TMDs: Towards a Synergy between Lattice QCD and Global Analyses
June 21, 2023

Global extraction of Transverse Momentum Distributions

Chiara Bissolotti
Argonne National Laboratory

on behalf of the MAP Collaboration

HAS QCD
HADRONIC STRUCTURE AND QUANTUM CHROMODYNAMICS

3DSPIN
MAPPING THE PROTON IN 3D

INFN
Istituto Nazionale di Fisica Nucleare

UNIVERSITÀ DI PAVIA

Extraction of unpolarized quark TMDs

What's new?

JHEP 10 (2022) 127

- **GLOBAL ANALYSIS** of Drell-Yan and Semi-Inclusive DIS data sets → **2031** data points

- Perturbative accuracy: $\mathrm{N^3LL^-}$

- **Normalization** of SIDIS multiplicities beyond NLL

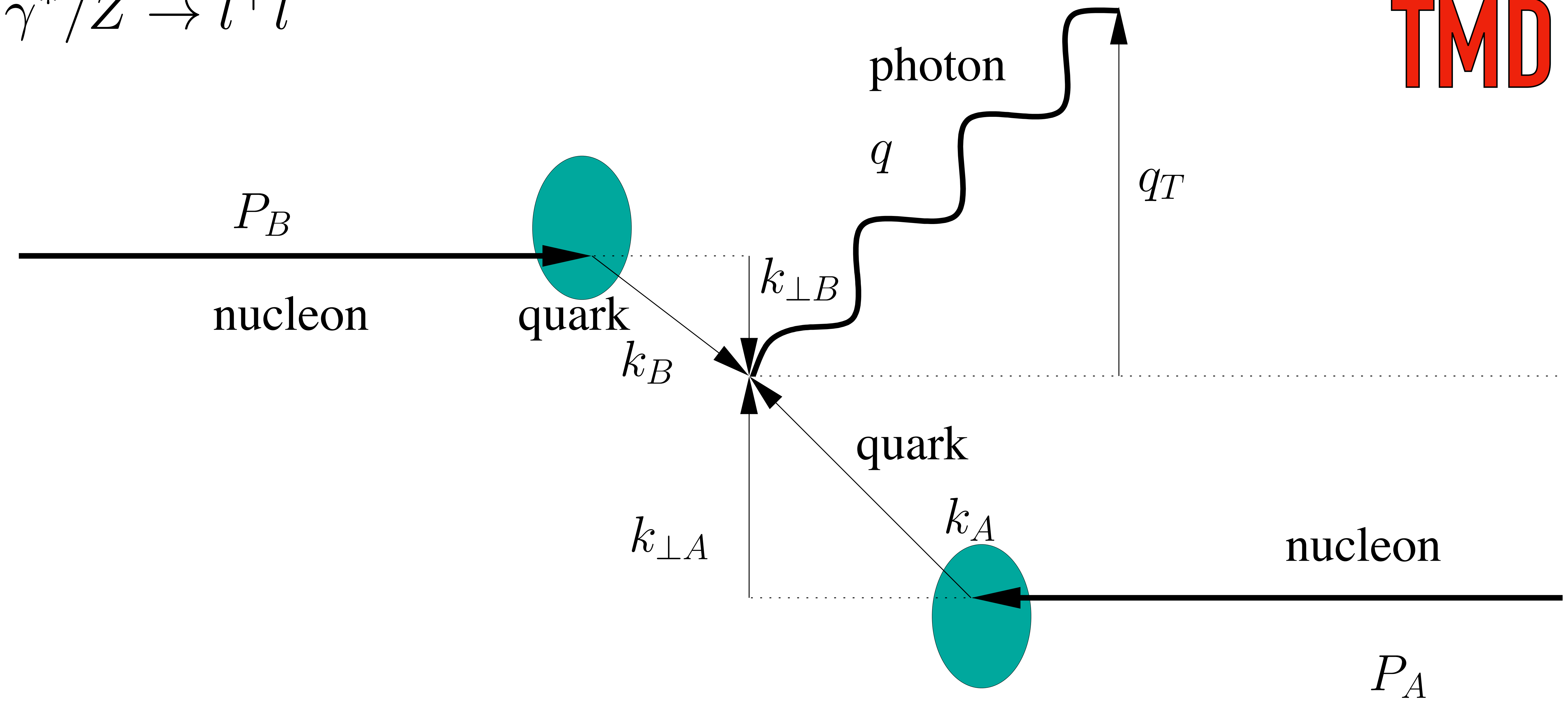
- Extremely good description: $\chi^2/N_{\text{data}} = 1.06$

Drell-Yan

$N(P_A) + N(P_B) \to \gamma^*/Z \to l^+ l^-$

for $q_T \ll Q$

TMD factorization

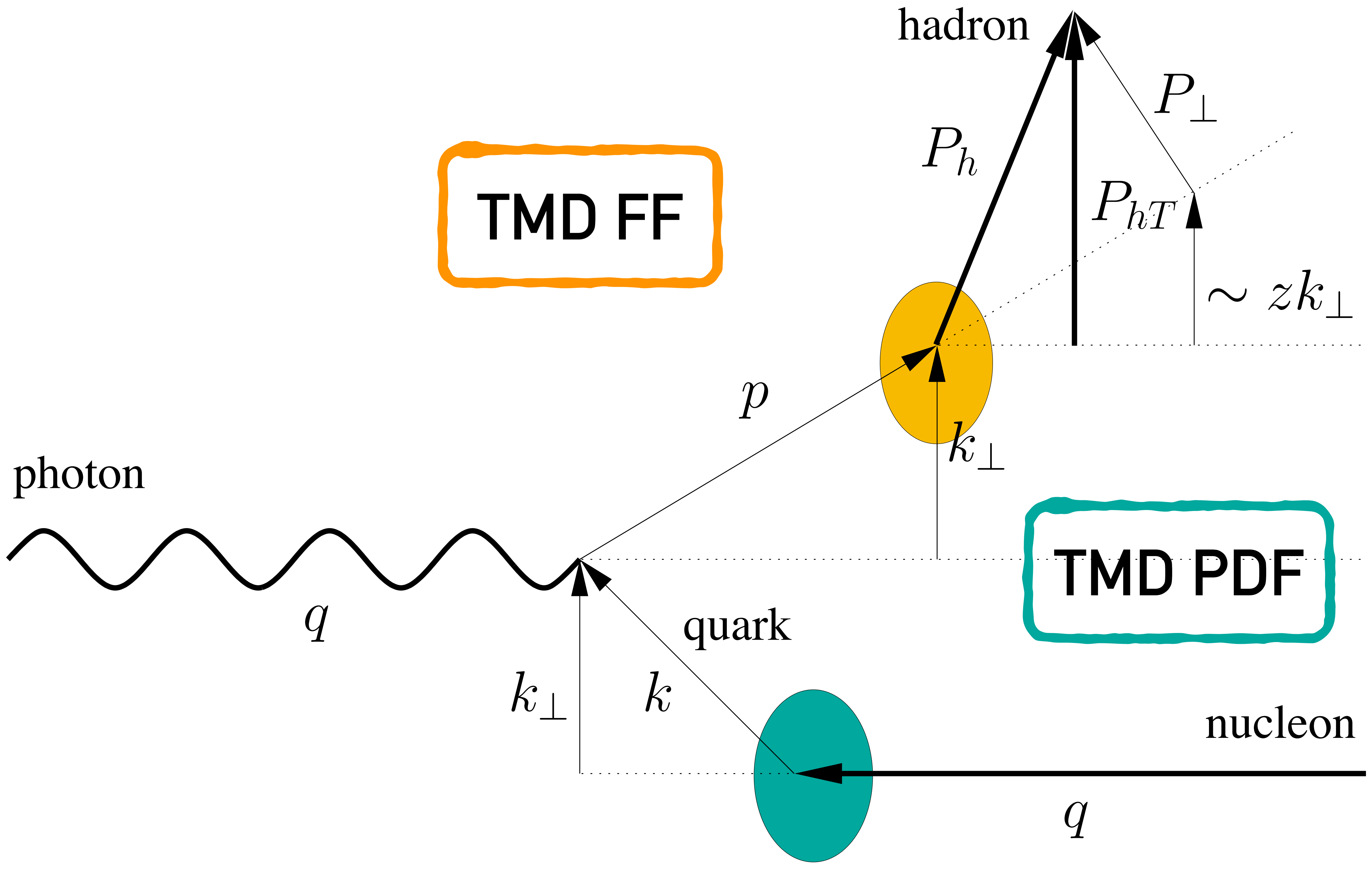


$$F_{UU}^1(x_A, x_B, \boldsymbol{q}_T^2, Q^2)$$
$$= \sum_a \mathcal{H}_{UU}^{1a}(Q^2, \mu^2) \int d^2\boldsymbol{k}_{\perp A}\, d^2\boldsymbol{k}_{\perp B}\, f_1^a(x_A, \boldsymbol{k}_{\perp A}^2; \mu^2)\, f_1^{\bar{a}}(x_B, \boldsymbol{k}_{\perp B}^2; \mu^2)\, \delta^{(2)}(\boldsymbol{k}_{\perp A} - \boldsymbol{q}_T + \boldsymbol{k}_{\perp B})$$
$$+ Y_{UU}^1(Q^2, \boldsymbol{q}_T^2) + \mathcal{O}(M^2/Q^2)$$

SIDIS

Semi-Inclusive Deep Inelastic Scattering

$\ell(l) + N(p) \to \ell(l') + h(P_h) + X$



TMD factorization

$$P_{hT}^2 \ll Q^2$$

$$F_{UU,T}(x\,.\,z;\mu_F,\mathbf{P}_{hT}^2,Q^2) = x\sum_a H_{UU,T}^a(Q^2,\mu^2)\int d^2\mathbf{k}_\perp \mathbf{d}^2\mathbf{P}_\perp f_1^a(x,\mathbf{k}_\perp^2;\mu^2) D_1^{a\to h}(z,\mathbf{P}_\perp^2;\mu^2)\delta^{(2)}(z\mathbf{k}_\perp - \mathbf{P}_{hT} + \mathbf{P}_\perp)$$

$$+Y_{UU,T}(Q^2,\mathbf{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

Bacchetta, Diehl, et al., JHEP 02 (2007)

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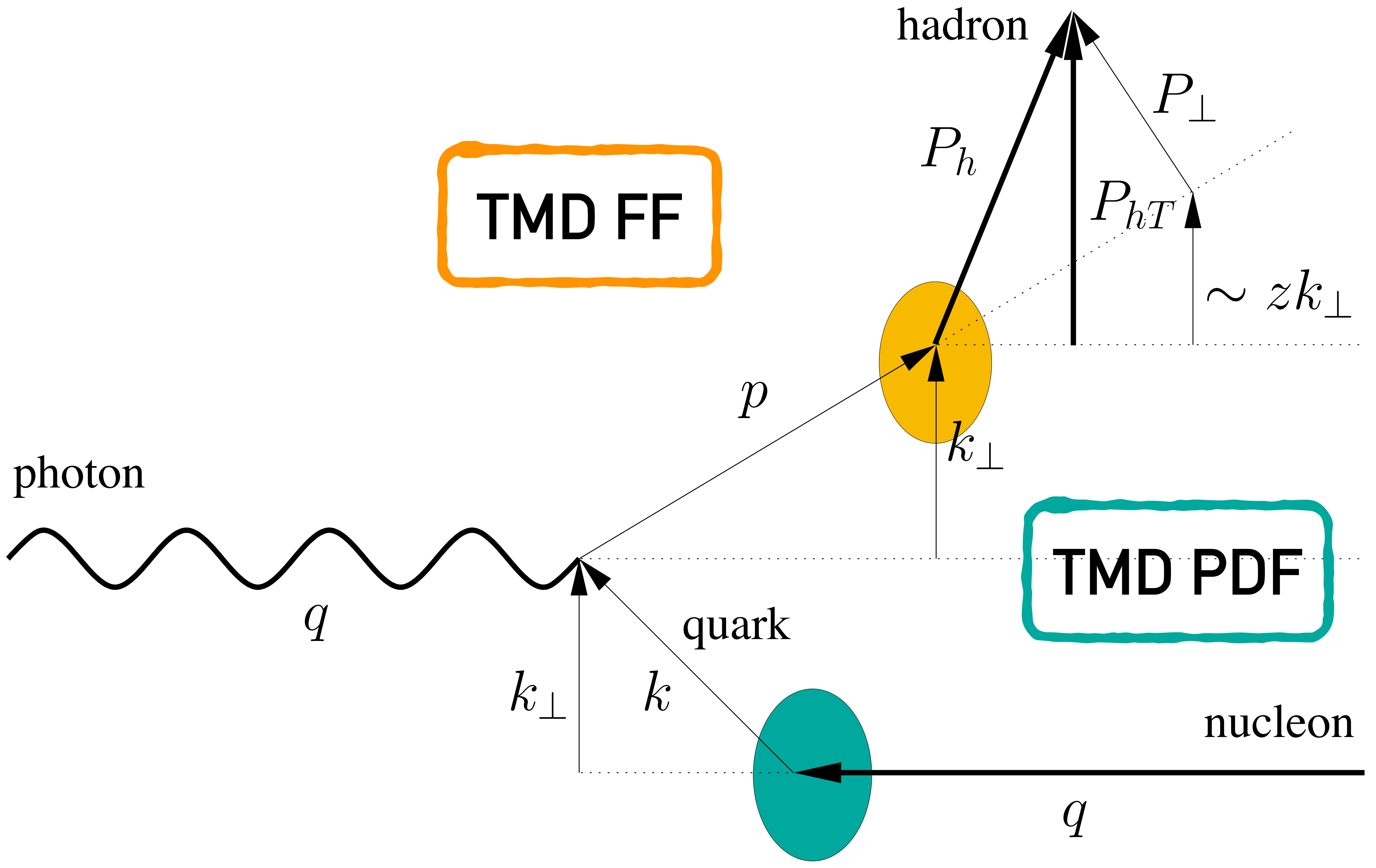

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$$+Y_{UU,T}(Q^2,\mathbf{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

W term

Y term

Bacchetta, Diehl, et al., JHEP 02 (2007)

SIDIS

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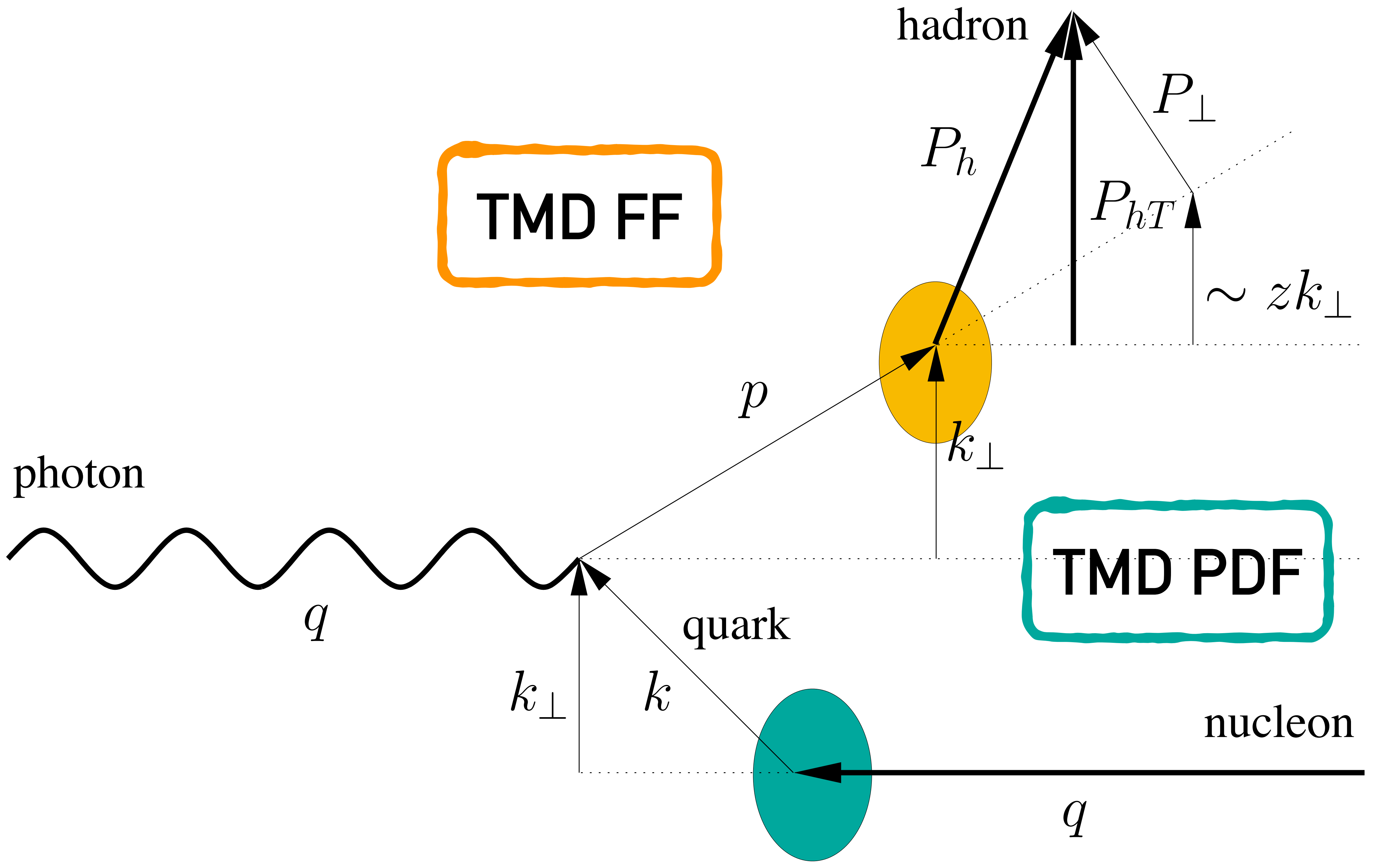
$$\ell(l) + N(p) \to \ell(l') + h(P_h) + X$$



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$$+Y_{UU,T}(Q^2,\mathbf{P}_{hT}^2) + \mathcal{O}(M^2/Q^2)$$

W term

Y term

- W term dominates in the region where $q_T \ll Q$
- Y term not included in the MAP analyses

Bacchetta, Diehl, et al., JHEP 02 (2007)

unpolarized Transverse Momentum Dependent Parton Distribution Function

TMD PDFs

$$f_1^q(x, b; \mu, \zeta) = \sum_j \left(C_j \otimes f^j\right)(x, b_*; \mu_b) e^{R(b_*;\mu_b,\mu)} f_{\rm NP}(x, b)$$

unpolarized Transverse Momentum Dependent Parton Distribution Function

TMD PDFs

collinear PDFs

$$f_1^q(x, b; \mu, \zeta) = \sum_j \left(C_j \otimes f^j\right)(x, b_*; \mu_b) e^{R(b_*;\mu_b,\mu)} f_{\rm NP}(x, b)$$

unpolarized Transverse Momentum Dependent Parton Distribution Function

TMD PDFs

matching to the collinear region

collinear PDFs

$$f_1^q(x, b; \mu, \zeta) = \sum_j \left(C_j \otimes f^j\right)(x, b_*; \mu_b) e^{R(b_*; \mu_b, \mu)} f_{\rm NP}(x, b)$$

perturbative expansion in $\alpha_s(\mu)$

perturbative evolution

unpolarized Transverse Momentum Dependent Parton Distribution Function

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perturbative expansion in $\alpha_s(\mu)$

perturbative evolution

resummation of large logarithms

$$L = \ln \frac{Q^2}{\mu_b^2}$$

log ordering

state of the art: N3LL

next-to-next-to-next leading log

unpolarized Transverse Momentum Dependent Parton Distribution Function

TMD PDFs

matching to the collinear region

collinear PDFs

$$f_1^q(x,b;\mu,\zeta) = \sum_j \left(C_j \otimes f^j\right)(x,b_*;\mu_b)\, e^{R(b_*;\mu_b,\mu)}\, f_{\mathrm{NP}}(x,b)$$

perturbative expansion in $\alpha_s(\mu)$

perturbative evolution

non perturbative transverse content

resummation of large logarithms

$$L = \ln \frac{Q^2}{\mu_b^2}$$

log ordering

state of the art: N3LL

next-to-next-to-next leading log

parametrized and fitted to data

Perturbative accuracy: N^3LL^-

Orders in powers of α_s

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^3LL^-	2	3	4	NLO (FF only)
N^3LL	2	3	4	NNLO
N^3LL'	3	3	4	N^3LO

Perturbative accuracy: N^3LL^-

Orders in powers of α_s

Hard factor and matching coefficient

Ingredients in perturbative Sudakov form factor

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evol.
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^3LL^-	2	3	4	NLO (FF only)
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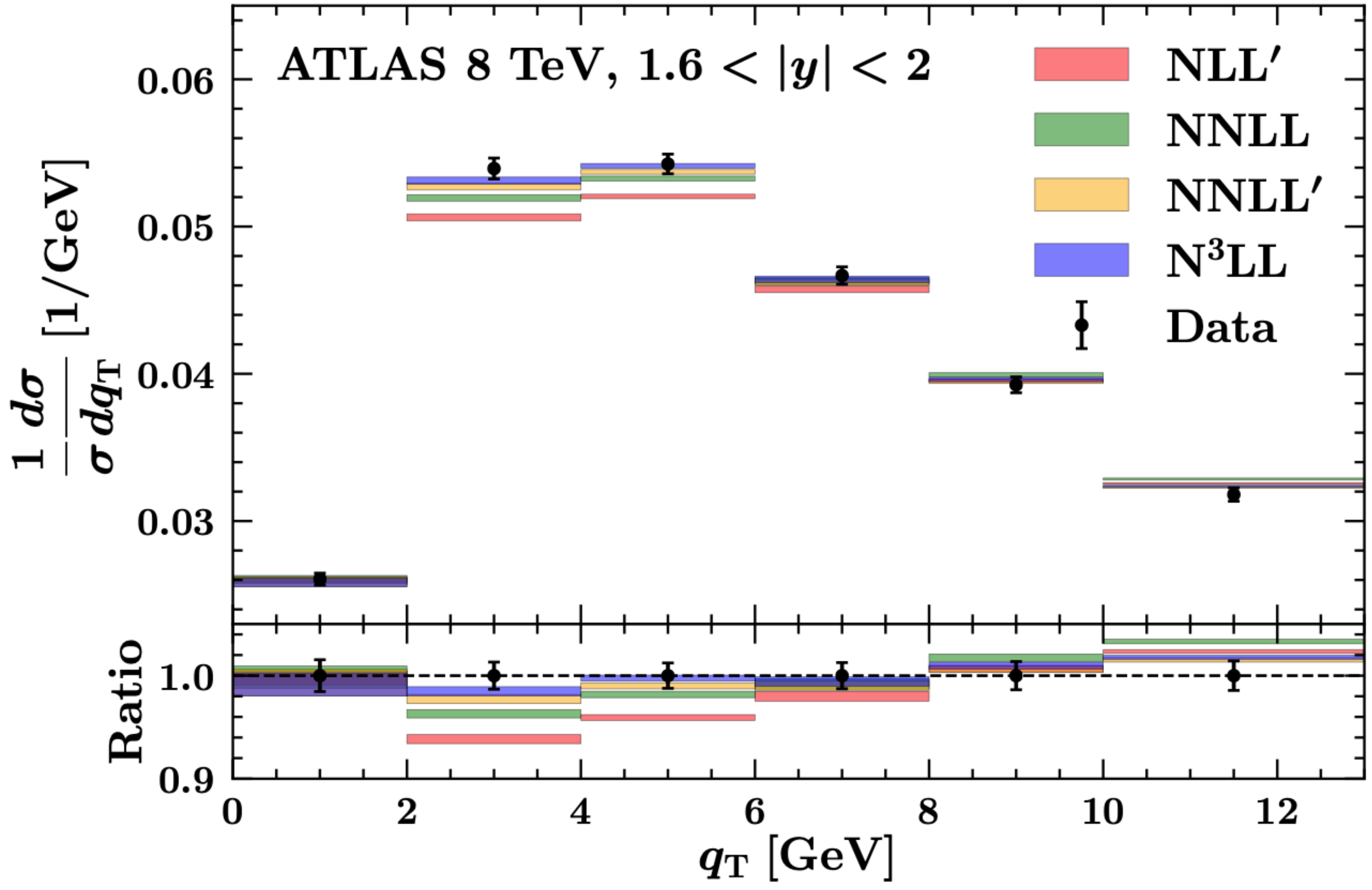
N^3LL'	3	3	4	N^3LO

Normalization of SIDIS multiplicities

High-Energy Drell-Yan beyond NLL

$Q \sim 100$ GeV

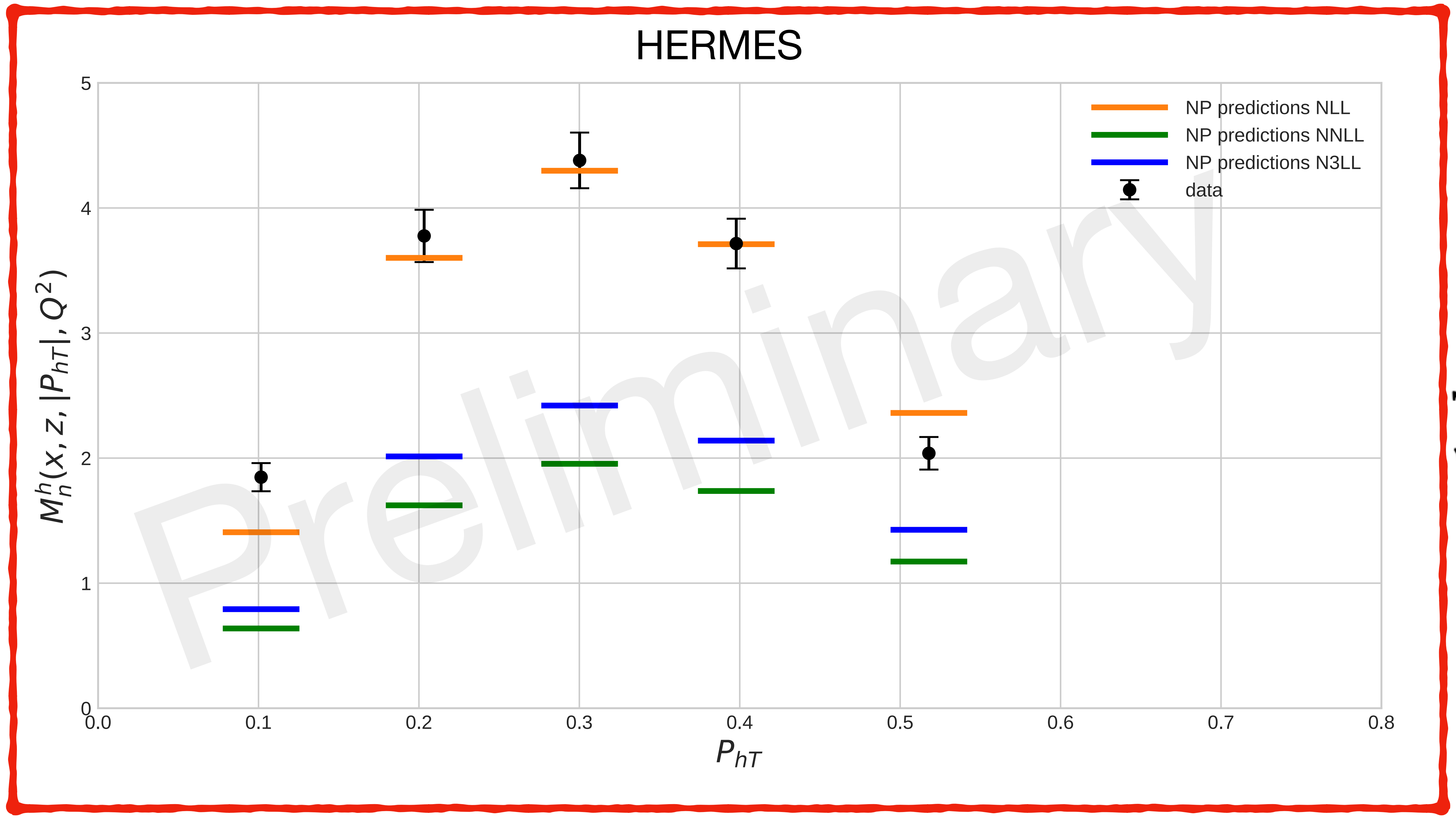


Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

Normalization of SIDIS multiplicities

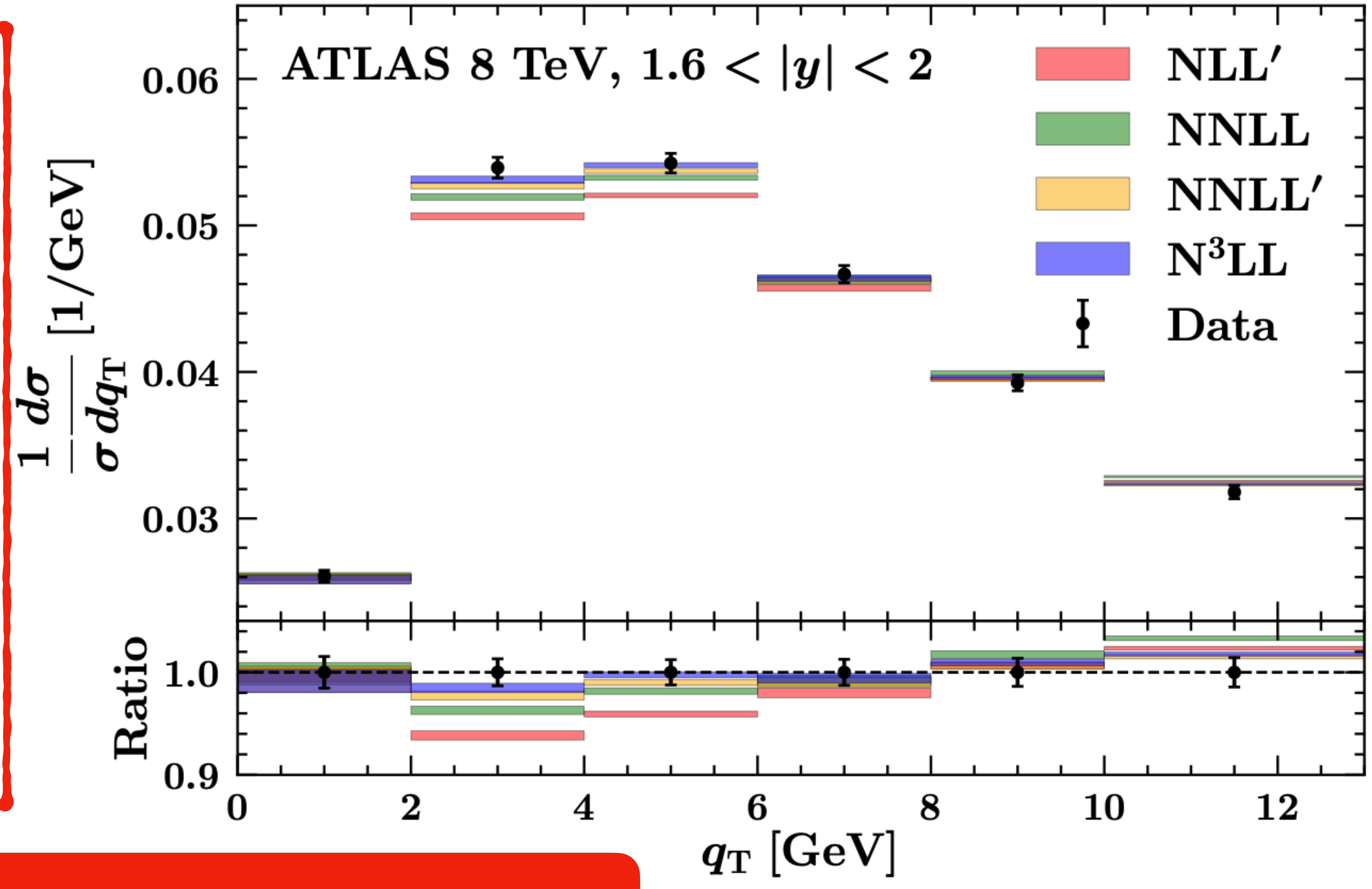
SIDIS multiplicities beyond NLL

$Q \sim 2$ GeV



High-Energy Drell-Yan beyond NLL

$Q \sim 100$ GeV



description considerably worsens at higher orders

Bacchetta, Bertone, Bissolotti, Bozzi, Delcarro, Piacenza, Radici, arXiv:1912.07550

Normalization of SIDIS multiplicities

Introduction of a normalization prefactor

SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \Big/ \frac{d\sigma}{dx dQ}$$

Normalization of SIDIS multiplicities

Introduction of a normalization prefactor

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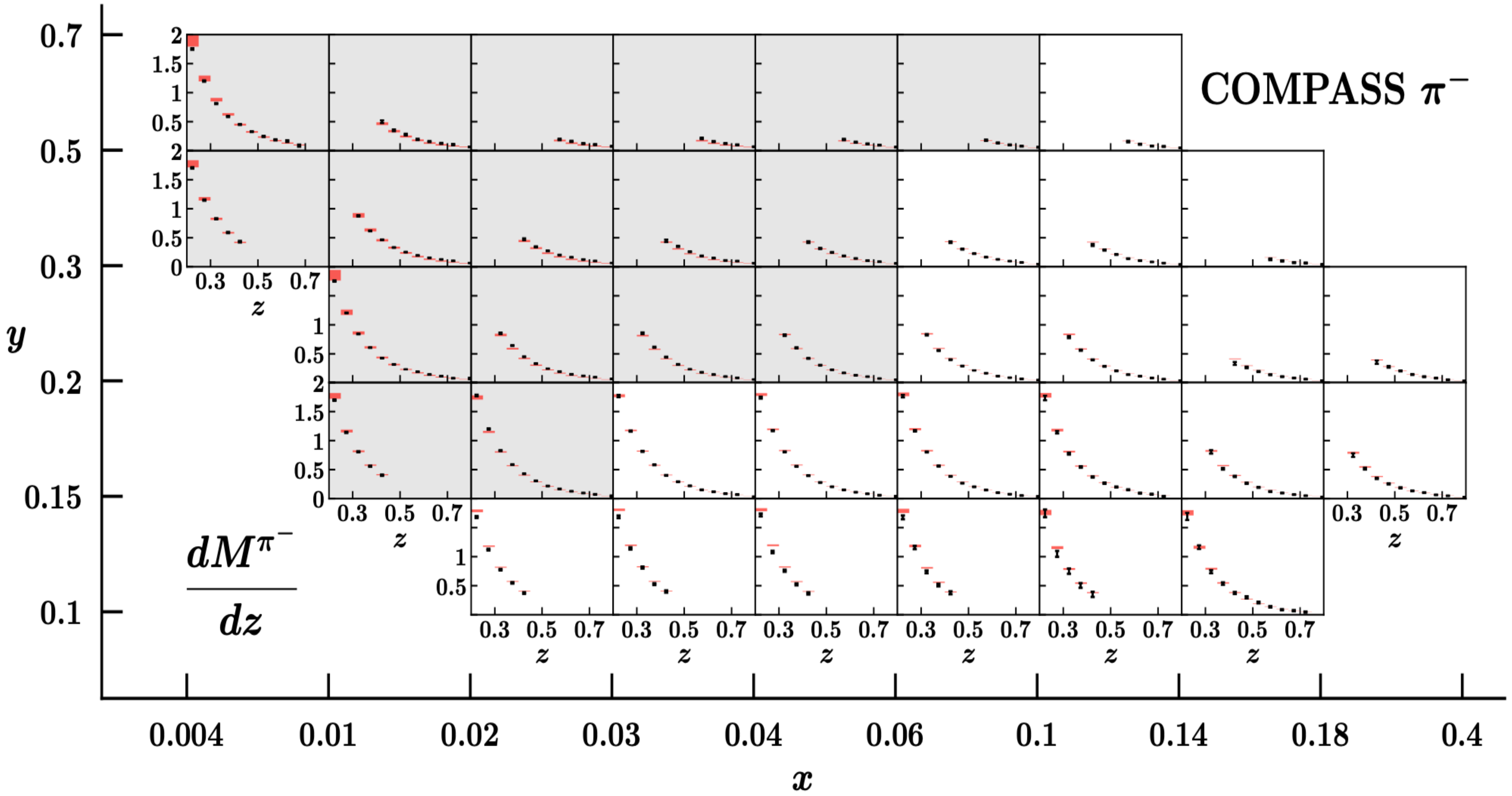
$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \Big/ \frac{d\sigma}{dx dQ}$$

Collinear SIDIS cross section $\frac{d\sigma}{dx dQ dz}$

No normalization problems

MAPFF1.0

MAP Collaboration collinear FFs fit



Khalek, Bertone, Nocera, arXiv: 2105.08725

Normalization of SIDIS multiplicities

Introduction of a normalization prefactor

SIDIS multiplicity

$$M(x, z, P_{hT}, Q) = \frac{d\sigma}{dx dQ dz dP_{hT}} \Big/ \frac{d\sigma}{dx dQ}$$

$$w(x, z, Q) = \frac{d\sigma}{dx dQ dz} \Big/ \int dP_{hT} \frac{d\sigma}{dx dQ dz dP_{hT}}$$

computed a priori, before the fit

- **Depends on the collinear PDFs**
- **independent of the fitting parameters**

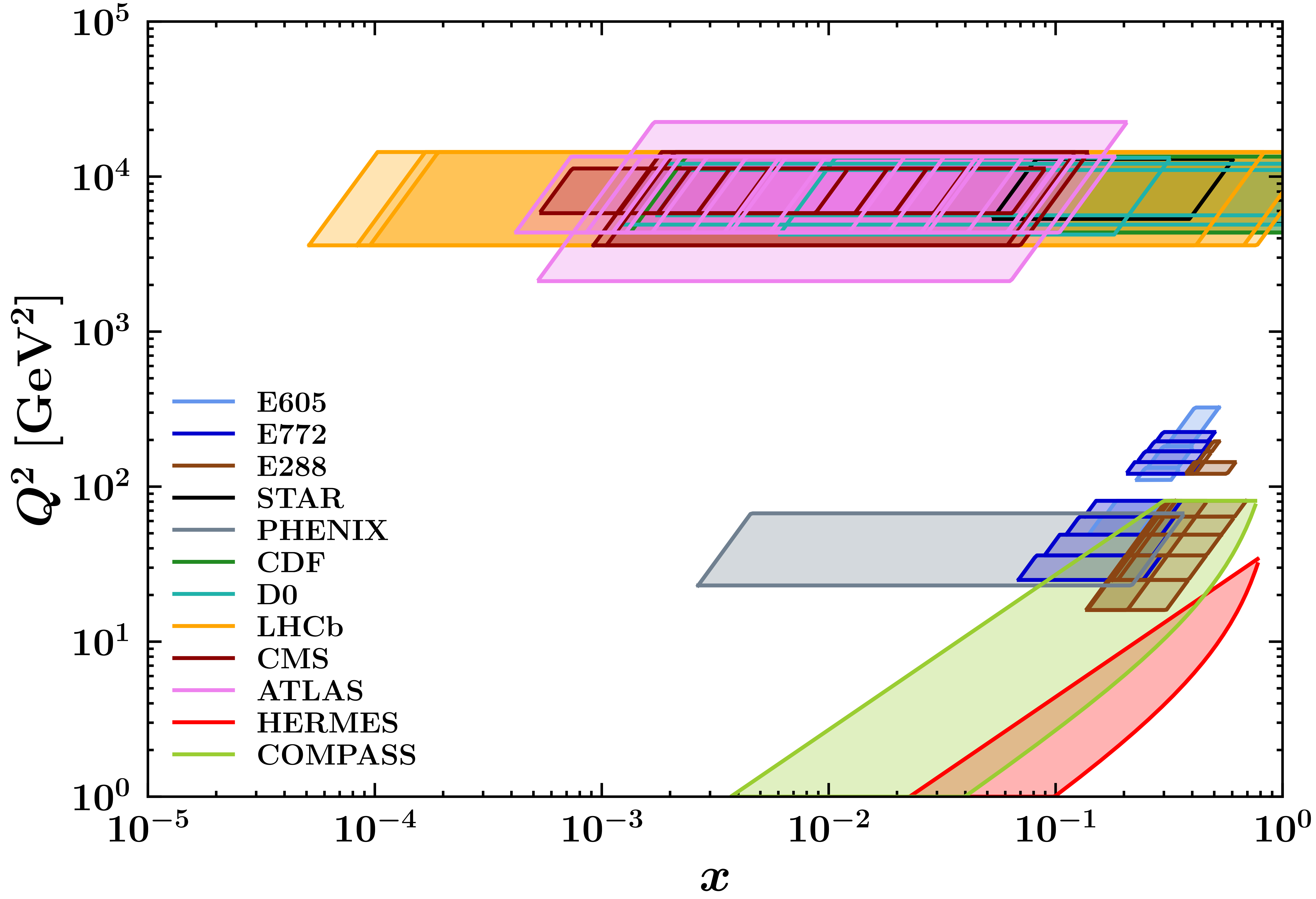
$$M(x, z, P_{hT}, Q) = w(x, z, Q) \frac{d\sigma}{dx dQ dz dP_{hT}} \Big/ \frac{d\sigma}{dx dQ}$$

Global analysis of DY and SIDIS data sets

Cuts on kinematics

$\langle Q \rangle > 1.3$ GeV

$0.2 < \langle z \rangle < 0.7$



DY

$$q_T|_{\max} = 0.2Q$$

SIDIS

$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3 \text{ GeV}, zQ]$$

Total number of points = 2031

Non-perturbative part of TMDs

TMD PDF

$$f_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

$$g_1(x) = N_1 \, \frac{(1-x)^\alpha \, x^\sigma}{(1-\hat{x})^\alpha \, \hat{x}^\sigma}$$

TMD FF

$$D_{1\mathrm{NP}}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{P_\perp^2}{g_{3A}}} + \lambda_{FB} k_\perp^2 e^{-\frac{P_\perp^2}{g_{3B}}} \right)$$

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NP evolution

$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

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Gaussians

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Gaussians

weighted Gaussians

TMD FF

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Gaussians

weighted Gaussians

TMD FF

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NP evolution

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Non-perturbative part of TMDs

TMD PDF

$$f_{1NP}(x, b_T^2) \propto \text{F.T. of} \left(e^{-\frac{k_\perp^2}{g_{1A}}} + \lambda_B k_\perp^2 e^{-\frac{k_\perp^2}{g_{1B}}} + \lambda_C e^{-\frac{k_\perp^2}{g_{1C}}} \right)$$

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Gaussians

weighted Gaussians

TMD FF

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NP evolution

$$g_K(b_T^2) = -g_2^2 \frac{b_T^2}{4}$$

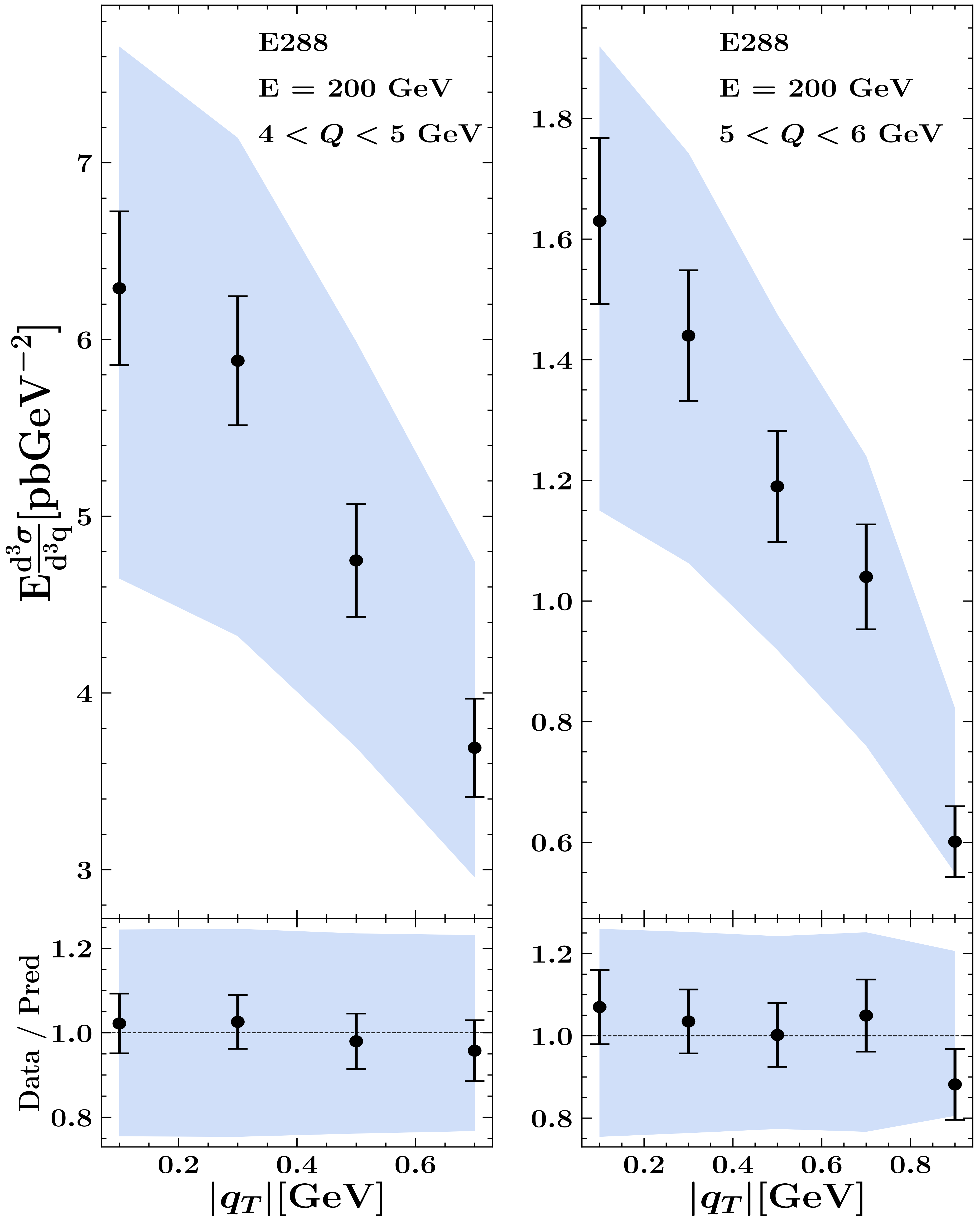
11 parameters for TMD PDF
+ 1 for NP evolution + 9 for TMD FF
= 21 free parameters

Comparison with data

N3LL⁻

Drell-Yan

Data set	N_{dat}	χ^2_D/N_{dat}	$\chi^2_\lambda/N_{\text{dat}}$	χ^2_0/N_{dat}
DY collider total	251	1.86	0.2	2.06
DY fixed-target total	233	0.85	0.4	1.24
SIDIS total	1547	0.59	0.28	0.87
Total	**2031**	**0.77**	**0.29**	**1.06**



Comparison with data

N3LL⁻

Drell-Yan

Good agreement
DY low energy

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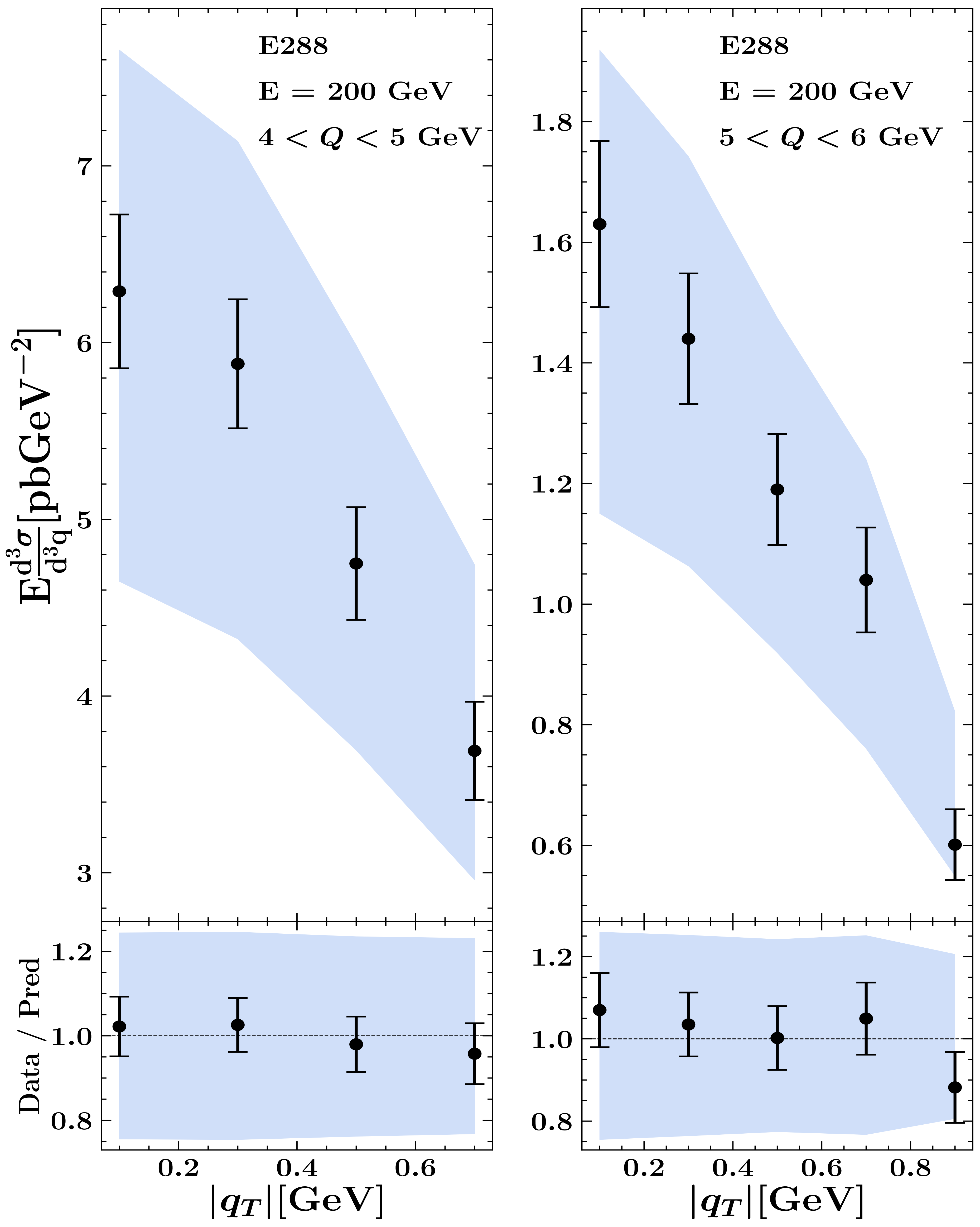
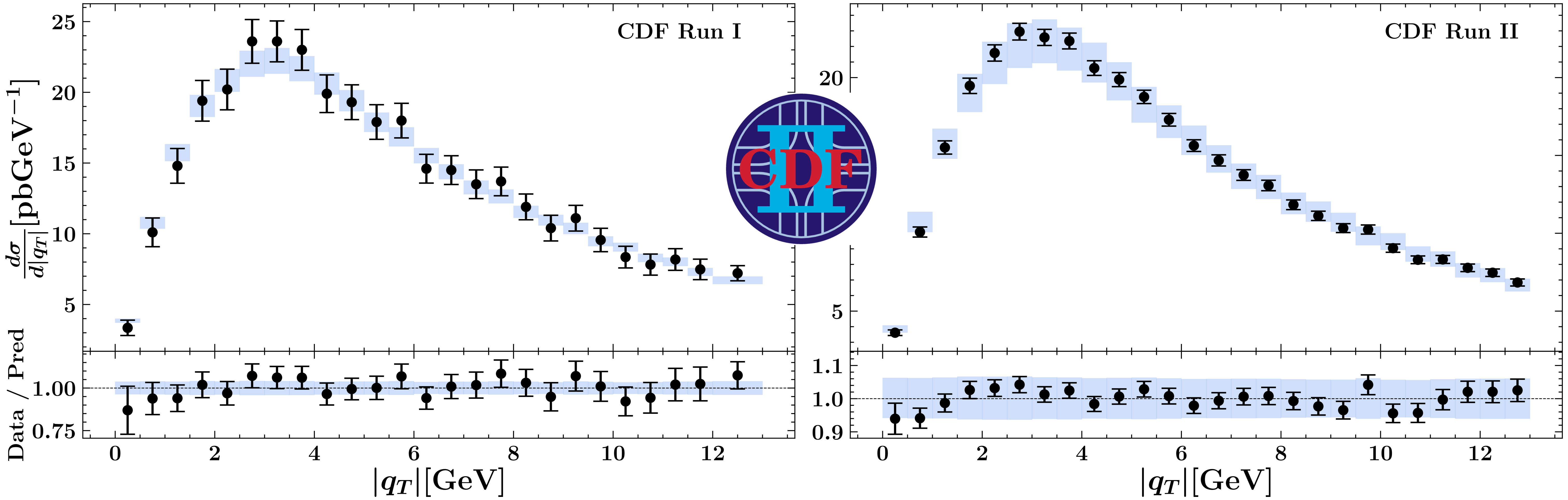
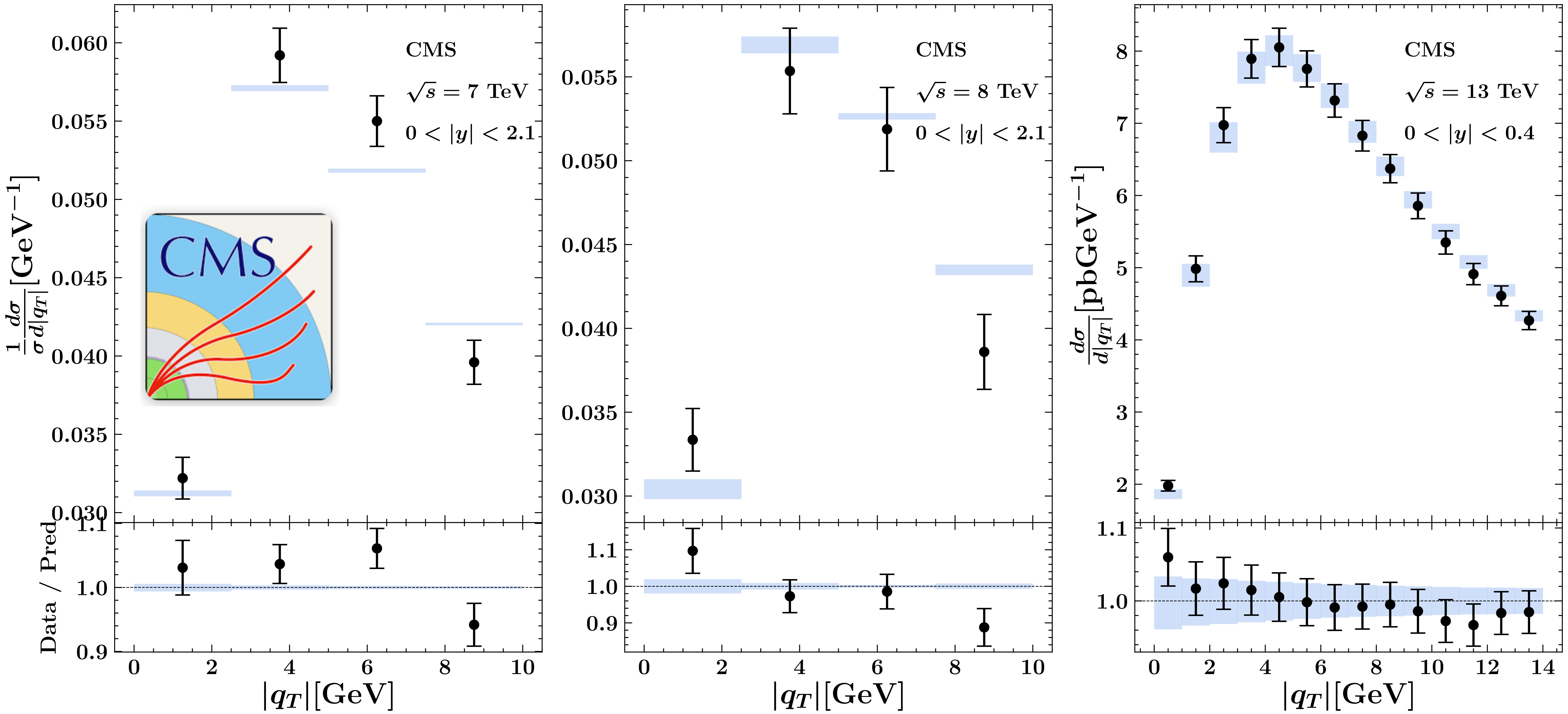


Comparison with data Drell-Yan

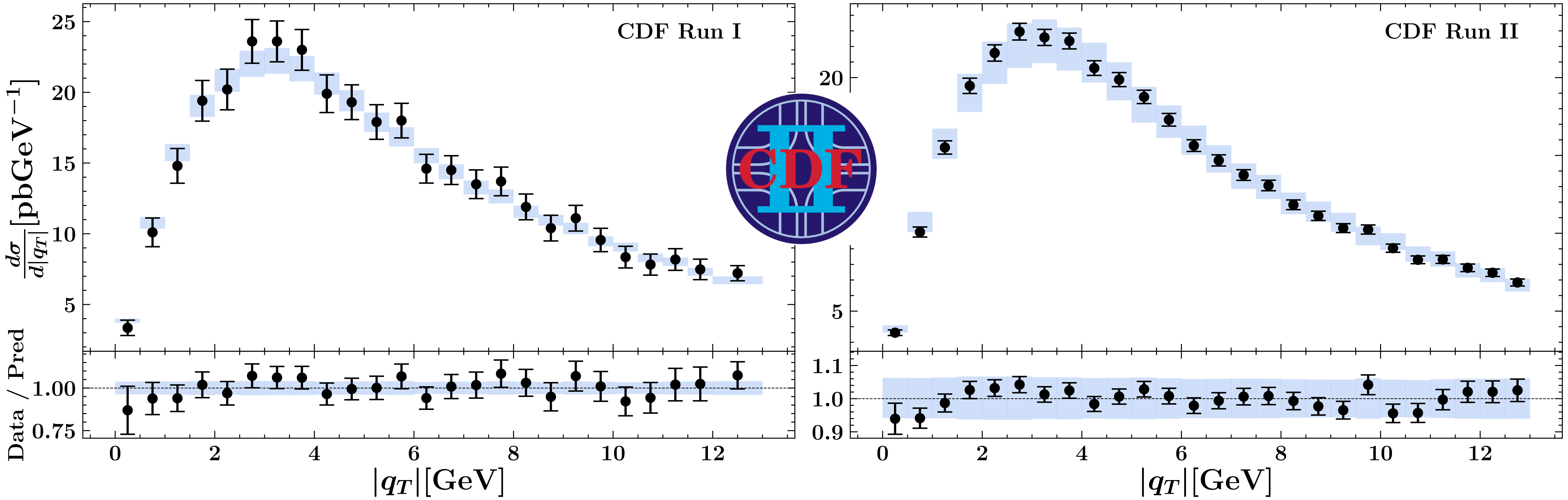
N3LL'



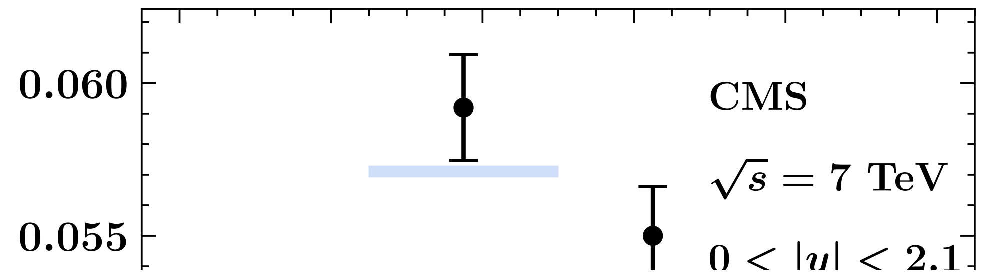


Comparison with data Drell-Yan

N3LL⁻



Fairly good agreement (DY high-energy)



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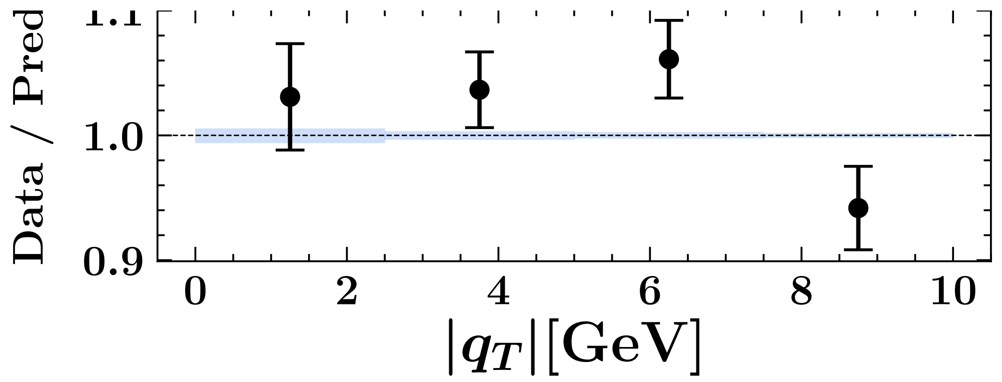
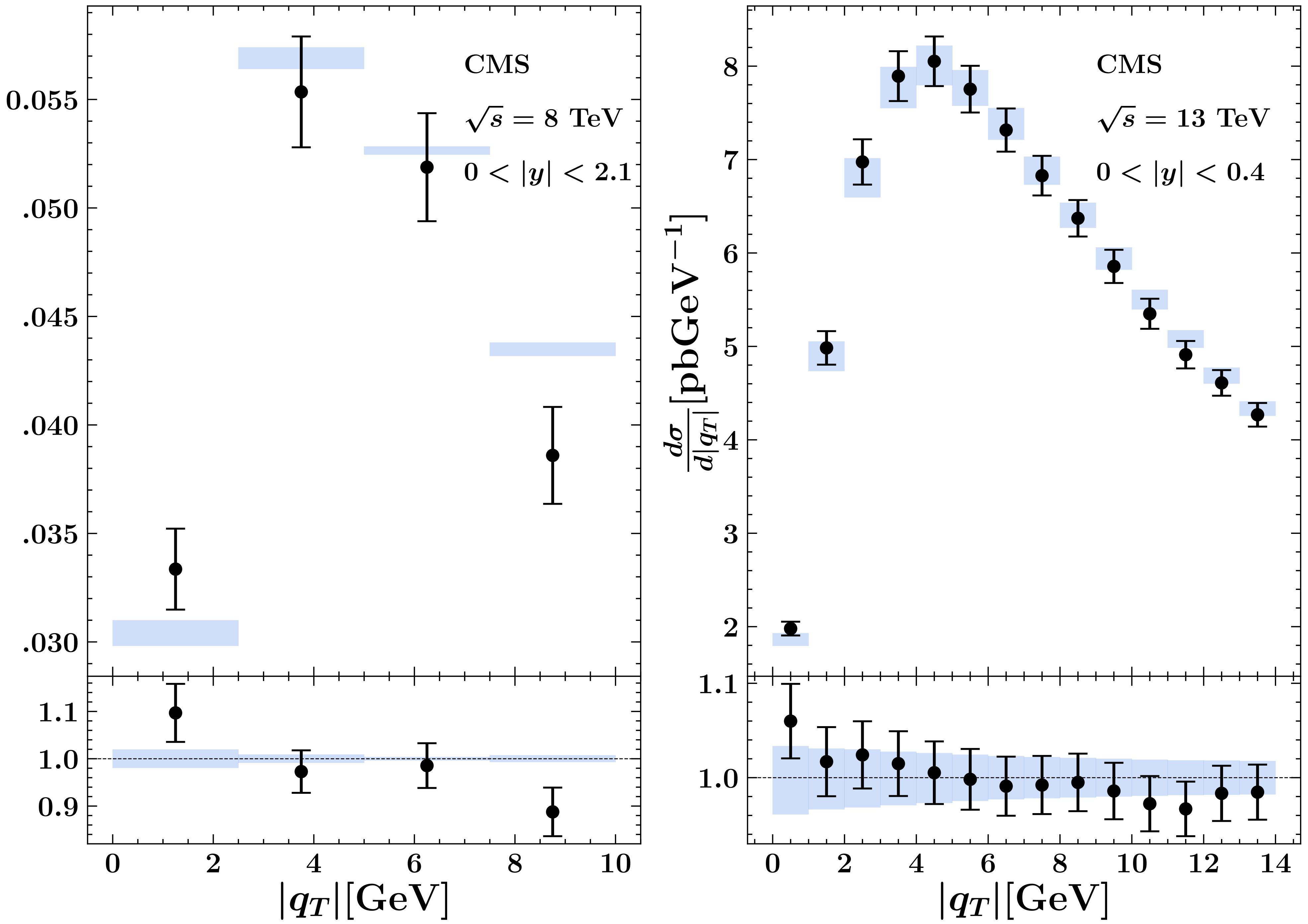
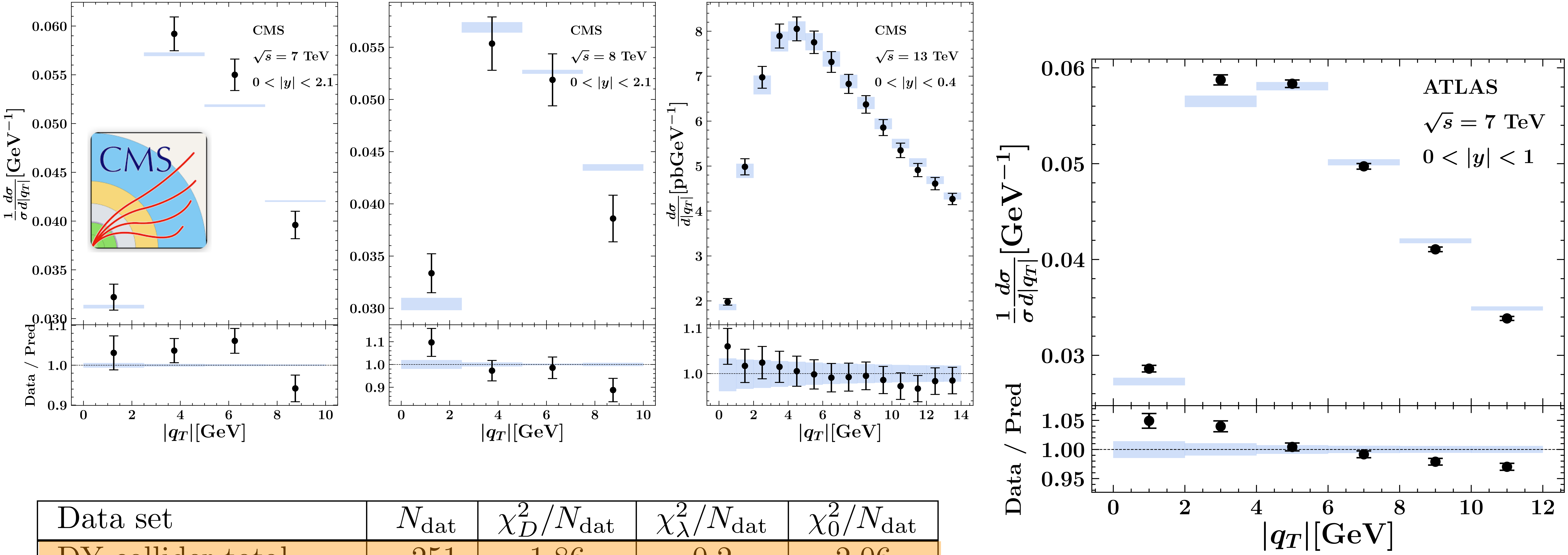




Comparison with data Drell-Yan

N3LL$^-$



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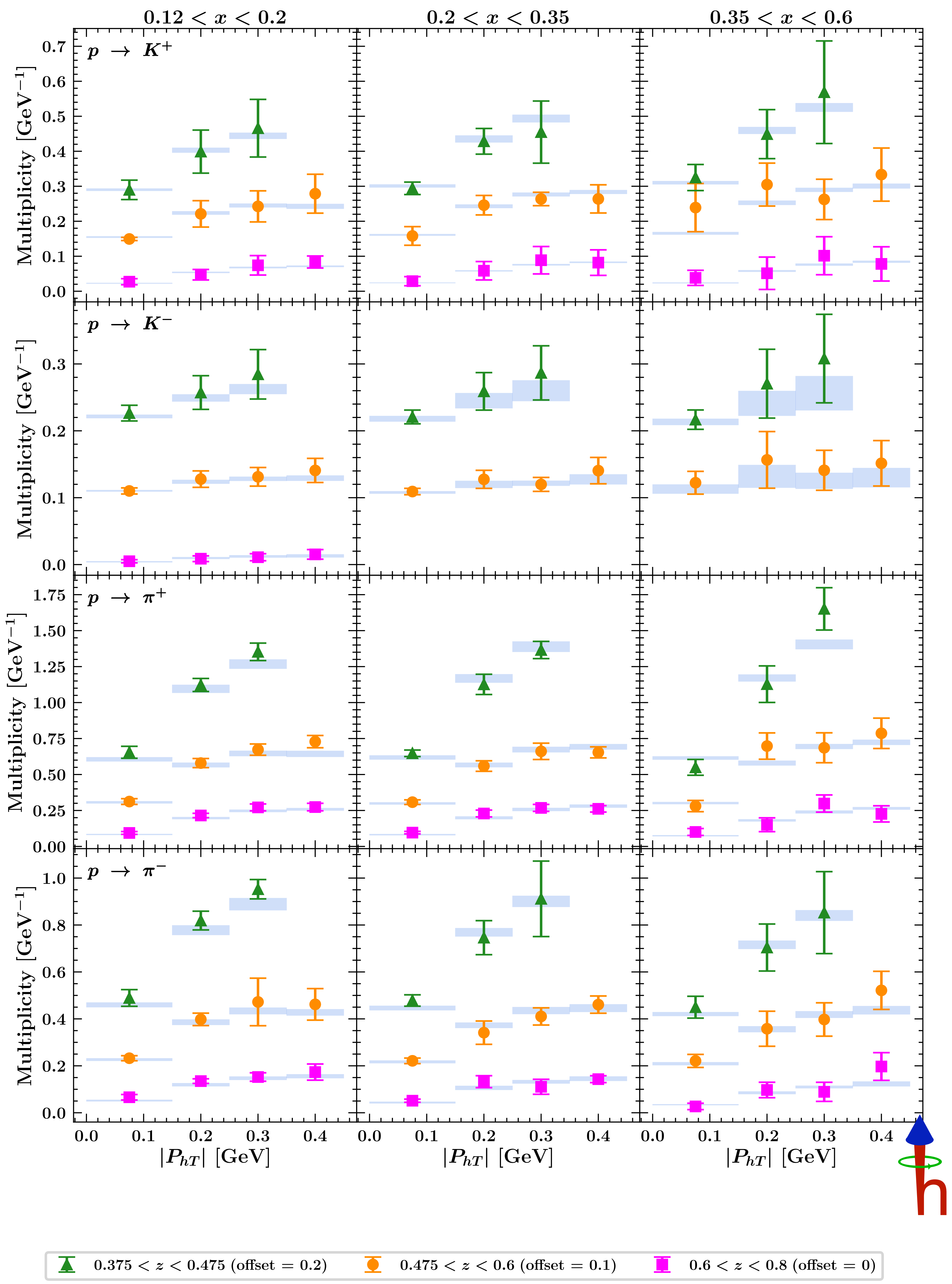
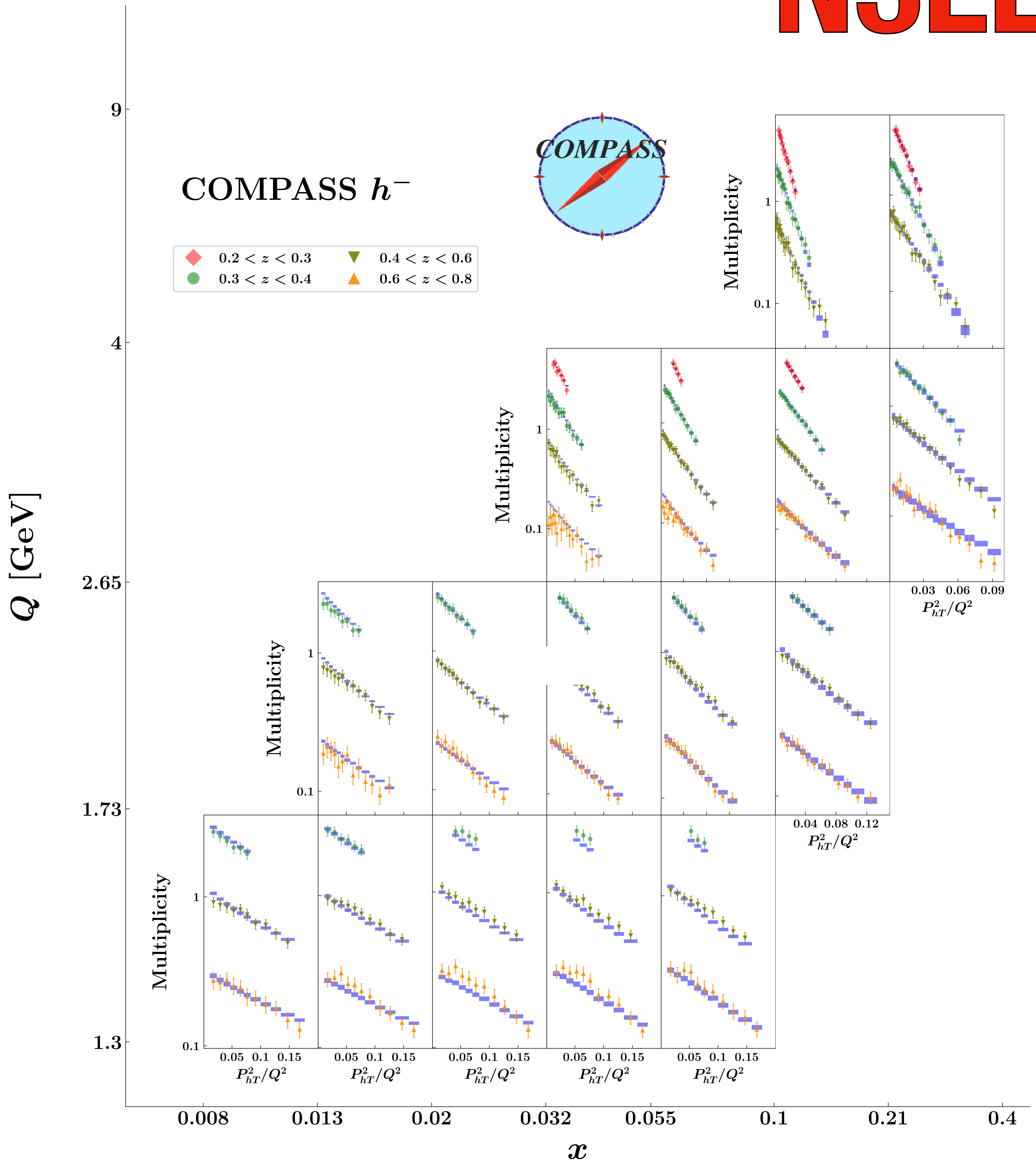
χ^2/N_{data} = 5.05 (DY ATLAS)

Comparison with data

N3LL$^-$

SIDIS

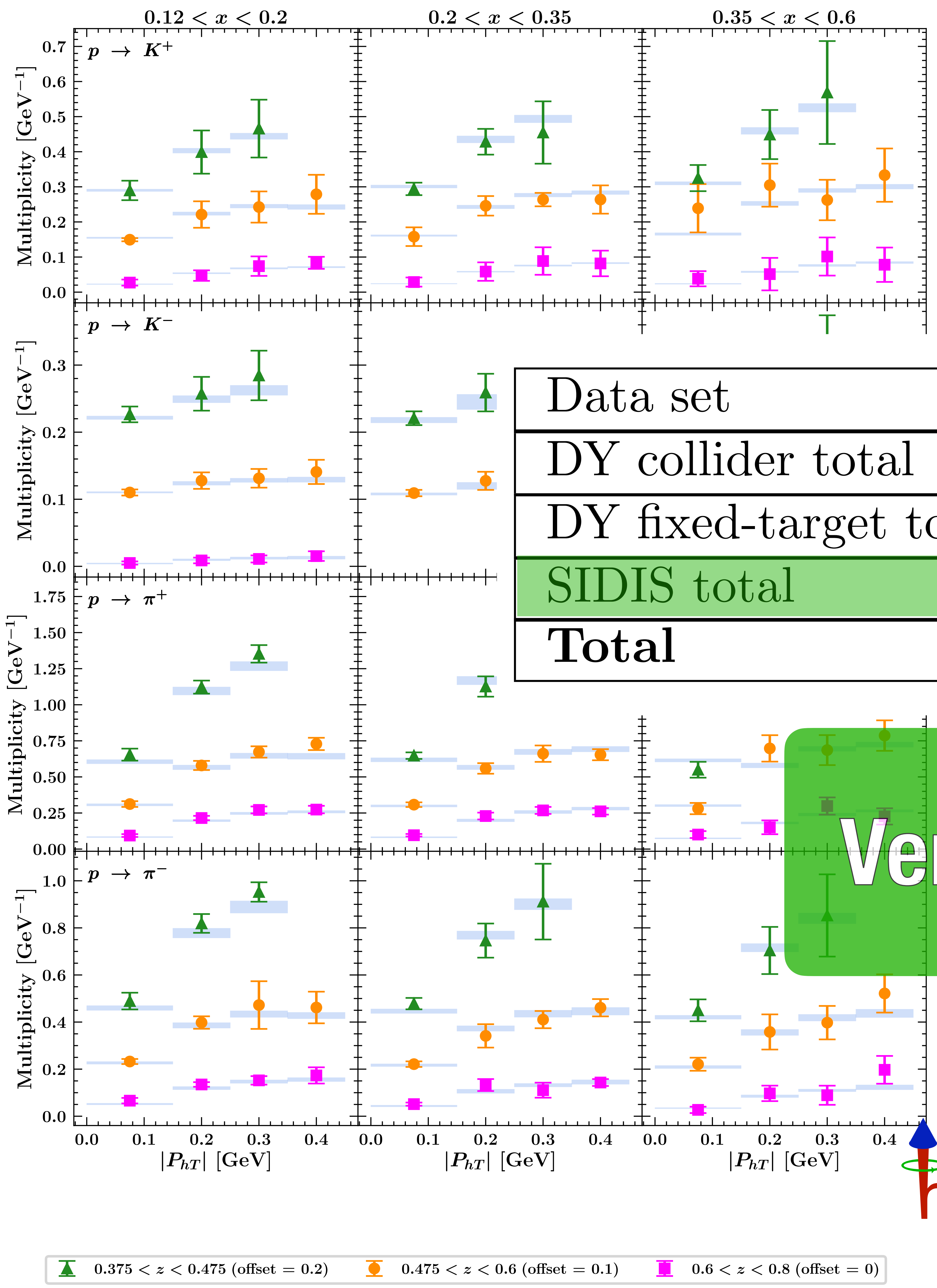


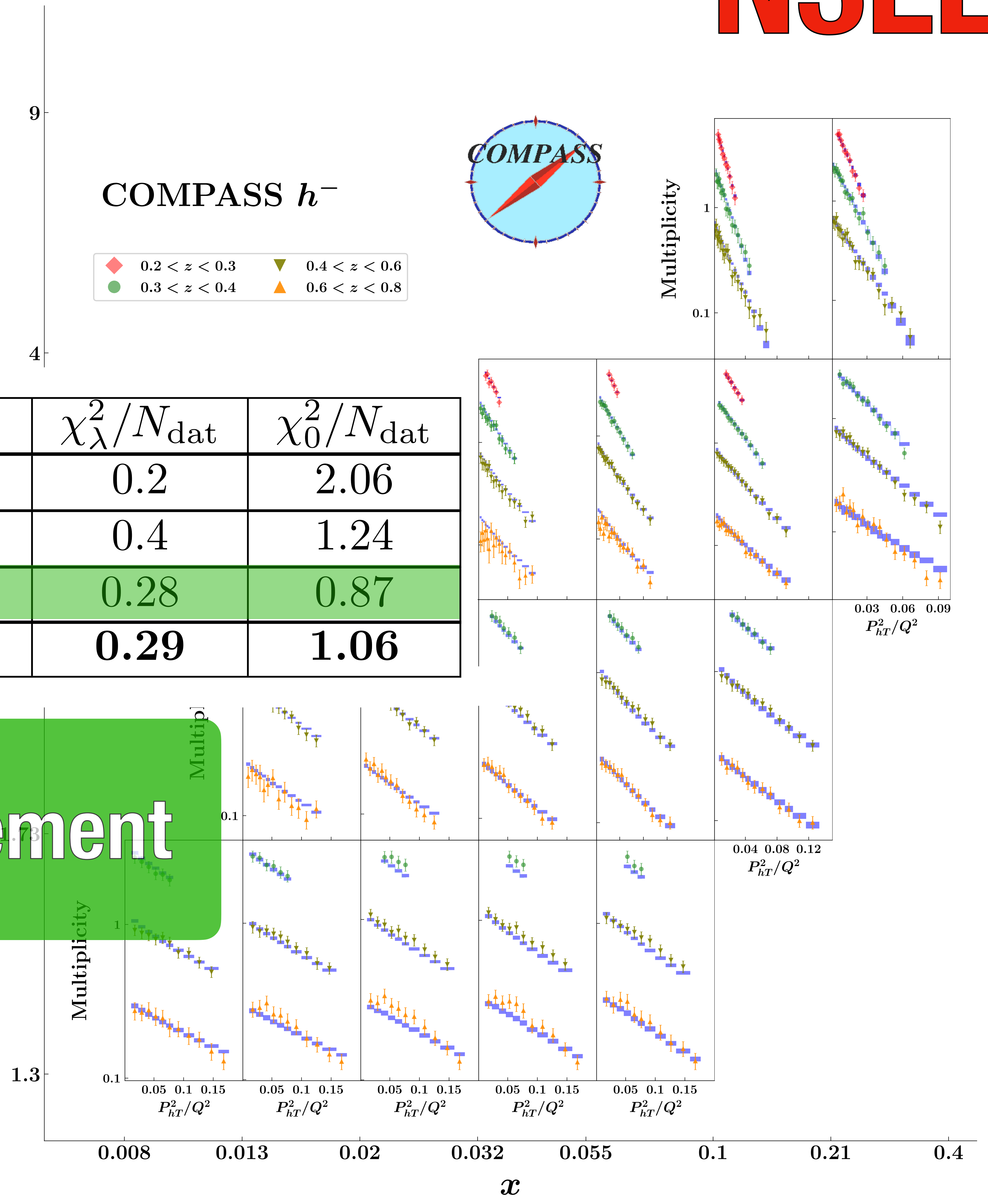


Comparison with data

N3LL^{-}

SIDIS





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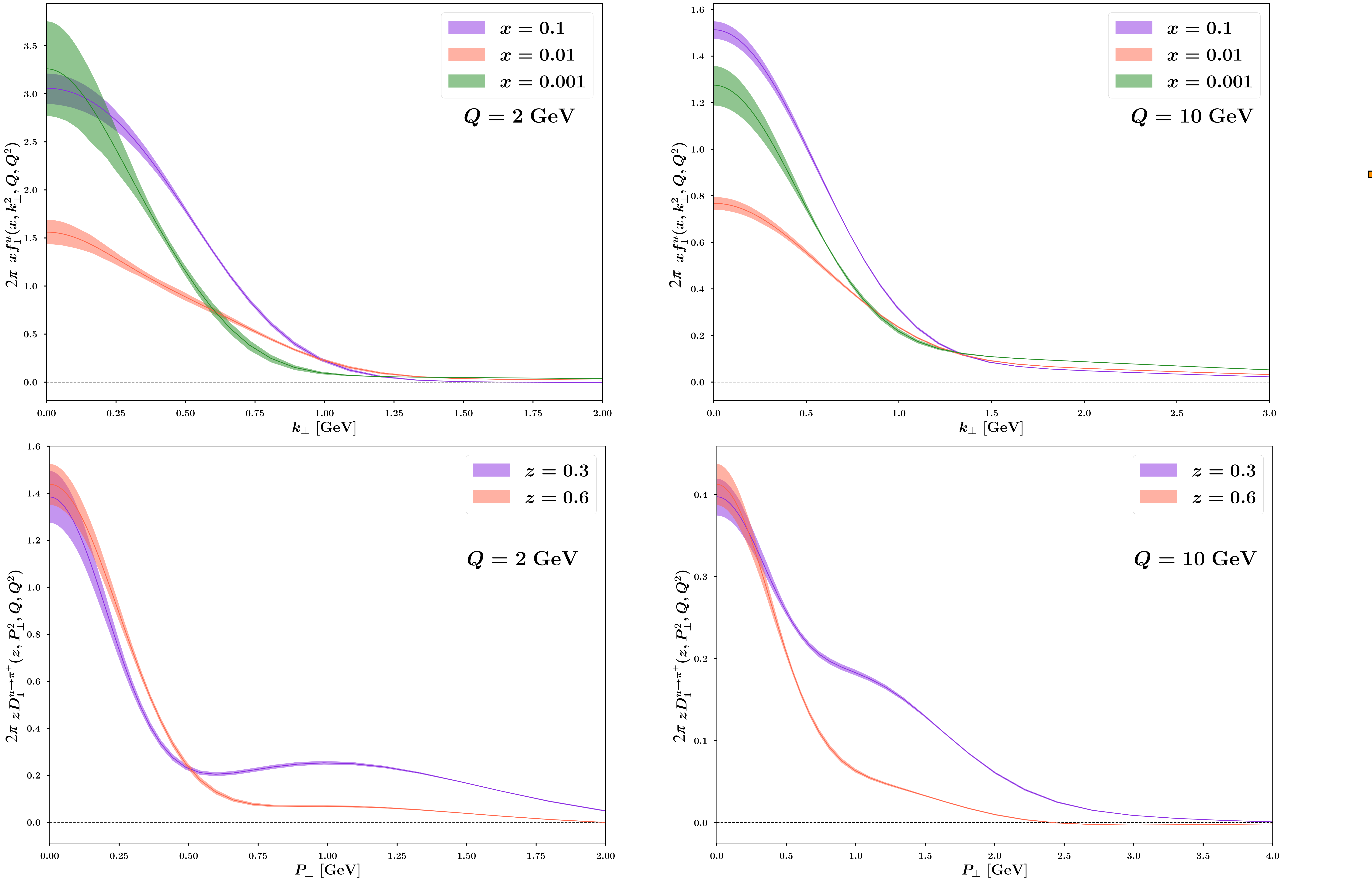
Very good agreement

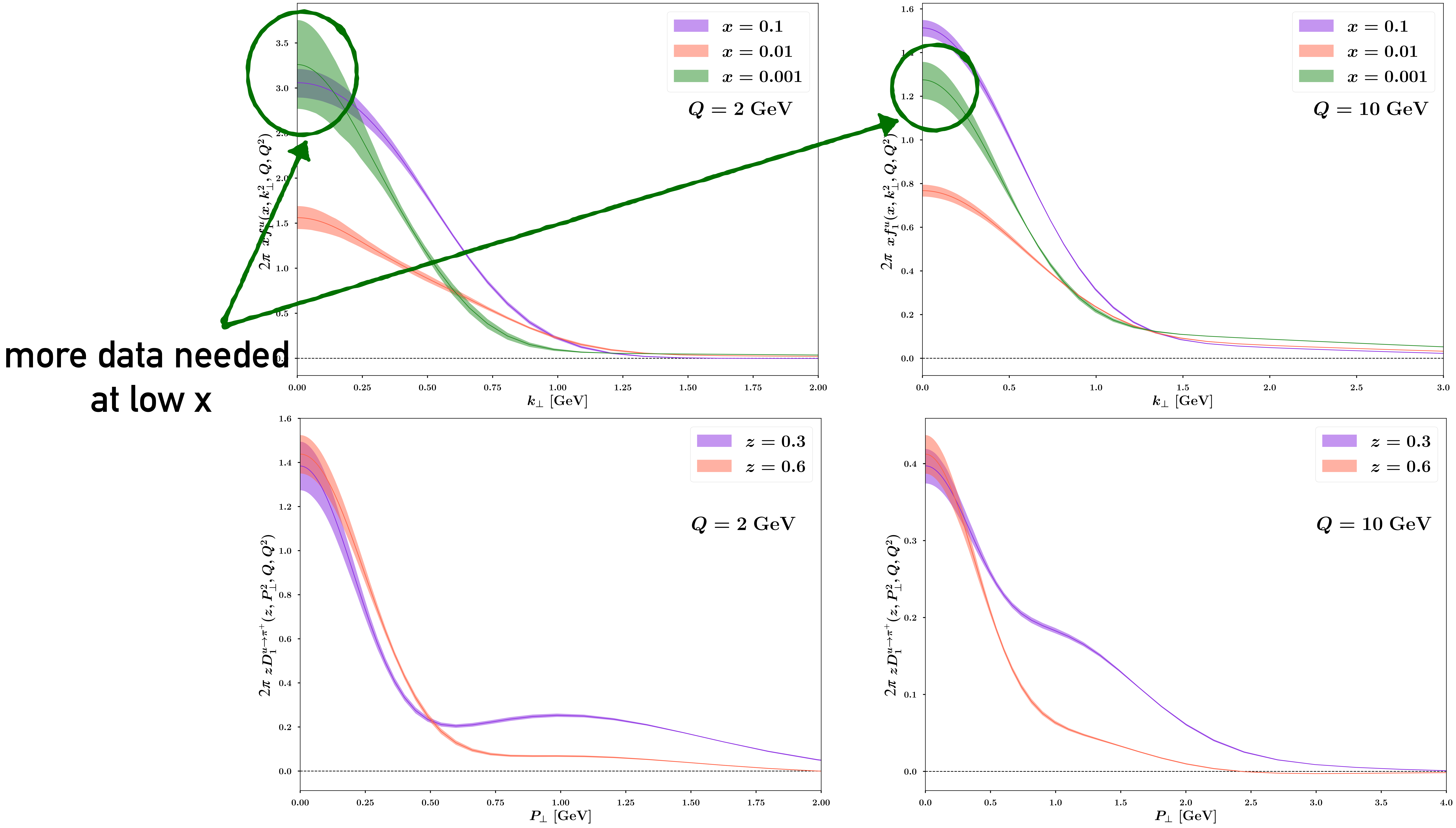
Fit results - TMDs



TMD PDFs

TMD FFs

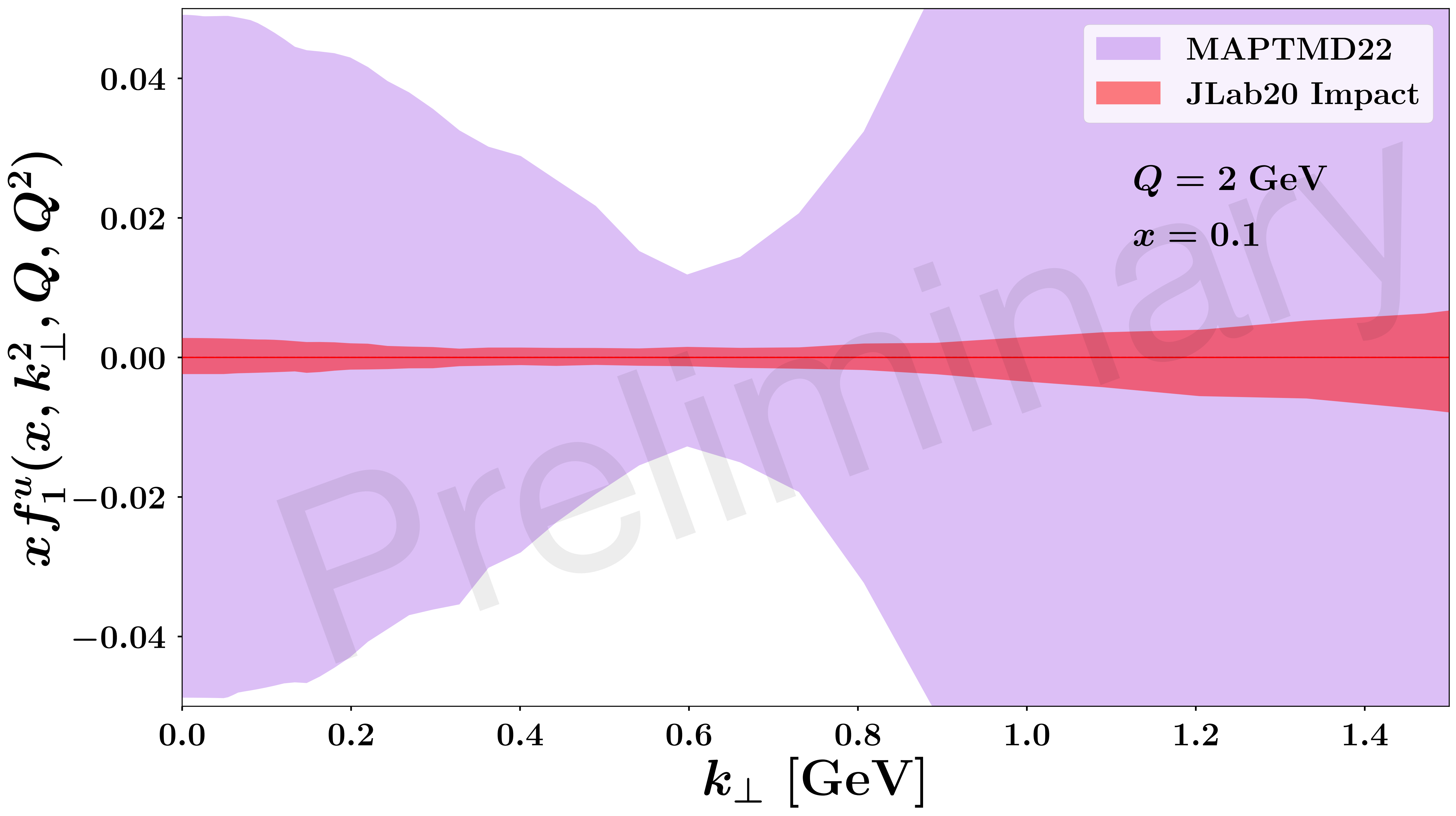
Fit results - TMDs

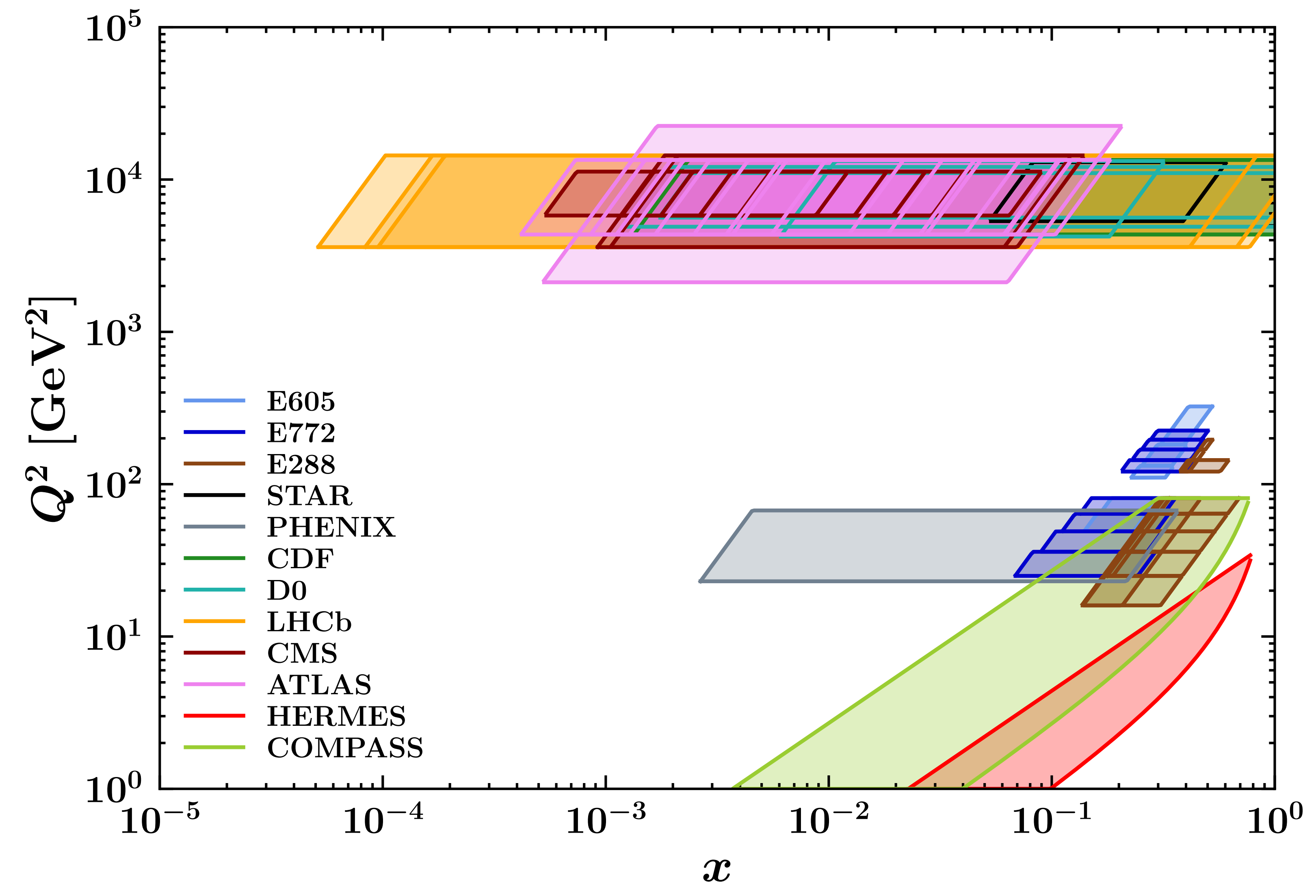


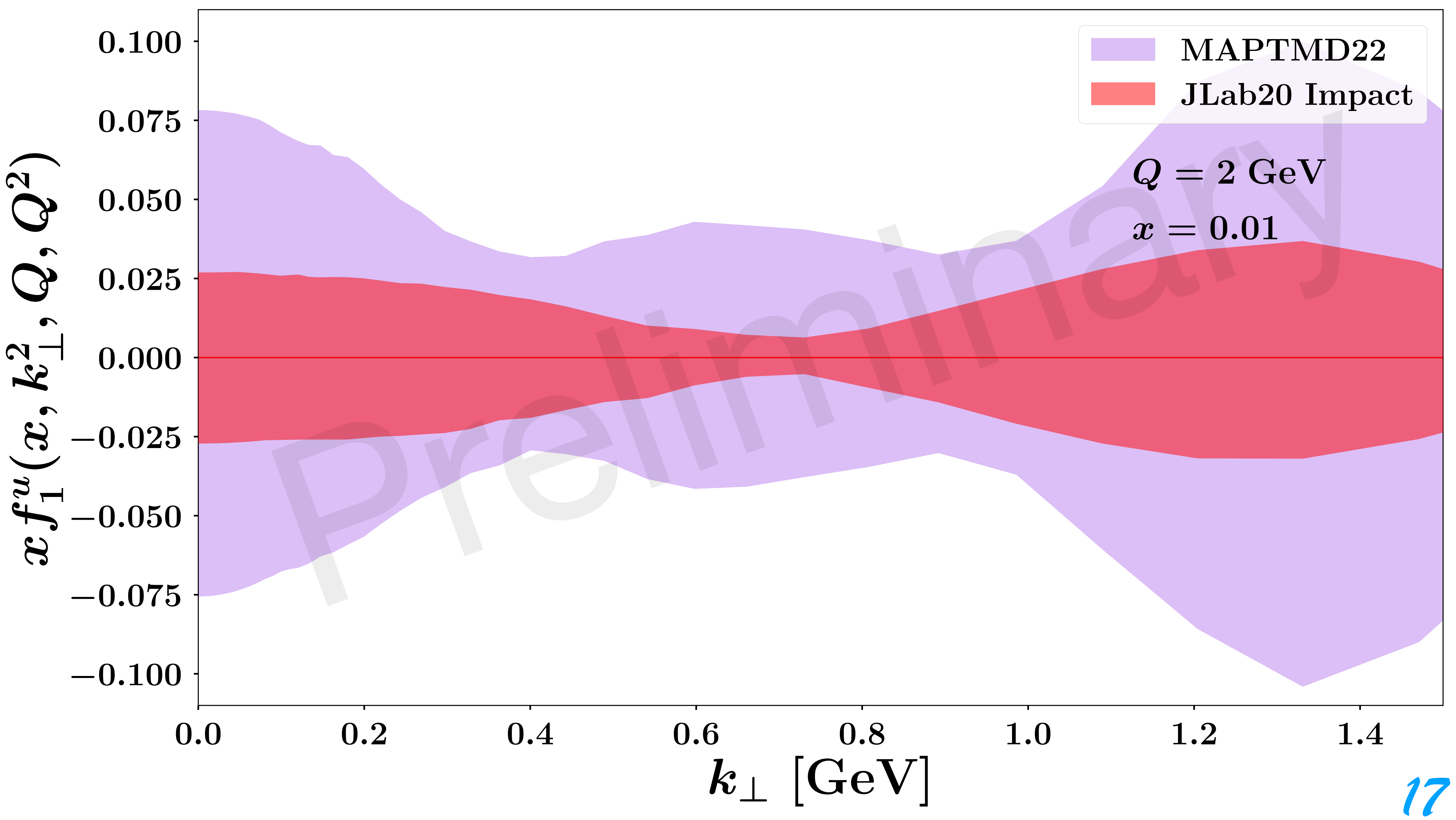
TMD PDFs

TMD FFs

Impact studies - JLab 20+

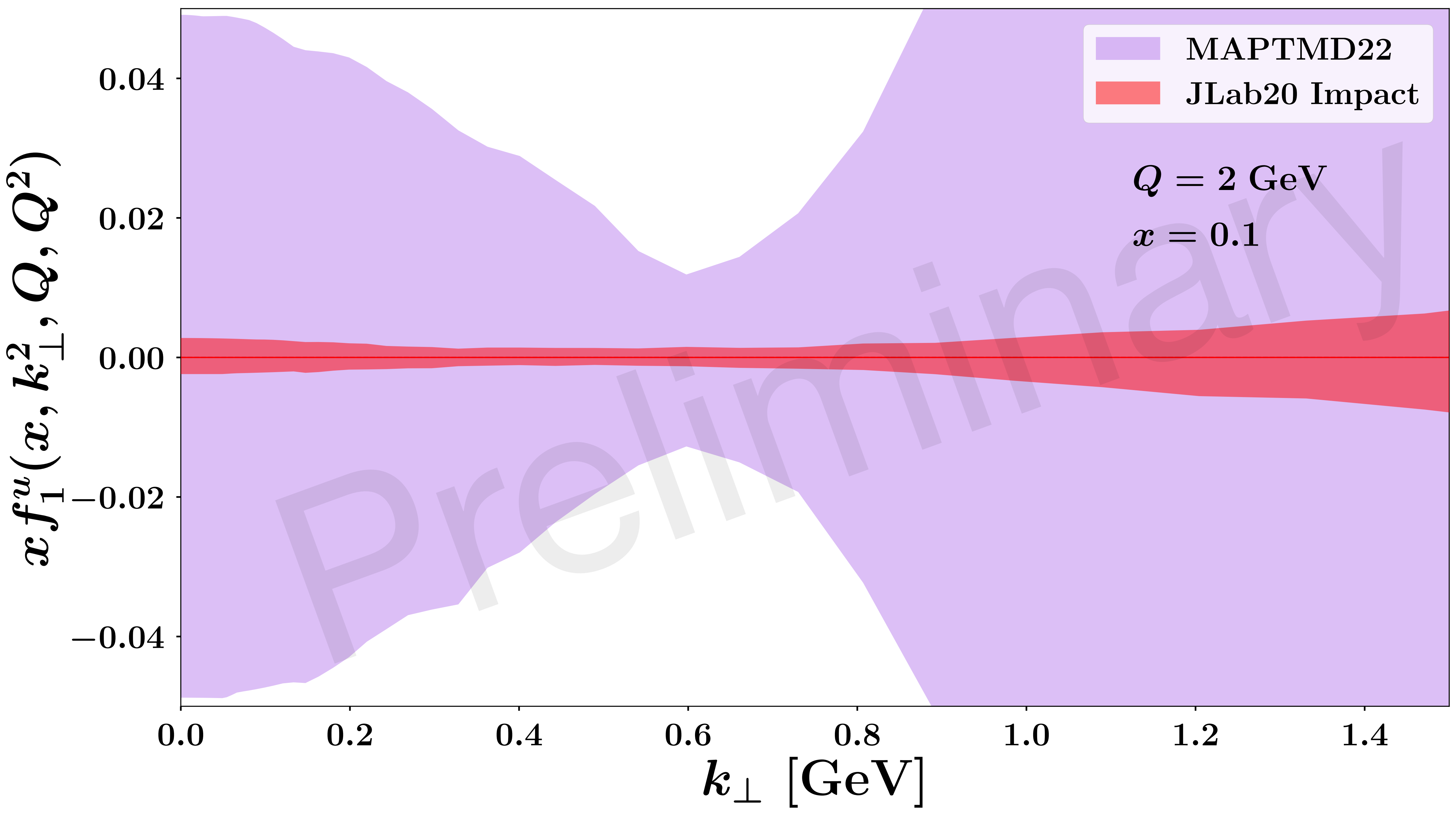


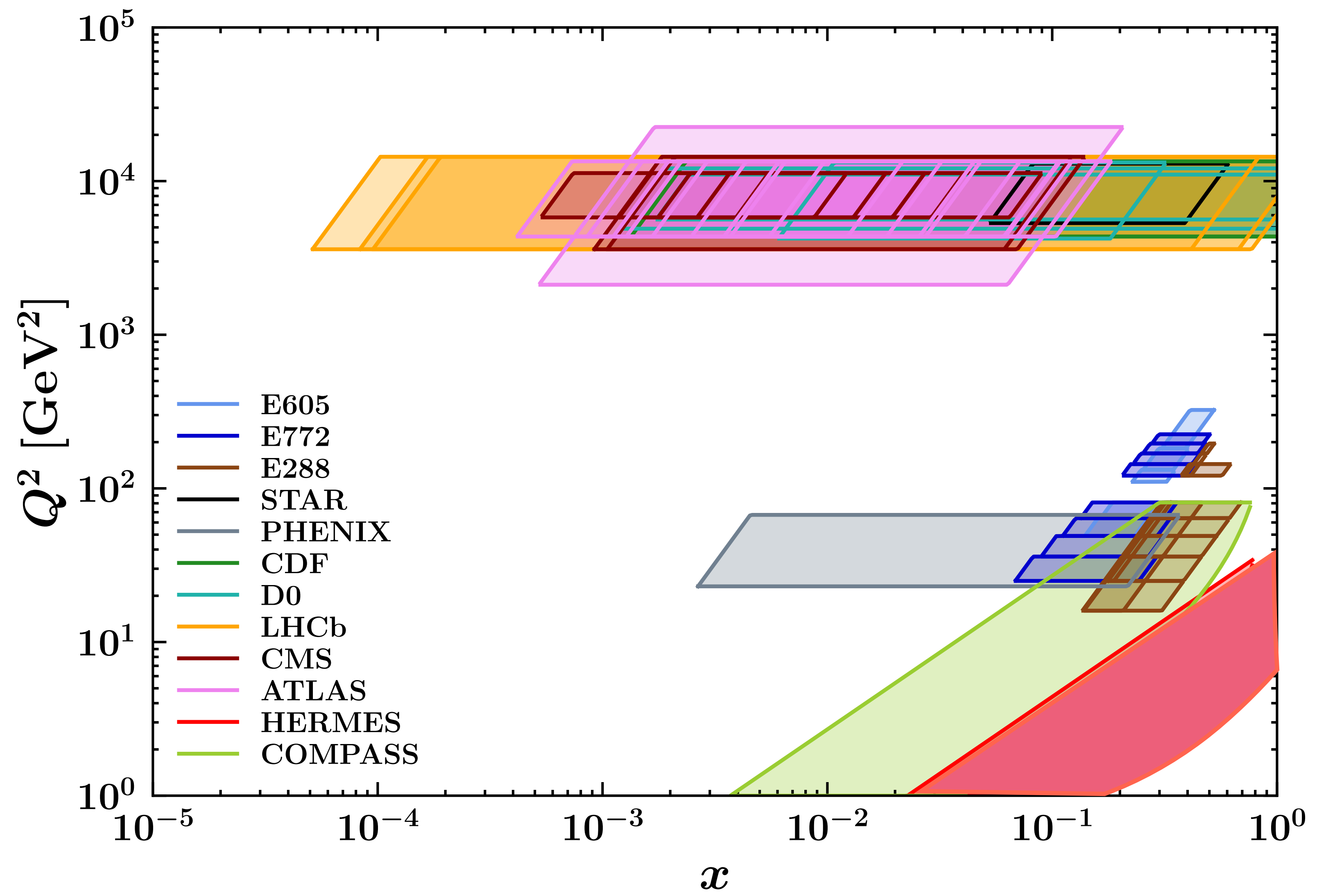




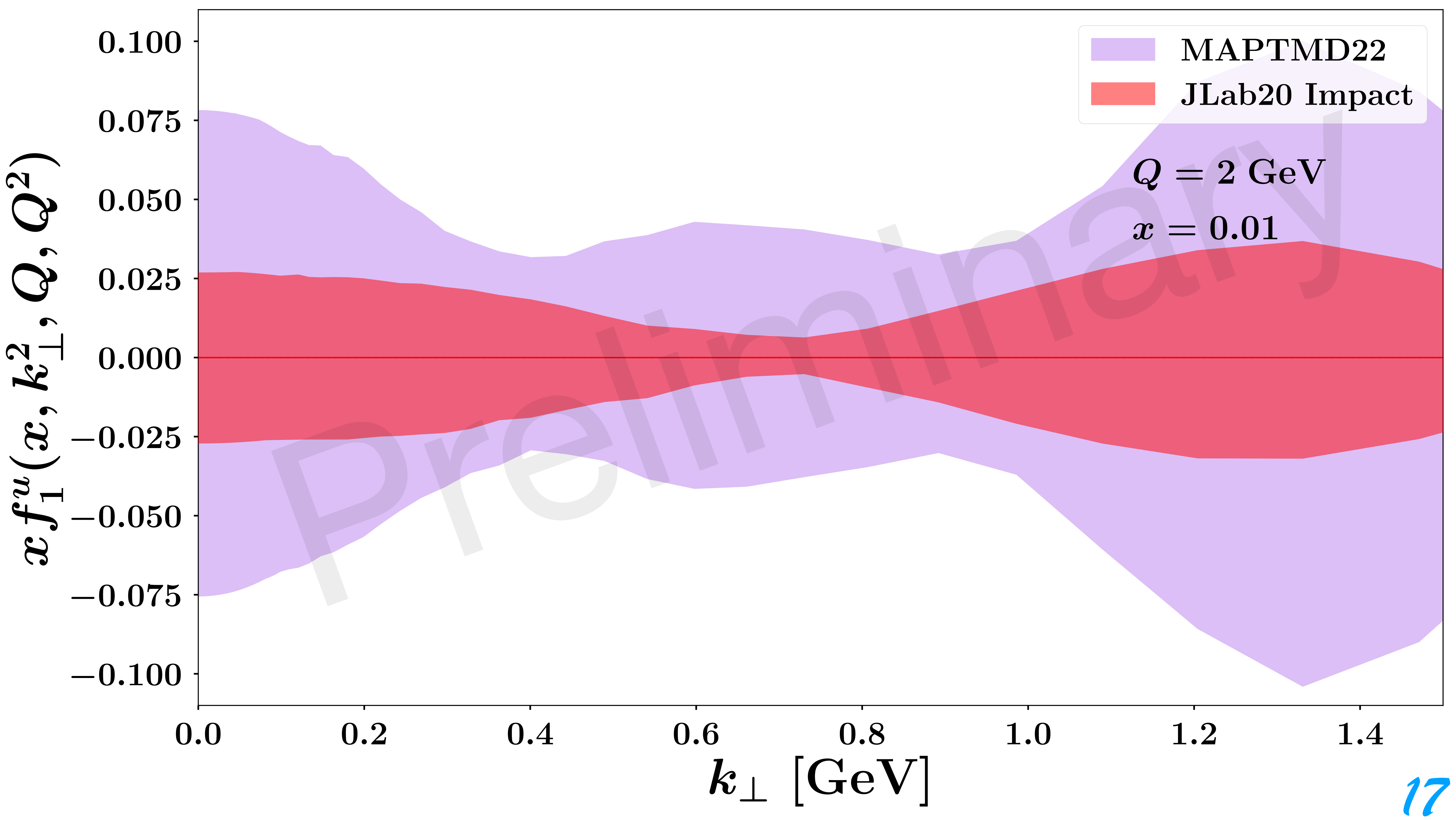
better constrain at high x and low Q

Impact studies - JLab 20+



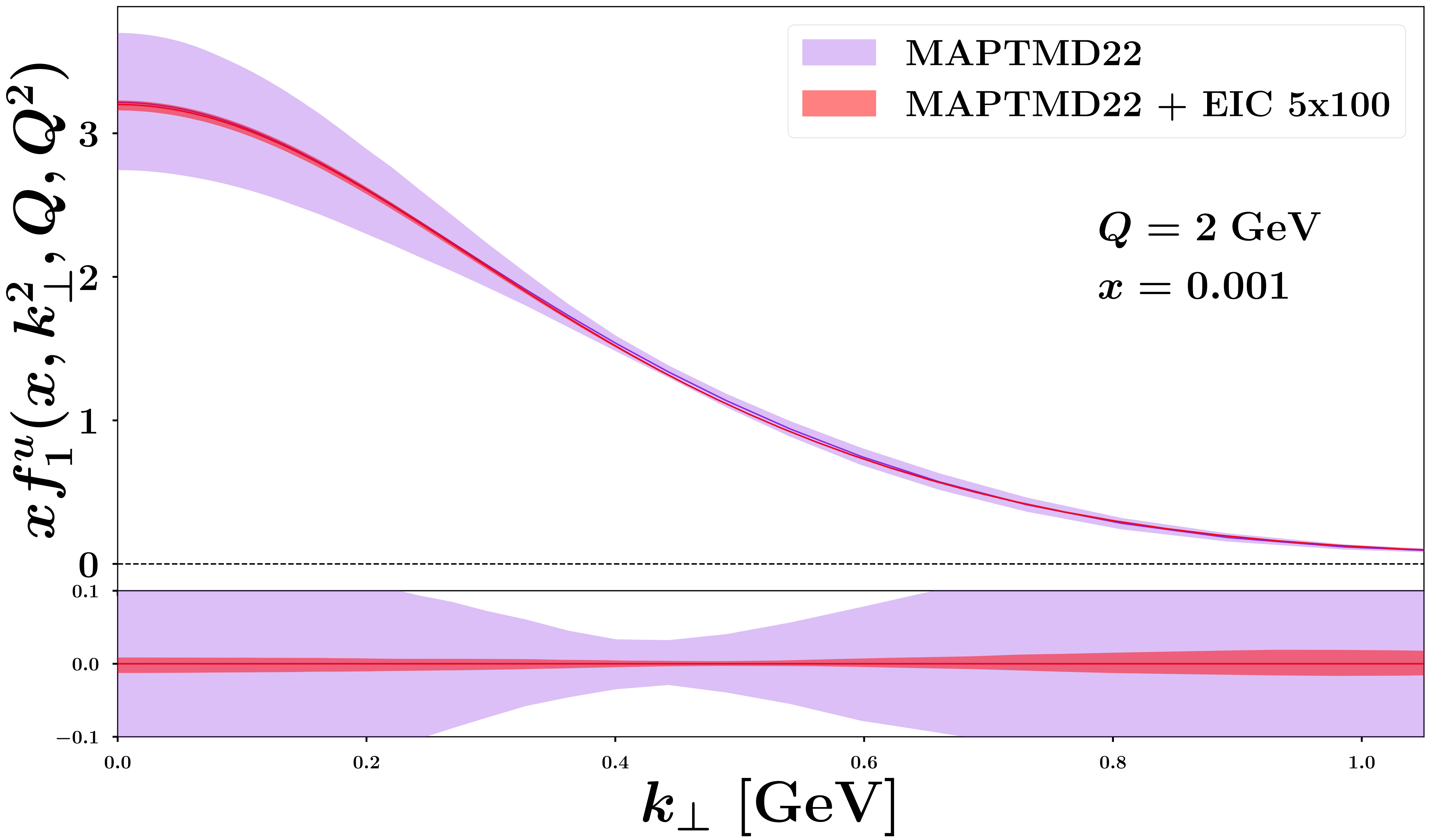


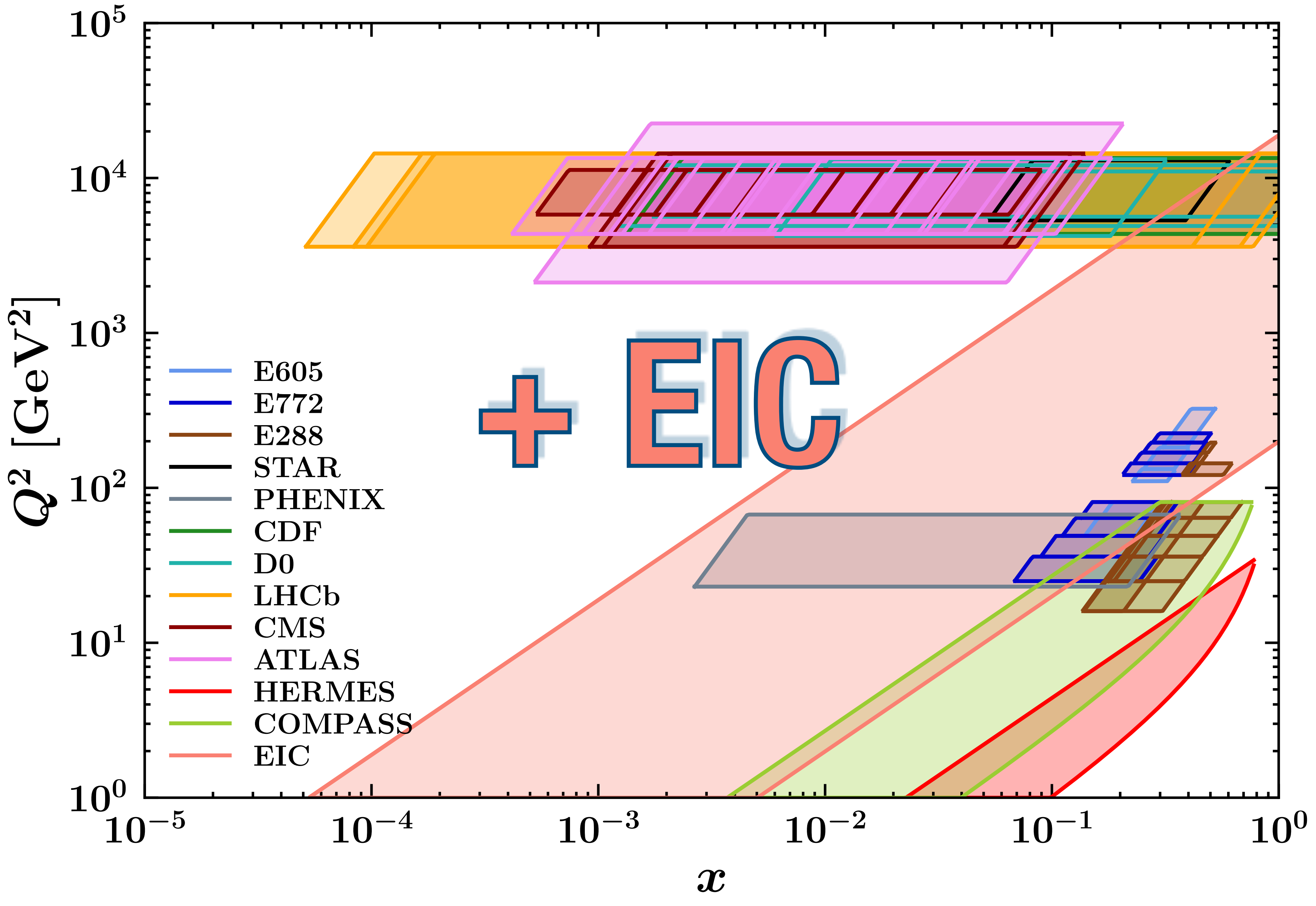
+ JLAB 20

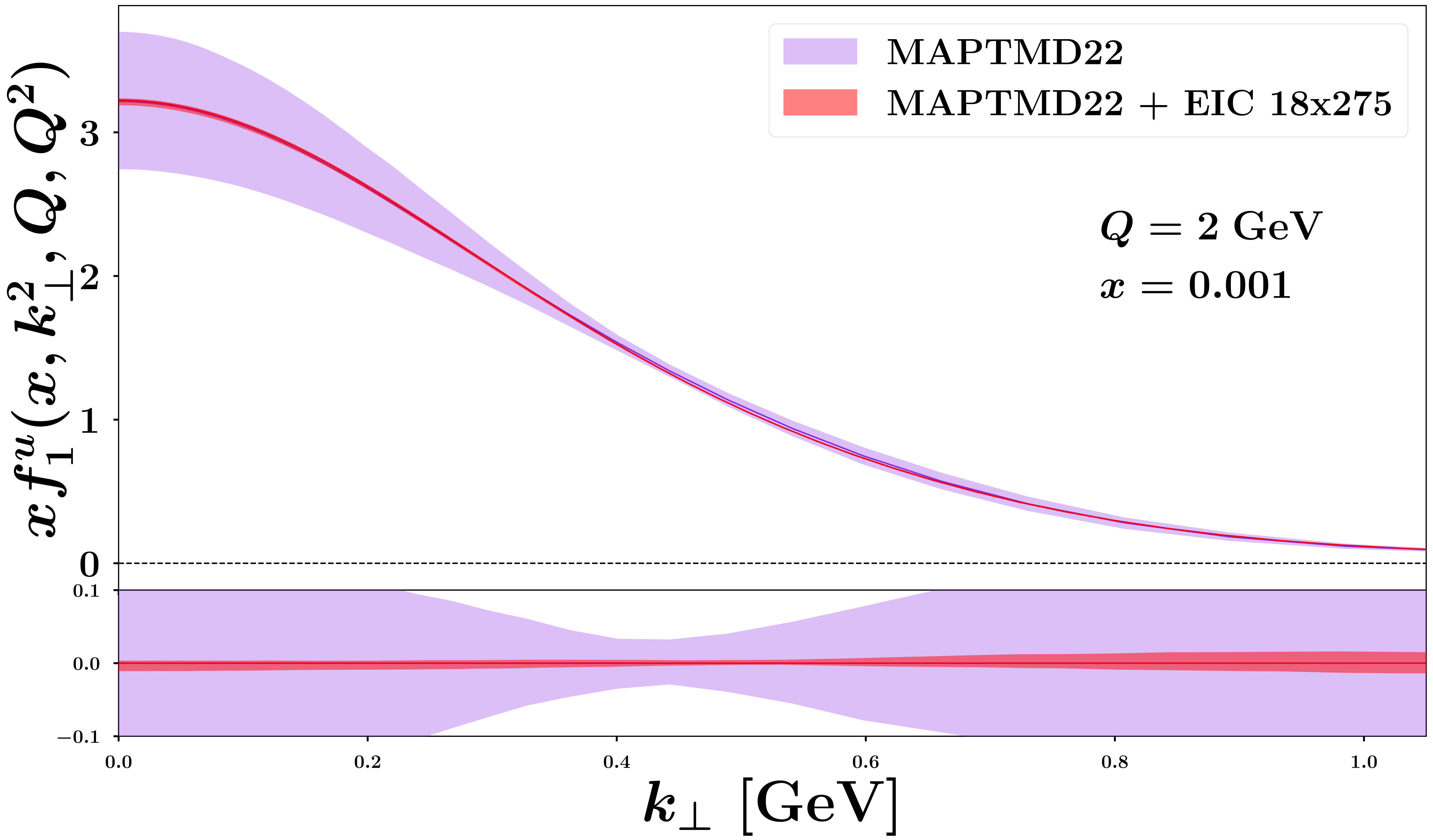


- better constrain at high x and low Q

Impact studies - EIC



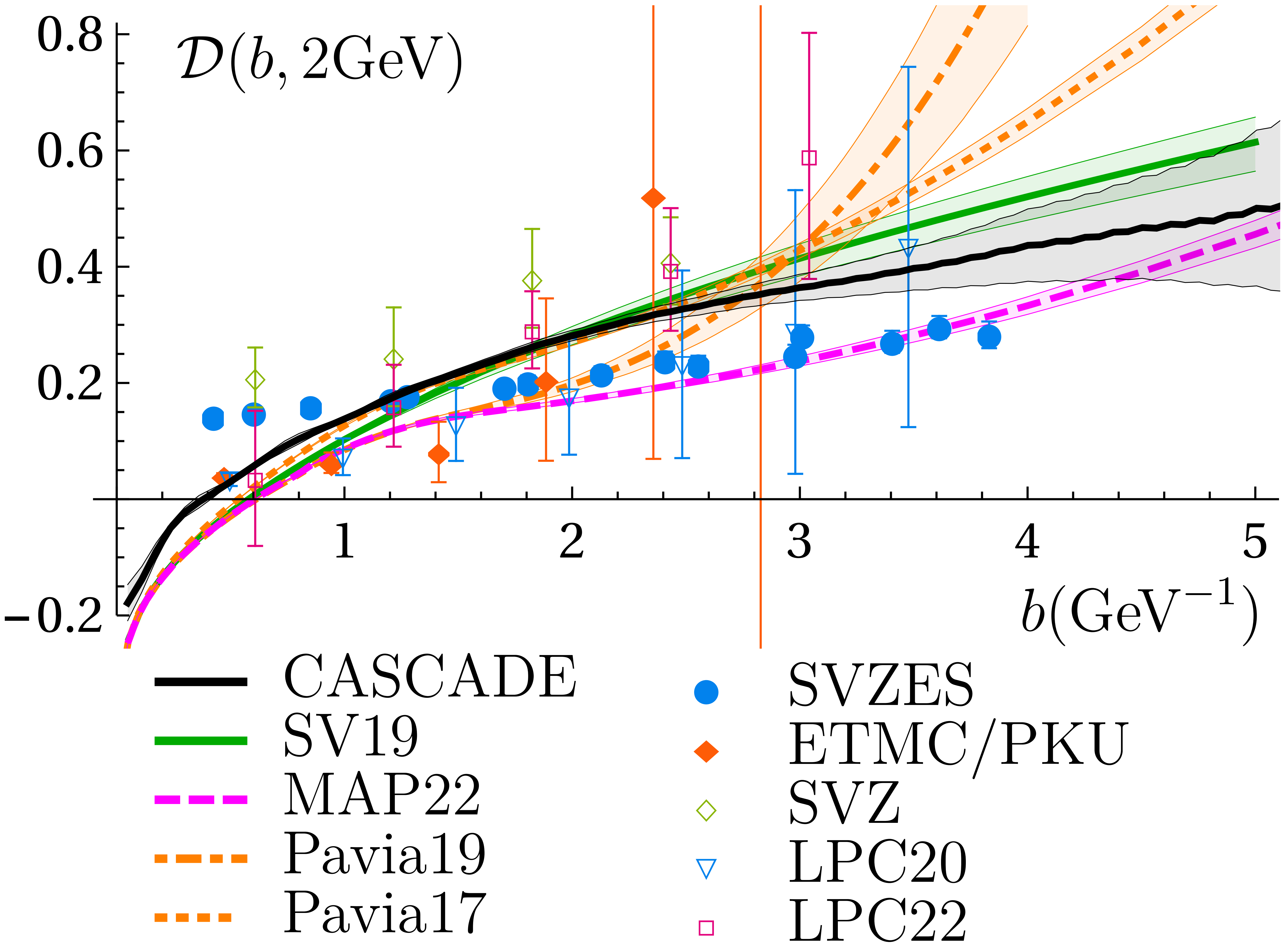




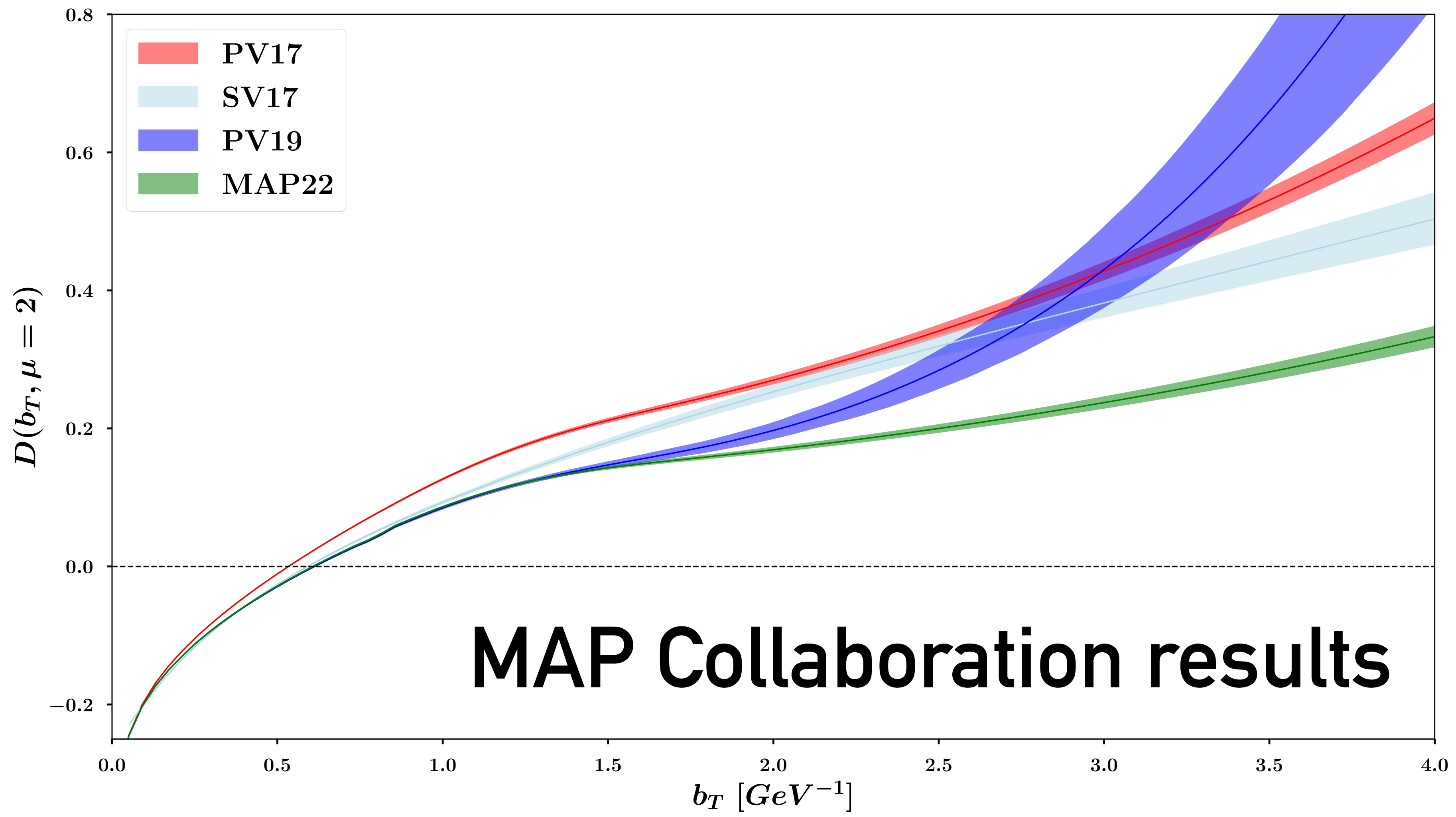
- general reduction of the bands

Collins-Soper Kernel

Comparison with lattice

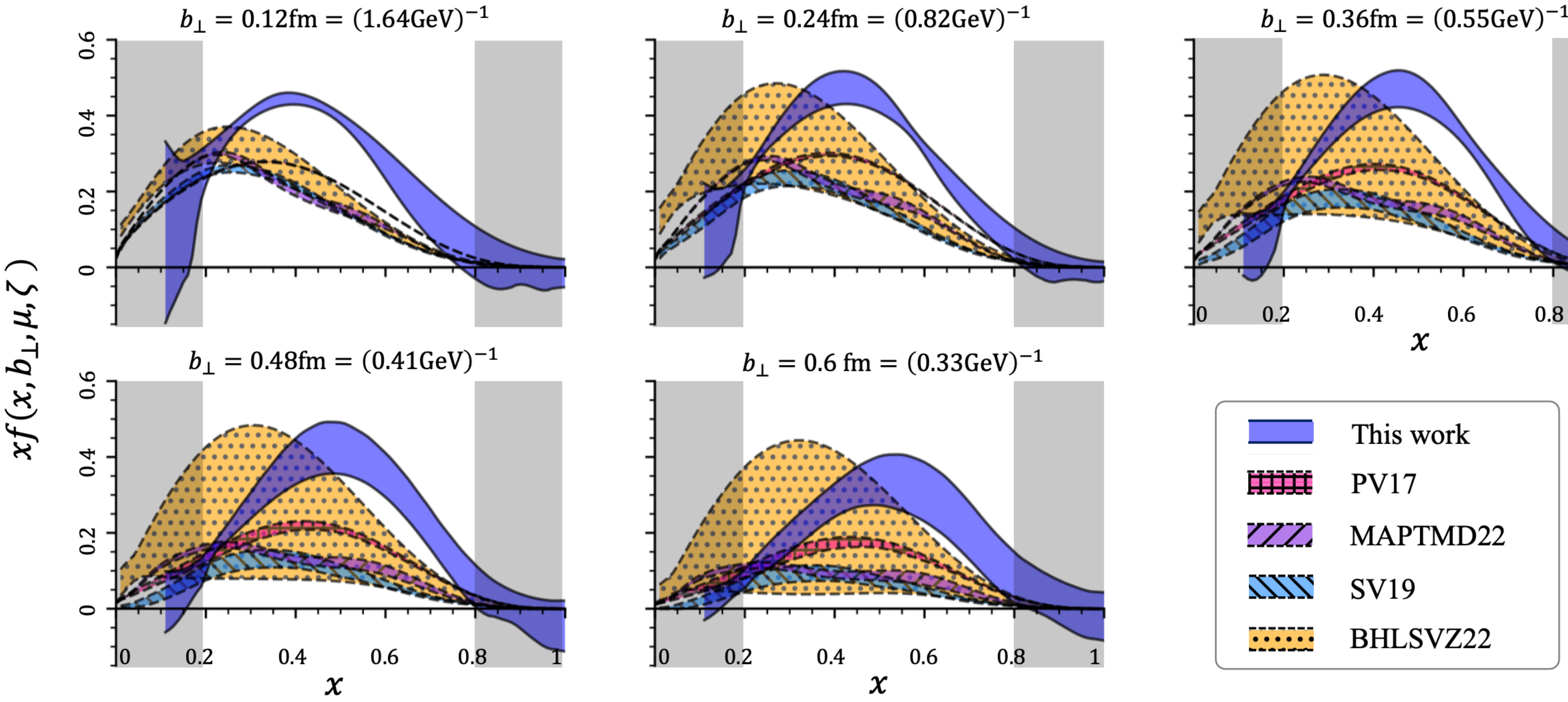


Martinez, Vladimirov, arXiv:2206.01105



TMD comparisons
in bT space

- in the shaded grey regions LaMET predictions are not reliable



Lattice Parton Collaboration, arXiv: 2211.02340

peak positions are not exactly the same

MAP22 global fit

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- **Global analysis** of Drell-Yan and Semi-Inclusive DIS data sets → **2031** data points
- Perturbative accuracy: **N^3LL^-**
- **Normalization** of SIDIS multiplicities beyond NLL
- Number of parameters: **21**
- Extremely good description: $\chi^2/N_{\text{data}} = 1.06$

Backup

Source of W term suppression

Hard factor

$$\mathcal{H}_{ab}^{\text{SIDIS}}(Q,Q) = e_a^2 \delta_{ab} \left(1 + \frac{\alpha_S}{4\pi} C_F \left(-16 + \frac{\pi^2}{3} \right) \right)$$

Source of W term suppression

Hard factor

$$\mathcal{H}_{ab}^{\text{SIDIS}}(Q,Q) = e_a^2\delta_{ab}\left(1 + \frac{\alpha_S}{4\pi}C_F\left(-16 + \frac{\pi^2}{3}\right)\right)$$

- introducing $\mathcal{O}(\alpha_s)$ terms

→ reduces the structure function to about 60% of its original value.

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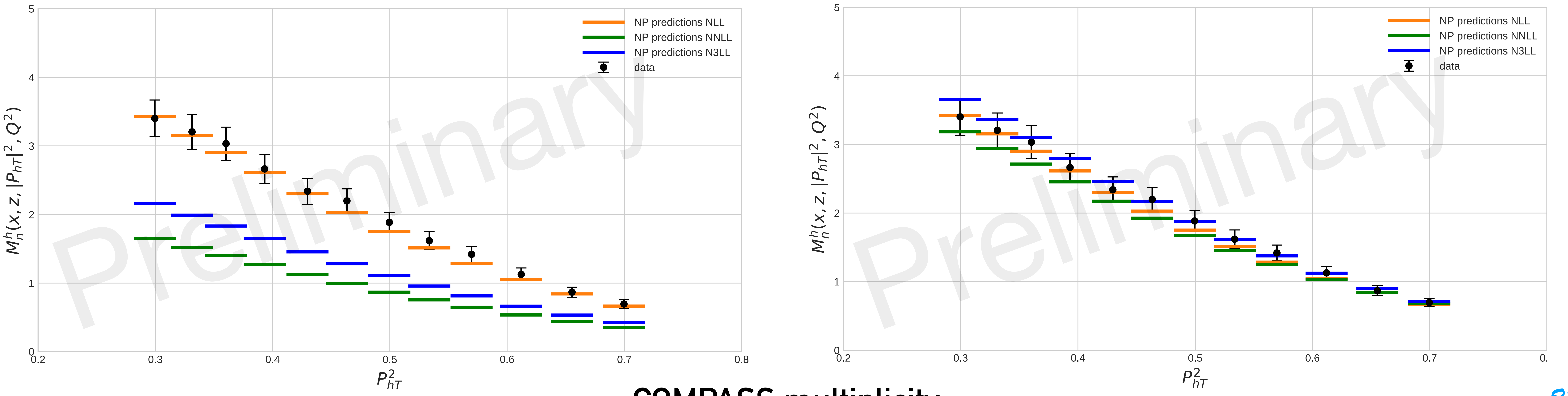
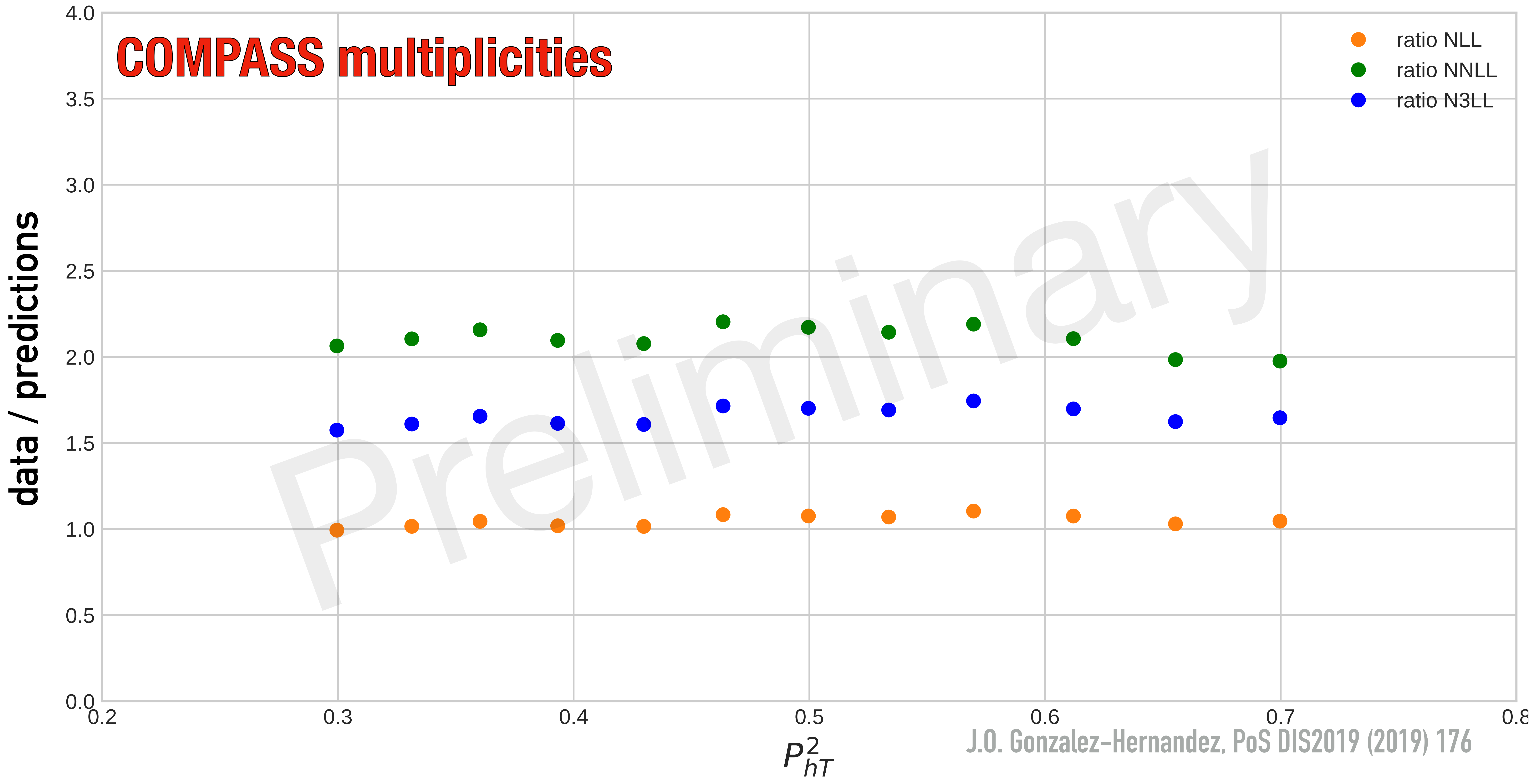
Full Hard Factor

Hard Factor = 1



COMPASS multiplicity

Normalization of SIDIS multiplicities



The discrepancy amounts to an almost **constant factor**

Normalization of SIDIS multiplicities

Introduction of a normalization prefactor

(at low P_{hT})

$$\int W \Big|_{O(\alpha_S)} \quad \textbf{should} \sim \quad \frac{d\sigma^h}{dx\,dQ^2\,dz}\Big|_{O(\alpha_S)}$$

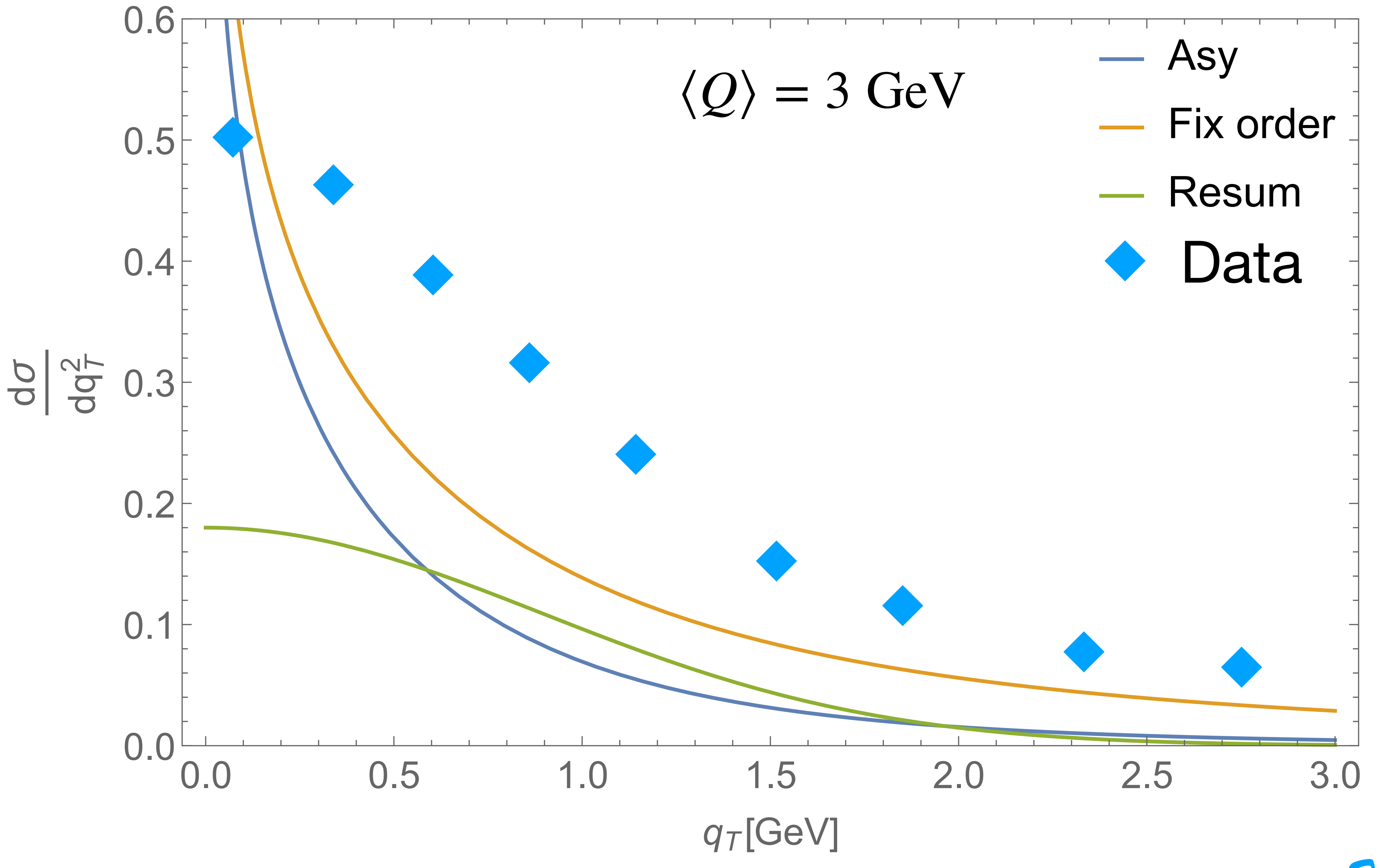
integral of the TMD formula — collinear cross section

BUT

this is not the case in the experimental regions under consideration

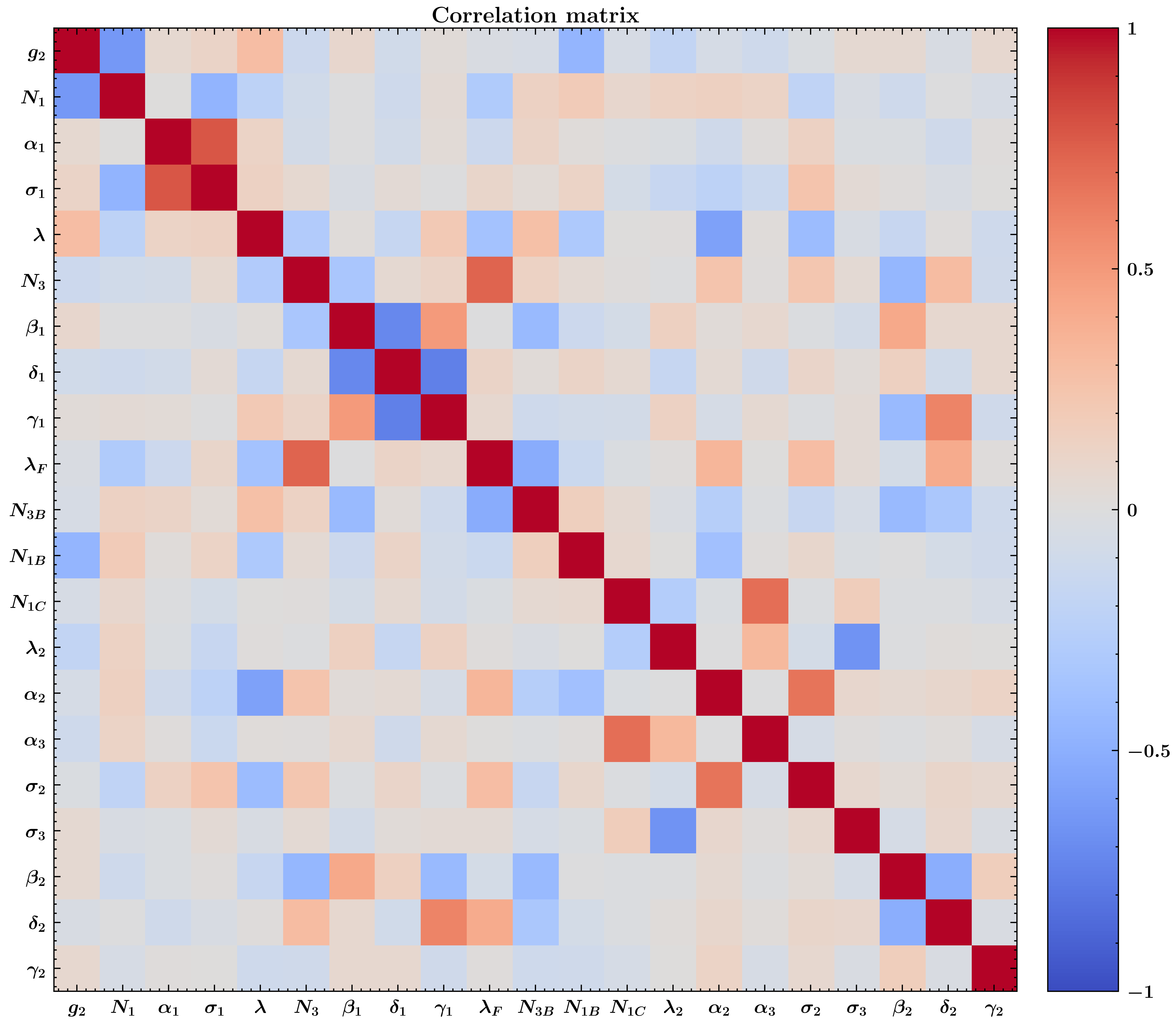
and we see

$$\int W \Big|_{O(\alpha_S)} \ll \frac{d\sigma^h}{dx\,dQ^2\,dz}\Big|_{O(\alpha_S)}$$



Fit results: correlation matrix

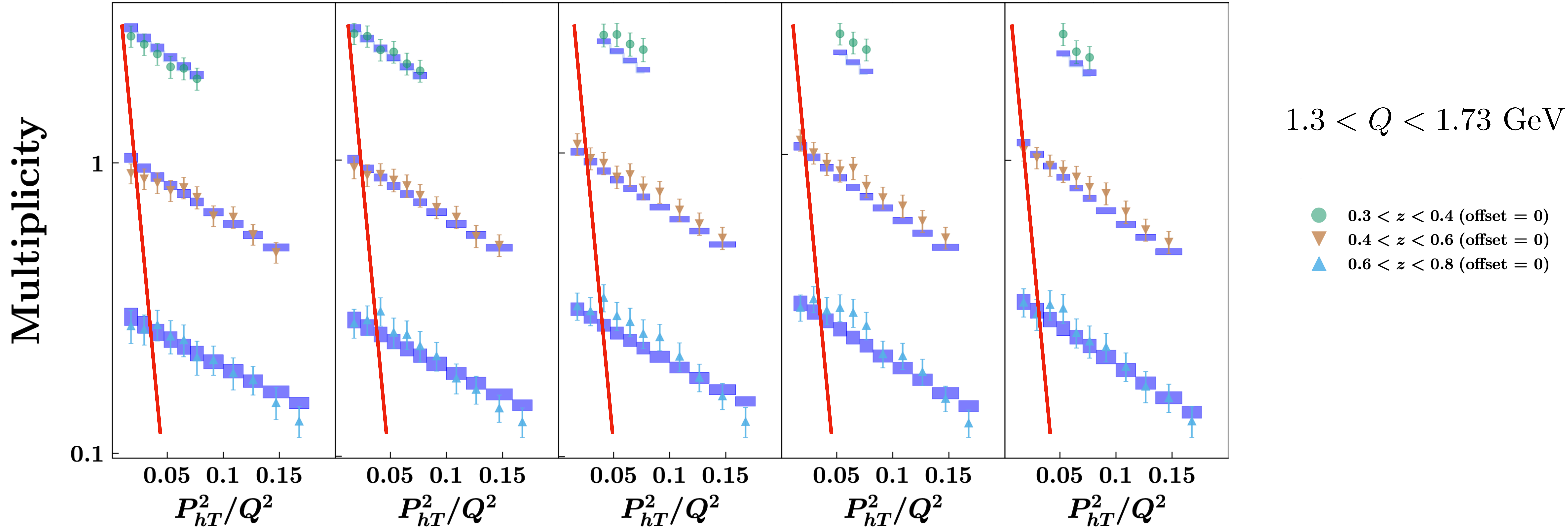
250 Montecarlo replicas



- **21 parameters**
- ideal situation: red diagonal, gray off-diagonal

SIDIS cut for data selection

COMPASS multiplicities



$$P_{hT}|_{\max} = \min[\min[0.2Q, 0.5zQ] + 0.3\text{ GeV}, zQ]$$

vs

$$\left.\frac{P_{hT}}{zQ}\right|_{\max} = 0.25$$

Cut qT/Q for SIDIS dataset