Lattice QCD calculations of transverse momentum-dependent (TMD) observables

Michael Engelhardt

New Mexico State University

In collaboration with:
B. Musch, P. Hägler, J. Negele, A. Schäfer
J. R. Green, N. Hasan, J. Peyton, C. Kallidonis, S. Krieg, S. Meinel, A. Pochinsky, G. Silvi, S. Syritsyn
T. Bhattacharya, R. Gupta, B. Yoon
T. Izubuchi

Fundamental TMD correlator

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots) \equiv \frac{1}{2}\langle P,S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0,\ldots,b]\ q(b)\ |P,S\rangle$$

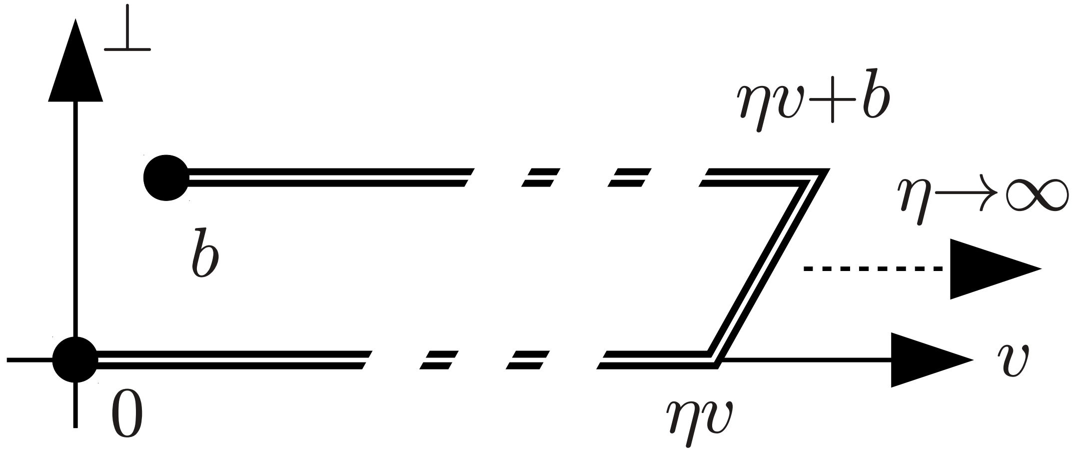
$$\Phi^{[\Gamma]}(x,k_T,P,S,\ldots) \equiv \int \frac{d^2b_T}{(2\pi)^2} \int \frac{d(b\cdot P)}{(2\pi)P^+} \exp\left(ix(b\cdot P) - ib_T\cdot k_T\right) \left.\frac{\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S,\ldots)}{\widetilde{\mathcal{S}}(b^2,\ldots)}\right|_{b^+=0}$$

- "Soft factor" $\widetilde{\mathcal{S}}$ required to subtract divergences of gauge link $\mathcal{U}$
- Will eventually consider ratios in which soft factors cancel

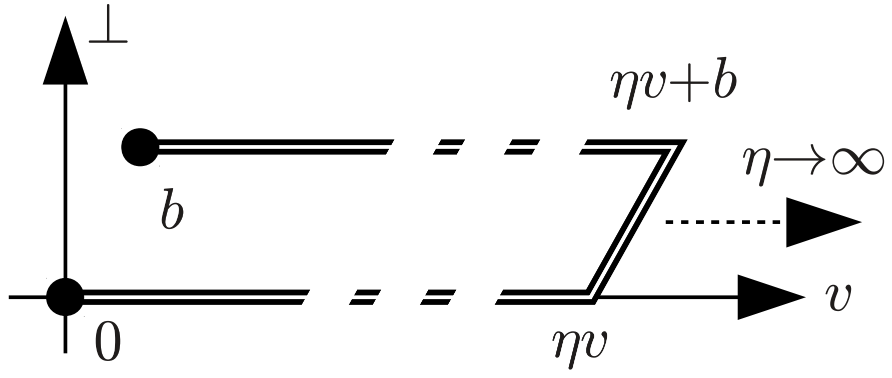
Gauge link structure motivated by factorization of physical process

SIDIS and DY: Staple-shaped gauge link $\mathcal{U}[0, \eta v, \eta v + b, b]$

- Accounts for initial/final state interactions
- Further regularization required!



Gauge link structure motivated by factorization of physical process



Beyond tree level: Rapidity divergences suggest taking staple direction slightly off the light cone. Approach of Aybat, Collins, Qiu, Rogers makes v space-like. Parametrize in terms of Collins-Soper type parameter

$$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

Light-like staple for $\hat{\zeta} \to \infty$. Control power corrections for large $\hat{\zeta}$.

“Modified universality”, $f^{\text{T-odd, SIDIS}} = -f^{\text{T-odd, DY}}$

Fundamental TMD correlator

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots) \equiv \frac{1}{2} \langle P, S | \ \bar{q}(0) \ \Gamma \ \mathcal{U}[0, \eta v, \eta v + b, b] \ q(b) \ | P, S \rangle$$

$$\Phi^{[\Gamma]}(x, k_T, P, S, \ldots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{(2\pi) P^+} \exp\left(ix(b \cdot P) - ib_T \cdot k_T\right) \frac{\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \ldots)}{\widetilde{\mathcal{S}}(b^2, \ldots)} \Bigg|_{b^+ = 0}$$

- "Soft factor" $\widetilde{\mathcal{S}}$ required to subtract divergences of gauge link $\mathcal{U}$
- Will eventually consider ratios in which soft factors cancel

Decomposition of Φ into TMDs

All leading twist structures:

$$\Phi^{[\gamma^+]} = f_1 - \left[\frac{\epsilon_{ij} k_i S_j}{m_H} f_{1T}^{\perp}\right] \text{odd}$$

$$\Phi^{[\gamma^+\gamma^5]} = \Lambda g_1 + \frac{k_T \cdot S_T}{m_H} g_{1T}$$

$$\Phi^{[i\sigma^{i+}\gamma^5]} = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij}) S_j}{2m_H^2} h_{1T}^{\perp} + \frac{\Lambda k_i}{m_H} h_{1L}^{\perp} + \left[\frac{\epsilon_{ij} k_j}{m_H} h_1^{\perp}\right] \text{odd}$$

TMD Classification

All leading twist structures:

$q \rightarrow$ H $\downarrow$	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

$\longleftarrow$ Boer-Mulders
(T-odd)

$\uparrow$
Sivers (T-odd)

Decomposition of $\widetilde{\Phi}$ into amplitudes

$$\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$$

Decompose in terms of invariant amplitudes; at leading twist,

$$\begin{aligned}
\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+]}_{\text{unsubtr.}} &= \widetilde{A}_{2B} + im_H\epsilon_{ij}b_iS_j\widetilde{A}_{12B} \\
\frac{1}{2P^+}\widetilde{\Phi}^{[\gamma^+\gamma^5]}_{\text{unsubtr.}} &= -\Lambda\widetilde{A}_{6B} + i[(b\cdot P)\Lambda - m_H(b_T\cdot S_T)]\widetilde{A}_{7B} \\
\frac{1}{2P^+}\widetilde{\Phi}^{[i\sigma^{i+}\gamma^5]}_{\text{unsubtr.}} &= im_H\epsilon_{ij}b_j\widetilde{A}_{4B} - S_i\widetilde{A}_{9B} \\
&\quad -im_H\Lambda b_i\widetilde{A}_{10B} + m_H[(b\cdot P)\Lambda - m_H(b_T\cdot S_T)]b_i\widetilde{A}_{11B}
\end{aligned}$$

(Decompositions analogous to work by Metz et al. in momentum space)

Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \ldots) \equiv \int d^2k_T \, \exp(ib_T \cdot k_T) f(x, k_T^2, \ldots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \ldots) \equiv n! \left(-\frac{2}{m_H^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \ldots)$$

Formally, in limit $|b_T| \to 0$, recover k_T-moments:

$$\tilde{f}^{(n)}(x, 0, \ldots) \equiv \int d^2k_T \left(\frac{k_T^2}{2m_H^2} \right)^n f(x, k_T^2, \ldots) \equiv f^{(n)}(x)$$

CAREFUL: Ill-defined for large k_T, so, for now, will not attempt to extrapolate to $b_T = 0$, but give results at finite $|b_T|$.

$\longrightarrow$ Bessel-weighted asymmetries (Boer, Gamberg, Musch, Prokudin, JHEP 1110 (2011) 021)

Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \ldots) \equiv \int d^2k_T \, \exp(ib_T \cdot k_T) f(x, k_T^2, \ldots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \ldots) \equiv n! \left(-\frac{2}{m_H^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \ldots)$$

Also, for now, concentrate mostly on first Mellin moments (accessible at $b \cdot P = 0$), rather than scanning range of $b \cdot P$:

$$f^{[1]}(k_T^2, \ldots) \equiv \int_{-1}^{1} dx \, f(x, k_T^2, \ldots)$$

AGAIN, CAREFUL: Matching factors between unsubtracted/renormalized TMDs may depend on x

Relation between Fourier-transformed TMDs and invariant amplitudes $\widetilde{A}_i$

Invariant amplitudes directly give selected x-integrated TMDs in Fourier (b_T) space (showing just the ones relevant for Sivers, Boer-Mulders shifts), up to soft factors:

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{f}_{1T}^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = -2\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

$$\tilde{h}_1^{\perp[1](1)}(b_T^2, \hat{\zeta}, \ldots, \eta v \cdot P) = 2\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\widetilde{S}(b^2, \ldots)$$

Etc.

Generalized shifts

Form ratios in which soft factors, (Γ-independent) multiplicative renormalization factors cancel

Sivers shift:

$$\langle k_y \rangle_{TU} \equiv m_N \frac{f_{1T}^{\perp [1](1)}}{f_1^{[1](0)}} = \frac{\int dx \int d^2 k_T \, k_y \Phi^{[\gamma^+]}(x, k_T, S_T = (1,0))}{\int dx \int d^2 k_T \, \Phi^{[\gamma^+]}(x, k_T, S_T = (1,0))}$$

Average transverse momentum of unpolarized ("U") quarks orthogonal to the transverse ("T") spin of nucleon; normalized to the number of valence quarks. "Dipole moment" in $b_T^2 = 0$ limit, "shift".

Issue: k_T-moments in this ratio singular; generalize to ratio of Fourier-transformed TMDs at *nonzero* b_T^2,

$$\langle k_y \rangle_{TU}(b_T^2, \ldots) \equiv m_N \frac{\tilde{f}_{1T}^{\perp [1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)}$$

(remember singular $b_T \to 0$ limit corresponds to taking k_T-moment). "Generalized shift".

Generalized shifts from amplitudes

Now, can also express this in terms of invariant amplitudes:

$$\langle k_y \rangle_{TU}(b_T^2, \ldots) \equiv m_N \frac{\tilde{f}_{1T}^{\perp [1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)} = -m_N \frac{\widetilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Analogously, Boer-Mulders shift:

$$\langle k_y \rangle_{UT}(b_T^2, \ldots) = m_N \frac{\widetilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

g_{1T} worm-gear shift:

$$\langle k_x \rangle_{TL}(b_T^2, \ldots) = -m_N \frac{\widetilde{A}_{7B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Generalized tensor charge (no k-weighting) :

$$\frac{\tilde{h}_1^{[1](0)}}{\tilde{f}_1^{[1](0)}} = -\frac{\widetilde{A}_{9B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P) - (m_N^2 b^2/2)\widetilde{A}_{11B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Generalized shifts from amplitudes

$h_{1L}^{\perp}$ worm gear shift:

$$\langle k_x \rangle_{LT}(b_T^2, \ldots) \equiv m_N \frac{\tilde{h}_{1L}^{\perp[1](1)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)} = -m_N \frac{\widetilde{A}_{10B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

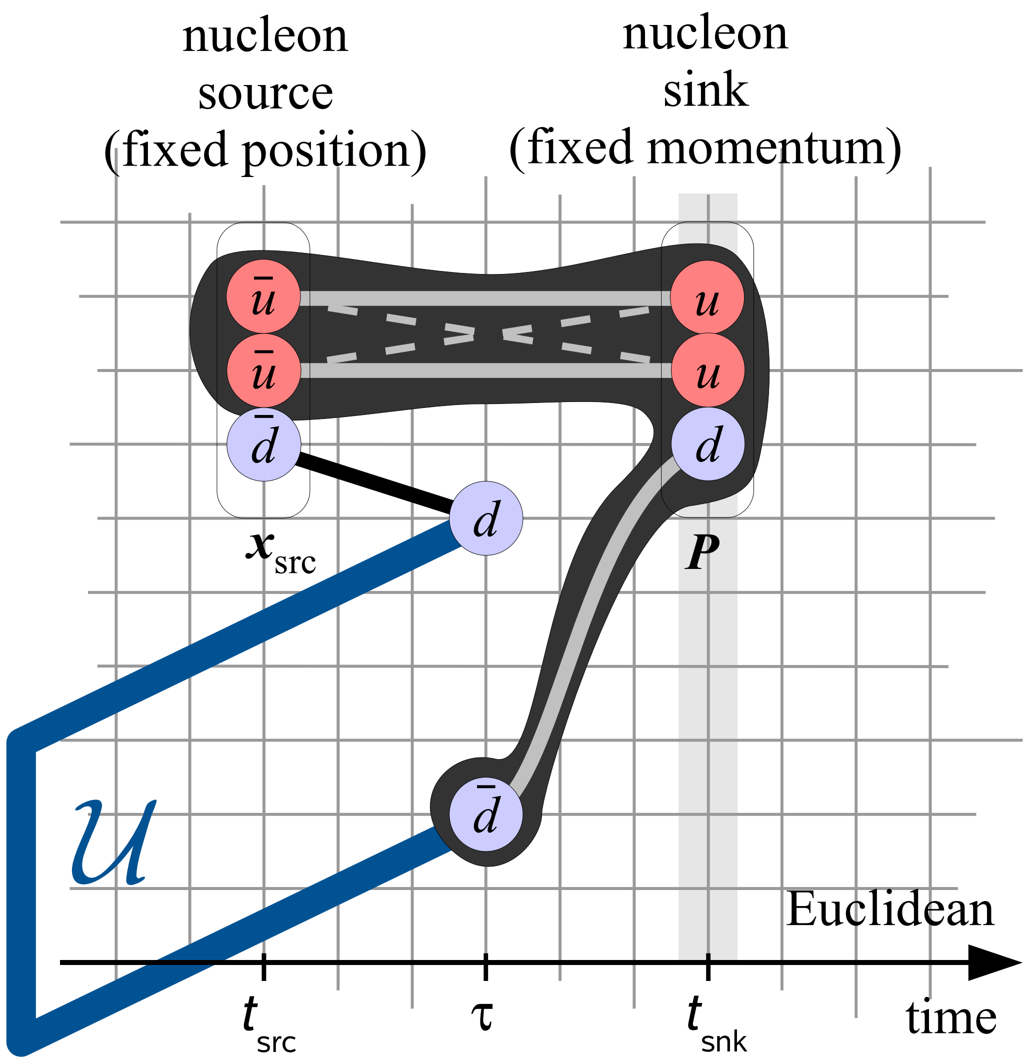
Generalized axial charge (no k_T weighting) :

$$\Delta\Sigma(b_T^2, \ldots) = \frac{\tilde{g}_1^{[1](0)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)} = -\frac{\widetilde{A}_{6B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Twist-3 generalized scalar charge (no k_T weighting) :

$$\frac{\tilde{e}^{[1](0)}(b_T^2, \ldots)}{\tilde{f}_1^{[1](0)}(b_T^2, \ldots)} = \frac{\widetilde{A}_1(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\widetilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Lattice setup



- Evaluate directly $\widetilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b, P, S, \hat{\zeta}, \mu)$

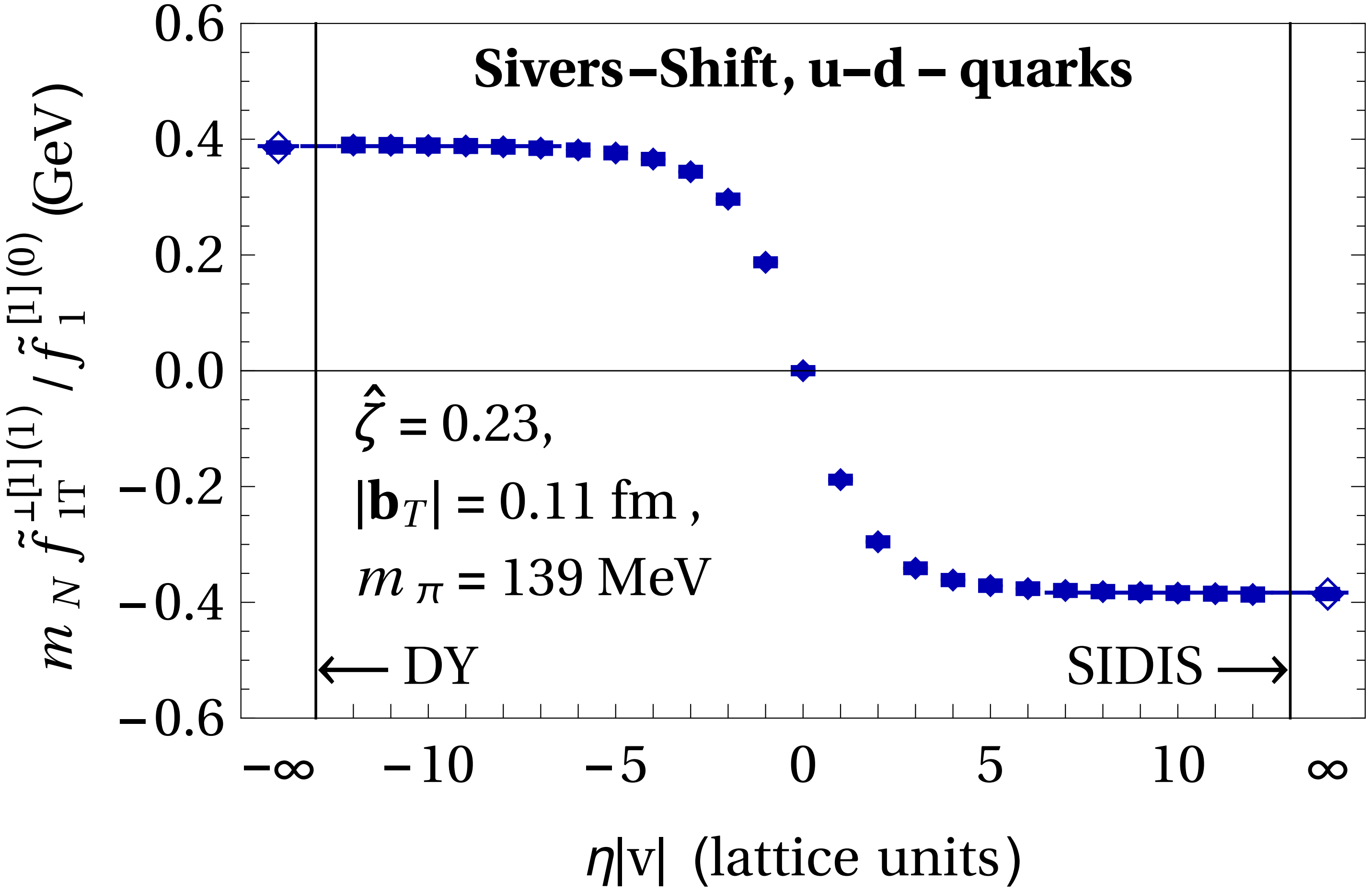
  $$\equiv \frac{1}{2}\langle P, S|\ \bar{q}(0)\ \Gamma\ \mathcal{U}[0, \eta v, \eta v + b, b]\ q(b)\ |P, S\rangle$$

- Euclidean time: Place entire operator at one time slice, i.e., b, ηv purely spatial
- Since generic b, v space-like, no obstacle to boosting system to such a frame!
- Parametrization of correlator in terms of $\widetilde{A}_i$ invariants permits direct translation of results back to original frame; form desired $\widetilde{A}_i$ ratios.
- Use variety of P, b, ηv; here $b \perp P$, $b \perp v$ (lowest x-moment, kinematical choices/constraints)
- Extrapolate $\eta \to \infty$, $\hat{\zeta} \to \infty$ numerically.

Data in the following obtained using a domain wall fermion ensemble at the physical pion mass

Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



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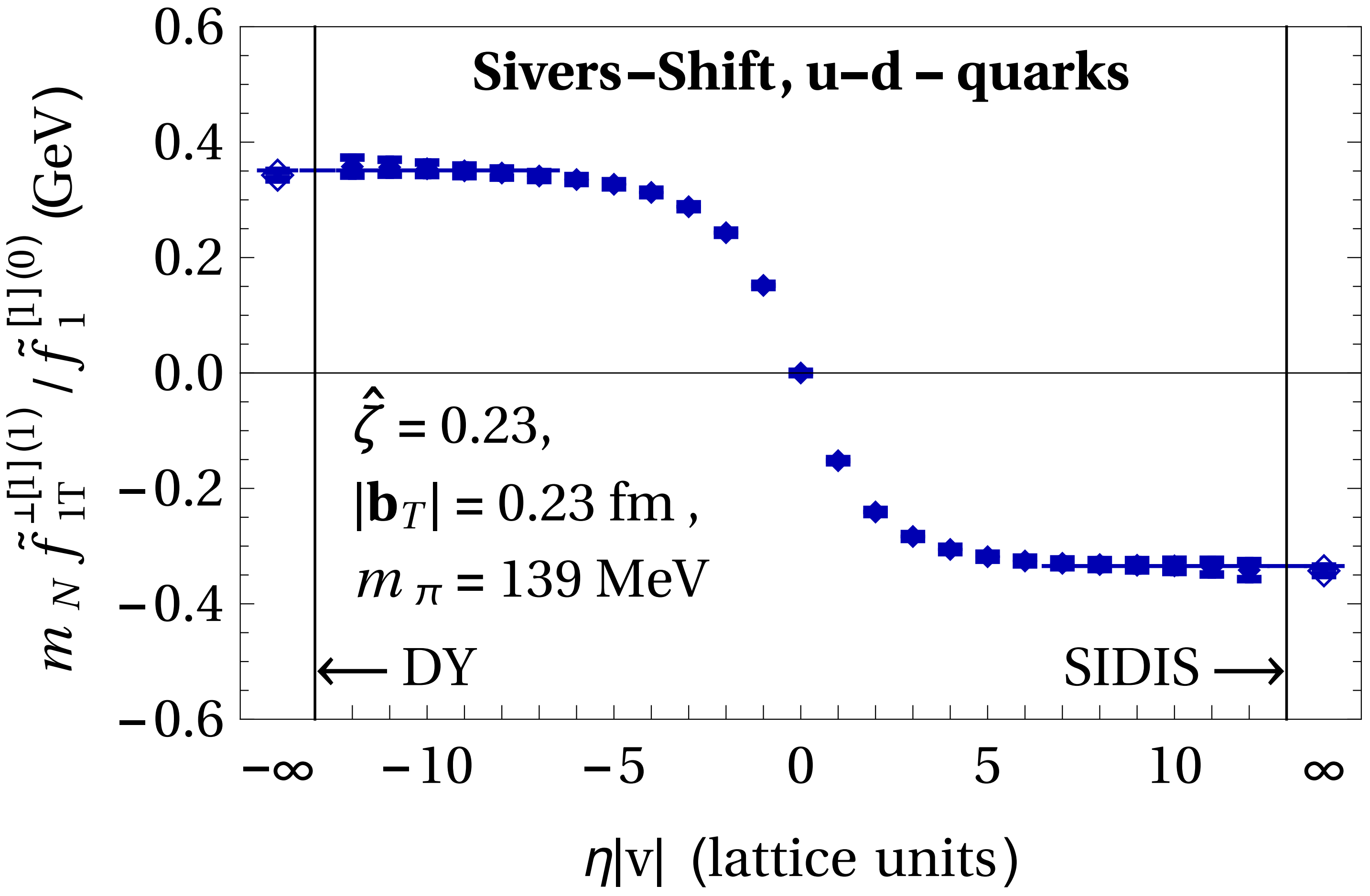

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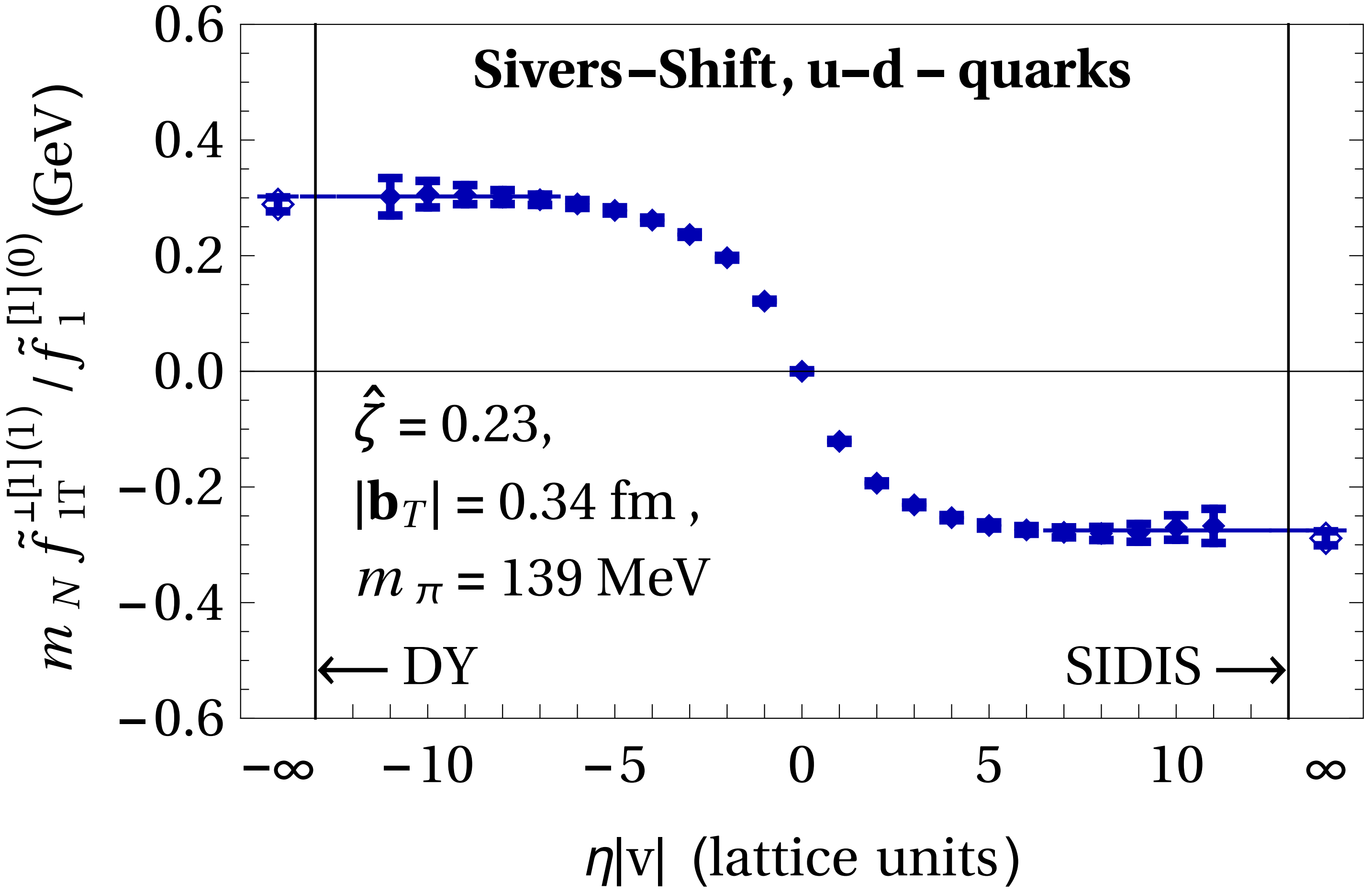

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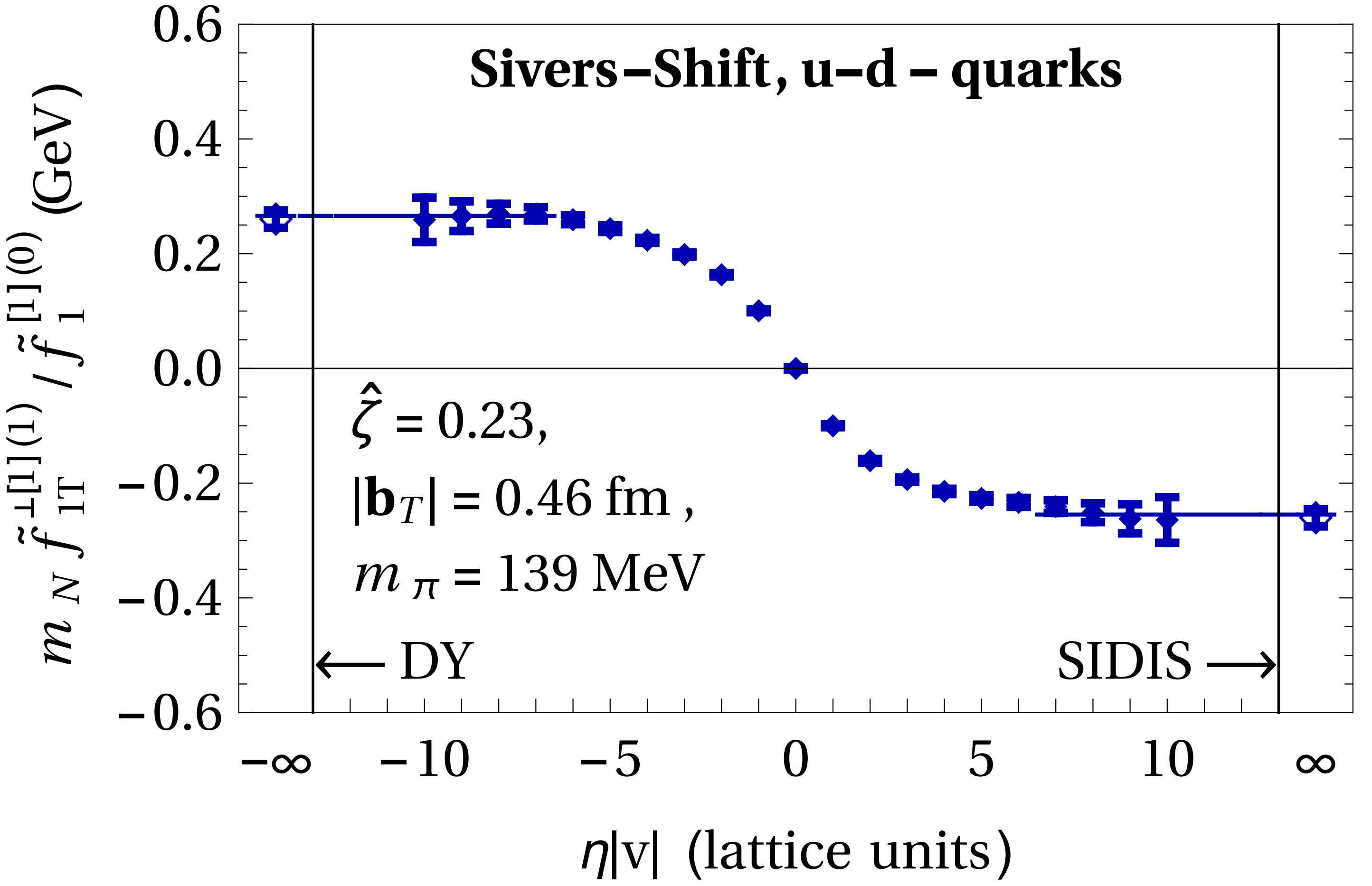
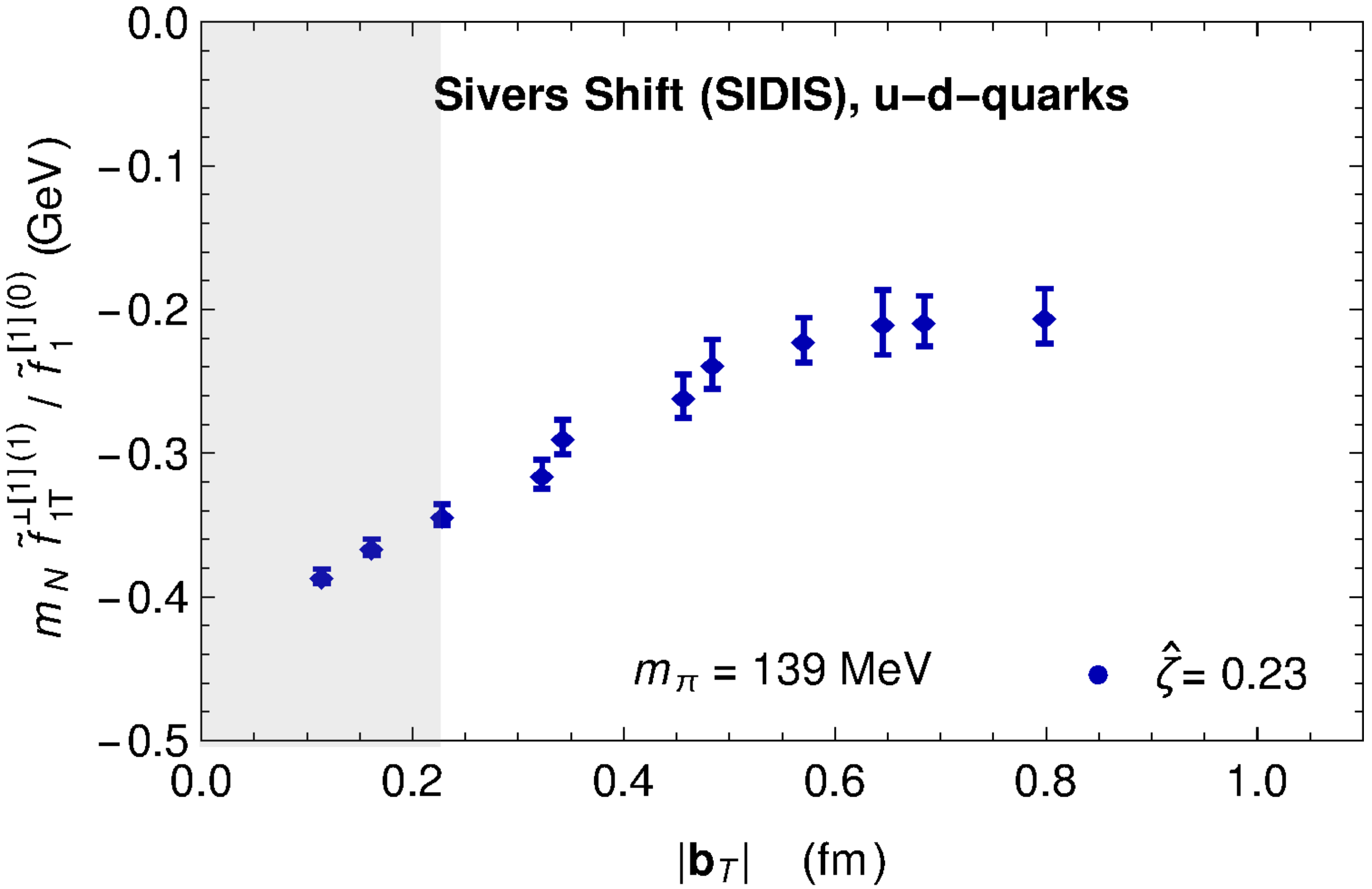


Results: Sivers shift

Dependence of SIDIS limit on $|b_T|$



Results: Sivers shift

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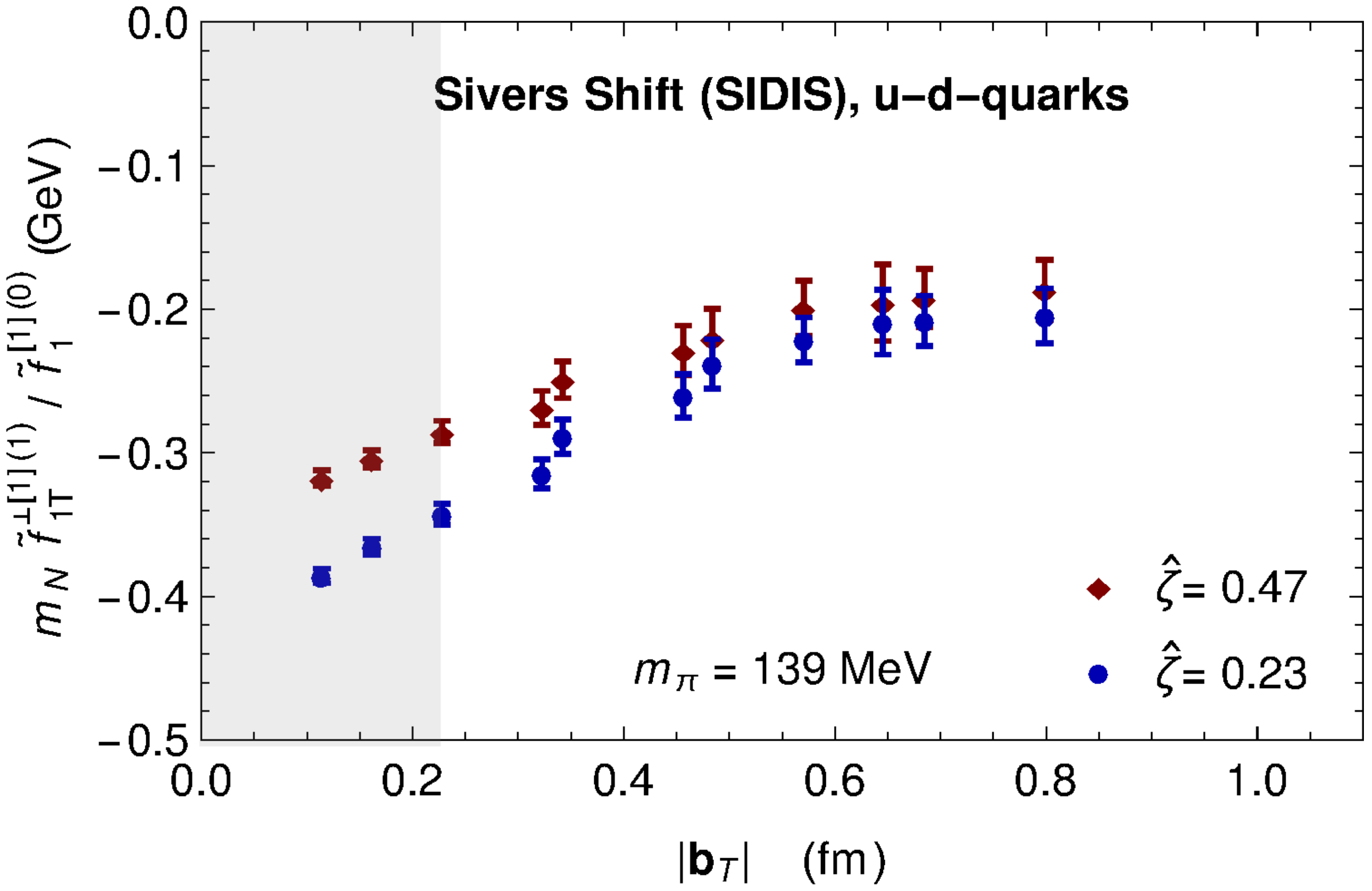
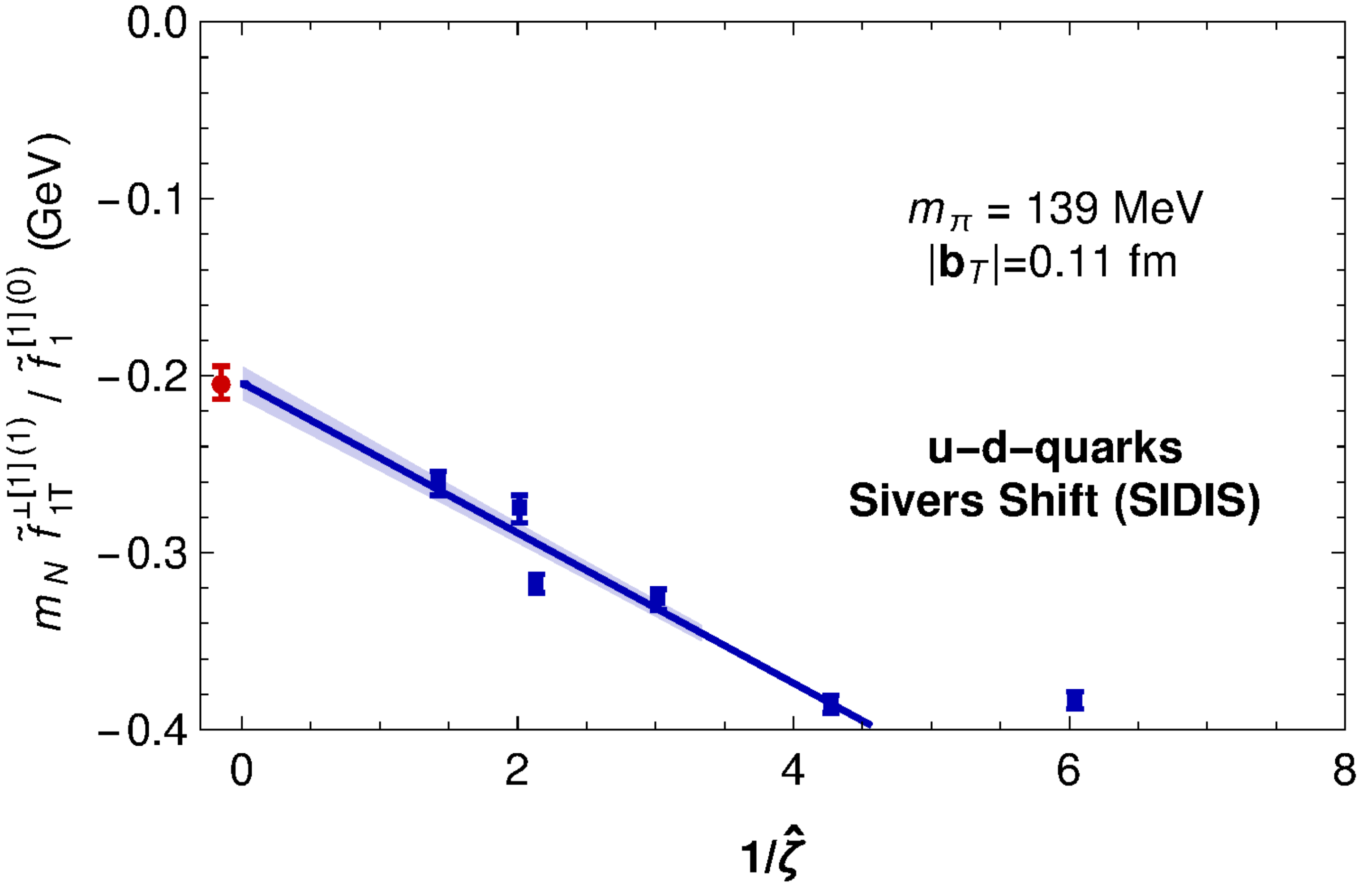


Results: Sivers shift

Extrapolation in $\hat{\zeta}$ for given $|b_T|$ – SIDIS limit



Results: Sivers shift

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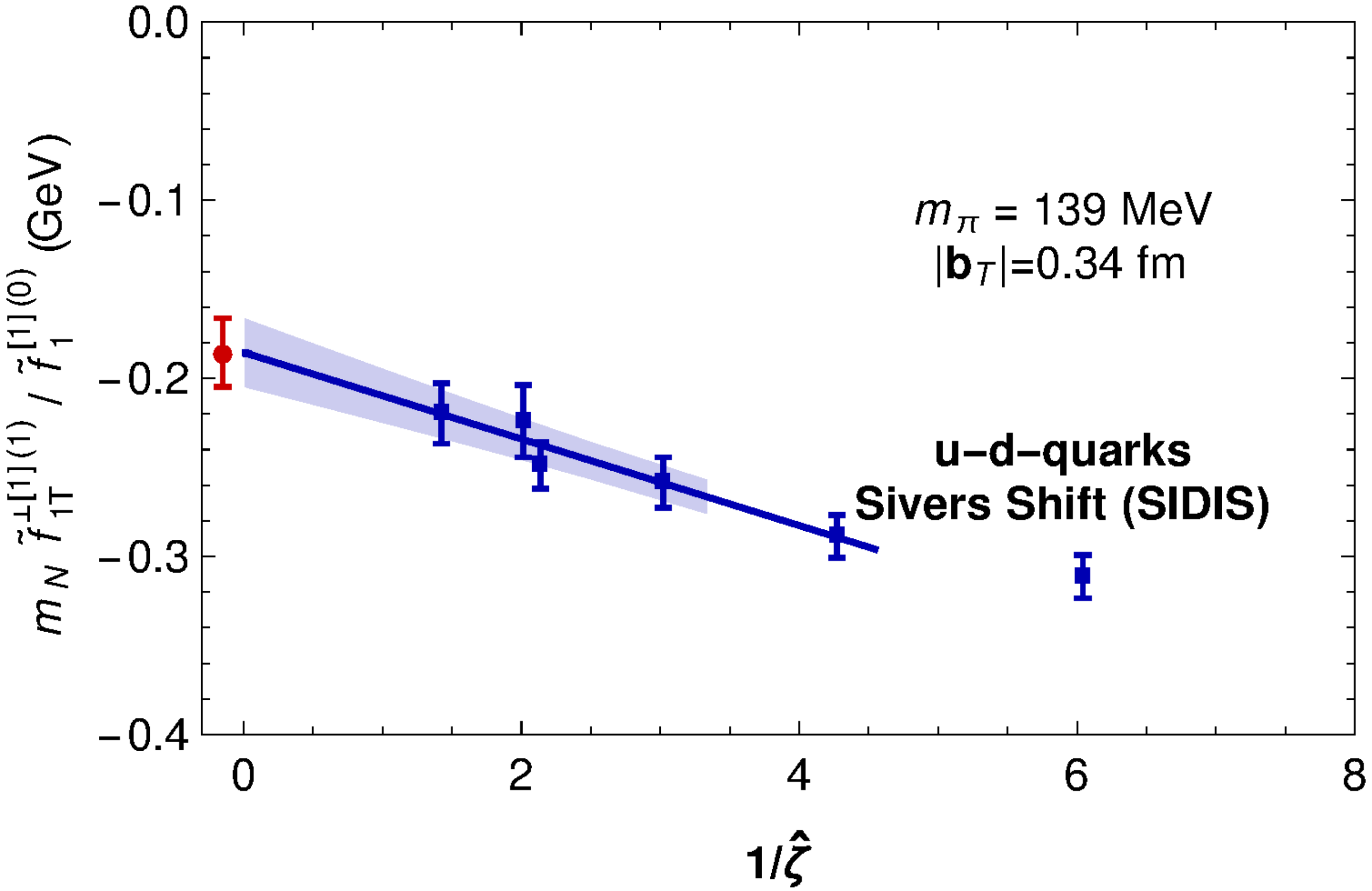

Results: Sivers shift

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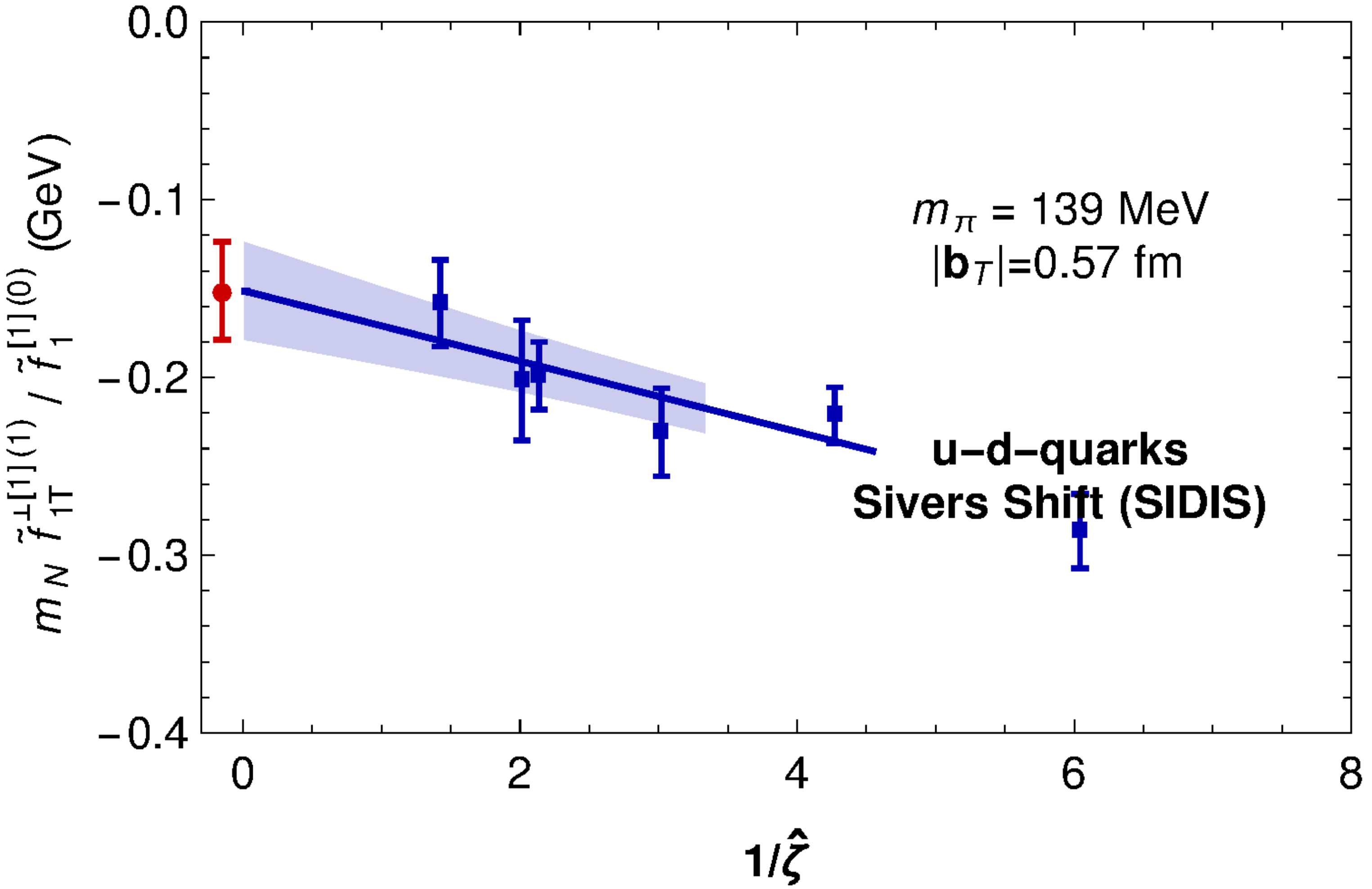

Results: Sivers shift

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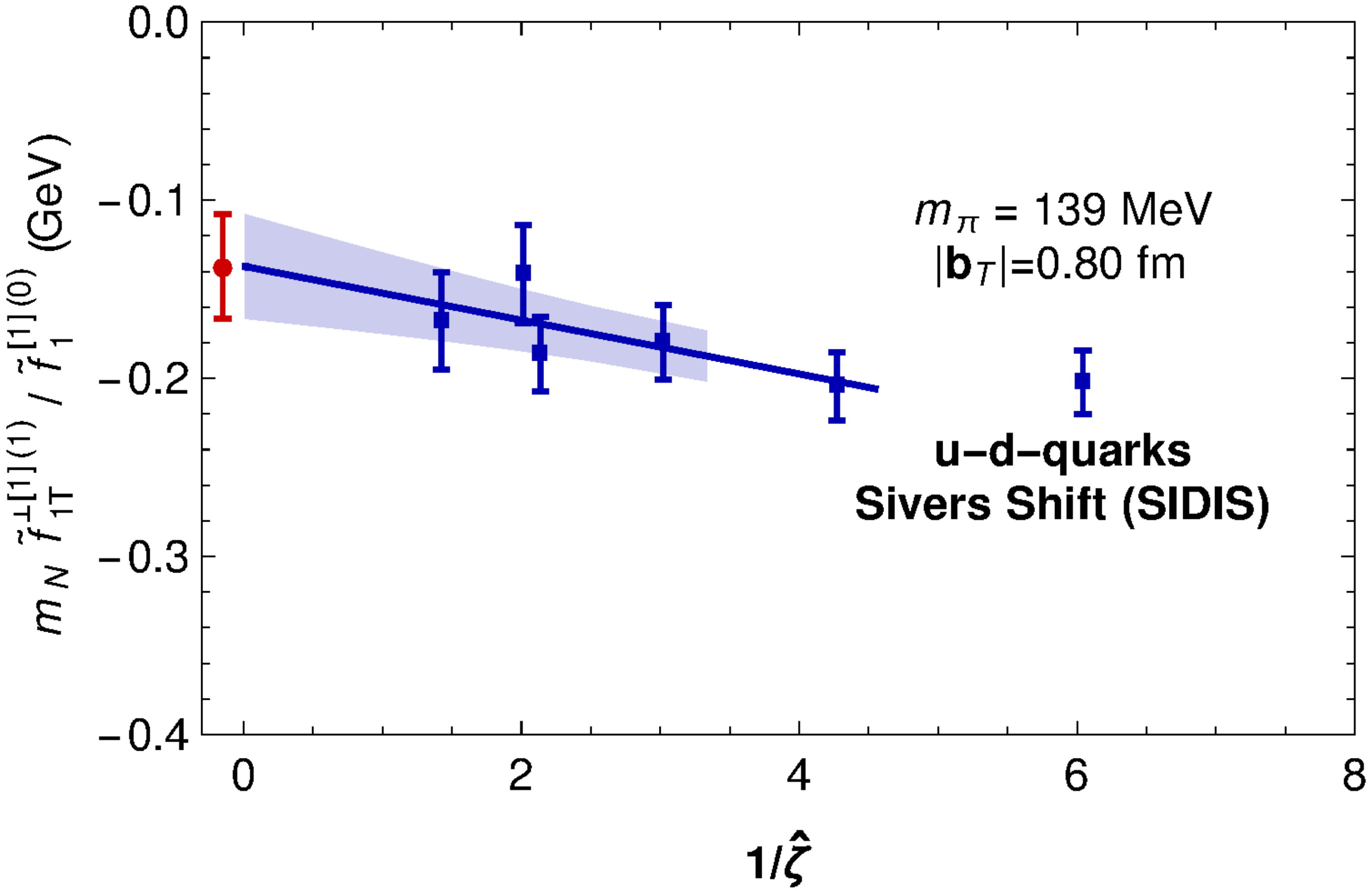

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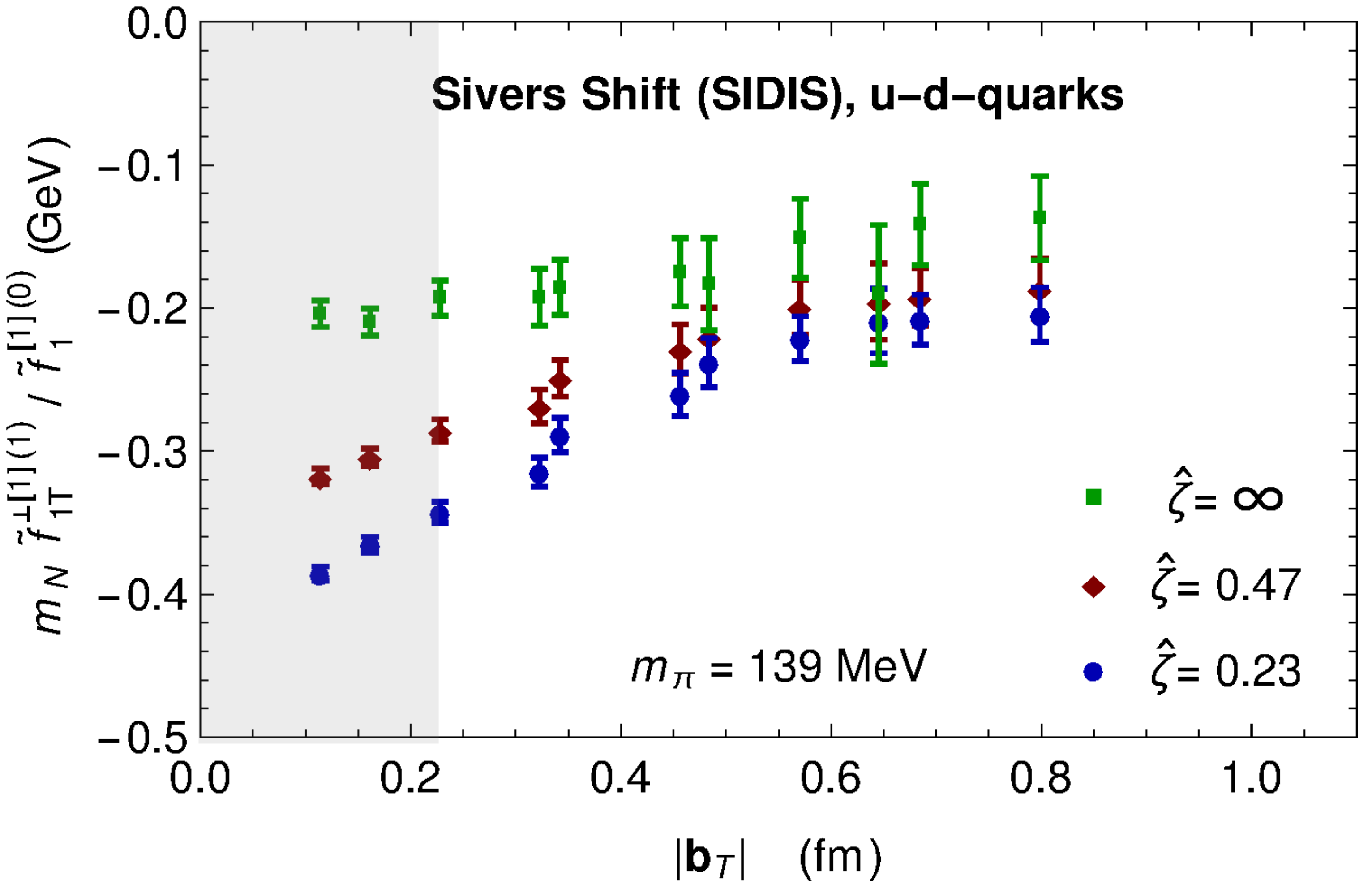
Dependence of SIDIS limit on $|b_T|$

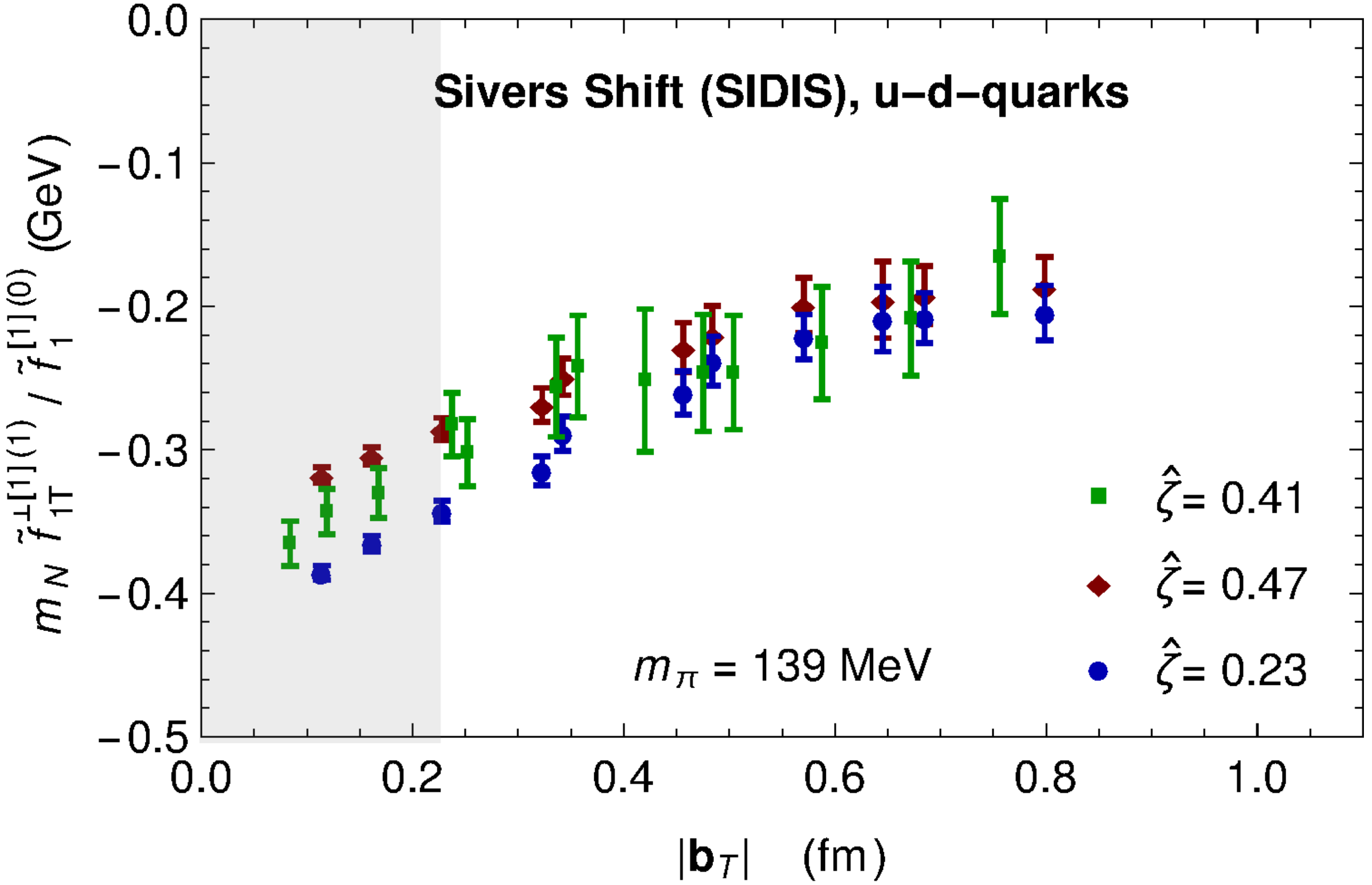


Experimental value from global fit to HERMES, COMPASS and JLab data,
M. Echevarria, A. Idilbi, Z.-B. Kang and I. Vitev, Phys. Rev. D 89 (2014) 074013:

$$\langle k_y \rangle_{TU} = -0.146(49)$$

Results: Sivers shift

Digression: Comparison with result at heavier quark mass – Dependence of SIDIS limit on $|b_T|$

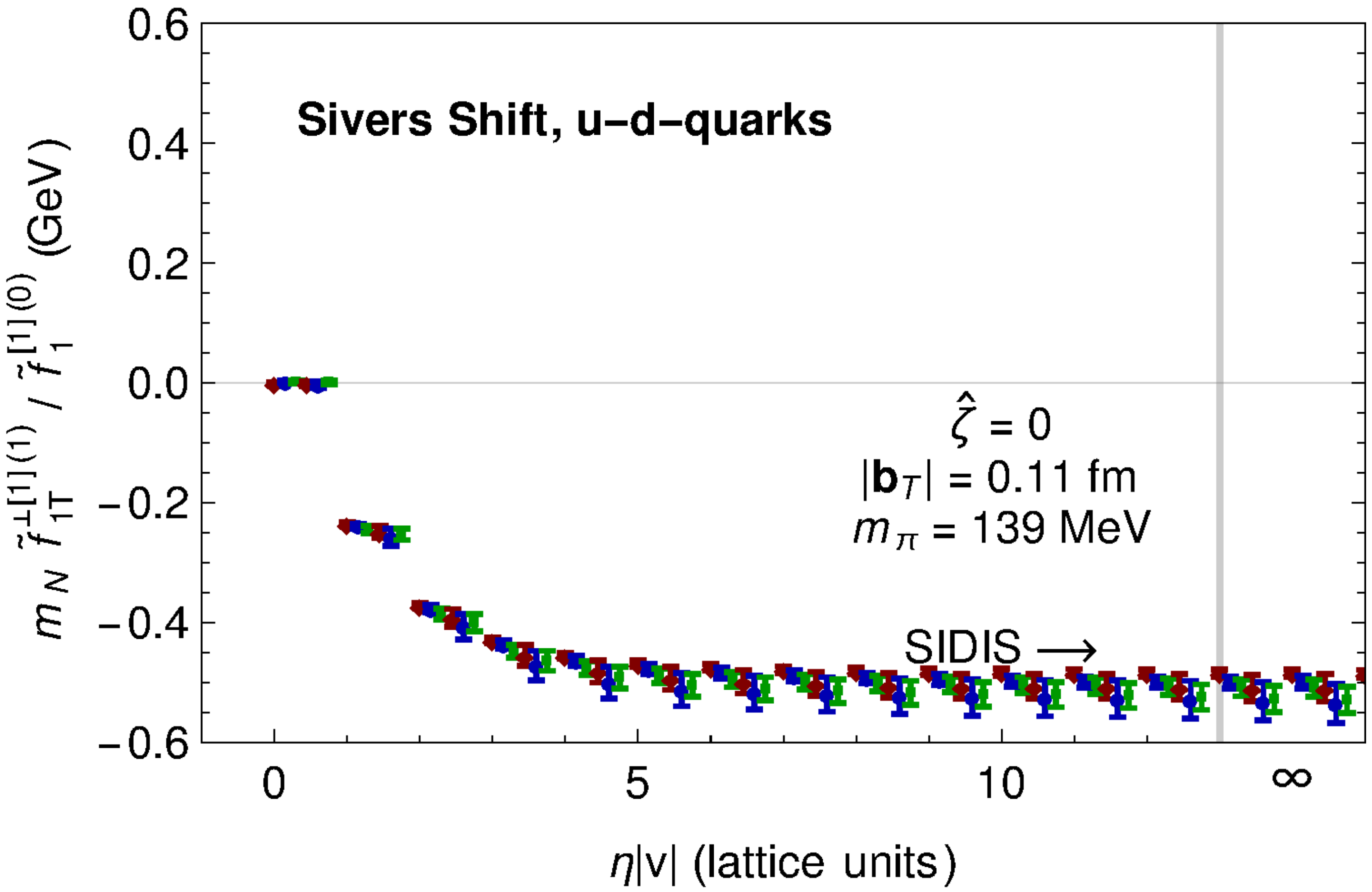


(Green data points are calculated at $m_\pi = 300\,\mathrm{MeV}$)

Results: Sivers shift

Digression: Excited state contaminations – Dependence on staple extent

Data obtained for $t_{snk} - t_{src} = 8, 9, 10, 11, 12$, and two-state fit



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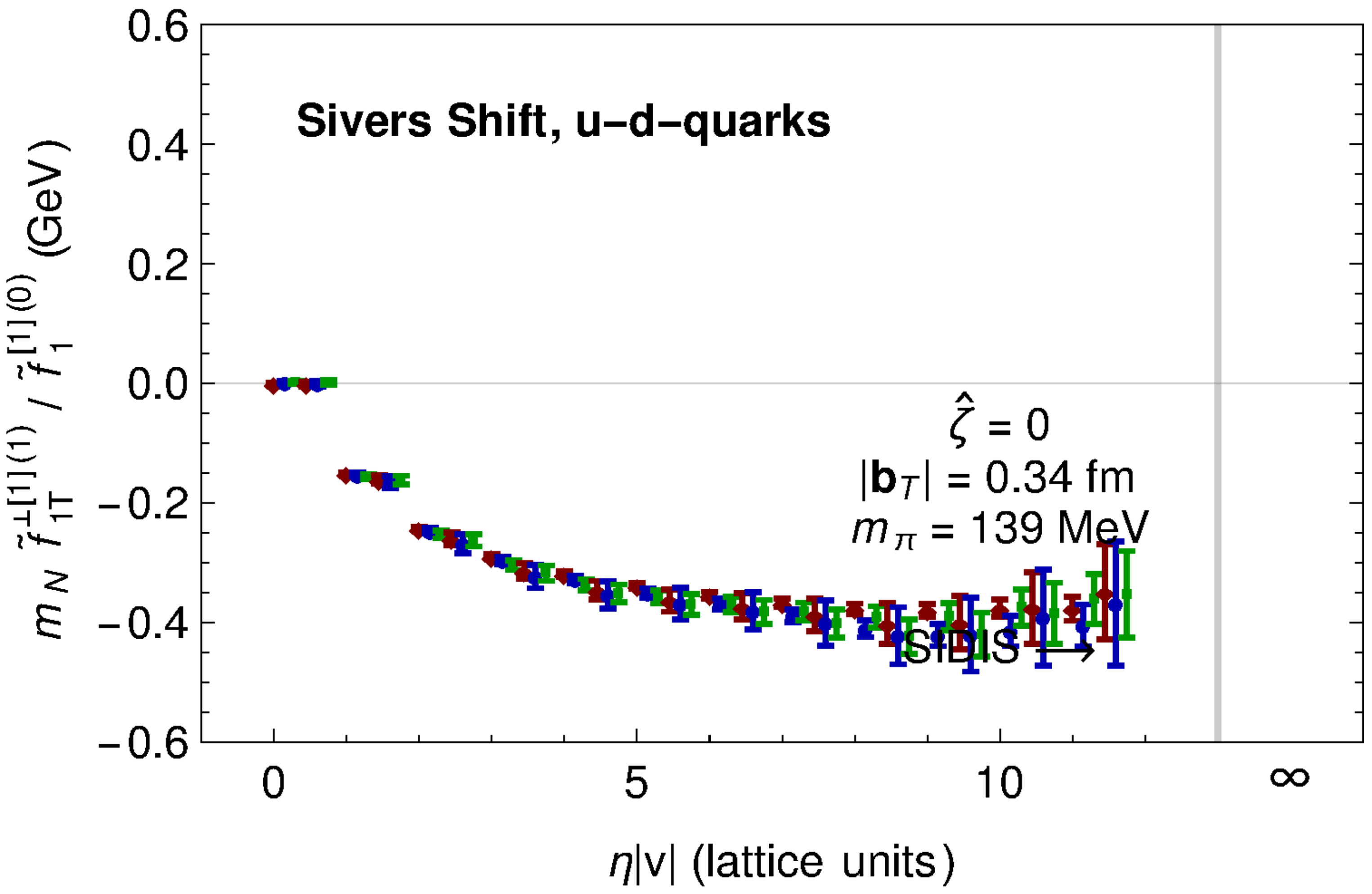

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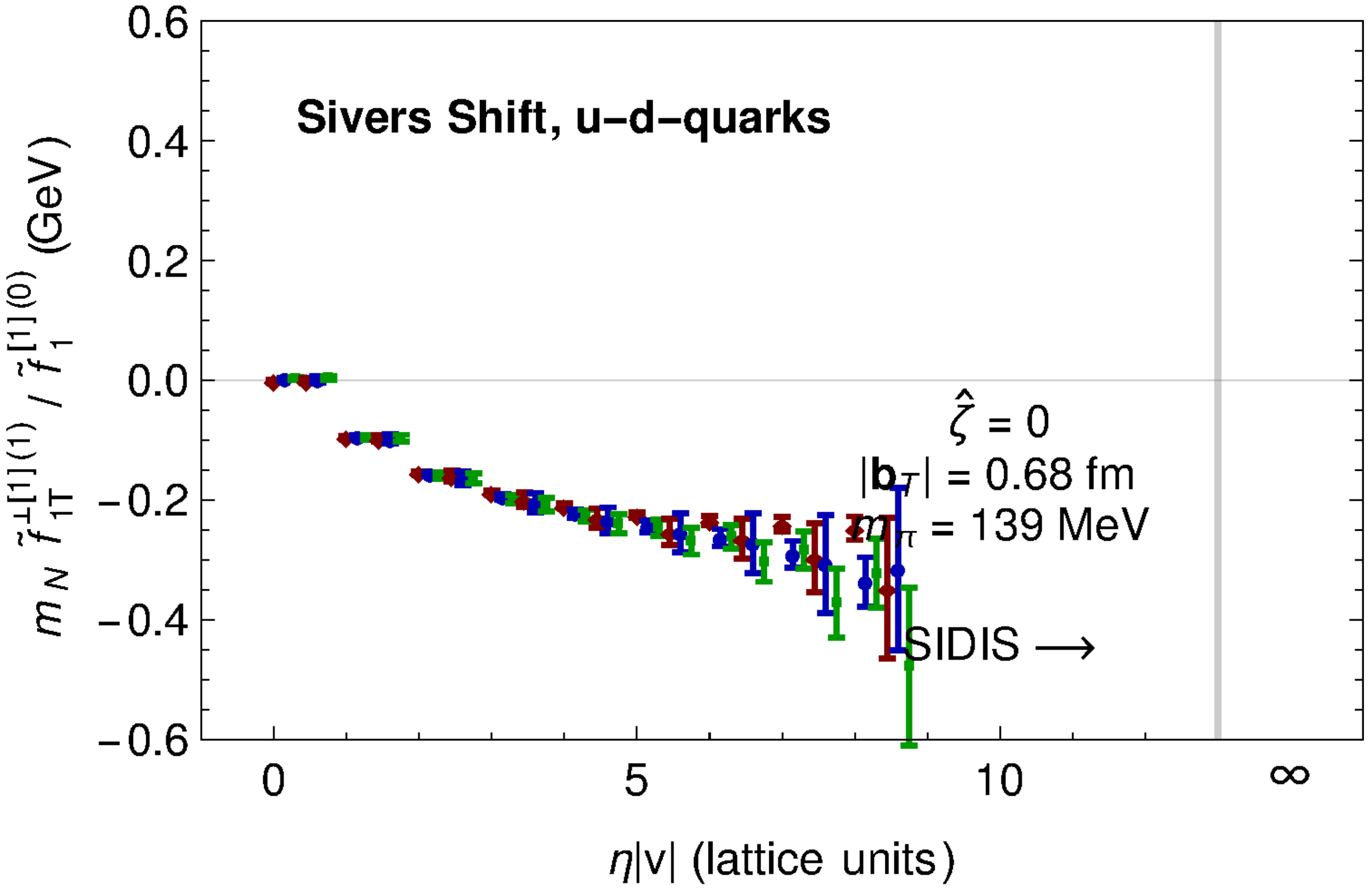
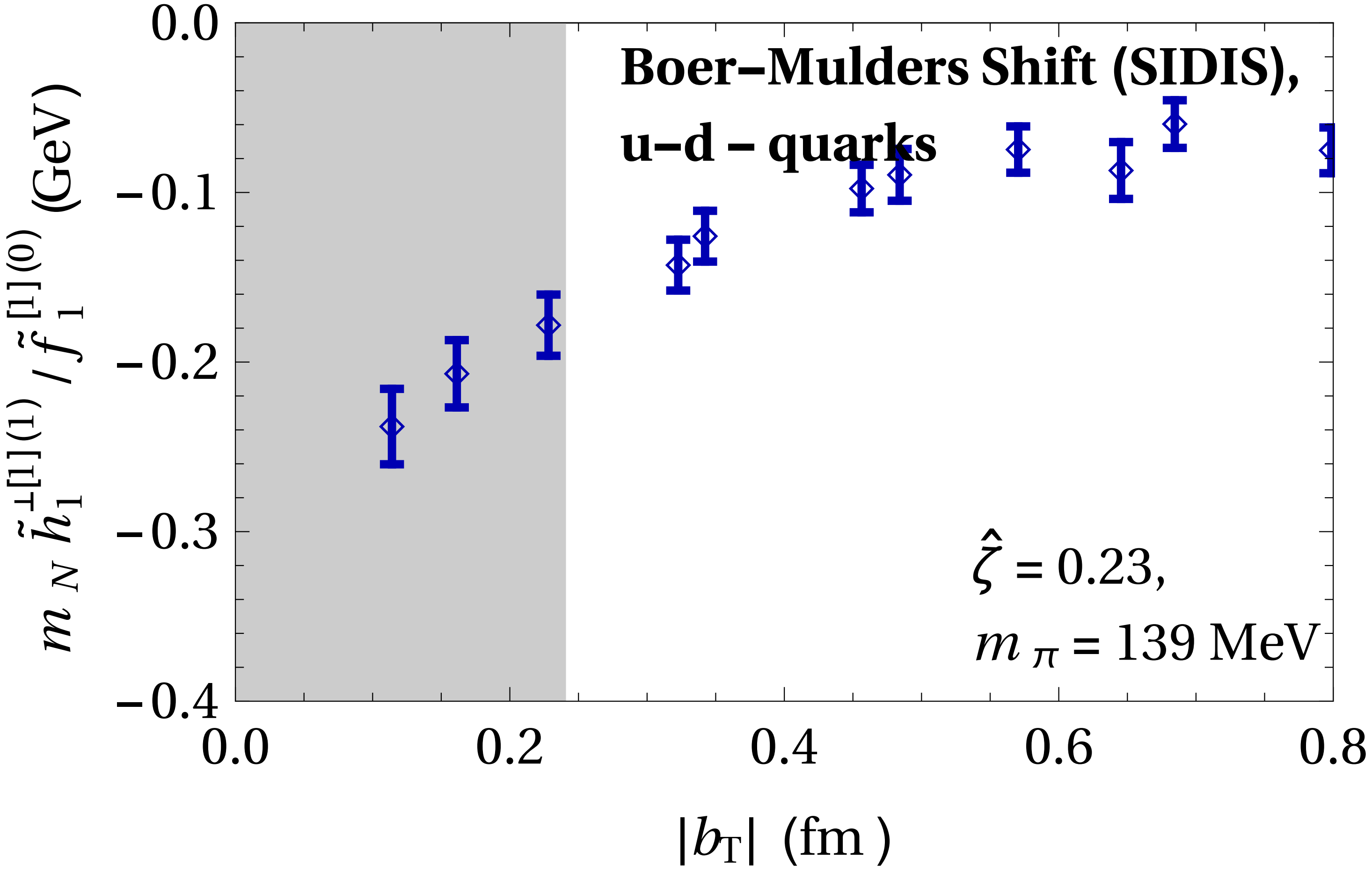


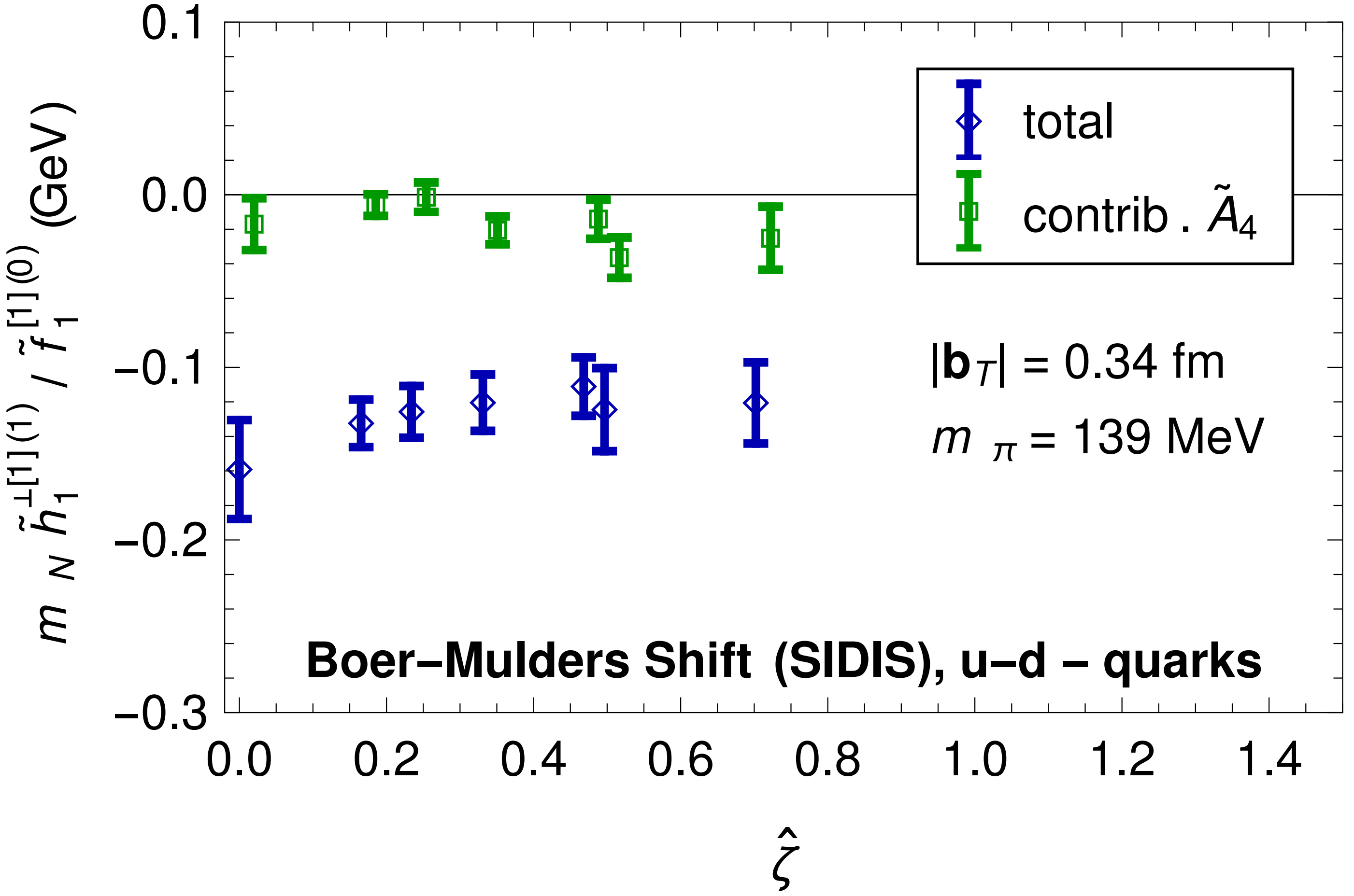
Results: Boer-Mulders shift

Dependence of SIDIS limit on $|b_T|$



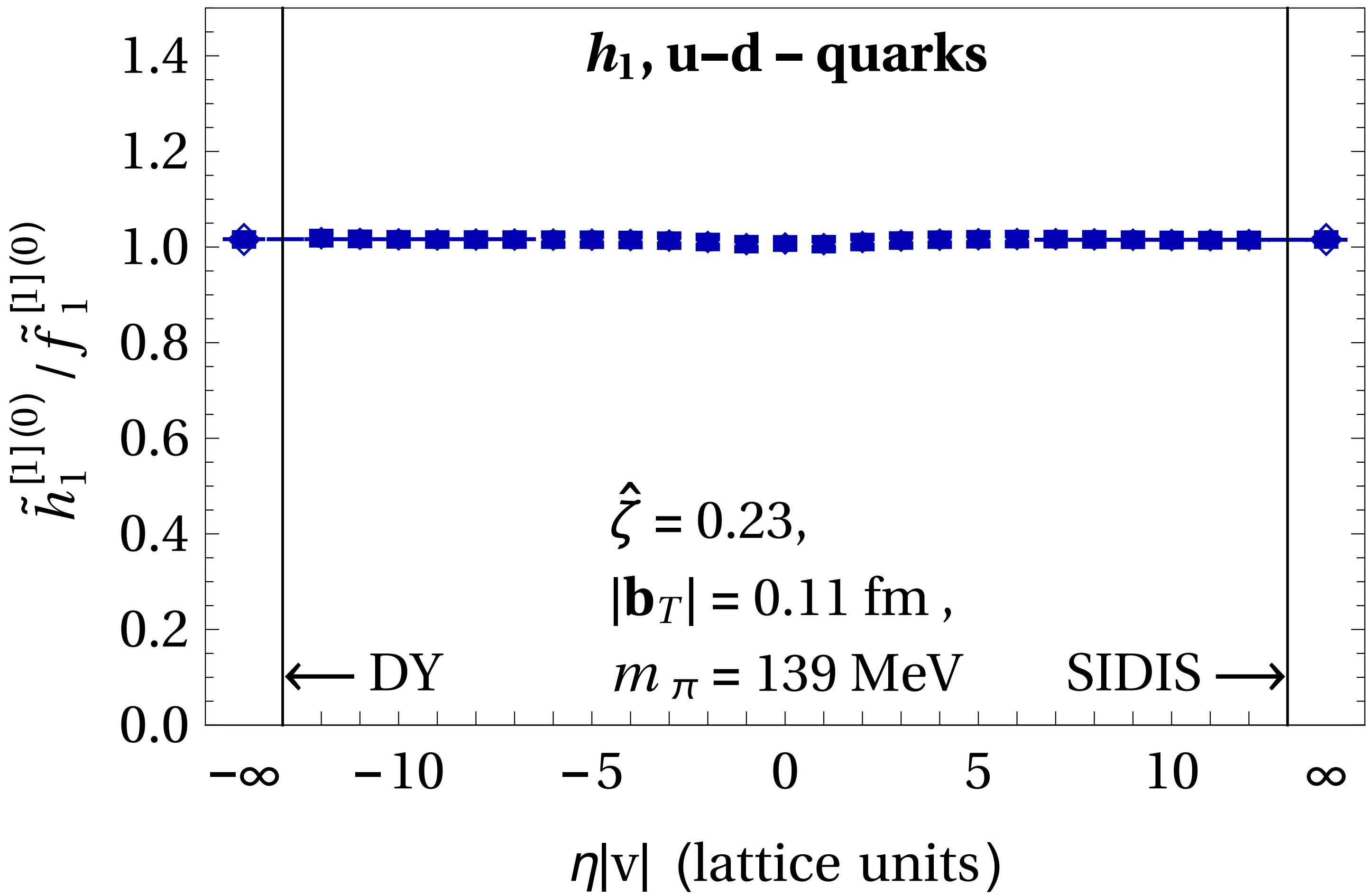
Results: Boer-Mulders shift

Dependence of SIDIS limit on $\hat{\zeta}$



Results: Transversity

Dependence on staple extent; sequence of panels at different $|b_T|$



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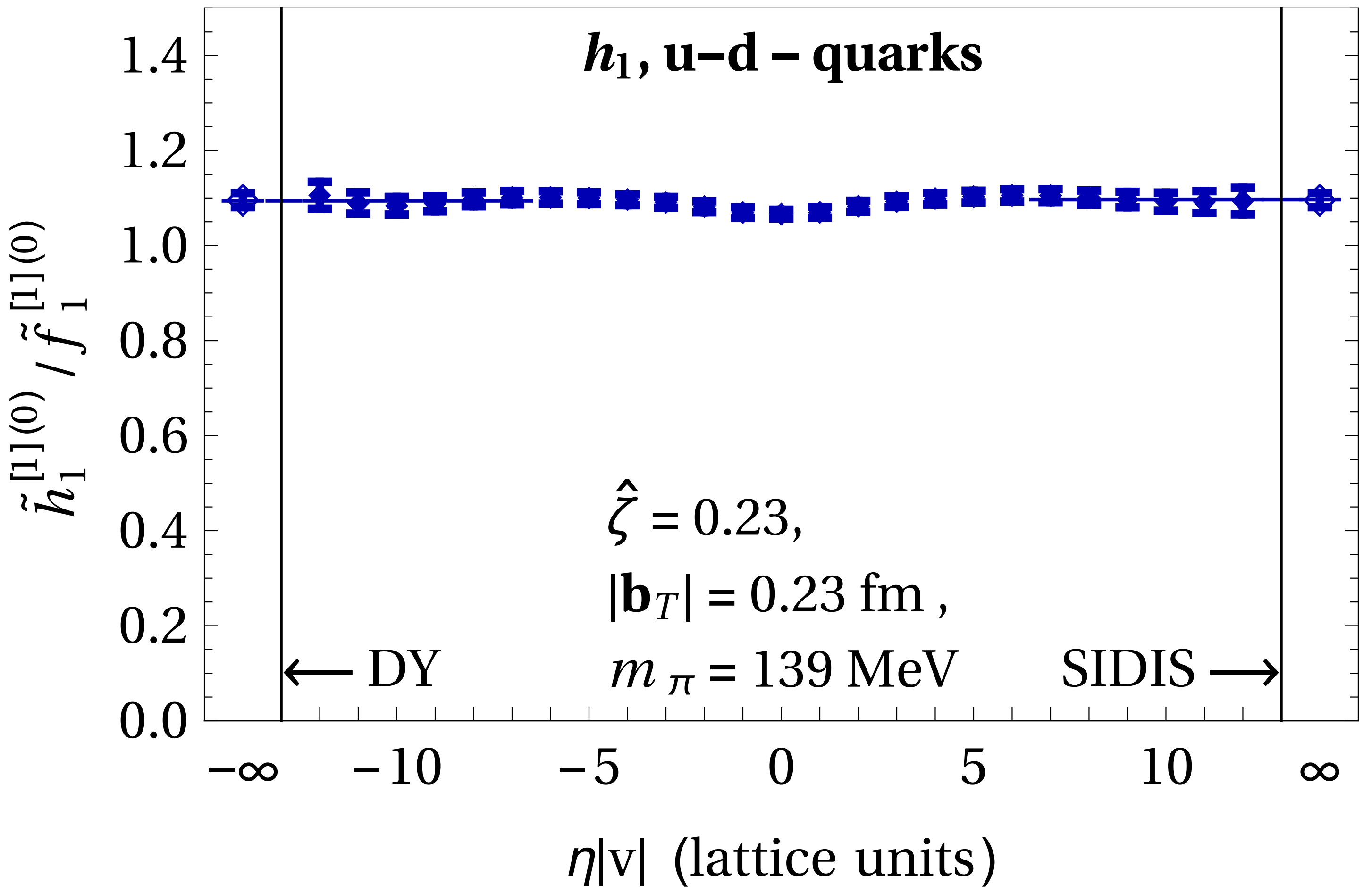

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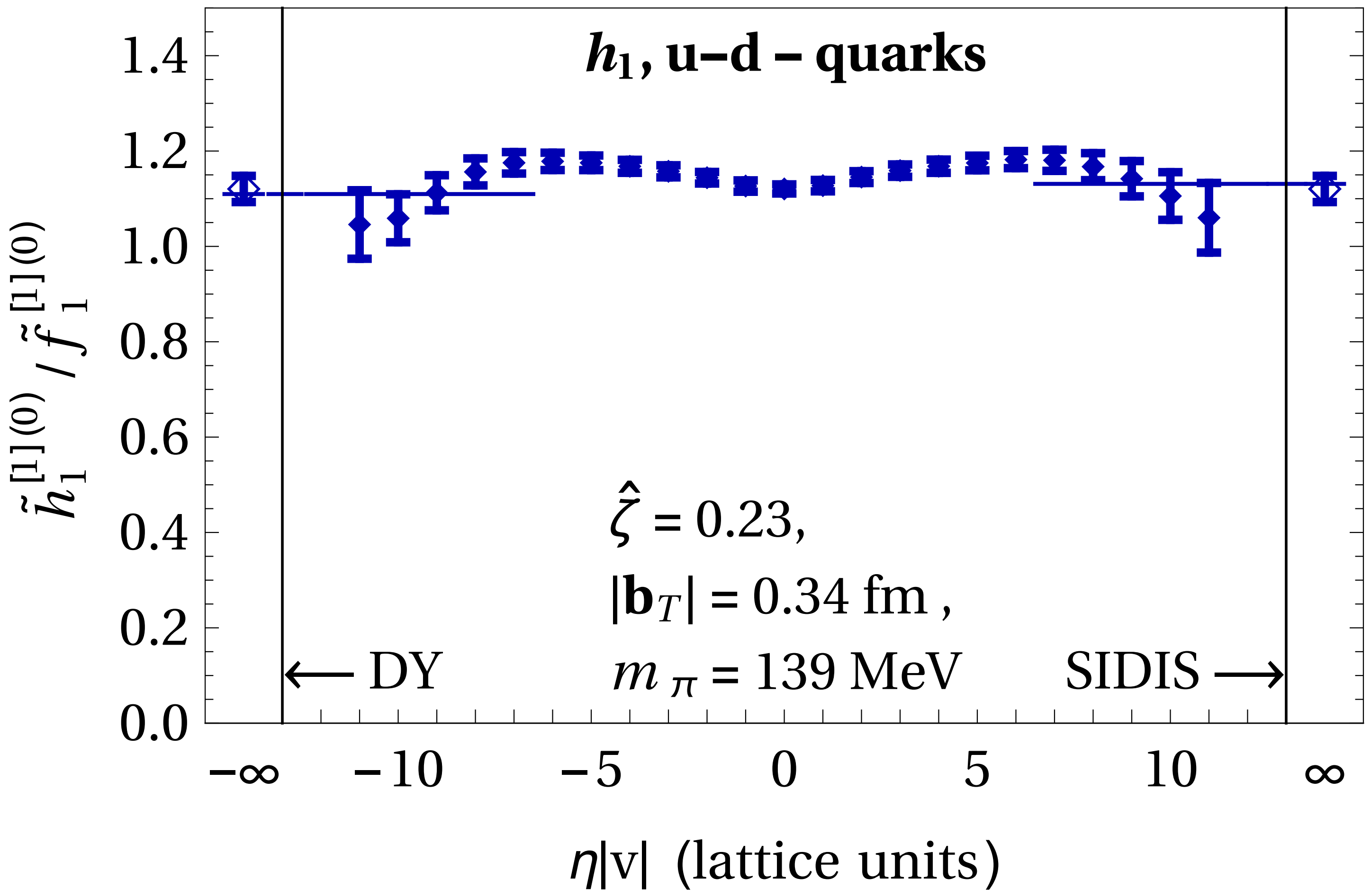

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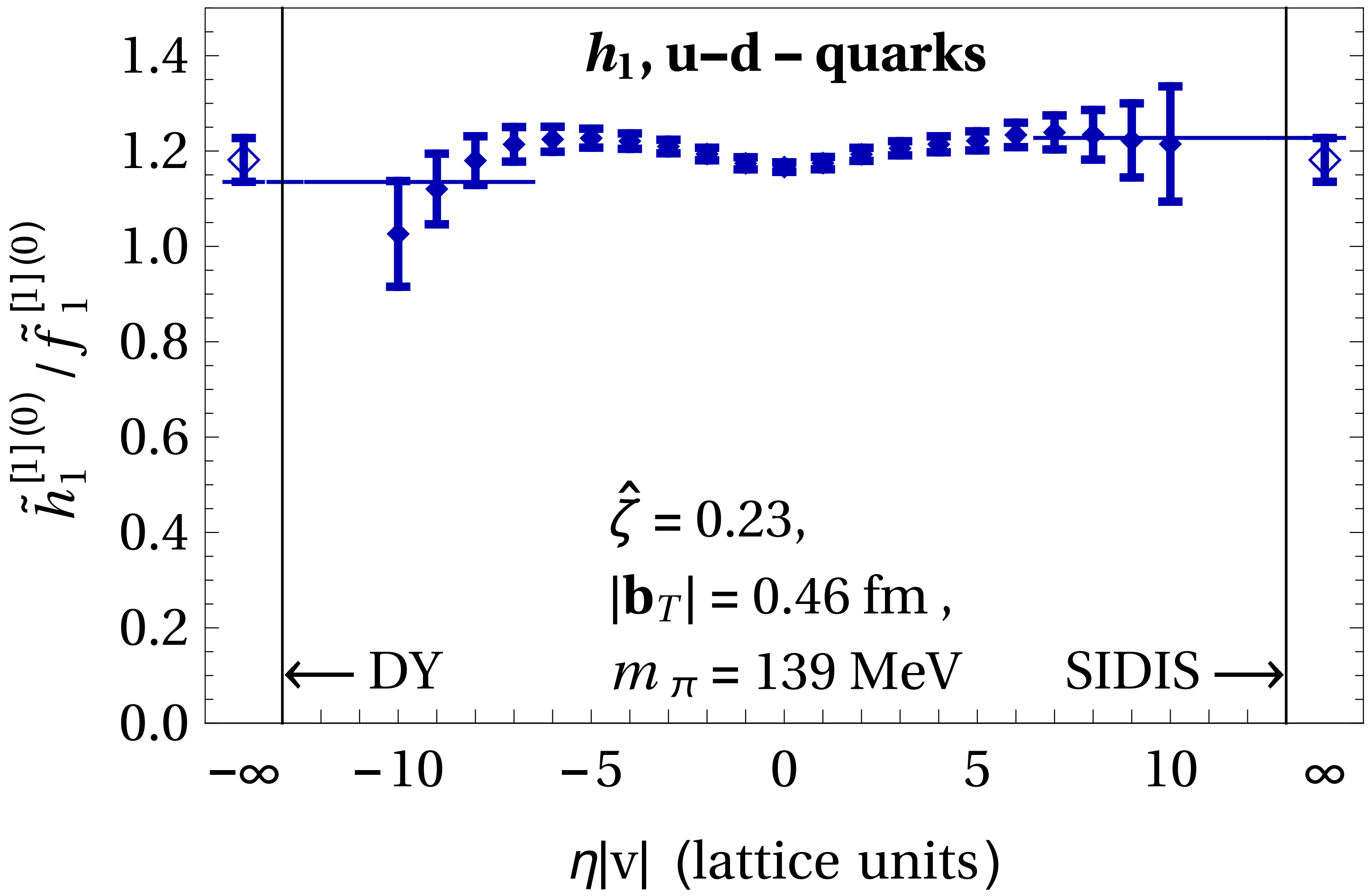
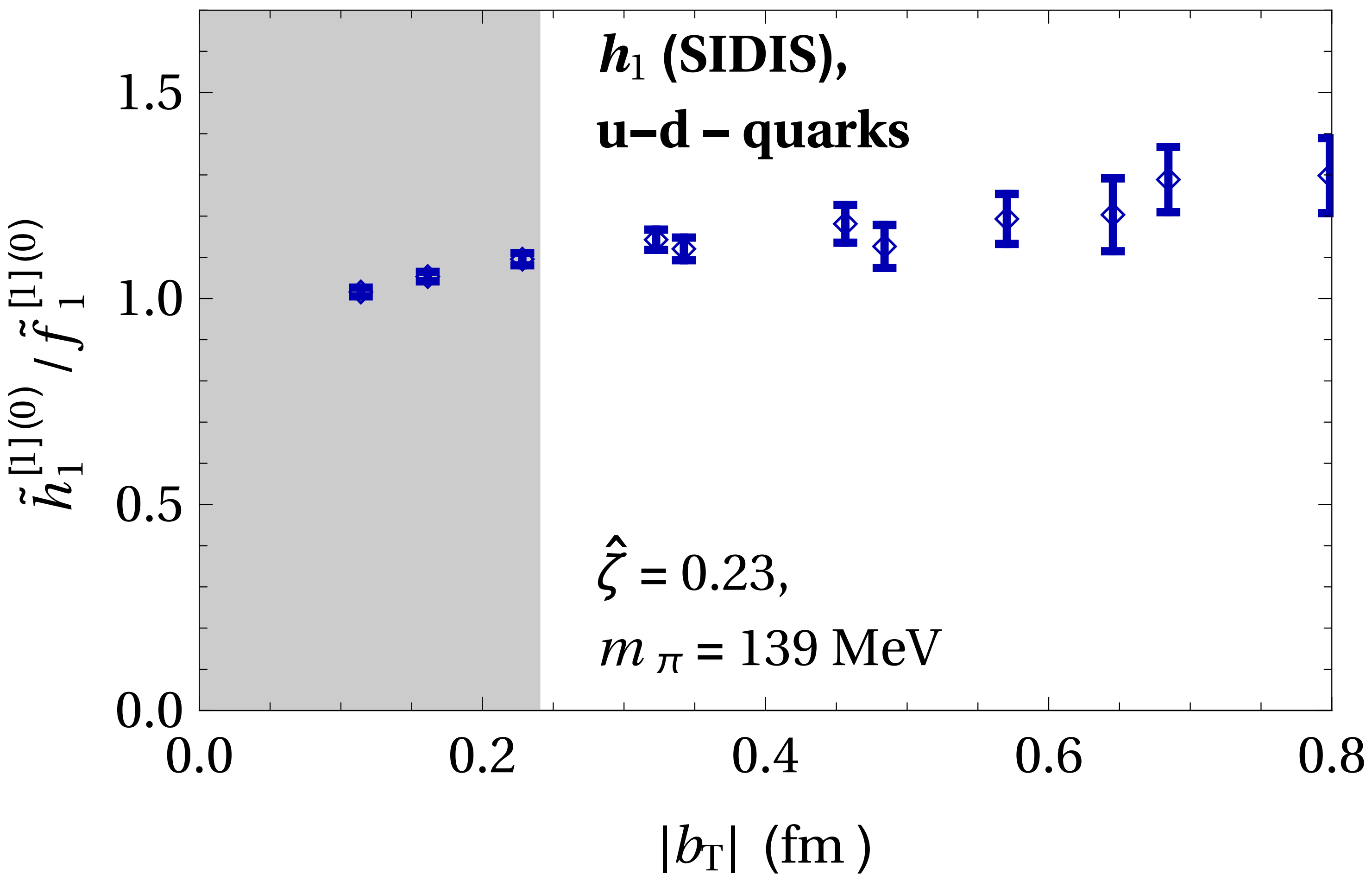


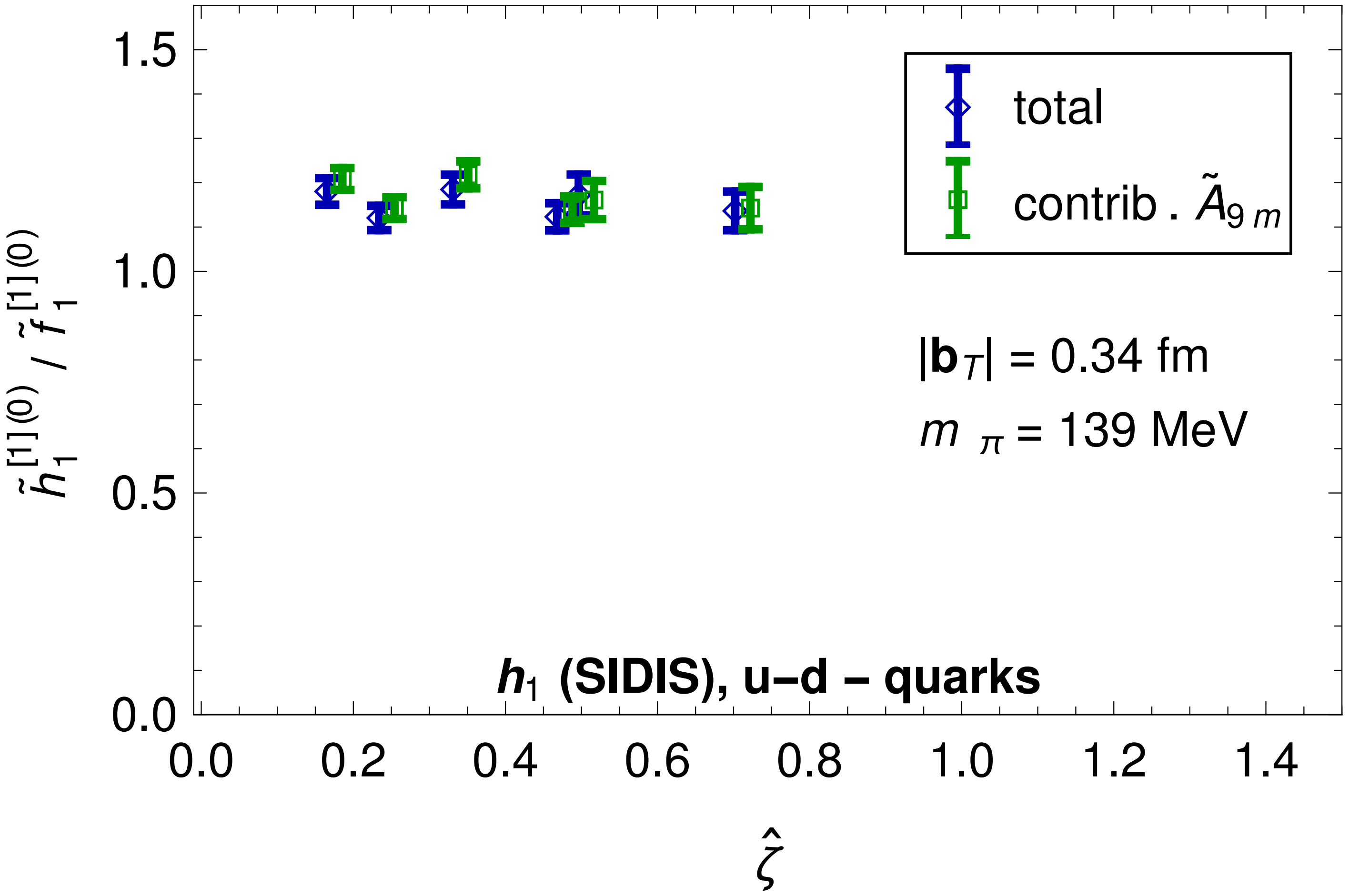
Results: Transversity

Dependence of SIDIS/DY limit on $|b_T|$



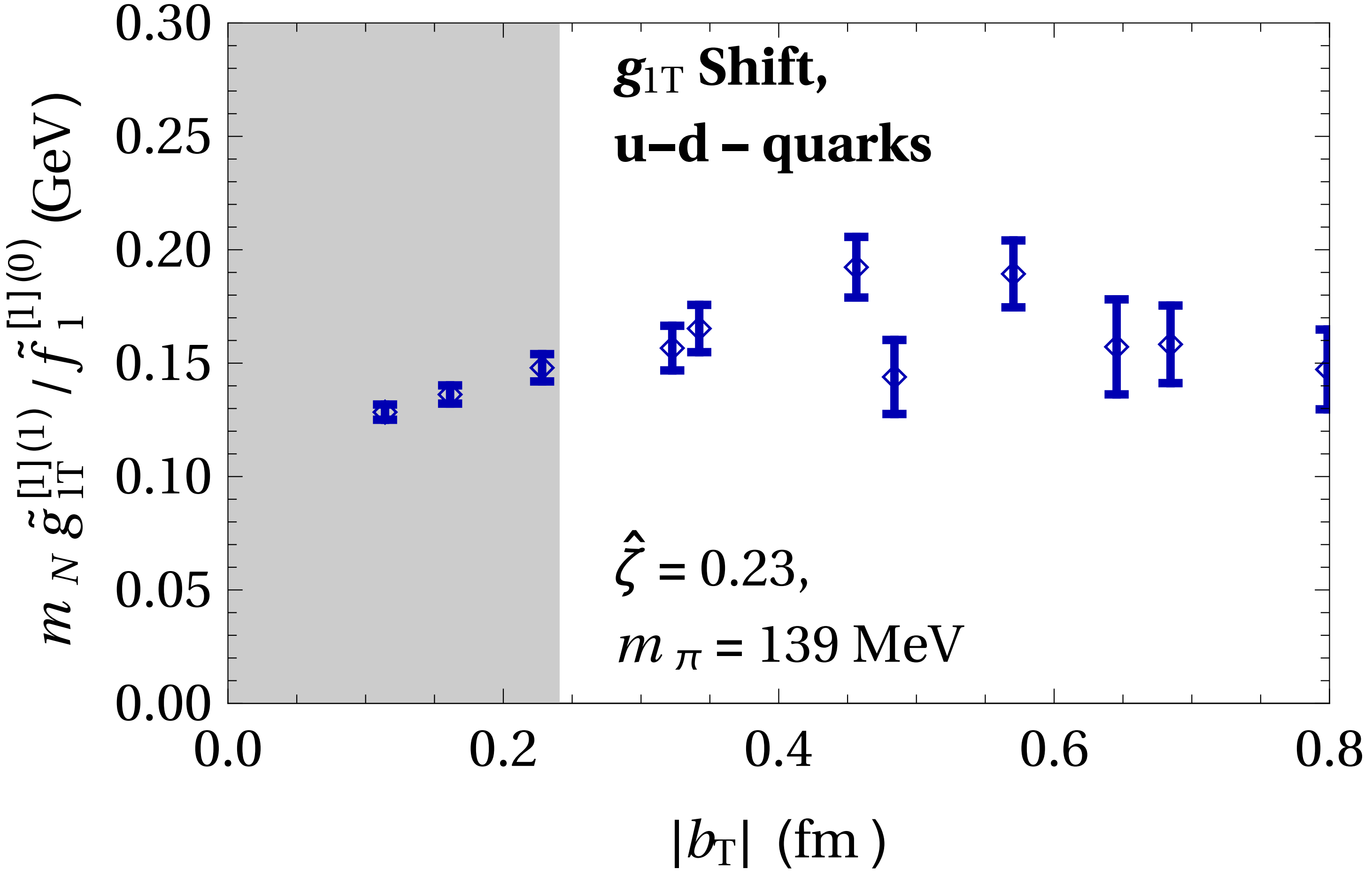
Results: Transversity

Dependence of SIDIS/DY limit on $\hat{\zeta}$



Results: g_{1T} worm gear shift

Dependence of SIDIS/DY limit on $|b_T|$



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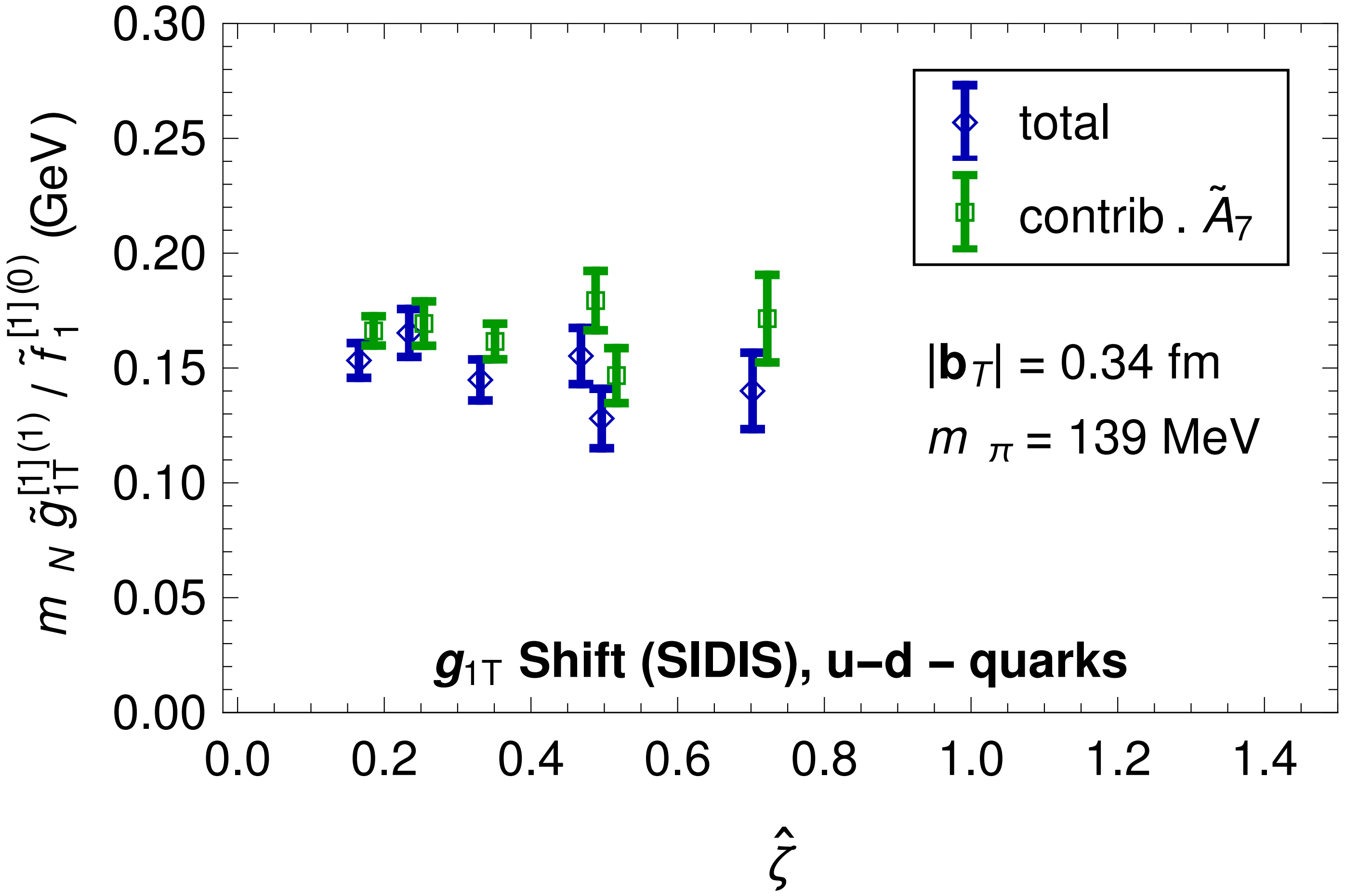
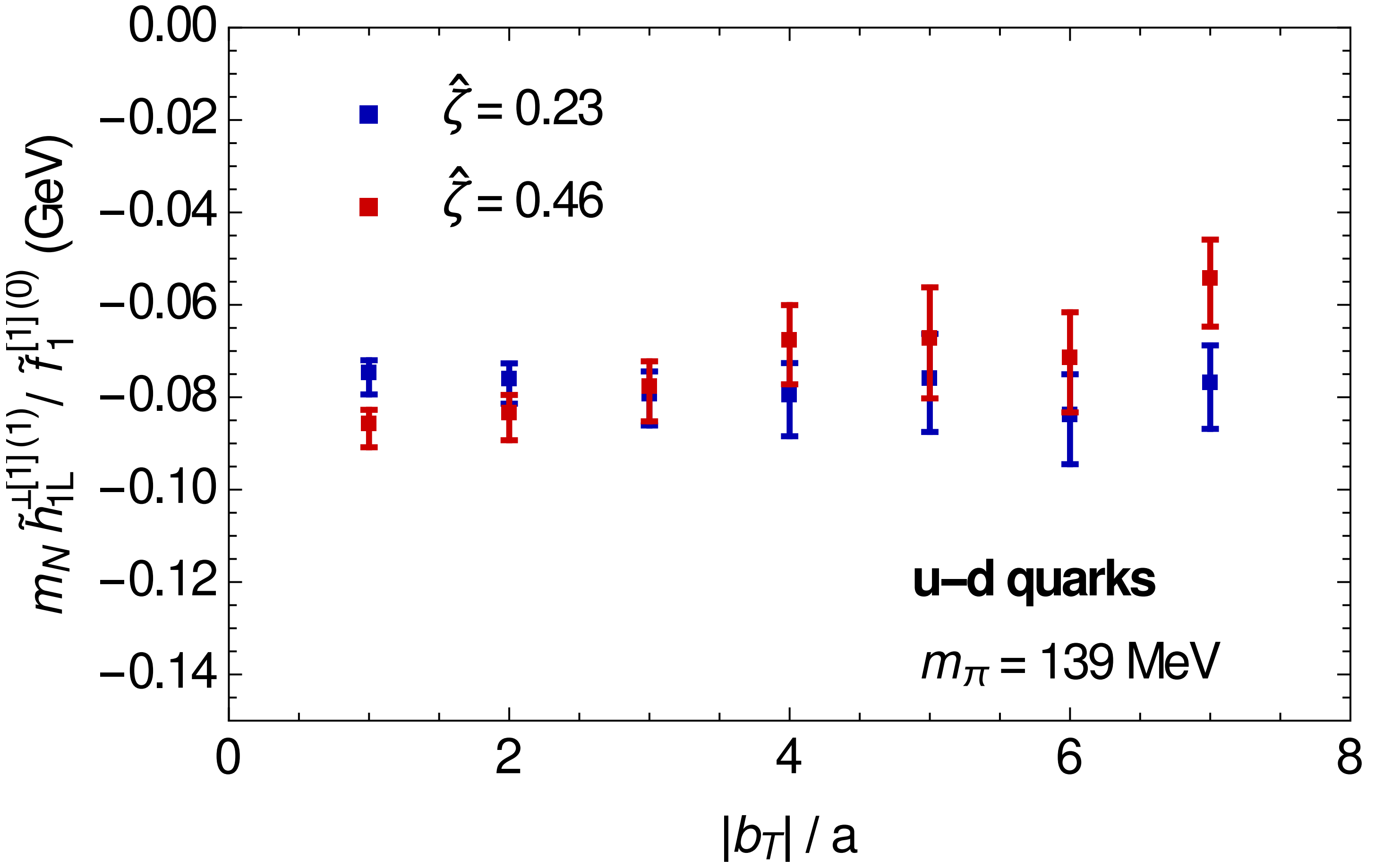


Results: $h_{1L}^{\perp}$ worm gear shift

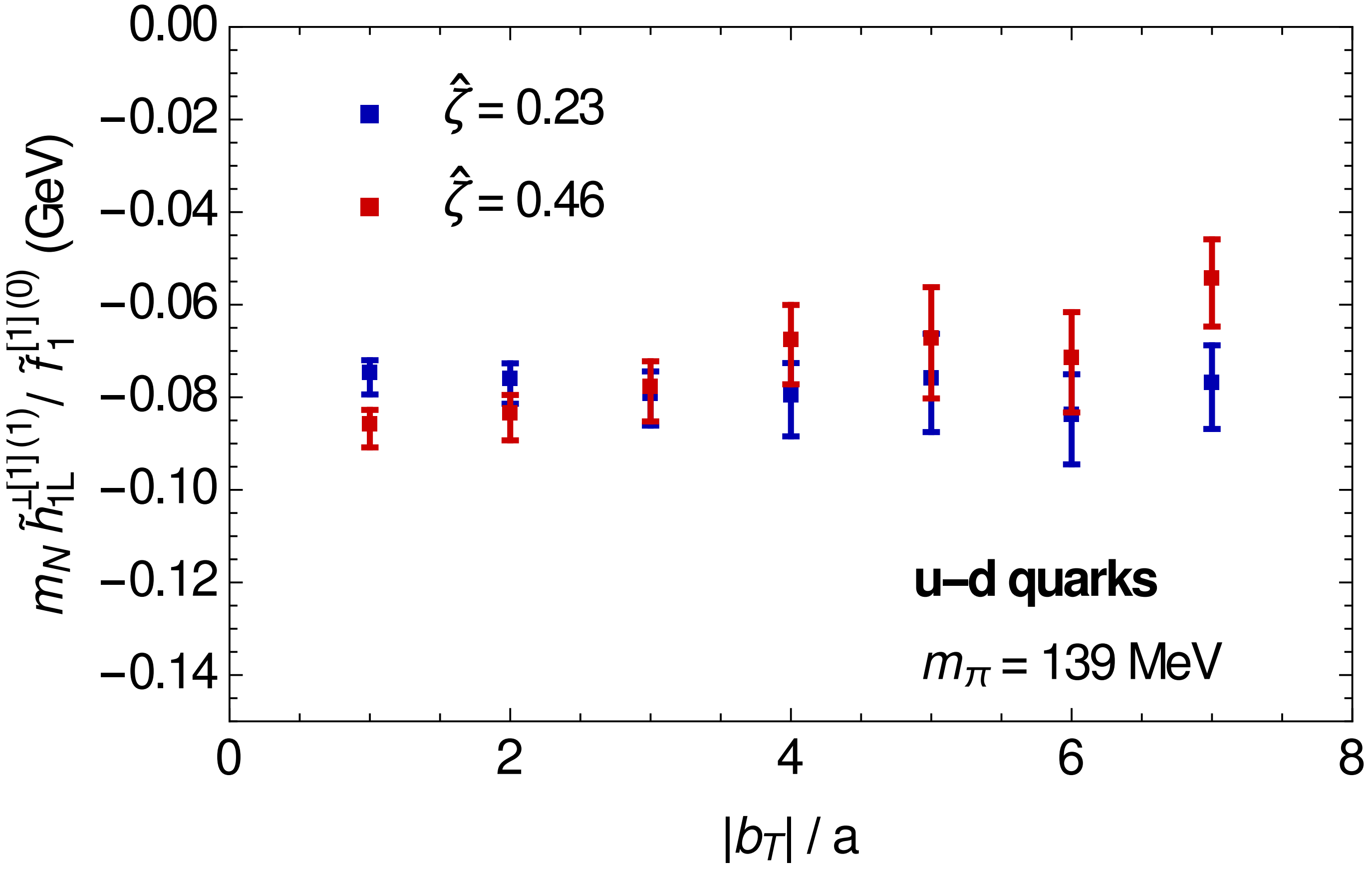
Dependence of SIDIS/DY limit on $|b_T|$

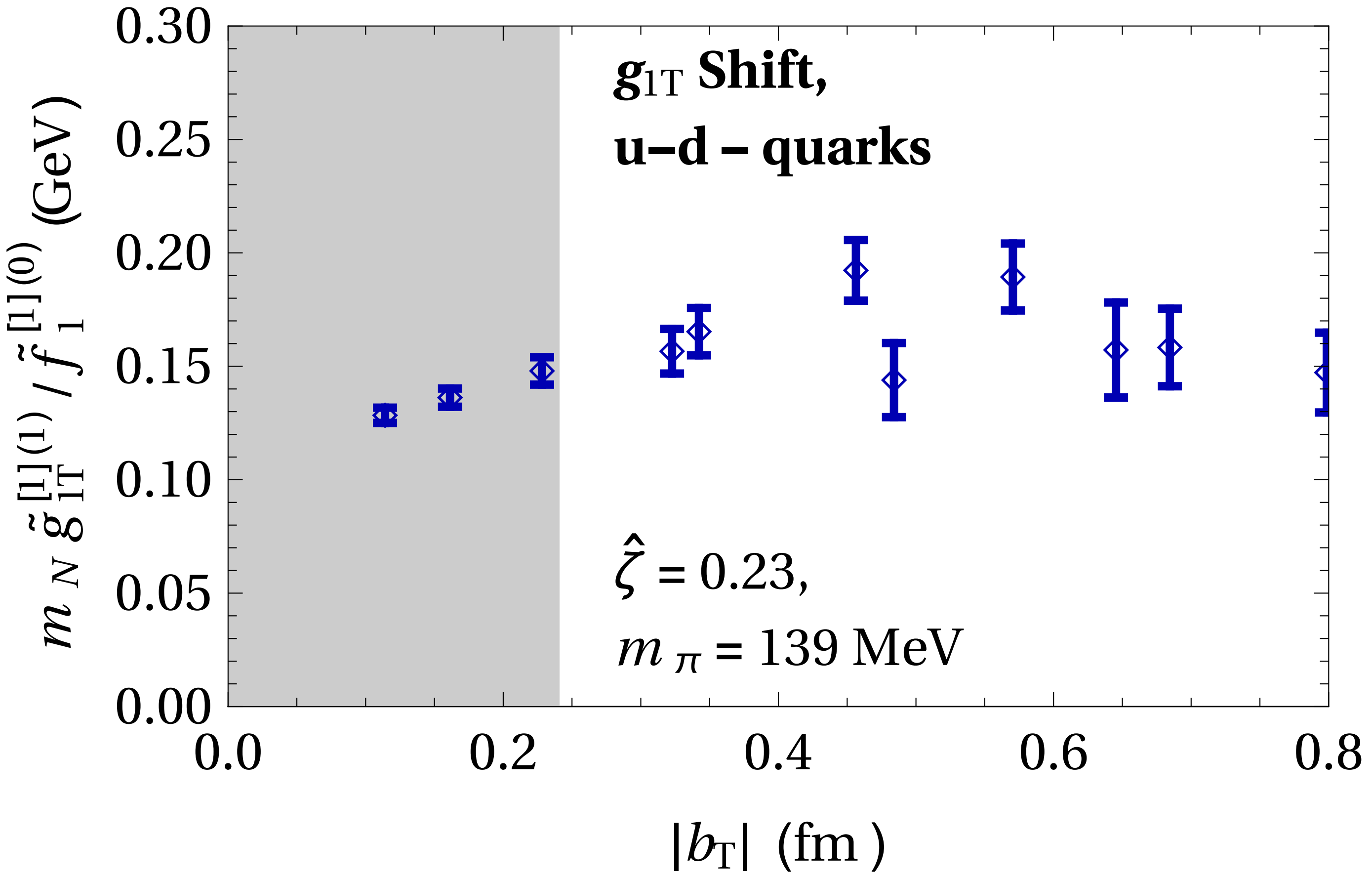


WARNING: Longitudinal polarization data obtained at one fairly small source-sink separation, $8a = 0.91\,\text{fm}$

Comparison: $h_{1L}^{\perp}$ vs. g_{1T} worm gear shift

SIDIS/DY limit as a function of $|b_T|$

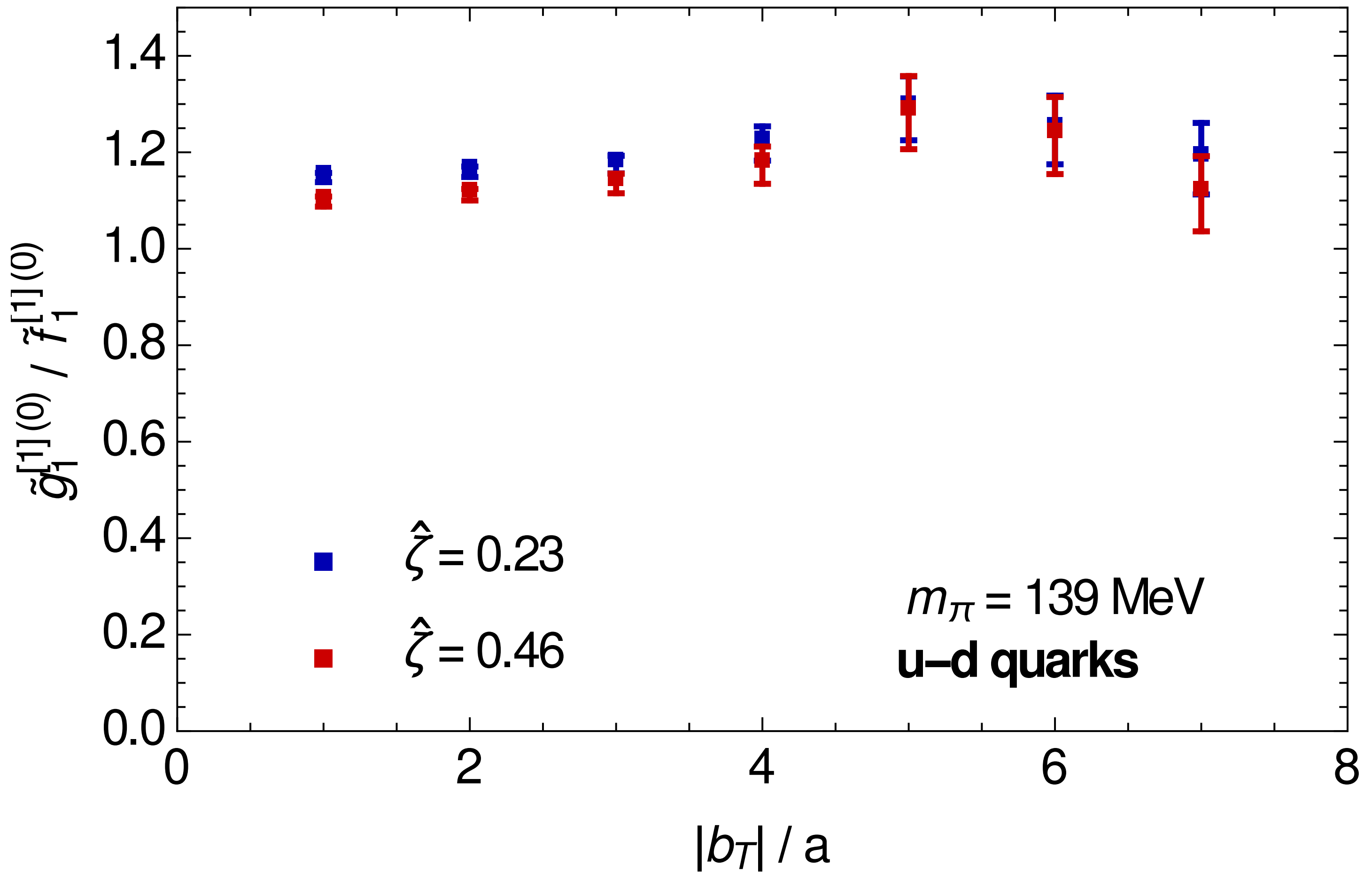




A wide variety of models predicts $h_{1L}^{\perp}$ and g_{1T} to have the same magnitude (and opposite sign):
Spectator model, light-front constituent quark model, covariant parton model, bag model, light-front quark-diquark model, light-front version of the chiral quark-soliton model, nonrelativistic quark model …
Significant QCD effects not captured by these models.

Results: Generalized axial charge

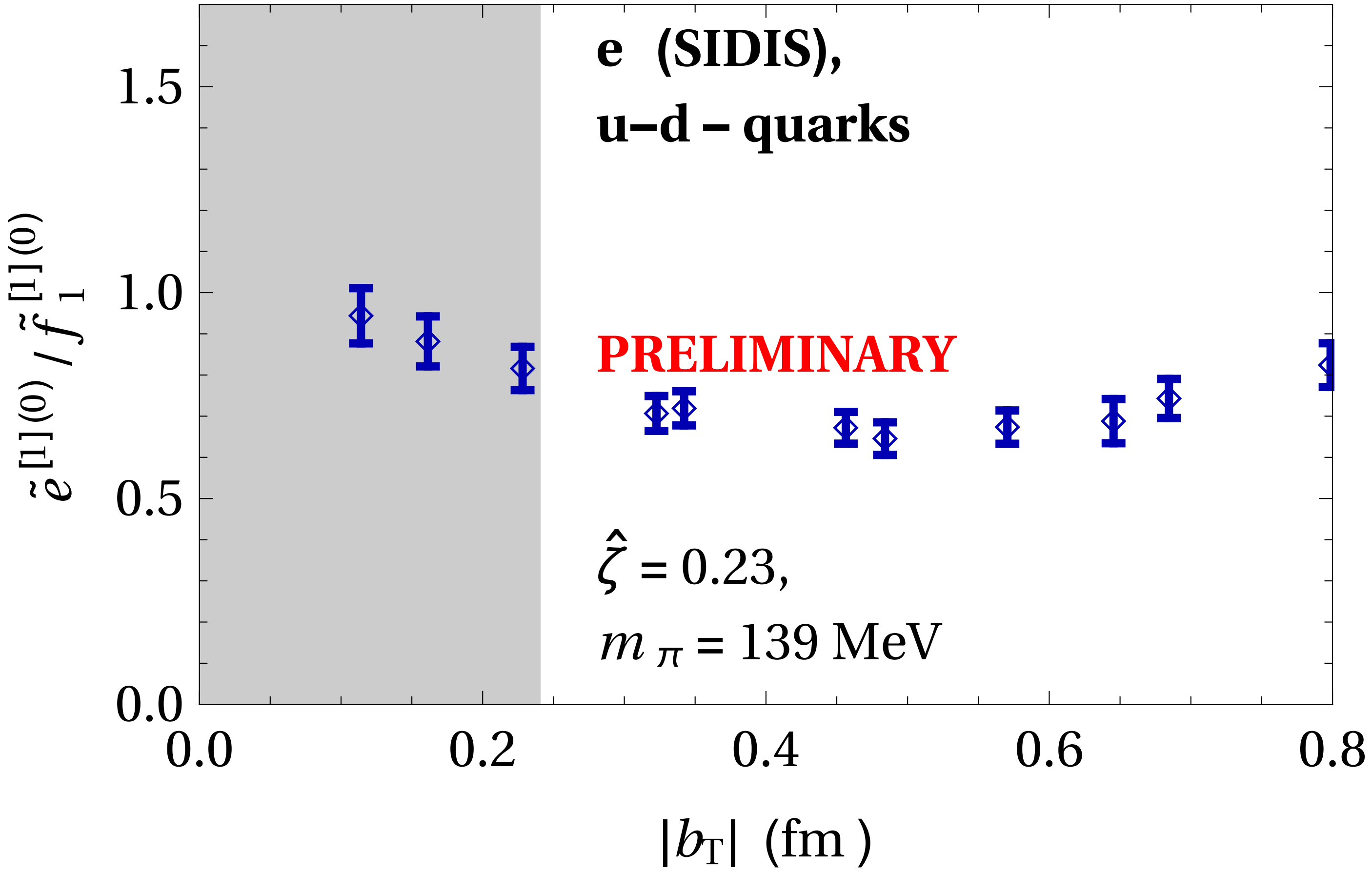
Dependence of SIDIS/DY limit on $|b_T|$



WARNING: Longitudinal polarization data obtained at one fairly small source-sink separation, $8a = 0.91\,\mathrm{fm}$

Results: Generalized scalar charge

Dependence of SIDIS/DY limit on $|b_T|$



Status summary

- Calculations of TMD observables using bilocal quark operators with staple-shaped gauge link structures have reached the physical pion mass.
- To avoid soft factors, multiplicative renormalization constants, considered appropriate ratios of Fourier-transformed TMDs (“shifts”, generalized charges).
- Increasing control over systematics – chiral symmetry, excited state contaminations, power corrections ($\hat{\zeta} \to \infty$ limit).
- Full twist-2 sector explored, as well as selected twist-3 TMDs.
- First contacts with phenomenology possible and encouraging; discrimination between models.

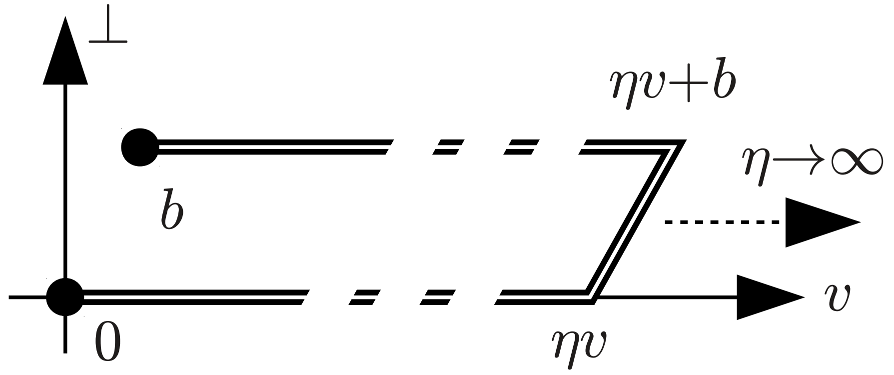
Preliminary sketch: x-dependence of Sivers shift

Sivers shift: Average transverse momentum of unpolarized quarks in a nucleon polarized in the other transverse direction

$$\frac{1}{2}\langle P,S|\ \bar{q}(0)\ \gamma^{+}\ \mathcal{U}[0,\ldots,b]\ q(b)\ |P,S\rangle = 2P^{+}\left(\widetilde{A}_{2B} + im_N\epsilon_{ij}b_iS_j\widetilde{A}_{12B}\right)$$

$$\langle k_T\rangle_{TU}(b_T^2,x,\ldots) = m_N\frac{\widetilde{f}_{1T}^{\perp(1)}(b_T^2,x,\ldots)}{\widetilde{f}_{1}^{(0)}(b_T^2,x,\ldots)} = -m_N\frac{\int d(b\cdot P)\exp(ixb\cdot P)\widetilde{A}_{12B}(-b_T^2,b\cdot P,\hat{\zeta},\eta v\cdot P)}{\int d(b\cdot P)\exp(ixb\cdot P)\widetilde{A}_{2B}(-b_T^2,b\cdot P,\hat{\zeta},\eta v\cdot P)}$$

With a grain of salt, soft factors do not depend on $b\cdot P$ – can be factored outside the Fourier transform



Preliminary sketch: x-dependence of Sivers shift

Phenomenological frame: $P_T = v_T = 0$, $b^+ = 0$

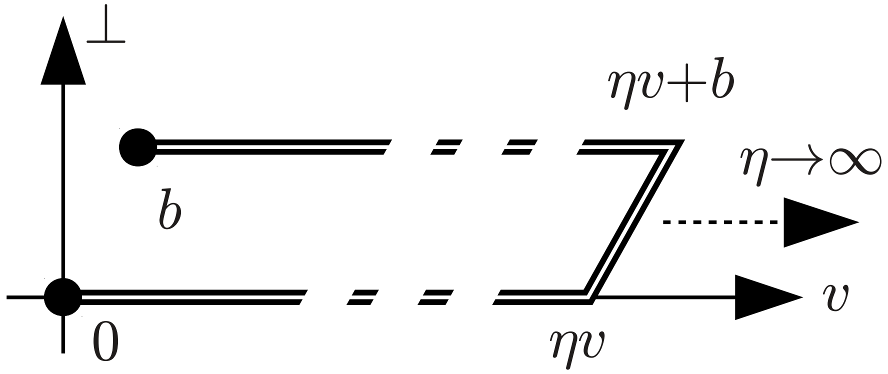
Expressed in Lorentz-invariant fashion: $\frac{v\cdot b}{v\cdot P} = \frac{b\cdot P}{m_N^2}\left(1 - \sqrt{1 + 1/\hat{\zeta}^2}\right)$

Lattice frame: b, v purely spatial

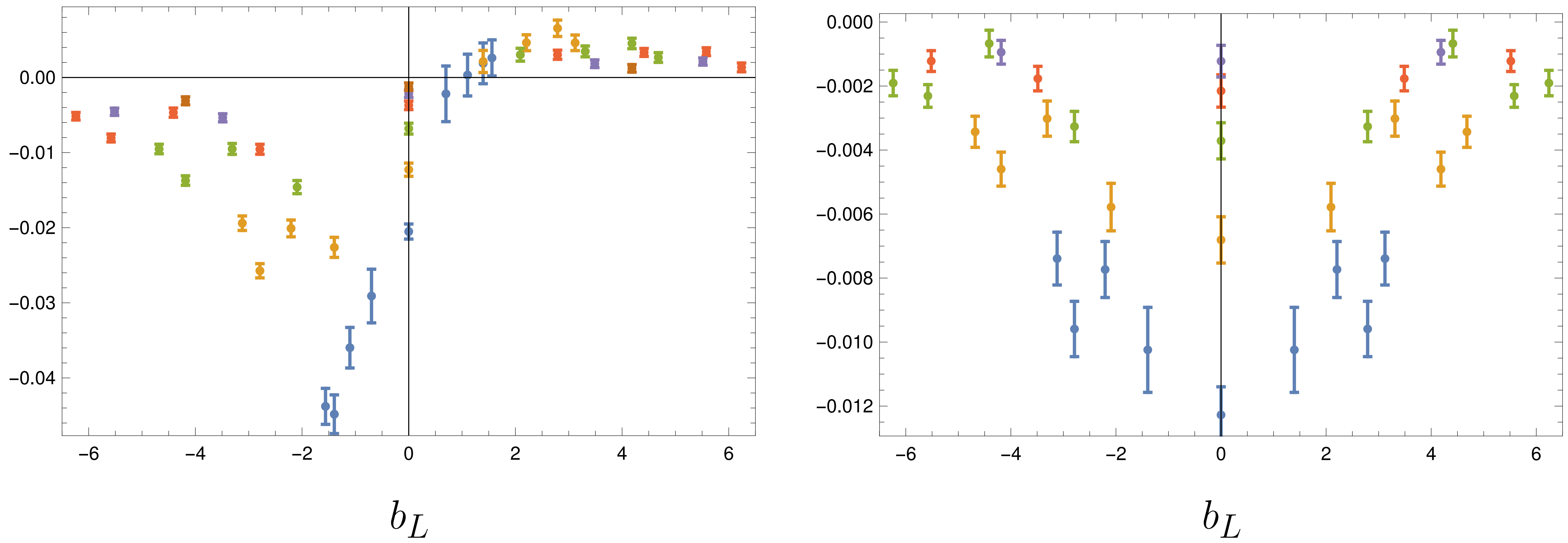
Constraint forces the use of general off-axis directions

Lorentz transformation between phenomenological and lattice frames is not pure boost, contains rotation

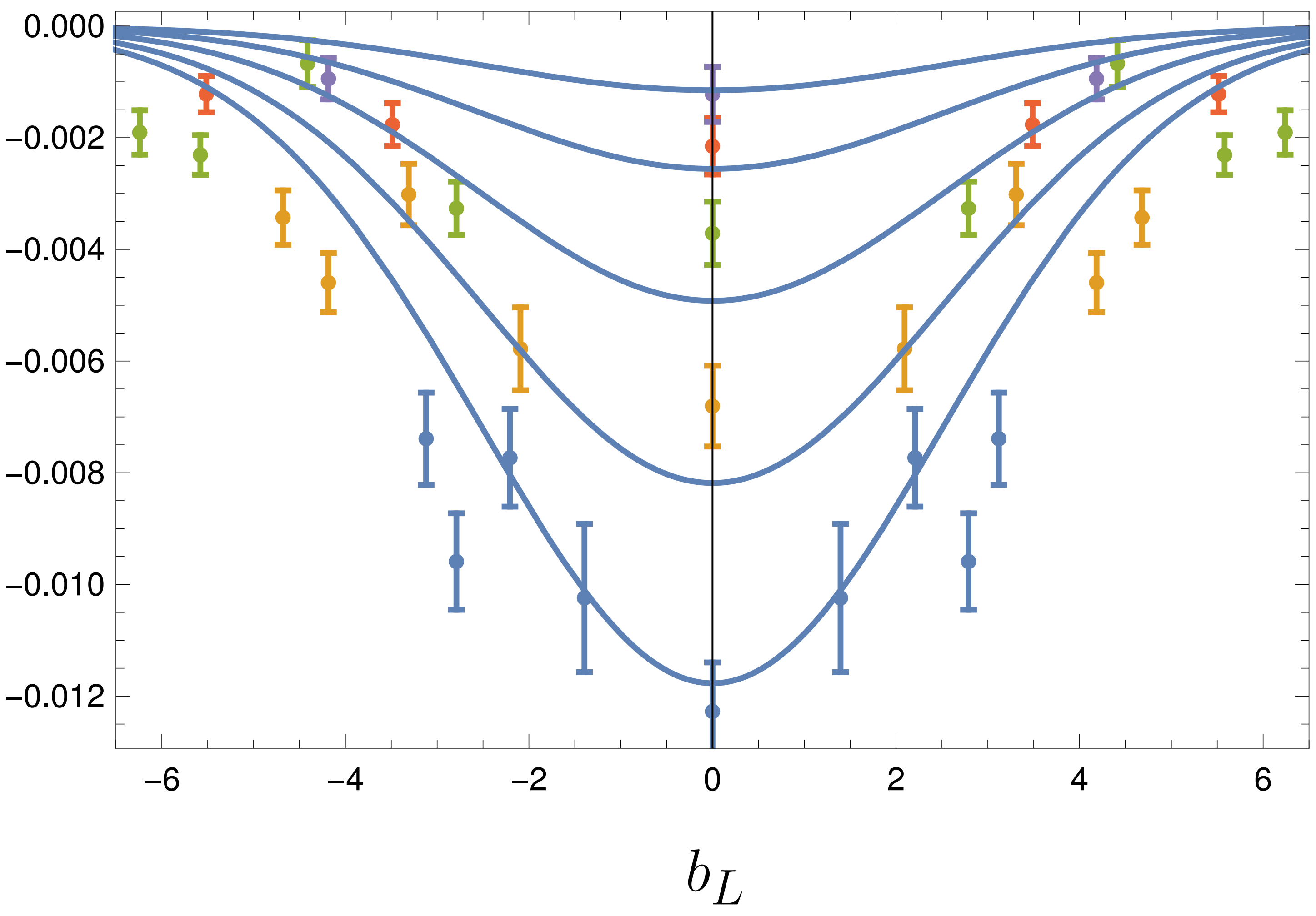
Perform analysis at large staple length



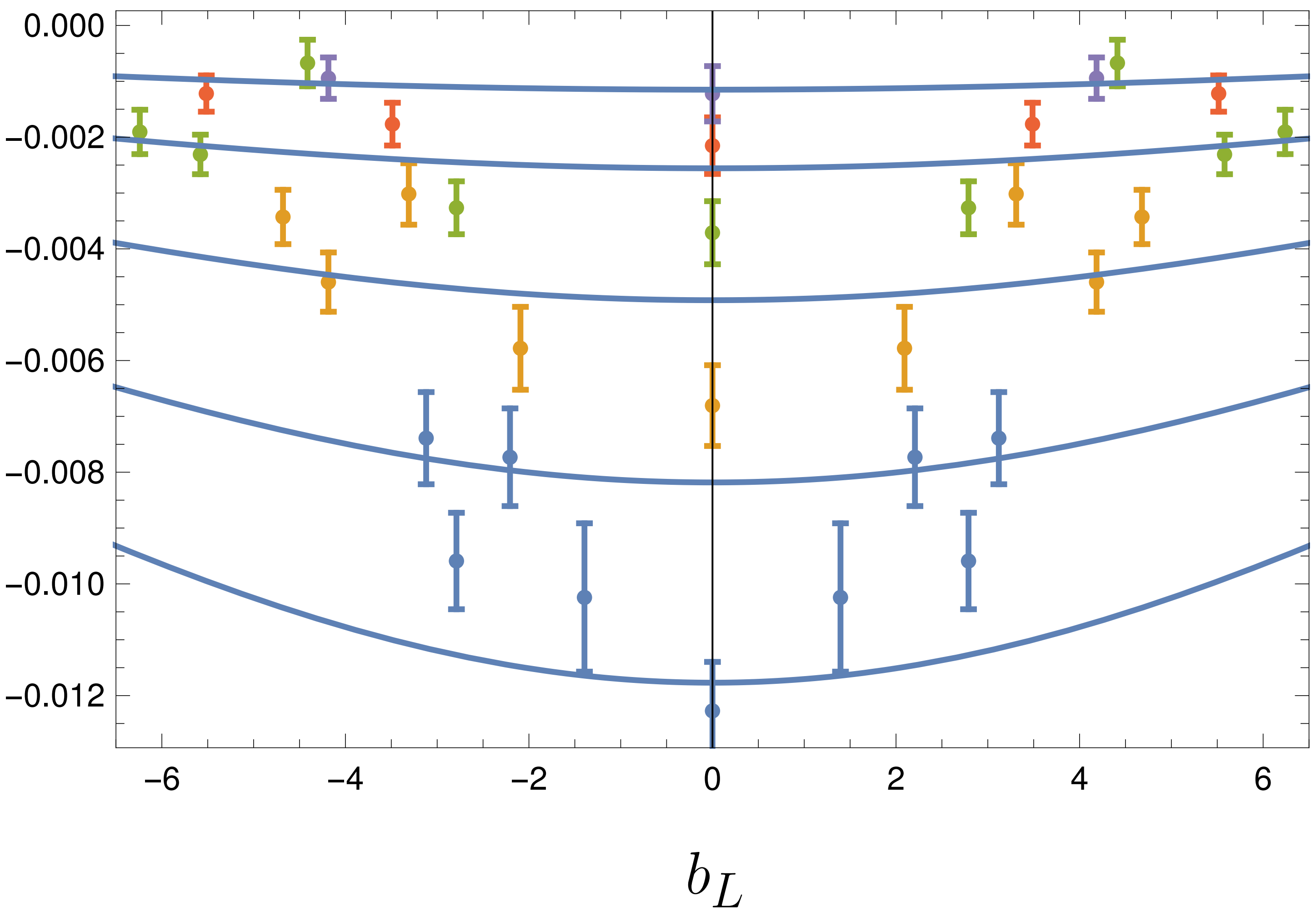
Extract b_L-even component of imaginary part of γ^+ correlator



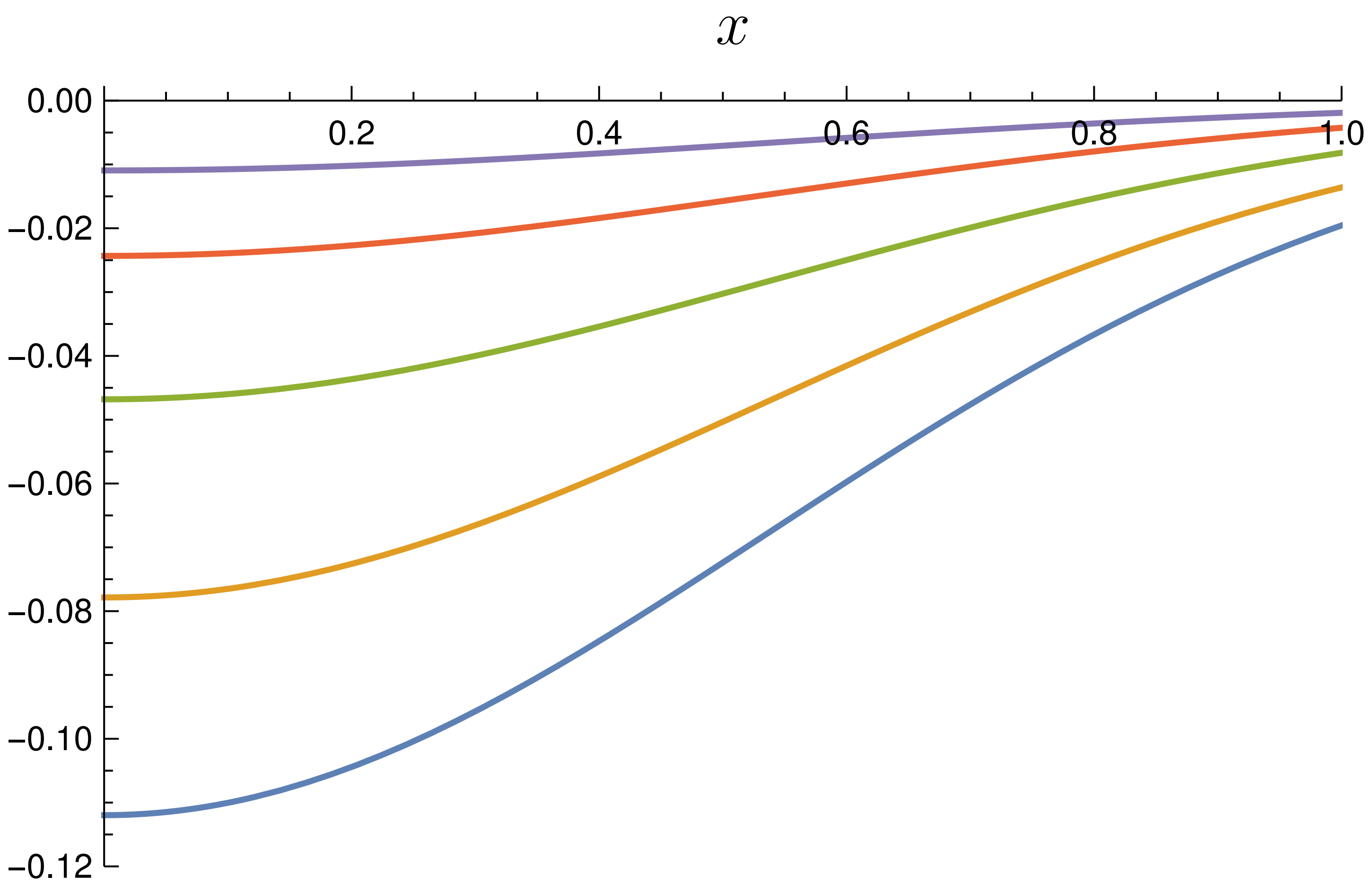
Fit dependence in b_L, $|b_T|$ space



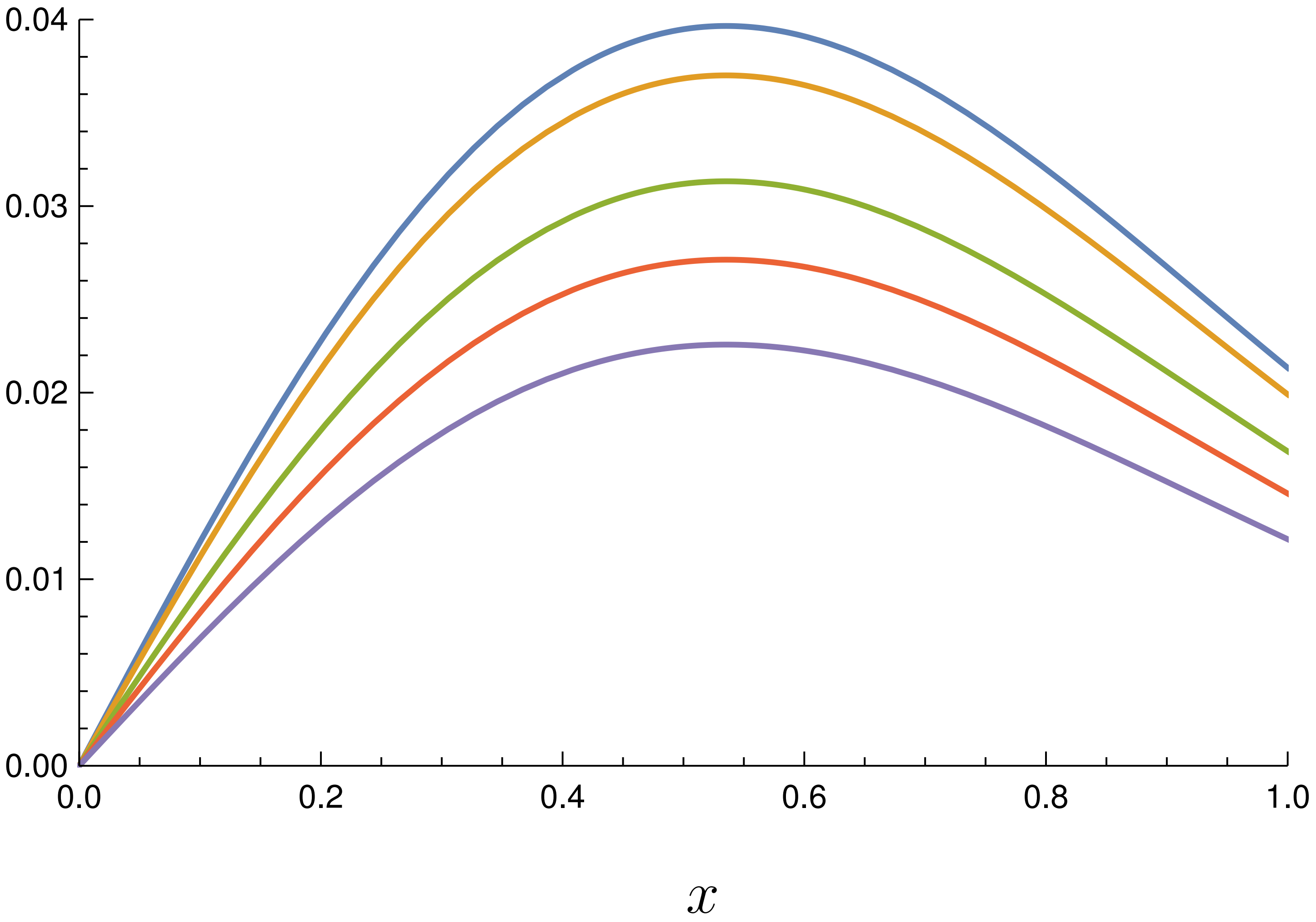
Cast in $b \cdot P$, b^2 space



Fourier transform $b \cdot P \longrightarrow x$



Normalize to x-integrated Sivers shift, multiply by x



Eyeball error