Disentangling Long and Short Distances in Momentum-Space TMDs

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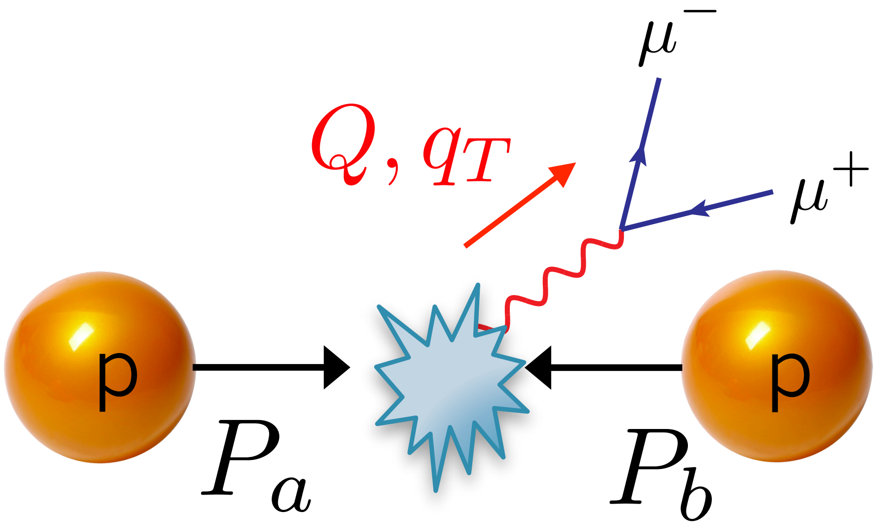
Massachusetts Institute of Technology

TMD Collaboration

TMD PDFs



- Factorization of Drell-Yan cross section:

$$\frac{d\sigma}{dQ\,dY\,d^2q_T} = H(Q,\mu)\sum_i \int d^2\vec{b}_T\, e^{i\vec{q}_T\cdot\vec{b}_T}\, f_i(x_a, b_T, \mu, \zeta_a)\, f_{\bar{i}}(x_b, b_T, \mu, \zeta_b) \times \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)\right]$$

Hard virtual corrections

Describe transverse momentum of the partons

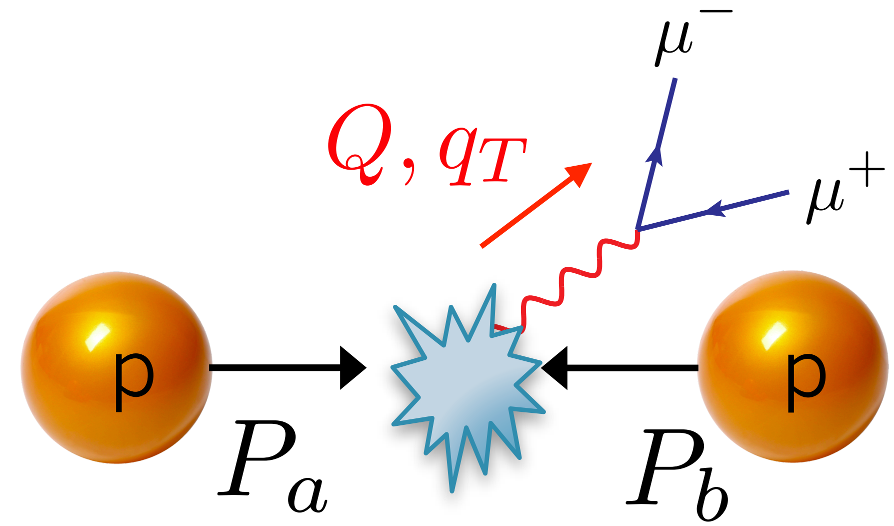
- Most easily written in position space

μ = Renormalization scale

ζ = Collins-Soper parameter

$\zeta_a \zeta_b = Q^4$

TMD PDFs

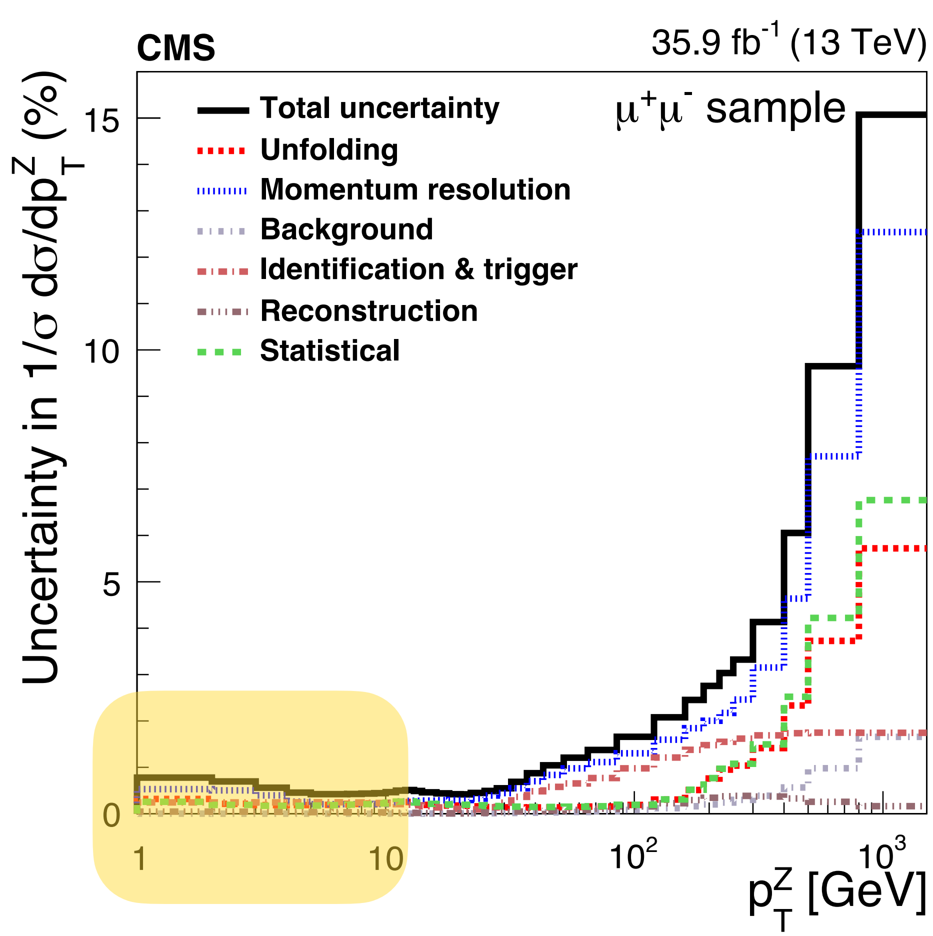


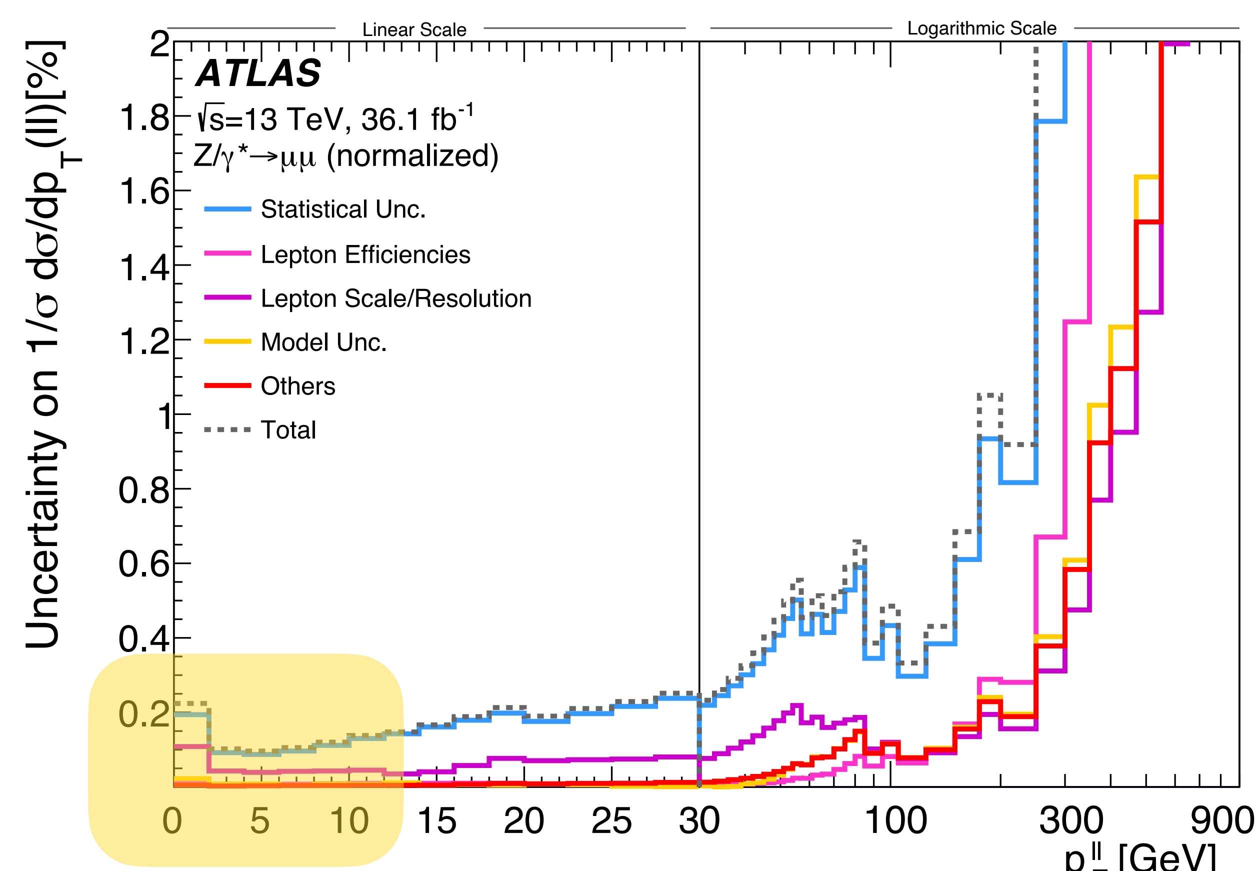
- Factorization of Drell-Yan cross section:

$$\frac{d\sigma}{dQ\,dY\,d^2q_T} = H(Q,\mu)\sum_i \int d^2\vec{b}_T\, e^{i\vec{q}_T\cdot\vec{b}_T}\, f_i(x_a, b_T, \mu, \zeta_a)\, f_{\bar{i}}(x_b, b_T, \mu, \zeta_b) \times \left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)\right]$$

- Measurements are done in momentum space!

CMS: 1909.04133
ATLAS: 1912.02844





(Non)perturbative TMD PDFs

- Challenging to use the nonperturbative info that lattice provides
- Modeling TMDPDFs with both perturbative and nonperturbative parts is usually done by introducing b^*:

$$f_i(x, b_T, \mu, \zeta) = f_{\text{pert, } i}(x, b^*(b_T), \mu, \zeta) \cdot f_{\text{NP}}(x, b_T, \zeta)$$

Calculated with expansion in $\alpha_s(1/b_T)$

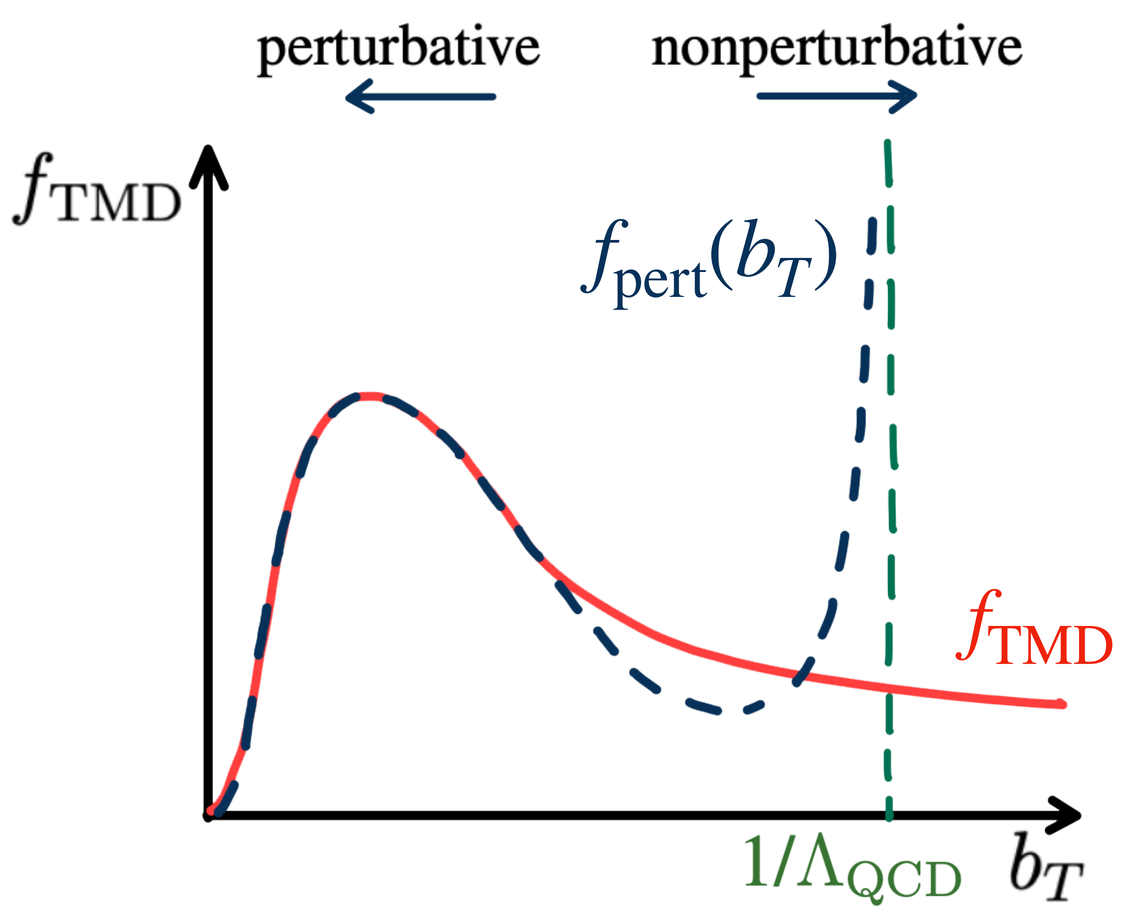
- The perturbative part can be computed with an operator product expansion (OPE):

$$f^{\text{TMD}}_{\text{pert, } i}(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{dz}{z} C_{ij}(\frac{x}{z}, b_T, \mu, \zeta) f_j^{\text{coll}}(z, \mu)$$
$$= f_i^{\text{coll}}(x, \mu) + \alpha_s C_{ij}^{(1)} \otimes f_j^{\text{coll}}(x, \mu) + \mathcal{O}(\alpha_s^2)$$

Modeling TMD PDFs

- Modeling TMD PDFs with both perturbative and nonperturbative parts is usually done by introducing b^*:

$$f_i(x, b_T, \mu, \zeta) = \underbrace{f_{\text{pert},\, i}(x, b^*(b_T), \mu, \zeta)}_{\text{Calculated with expansion in } \alpha_s(1/b_T)} \cdot \underbrace{f_{\text{NP}}(x, b_T, \zeta)}_{\text{Has to be } 1+\mathcal{O}(b_T^2)}$$



- $b^*(b_T)$ shields the Landau pole
- $b_T \ll 1/\Lambda_{\text{QCD}}$: $b^*(b_T) \to b_T$, $f_{\text{NP}} \to 1$

  f_{pert} dominates
- $b_T \gg 1/\Lambda_{\text{QCD}}$: $b^*(b_T) \to$ constant

  f_{NP} dominates

Modeling TMDPDFs

- Different models of $f_{\rm NP}$ are used for fitting to data
- $b*(b_T)$ shields the Landau pole and is coupled to $f_{\rm NP}$

$$
\begin{aligned}
f_{\rm TMD}(x, b_T, \mu, \zeta) &= f_{\rm pert}(x, b_A^*(b_T), \mu, \zeta) \cdot f_{\rm NP}^A(x, b_T, \zeta) \\
&= f_{\rm pert}(x, b_B^*(b_T), \mu, \zeta) \cdot f_{\rm NP}^B(x, b_T, \zeta)
\end{aligned}
$$

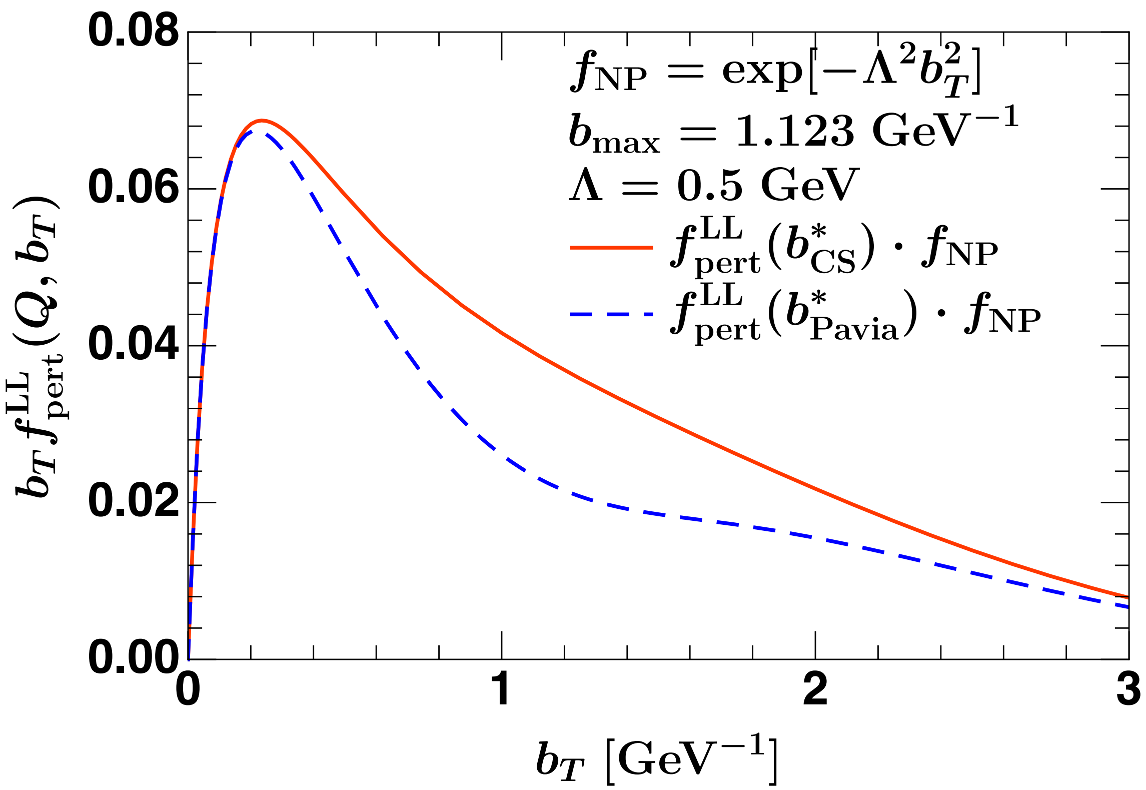
$$
b_A^*(b_T) \neq b_B^*(b_T) \quad \Rightarrow \quad f_{\rm NP}^A(x, b_T) \text{ and } f_{\rm NP}^B(x, b_T) \text{ are not comparable!}
$$

- The perturbative and nonperturbative effects are mixed up!

Modeling TMDPDFs

- b^* prescriptions makes different $f_{\rm NP}$ not comparable
- For example, take the same $f_{\rm NP}(b_T) = e^{-(0.5{\rm GeV}\ b_T)^2}$,

  use either $b^*_{\rm CS}(b_T)$ or $b^*_{\rm Pavia}(b_T)$:



$$b^*_{\rm CS}(b_T) = \frac{b_T}{\sqrt{1 + (b_T/b_{\rm max})^2}} = b_T\,(1 + \mathcal{O}(b_T^2))$$

Collins+Soper (1982)

$$b^*_{\rm Pavia}(b_T) = b_{\rm max}\left(1 - \exp(-\frac{b_T^4}{b_{\rm max}^4})\right)^{\frac{1}{4}} = b_T\,(1 + \mathcal{O}(b_T^4))$$

Pavia: 1703.10157

- Goal: extract nonperturbative physics without b^* contamination

Momentum Space

- Measurements are in q_T space: Fourier transform

$$\frac{d\sigma}{dq_T} = 2\pi q_T \int_0^\infty \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T\cdot\vec{b}_T}\sigma(b_T)$$

q_T spectrum

$$= q_T \int_0^\infty db_T\, b_T \int_0^{2\pi} \frac{d\phi}{2\pi}\, e^{iq_T b_T \cos\phi}\sigma(b_T) = q_T \int_0^\infty db_T\, b_T\, J_0(q_T b_T)\sigma(b_T)$$

  - For perturbative q_T, integral still includes nonperturbative b_T !

- Intuition: perturbative q_T should be dominated by perturbative $b_T \sim 1/q_T$

Momentum Space

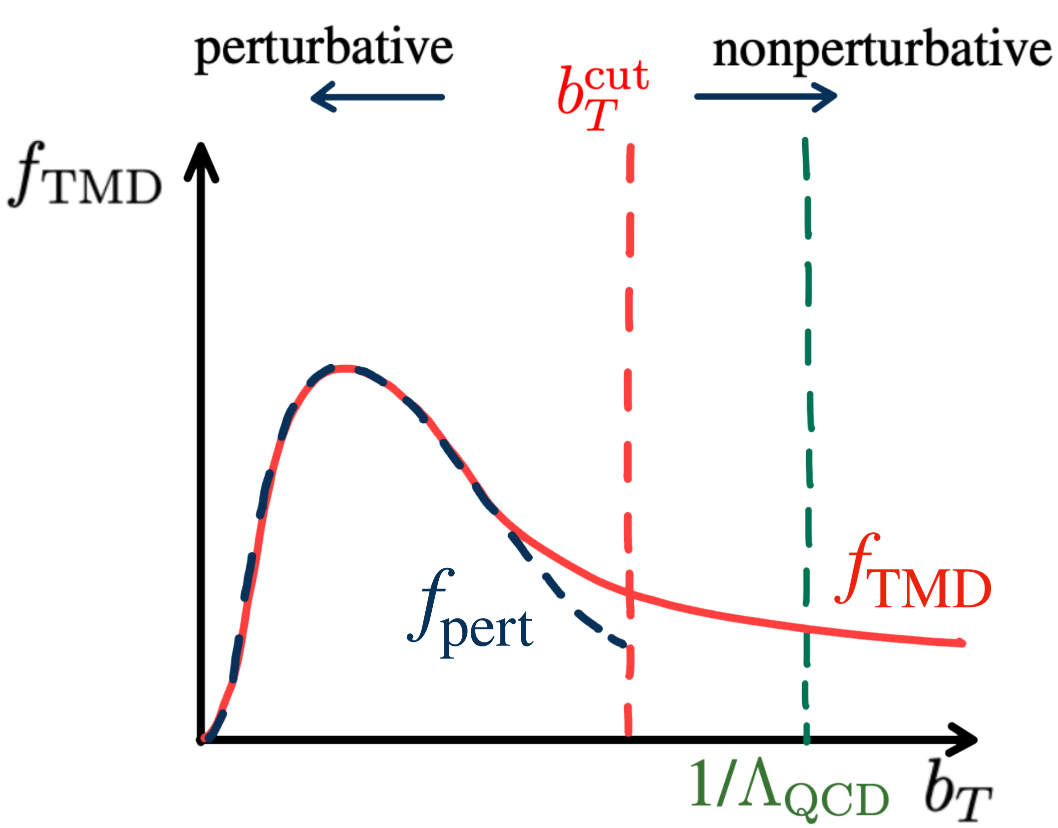
- Intuition: perturbative q_T should be dominated by perturbative b_T
- Goal: make this intuition manifest
- Solution: introducing b_T^{cut}

Can use perturbative OPE

Nonperturbative physics

$$S[f](q_T) \equiv q_T \int_0^\infty db_T\, b_T J_0(q_T b_T) f(b_T) = S_<[f](q_T) + S_>[f](q_T)$$

full spectrum



$$S_<[f](q_T) \equiv q_T \int_0^{b_T^{\mathrm{cut}}} db_T\, b_T J_0(q_T b_T) f(b_T),$$

$$S_>[f](q_T) \equiv q_T \int_{b_T^{\mathrm{cut}}}^\infty db_T\, b_T J_0(q_T b_T) f(b_T)$$

Truncated Functionals

- Want to approximate $S[f]$ using perturbative $b_T \leq b_T^{\rm cut}$
- Can use $S_<[f]$, but need to systematically account for $S_>[f]$

$$S_>[f](q_T, b_T^{\rm cut}) = q_T \int_{b_T^{\rm cut}}^{\infty} db_T\, b_T J_0(q_T b_T) f(b_T)$$

$$= -\, b_T^{\rm cut} \underbrace{J_1(q_T b_T^{\rm cut})} f(b_T^{\rm cut}) - \int_{b_T^{\rm cut}}^{\infty} db_T\, b_T J_1(q_T b_T) \underbrace{f'(b_T)}_{< b_T^{-\rho-1}}$$

Assumption:

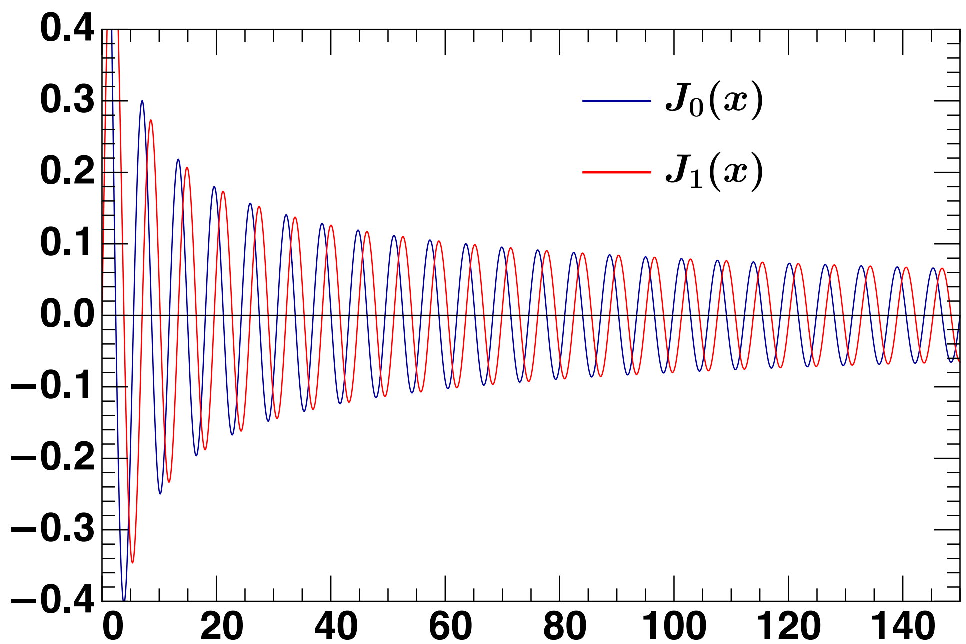
a) $f(b_T \to \infty) < b_T^{-\rho}$, $\rho > \dfrac{1}{2}$

b) $f(b_T)$ differentiable at $b_T^{\rm cut}$

asymptotic form

$q_T \gg 1/b_T^{\rm cut} > \Lambda_{\rm QCD}$

$$= \sqrt{\frac{2 b_T^{\rm cut}}{\pi q_T}} \cos\left(q_T b_T^{\rm cut} + \frac{\pi}{4}\right) f(b_T^{\rm cut}) + \mathcal{O}[(b_T^{\rm cut} q_T)^{-\frac{3}{2}}]$$



$$J_0(x \to \infty) = \sqrt{\frac{2}{\pi x}} \cos(x - \frac{\pi}{4}) + \mathcal{O}(x^{-\frac{3}{2}})$$

$$J_1(x \to \infty) = -\sqrt{\frac{2}{\pi x}} \cos(x + \frac{\pi}{4}) + \mathcal{O}(x^{-\frac{3}{2}})$$

Truncated Functionals

- Define a systematic series to approximate $S[f]$ using $b_T \leq b_T^{\text{cut}}$

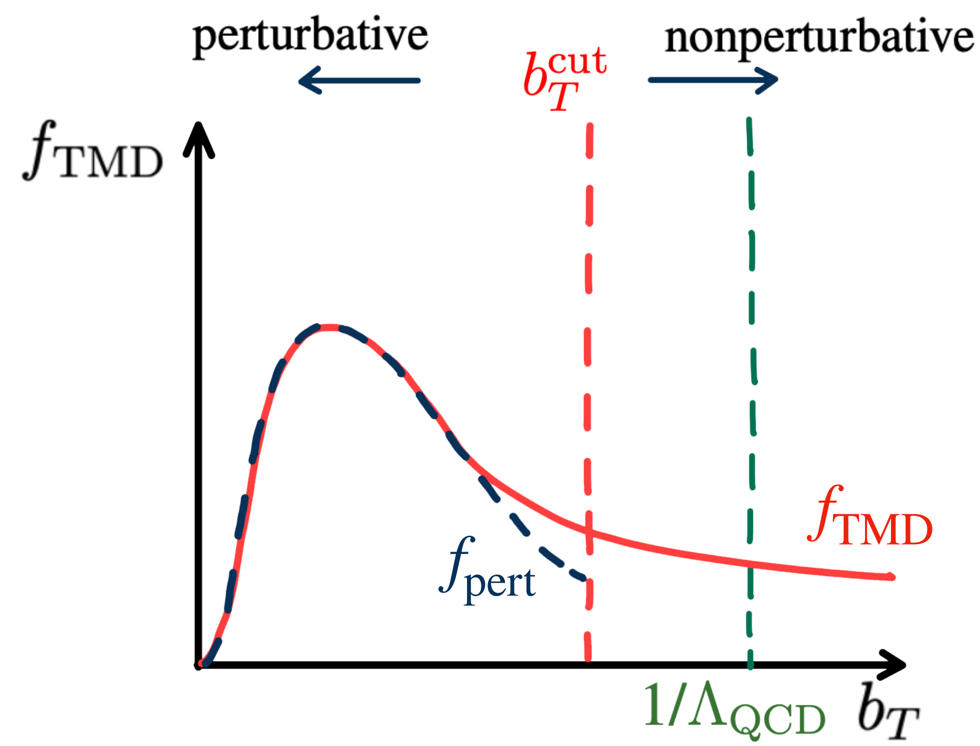
$$S^{(0)}[f](q_T) \equiv S_<[f](q_T) = q_T \int_0^{b_T^{\text{cut}}} db_T\, b_T J_0(q_T b_T) f(b_T)$$

- Define $S^{(1)}[f]$ to include leading boundary contribution from $S_>[f]$

$$S^{(1)}[f](q_T) \equiv S^{(0)}[f] + \sqrt{\frac{2b_T^{\text{cut}}}{\pi q_T}} \cos\left(q_T b_T^{\text{cut}} + \frac{\pi}{4}\right) f(b_T^{\text{cut}}) \longleftarrow \text{First correction!}$$

$$\boxed{S[f](q_T) = S^{(1)}[f](q_T, b_T^{\text{cut}}) + \frac{1}{q_T}\mathcal{O}[(b_T^{\text{cut}} q_T)^{-\frac{1}{2}}]}$$

Truncated Functionals



- Systematically add on power corrections

so $S^{(n)}[f] \to S[f]$

$$S^{(0)}[f](q_T, b_T^{\rm cut}) = \int_0^{b_T^{\rm cut}} db_T\, b_T J_0(q_T b_T) f(b_T)$$

$$S^{(1)}[f](q_T, b_T^{\rm cut}) = S^{(0)}[f] + \sqrt{\frac{2b_T^{\rm cut}}{\pi q_T}} f(b_T^{\rm cut}) \cdot \cos(b_T^{\rm cut} q_T + \frac{\pi}{4})$$

$$S^{(2)}[f](q_T, b_T^{\rm cut}) = S^{(1)}[f] - \sqrt{\frac{2b_T^{\rm cut}}{\pi q_T}} \left(\frac{3\, f(b_T^{\rm cut})}{8\, b_T^{\rm cut} q_T} + \frac{f'(b_T^{\rm cut})}{q_T} \right) \cdot \cos(b_T^{\rm cut} q_T - \frac{\pi}{4})$$

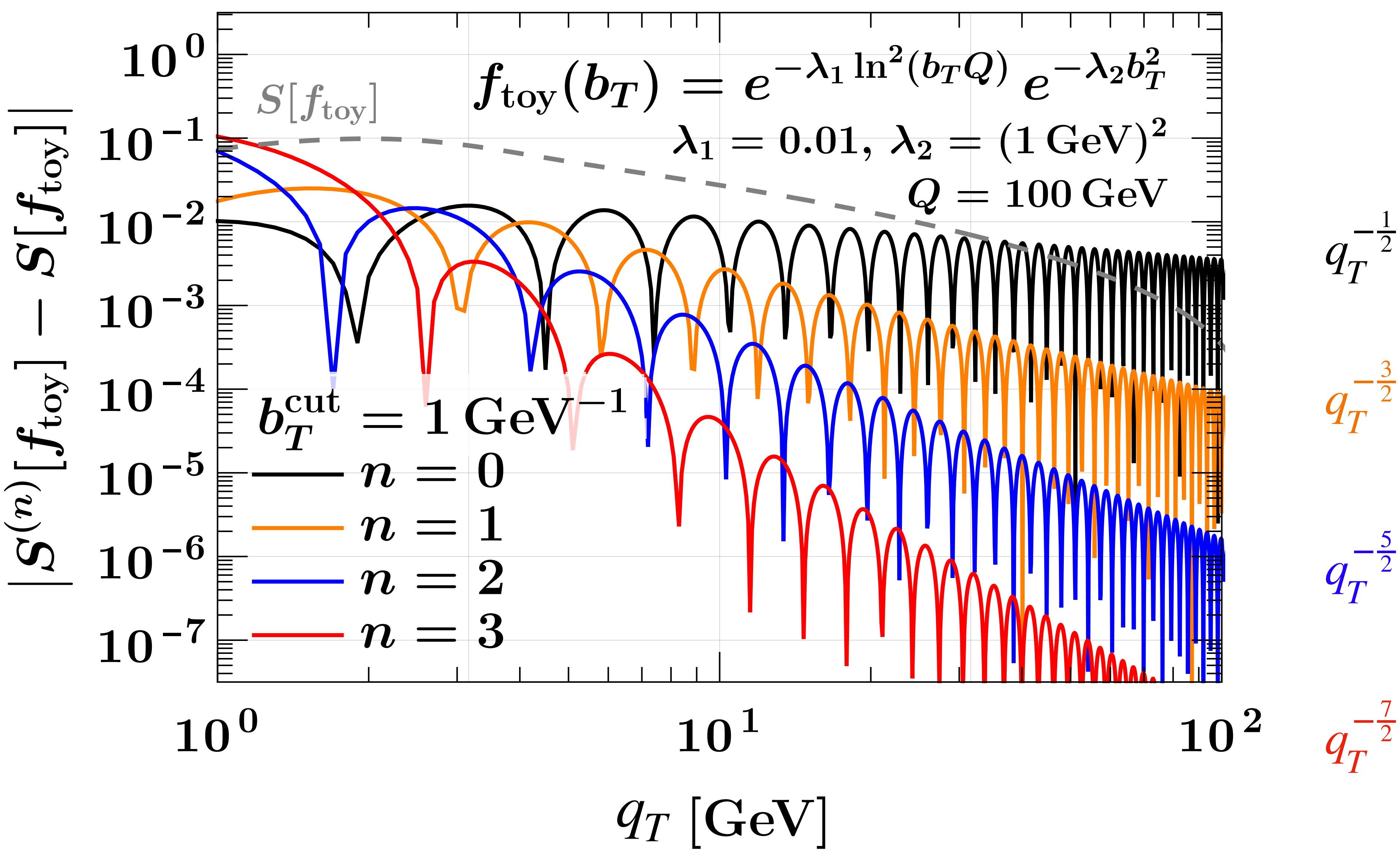
$$S^{(3)}[f](q_T, b_T^{\rm cut}) = S^{(2)}[f] + \sqrt{\frac{2b_T^{\rm cut}}{\pi q_T}} \left(\frac{15\, f(b_T^{\rm cut})}{128\, {b_T^{\rm cut}}^2 q_T^2} - \frac{7\, f'(b_T^{\rm cut})}{8\, b_T^{\rm cut} q_T^2} - \frac{f''(b_T^{\rm cut})}{q_T^2} \right) \cdot \cos(b_T^{\rm cut} q_T + \frac{\pi}{4})$$

$$\boxed{S[f](q_T) = S^{(n)}[f] + \frac{1}{q_T} \mathcal{O}[(b_T^{\rm cut} q_T)^{-n+\frac{1}{2}}]}$$

Power Correction to Functionals

- Toy function $f = \exp[-\lambda_1 \ln^2(b_T Q)]\exp[-\lambda_2 b_T^2]$
- Errors of truncated functionals follow expected power law

$$S[f](q_T) = S^{(n)}[f] + \frac{1}{q_T}\mathcal{O}[(b_T^{\text{cut}} q_T)^{-n+\frac{1}{2}}]$$

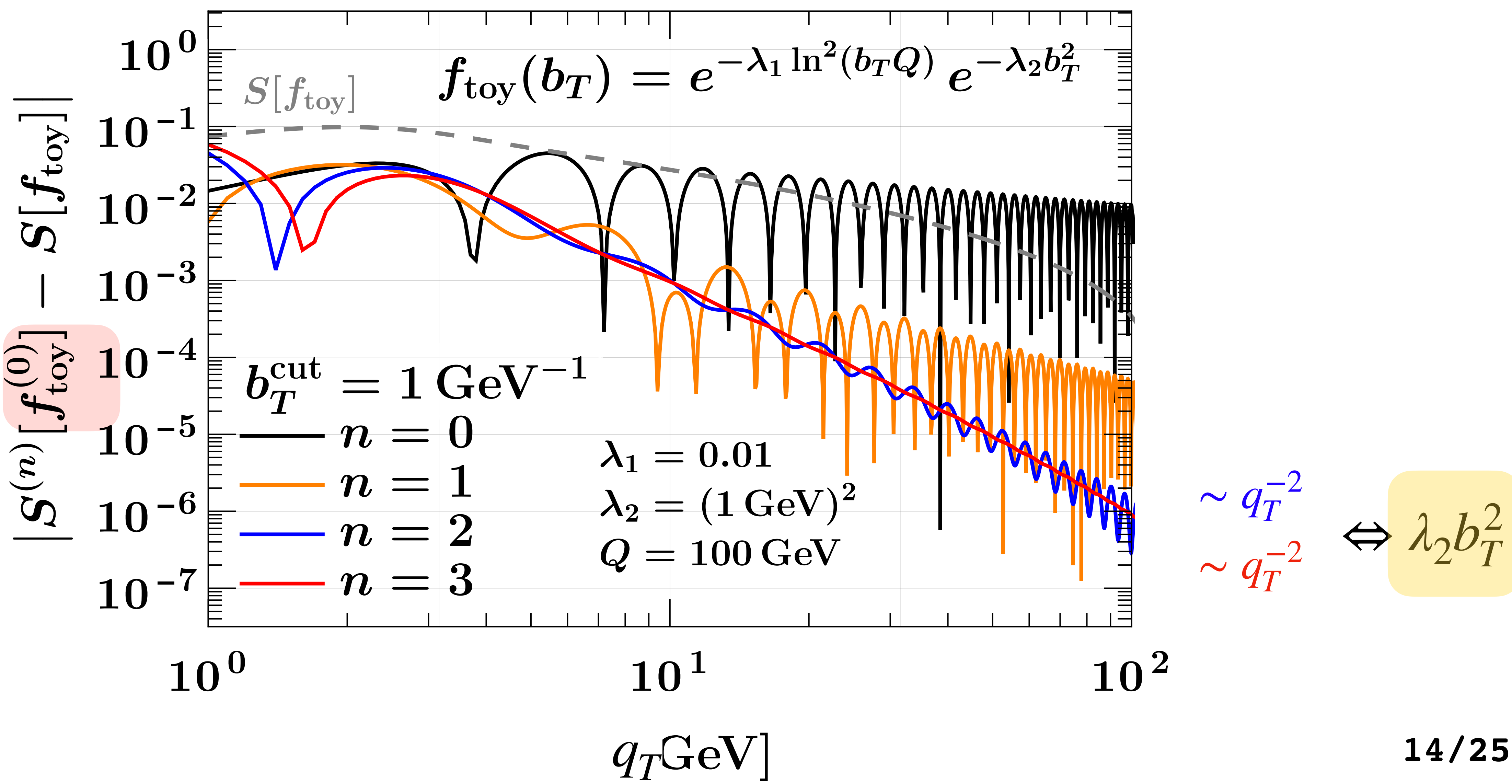


Perturbative Input

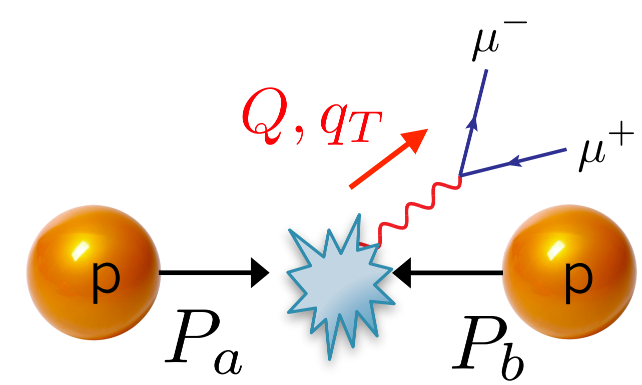
- Power expand toy function and use only “perturbative” input $f^{(0)}$

$$f = \underbrace{\exp[-\lambda_1 \ln^2(b_T Q)]}_{f^{(0)}}(1 - \lambda_2\, b_T^2 + \mathcal{O}(b_T^4))$$

- “Errors" of truncated functionals identify missing quadratic term



Drell-Yan Cross Section



- Leading NP correction for cross section in b_T space:

$$\sigma(b_T) = \sigma^{(0)}(b_T)\Big\{1 + b_T^2\Big(2\overline{\Lambda}^{(2)} + \gamma_{\zeta,q}^{(2)} L_{Q^2}\Big)\Big\} + \mathcal{O}\big[(\Lambda_{\mathrm{QCD}} b_T)^4\big]$$

Perturbative computed to N3LL

Intrinsic to TMDPDF

From evolution

- Pair with the $S[f]$ functionals to get momentum space spectrum

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T} = S^{(3)}\big[\sigma^{(0)}(b_T)\big](q_T) + 2\overline{\Lambda}^{(2)} S^{(3)}\big[b_T^2\sigma^{(0)}(b_T)\big](q_T) + \gamma_{\zeta,q}^{(2)} S^{(3)}\big[b_T^2 L_{Q^2}\sigma^{(0)}(b_T)\big](q_T)$$
$$+ \frac{1}{q_T}\mathcal{O}\Big[(q_T b_T^{\mathrm{cut}})^{-\frac{5}{2}}, \Big(\frac{\Lambda_{\mathrm{QCD}}}{q_T}\Big)^4\Big]$$

Truncating functional

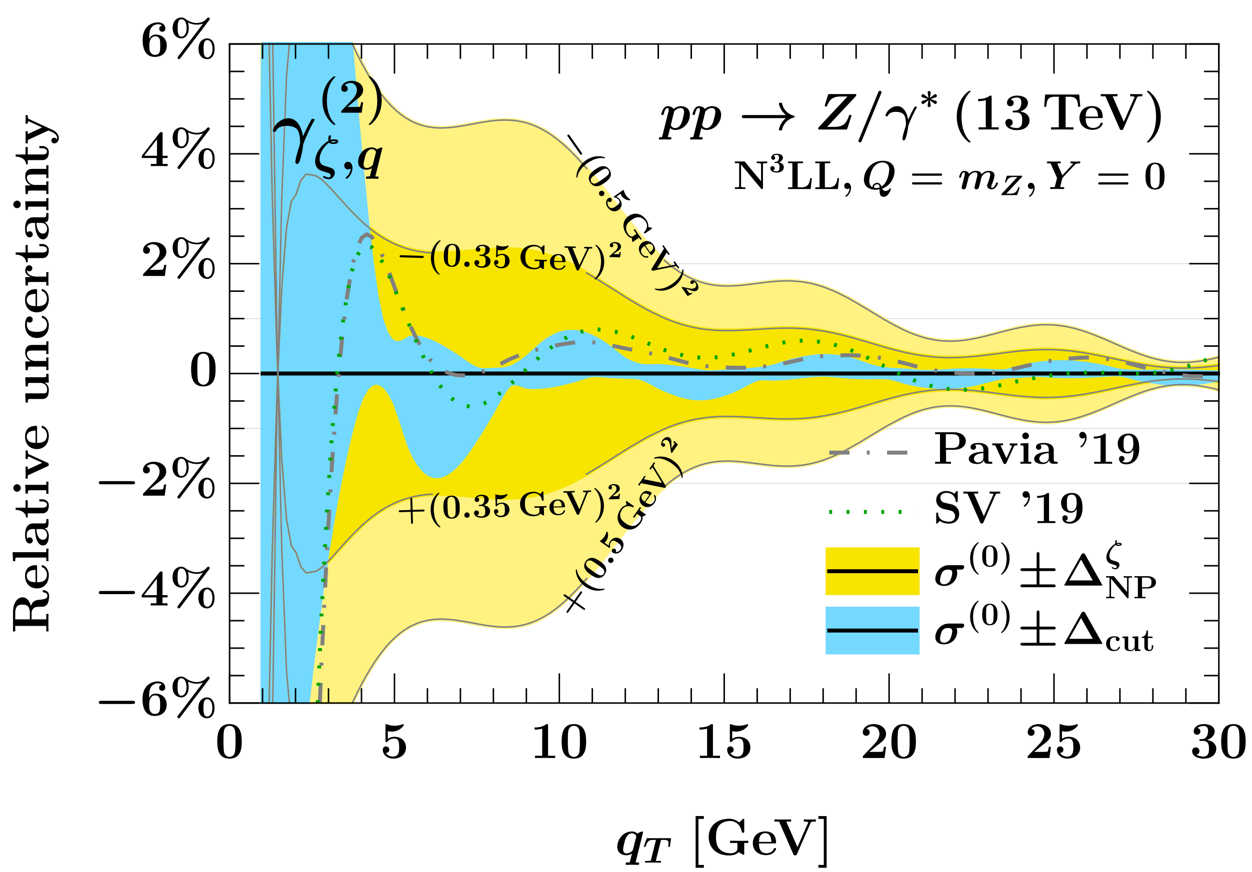
Truncating OPE

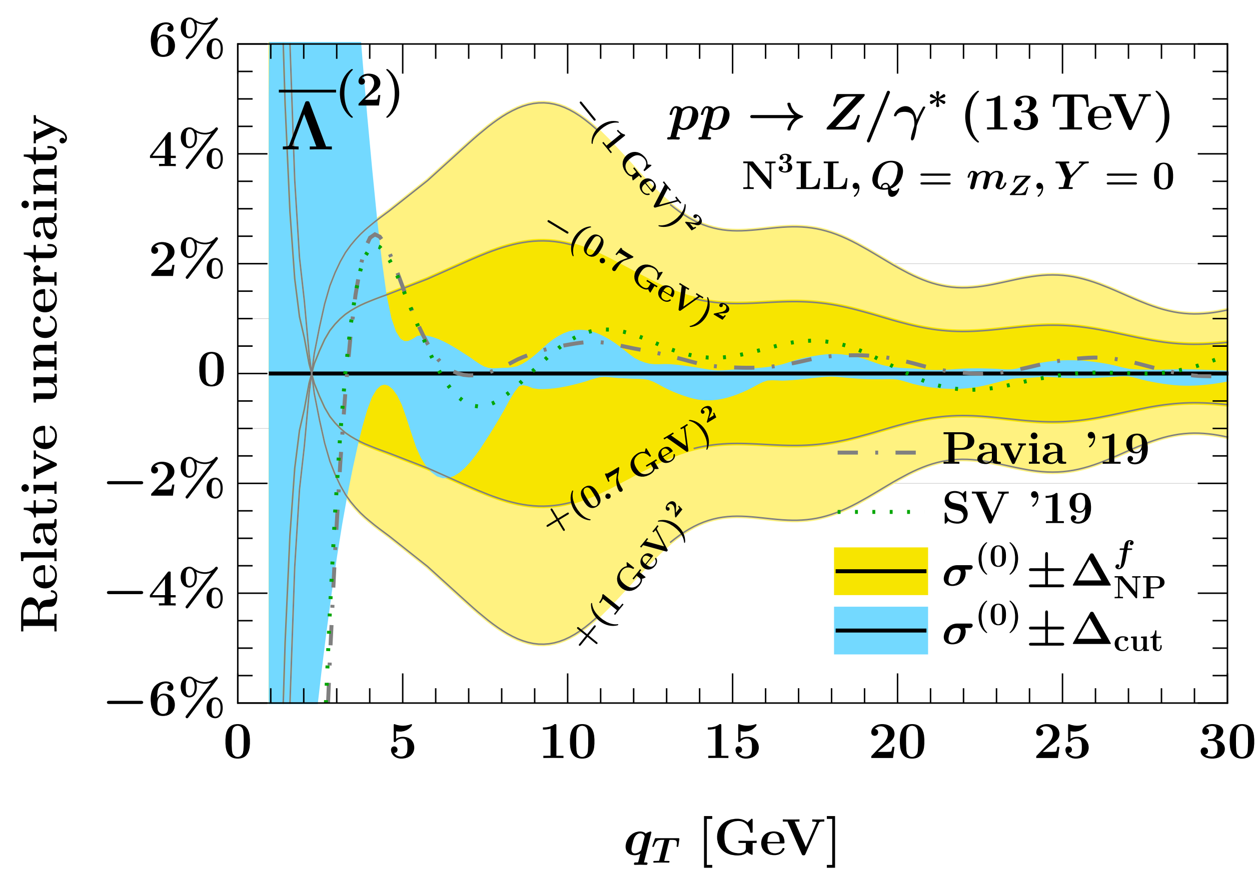
Linearity for the win!

$\Rightarrow$ Useful for estimating uncertainties or putting model-independent constraints on $\gamma_{\zeta,q}^{(2)}$ and $\overline{\Lambda}^{(2)}$

Extracting NP Parameter

- Evaluating impact from each NP parameter individually
- Compare with SV and Pavia global fits
- Can compare with NP parameters from LQCD extraction



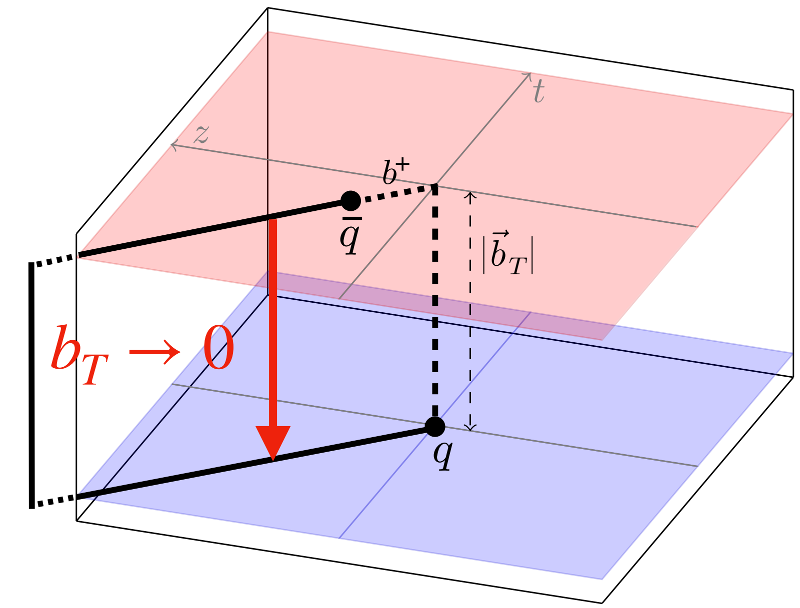


Cumulative Functionals

- What's the normalization of the TMDPDFs?

$$\int d^2\vec{k}_T\, f^{\rm TMD}(x,k_T,\mu,\zeta) \stackrel{?}{=} f^{\rm coll}(x,\mu)$$

naively yes :)



- Renormalization breaks the naive expectation

$$\mu\frac{d}{d\mu}\int d^2\vec{k}_T\, f^{\rm TMD}(x,k_T,\mu,\zeta) \neq \mu\frac{d}{d\mu} f^{\rm coll}(x,\mu)$$

renormalization says no :(

Cumulative Functionals

- Investigate $\int d^2\vec{k}_T \, f^{\mathrm{TMD}}(x, k_T, \mu, \zeta) \stackrel{?}{=} f^{\mathrm{coll}}(x, \mu)$

- Cumulative distribution with a UV cutoff:

$$\int_{|k_T| \le k_T^{\mathrm{cut}}} d^2\vec{k}_T \, f(k_T) = \int_{|k_T| \le k_T^{\mathrm{cut}}} d^2\vec{k}_T \int \frac{d^2 b_T}{(2\pi)^2} \, e^{+i\vec{k}_T \cdot \vec{b}_T} f(b_T)$$

$$= \int^{k_T^{\mathrm{cut}}} dk_T \, k_T \int_0^\infty db_T \, b_T J_0(b_T k_T) f(b_T)$$

$$= \underbrace{k_T^{\mathrm{cut}} \int_0^\infty db_T \, J_1(b_T k_T^{\mathrm{cut}}) \, f(b_T)}_{K[f](k_T^{\mathrm{cut}})}$$

- Approximate using perturbative region:

$$K^{(0)}[f](k_T^{\mathrm{cut}}, b_T^{\mathrm{cut}}) = k_T^{\mathrm{cut}} \int_0^{b_T^{\mathrm{cut}}} db_T \, J_1(b_T k_T^{\mathrm{cut}}) \, f(b_T)$$

Cumulative Functionals

- Systematically add on power corrections so $K^{(n)}[f] \to K[f]$

$$K^{(0)}[f](k_T^{\rm cut}, b_T^{\rm cut}) = k_T^{\rm cut} \int_0^{b_T^{\rm cut}} db_T \, J_1(b_T k_T^{\rm cut})\, f(b_T)$$

$$K^{(1)}[f](k_T^{\rm cut}, b_T^{\rm cut}) = K^{(0)}[f] + f(b_T^{\rm cut}) \cdot \frac{\cos(b_T^{\rm cut} k_T^{\rm cut} - \frac{\pi}{4})}{\sqrt{\frac{\pi}{2}}(b_T^{\rm cut} k_T^{\rm cut})^{1/2}}$$

$$K^{(2)}[f](k_T^{\rm cut}, b_T^{\rm cut}) = K^{(1)}[f] - \left(\frac{f(b_T^{\rm cut})}{8\, b_T^{\rm cut} k_T^{\rm cut}} - \frac{f'(b_T^{\rm cut})}{k_T^{\rm cut}}\right) \cdot \frac{\cos(b_T^{\rm cut} k_T^{\rm cut} + \frac{\pi}{4})}{\sqrt{\frac{\pi}{2}}(b_T^{\rm cut} k_T^{\rm cut})^{1/2}}$$

$$K^{(3)}[f](k_T^{\rm cut}, b_T^{\rm cut}) = K^{(2)}[f] - \left(\frac{9 f(b_T^{\rm cut})}{128\, b_T^{{\rm cut}\,2} k_T^{{\rm cut}\,2}} - \frac{5 f'(b_T^{\rm cut})}{8\, b_T^{\rm cut} k_T^{{\rm cut}\,2}} + \frac{f''(b_T^{\rm cut})}{k_T^{{\rm cut}\,2}}\right) \cdot \frac{\cos(b_T^{\rm cut} k_T^{\rm cut} - \frac{\pi}{4})}{\sqrt{\frac{\pi}{2}}(b_T^{\rm cut} k_T^{\rm cut})^{1/2}}$$

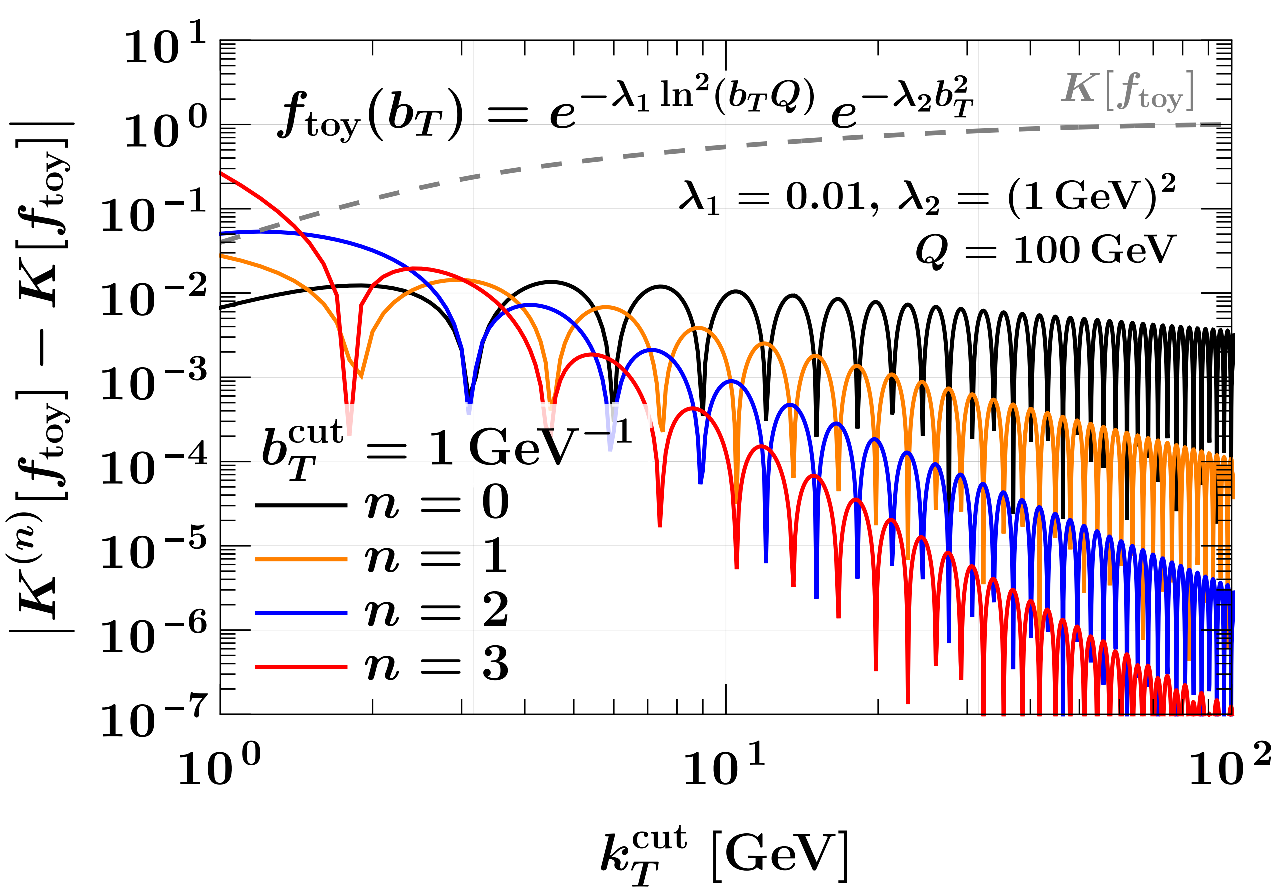
$$\boxed{K[f](k_T^{\rm cut}) = K^{(n)}[f](k_T^{\rm cut}, b_T^{\rm cut}) + \mathcal{O}[(b_T^{\rm cut} k_T^{\rm cut})^{-n-\frac{1}{2}}]}$$

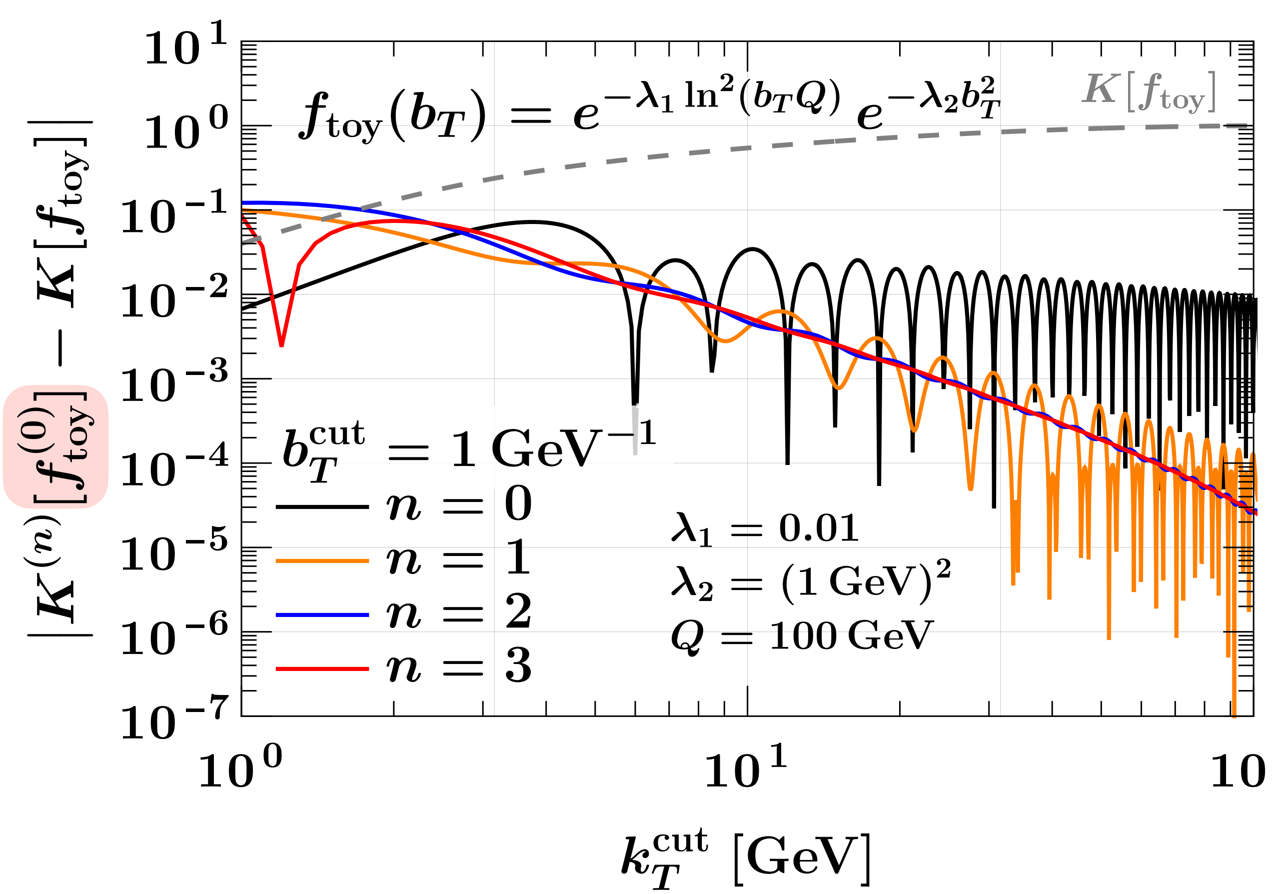
Power Correction of $K^{(n)}$

- Same toy function $\quad f = \exp[-\lambda_1 \ln^2(b_T Q)]\,\exp[-\lambda_2\, b_T^2]$

- Errors of truncated functionals follow expected power law

$$K[f](k_T^{\text{cut}}) = K^{(n)}[f](k_T^{\text{cut}}, b_T^{\text{cut}}) + \mathcal{O}[(b_T^{\text{cut}} k_T^{\text{cut}})^{-n-\frac{1}{2}}]$$





Perturbative Baseline

- Leading NP correction for TMDPDF:

$L_\zeta = \ln(b_T^2\zeta/b_0^2)$

$$f_i(x,b_T,\mu,\zeta) = \underbrace{f_i^{(0)}(x,b_T,\mu,\zeta)}_{\text{Perturbative, use matching computed to N3LL}}\Big\{1 + b_T^2\Big(\underbrace{\Lambda_i^{(2)}(x)}_{\text{Intrinsic}} + \frac{1}{2}\underbrace{\gamma_{\zeta,i}^{(2)}}_{\text{From NP evolution}} L_\zeta\Big)\Big\} + \mathcal{O}\big[(\Lambda_{\mathrm{QCD}} b_T)^4\big]$$

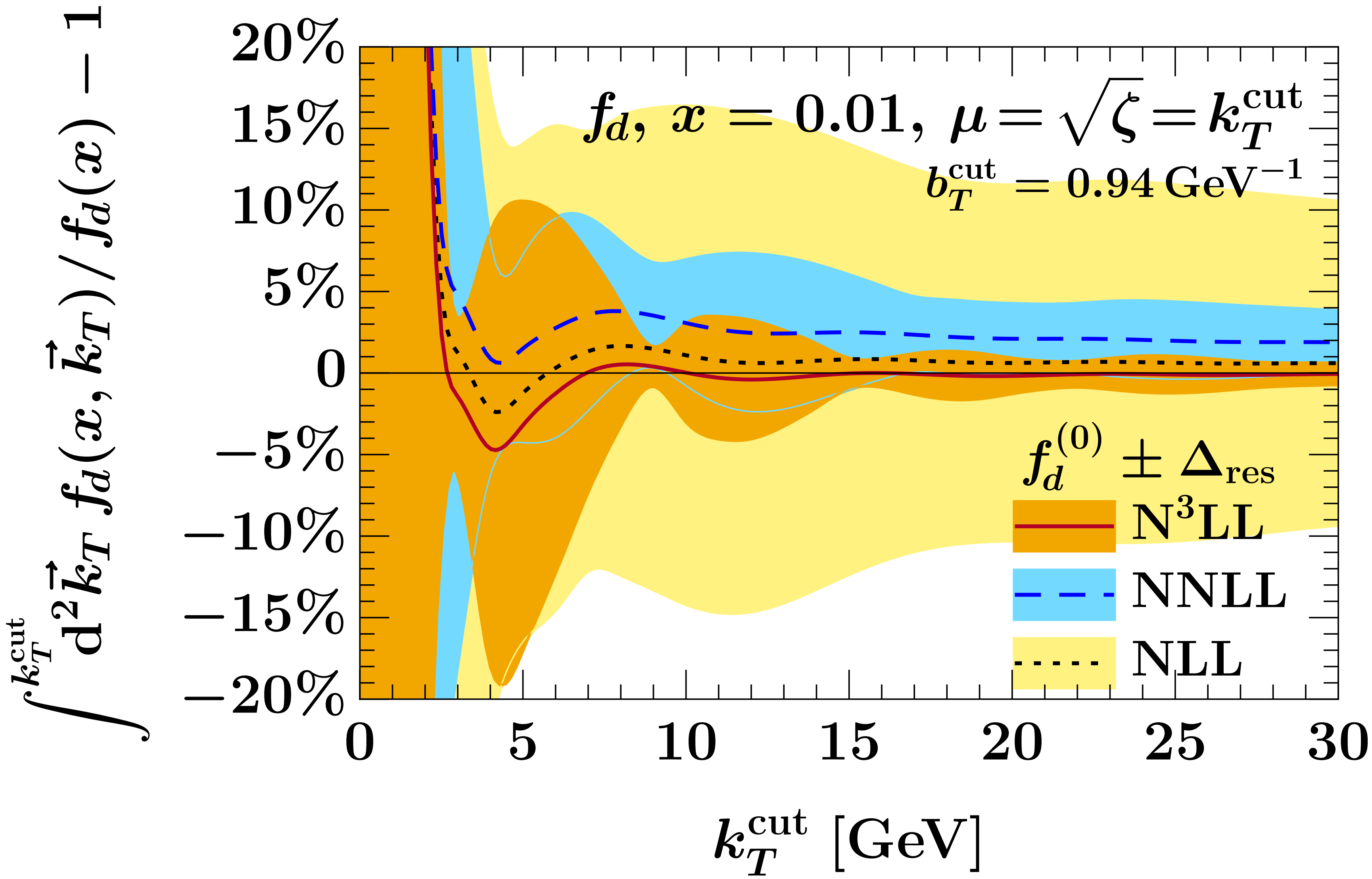
- Pair with the $K[f]$ functionals to get the TMDPDF cumulant

$$\int^{k_T^{\mathrm{cut}}} d^2\vec{k}_T\, f_i(k_T) = \underbrace{\boxed{K^{(3)}[f_i^{(0)}](k_T^{\mathrm{cut}})}}_{\text{Model-independent, fully perturbative}} + \Lambda_i^{(2)} K^{(3)}[b_T^2 f_i^{(0)}] + \frac{1}{2}\gamma_{\zeta,i}^{(2)} K^{(3)}[b_T^2 L_\zeta f_i^{(0)}]$$

$$+\, \mathcal{O}\Big[\underbrace{\big(k_T^{\mathrm{cut}} b_T^{\mathrm{cut}}\big)^{-\frac{7}{2}}}_{\text{Truncating functional}},\, \underbrace{\Big(\frac{\Lambda_{\mathrm{QCD}}}{k_T^{\mathrm{cut}}}\Big)^4}_{\text{Truncating OPE}}\Big]$$

Cumulant of TMDPDFs

- Approximate the cumulant using $K^{(3)}[f^{(0)}_{\rm TMD}]$ and normalize to $f^{\rm coll}$
- Deviation is small! $\int^{k_T^{\rm cut}} d^2\vec{k}_T\, f^{\rm TMD}(x, k_T, \mu = \sqrt{\zeta} = k_T^{\rm cut}) = f^{\rm coll}(x, \mu = k_T^{\rm cut})$ YES!



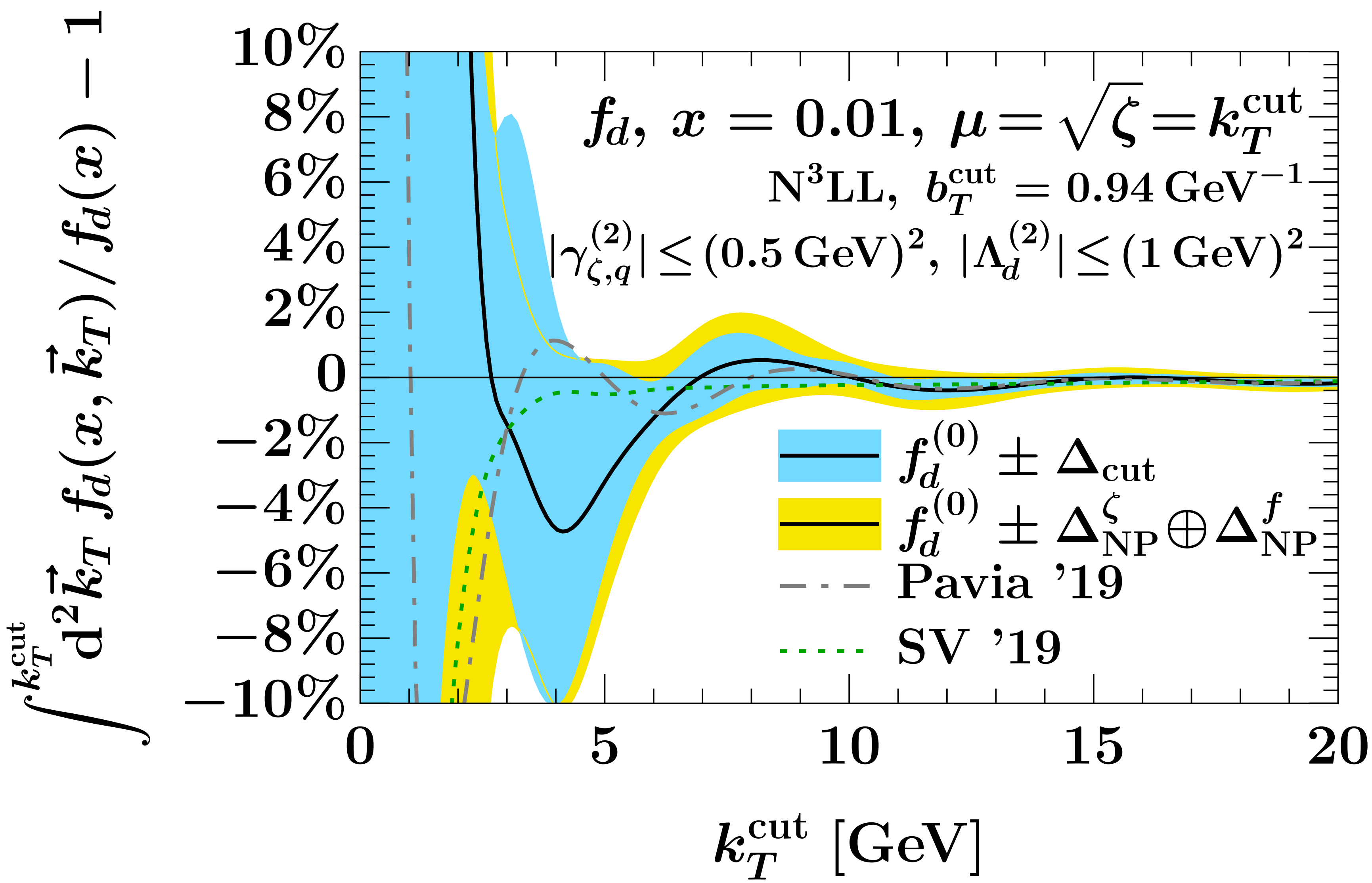
- Model-independent confirmation of previous results at $\mu = k_T^{\rm cut}$

Bacchetta & Prokudin (1303.2129)

- Central value consistent with intuitive result within percents
- $\Delta_{\rm res}$ is perturbative uncertainties estimated by scale variations

Cumulant of TMDPDFs

- Deviation is small! $\int^{k_T^{\rm cut}} d^2\vec{k}_T\, f^{\rm TMD}(x, k_T, \mu = \sqrt{\zeta} = k_T^{\rm cut}) = f^{\rm coll}(x, \mu = k_T^{\rm cut})$ YES!
- NP uncertainty much smaller than perturbative uncertainty



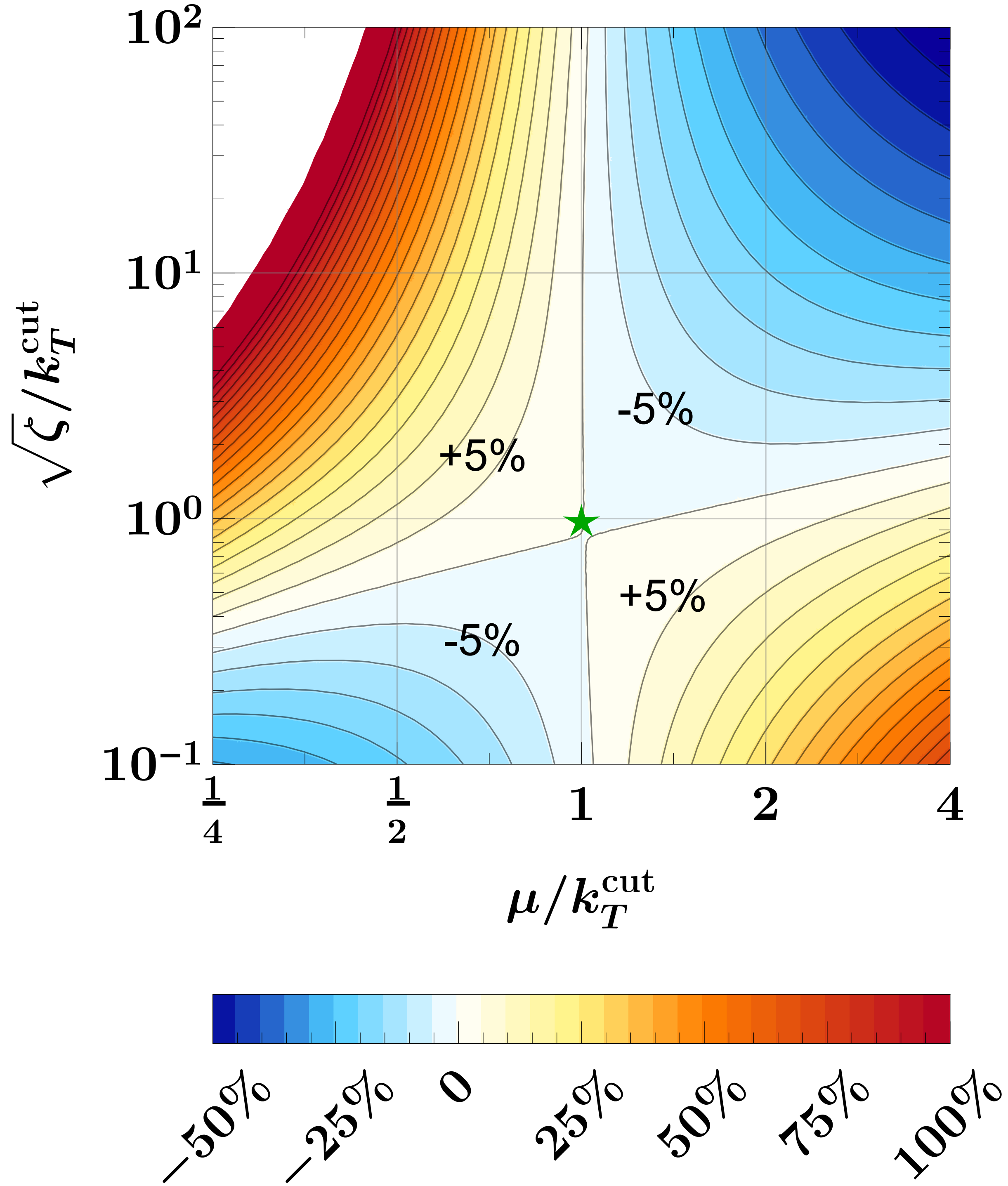
- $\Delta_{\rm cut}$ from varying $b_T^{\rm cut}$
- $\Delta_{\rm NP}$ from varying $\gamma_\zeta^{(2)}$ and $\Lambda^{(2)}$
- Small deviation supported by SV and Pavia model-based global fits

Scimemi & Vladimirov (1912.06532)
Bacchetta et al: 1912.07550

Impact of Evolution Effects

$$\int^{k_T^{\rm cut}} \mathrm{d}^2\vec{k}_T\, f_d(x, \vec{k}_T, \mu, \zeta)/f_d(x,\mu) - 1$$

$x = 0.01,\, k_T^{\rm cut} = 10\,\mathrm{GeV}$



- Intuitive expectation is robust in the vicinity of $\mu = \sqrt{\zeta} = k_T^{\rm cut}$
- For $\mu = k_T^{\rm cut}$, the ζ evolution is negligible*
- Sizable corrections from evolution away from these regions, due to the cusp anomalous dimension
- Evolution effect matters, but at the natural scale $\mu = k_T^{\rm cut}$ the intuition is valid

$$\boxed{\int^{k_T^{\rm cut}} d^2\vec{k}_T\, f^{\rm TMD}(x, k_T, \mu = k_T^{\rm cut}, \zeta) = f^{\rm coll}(x, \mu = k_T^{\rm cut})}$$

Conclusions

- Disentangling perturbative and nonperturbative physics in TMD PDFs helps make use of lattice data

- Truncated functionals provide a model-independent and systematically improvable method to map perturbative results from position to momentum space

- Developed model-independent method to assess the impact of non-perturbative effects (OPE coefficients) in momentum space, can be compared to lattice extraction

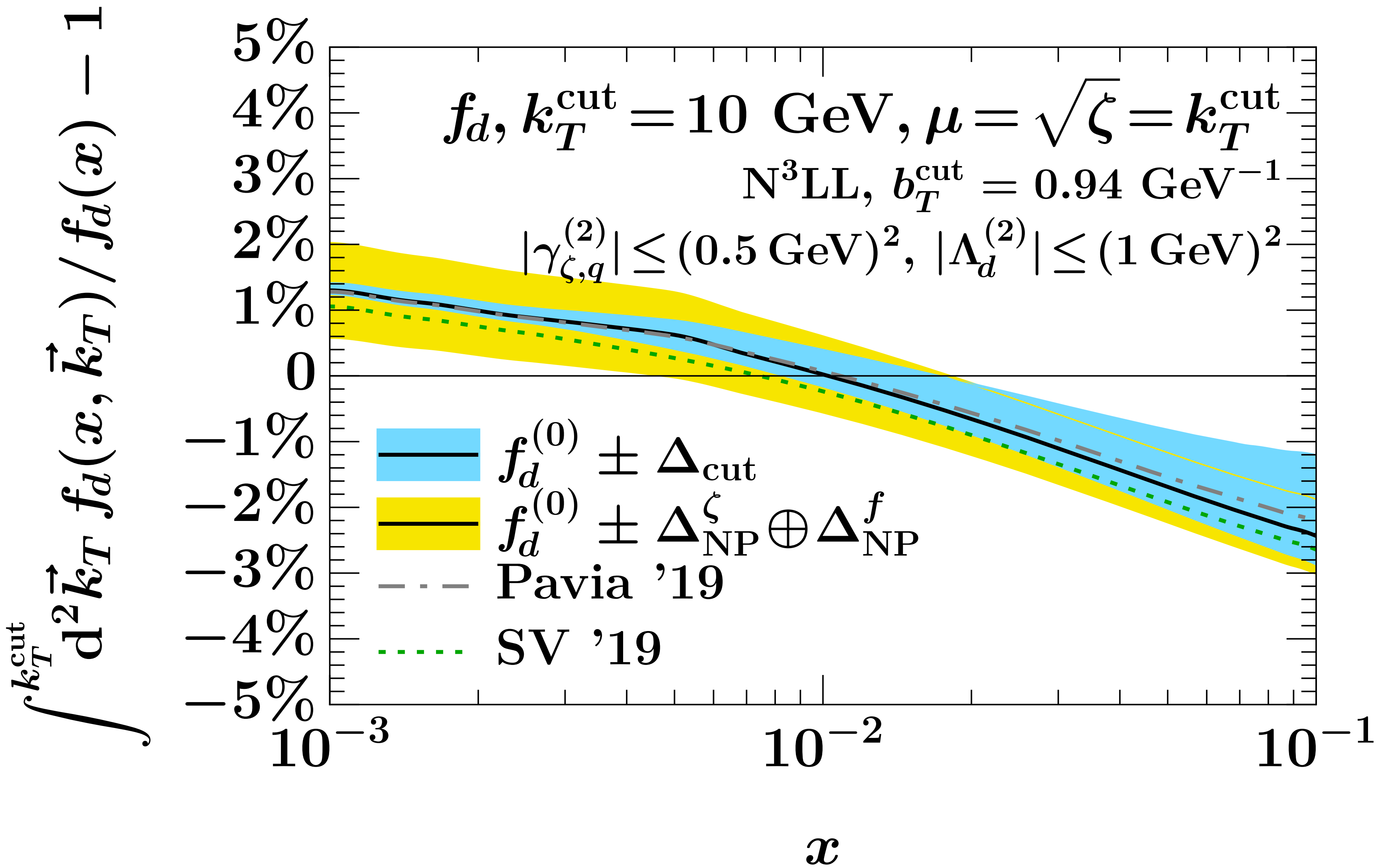
- Demonstrated that integrating the unpolarized TMD PDF over $[0,\ k_T^{\mathrm{cut}}]$ gives the collinear PDF to the percent level (when renormalization scale $\mu = k_T^{\mathrm{cut}}$)

Thank you!!!

Back-up Slides

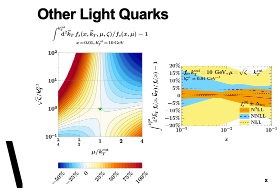
Cumulant of TMDPDFs

- Deviation is small! $\int^{k_T^{\text{cut}}} d^2\vec{k}_T\, f^{\text{TMD}}(x, k_T, \mu = \sqrt{\zeta} = k_T^{\text{cut}}) = f^{\text{coll}}(x, \mu = k_T^{\text{cut}})$ YES!
- Test the observation as a function of x and keep k_T^{cut} fixed



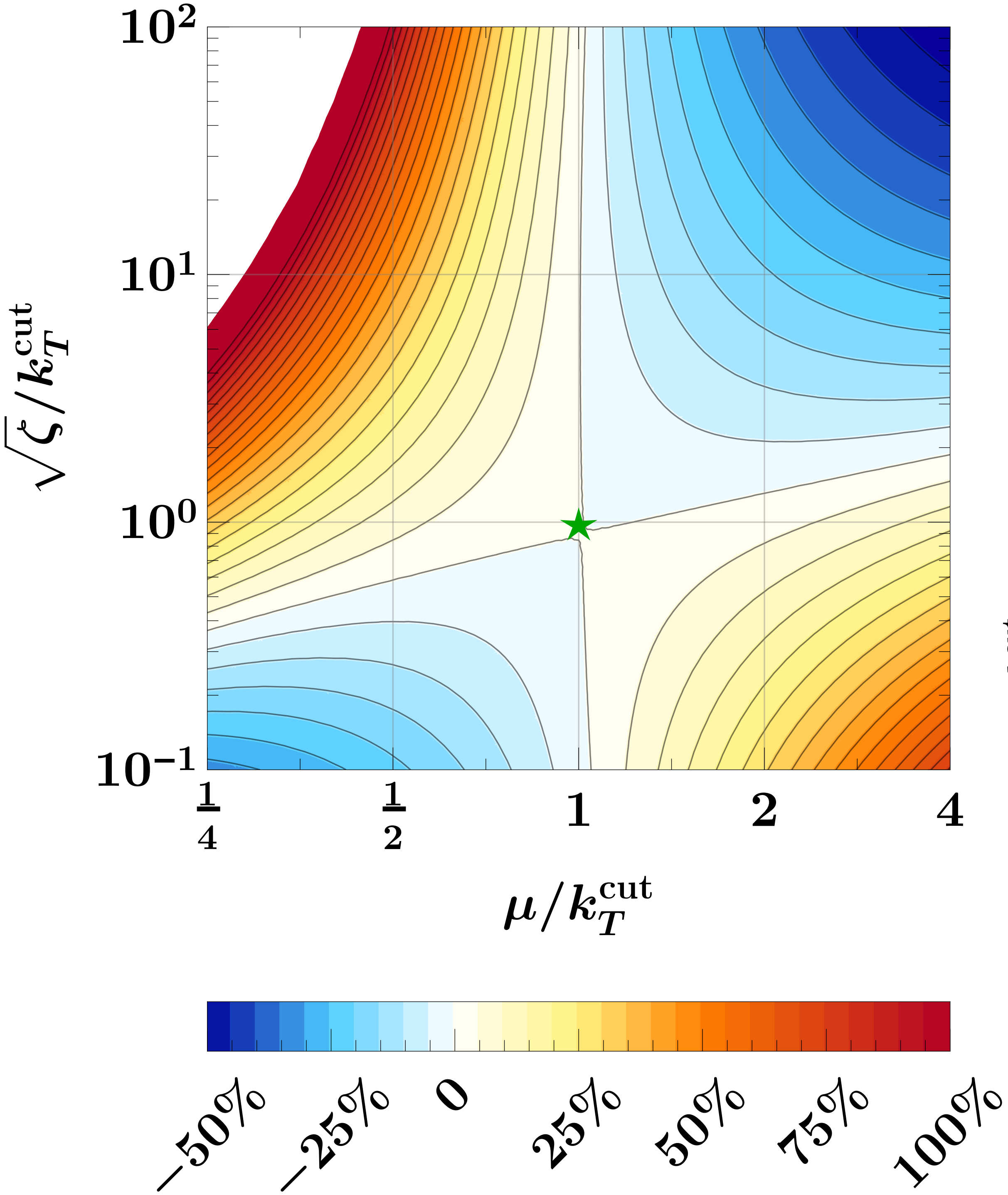
- Central value can differ from zero by 2%
- Small deviation supported by model-based global fits

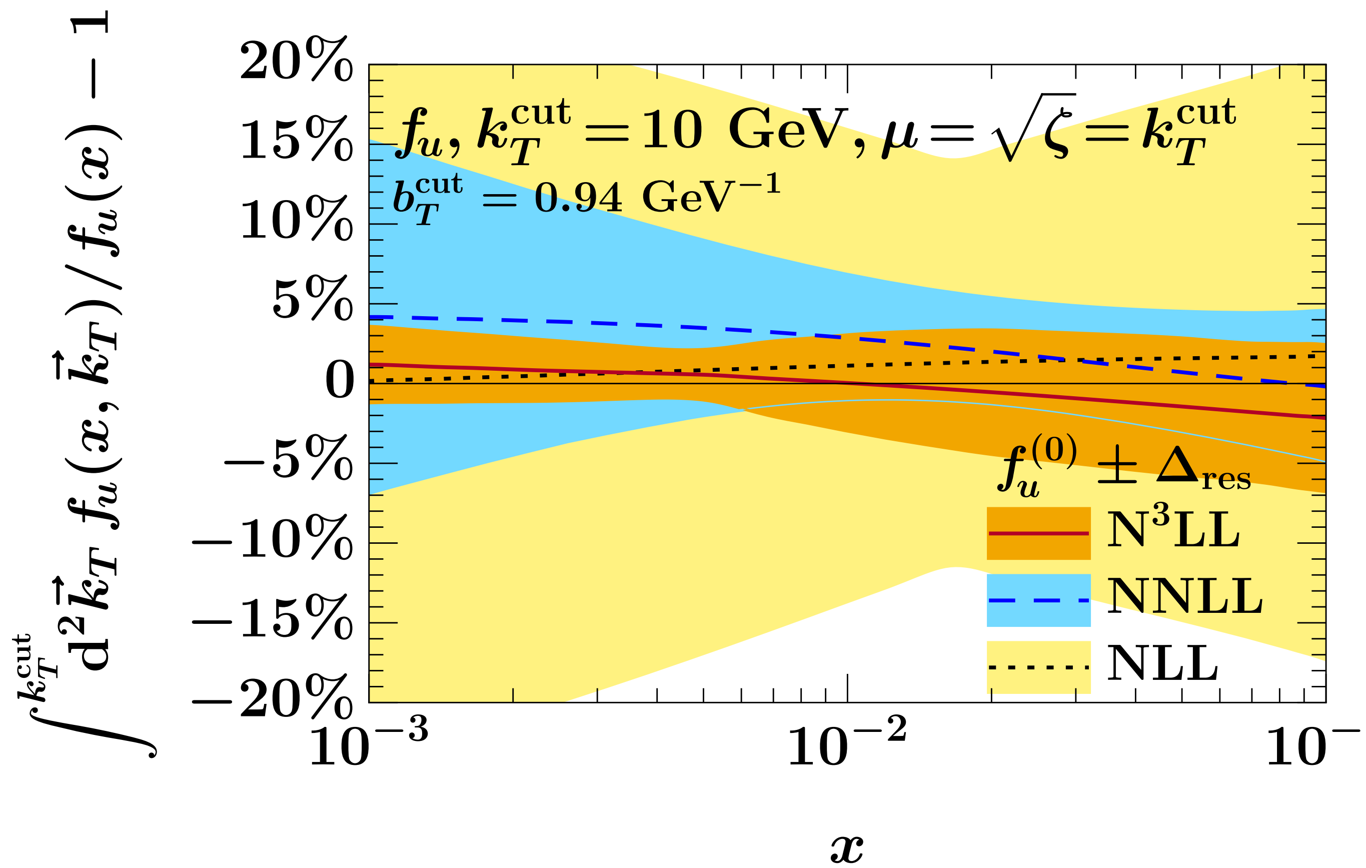
Scimemi & Vladimirov (1912.06532)
Bacchetta et al: 1912.07550



$$\int^{k_T^{\rm cut}} \mathrm{d}^2\vec{k}_T\, f_u(x,\vec{k}_T,\mu,\zeta)/f_u(x,\mu) - 1$$

$$x = 0.01,\, k_T^{\rm cut} = 10\,\mathrm{GeV}$$

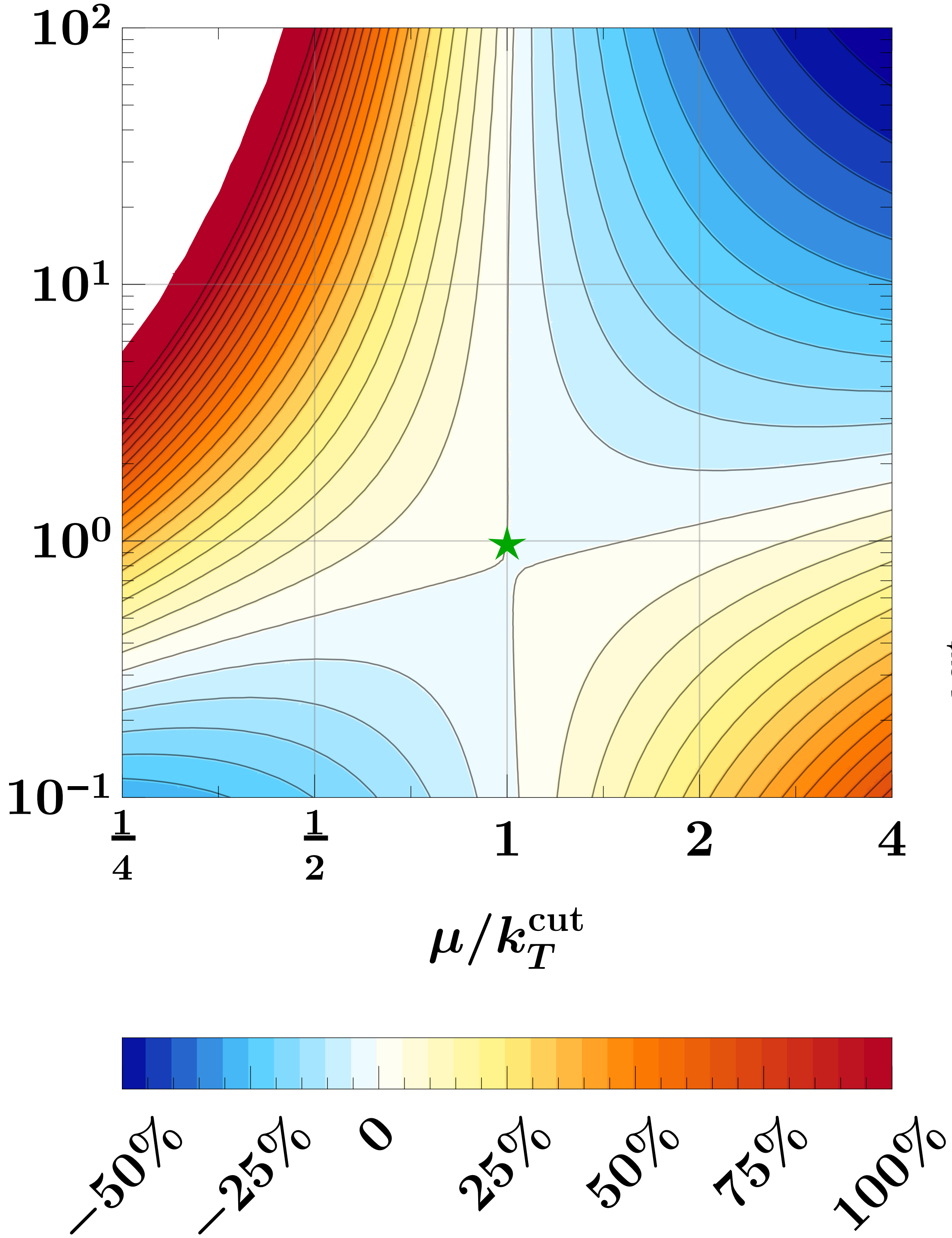


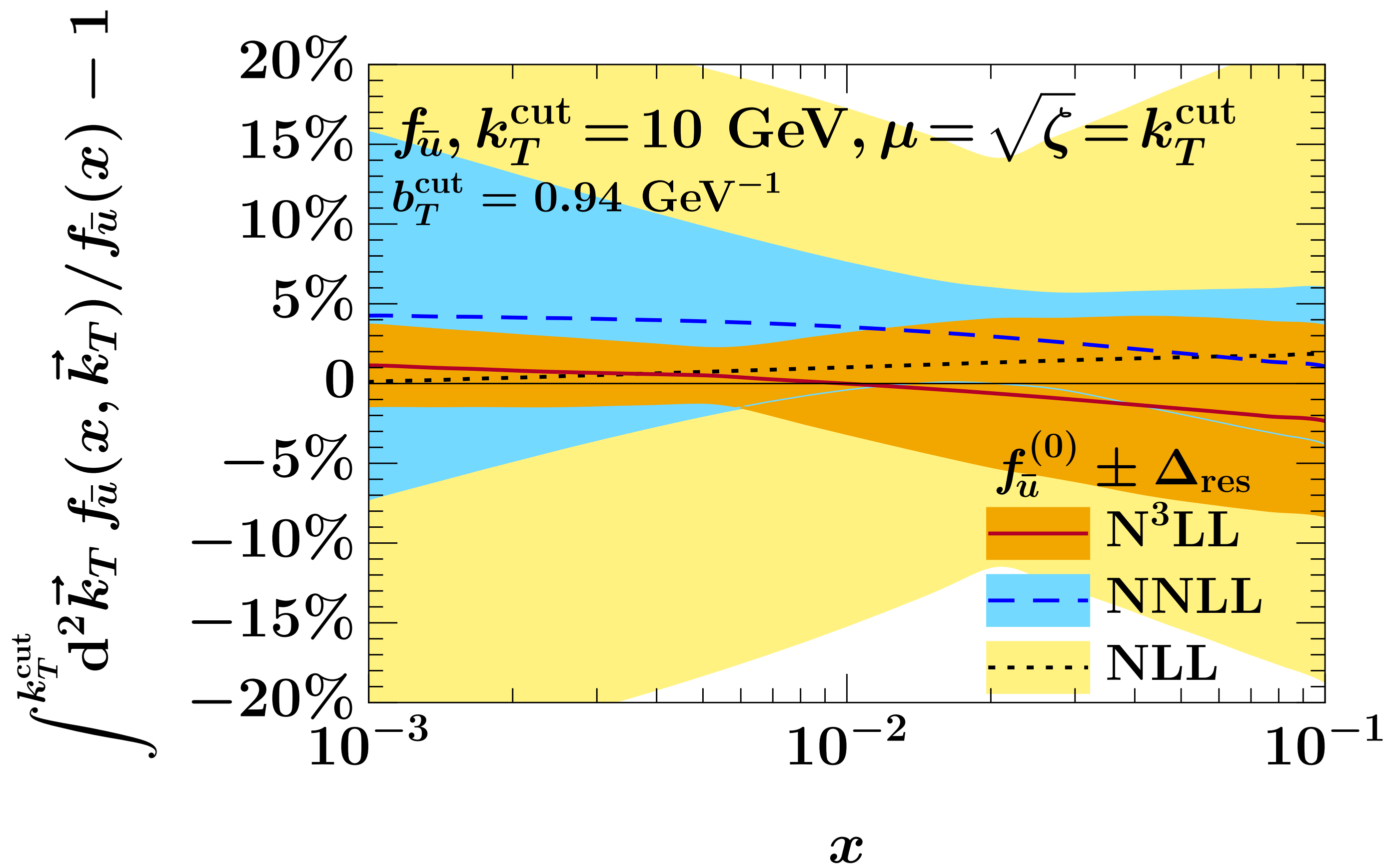


Other Light Quarks

$$\int^{k_T^{\rm cut}} \mathrm{d}^2\vec{k}_T\, f_{\bar{u}}(x,\vec{k}_T,\mu,\zeta)/f_{\bar{u}}(x,\mu) - 1$$

$$x = 0.01,\, k_T^{\rm cut} = 10\,\mathrm{GeV}$$

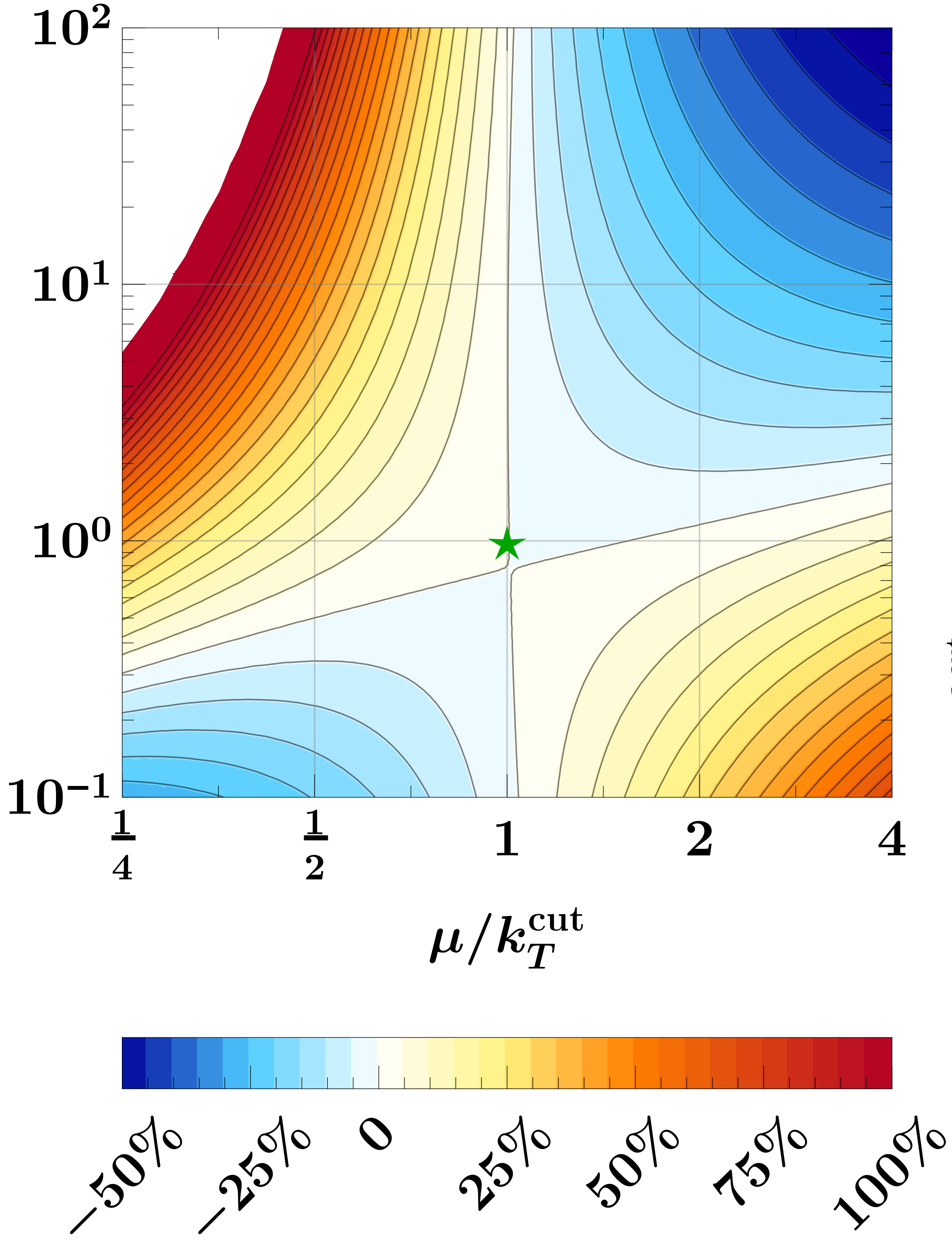


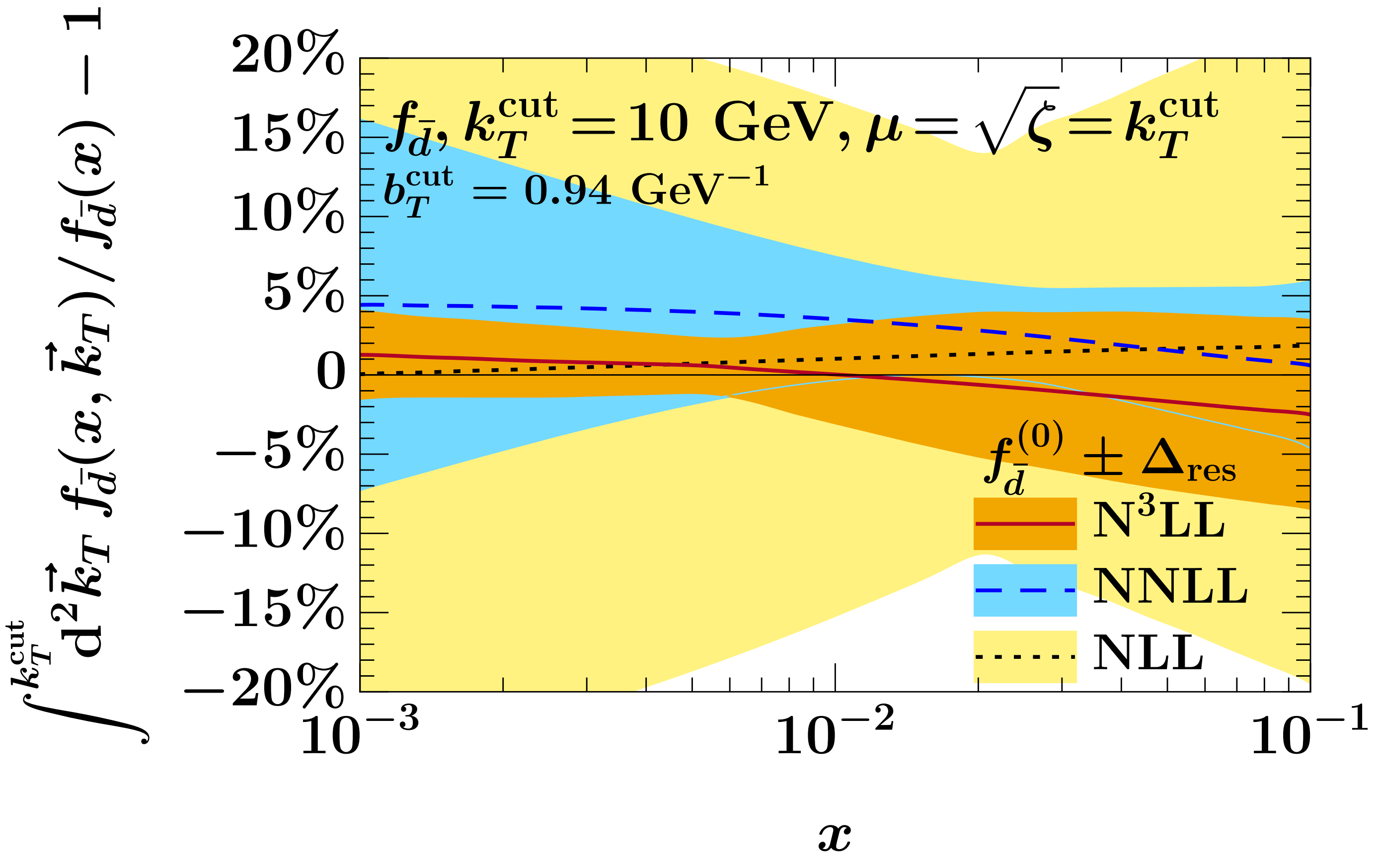


Other Light Quarks

$$\int^{k_T^{\text{cut}}} \mathrm{d}^2\vec{k}_T\, f_{\bar{d}}(x, \vec{k}_T, \mu, \zeta) / f_{\bar{d}}(x, \mu) - 1$$

$x = 0.01,\, k_T^{\text{cut}} = 10\,\text{GeV}$





Other Light Quarks

$$\int^{k_T^{\rm cut}} \mathrm{d}^2\vec{k}_T\, f_s(x,\vec{k}_T,\mu,\zeta)/f_s(x,\mu) - 1$$

$$x = 0.01,\; k_T^{\rm cut} = 10\,\mathrm{GeV}$$