TMDs from Collins-Soper-Sterman Resummation

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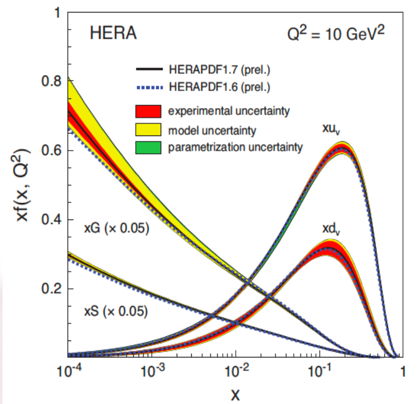
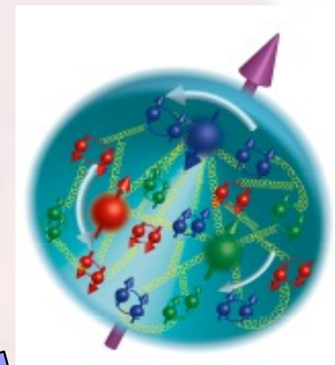
TMDs: center piece of nucleon structure

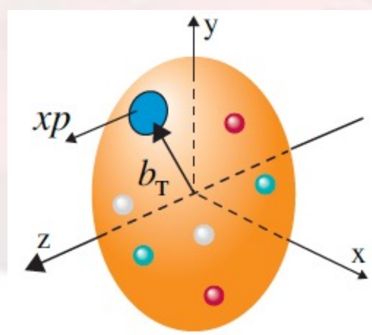


Long. Momentum distributions

QCD:
Factorization,
Universality,
Evolution,
Lattice, …



Nucleon Spin



3D imaging

Transverse-momentum-dependent and Generalized PDFs

Transverse-momentum-dependent (TMD) Parton distributions

- Generalize Feynman parton distribution q(x) by including the transverse momentum dependence

$$q(x,k_T)$$

- At small k_T, the transverse-momentum dependence is generated by soft non-perturbative physics.
- At large k_T, the k-dependence can be calculated in perturbative QCD and falls like powers of $1/k_T^2$

Where can we learn TMDs: two scales

- Semi-inclusive hadron production in deep inelastic scattering (SIDIS)
- Drell-Yan lepton pair, photon pair productions in pp scattering
- Dijet correlation in DIS
- Relevant e+e- annihilation processes
- …

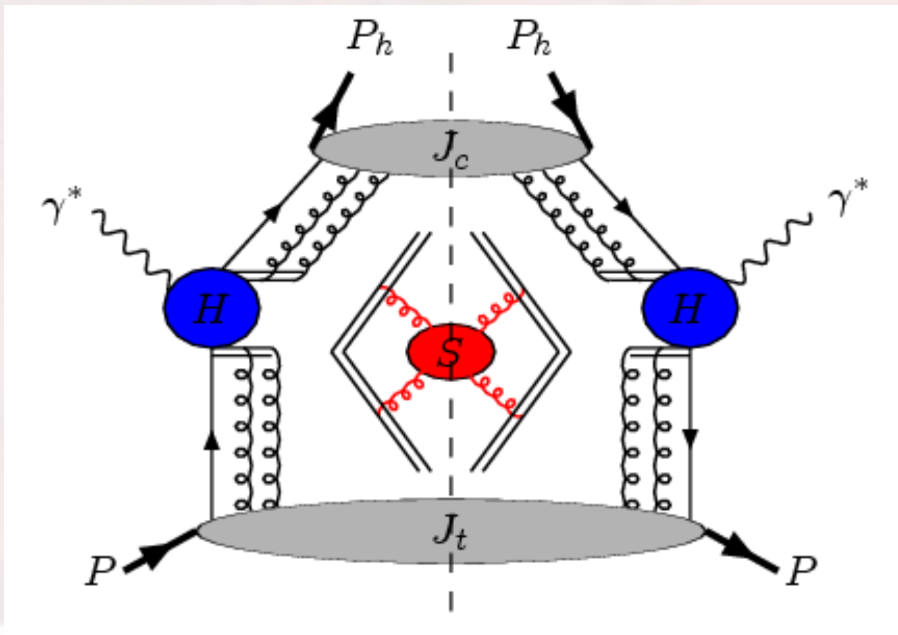
Except: Friday morning

Collinear vs TMD factorization

- TMD factorization is an extension and simplification to the collinear factorization
- Extends to the region where collinear fails
- Simplifies the kinematics
  - Power counting, correction 1/Q neglected

    $\sigma(P_T,Q)=H(Q)\, f_1(k_{1T},Q)\, f_2(k_{2T}, Q)\, S(\lambda_T)$

TMD factorization: a nutshell



TMD predictions rely on

- Non-perturbative TMDs constrained from experiments
- QCD evolutions, in particular, respect to the hard momentum scale Q
  - Strong theory/phenomenological efforts in the last few years
  - Need more exp. data/lattice calculations

Tremendous progress has been made!!

Soft gluon radiation leads to Sudakov Logarithms

Sudakov, 1956; Collins-Soper-Sterman 1985

- Differential cross section depends on $Q_1 = q_T$, where $Q^2 \gg Q_1^2 \gg \Lambda^2_{QCD}$

$$\frac{d\sigma}{dQ_1^2} = \frac{1}{Q_1^2} f_1 \otimes f_2 \otimes \sum_i \alpha_s^i \ln^{2i-1} \frac{Q^2}{Q_1^2} + \cdots$$

- Resummation of these large logs
  - In terms of transverse momentum dependent parton distributions and fragmentation functions and apply to
  - Semi-inclusive hadron production in DIS, Drell-Yan type of hard processes in pp collisions, e.g., Higgs, Z/W boson, …

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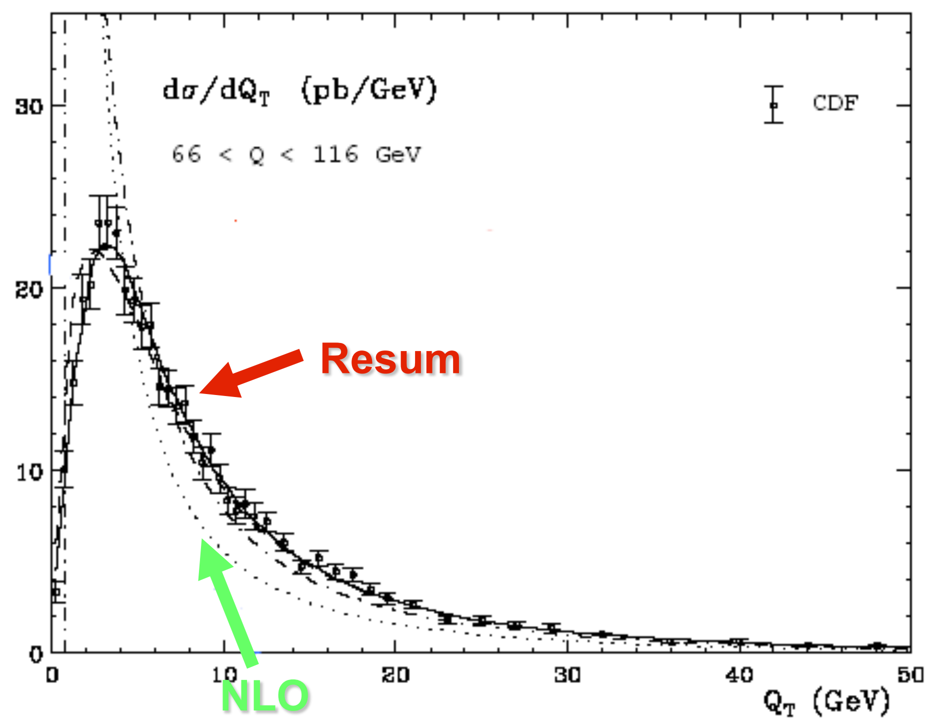
How Large of the Resummation effects



Kulesza, Sterman, Vogelsang, 02

Collins-Soper-Sterman Resummation

- $\sigma(P_T,Q)=H(Q)\, f_1(k_{1T},Q)\, f_2(k_{2T}, Q)\, S(\lambda_T)$
- Large Logs are resummed by solving the energy evolution equation of the TMDs

$$\frac{\partial}{\partial \ln Q} f(k_\perp, Q) = (K(q_\perp,\mu) + G(Q,\mu)) \otimes f(k_\perp, Q)$$

- K and G obey the renormalization group eq.

$$\frac{\partial}{\partial \ln \mu} K = -\gamma_K = \frac{\partial}{\partial \ln \mu} G$$

(Collins-Soper 81, Collins-Soper-Sterman 85)

CSS Formalism (II)

- The large logs will be resummed into the exponential form factor

$$W(Q,b) = e^{-\int_{1/b}^{Q}\frac{d\mu}{\mu}\left(\ln\frac{Q}{\mu}A+B\right)} C\otimes f_1 C\otimes f_2$$

  - A,B,C functions are perturbative calculable
  - f_1,f_2 are integrated PDFs
  - all are scheme-independent
  - Collins 2011 is slightly different, but final results are the same

(Collins-Soper-Sterman 85)

Non-perturbative input: b* prescription

$$W(Q,b) = e^{-\int_{1/b}^{Q} \frac{d\mu}{\mu}\left(\ln\frac{Q}{\mu}A+B\right)} C \otimes f_1 C \otimes f_2$$

- b* always in perturbative region

$$b \Rightarrow b_* = b/\sqrt{1+b^2/b_{max}^2}\ , \quad b_{max} < 1/\Lambda_{QCD}\ ,$$

- This will introduce a non-perturbative form factors

$$\mathcal{S}_{sud} \Rightarrow \mathcal{S}_{pert}(Q;b_*) + S_{NP}(Q;b)$$

- Generic behavior

$$S_{NP} = g_2(b)\ln Q + g_1(b)$$

Collins-Soper-Sterman 85

CSS Resummation Phenomenology

- Phenomenogical applications of the QCD resummation to the P_T spectrum of EW bosons production have been very successful

  Yuan, Nadolsky, Ladinsky, Landry,

  Qiu, Zhang, Berger, Li,

  Laenen, Sterman, Vogelsang, Kulesza,

  Bozzi, Catani, deFlorian,

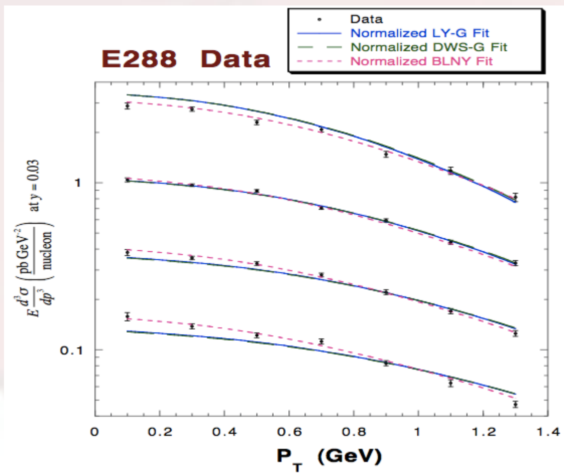
  Kulesza, Stirling, and many others, …

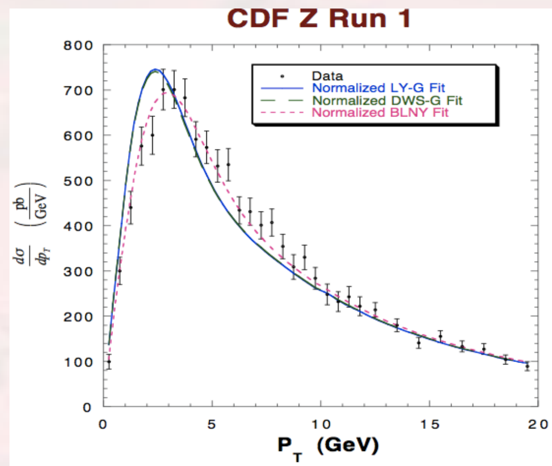
around and before 2000

BLNY form factors and ResBos

- Fit to Drell-Yan and W/Z boson production

$$S_{NP} = g_1 b^2 + g_2 b^2 \ln(Q/3.2) + g_1 g_3 b^2 \ln(100 x_1 x_2)$$





$$g_1 = 0.21^{+0.01}_{-0.01}\ \mathrm{GeV}^2\ ,\ g_2 = 0.68^{+0.01}_{-0.02}\ \mathrm{GeV}^2\ ,\ g_3 = -0.6^{+0.05}_{-0.04}.$$

b_{max}=0.5GeV^{-1}

Phys. Rev. D **67**, 073016 (2003)

CSS Formalism (III): Universality

Catani-de Florian-Grazzini 2000

- Apply the renormalization group equation,

$$g_1(\alpha_S(Q^2)) = \exp\left\{ \int_{b_0^2/b^2}^{Q^2} \frac{dq^2}{q^2}\, g_2(\alpha_S(q^2)) \right\} g_1(\alpha_S(b_0^2/b^2))$$

$$g_2(\alpha_S) = \beta(\alpha_S)\frac{d\ln g_1(\alpha_S)}{d\ln \alpha_S}$$

- Process-dependence in hard factor

$$W_{ab}^F(s;Q,b) = \sum_c \int_0^1 dz_1 \int_0^1 dz_2\, C_{ca}\left(\alpha_S\left(b_0^2/b^2\right), z_1\right) C_{\bar{c}b}\left(\alpha_S\left(b_0^2/b^2\right), z_2\right)$$
$$\times\, \delta\left(Q^2 - z_1 z_2 s\right) \sigma_{c\bar{c}}^F\left(Q^2, \alpha_S\left(Q^2\right)\right) S_c(Q,b)$$

$$\to \sigma_{c\bar{c}}^{(\mathrm{LO})\,F}\left(Q^2\right) H_c^F\left(\alpha_S\left(Q^2\right)\right)$$

CSS Formalism (IV): TMD Interpretation

- Factorized in terms of subtracted TMDs

$$\mathcal{H}^{\rm TMD}_{\rm (DY)}(Q;\mu)\sum_{q=q,\bar{q}} e_q^2\, \tilde{f}^{\rm (sub)}_{q/A}(x_1,b;Q,\mu)\,\tilde{f}^{\rm (sub)}_{\bar{q}/B}(x_2,b;Q,\mu)$$

$$\tilde{f}^{\rm (sub)}_{q(JCC)}(x,b;\zeta_c^2=Q^2,\mu_F=Q)=e^{-S^q_{pert}(Q,b_*)-S^q_{\rm NP}(Q,b)}\,\tilde{\mathcal{F}}^{\rm JCC}_q(\alpha_s(Q))\times\sum C^{(TMD)}_{q\leftarrow i}\otimes f_i(x,\mu_b)\,,$$

- Hard factor in the CSS resummation written as

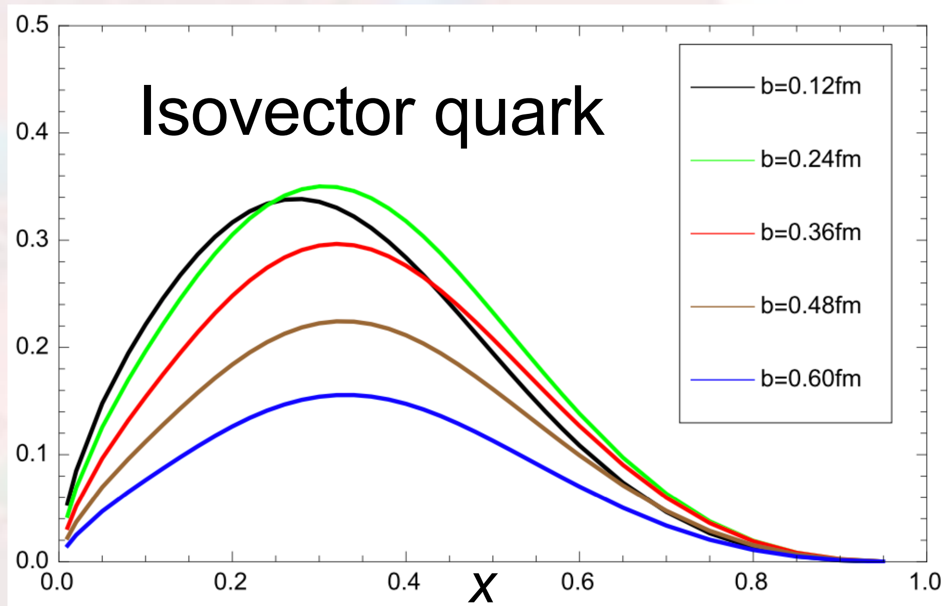
$$H^{\rm (DY)}_{\rm TMD}(\alpha_s(Q))=\tilde{\mathcal{F}}_q(\alpha_s(Q))\times\tilde{\mathcal{F}}_{\bar{q}}(\alpha_s(Q))\times\mathcal{H}^{\rm TMD}_{\rm (DY)}(Q;Q)$$

Prokudin-Sun-Yuan 2015

SIYY parameterization and TMDs

$$g_1 b^2 + g_2 \ln(b/b_*) \ln(Q/Q_0) + g_3 b^2 \left((x_1/x_0)^\lambda + (x_2/x_0)^\lambda\right)$$

- Log(Q) term motivated by matching to perturbative form factors
  - Collins-Soper 87
- Small-x behavior motivated by saturation model suggestions



Sun-Issac-Yuan-Yuan, 2014; Prokudin-Sun-Yuan, 2014

Comments

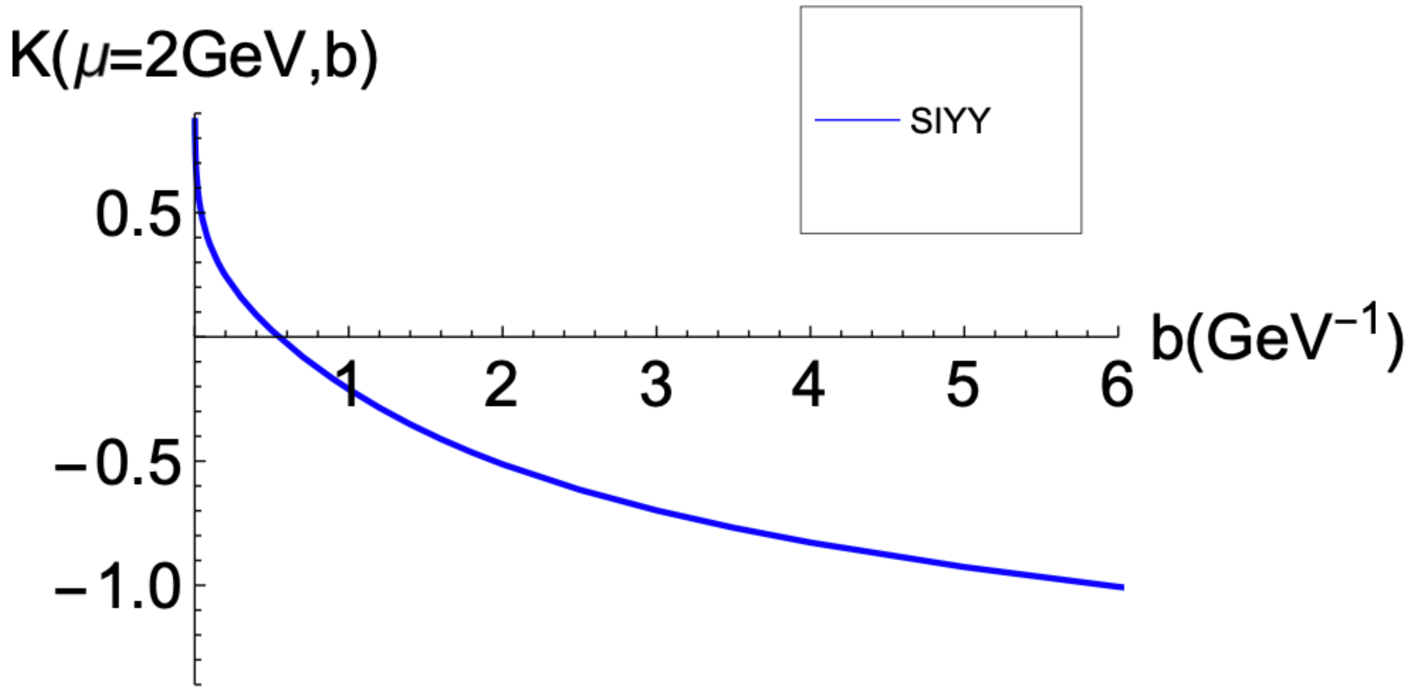
- Resummation in terms of the integrated PDFs, the final result is scheme-independent
  - Universality of the TMD distributions
- It will be important to compare the universal TMD distribution extracted from exp. with the Lattice calculation at the same factorization scale with full resummation effects

Energy evolution: Collins-Soper Kernel

- CS kernel is scheme-dependent
  - JMY/Lattice scheme, it contains both hard and soft contributions: **K+G**
  - JCC/SCET schemes, it only has the soft term: **K**
- Therefore, one has to be careful when we compare the CS kernel from different schemes, lattice, and phenomenology

CS Kernel from CSS resummation

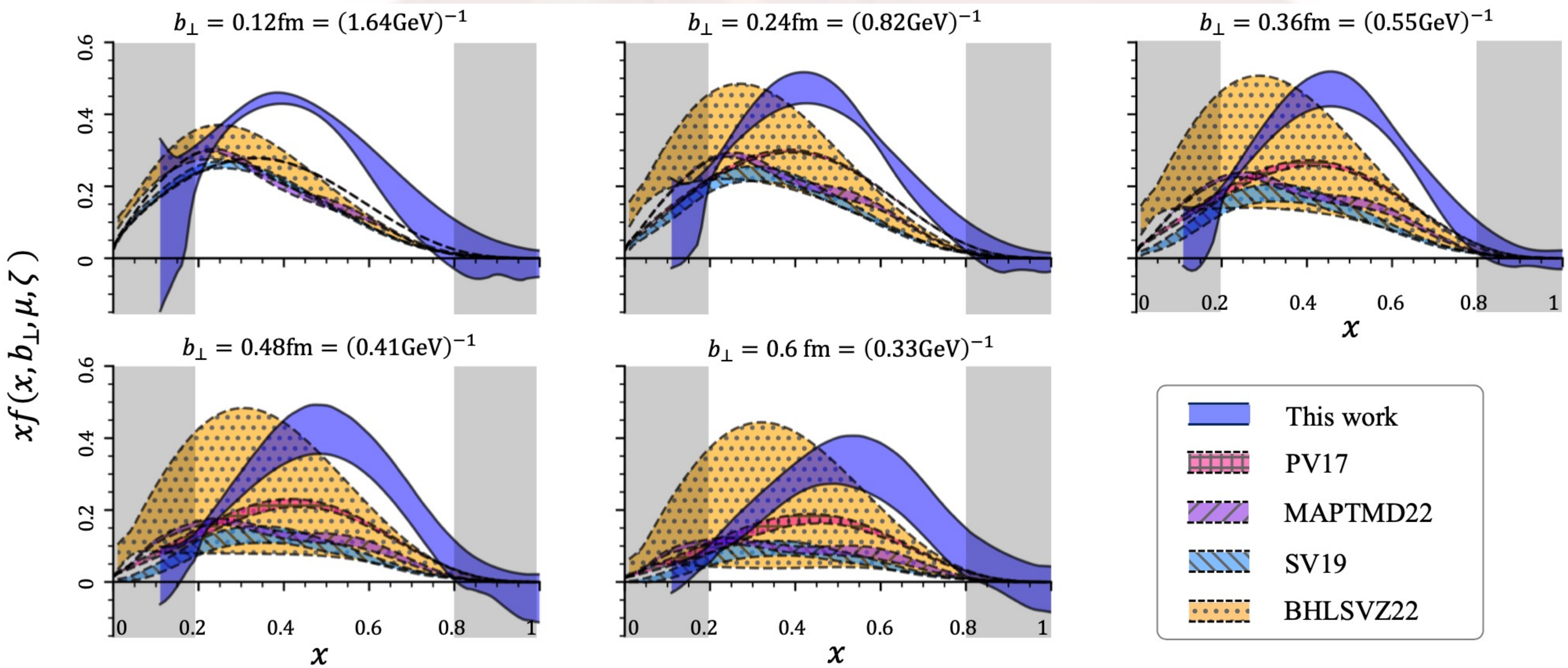
$$K(\mu, b) = -2\int_{\mu_{b*}}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}A(\alpha_s(\bar{\mu})) - \frac{1}{2}g_2\ln(b/b_*)$$



Discussions

- We should unify the TMD interpretation extracted from the global analyses
  - Universal TMDs suggested by Catani-de Florian-Grazzini is a good starting point
- When we compare to Lattice simulations on these TMDs and Collins-Soper kernel, we need to be careful, including all resummation effects

TMD quark distributions compared to global analyses



Jin-Chen He, et al, arXiv:2211.02340

Collins-Soper Kernel in LaMET

$$\ln \frac{\tilde{f}^{(sub)}_{q(\mathrm{LaMET})}(x,b;\zeta^2=Q_1^2,\mu)}{\tilde{f}^{(sub)}_{q(\mathrm{LaMET})}(x,b;\zeta^2=Q_2^2,\mu)}$$

$$= -\ln\frac{Q_1^2}{Q_2^2}\left[\int_{\mu_{b*}}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}A(\alpha_s(\bar{\mu}))+\frac{1}{4}g_2\ln(b/b_*)\right]$$

$$-\int_{\mu}^{Q_1}\frac{d\bar{\mu}}{\bar{\mu}}\left(\ln\frac{Q_1^2}{\bar{\mu}^2}A(\alpha_s(\bar{\mu}))+B(\alpha_s(\bar{\mu}))+2\gamma_F(\bar{\mu})\right)$$

$$+\int_{\mu}^{Q_2}\frac{d\bar{\mu}}{\bar{\mu}}\left(\ln\frac{Q_2^2}{\bar{\mu}^2}A(\alpha_s(\bar{\mu}))+B(\alpha_s(\bar{\mu}))+2\gamma_F(\bar{\mu})\right)$$

Matching coeff.
Need resummatioı