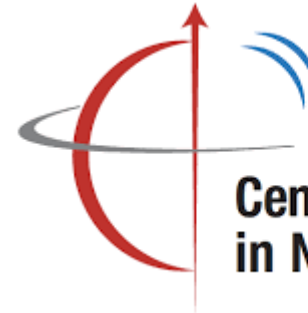




Fondazione  
di Sardegna



**WORKSHOP ON**  
***TMDs: Towards a Synergy between***  
***Lattice QCD and Global Analyses***



**Center for Frontiers  
in Nuclear Science**

**TRANSVERSE  $\Lambda$  POLARIZATION  
IN  $e^+e^-$  ANNIHILATION AND  
SIDIS PROCESSES  
WITHIN A TMD FRAMEWORK**

**Umberto D'Alesio**

in coll. with

*L. Gamberg, F. Murgia & M. Zaccheddu*

JHEP 12 (2022) 074

arXiv:2306.xxxxx

# OUTLINE

## □ Motivations

### □ $e^+e^- \rightarrow h_1 h_2 + X$ :

- ✓ TMD formulation and fit of transverse  $\Lambda$  polarization Belle data
- ✓ Extraction of the polarizing FF
- ✓  $SU(2)$  symmetry and role of charm contribution

## □ SIDIS

- ✓ TMD formalism
- ✓ Predictions for EIC

## □ Concluding remarks



# MOTIVATIONS AND MORE

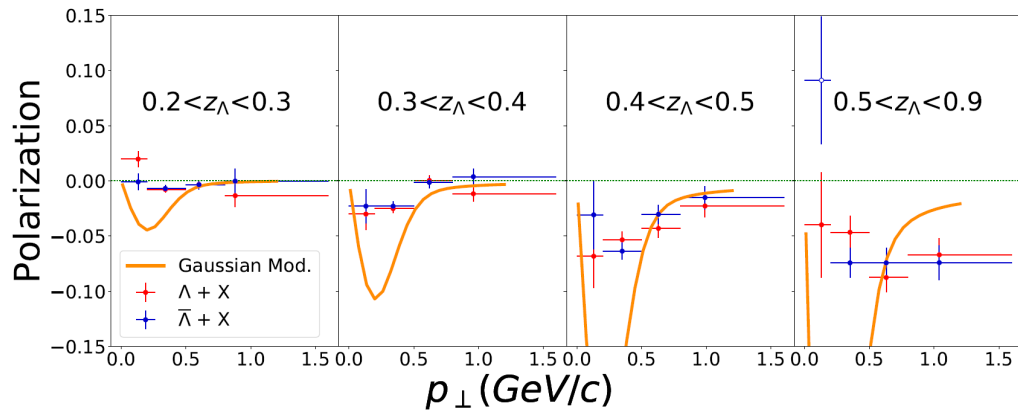
- Transverse  $\Lambda$  polarization in *unpolarized*  $pp$  collisions: [Bunce *et al.* 1976]  
sizeable, rising with  $x_F, p_T$  plateau...**still challenging**
- First attempt within a TMD scheme: [Anselmino, Boer, UD, Murgia 2001]  
good description ... no formal theory behind
- Breakthrough: Belle  $e^+e^-$  data [Guan *et al.* (Belle Coll.) 2019]
- $e^+e^-$  collisions:  
 $2h$ , TMD fact. [Collins 2011; Echevarria, Idilbi, Scimemi 2012]  
 $1h$ : new TMD scheme? [Kang, Shao, Zhao 2020; Boglione, Simonelli 2021, 2022]
- First analysis of Belle data: extraction of the polarizing FF (fixed scale)  
2h and 2h+1h fits [UD, Murgia, Zaccheddu 2020]  
2h fit [Callos, Kang, Terry 2020]
- Analysis w/ CSS scheme [Li, Wang, Yang, Lu 2021; Gamberg, Kang, Shao, Terry, Zhao 2021;  
UD, Gamberg, Murgia, Zaccheddu 2022]
- $SU(2)$  issue: [Chen, Liang, Pan, Song, Wei 2021; Chen, Liang, Song, Wei 2022]



# OPEN ISSUES (THE LAST ANALYSIS)

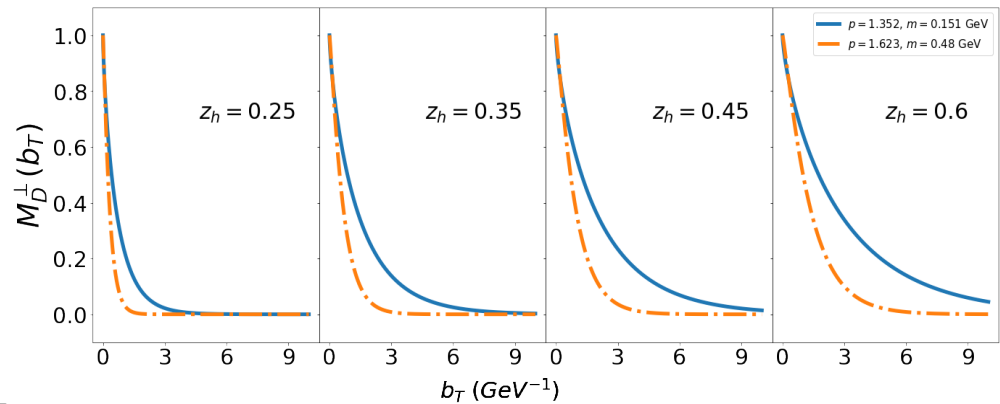
[UD, Gamberg, Murgia, Zaccheddu 2022]

- Tension between  $2h$  &  $1h$  data description



$1h$  estimates from  $2h$ -fit

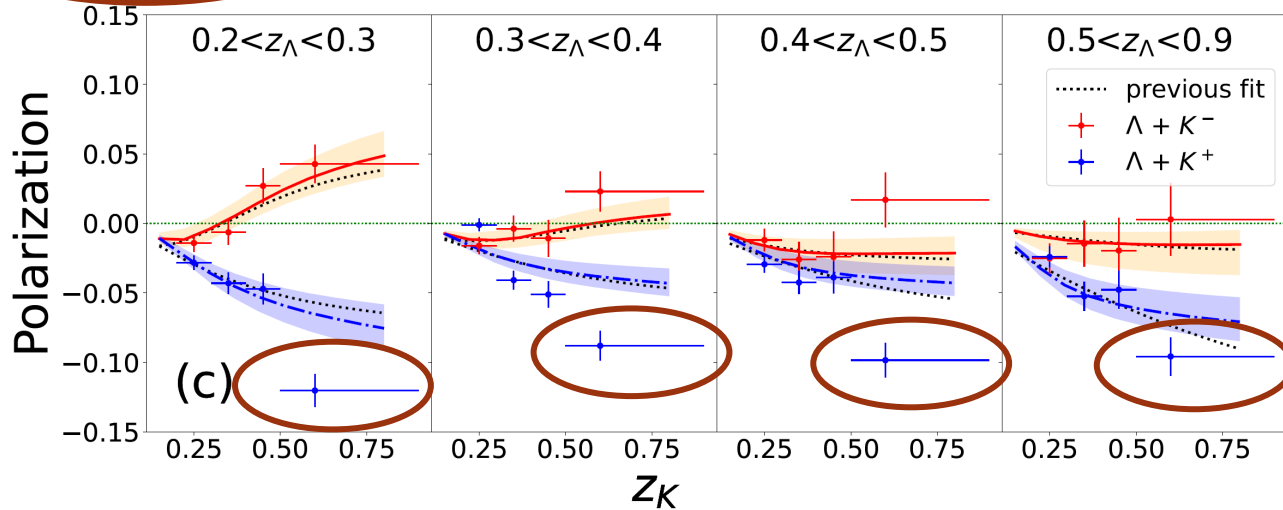
Non perturbative models  
to fit  $2h$  and  $1h$  data



Maybe two factorization schemes



- Large- $z$   $K\Lambda$  data: failure w/ or w/o proper TMD framework

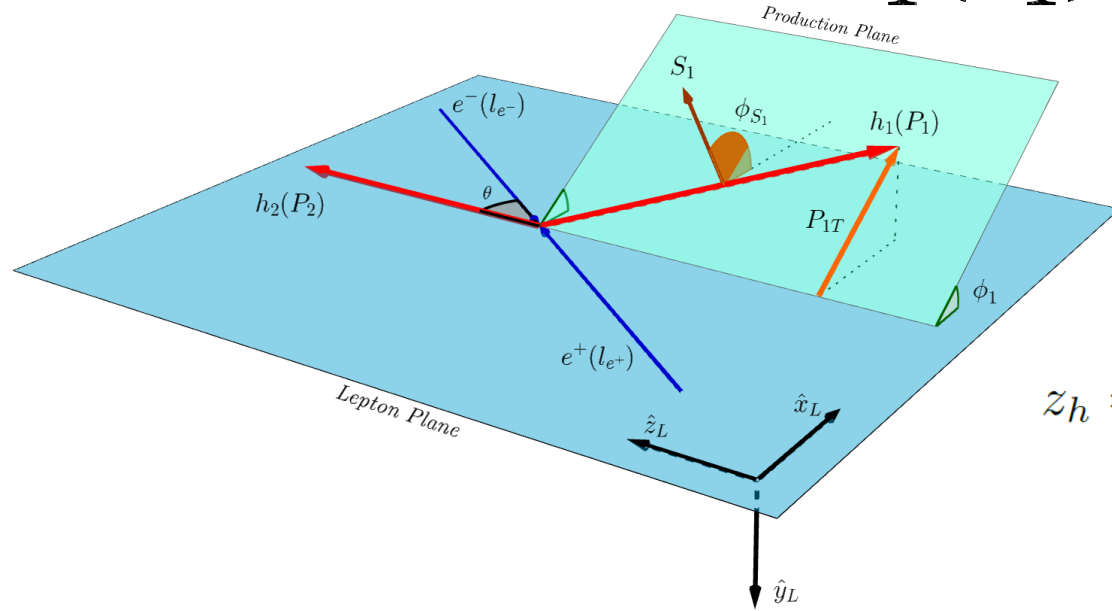


- Free three-flavor ( $u, d, s$ ) fit  $\rightarrow$  strong  $SU(2)$  violation ( $d \cong -u$ )

HERE, focus on:  $SU(2)$  symmetry and role of charm



$$e^+ e^- \rightarrow h_1(P_1) h_2(P_2) + X$$



$$z_p = \frac{2|P_h|}{Q} \simeq z \left( 1 - \frac{M_h^2}{z^2 Q^2} \right)$$

$$z_h = \frac{2P_h \cdot q}{Q^2} = \frac{2E_h}{Q} \simeq z \left( 1 + \frac{M_h^2}{z^2 Q^2} \right)$$

$$\frac{d\sigma^{e^+e^- \rightarrow h_1(S_1)h_2 X}}{2dy dz_{h_1} dz_{h_2} d^2\mathbf{q}_T} = \sigma_0^{e^+e^-} \left[ F_{UU} - |S_{1T}| \sin(\phi_1 - \phi_{S_1}) F_{TU}^{\sin(\phi_1 - \phi_{S_1})} + \dots \right]$$

$$F_{UU} = z_{p_1}^2 z_{p_2}^2 \mathcal{H}^{(e^+e^-)}(Q) \mathcal{F}[D_1 \bar{D}_1] \quad \mathcal{H}^{(e^+e^-)}(Q)|_{\text{LO}} = 1$$

$$F_{TU}^{\sin(\phi_1 - \phi_{S_1})} = z_{p_1}^2 z_{p_2}^2 \mathcal{H}^{(e^+e^-)}(Q) \mathcal{F} \left[ \frac{\hat{\mathbf{h}} \cdot \mathbf{k}_T}{M_1} D_{1T}^\perp \bar{D}_1 \right]$$

[Boer, Jakob, Mulders 1997; Pitonyak, Schlegel, Metz 2014; UD, Murgia, Zaccheddu 2021]



$$e^+ e^- \rightarrow h_1(P_1) h_2(P_2) + X$$

Convolution in  $k_T$  space

$$\mathcal{F}[\omega D \bar{D}] = \sum_q e_q^2 \int d^2 \mathbf{k}_T d^2 \mathbf{p}_T \delta^{(2)}(\mathbf{k}_T + \mathbf{p}_T - \mathbf{q}_T) \omega(\mathbf{k}_T, \mathbf{p}_T) D(z_1, \mathbf{k}_\perp) \bar{D}(z_2, \mathbf{p}_\perp)$$

CSS approach...move to  $b_T$  space

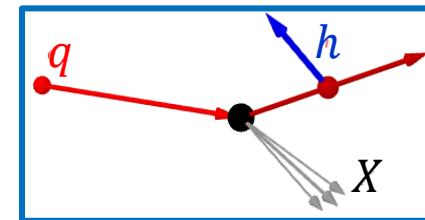
$$F_{UU} = z_{p_1}^2 z_{p_2}^2 \mathcal{B}_0 \left[ \tilde{D}_1 \tilde{\bar{D}}_1 \right] = z_{p_1}^2 z_{p_2}^2 \sum_q e_q^2 \int \frac{db_T}{2\pi} b_T J_0(b_T q_T) \tilde{D}_1(z_1, b_T) \tilde{\bar{D}}_1(z_2, b_T)$$

$$F_{TU}^{\sin(\phi_1 - \phi_{S_1})} = M_1 z_{p_1}^2 z_{p_2}^2 \mathcal{B}_1 \left[ \tilde{D}_{1T}^{\perp(1)} \tilde{\bar{D}}_1 \right]$$

$$= M_1 z_{p_1}^2 z_{p_2}^2 \sum_q e_q^2 \int \frac{db_T}{2\pi} b_T^2 J_1(b_T q_T) \tilde{D}_{1T}^{\perp(1)}(z_1, b_T) \tilde{\bar{D}}_1(z_2, b_T)$$

1st moment of the pFF

$$\Delta^N D_{h^\uparrow/q}(z, k_\perp) = \frac{k_\perp}{zM_h} D_{1T}^\perp(z, k_\perp) \quad \text{polarizing FF}$$



$$e^+ e^- \rightarrow h_1(P_1) h_2(P_2) + X$$

Polarization along  $\mathbf{n}$  ( $\perp$  to the production plane)

$$P_n^{h_1}(z_{h_1}, z_{h_2}) = \frac{M_1 \int dq_T q_T d\phi_1 \mathcal{B}_1 \left[ \tilde{D}_{1T}^{\perp(1)} \tilde{D}_1 \right]}{\int dq_T q_T d\phi_1 \mathcal{B}_0 \left[ \tilde{D}_1 \tilde{D}_1 \right]} \quad q_T < 0.27 Q$$

TMD fact.

Denominator (*known*)

(*small- $b_T$  matching onto*  
collinear FFs

$$\mathcal{B}_0 \left[ \tilde{D}_1 \tilde{D}_1 \right] = \frac{1}{z_1^2 z_2^2} \sum_q e_q^2 \int \frac{db_T}{2\pi} b_T J_0(b_T q_T) d_{h_1/q}(z_1; \bar{\mu}_b) d_{h_2/\bar{q}}(z_2; \bar{\mu}_b)$$

$$\times M_{D_1}(b_c(b_T), z_1) M_{D_2}(b_c(b_T), z_2) e^{-g_K(b_c(b_T); b_{\max}) \ln \left( \frac{Q^2 z_1 z_2}{M_1 M_2} \right) - S_{\text{pert}}(b_*; \bar{\mu}_b)}$$

Nonpert. funct.s
Nonpert. CS Kernel
Perturbative Sudakov factor





$$S_{\text{pert}}(b_*; \bar{\mu}_b) = -\tilde{K}(b_*; \bar{\mu}_b) \ln \frac{Q^2}{\bar{\mu}_b^2} - \int_{\bar{\mu}_b}^Q \frac{d\mu'}{\mu'} \left[ 2\gamma_D(g(\mu'); 1) - \gamma_K(g(\mu')) \ln \frac{Q^2}{\mu'^2} \right]$$

$$\bar{\mu}_b = \frac{C_1}{b_*(b_T)}$$

**Perturb.  
CS Kernel**

**RG  
anomalous dimensions**

$$C_1 = 2e^{-\gamma_E}$$

**Nonperturbative functions from Bacchetta, Delcarro, Pisano, Radici, Signori 2017**

$$M_D(b_T, z) = \frac{g_3 e^{-b_T^2 \frac{g_3}{4z^2}} + \frac{\lambda_F}{z^2} g_4^2 \left(1 - g_4 \frac{b_T^2}{4z^2}\right) e^{-b_T^2 \frac{g_4}{4z^2}}}{g_3 + \frac{\lambda_F}{z^2} g_4^2}$$

$$b_* \equiv b_*(b_T; b_{\min}, b_{\max}) = b_{\max} \left( \frac{1 - e^{-b_T^4/b_{\max}^4}}{1 - e^{-b_T^4/b_{\min}^4}} \right)^{1/4} \quad b_*(b_c(b_T)) \rightarrow \begin{cases} b_{\min} & b_T \ll b_{\min} \\ b_T & b_{\min} \ll b_T \ll b_{\max} \\ b_{\max} & b_T \gg b_{\max} \end{cases}$$

$$b_{\min} = 2e^{-\gamma_E}/Q \text{ and } b_{\max} = 0.6 \text{ GeV}^{-1} \quad g_K(b_T; b_{\max}) = \frac{g_2 b_T^2}{2}; \quad g_2 = 0.13 \text{ GeV}^2$$

**Coll. FF sets: for  $\pi/K$  de Florian, Sassot, Stratmann 2007; for  $\Lambda$  Albino, Kniehl, Kramer 2008**



# Numerator of $P_n$

$$\mathcal{B}_1 \left[ \tilde{D}_{1T}^{\perp(1)} \tilde{D}_1 \right] = \frac{1}{z_1^2 z_2^2} \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T^2 J_1(b_T q_T) D_{1T}^{\perp(1)}(z_1; \bar{\mu}_b) d_{h_2/\bar{q}}(z_2; \bar{\mu}_b) \\ \times M_{D_1}^{\perp}(b_c(b_T), z_1) M_{D_2}(b_c(b_T), z_2) e^{-g_K(b_c(b_T); b_{\max}) \ln \left( \frac{Q^2 z_1 z_2}{M_1 M_2} \right) - S_{\text{pert}}(b_*; \bar{\mu}_b)}$$

- Polarizing FF first moment

$$q = u, d, s, \bar{u}, \bar{d}, \bar{s}$$

$$D_{1T, \Lambda/q}^{\perp(1)}(z; \mu_b) = \mathcal{N}_q^p(z) d_{\Lambda/q}(z; \mu_b)$$

$$\mathcal{N}_q^p(z) = N_q z^{a_q} (1-z)^{b_q} \frac{(a_q + b_q)^{(a_q + b_q)}}{a_q^{a_q} b_q^{b_q}}$$

- nonperturbative pFF

$$M_{D, \Lambda}^{\perp}(b_T, z) = \exp \left( - \frac{\langle p_{\perp}^2 \rangle_P b_T^2}{4z_p^2} \right)$$



# FIT OF BELLE DATA [NLL]

- Data selection:  $\Lambda + \pi/K$ :  $z_{\pi,K}$  [0.5-0.9] bin excluded  $\rightarrow$  96 data points (128)

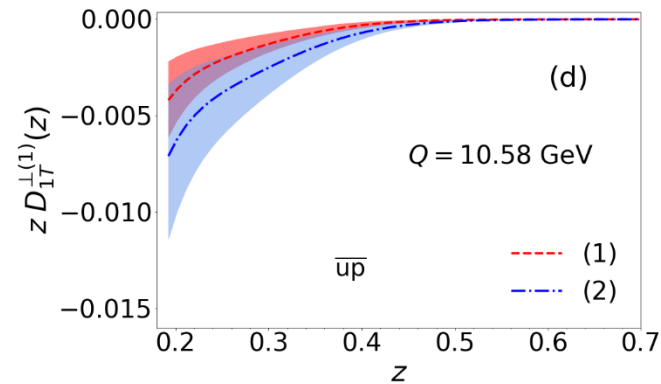
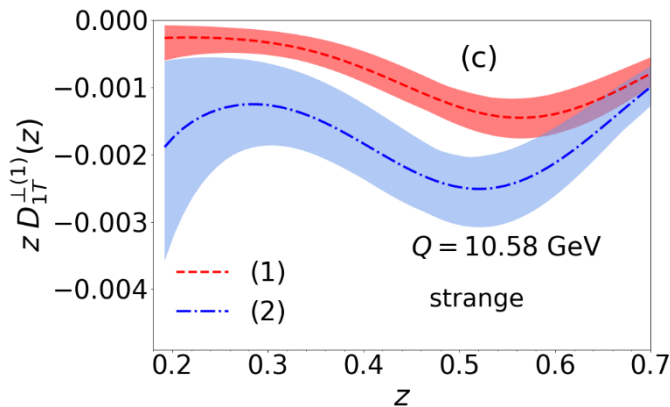
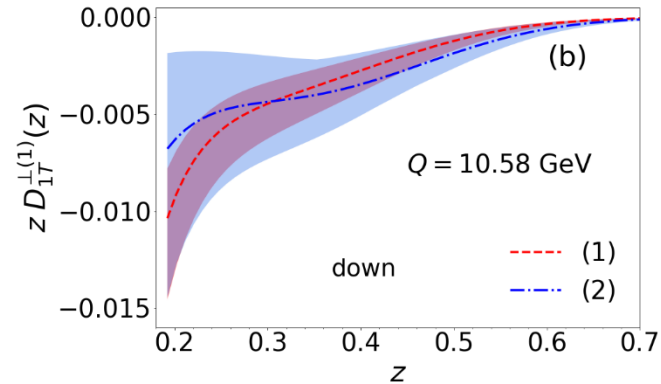
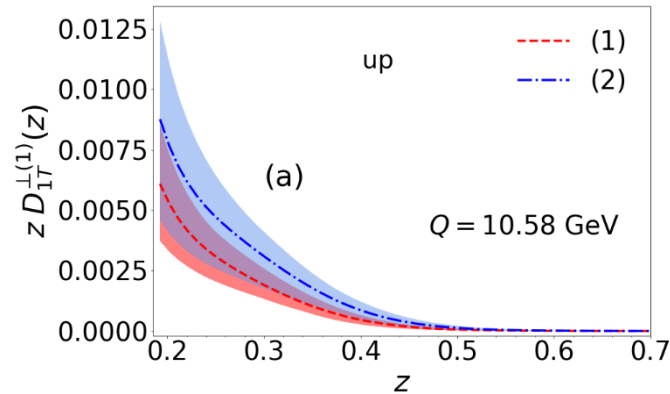
Scenarios considered:

1. Only light quarks & No  $SU(2)$  sym.  
pFFs for  $u, d, s, sea$ ; [8 par]
2. Inclusion of charm **in unpol. xsec**, & No  $SU(2)$  sym.  
pFFs for  $u, d, s, sea$ ; [9 par]
3. Inclusion of charm **in unpol. xsec**,  $SU(2)$  sym.  
pFFs for  $d = u, \bar{d} = \bar{u}, s, \bar{s}$  [9 par]

$\chi_{dof}^2$	$\chi_{dof}^2$
96 pts	128 pts
1.17	1.90
1.26	1.62
1.36	1.64



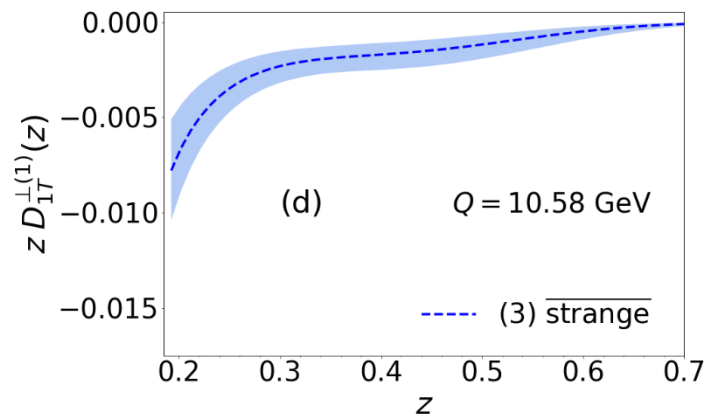
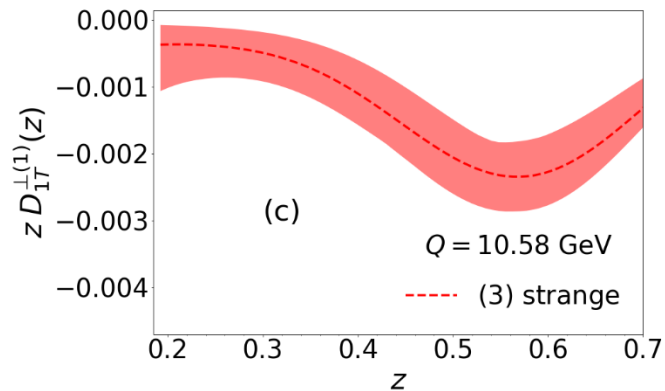
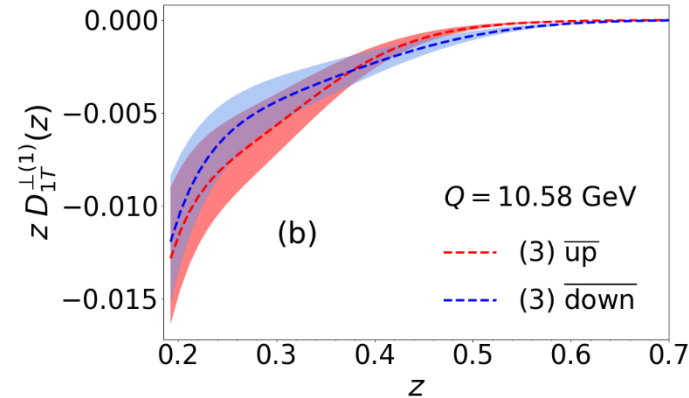
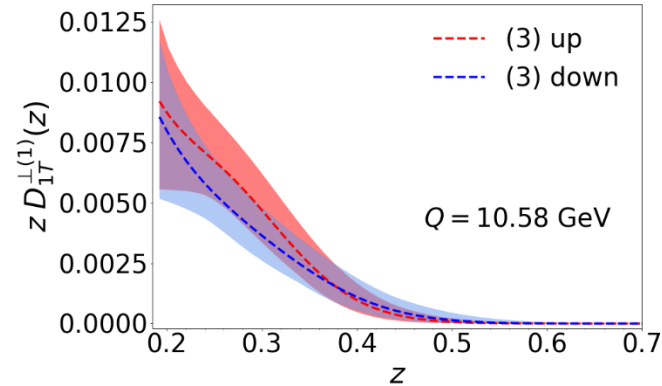
# FIRST MOMENTS: SC 1, 2 NO $SU(2)$ SYMM.



- different magnitudes, Sc 2 bigger in size: charm contribution;
- $u$  pFF positive;  $d$  pFF negative **STRONG  $SU(2)$  violation**
- Sc 1 and 2 compatible, except for strange
- Similar size for the Gaussian width.



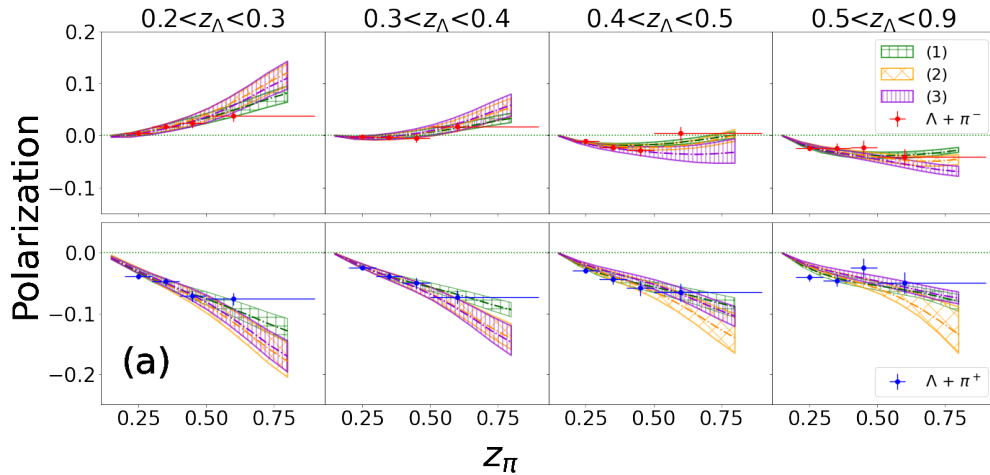
# FIRST MOMENTS: SC 3 WITH $SU(2)$ SYMM.



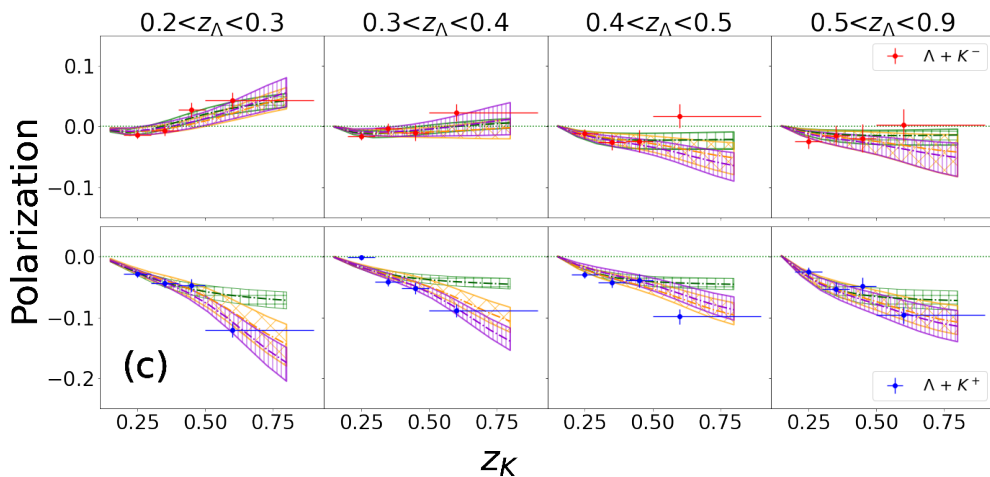
- $u, d$  pFFs **positive**; all others pFFs negative
- sea contribution larger in size w.r.t. Sc. 1 & 2



# DATA DESCRIPTION



- Good agreement for  $\Lambda\pi^\pm, (\overline{\Lambda}\pi^\pm)$  data, Sc. 1,2,3;



- Good agreement for  $\Lambda K^-(\overline{\Lambda}K^+)$  data, Sc. 1,2,3;
- Sc. 1 (no charm/no  $SU(2)$ ) cannot describe  $\Lambda K^+(\overline{\Lambda}K^-)$  data with  $z_K > 0,5$  ;
- Inclusion of charm allows for similar good fits (w/, w/o  $SU(2)$ )



# A LOOK INTO $\Lambda K^+$ VS. $\Lambda \pi^+$ AT LARGE $z_{K/\pi}$

$$P_n(\Lambda K^+) \simeq 4D_{K^+/u} \Delta^N D_{\Lambda^\uparrow/\bar{u}} + D_{K^+/\bar{s}} \Delta^N D_{\Lambda^\uparrow/s}$$

$$P_n(\Lambda \pi^+) \simeq 4D_{\pi^+/u} \Delta^N D_{\Lambda^\uparrow/\bar{u}} + D_{\pi^+/\bar{d}} \Delta^N D_{\Lambda^\uparrow/d}$$

- Both negative and increasing at large  $z$
- Sc 1: **down pFF < 0** ...enough for  $\Lambda \pi^+$
- Sc 2: inclusion of charm implies larger pFFs (**down pFF stable**):  
negative **strange** and the **sea pFF** larger
- Sc 3: inclusion of charm &  $SU(2)$  symmetry (**down pFF > 0**)  
even larger size of the **sea pFF** (**strange pFF stable w.r.t. Sc 2**)



# GENERAL REMARKS FROM $e^+e^-$

- Charm in unpol. xsec improves the quality of the fit (kaons)
- Polarizing FF for charm: tried but without any improvement
- Sc 2 and 3 (charm, w/ or w/o  $SU(2)$  symmetry) equally good
- If  $SU(2)$  not imposed, fits favor opposite pFFs for  $u, d$

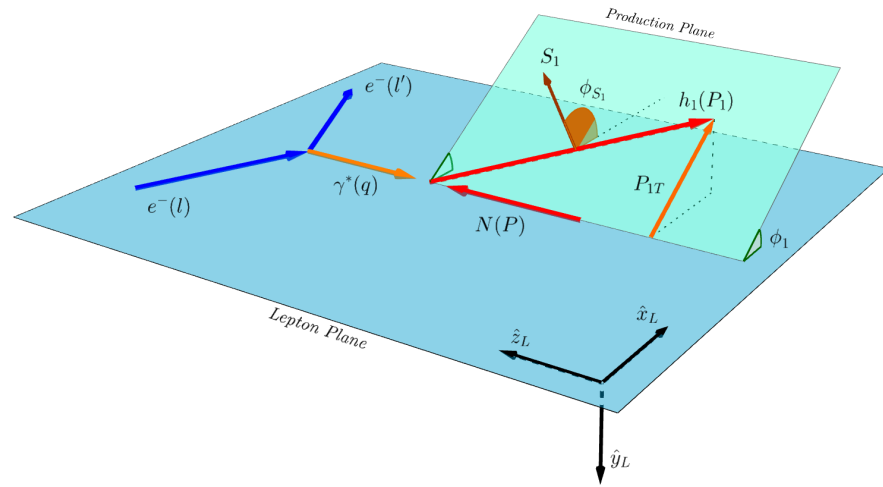
**STRONG  $SU(2)$  symmetry violation**

- Where/how can we check it?
- Different energies in  $e^+e^-$  do not help.





# SIDIS



$$P_n^{h1}(x_B, z_h) = \frac{M_1 \int dq_T q_T d\phi_1 \mathcal{B}_1 \left[ \tilde{f}_1 \tilde{D}_{1T}^{\perp(1)} \right]}{\int dq_T q_T d\phi_1 \mathcal{B}_0 \left[ \tilde{f}_1 \tilde{D}_1 \right]}$$

$$M_{f_1}(b_T, x) = \frac{1}{2\pi} e^{-g_1 \frac{b_T^2}{4}} \left( 1 - \frac{\lambda g_1^2}{1 + g_1} \frac{b_T^2}{4} \right)$$

Bacchetta, Delcarro, Pisano, Radici, Signori 2017

$$\mathcal{B}_0 \left[ \tilde{f}_1 \tilde{D}_1 \right] = \frac{1}{z^2} \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T J_0(b_T q_T) f_{q/N}(x; \bar{\mu}_b) d_{h/q}(z; \bar{\mu}_b) \\ \times M_{f_1}(b_c(b_T), x) M_{D_h}(b_c(b_T), z) e^{-g_K(b_c(b_T); b_{\max}) \ln \left( \frac{Q^2 z}{x M_P M_h} \right) - S_{\text{pert}}(b_*; \bar{\mu}_b)}$$

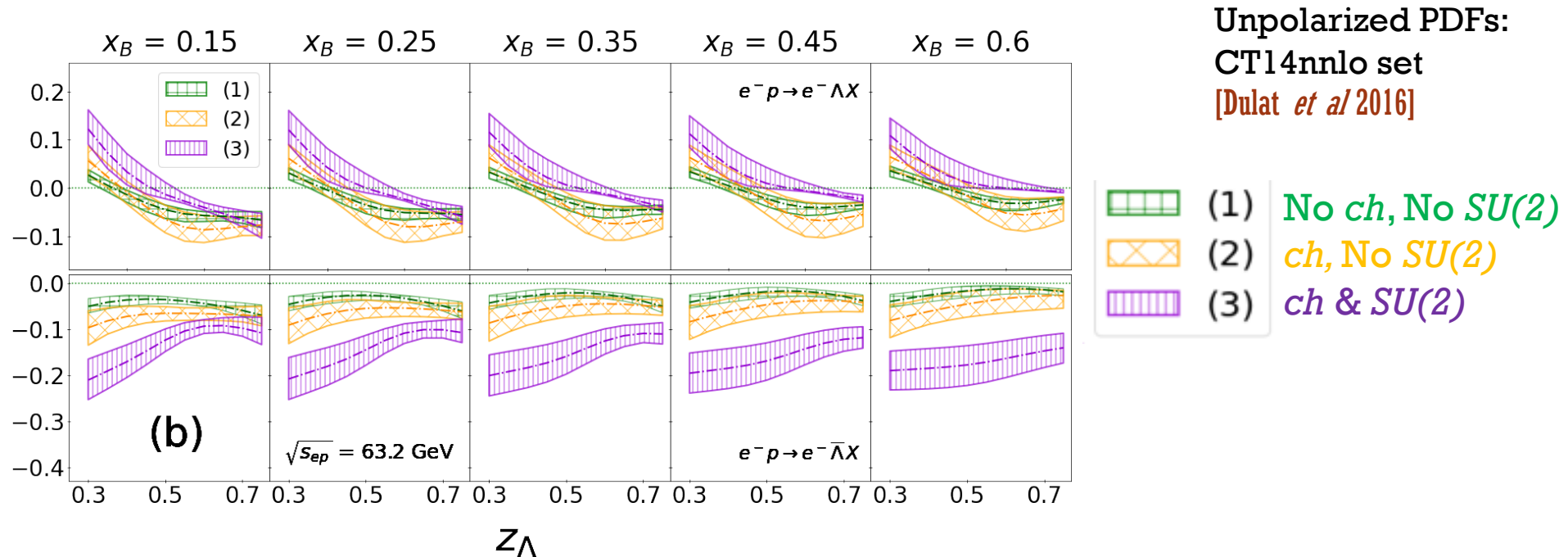
Coll PDF

$$\mathcal{B}_1 \left[ \tilde{f}_1 \tilde{D}_{1T}^{\perp(1)} \right] = \frac{1}{z^2} \sum_q e_q^2 \int \frac{db_T}{(2\pi)} b_T^2 J_1(b_T q_T) f_{q/N}(x; \bar{\mu}_b) D_{1T,q}^{\perp(1)}(z; \bar{\mu}_b) \\ \times M_{f_1}(b_c(b_T), x) M_{D_1}^{\perp}(b_c(b_T), z) e^{-g_K(b_c(b_T); b_{\max}) \ln \left( \frac{Q^2 z}{x M_P M_h} \right) - S_{\text{pert}}(b_*; \bar{\mu}_b)}$$

Nonpert funct.



# PREDICTIONS FOR EIC

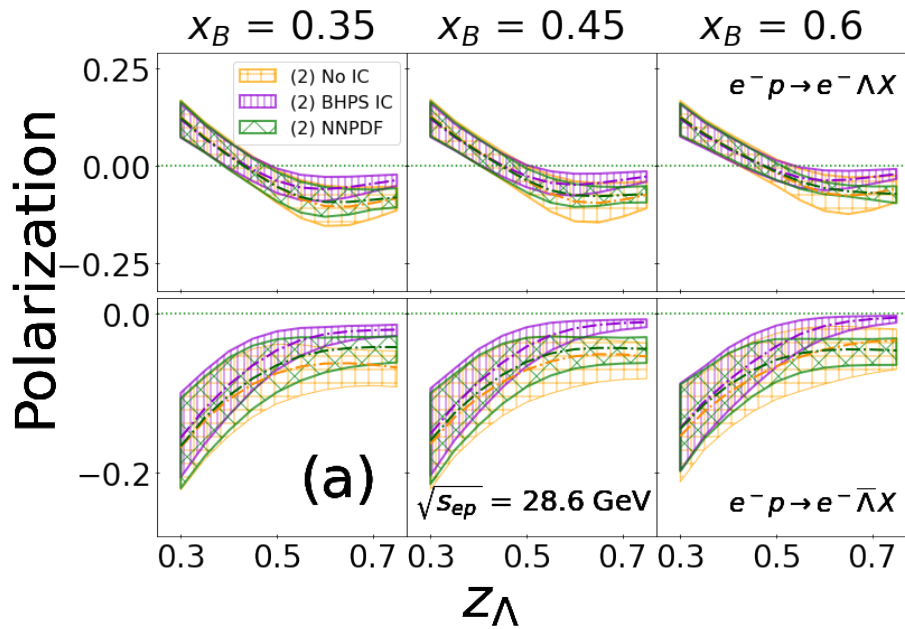


- Sc. 1 & 2: no significant differences
- $\Lambda$  pol. decreases and becomes negative
- $\bar{\Lambda}$  is always negative
- Sc. 3: similar size
- $\Lambda$  pol. slightly greater
- $\bar{\Lambda}$  most significant difference

Impact study in SIDIS @ EIC (Sc 1 at L0) Kang, Terry, Vossen, Xu, Zhang 2022



# INTRINSIC CHARM IN THE PROTON WITHIN SCENARIO 2

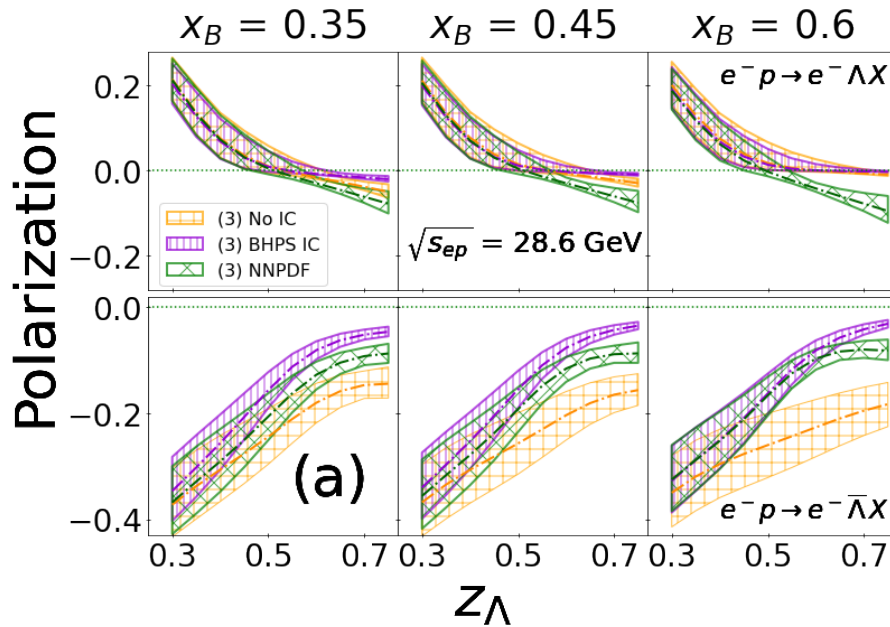


- CT14 nnlo IC set with BHPS model [T.-J. Hou *et al.*, 2018]
- NNPDF4.0 nnlo set [NNPDF Coll. 2022]

- BHPS and NNPDF: similar polarization values
- compatible with no intrinsic charm case



# INTRINSIC CHARM IN THE PROTON WITHIN SCENARIO 3



- CT14 nnlo IC set with BHPS model [T.-J. Hou *et al.*, 2018]
- NNPDF4.0 nnlo set [NNPDF Coll. 2022]

- Estimates vary significantly as  $x_B$  increases;
- $\bar{\Lambda}$  estimates different w.r.t. the no IC case;
- Similar behavior with perturbative charm contribution



# CONCLUDING REMARKS

- ❑ New fit of Belle  $e^+e^- \rightarrow h_1 h_2 + X$  data on transverse  $\Lambda$  polarization within a TMD factorization at NLL accuracy
- ❑ Extraction of the polarizing FFs within three scenarios
- ❑ Charm contribution (seems) relevant
- ❑  $SU(2)$  symmetry issue: no clear conclusion from  $e^+e^-$  data
- ❑ Predictions for SIDIS@EIC
  - ✓  $SU(2)$  symmetry could be tested
  - ✓  $\bar{\Lambda}$  special role [ $\Lambda$  dominated by  $u$  and  $d$ ]
  - ✓ check of the universality and scale evolution of the pFFs

THANKS for the ATTENTION



# BACK-UP SLIDES

