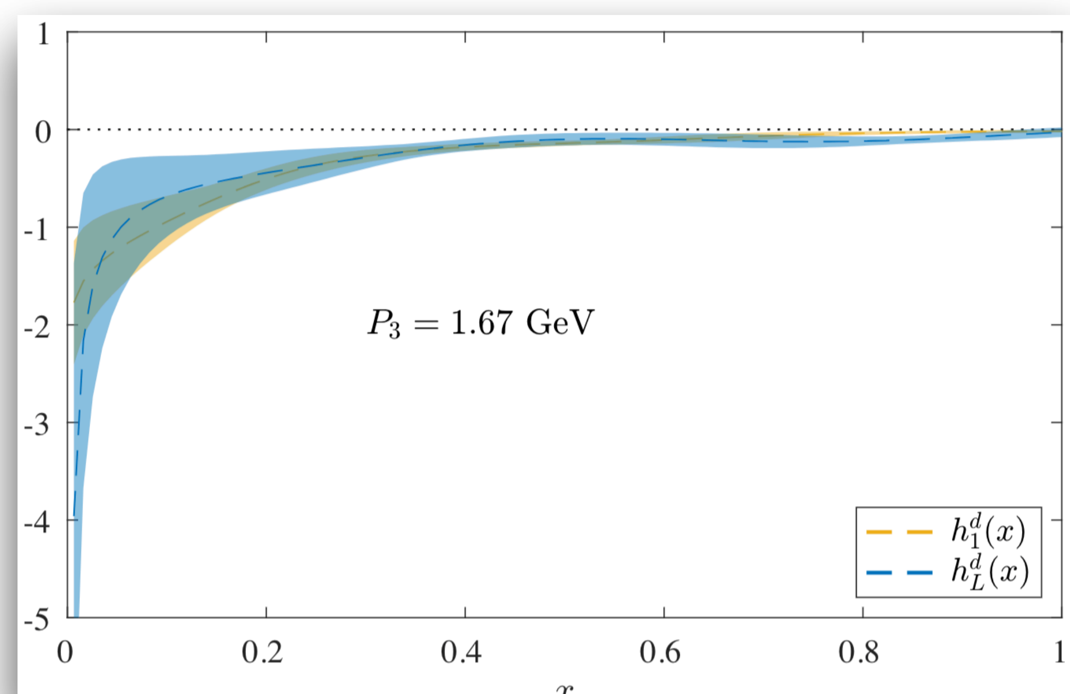


Disconnected contributions very small

★ Role of up and down quarks:





★ Up-quarks dominant in both h_L, h_1 x-dependence: $h^u_{1,L} \sim 2h^d_{1,L}$

Twist-3 $h_L(x)$ PDF

Burkhardt-Cottingham (quasi-h_L)

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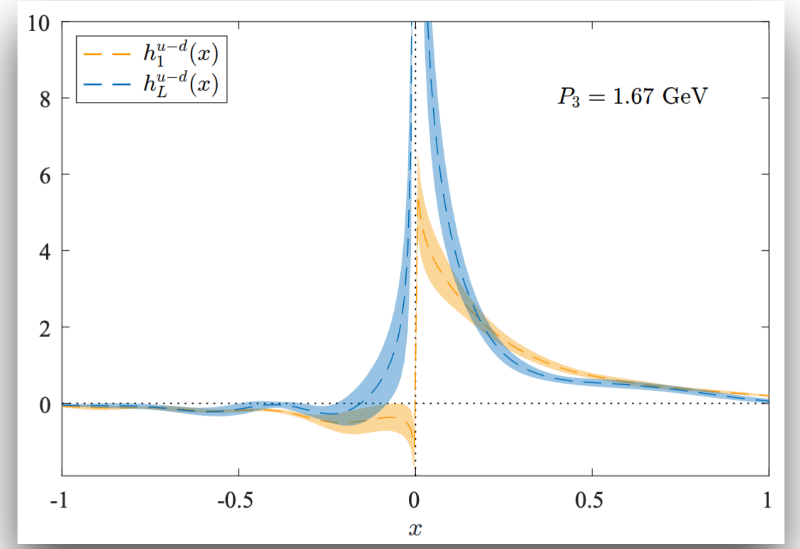

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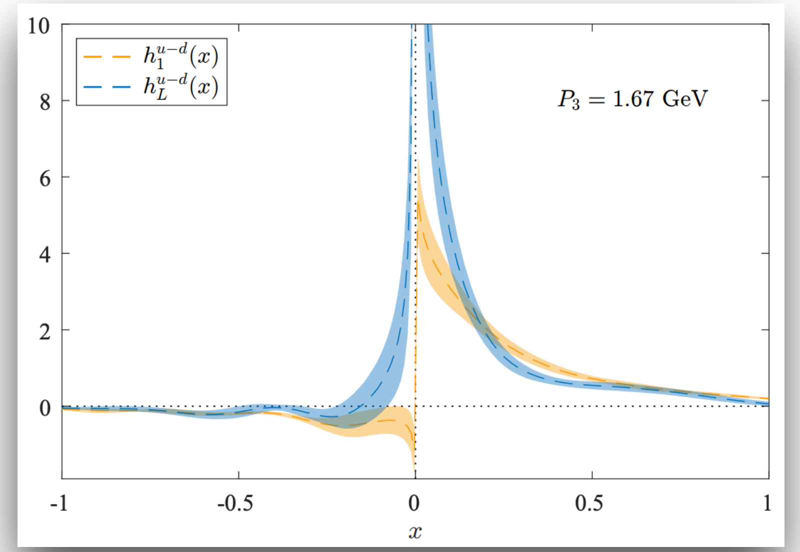

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$$h_L(x) = h_L^{\text{WW}}(x) + h_L^{\text{twist}-3}(x) = 2x \int_x^1 dy\, \frac{h_1(y)}{y^2} + h_L^{\text{twist}-3}(x)$$

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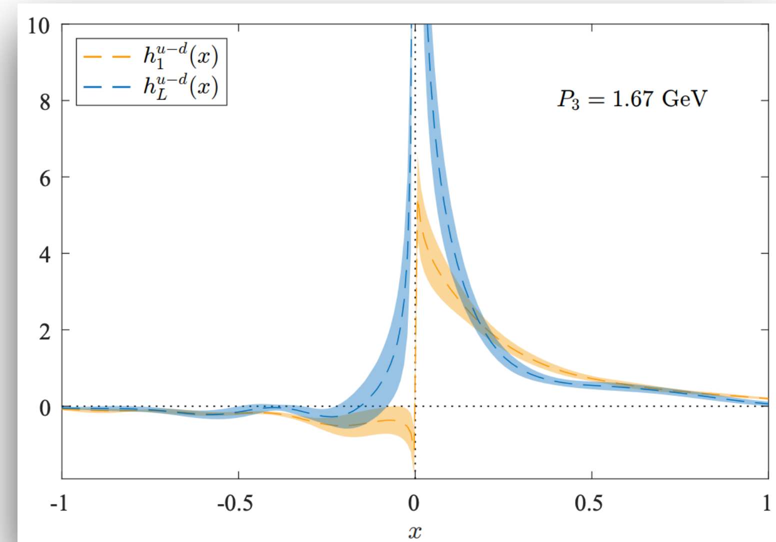

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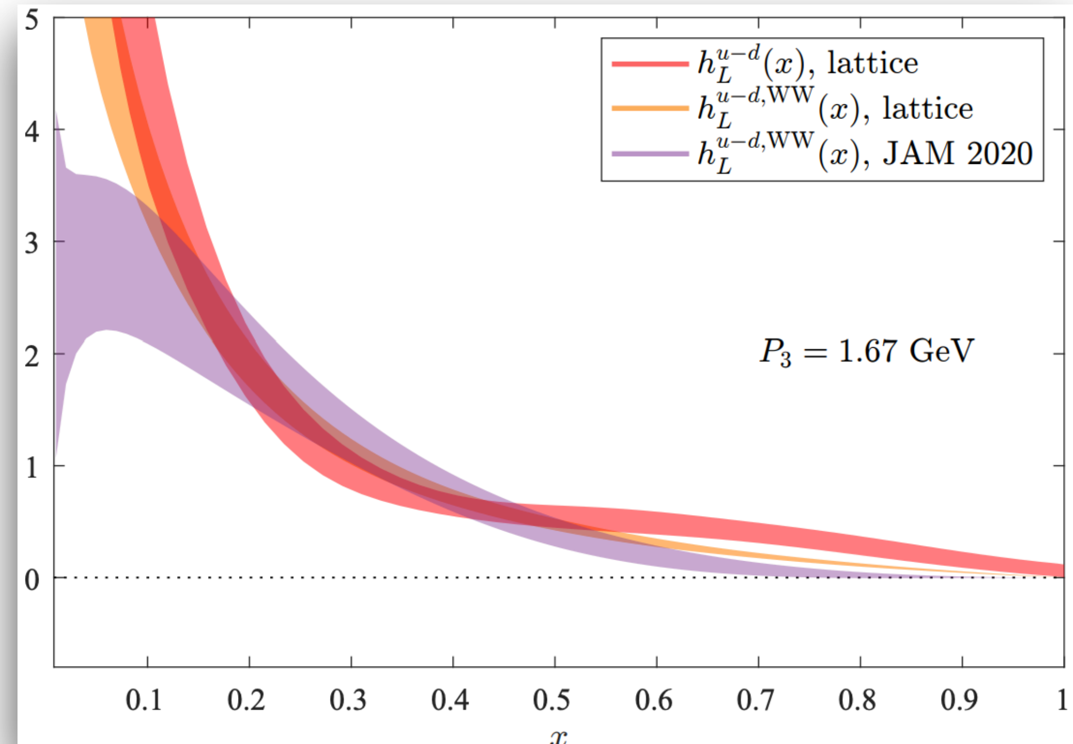

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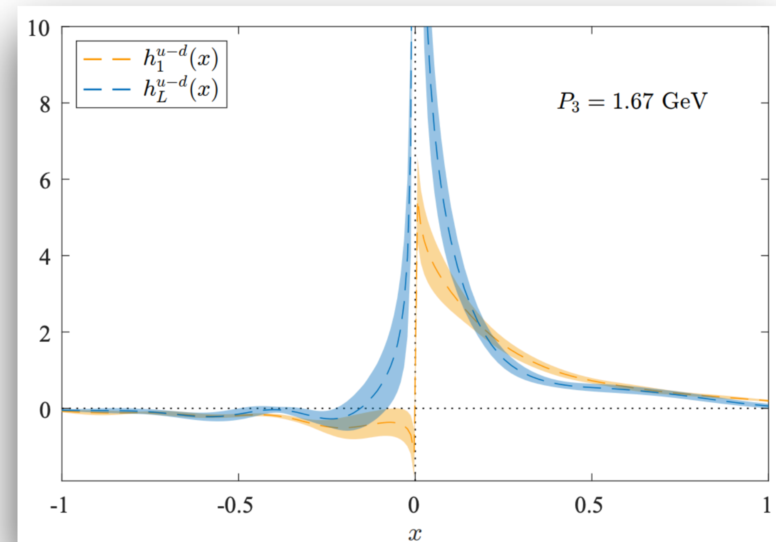

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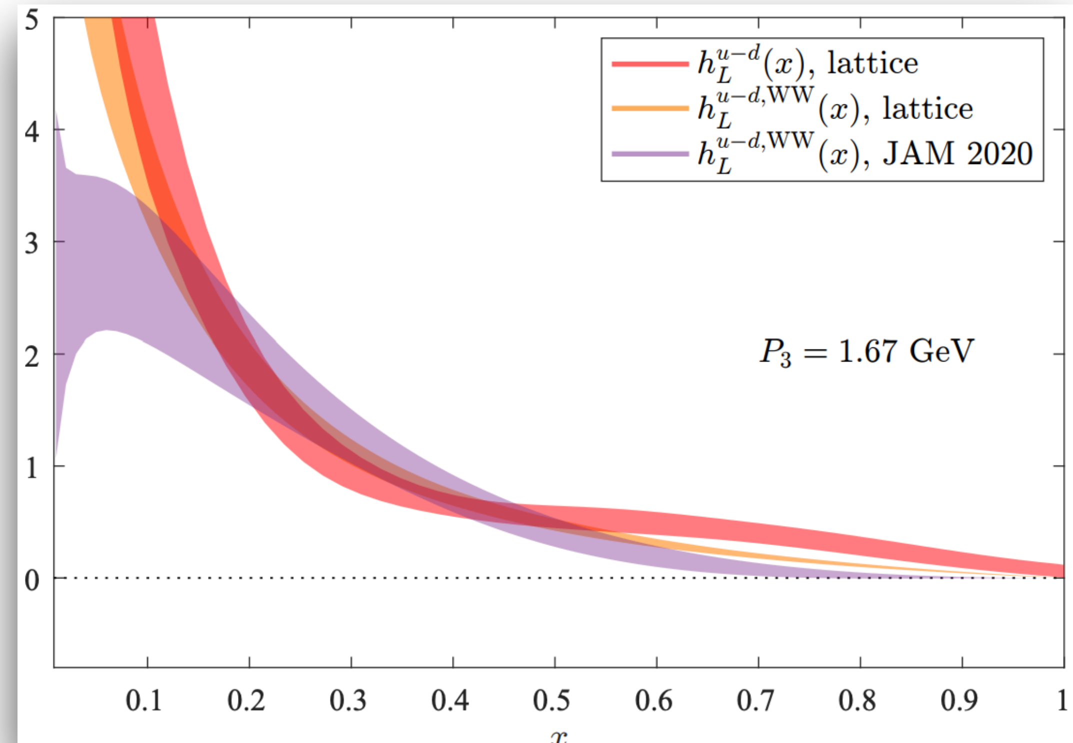

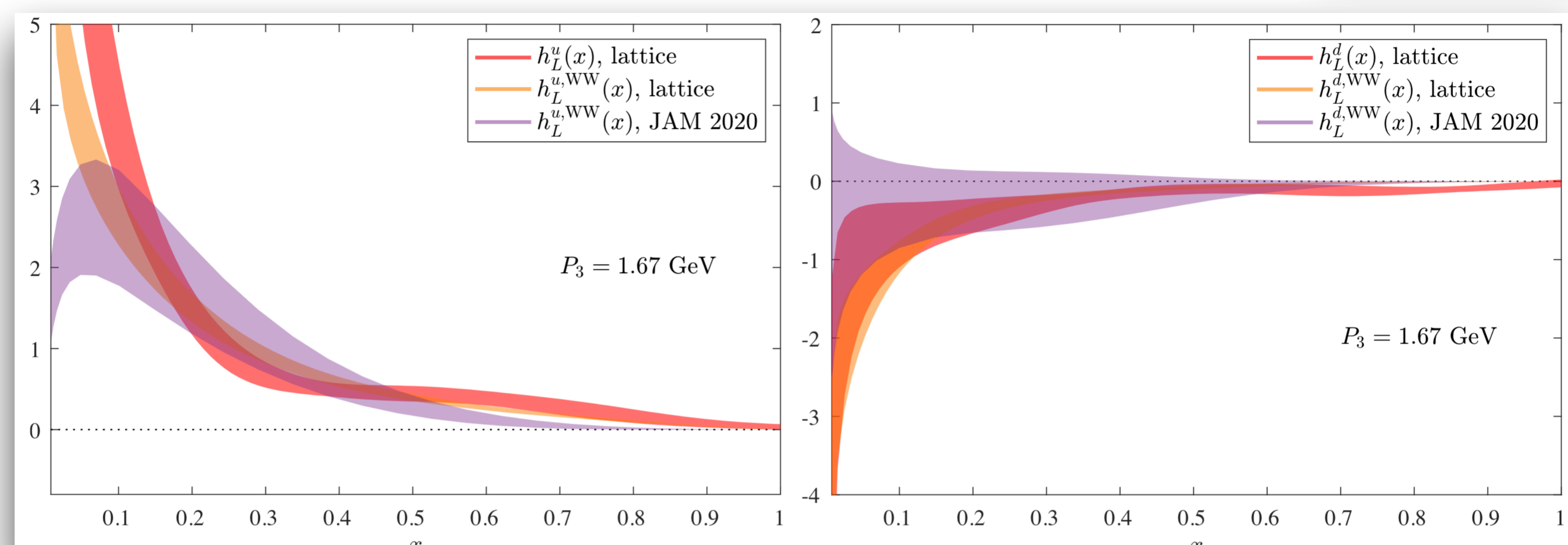

- h_L^u dominant - diff. between h_L & $h_L^{\rm WW}$
- h_L^d <0 and decays faster then h_L^u

Extension to twist-3 tensor GPDs

arXiv:2306.05533v1 [hep-lat] 8 Jun 2023

Chiral-even axial twist-3 GPDs of the proton from lattice QCD

Shohini Bhattacharya[1,2], Krzysztof Cichy[3], Martha Constantinou[1], Jack Dodson[1], Andreas Metz[1], Aurora Scapellato[1], Fernanda Steffens[4]

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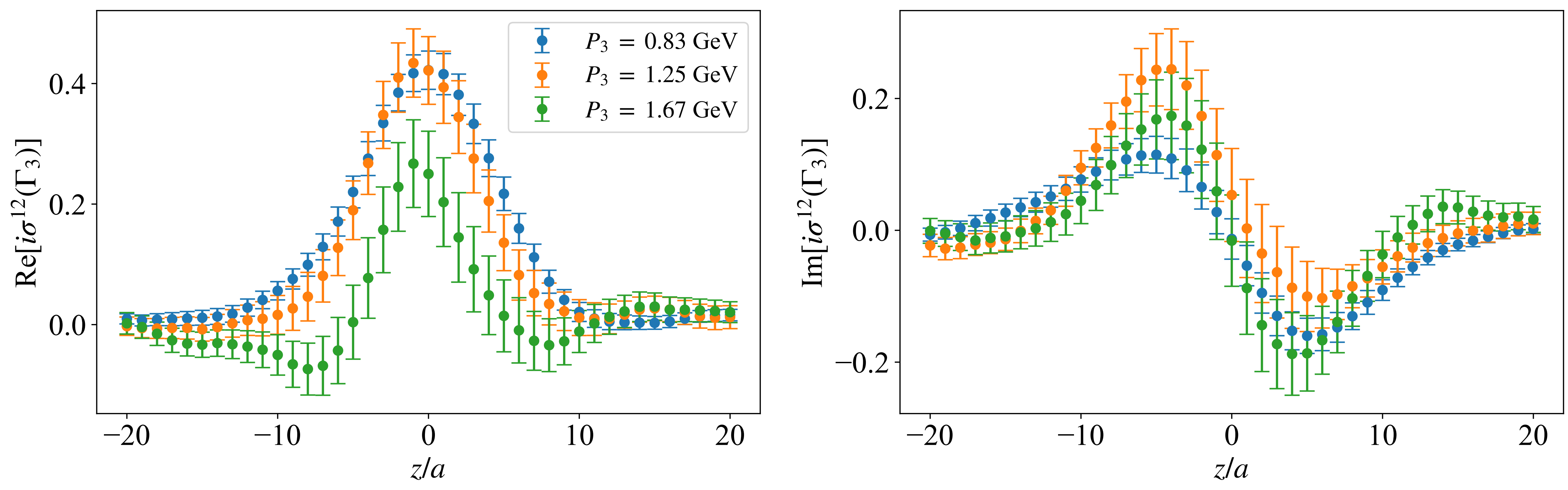

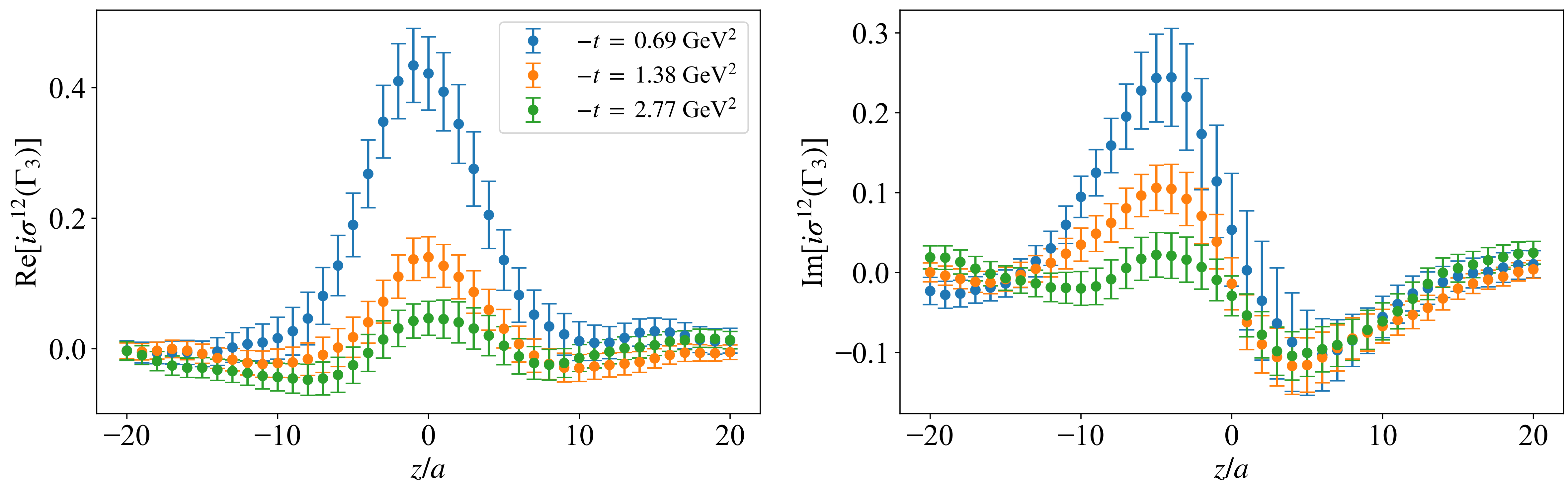

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PHENOMENOLOGY
LATTICE QCD
THEORY

QUARK-GLUON TOMOGRAPHY COLLABORATION

U.S. DEPARTMENT OF ENERGY | Office of Science

Award Number: DE-SC0023646

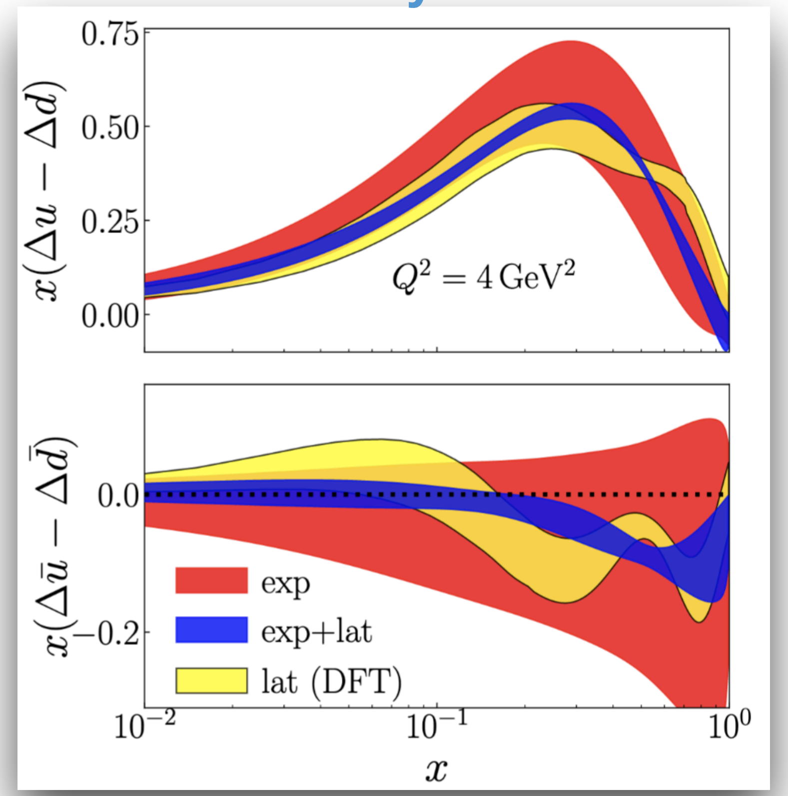
1. Theoretical studies of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties

2. Lattice QCD calculations of GPDs and related structures

3. Global analysis of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

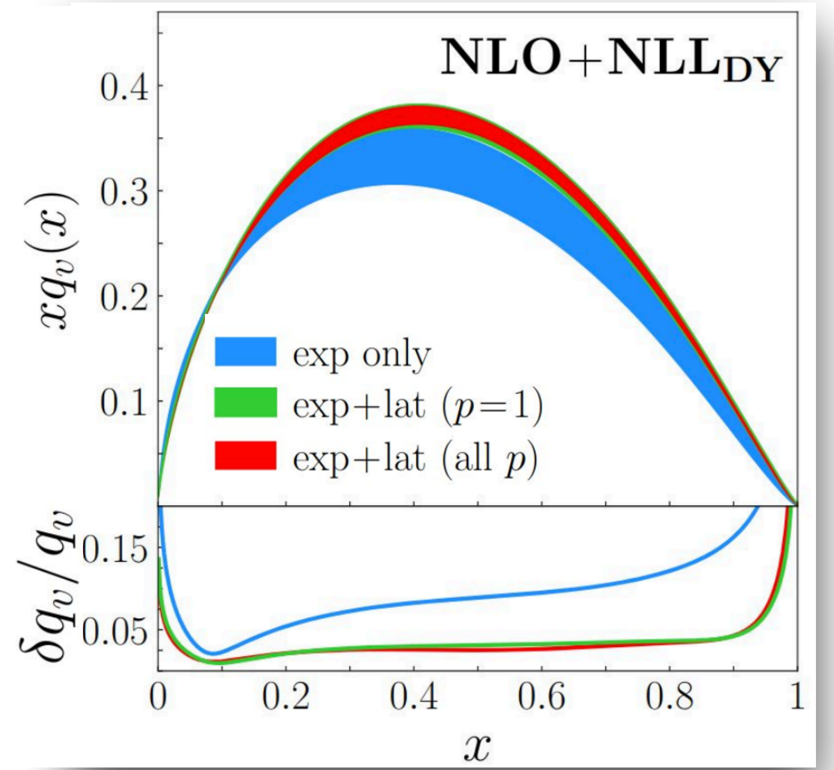
Synergies: constraints & predictive power of lattice QCD

helicity PDF



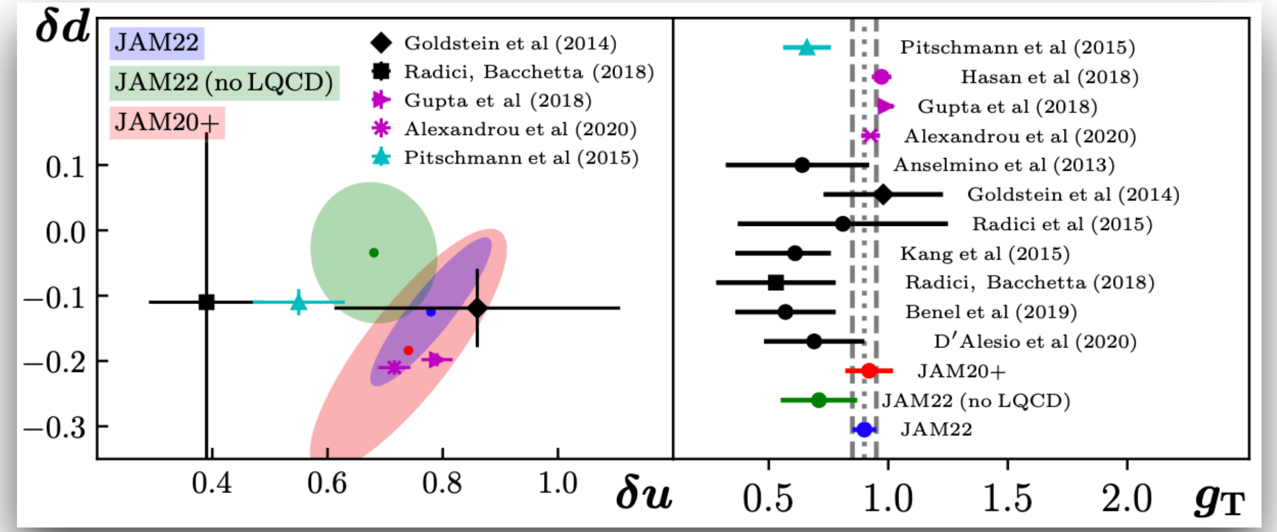
[JAM & ETMC, PRD 103 (2021) 016003]

pion PDF



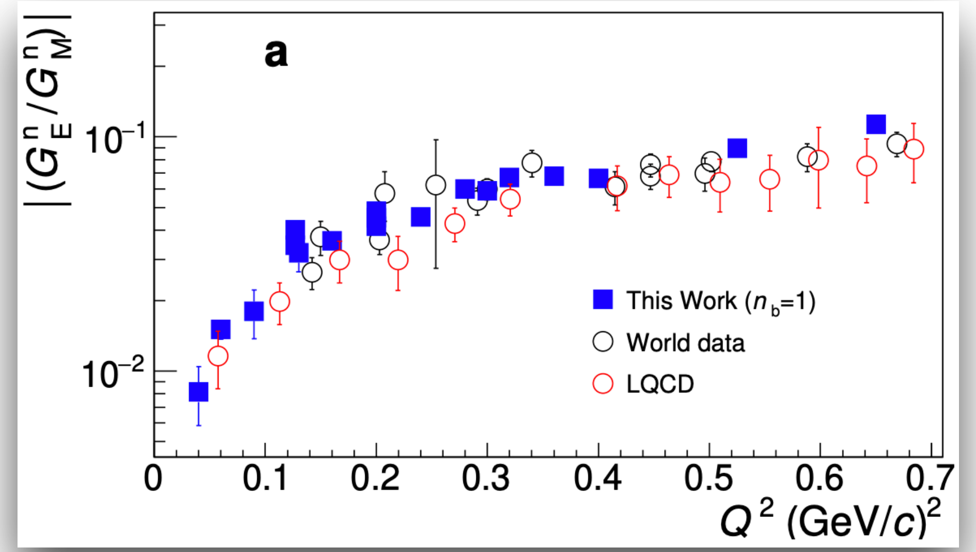
[JAM/HadStruc, PRD105 (2022) 114051]

transversity PDF



[JAM, PRD 106 (2022) 3, 034014]

proton & neutron radius



[Atac et al., Nature Comm. 12, 1759 (2021)]

Summary

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- significant reduction of computational cost
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Thank you

U.S. DEPARTMENT OF ENERGY INCITE LEADERSHIP COMPUTING

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DOE Early Career Award (NP)
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