Semi-inclusive diffractive DIS at small-x

Yoshitaka Hatta

BNL & RIKEN BNL

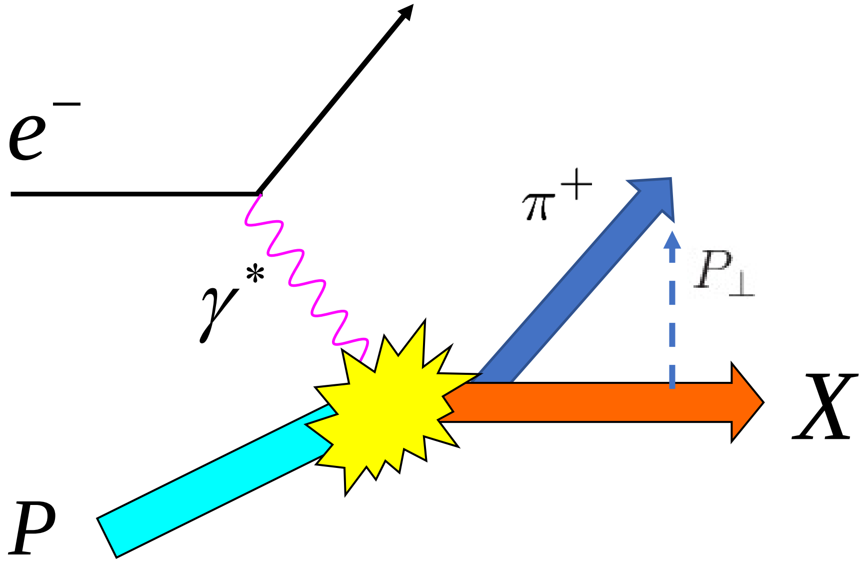
with Feng Yuan and Bowen Xiao, PRD106 (2022) 094015

Semi-inclusive DIS (SIDIS)

Tag one hadron species
with fixed transverse momentum $P_\perp$

When $P_\perp$ is small, TMD factorization

Collins, Soper, Sterman; Ji, Ma, Yuan;…



$$\frac{d\sigma}{dP_\perp} = H \otimes f(x, k_\perp) \otimes D(z, q_\perp)$$

TMD PDF TMD FF

Open up a new class of observables where perturbative QCD is applicable.
Variety of novel phenomena due to intrinsic transverse momentum
3D imagining of partons in momentum space

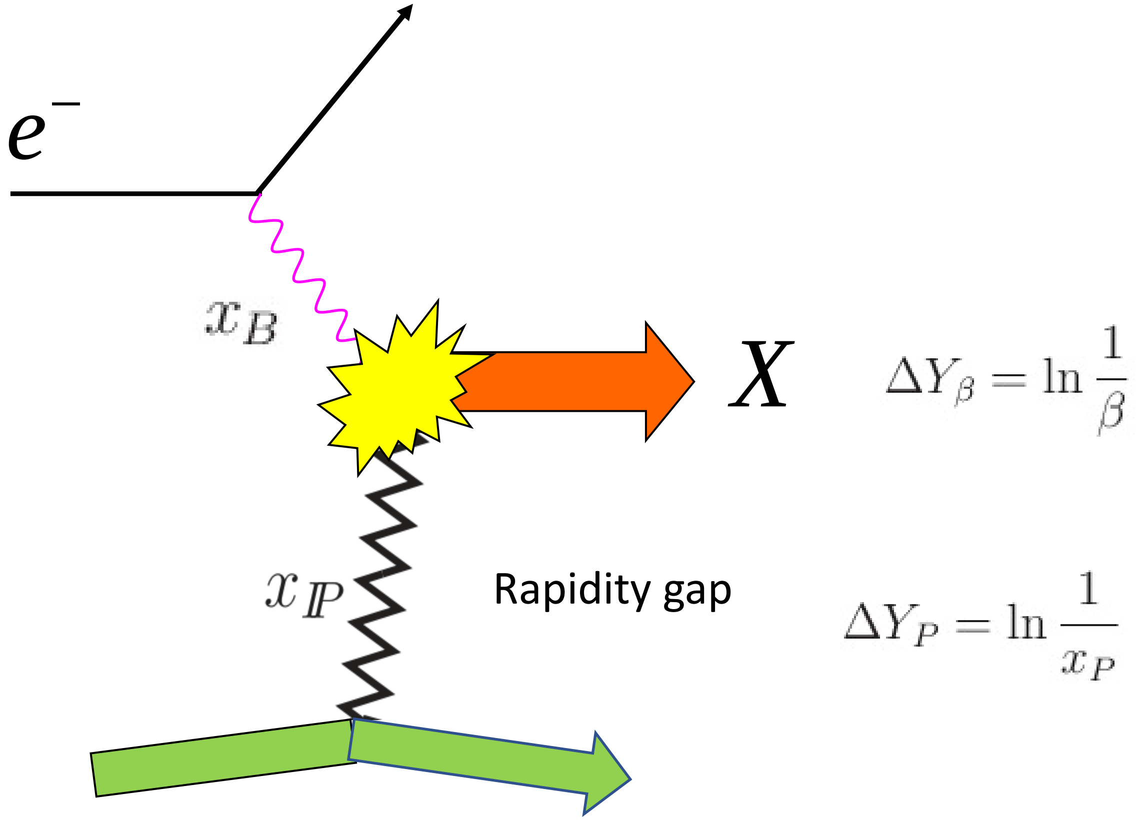
Diffractive DIS

~10% of HERA events

probe of BFKL/gluon saturation at small-x

Factorization theorem in terms of diffractive PDF

$$F_{2/L}^{D(3)}(\beta, Q^2, x_{\mathbb{P}}) = \sum_i \int_\beta^1 \frac{dz}{z} C_{2/L,i}\left(\frac{\beta}{z}\right) f_i^D(z, x_{\mathbb{P}}; Q^2)$$



$$2E_{P'} \frac{df_q^D(x, x_P, t)}{d^3P'} = \int \frac{d\xi^-}{2(2\pi)^4} e^{-ix\xi^- P^+} \langle PS|\bar{\psi}(\xi)\gamma^+|P'X\rangle\langle P'X|\psi(0)|PS\rangle$$

$$\beta = \frac{Q^2}{Q^2 + M_X^2} = \frac{x_B}{x_P}$$

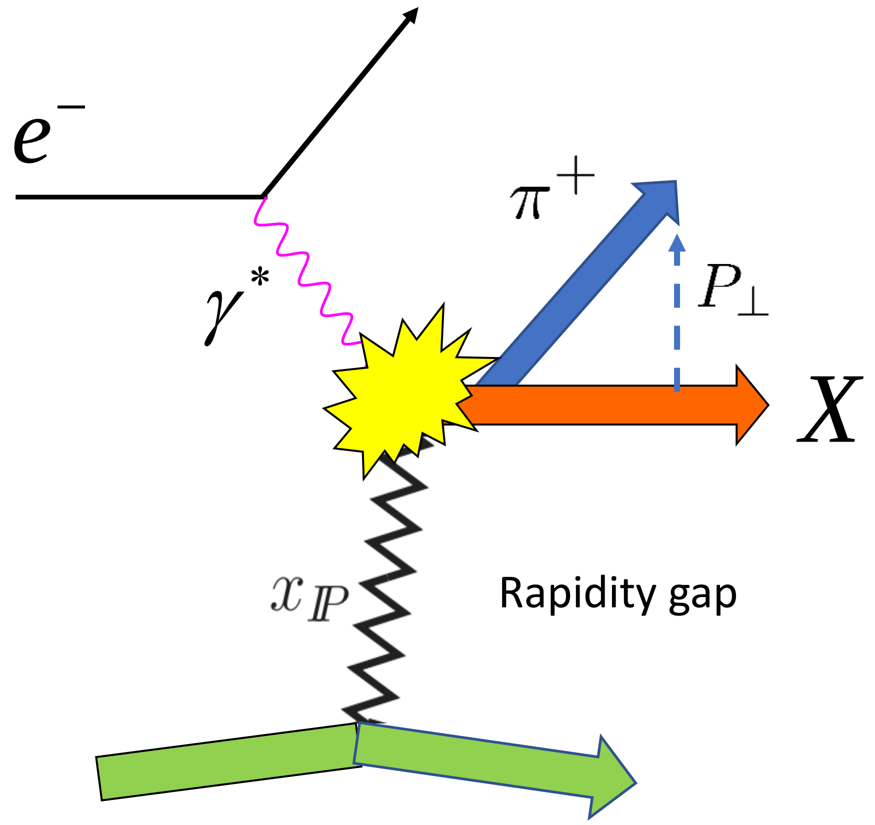
Momentum fraction of the `Pomeron' carried by partons

Semi-inclusive diffractive DIS (SIDDIS)

$$\frac{d\sigma^{\text{SIDDIS}}(\ell p \to \ell' p' q X)}{dx_B dy d^2k_\perp dY_{IP} dt} = \sigma_0 e_q^2 x_B \frac{df_q^D(\beta, k_\perp; x_{IP})}{dY_{IP} dt}$$

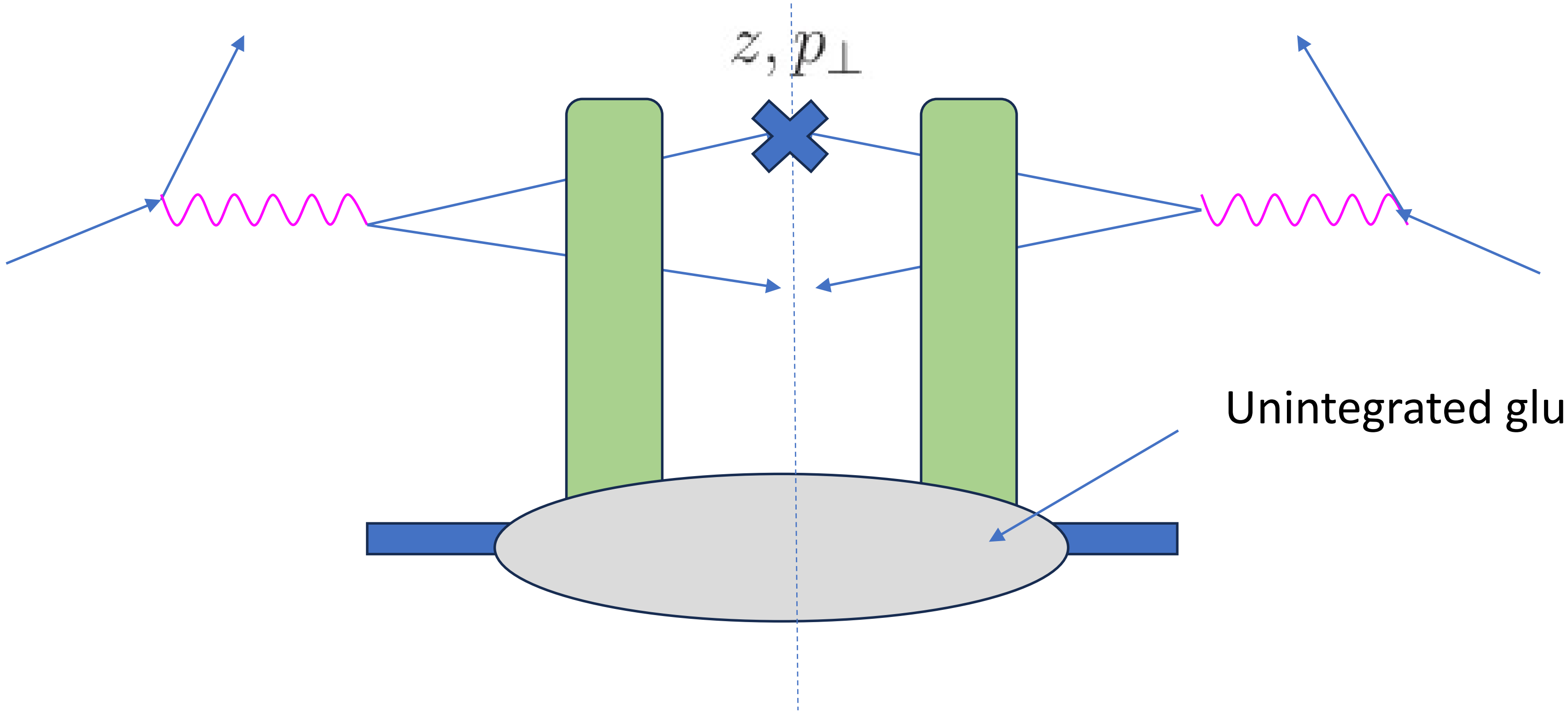
TMD version of diffractive PDF

$$2E_{P'} \frac{df_q^D(x, k_\perp; x_{IP}, t)}{d^3P'}$$
$$= \int \frac{d\xi^- d^2\xi_\perp}{2(2\pi)^6} e^{-ix\xi^- P^+ + i\vec{\xi}_\perp \cdot \vec{k}_\perp}$$
$$\times \langle PS|\bar{\psi}(\xi)\mathcal{L}_n^\dagger(\xi)\gamma^+|P'X\rangle\langle P'X|\mathcal{L}_n(0)\psi(0)|PS\rangle$$



SIDIS at small-x

- TMD factorization well established at large to medium x
- Challenging to include small-x resummation effects → talk by Ian
- Small-x people have developed their own formalism (BFKL/kt factorization/color dipole/CGC/rapidity factorization)



Unintegrated gluon distribution $\stackrel{??}{=}$ gluon TMD

Gluon TMD and color dipole

'dipole' type

Start with the gluon TMD

$$F(x,k_\perp) \quad = \quad \frac{2}{p^+}\int\frac{dz^-d^2z_\perp}{(2\pi)^3}e^{ixp^+z^- - ik_\perp\cdot z_\perp}\langle p|\mathrm{Tr}[F^{+i}(0)WF^{+i}(z^-,z_\perp)W]|p\rangle$$

Take the formal Regge limit $x \to 0 \qquad e^{ixp^+z^-} \approx 1$

$$F(x,k_\perp) \approx \frac{2N_ck_\perp^2}{\alpha_s}\int\frac{d^2b_\perp}{(2\pi)^2}\int\frac{d^2r_\perp}{(2\pi)^2}e^{-ik_\perp\cdot r_\perp}\frac{\langle p|\frac{1}{N_c}\mathrm{Tr}[U(x_\perp)U^\dagger(y_\perp)]|p\rangle}{\langle p|p\rangle}$$

Dipole S-matrix

$$U(z_\perp) = P\exp\left(ig\int_{-\infty}^{\infty}dz^-A^+(z^-,z_\perp)\right)$$

x-dependence restored by tilting the Wilson line, evolution equation with respect to the slope Balitsky (1996)

To make connection to the (usual) TMD, use a different regularization of the rapidity divergence for the latter Balitsky, Tarasov (2016)

Quark TMD at small-x

McLerran, Venugopalan (1994)
Mueller (1999)
Marquet, Yuan, Xiao (2009)

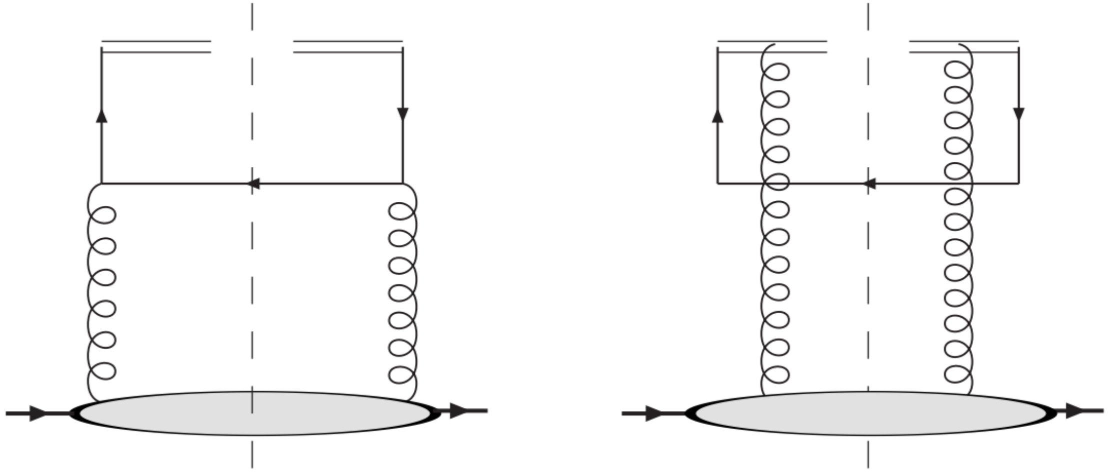
$$f(x,k_\perp) = \int \frac{d^3\xi}{2(2\pi)^3} e^{-ixP^+\xi^- + ik_\perp\cdot\xi_\perp} \langle P|\bar{\psi}(\xi^-,\xi_\perp)\mathcal{L}\gamma^+\psi(0)|P\rangle$$

$$= \frac{T_R}{4\pi^4} S_\perp N_c \int d^2k_{g\perp} \int_x \frac{dx_g}{x_g^2} \left(\frac{\vec{k}_\perp |k_\perp - k_{g\perp}|}{\hat{x}(k_{g\perp} - k_\perp)^2 + (1-\hat{x})k_\perp^2} - \frac{\vec{k}_\perp - \vec{k}_{g\perp}}{|k_\perp - k_{g\perp}|} \right)^2 \frac{\langle P|\frac{1}{N_c}\mathrm{tr}UU^\dagger(k_{g\perp})|P\rangle}{\langle P|P\rangle}$$

$$\sim \frac{1}{k_\perp^2}$$

SIDIS cross section

$$\frac{d\sigma}{dx_B dy d^2P_\perp} = \sigma_0 e_q^2 x_B f_q(x,k_\perp) \otimes D(z)$$



Geometric scaling

A priori, $F(x, k_\perp)$ depends separately on x and $k_\perp$

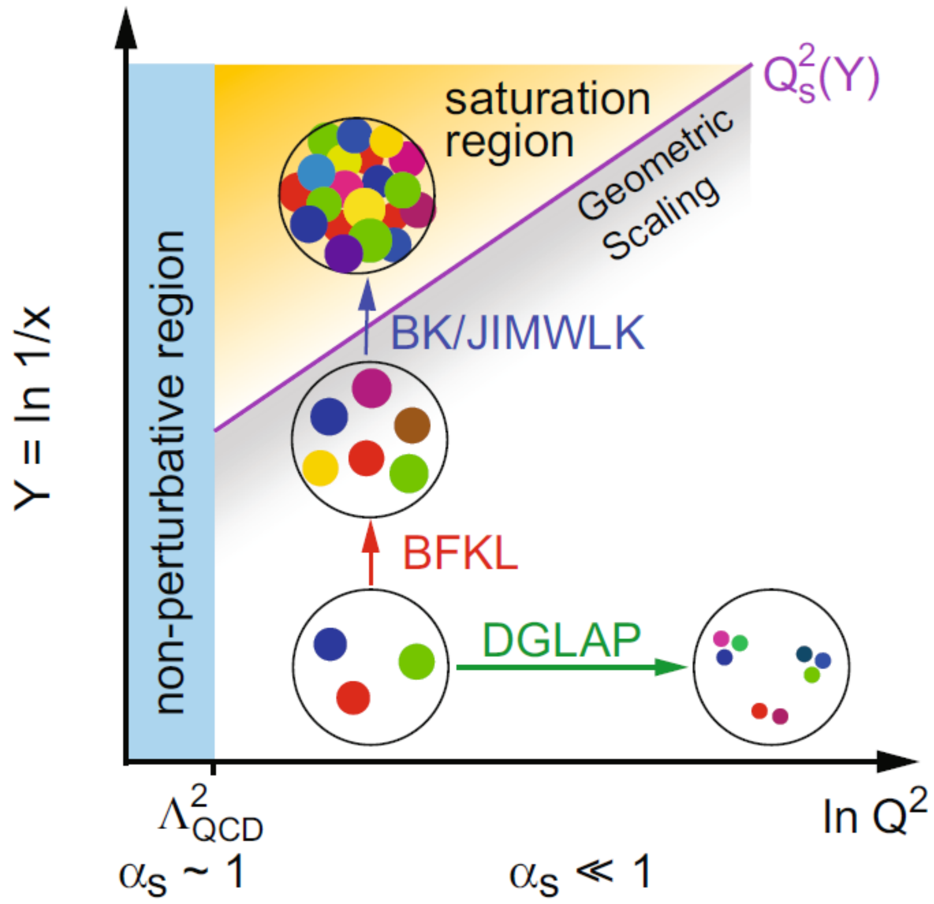
However, at small-x there is a dynamically generated scale called saturation momentum

$$Q_s(x) = \Lambda_{QCD} \left(\frac{1}{x}\right)^\gamma$$

$F(x, k_\perp)$ becomes a function only of the ratio in a certain kinematical window

$$F(x, Q_s) \sim \frac{1}{Q_s^2} f\left(\frac{k_\perp}{Q_s(x)}\right)$$

Necessary consequence of the BK equation, or more generally, the gluon saturation



Color dipole=Wigner (Mother) distribution

$$xW(x, \vec{q}_\perp, \vec{b}_\perp) \approx \frac{2N_c}{\alpha_s} \int \frac{d^2\vec{r}_\perp}{(2\pi)^2} e^{i\vec{q}_\perp \cdot \vec{r}_\perp} \left(\frac{1}{4}\vec{\nabla}_b^2 - \vec{\nabla}_r^2\right) \langle P'|\mathrm{Tr} U_{x_\perp} U^\dagger_{y_\perp}|P\rangle$$

YH, Xiao, Yuan (2016)

Unpol TMD, linearly polarized distribution,
gluon Sivers and other T-odd TMDs
gluon GPD H_g, E_g
transversity GPD

→ Application to diffractive PDF   Cf. Hautmann, Kunszt, Soper (1999)

Distributions associated with proton's longitudinal polarization is more difficult → Talk by Andrey

Quark diffractive TMD at small-x

YH, Xiao, Yuan (2022)

Start with the operator definition

$$x\frac{df_q^D(\beta,k_\perp;x_{IP})}{dY_{IP}dt} = \int d^2k_{1\perp}d^2k_{2\perp}\mathcal{F}_{x_{IP}}(k_{1\perp},\Delta_\perp) \times\mathcal{F}_{x_{IP}}(k_{2\perp},\Delta_\perp)\frac{N_c\beta}{2\pi}\frac{k'_{1\perp}\cdot k'_{2\perp}k_\perp^2}{[\beta k_\perp^2+(1-\beta)k'^2_{1\perp}][\beta k_\perp^2+(1-\beta)k'^2_{2\perp}]}+\cdots$$

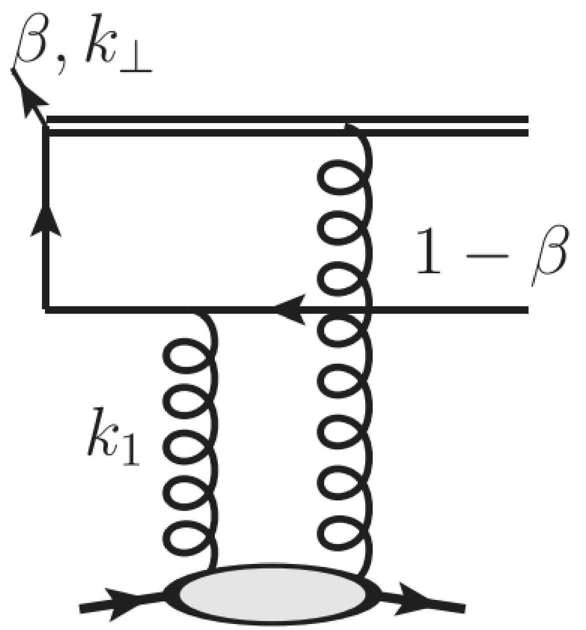
$$k'_{1\perp}=k_\perp-k_{1\perp}$$

color dipole

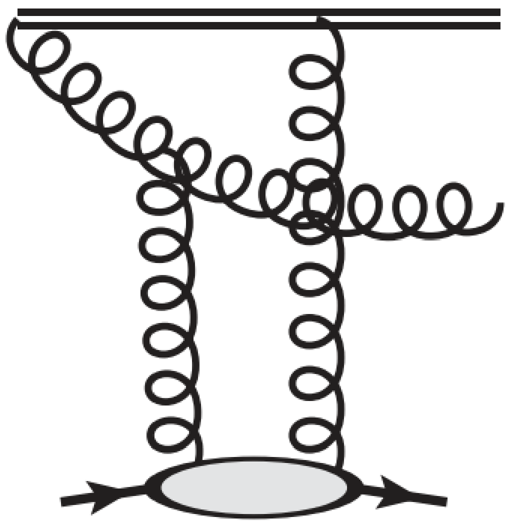
At large transverse momentum

$$\sim\frac{1}{k_\perp^4}(H_g(x_P,t))^2$$

Gluon GPD YH, Xiao, Yuan (2017)



Gluon diffractive TMD at small-x



$$x\frac{df_g^D(\beta,k_\perp;x_{IP})}{dY_{IP}dt}=\int d^2k_{1\perp}d^2k_{2\perp}\mathcal{G}_{x_{IP}}(k_{1\perp},\Delta_\perp)\mathcal{G}_{x_{IP}}(k_{2\perp},\Delta_\perp)$$
$$\times\frac{N_c^2-1}{\pi(1-\beta)}\frac{1}{[\beta k_\perp^2+(1-\beta)k_{1\perp}'^2]}\frac{1}{[\beta k_\perp^2+(1-\beta)k_{2\perp}'^2]}$$
$$\times\left[\beta(1-\beta)k_\perp^2\frac{k_{1\perp}'^2+k_{2\perp}'^2}{2}+(1-\beta)^2(k_{1\perp}'\cdot k_{2\perp}')^2+\beta^2\frac{(k_\perp^2)^2}{2}\right]+\cdots$$

$$\mathcal{G}_x(q_\perp,\Delta_\perp)=\int\frac{d^2b_\perp d^2r_\perp}{(2\pi)^4}e^{iq_\perp\cdot r_\perp+i\Delta_\perp\cdot b_\perp}\frac{1}{N_c^2-1}\left\langle\mathrm{Tr}\left[\tilde{U}\left(b_\perp+\frac{r_\perp}{2}\right)\tilde{U}^\dagger\left(b_\perp-\frac{r_\perp}{2}\right)\right]\right\rangle_x$$

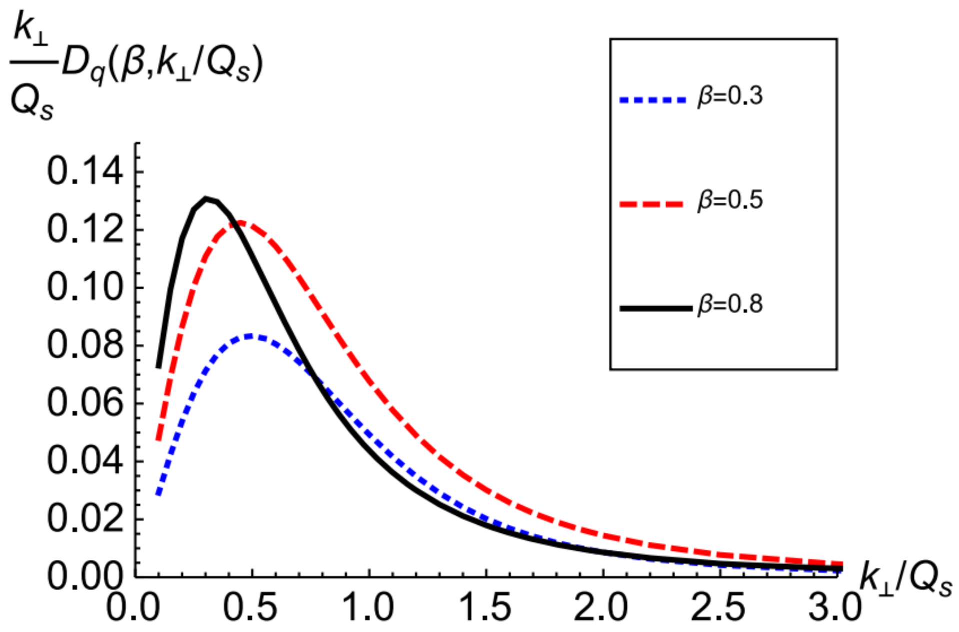
Color dipole (adjoint rep.)

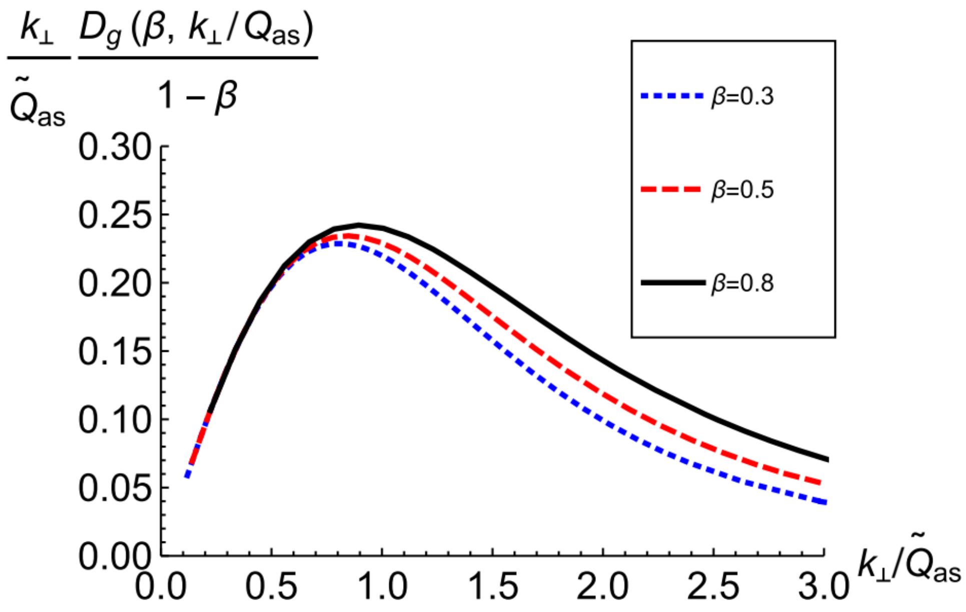
Modified geometric scaling

Iancu, Mueller, Triantafyllopoulos (2021)
YH, Xiao, Yuan (2022)

$$x\frac{df^D_{q,g}(\beta,k_\perp;x_{IP})}{dY_{IP}dt}=\mathcal{N}_{q,g}D_{q,g}\left(\beta,\frac{k_\perp}{Q_{s,as}}\right)\sim D_{q,g}\left(\frac{k_\perp}{\sqrt{1-\beta}Q_{s,as}(x_P)}\right)$$

Easily understood from an inspection of the propagator denominator $\frac{1}{[\beta k_\perp^2+(1-\beta)k_{1\perp}'^2]}$



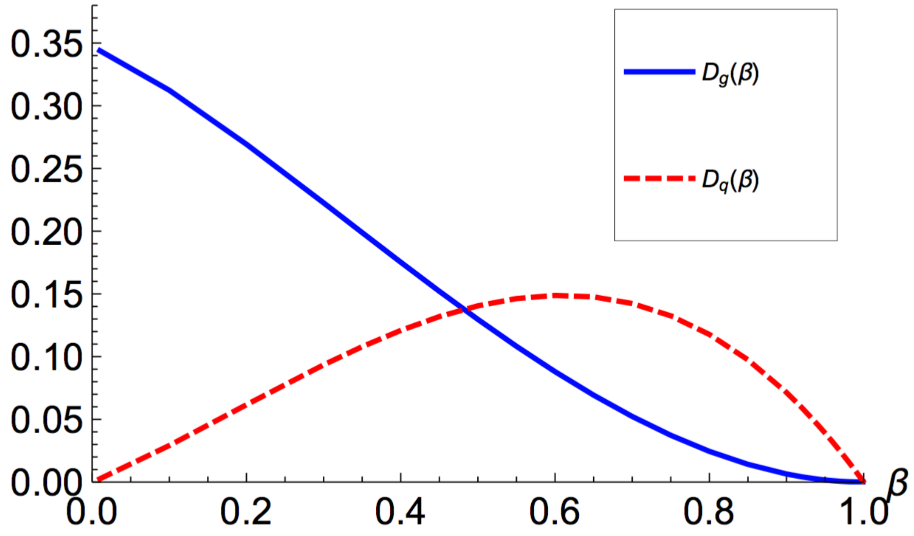


Collinear DPDF

Integrate over $k_\perp$

$$x\frac{df^D_{q,g}(\beta;x_{IP})}{dY_{IP}dt} = \mathcal{N}_{q,g}2\pi\mathcal{D}_{q,g}(\beta)Q^2_{s,as}$$

$$\mathcal{D}_q(\beta) = \beta\left(b_1(1-\beta)+b_2(1-\beta)^2\right) \qquad \mathcal{D}_g(\beta) = (a_0+a_1\beta)(1-\beta)^2$$



The end point behavior analytically computed for Gaussian models.

$$b_1 = \frac{3\pi^2}{16} - 1, \quad b_2 = \frac{20-3\pi^2}{16}$$

$$a_0 = \frac{\ln(2)}{2} \qquad a_1 = \frac{45\pi^2-272}{256} - \frac{\ln(2)}{2}$$

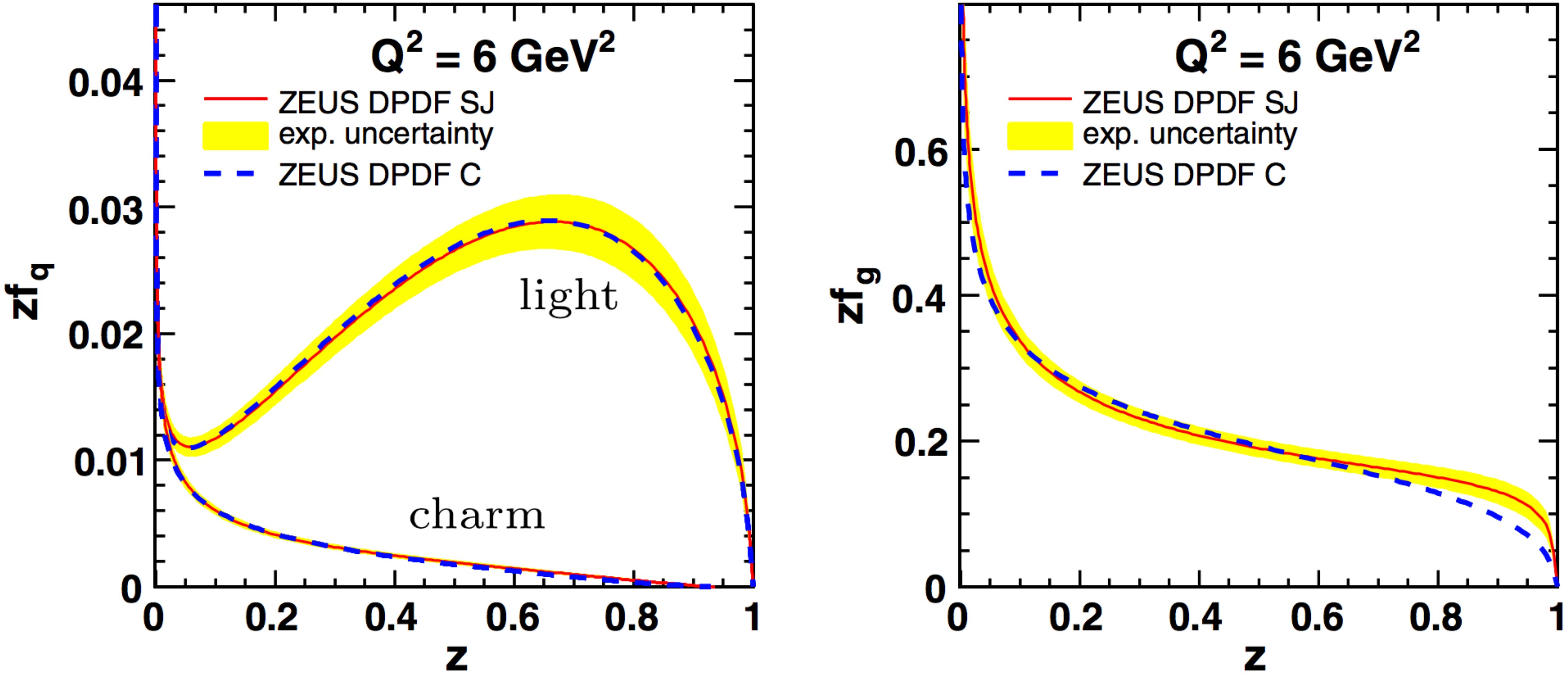
Buchmuller, Gehrmann, Hebecker (1999)

Initial condition for the DGLAP evolution

HERA data

ZEUS, NPB831, 1 (2010)

$$F_{2/L}^{D(3)}(\beta, Q^2, x_{\mathbb{P}}) = \sum_i \int_\beta^1 \frac{dz}{z} C_{2/L,i}\left(\frac{\beta}{z}\right) f_i^D(z, x_{\mathbb{P}}; Q^2)$$



Diffractive structure functions

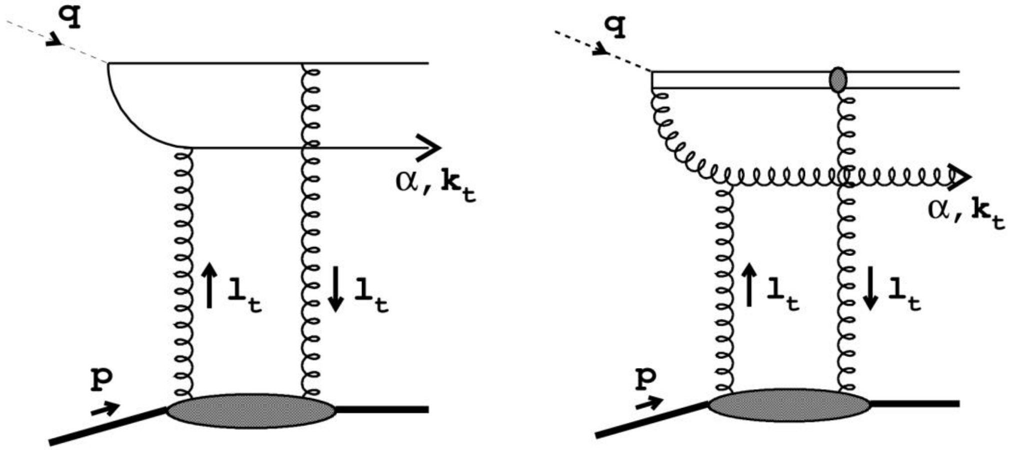
Wusthoff (1997)

Directly compute the cross section (diffractive structure functions)

At large- Q^2, one can identify TMD DPDF in the integrand (except for a factor of 2 discrepancy in the gluon case).

$$F^D_{\{t,q\bar{q}\}}(Q^2,\beta,x_{IP}) = Q^2\pi(1-\beta)\int_0^1 d\alpha(\alpha^2+(1-\alpha)^2)\,\frac{df_q^D(\beta,k_\perp;x_{IP})}{dY_{IP}}$$

$$x_{IP}F^D_{\{t,q\bar{q}g\}}(Q^2,\beta,x_{IP}) = \int_\beta^1 d\xi((1-\xi)^2+\xi^2)\int^{(1-\beta')Q^2}\frac{d^2k_\perp}{k_\perp^2}\frac{\alpha_s}{2\pi^2}\int^{k_\perp^2} d^2k'_\perp x'\,\frac{df_g(\beta',k'_\perp;x_{IP})}{dY_{IP}}$$



More complete calculation

Beuf, Hanninen, Lappi, Mulian, Mantysaari (2022)

2+1-jet production at the EIC

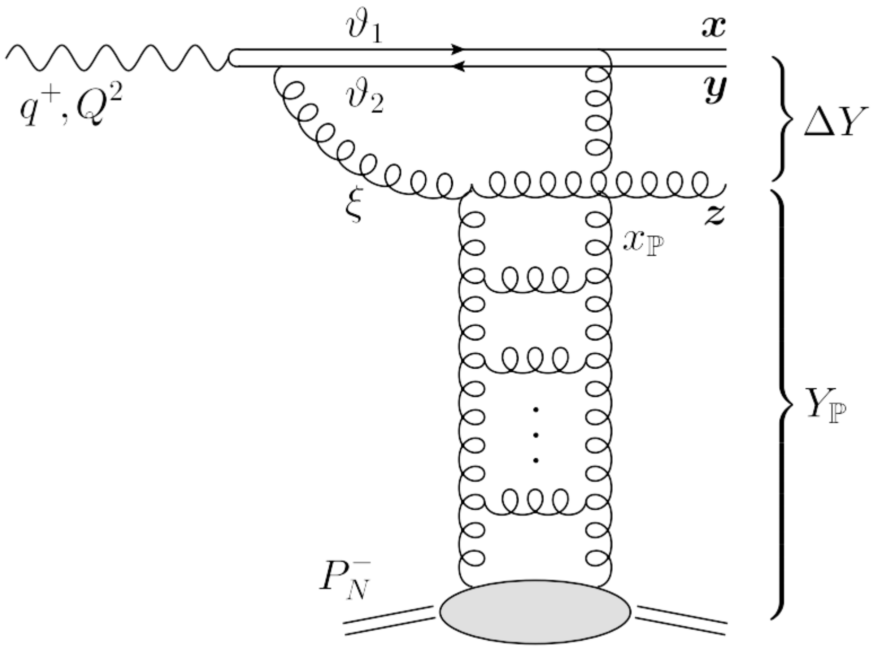
Iancu, Mueller, Triantafyllopoulos (2021)

Hard dijet plus a semi-hard jet production

$$P_\perp \gg K_\perp \sim Q_s$$

still sensitive to gluon saturation even though dijet pT is high.

Factorizes into diffractive gluon TMDPDF



$$\frac{\mathrm{d}\sigma_{\mathrm{D}}^{\gamma_T^* A \to q\bar{q}A' X}}{\mathrm{d}\vartheta_1 \mathrm{d}\vartheta_2 \mathrm{d}^2\boldsymbol{P} \mathrm{d}^2\boldsymbol{K} \mathrm{d}Y_{\mathbb{P}}} = H(x_{q\bar{q}}, Q^2, P_\perp^2) \frac{\mathrm{d}\, xG_{\mathbb{P}}(x, x_{\mathbb{P}}, K_\perp^2)}{\mathrm{d}^2\boldsymbol{K}}$$

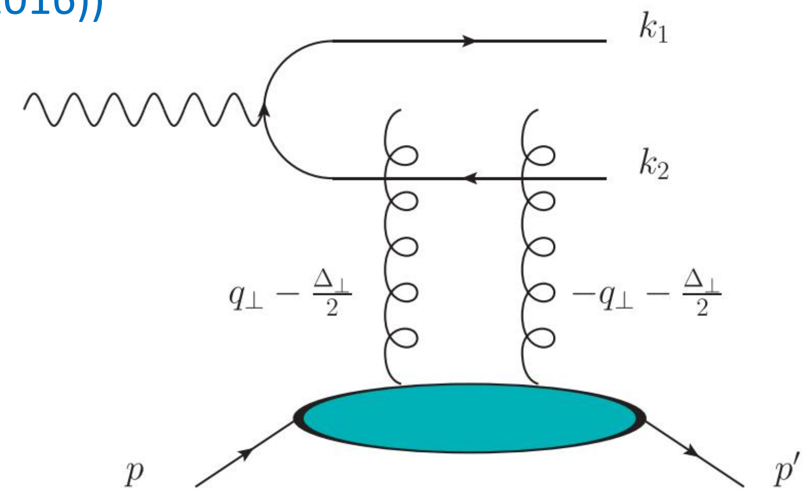
$$\int d^2 K_\perp$$

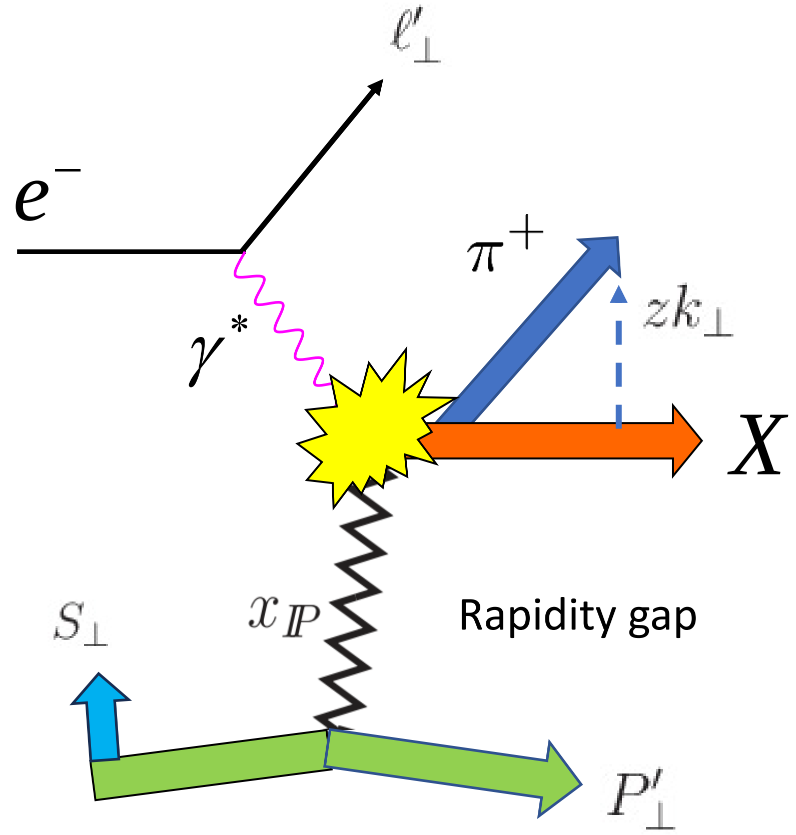
$$\frac{\mathrm{d}\sigma_{D}^{\gamma_T^* A \to q\bar{q}A' X}}{\mathrm{d}\vartheta_1 \mathrm{d}\vartheta_2 \mathrm{d}^2\boldsymbol{P} \mathrm{d}Y_{\mathbb{P}}} = H(x_{q\bar{q}}, Q^2, P_\perp^2)\, xG_{\mathbb{P}}(x, x_{\mathbb{P}}, P_\perp^2)$$

collinear factorization

Semi-inclusive diffractive DIS (SIDDIS)

Start with the cross section for diffractive dijet production
(YH, Xiao, Yuan (2016))





Integrate over the antiquark phase space to get

$$\frac{d\sigma^{\mathrm{SIDDIS}}(\ell p \to \ell' p' q X)}{dx_B dy d^2k_\perp dY_{IP} dt} = \sigma_0 e_q^2 x_B \frac{df_q^D(\beta, k_\perp; x_{IP})}{dY_{IP} dt}$$

Additional vector $P'_\perp$ compared to SIDIS. Can also add spin dependence $S_\perp$
Rich pattern of angular correlations between $P'_\perp, S_\perp, k_\perp, \ell'_\perp$

Conclusions

- Small-x expression of diffractive quark/gluon TMD from the operator definition.
- Modified geometric scaling in terms of $\tilde{Q}_s = \sqrt{1-\beta} Q_s$
- Semi-inclusive diffractive DIS (SIDDIS): new research avenue
- Additional vector $P'_\perp$ compared to SIDIS. Rich pattern of angular correlations between $P'_\perp, S_\perp, k_\perp, \ell'_\perp$