Lattice calculation of Collins-Soper Kernel

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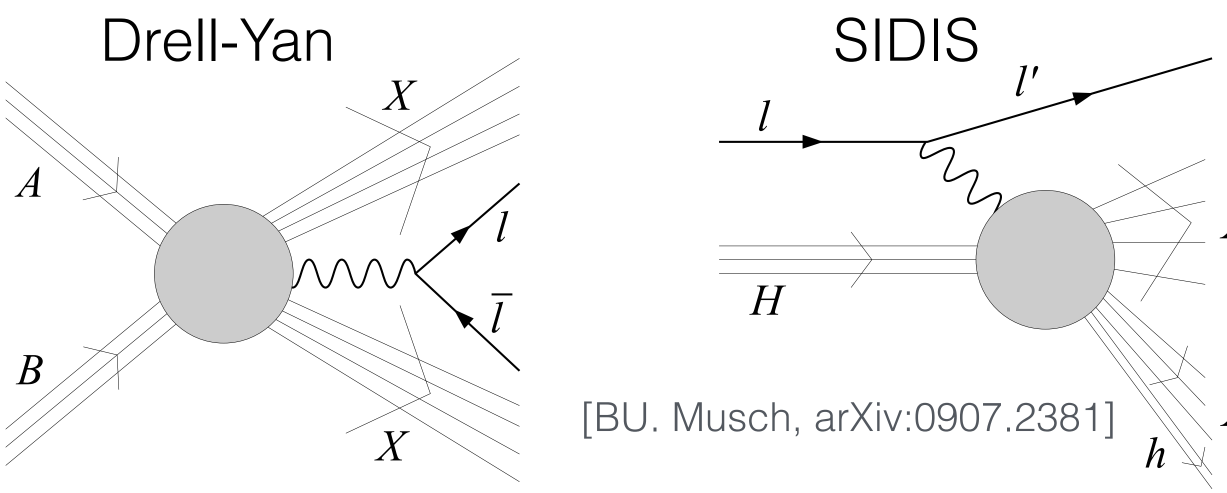
arXiv: 2302.06502
arXiv: 2302.09961
arXiv: 2306.06488

TMDs: Towards a Synergy between Lattice QCD and Global Analyses
June 21-23, 2023, Stony Brook University

From TMDs to Collins-Soper kernel

- TMDs universal in Drell-Yan process and semi-inclusive deep inelastic scattering



[BU. Musch, arXiv:0907.2381]

- CS kernel encodes the rapidity scaling properties of transverse momentum dependent parton distribution functions (TMDPDFs) / wave functions (TMDWFs):

$$2\zeta \frac{\mathrm{d}}{\mathrm{d}\zeta} \ln f^{\mathrm{TMD}}(x, b_\perp, \mu, \zeta) = K(b_\perp, \mu)$$

- Extraction of CS kernel on the lattice from quasi TMDs:

Part I: from ratio of quasi TMDWFs within LaMET [M. A. Ebert, L. W. Stewart and Y. Zhao, PRD 99, 034505]

Part II: from ratio of Mellin moments of the quasi TMDPDFs [A. A. Vladimirov and A. Schaefer, PRD 101, 074517]

- Application of CS kernel: ingredient from quasi TMDs to physical TMDs

$$\tilde{f}_\Gamma(x, b_\perp, \zeta_z, \mu) \sqrt{S_I(b_\perp, \mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right) K(b_\perp, \mu)} f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$

[X. Ji and Y. Liu, PRD105,076014]

Part I

CS kernel from q-TMDWFs within LaMET

- Construction a ratio of quasi TMDWFs at different momenta (but same zeta_z)

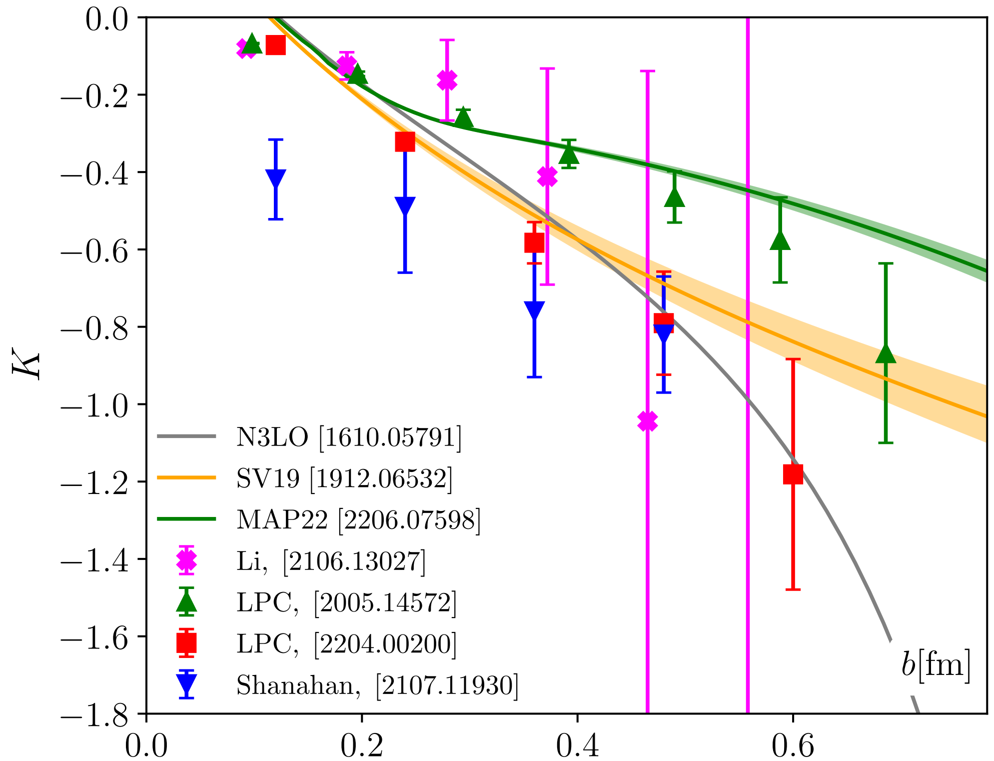
$$\tilde{f}_\Gamma(x, b_\perp, \zeta_z, \mu)\boxed{\sqrt{S_I(b_\perp, \mu)}} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right)K(b_\perp,\mu)} f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2\zeta_z}\right)$$

rapidity free intrinsic soft function cancels in the ratio

- Factorization formula via quasi TMDs available from lattice QCD:

$$K(b_\perp, \mu, x, P_1^z, P_2^z) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{H^\Gamma(xP_2^z, \mu)\tilde{f}(x, b_\perp, \mu, P_1^z)}{H^\Gamma(xP_1^z, \mu)\tilde{f}(x, b_\perp, \mu, P_2^z)}$$

- via quasi TMDPDFs:
  - Leading order matching:
    [P. Shanahan, et al, PRD 104 (2021) 114502] […]
- via quasi TMDWFs:
  - Leading order matching:
    [Q.A. Zhang et al., PRL125, 192001]
    [Yuan Li et al, PRL.128(2022)6,062002]
  - Next-to-leading order matching (MILC):
    [M.-H. Chu et al, PRD 106 (2022) 3, 034509]



- Lattice computations display qualitatively similar behavior
- The discrepancies can be understood as systematic effects

Goal of this talk:

A NLO determination on CLS ensemble

Hear more at physical pion mass with 2-loop matching from A. Avkhadiev's talk at 12:00pm

Calculation of quasi TMDWFs on the lattice

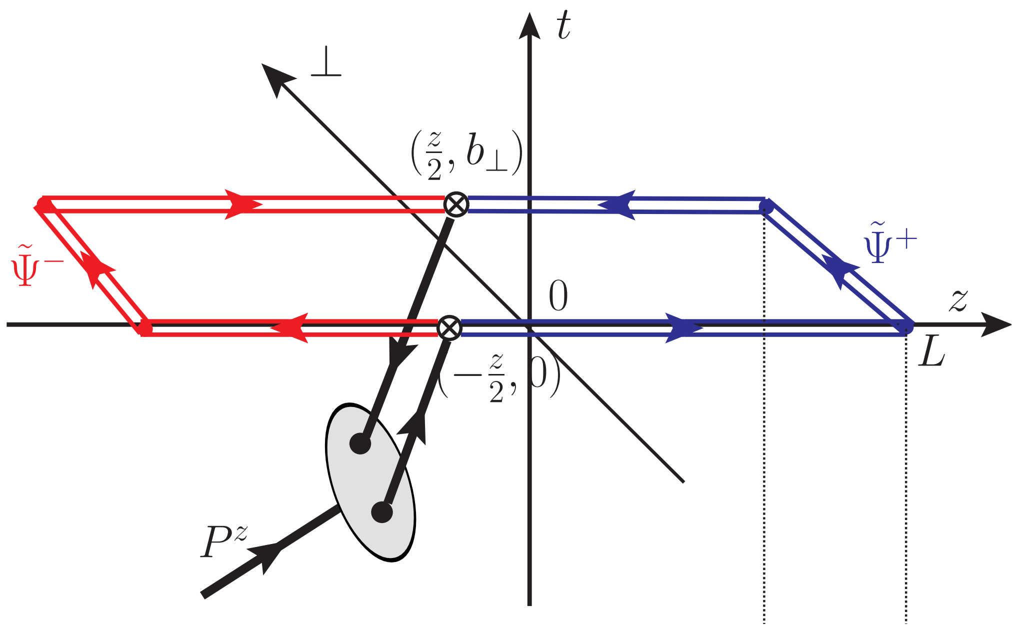
- Quasi TMD wave functions (in momentum space):

Renormalization: [LPC, PRL 129 (2022) 082002]

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z) = \lim_{L\to\infty} \int \frac{P^z dz}{2\pi} e^{ixzP^z} \frac{\tilde{\Psi}^{\pm}(z, b_\perp, \mu, P^z)}{\sqrt{Z_E(2L+|z|, b_\perp, \mu)} Z_O(1/a, \mu, \Gamma)}$$

- Quasi TMD wave functions (in position space):

$$\tilde{\Psi}^{-}(L, z, b_\perp, \mu, P^z) = \langle 0 | \bar{q}(b_\perp \hat{n}_\perp) \gamma^t \gamma_5 U_c q(z\hat{n}_z) | \pi(P^z) \rangle$$



- Quasi TMD wave functions from lattice data:

[M.-H. Chu et al, PRD 106 (2022) 3, 034509]

$$C_2^-(L, z, b_\perp, t, P^z) = \sum_{\vec{x}} e^{iP^z \vec{x}\cdot\hat{n}_z} \langle S_w^\dagger(\vec{x} + b_\perp \hat{n}_\perp, t) U_c \gamma^5 \Gamma S_w(\vec{x} + z\hat{n}_z, t) \rangle$$

$\Gamma = \gamma^t (\text{or } \gamma^z)$

$$\frac{C_2^-(L, z, b_\perp, t, P^z)}{C_2^-(L, z=0, b_\perp=0, t, P^z)} = \tilde{\Psi}^-(L, z, b_\perp, \zeta^z) \frac{1 + c_0(z, b_\perp, P^z, L) e^{-\Delta E t}}{1 + c_1 e^{-\Delta E t}}$$

Higher twists effects are <=5%, see

[Q.A. Zhang et al., PRL125, 192001]

Extract q-TMDWFs via joint fit of data at all l&b (sharing energies) for each P^z

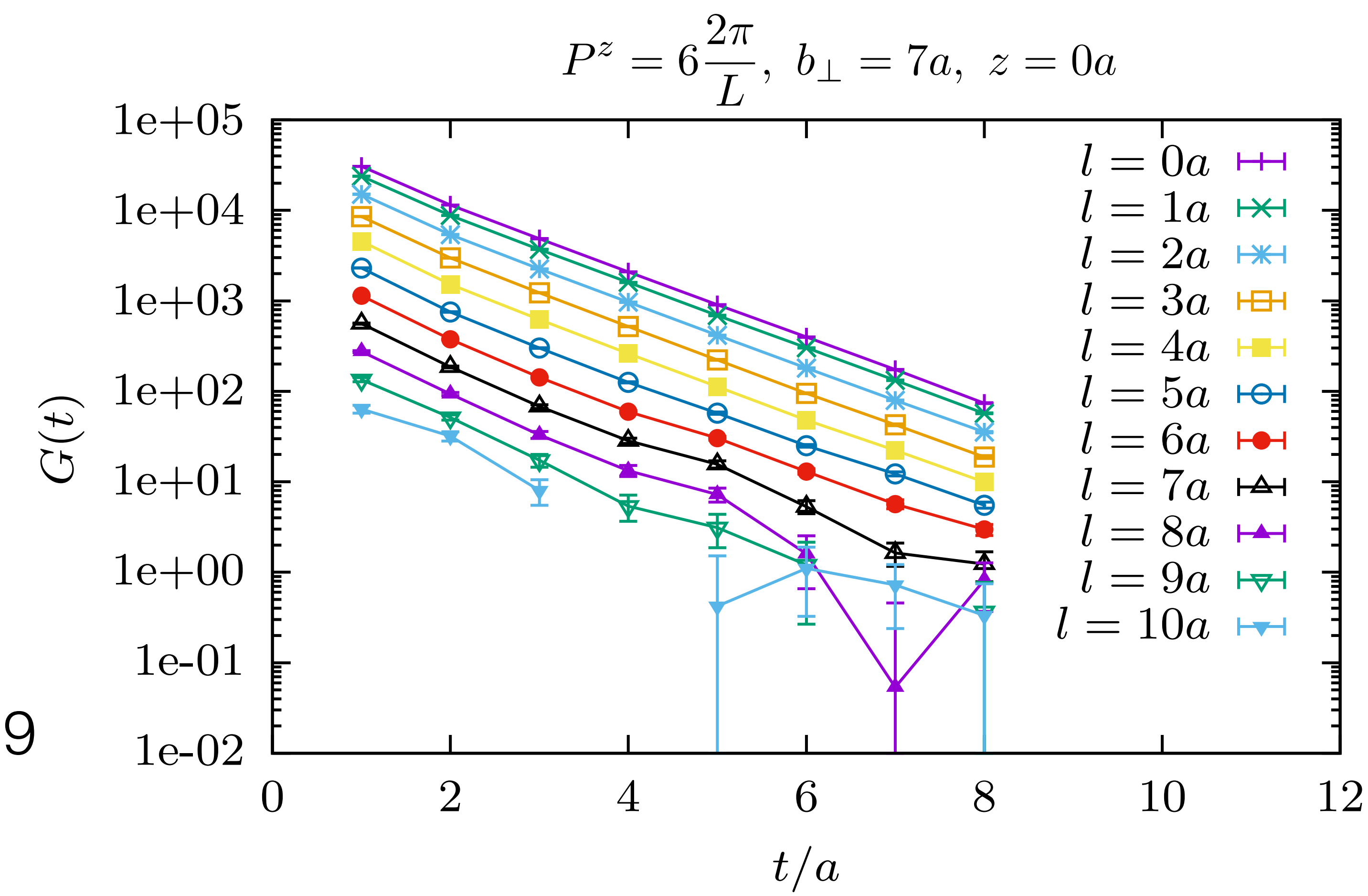
Lattice setup

- We use newly generated 2+1 flavor clover fermion CLS ensemble X650

β	$L^3 \times T$	a	m_π^{sea}	m_π^v	#conf
3.34	$48^3 \times 48$	0.098 fm	333 MeV	662 MeV	1000

- HYP smearing + Wall source
- 2 sources per configuration
- Transverse direction: x and y
- Momenta: P^z={0, 6,8,10,12} x 2pi/L

  (i.e. 0,1.6, 2.1, 2.6, 3.2 GeV)
- 0<=l/a<=10, 0<=b/a<=7, 0<=t/a<=9



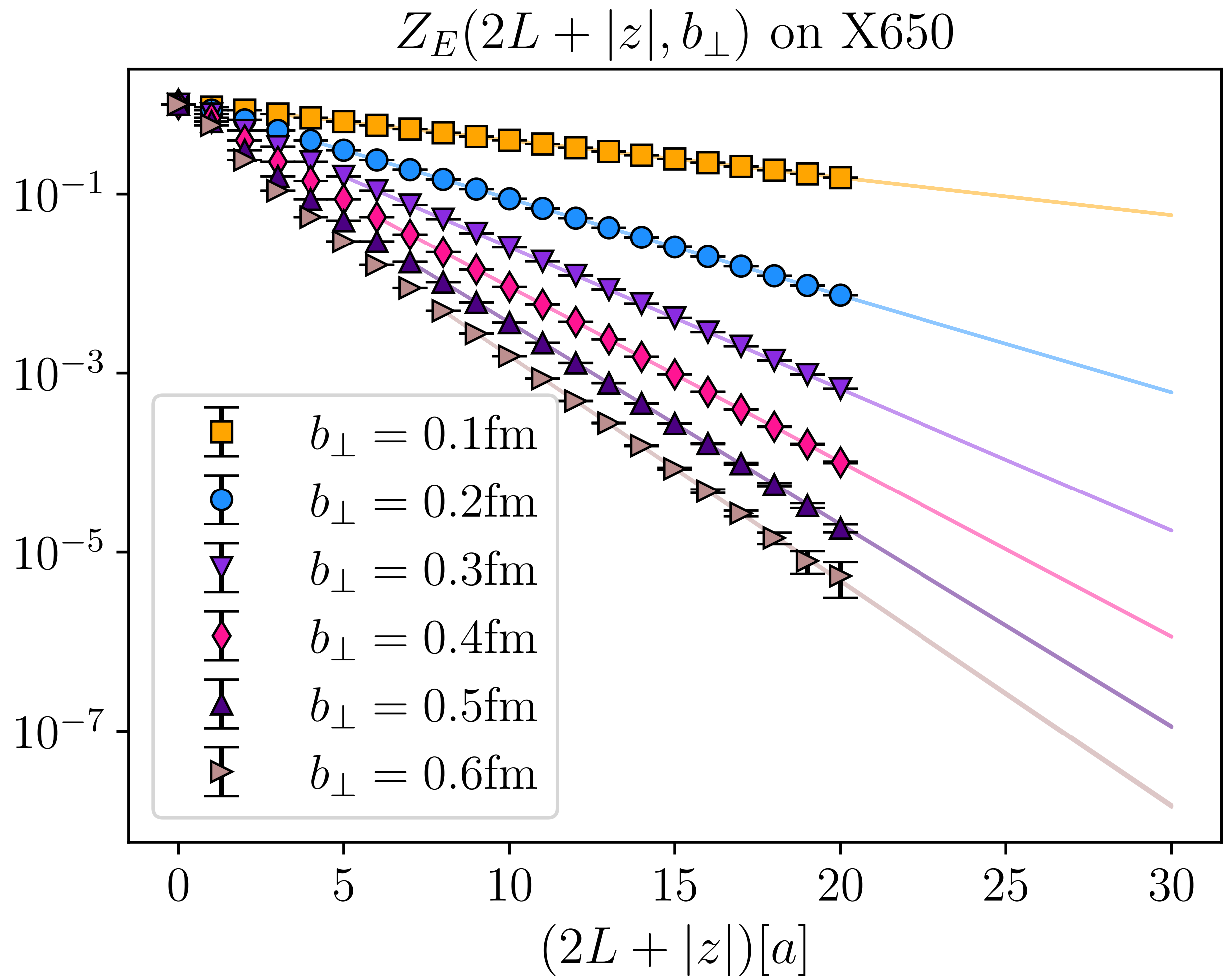
A glance at the two point functions

Renormalization of the quasi TMDWFs (I)

[LPC, PRL 129 (2022) 082002]

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z) = \lim_{L\to\infty} \int \frac{P^z dz}{2\pi} e^{ixzP^z} \frac{\tilde{\Psi}^{\pm}(z, b_\perp, \mu, P^z)}{\sqrt{Z_E(2L+|z|, b_\perp, \mu)} Z_O(1/a, \mu, \Gamma)}$$

- The linear divergence and heavy quark potential is removed by Wilson loop Z_E
- Residual logarithmic divergence is removed by Z_O



see, e.g. [X.-D. Ji, et al, PRL 120 (2018) 112001]

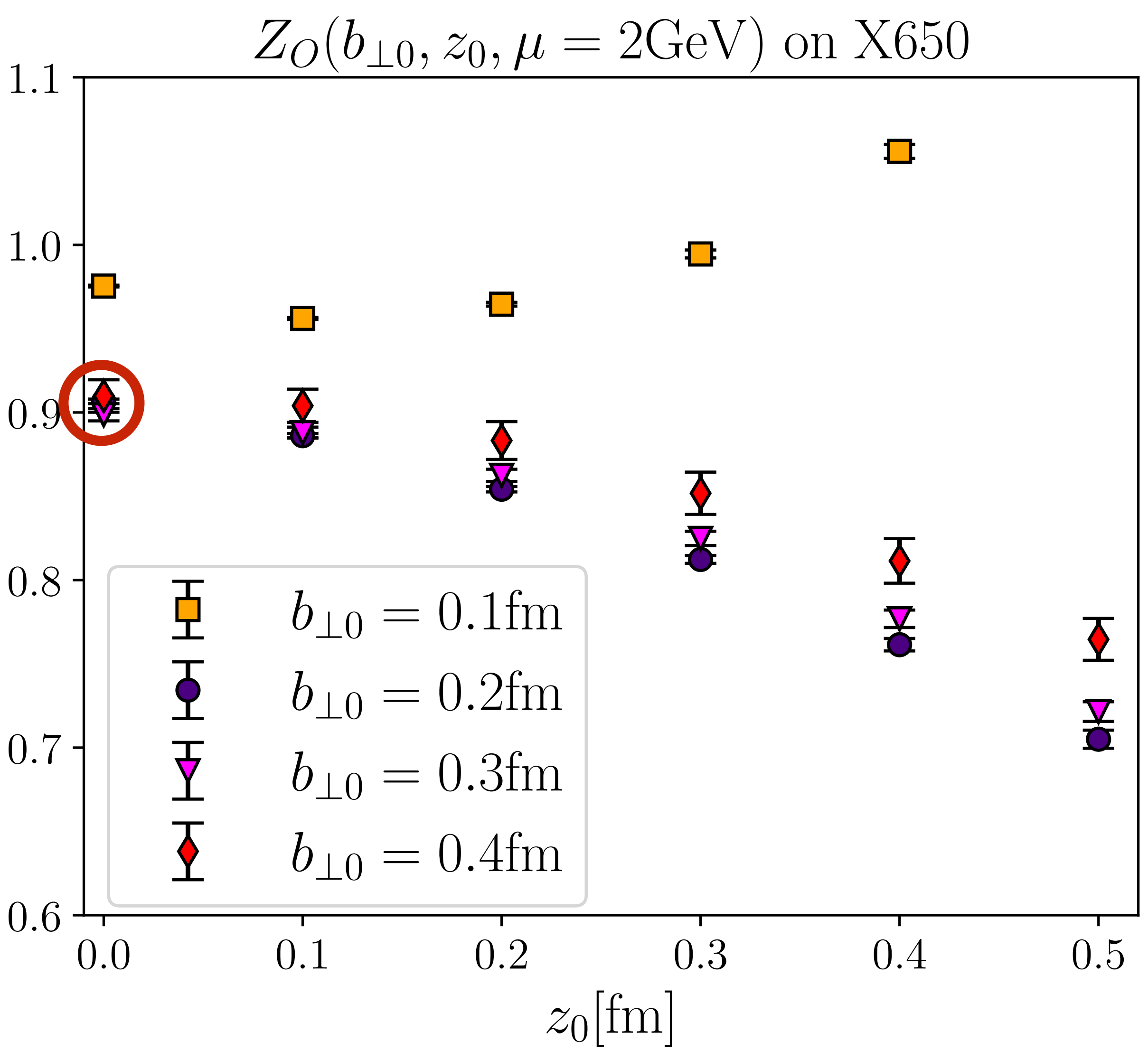
- Extrapolation to large length with 1-state fit

Renormalization of the quasi TMDWFs (II)

[LPC, PRL 129 (2022) 082002]

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z) = \lim_{L\to\infty} \int \frac{P^z dz}{2\pi} e^{ixzP^z} \frac{\tilde{\Psi}^{\pm}(z, b_\perp, \mu, P^z)}{\sqrt{Z_E(2L+|z|, b_\perp, \mu)} Z_O(1/a, \mu, \Gamma)}$$

- The linear divergence and heavy quark potential is removed by Wilson loop Z_E
- Residual logarithmic divergence is removed by Z_O [LPC, PRL 129 (2022) 082002]



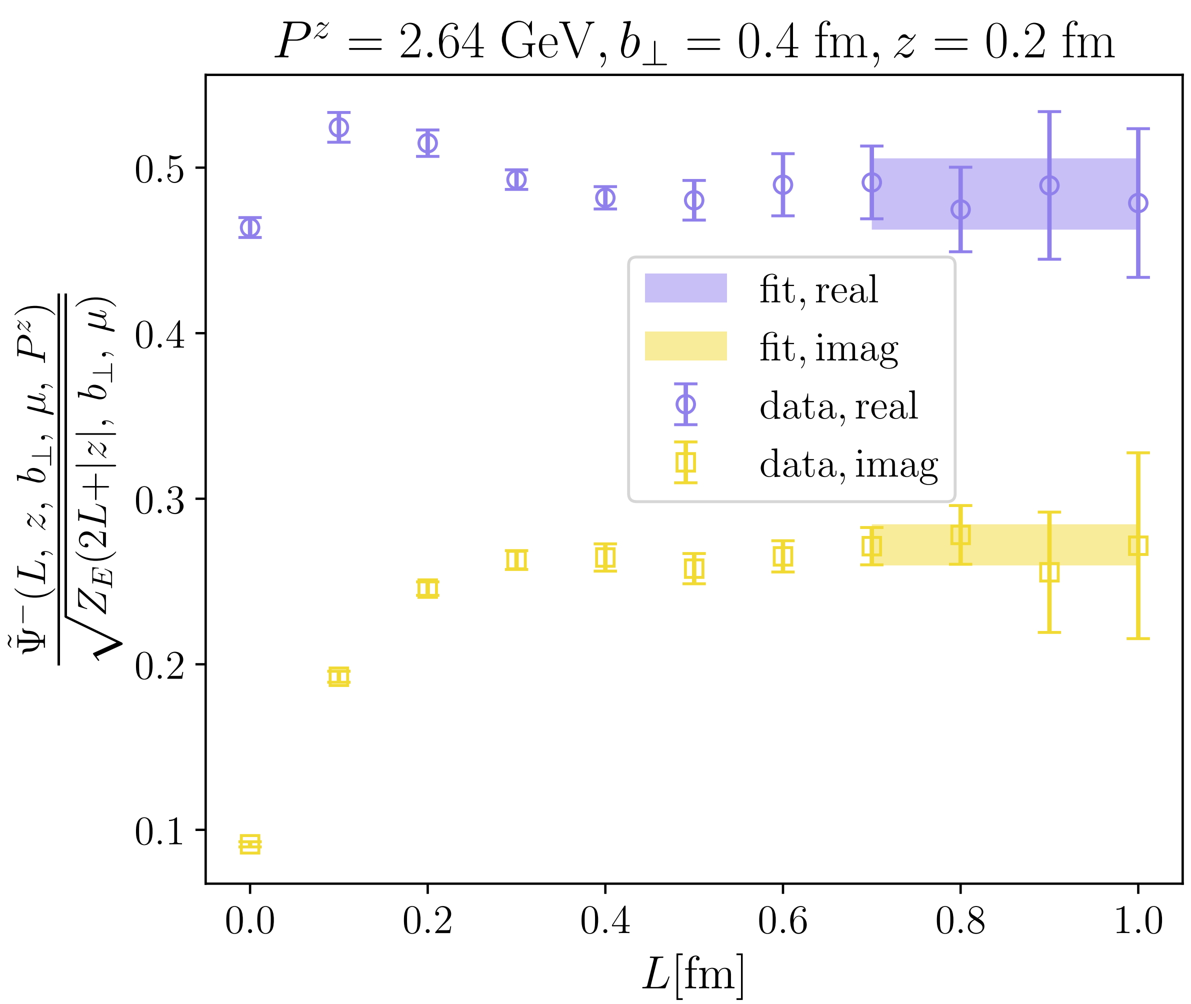
$$Z_O(1/a, \mu) = \frac{\tilde{\Psi}^{-}(z_0, b_{\perp 0}, P^z = 0)}{\tilde{\psi}^{\overline{\text{MS}}}(z_0, b_{\perp 0}, \mu)}$$

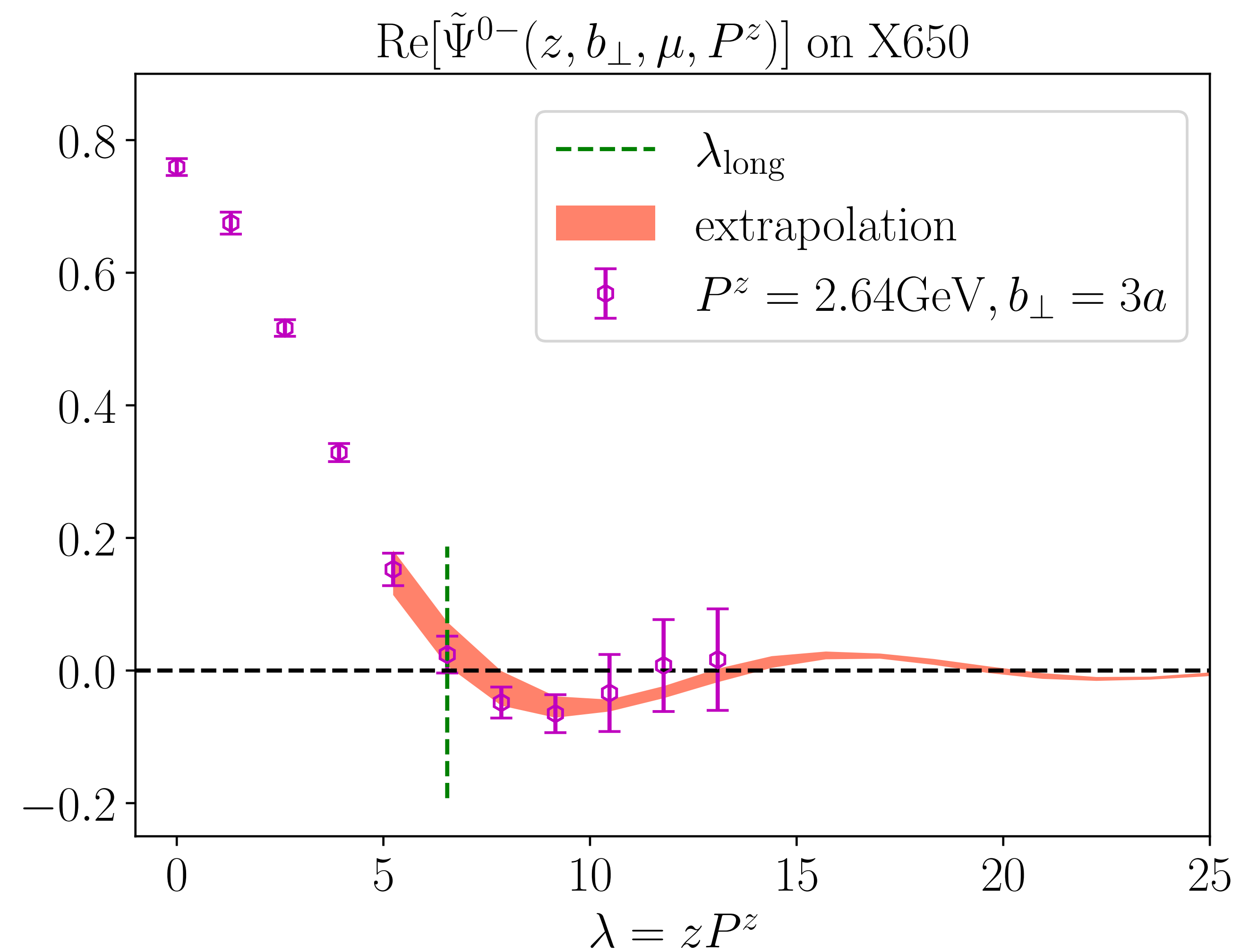
$$\tilde{\Psi}^{\overline{\text{MS}}}(z_0, b_{\perp 0}, \mu) = 1 + \frac{\alpha_s C_F}{2\pi}\left\{\frac{1}{2} + 3\gamma_E - 3\ln 2 + \frac{3}{2}\ln[\mu^2(b_{\perp 0}^2 + z_0^2)] - 2\frac{z_0}{b_{\perp 0}}\arctan\frac{z_0}{b_{\perp 0}}\right\} + \mathcal{O}(\alpha_s^2)$$

[LPC, PRL 129 (2022) 082002]

- Look for plateau in b where both discretization effects and higher twist contaminations are suppressed

From position space to momentum space



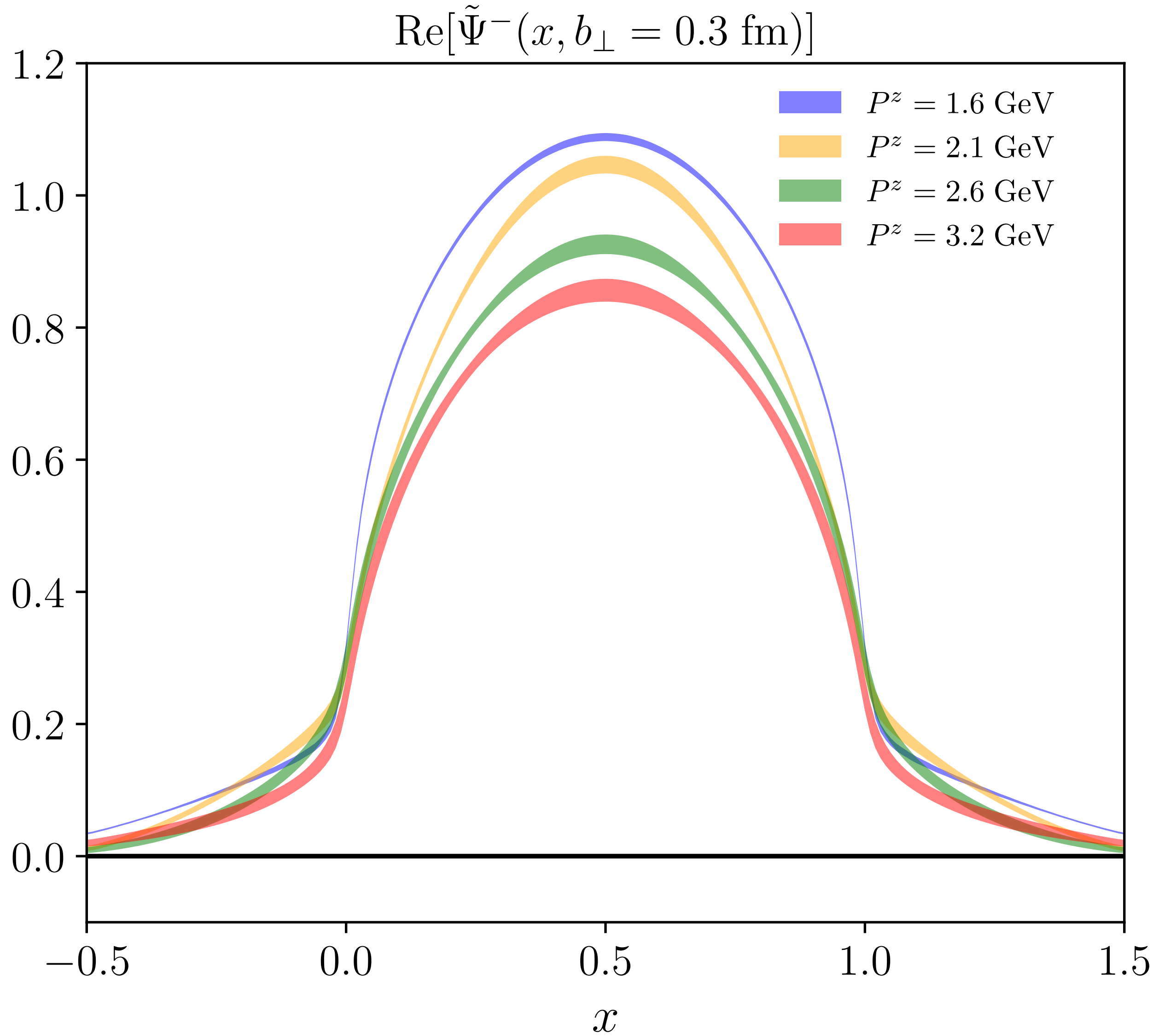


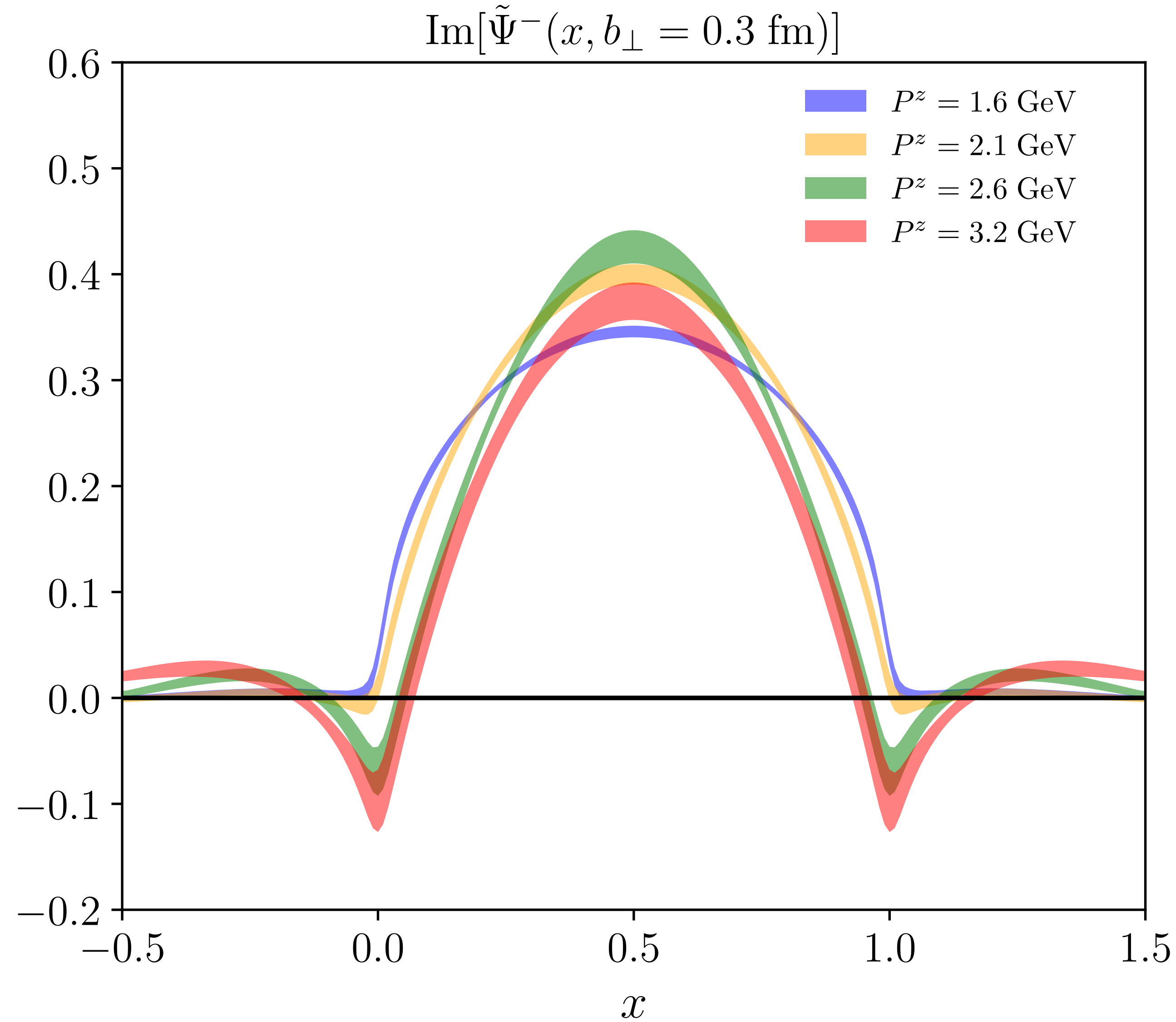
$$\tilde{\Psi}^{0-}_{\mathrm{extra}}(\lambda) = \left[\frac{m_1}{(-i\lambda)^{n_1}} + e^{i\lambda}\frac{m_2}{(i\lambda)^{n_2}}\right] e^{-\lambda/\lambda_0}$$

- Large L limit via constant fit in plateau range (L>~0.7 fm)
- Extrapolation for large lambda range using theoretically-inspired Ansatz [X.-D. Ji, et al., NPB 964 (2021) 115311]
- Fourier transformation going to momentum space

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z) = \lim_{L\to\infty} \int \frac{P^z dz}{2\pi} e^{ixzP^z} \cdots$$

Quasi TMDWFs in momentum space (II)





- Real part has larger amplitude at same momentum
- Clear dependence on momentum for both real and imaginary parts
- Stronger dependence on momentum for real part

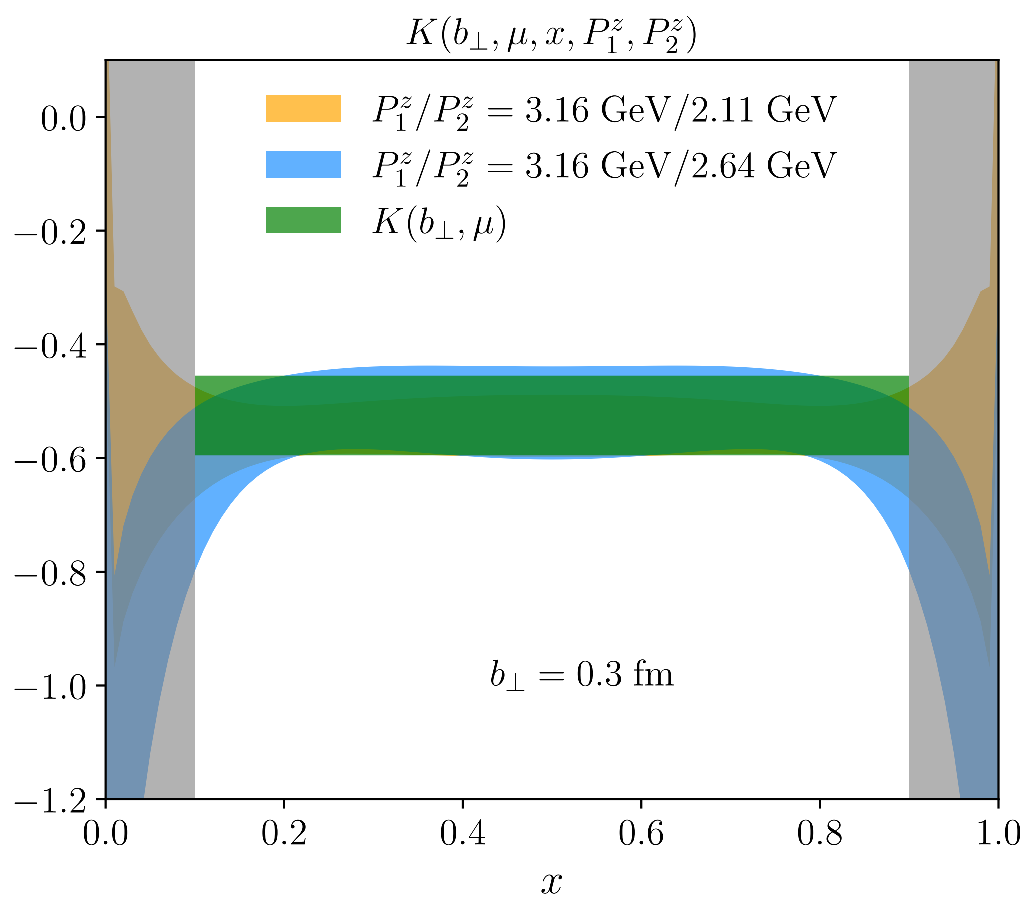
Collins-Soper kernel from LaMET

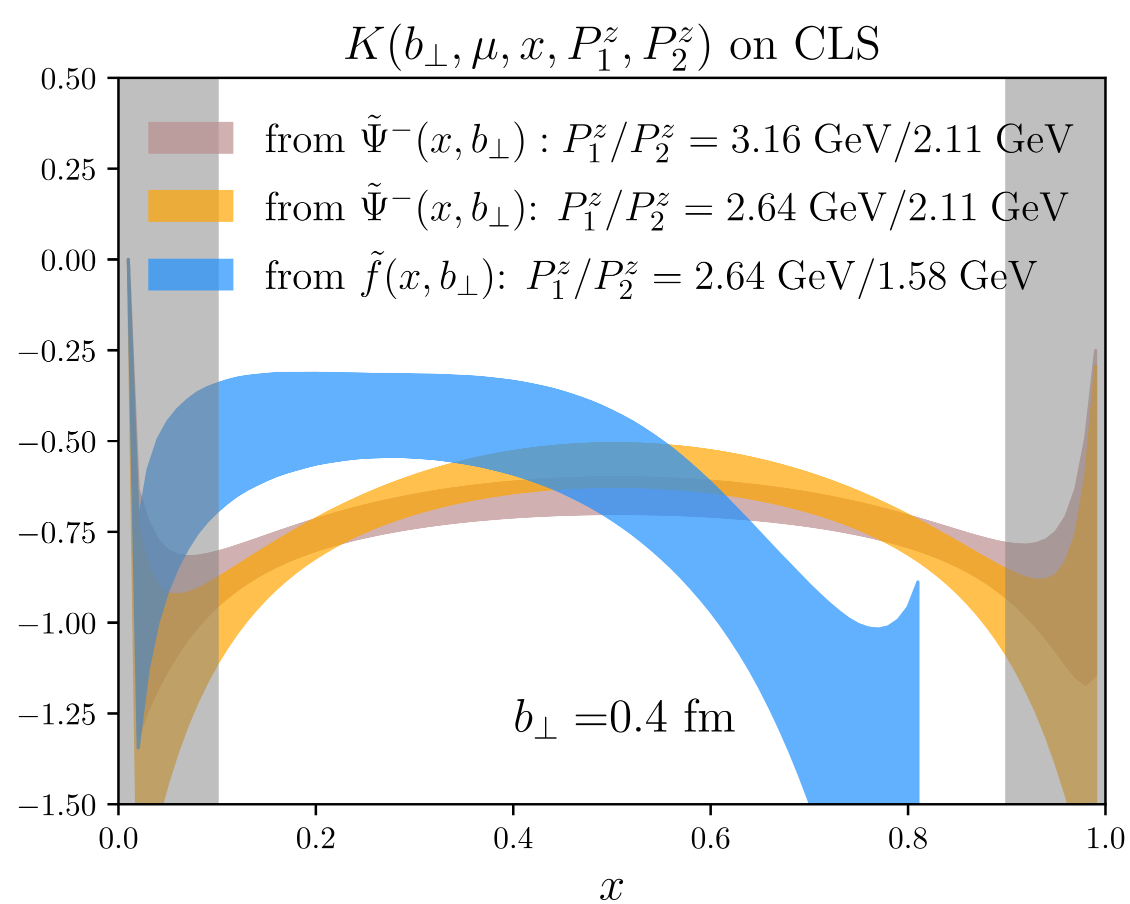
- 1-loop matching kernel H from: [X.-D. Ji and Y.-Z. Liu, PRD105, 076014]

$$K(b_\perp,\mu,x,P_1^z,P_2^z) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{H^-(xP_2^z,\mu)\tilde{\Psi}^-(x,b_\perp,\mu,P_1^z)}{H^-(xP_1^z,\mu)\tilde{\Psi}^-(x,b_\perp,\mu,P_2^z)}$$

- Extract leading power contribution via joint fit: [M.-H. Chu et al, PRD 106 (2022) 3, 034509]

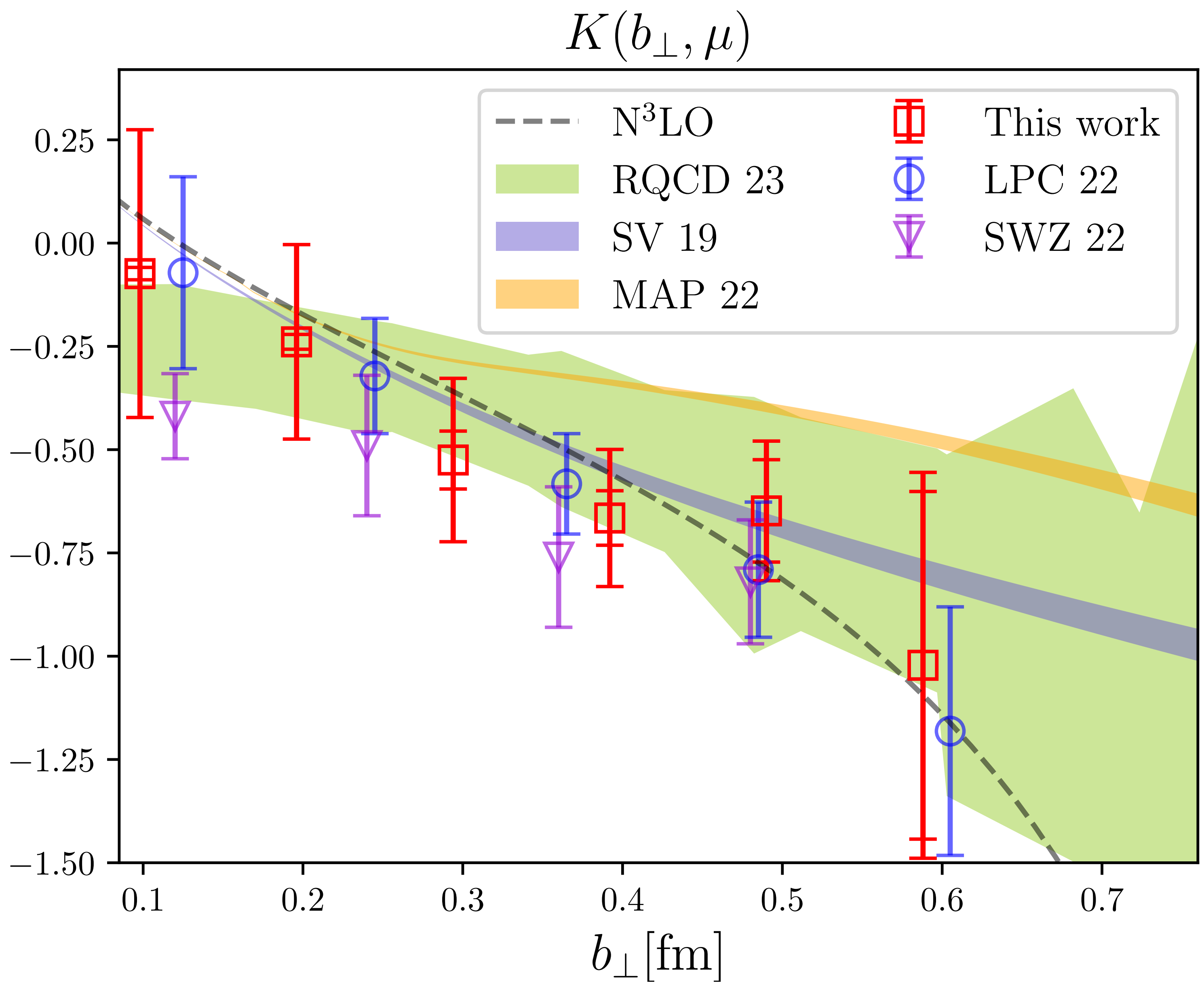
$$K(b_\perp,\mu,x,P_1^z,P_2^z) = K(b_\perp,\mu) + A\left[\frac{1}{x^2(1-x)^2(P_1^z)^2} - \frac{1}{x^2(1-x)^2(P_2^z)^2}\right]$$





- from quasi TMDWFs
- q-TMDWFs v.s. q-TMDPDFs

A comparison to literature



- Larger uncertainties in lattice extractions than in perturbative and phenomenal extractions
- Consistency among different lattice calculations
- Lattice determinations agree with 3-loop perturbative and SV19 phenomenal determinations

Part II

CS kernel from quasi TMDPDFs

- Leading power factorization of quasi TMDPDFs :

For theoretical intro. see
A. Vladimirov's talk at 9:30am

$$W^{[\Gamma]}_{f/h}(x;b;P;\mu) = \left(\frac{2|x|(P^+)^2}{\zeta}\right)^{K(b,\mu)/2} \mathbb{C}_H(xP^+,\mu)\Phi^{[\Gamma]}_{f/h}(x,b;\mu,\zeta) + \mathcal{O}(\lambda^2)$$

[S. Rodini and A. Vladimirov, arXiv:2211.04494]

- Evolve TMDPDFs to different momenta from the same momentum/rapidity along constant mu

$$2\zeta \frac{\mathrm{d}}{\mathrm{d}\zeta} \ln f^{\mathrm{TMD}}(x,b_\perp,\mu,\zeta) = K(b_\perp,\mu)$$

$W^{[\Gamma]}_{f/h}$ plays the role of $\tilde{f}$

$\mathbb{C}_H$ plays the role of H^Γ

$$\rightarrow \quad R^{[\Gamma]}(x,b,\mu;P_1,P_2) = \frac{W^{[\Gamma]}_{f/h}(x,b;P_1,S;\mu)}{W^{[\Gamma]}_{f/h}(x,b;P_2,S;\mu)} = \left(\frac{P_1^+}{P_2^+}\right)^{K(b,\mu)} \frac{\mathbb{C}_H(xP_1^+,\mu)}{\mathbb{C}_H(xP_2^+,\mu)} + \mathcal{O}(\lambda^2)$$

- Avoid difficulty in momentum space by letting z=0

[M. Schlemmer, et al., 10.1007/JHEP08(2021)004]

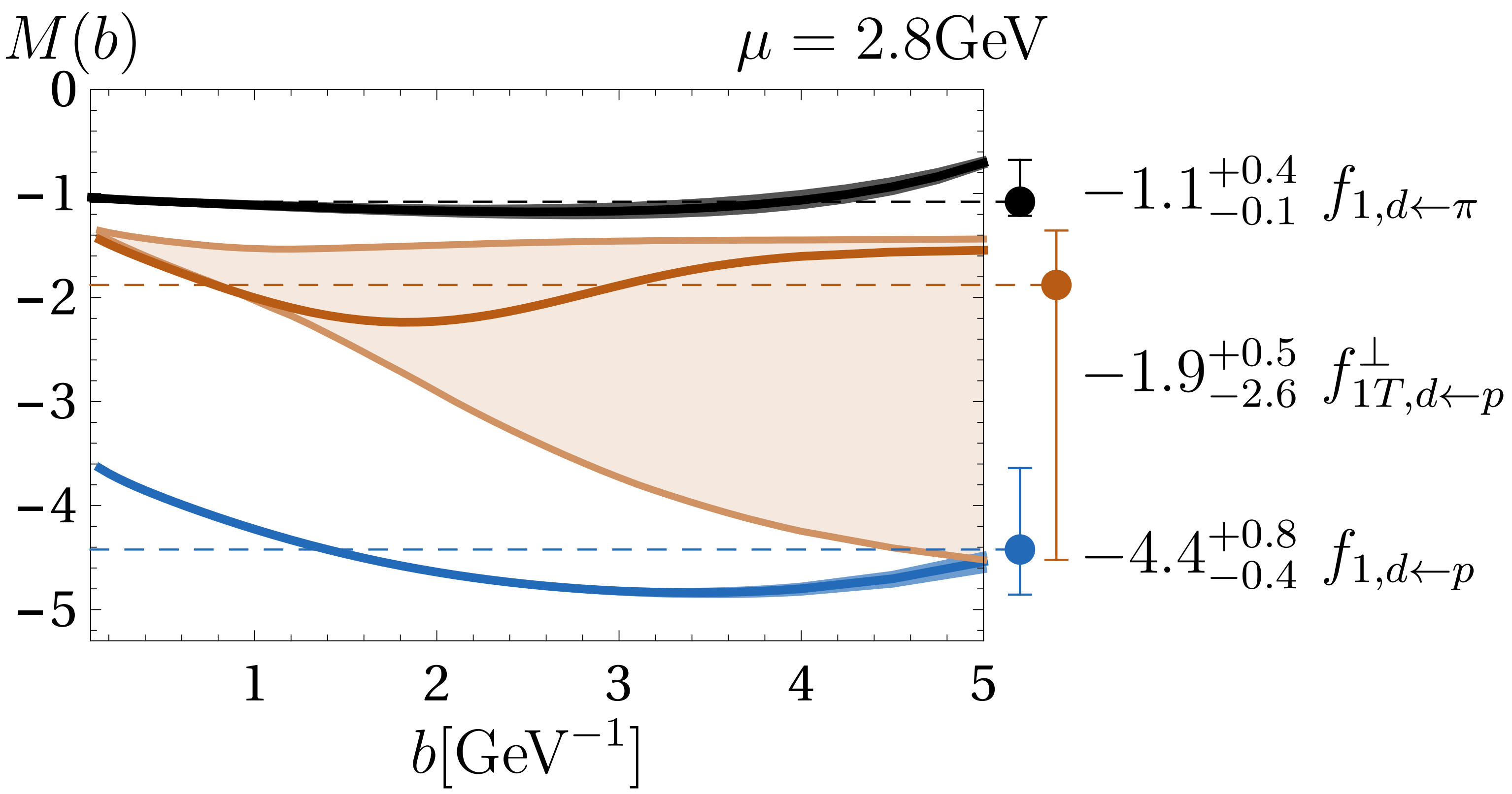
$$\frac{\mathbb{C}_H(xP_1^+,\mu)}{\mathbb{C}_H(xP_2^+,\mu)} \xrightarrow{z=0} \mathbf{r}^{[\Gamma]}(b,\mu;P_1,P_2) = 1 + 4C_F\frac{\alpha_s(\mu)}{4\pi}\ln\left(\frac{P_1^+}{P_2^+}\right)\left[1 - \ln\left(\frac{2P_1^+P_2^+}{\mu^2}\right) - 2\mathbf{M}^{[\Gamma]}(b,\mu)\right]$$

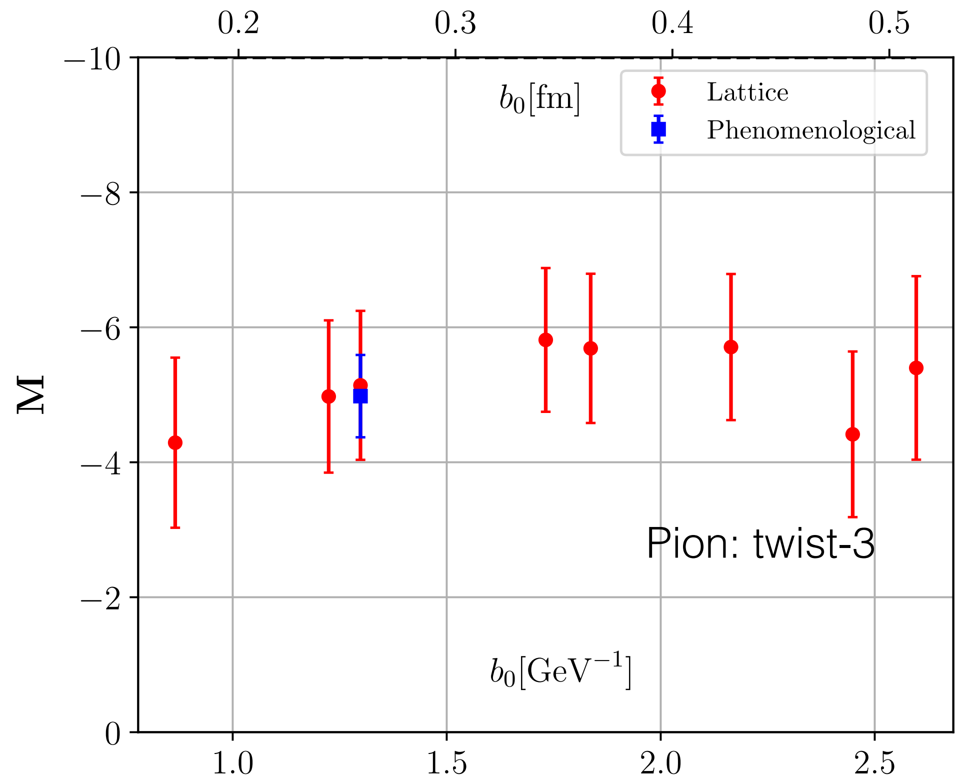
  - No need for renormalization
  - Consider only the first Mellin moments

  - Key argument: **M** constant in *b* !

Determination of M

- Phenomenological and lattice check of constant assumption for **M**





[I. Scimemi and A. Vladimirov, JHEP06(2020)137]
[A. Vladimirov, JHEP10(2019)090]
[M. Bury, et al., PRL126,112002(2021)]

[H.-T. Shu, et al., arXiv:2302.06502]

$$K(b,\mu) = \frac{1}{\ln(P_1^+/P_2^+)} \ln \frac{R^{[\Gamma]}}{\mathbf{r}^{[\Gamma]}}$$

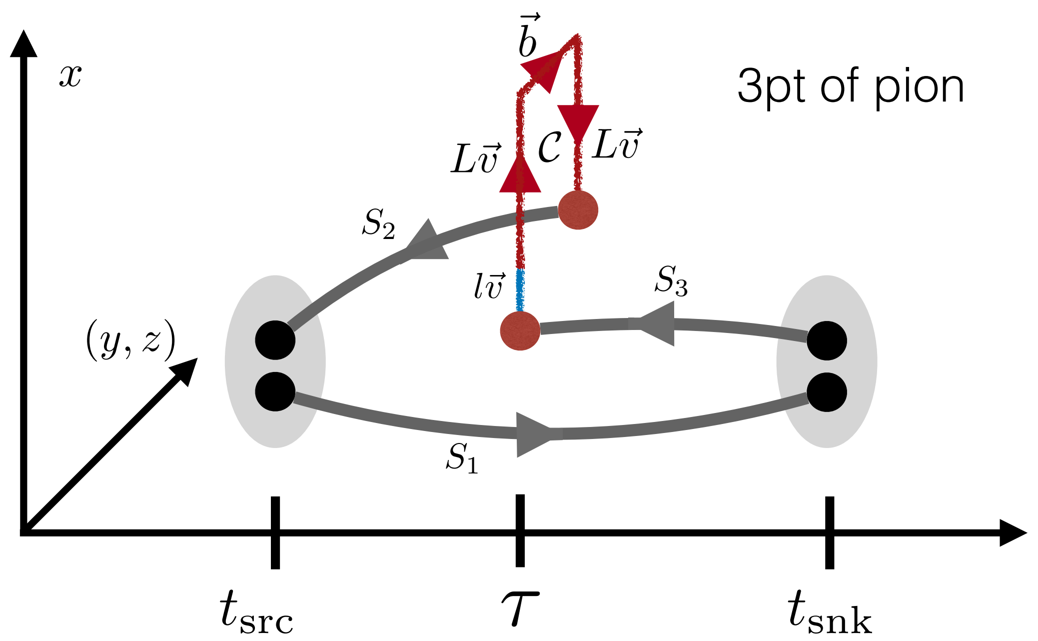
- Systematics better controlled

1. Calculate the ratio *R* of the quasi TMDPDF matrix elements
2. Determine **M** by comparing to the perturbative calculation at b_0
3. Use **M** determined at b_0 for other *b*
4. Fix CS kernel at other *b* based on the relative shift of the ratio to it at b_0

Quasi TMDPDFs in lattice calculation

- Construction of quasi TMDPDF matrix element: [A. Vladimirov and A. Schaefer, PRD 101, 074517]

$$W^{[\Gamma]}_{f/h}(b;\ell,L;v,P,S;\mu) = \langle h(P,S)|\bar{q}_f(b+\ell v)\Gamma\,\mathcal{U}[\mathcal{C}(\ell,v,b,L)]\;q_f(0)|h(P,S)\rangle$$



We consider both pion and proton.

- Large separation limit achieved by constant fit: $W^{[\Gamma]} = 2E_P \lim_{0\ll\tau\ll t} \frac{C_{3pt}(P,\mathcal{C},t,\tau,\Gamma)}{C_{2pt}(P,t)}$

- Parametrization via (Lorentz) invariant amplitudes: [B. U. Musch, et al, PRD 85, 094510]

$$W^{[\gamma^\mu]}(b;L;v,P,S) = P^\mu \tilde{A}_2 + \frac{M^2}{v\cdot P}v^\mu \tilde{B}_1 - iM\epsilon^{\mu\nu\alpha\beta}P_\nu b_\alpha S_\beta \tilde{A}_{12} - iM^3\epsilon^{\mu\nu\alpha\beta}b_\nu(Lv_\alpha)S_\beta\tilde{B}_8 - iM^2 b^\mu \tilde{A}_3 + \cdots$$

- Combine invariant amplitudes to build twist-2 and twist-3 TMDPDFs:

$$f_1(b^2,P^+) = P^+\big(\tilde{A}_2(b^2) + M^2\frac{v^+}{(v\cdot P)P^+}\tilde{B}_1(b^2)\big),$$
$$e(b^2,P^+) = P^+\tilde{A}_1(b^2)$$

We consider both twist-2 and twist-3.

Lattice setup

- Calculations on CLS ensemble H101

name	β	$L^3 \times T$	a[fm]	$\kappa_l = \kappa_s$	m_π	m_K	$m_\pi L$	L[fm]	conf
H101	3.4	$32^3 \times 96$	0.0854	0.13675962	422	422	5.8	2.7	2016

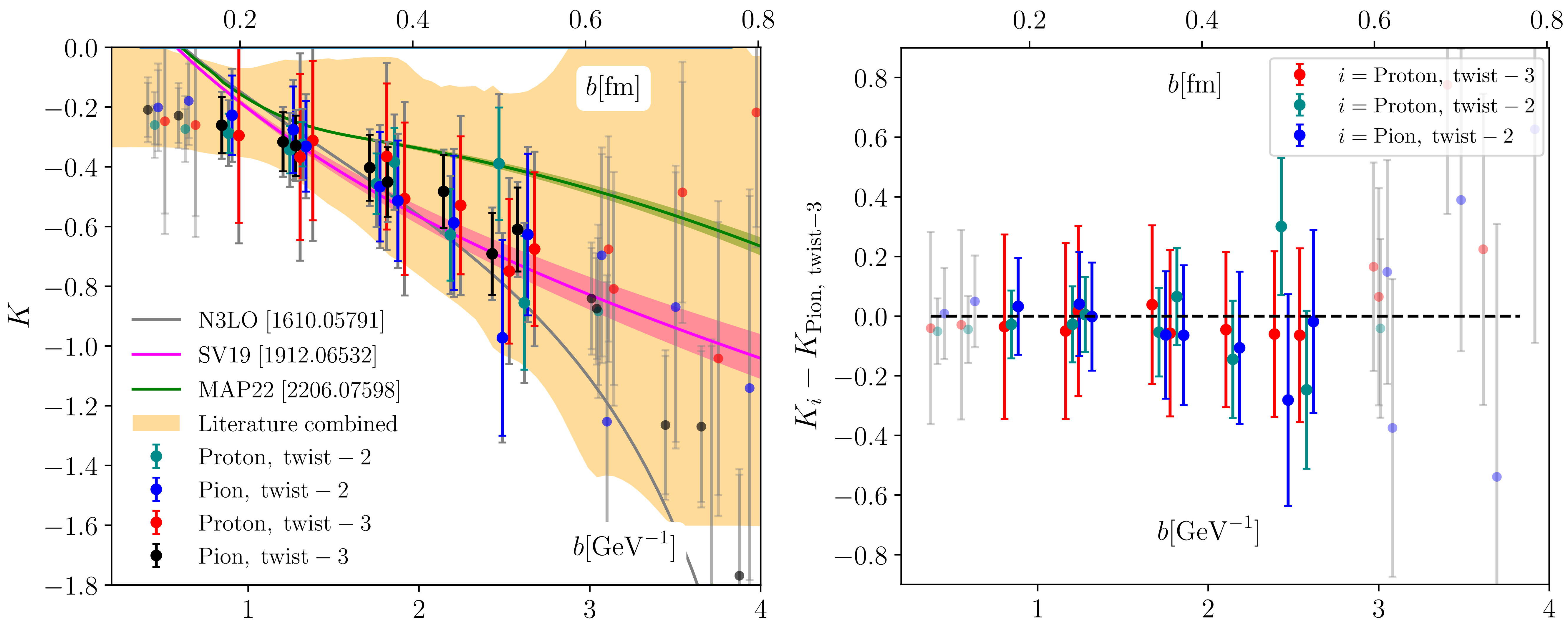
Table 1: Parameters of the H101 CLS ensemble used in the present study.

- The hierarchy needs to be respected for the factorization theorem

$$\frac{P^-}{P^+} \ll 1, \quad \frac{1}{|b|P^+} \ll 1, \quad \frac{|b|}{L} \ll 1, \quad \frac{\ell}{L} \ll 1, \quad \ell\Lambda_{\rm QCD} \ll 1$$

  - HYP smearing + Momentum smearing
  - Multiple source per configuration
  - Momenta: P^z={0, 1, 2, 3, 4, 5} x 2pi/L
  - l=0, 0<=L/a<=9~20, 0<=b/a<=8, 0<=t/a<=9/11

Collins-Soper kernel from P_1/P_2=3/2

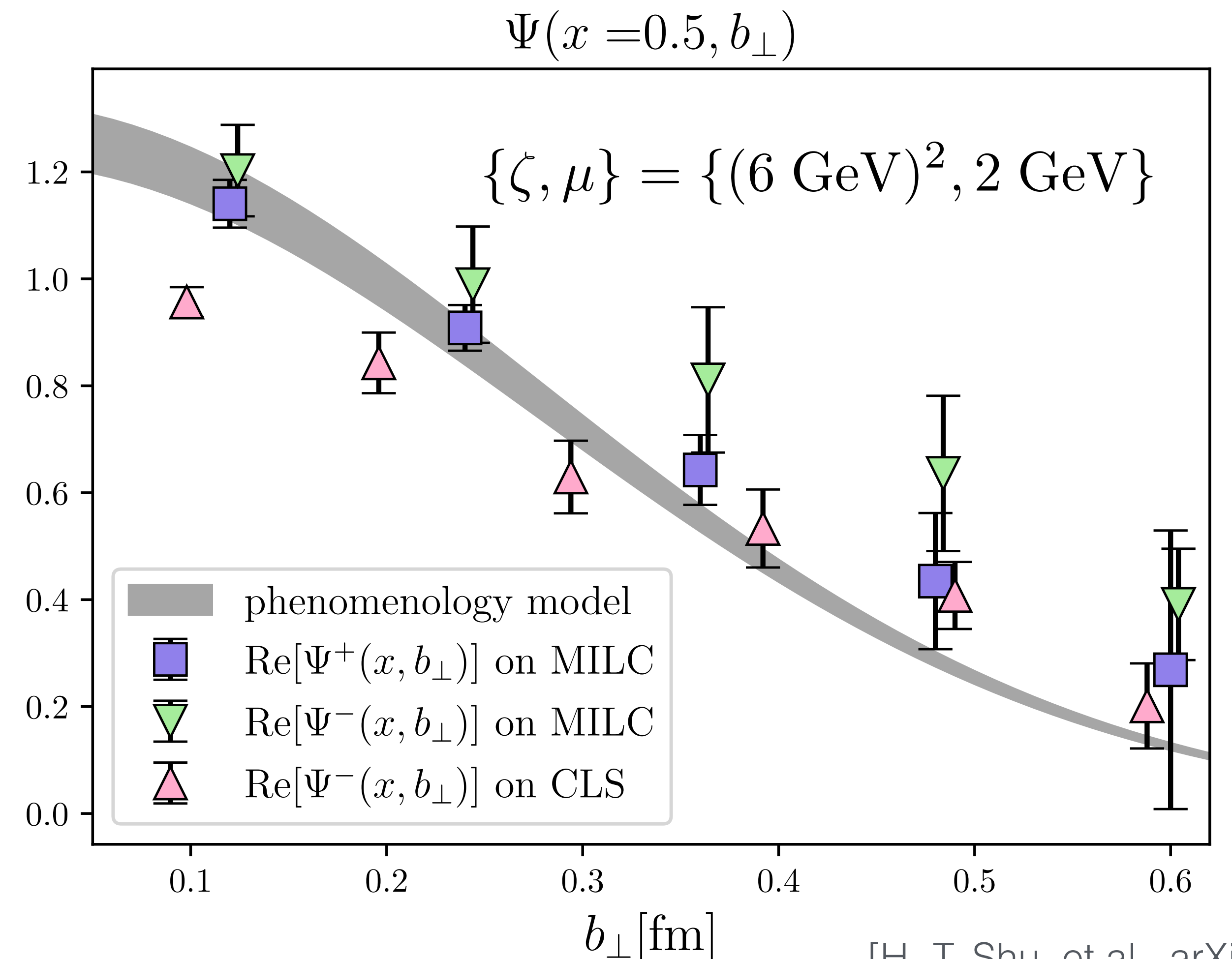


[H.-T. Shu, et al., arXiv:2302.06502]

- Sizable uncertainties but not exceed previous lattice determinations
- Agrees with SV19 phenomenal calculations and is lower than MAP22
- Consistent results for pion and proton, twist-2 and twist-3 -> Universality confirmed

Application of the CS kernel: physical TMDWFs

$$\tilde{f}_\Gamma(x, b_\perp, \zeta_z, \mu)\sqrt{S_I(b_\perp, \mu)} = H_\Gamma\left(\frac{\zeta_z}{\mu^2}\right) e^{\frac{1}{2}\ln\left(\frac{\zeta_z}{\zeta}\right)K(b_\perp,\mu)} f(x, b_\perp, \mu, \zeta) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{\zeta_z}, \frac{M^2}{(P^z)^2}, \frac{1}{b_\perp^2 \zeta_z}\right)$$



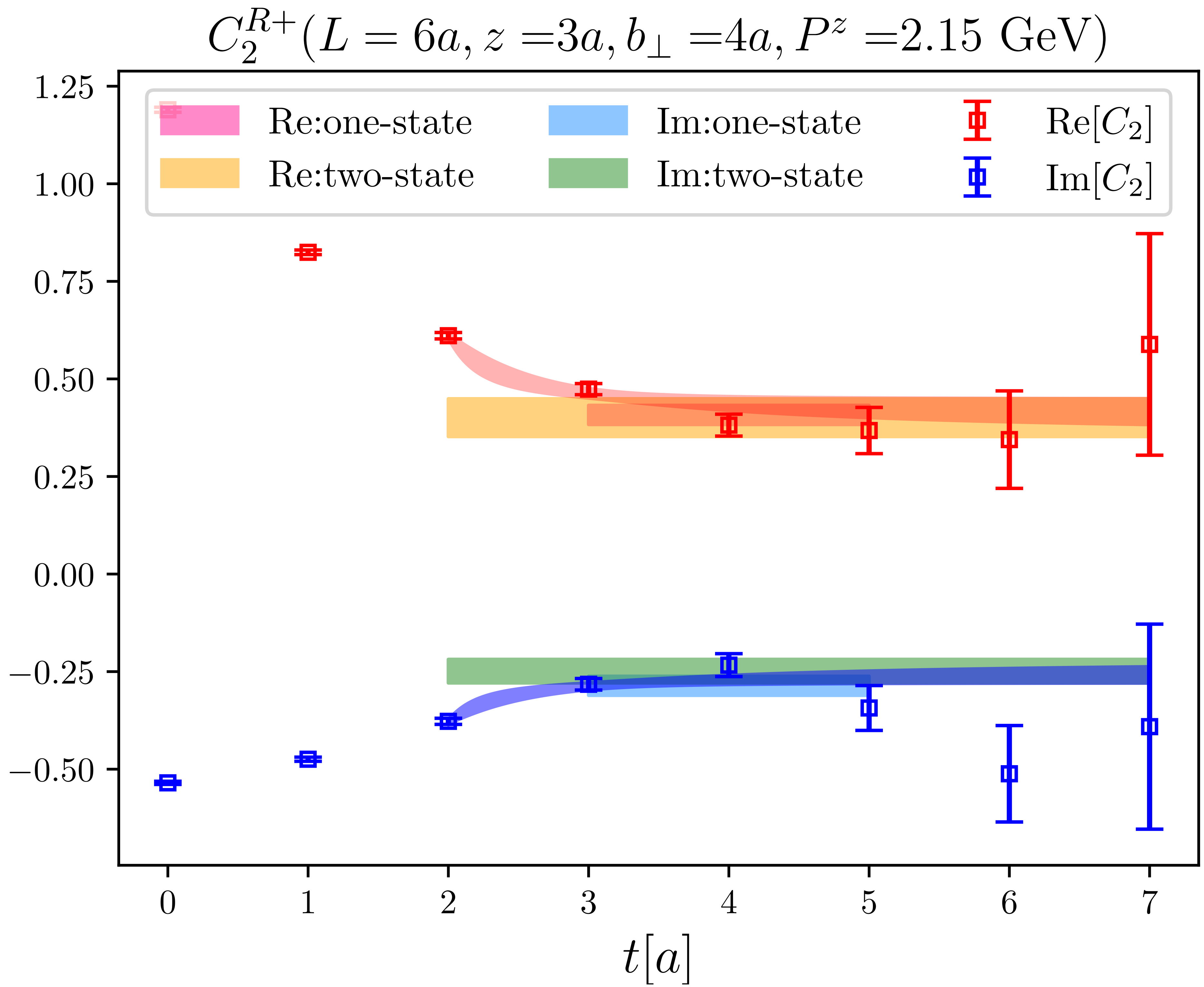
[H.-T. Shu, et al., arXiv:2302.09961]

- TMDWFs from both ensembles show consistent *b* dependence
- Visible differences at small *b* which might come from discretization errors
- Provide crucial theory inputs for predicting the exclusive processes

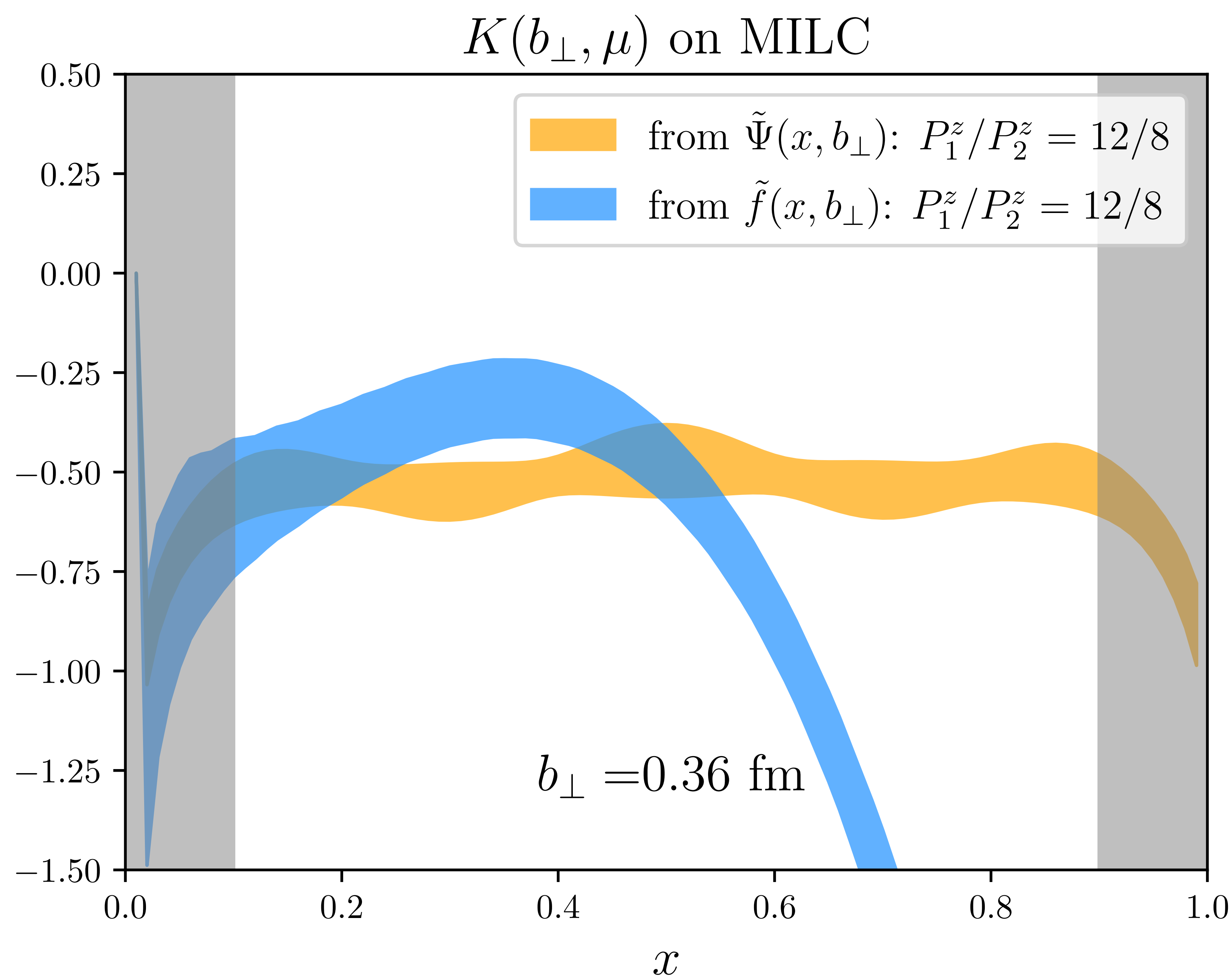
Summary

- CS kernel calculated using lattice QCD in different ways on two different CLS ensembles
- Good agreement among different lattice extractions
- Universality of the lattice-determined CS kernel was observed
- High-precision determination of CS kernel is in reach
- High-precision determination of physical TMDs can be expected

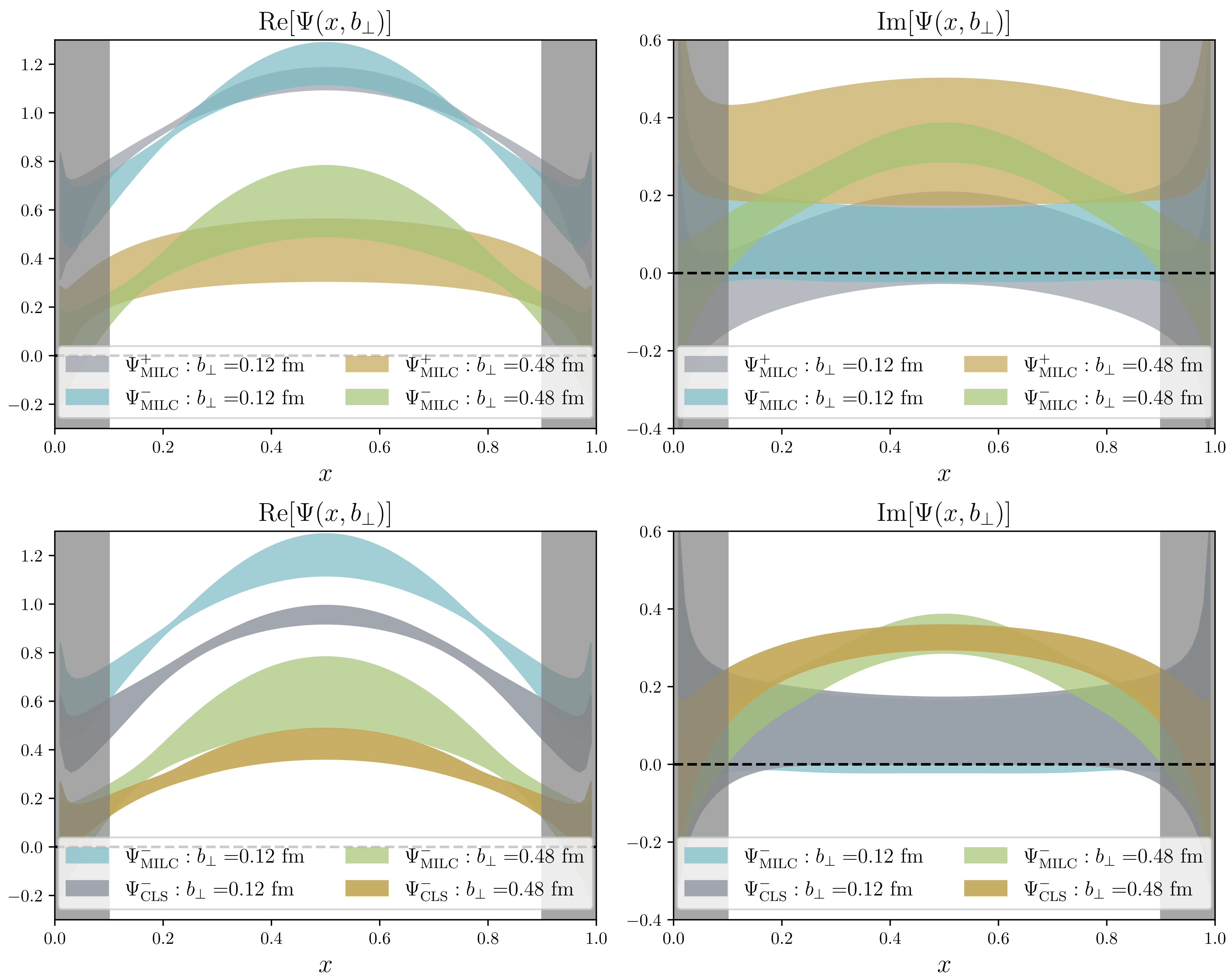
Backup: 2pt fit



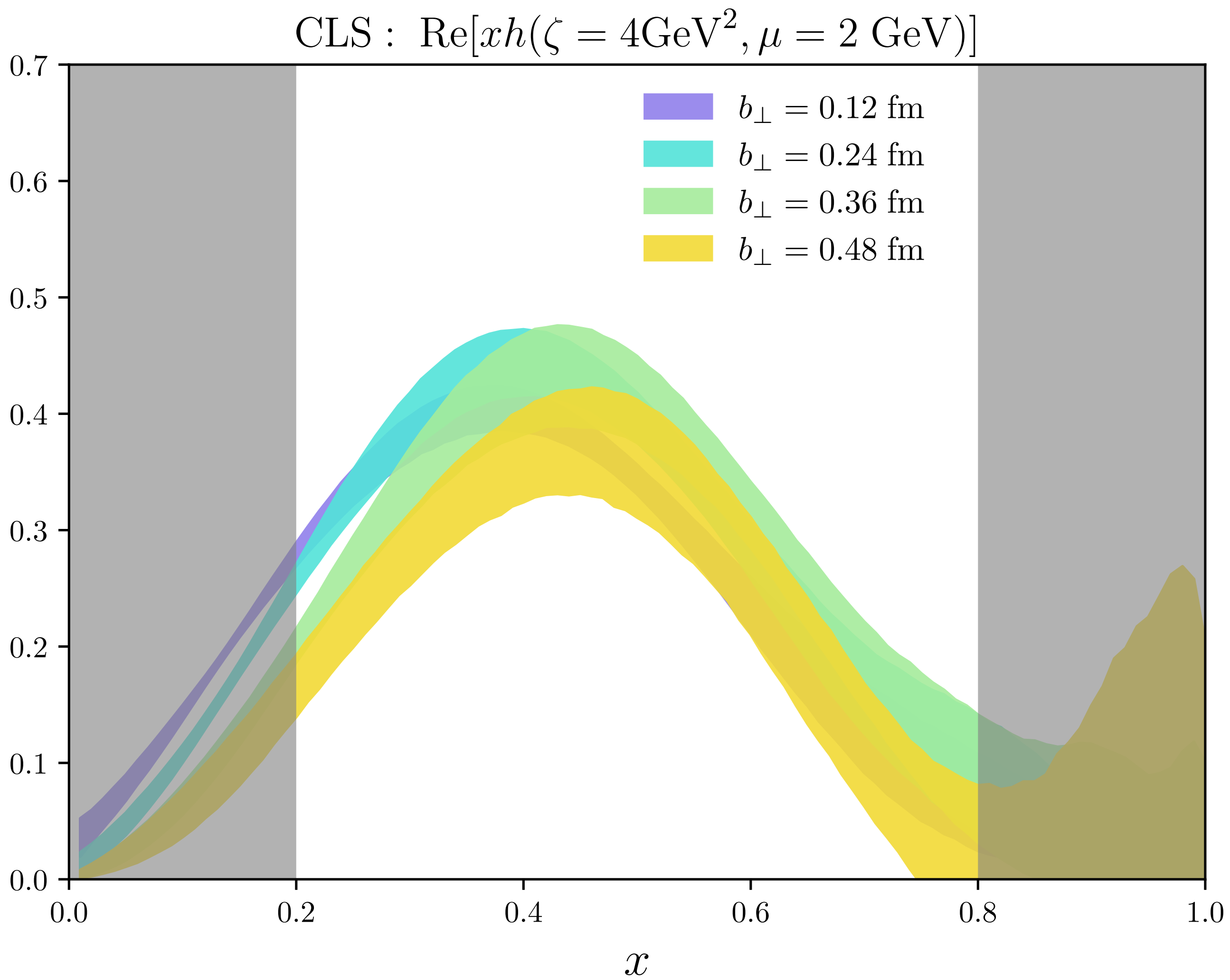
Backup: x-dependence on milc

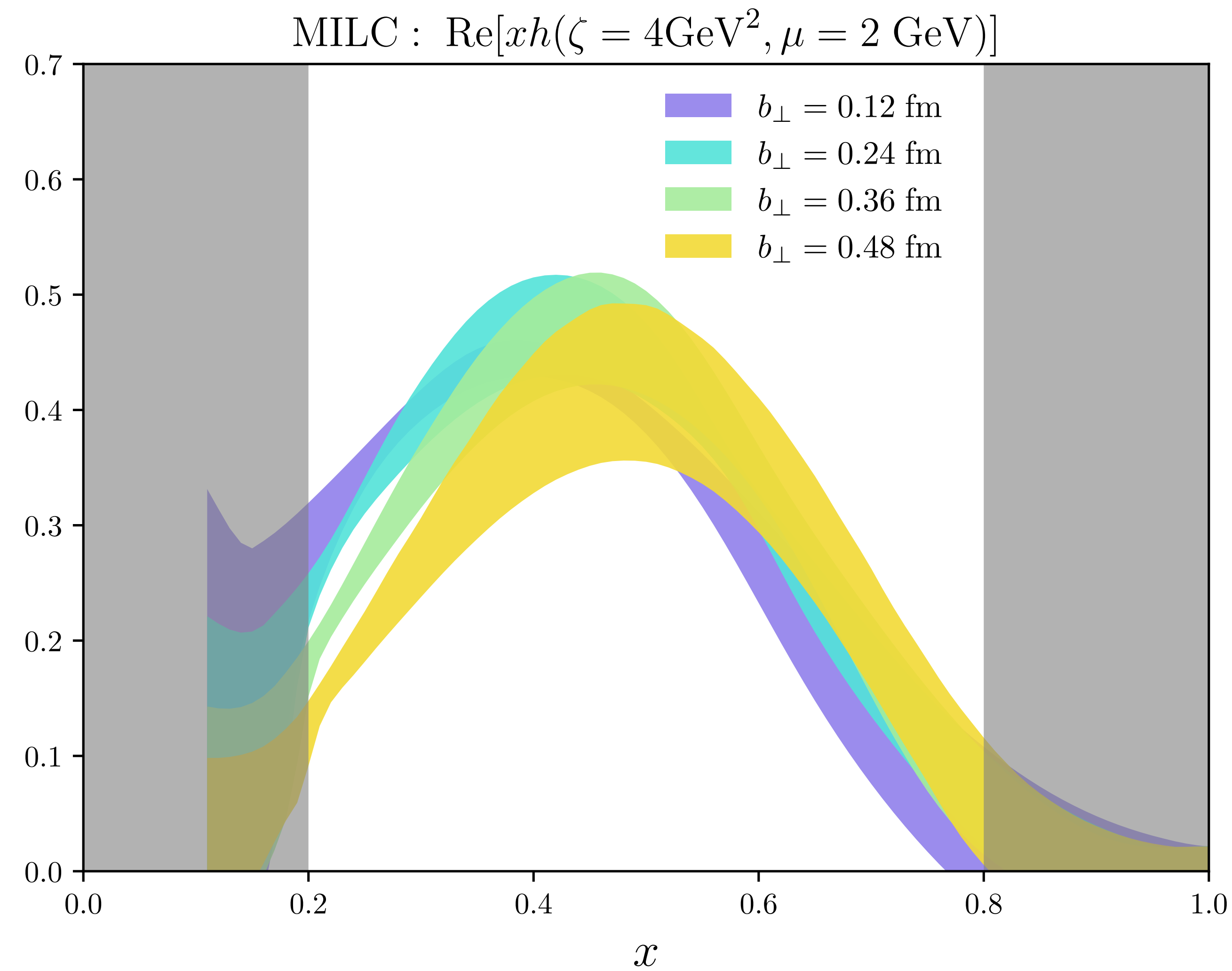


Backup: light cone TMDWFs

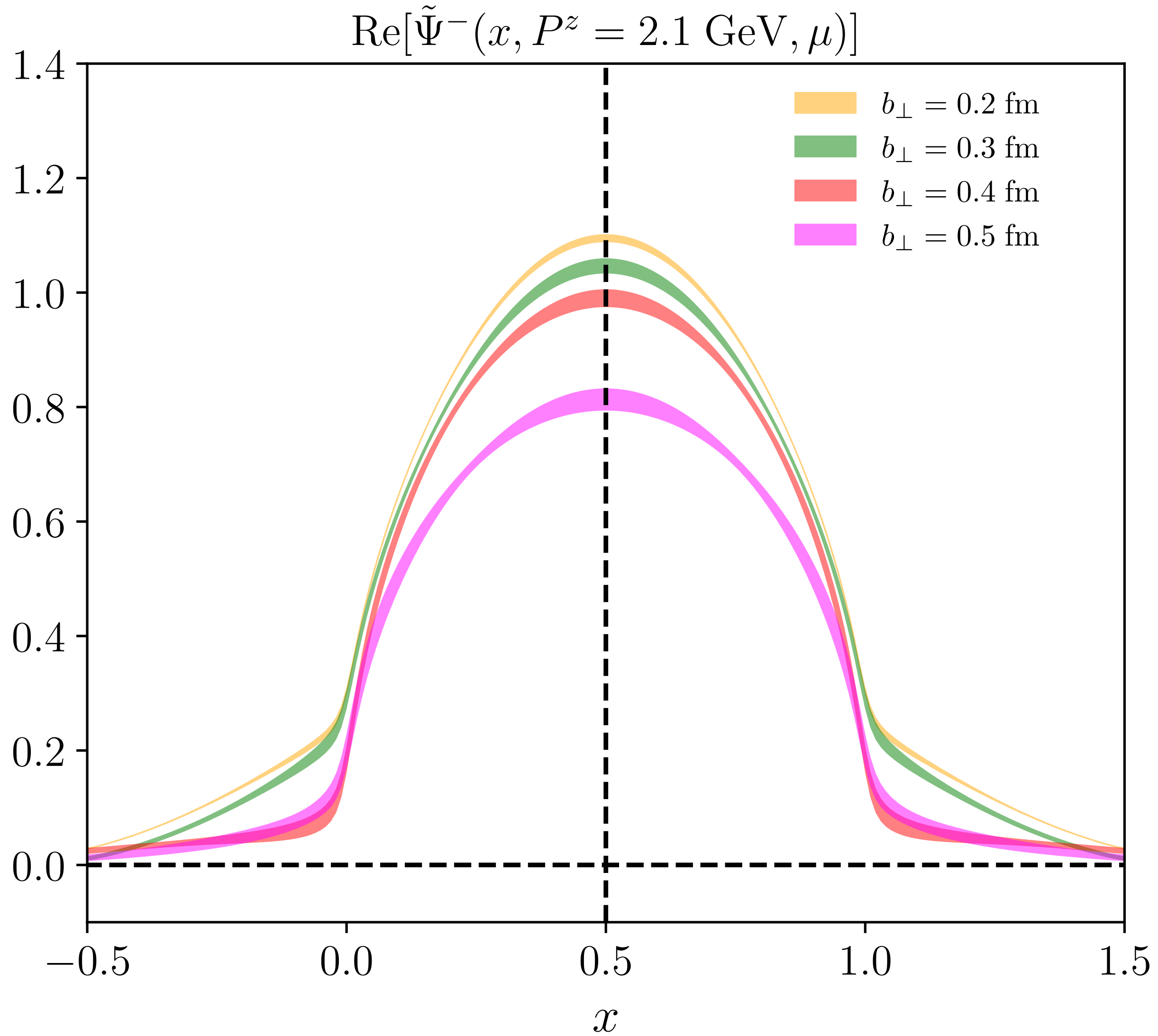


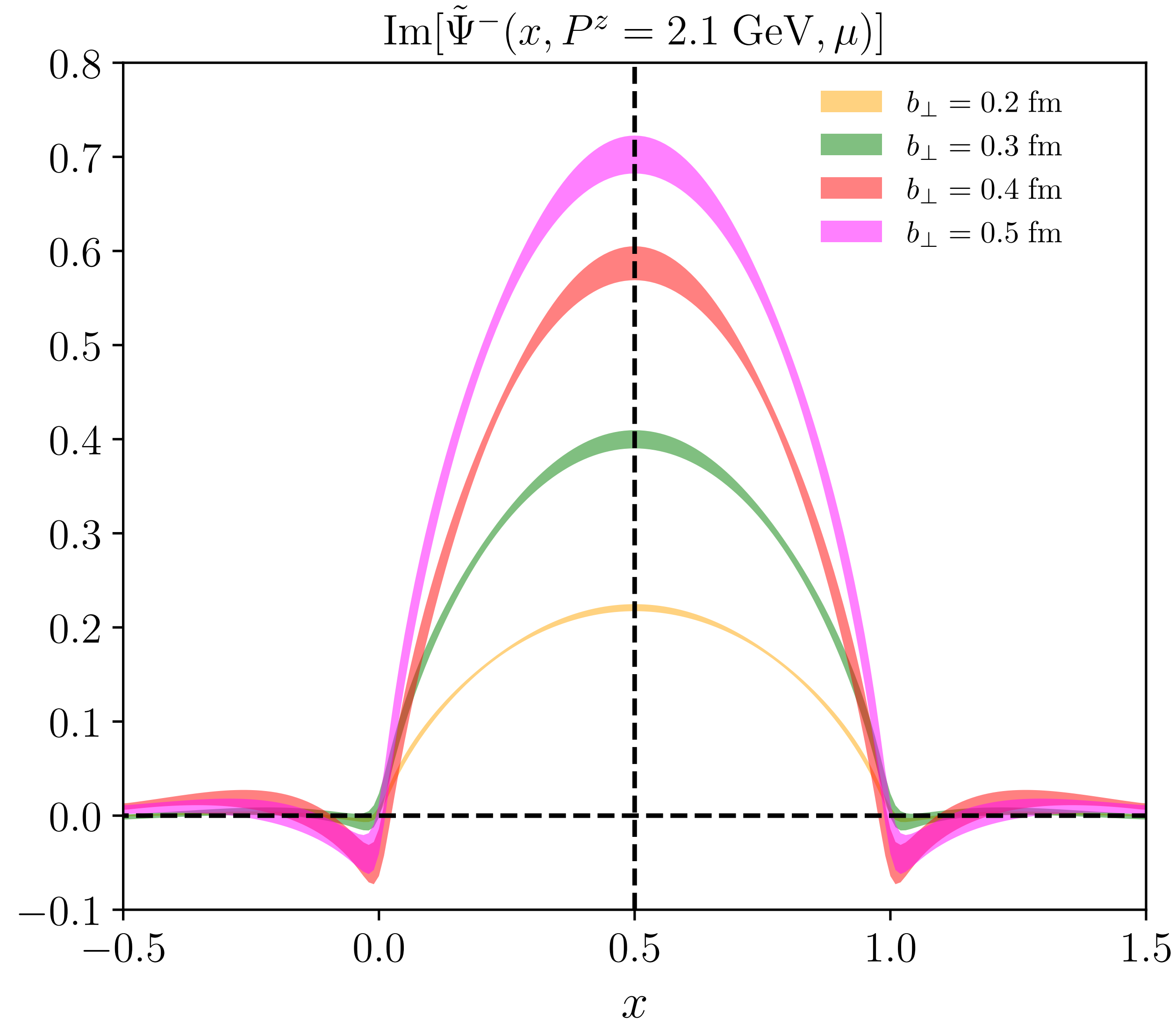
Backup: light cone TMDPDFs





Backup: Quasi TMDWFs in momentum space (I)



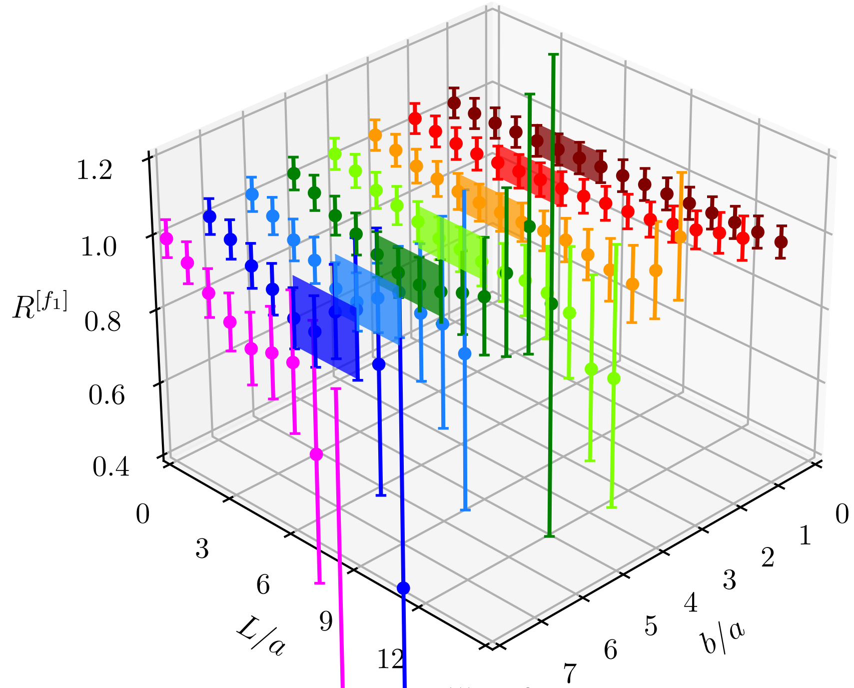


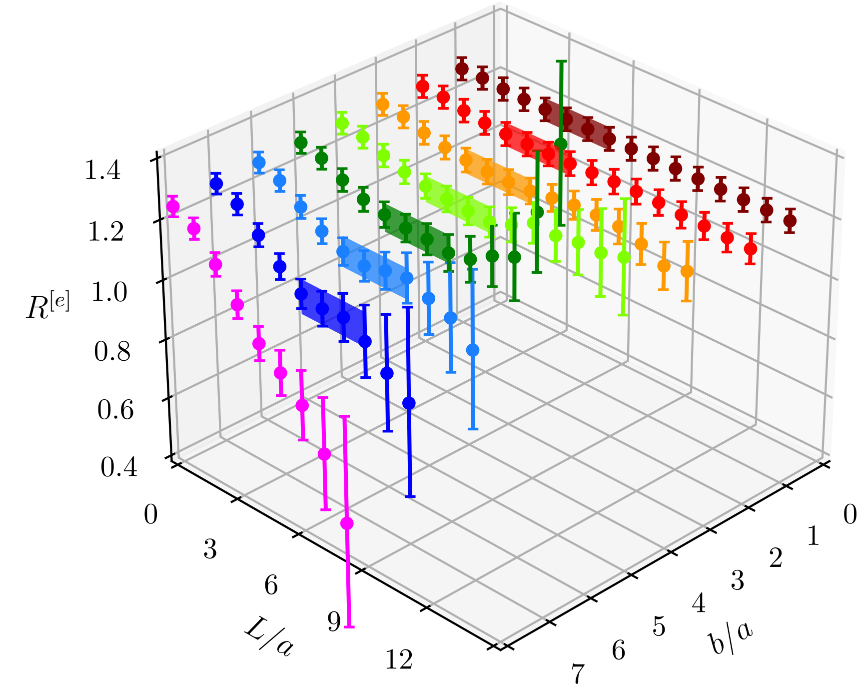
- Real part has larger amplitude at same b
- Clear dependence on b for both real and imaginary parts
- Stronger dependence on b for imaginary part

Backup: Constant fit for the ratio at P_1/P_2=3/2

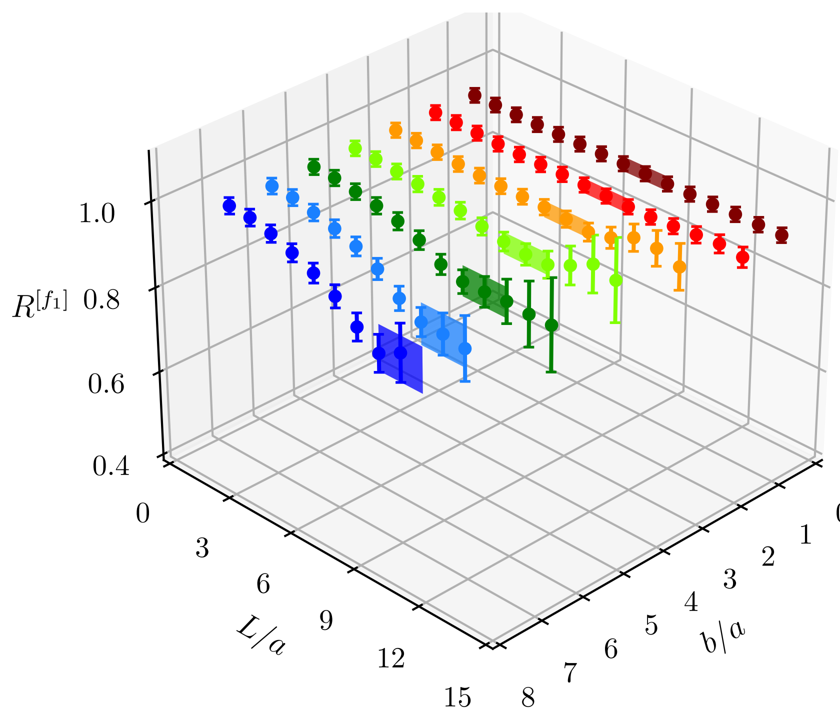
f_1: *e*:

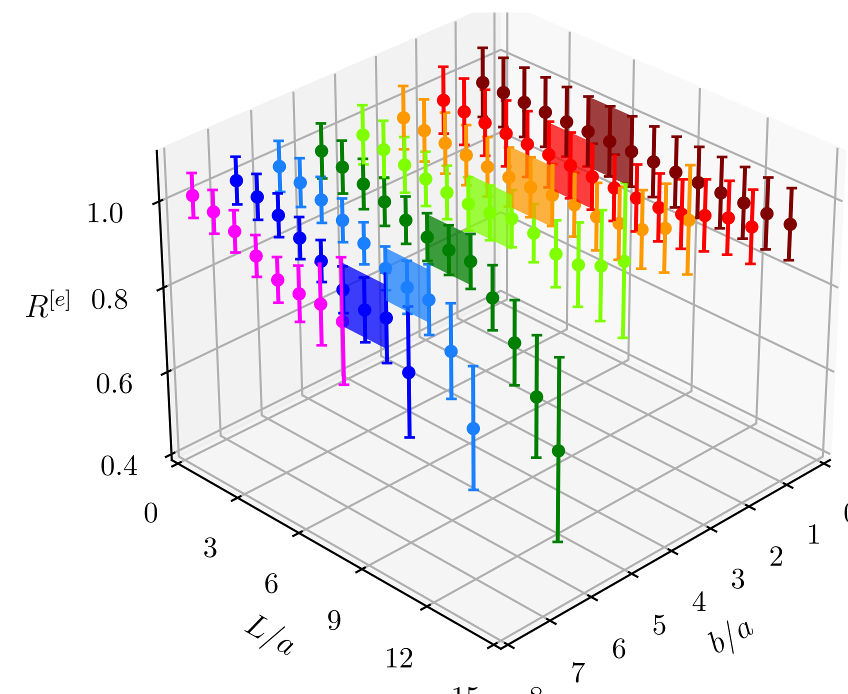
pion:





proton:

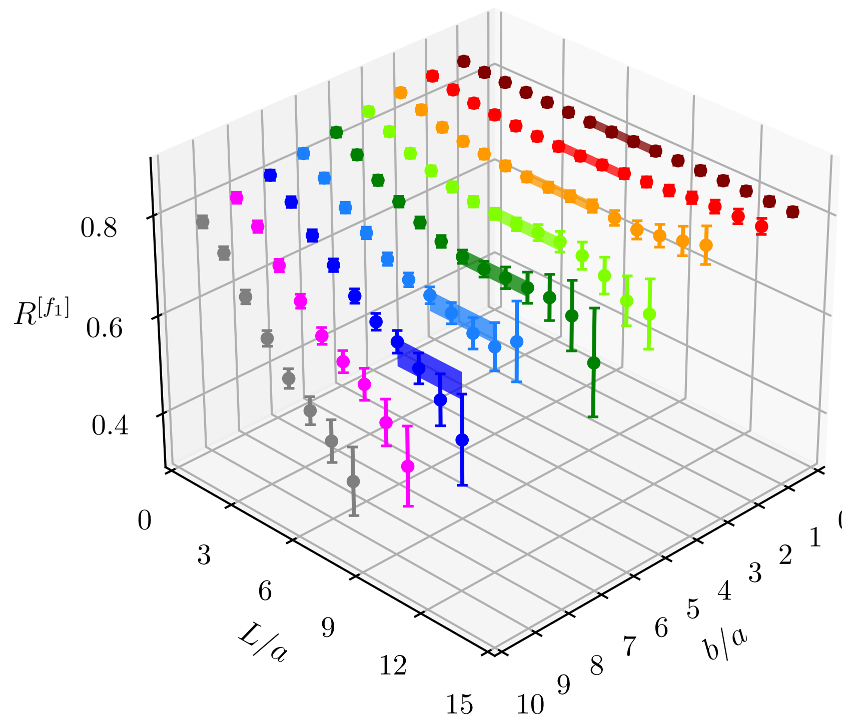


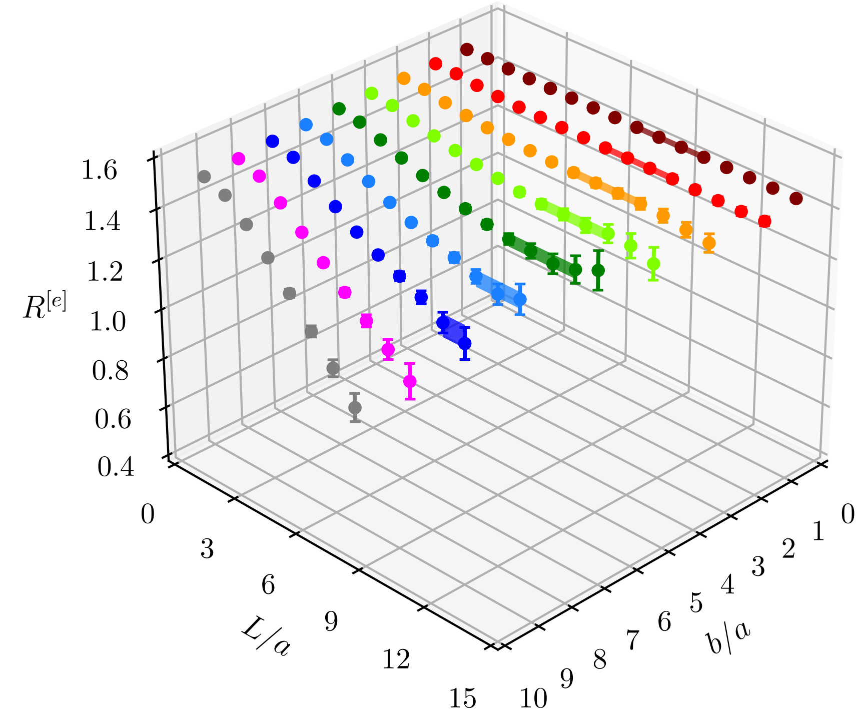


$$K(b,\mu) = \frac{1}{\ln(P_1^+/P_2^+)} \ln \frac{R^{[\Gamma]}}{\mathbf{r}^{[\Gamma]}}$$

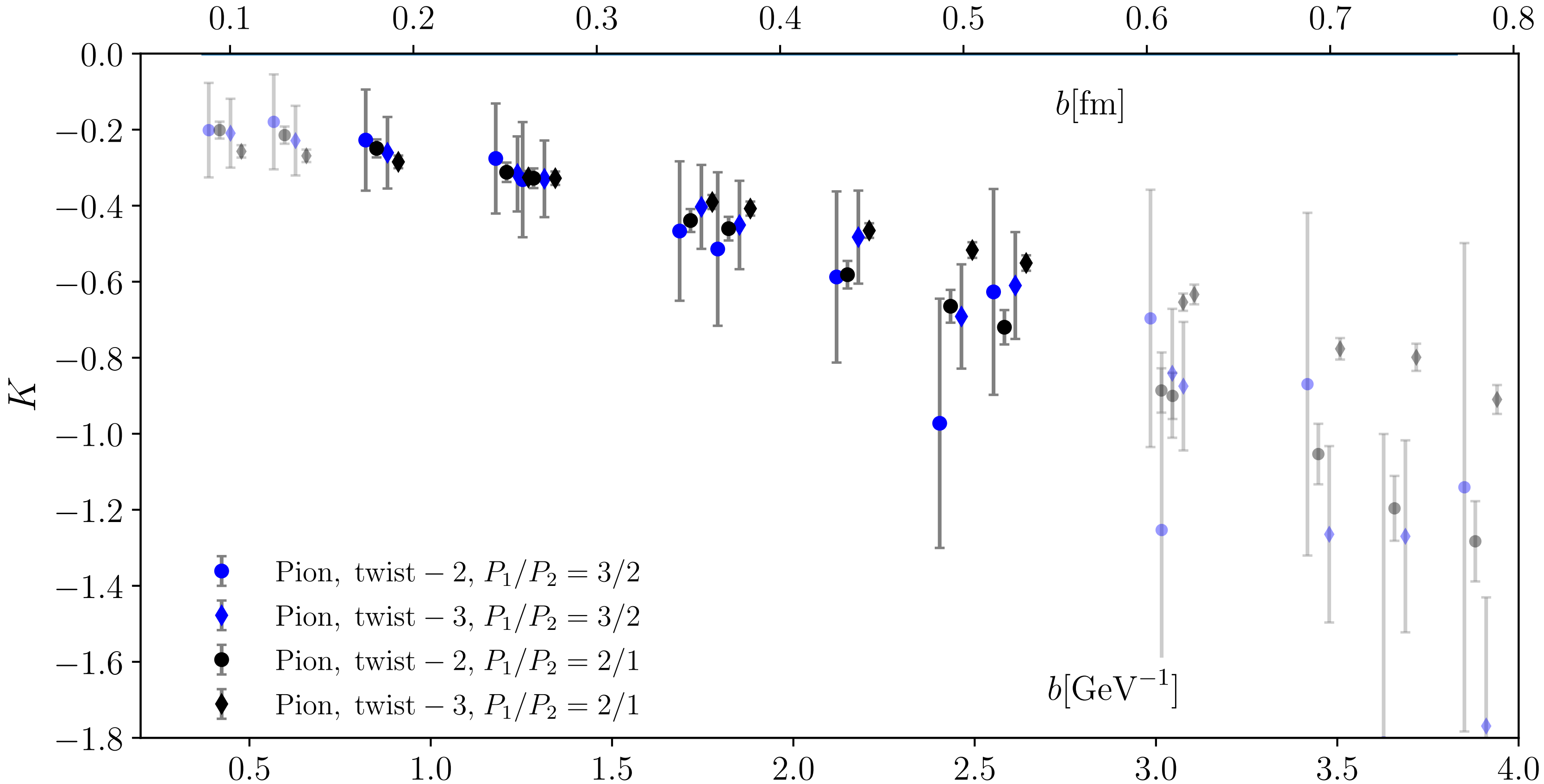
Backup: Collins-Soper kernel from P_1/P_2=2/1

f1



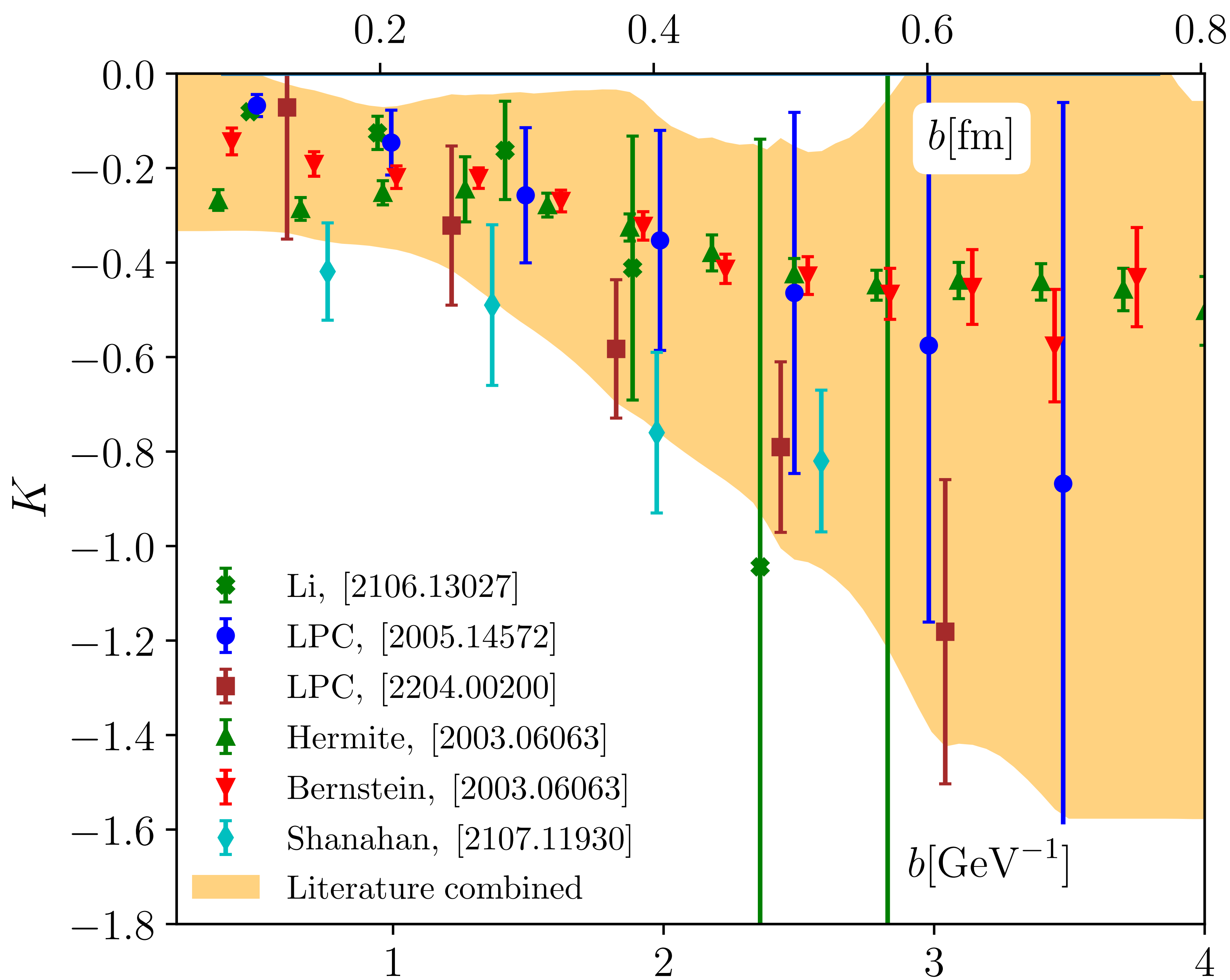


e



- Smaller statistical errors but probably larger systematical errors
- Better identified plateaus
- Very close central values for CS kernel from different momentum pairs (power corrections suppressed)

Backup: combine the literature



- Combine the literature using Gaussian bootstrap resampling

Backup: dispersion relation (X650)