Transversity PDFs of the proton from lattice QCD with physical quark masses

Xiang Gao
Argonne National Laboratory

In collaboration with: A. Hanlon, N. Karthik, S. Mukherjee,
P. Petreczky, Q. Shi, S. Syritsyn, and Y. Zhao

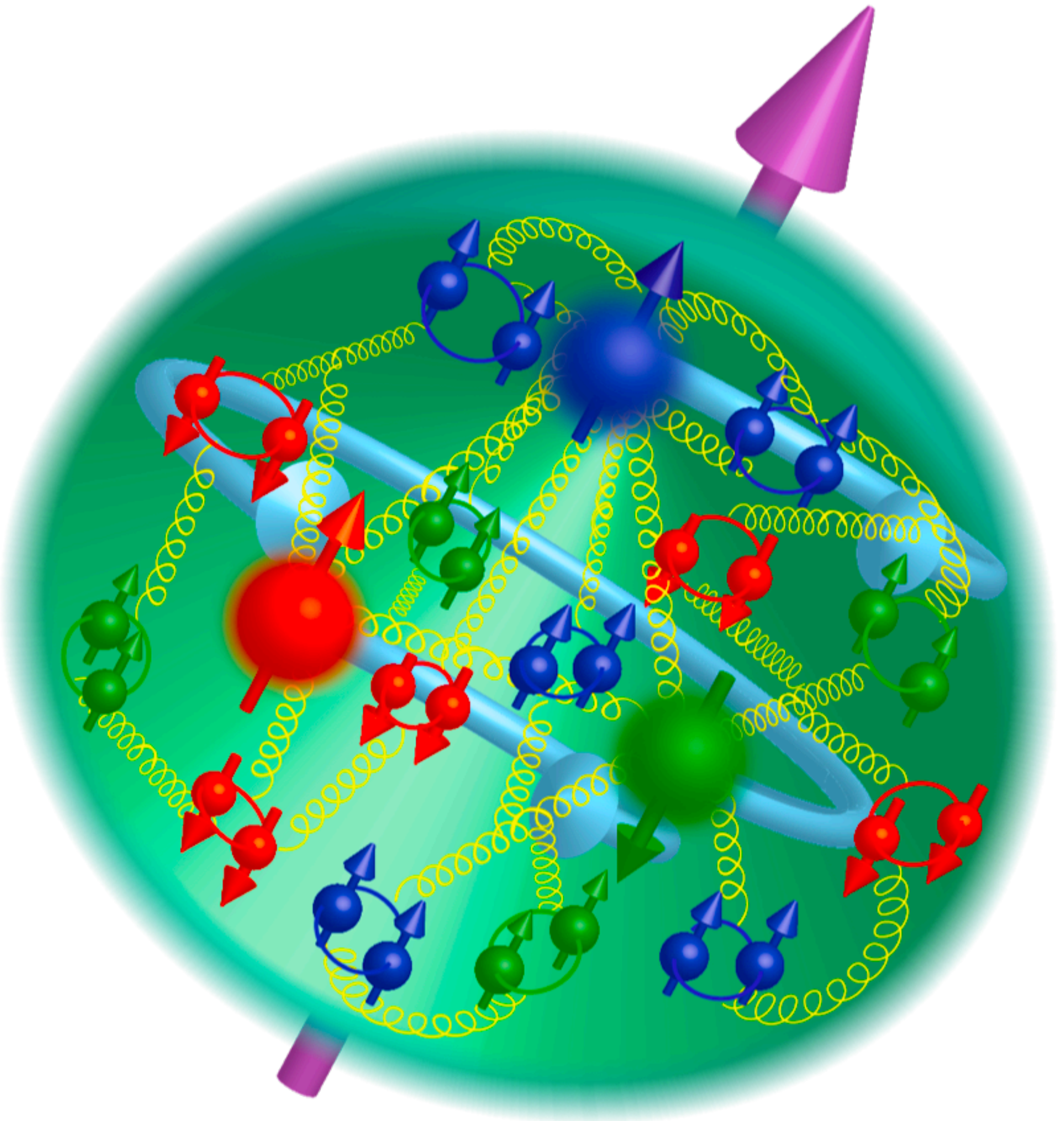
TMDs: Towards a Synergy between Lattice QCD and Global Analyses
21–23 Jun 2023

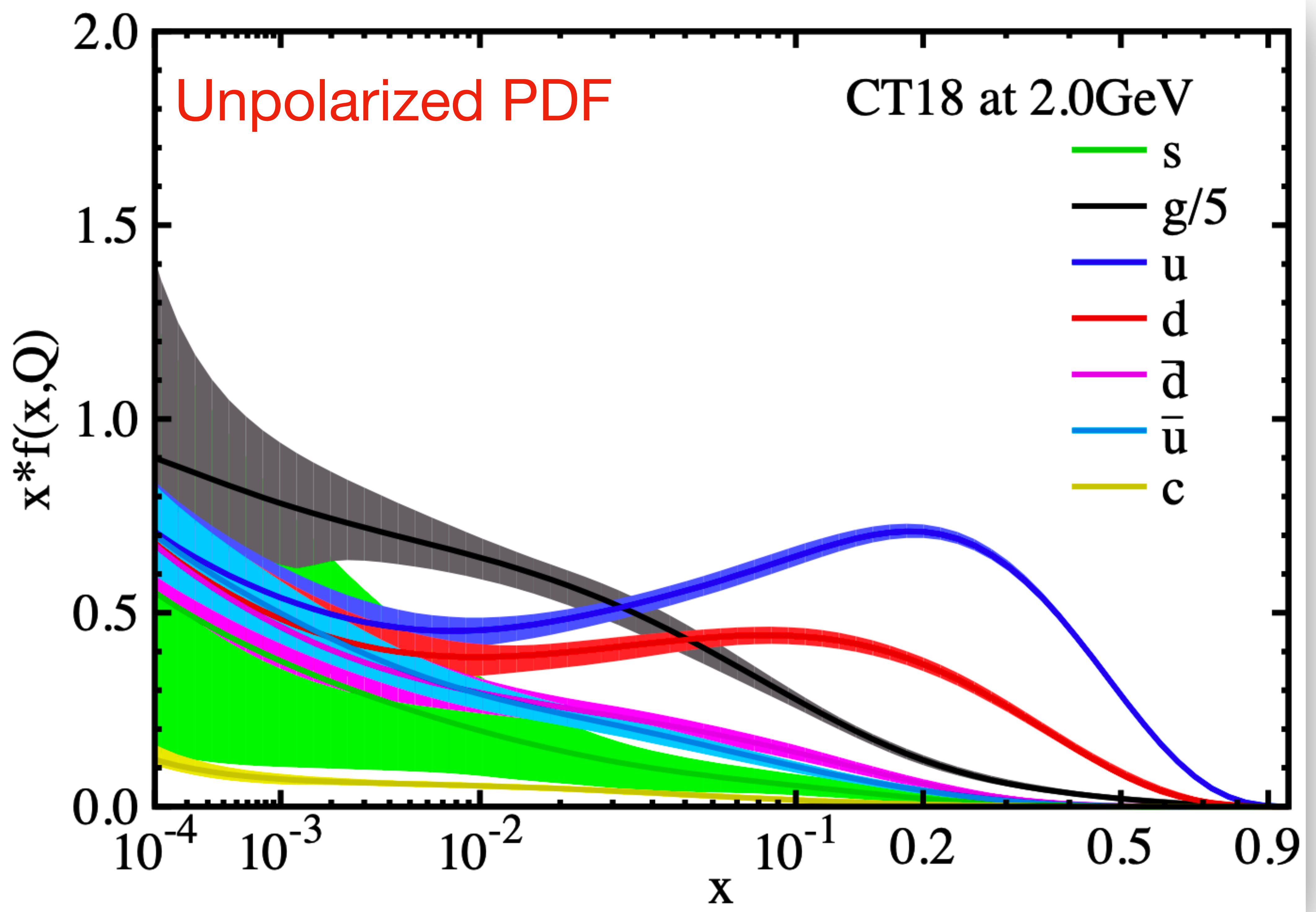
Parton distribution functions

At leading-twist, three collinear PDFs can be defined:

CTEQ: Phys.Rev.D 103 (2021) 1, 014013

- Unpolarized distribution.
- Helicity distribution.
- Transversity distribution.

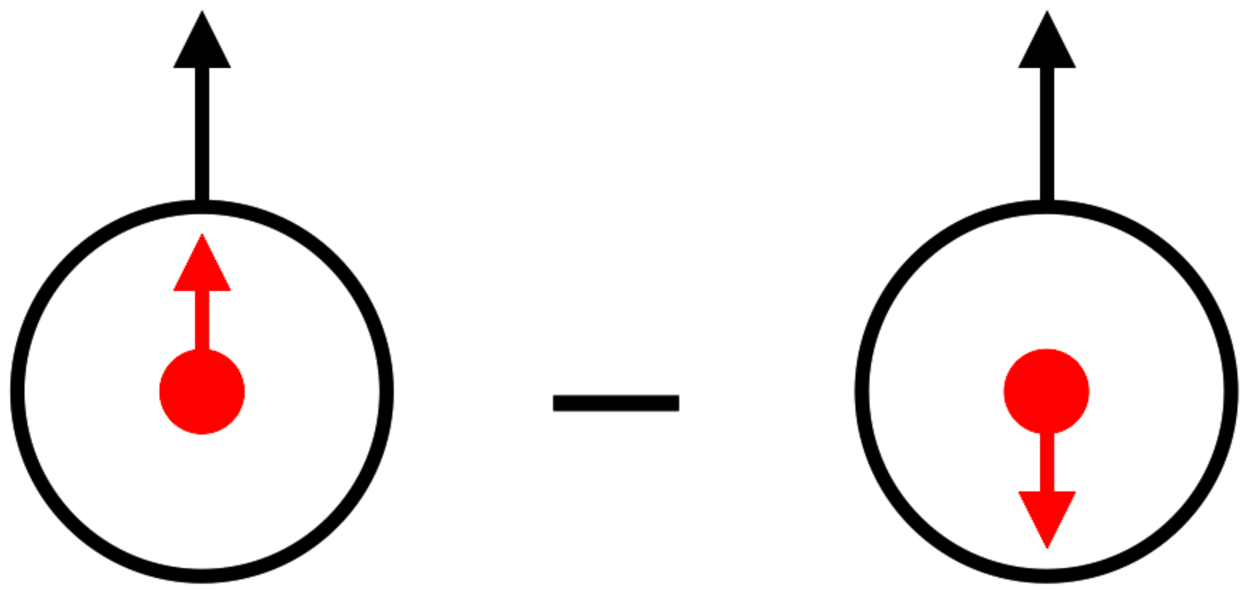


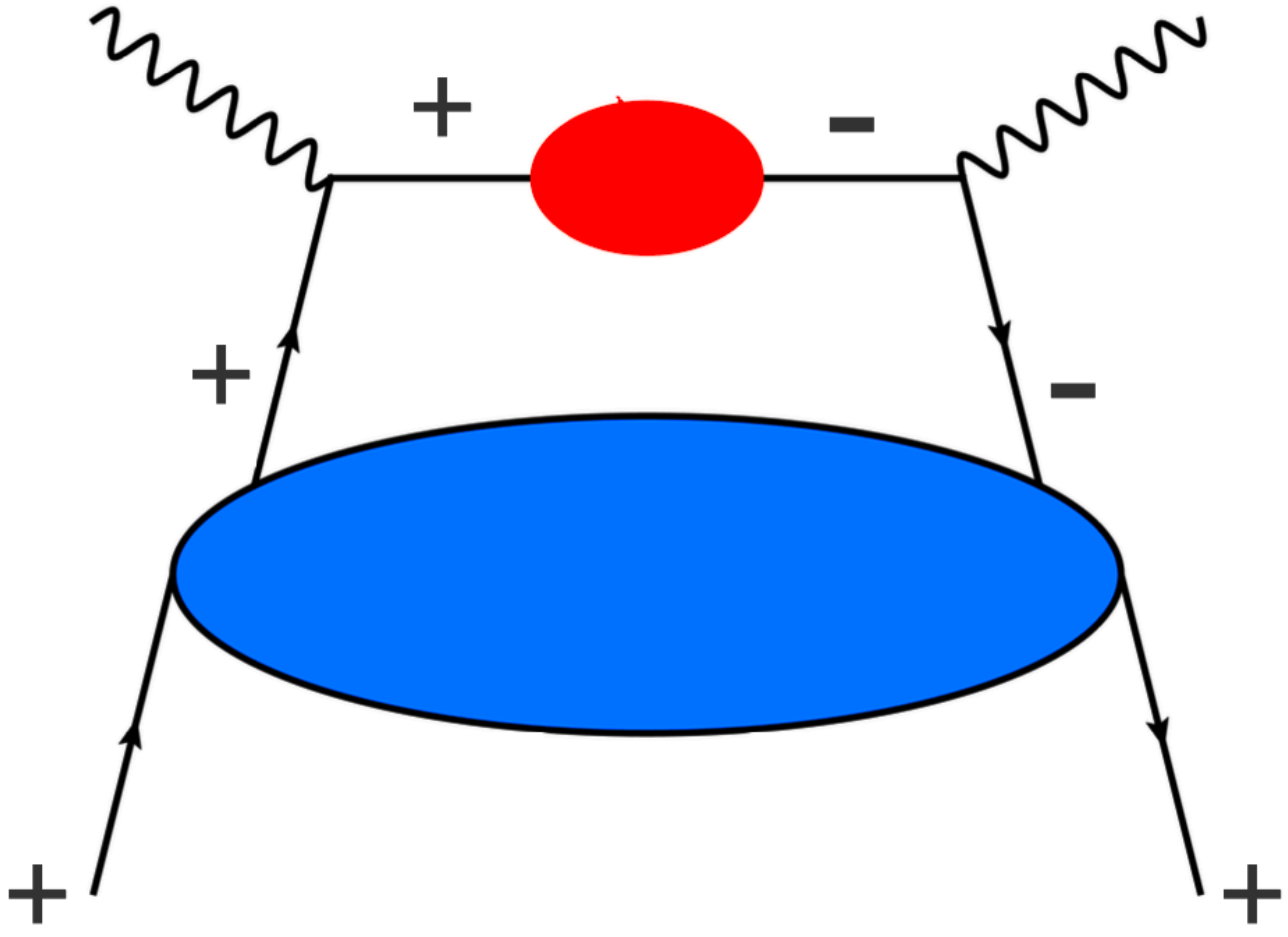


Global analysis: DIS, DY, vector boson production, single-inclusive jet production, W/Z bosons production, …

Parton distribution functions

- **Transversity distribution**



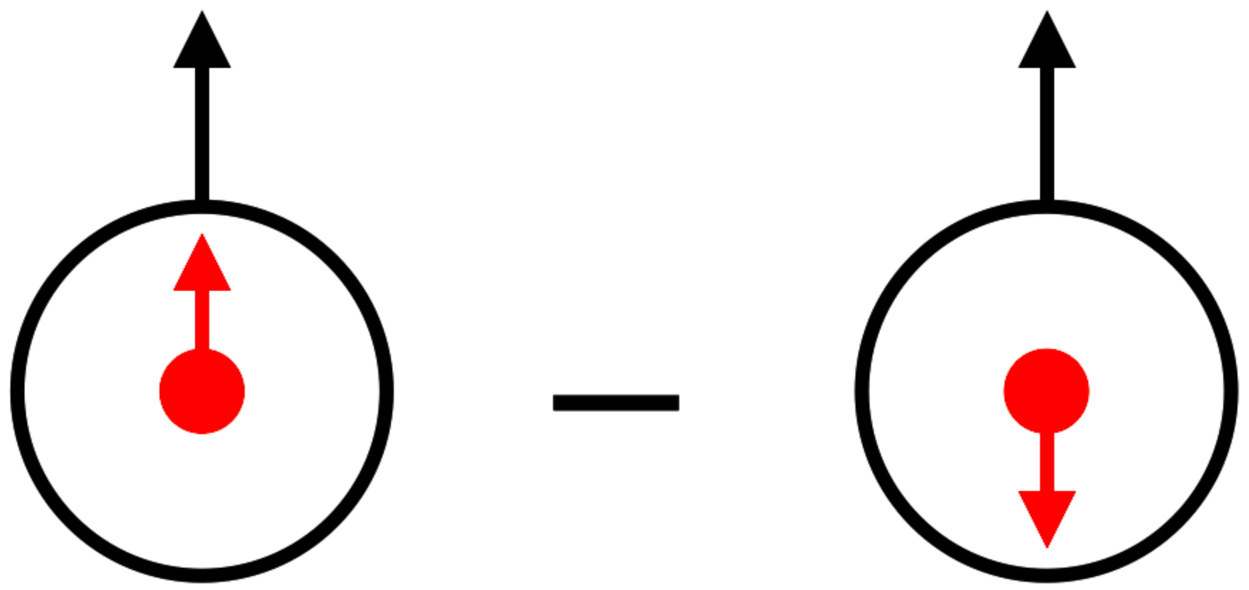


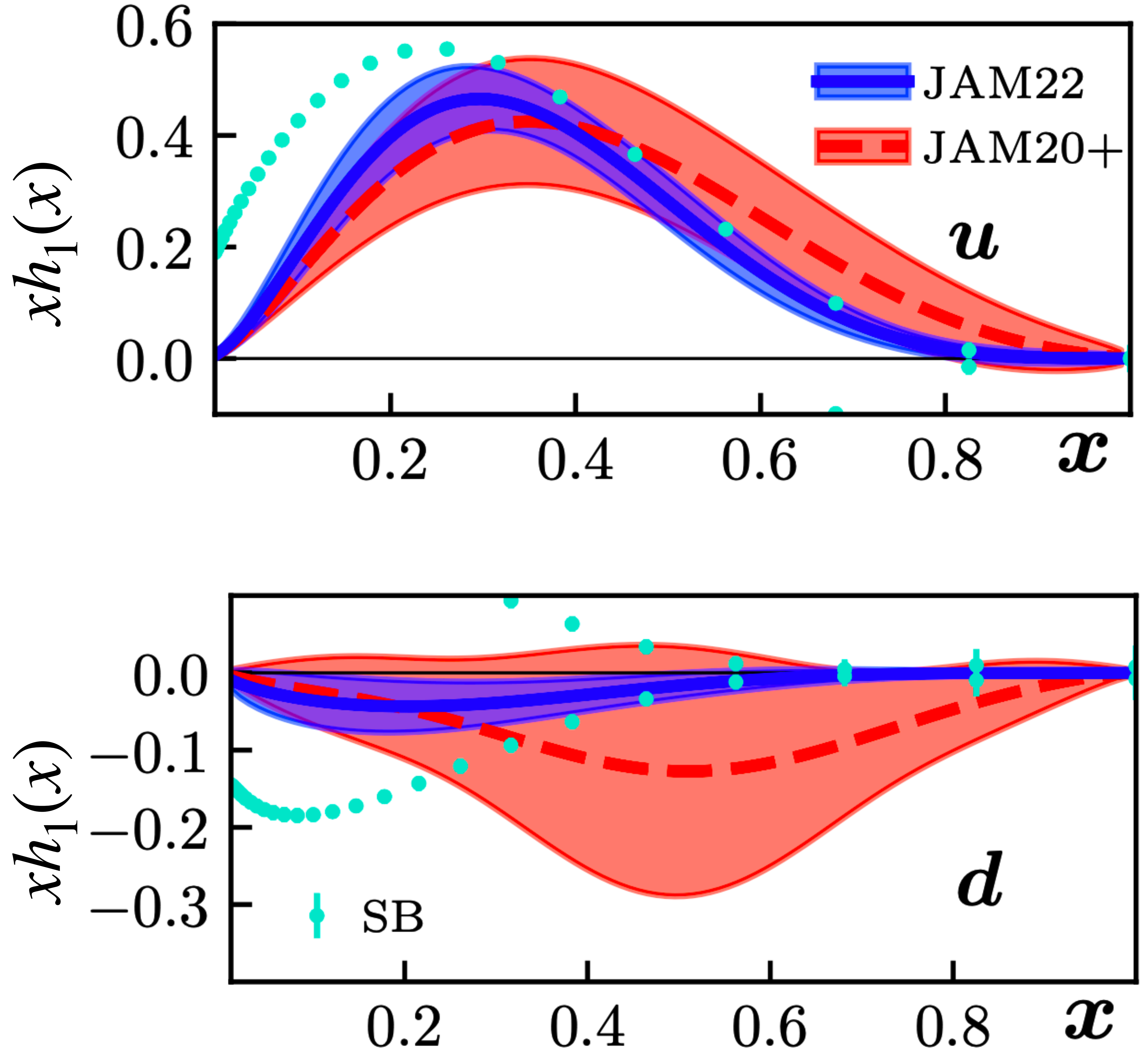
- $\bar{\psi}\sigma_{+\perp}i\gamma_5\psi$ flips quark helicity, chiral odd.
- must couple to another chiral odd process to be measured, e.g. fragmentation functions.
- TMD factorization, e.g. SIDIS,

$$\ell(l) + N(P, S_T) \to \ell(l') + h(P_h) + X$$

Parton distribution functions

- **Transversity distribution**





JAM: Phys.Rev.D 106 (2022) 3, 034014

- $\bar{\psi}\sigma_{+\perp}i\gamma_5\psi$ flips quark helicity, chiral odd.
- must couple to another chiral odd process to be measured, e.g. fragmentation functions.
- TMD factorization, e.g. SIDIS,

$$\ell(l) + N(P, S_T) \to \ell(l') + h(P_h) + X$$

Less constrained than the unpolarized case

Parton distribution functions

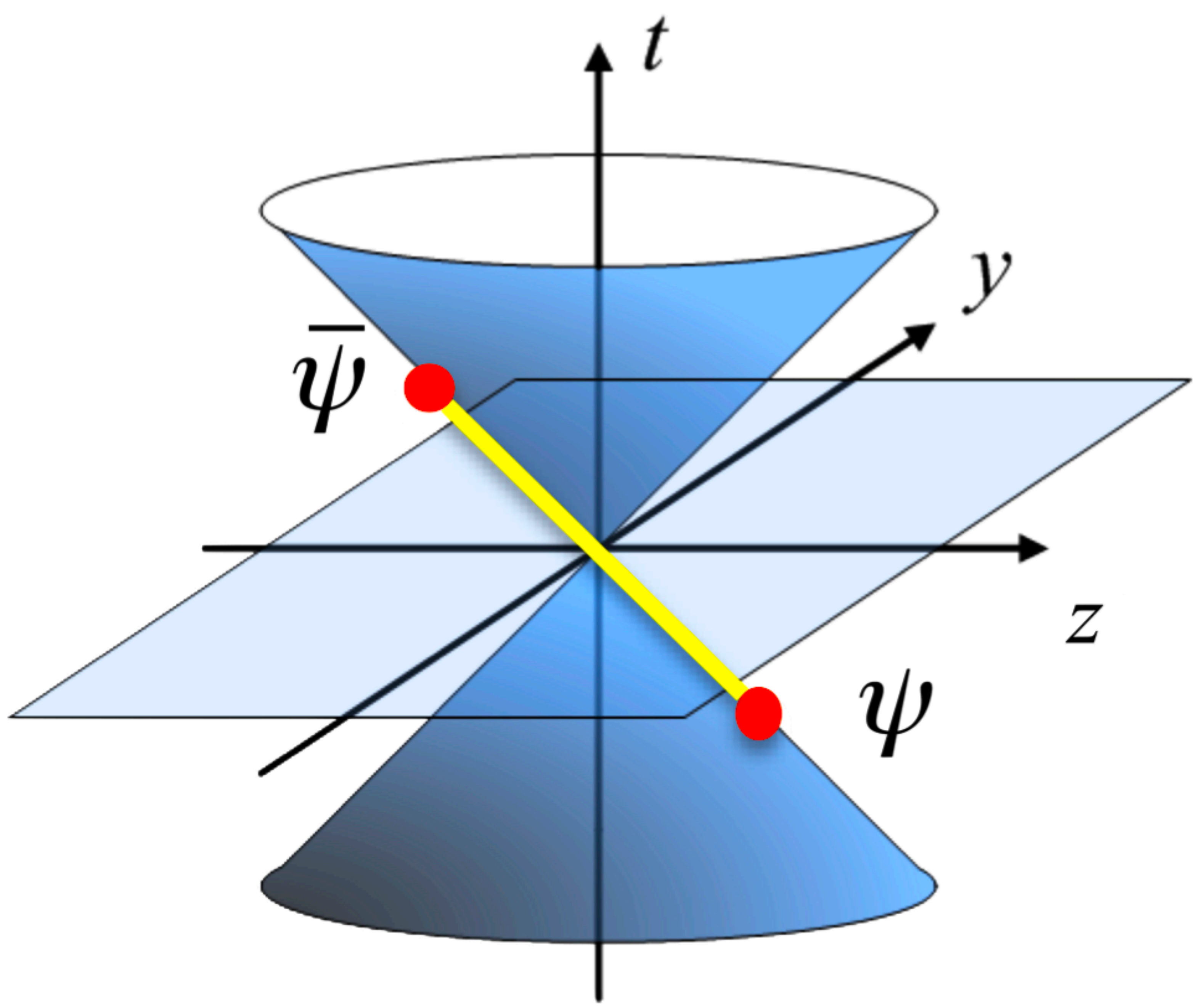
The collinear distributions are related to the target matrix elements of bilinear quark operators,

$$z + ct = 0, \quad z - ct \neq 0$$

$$f_1(x) = \frac{1}{2P^+}\int \frac{d\lambda}{2\pi} e^{iz^- P^+ x} \langle PS | \bar{q}(0)\gamma_+ q(z^-) | PS \rangle$$

$$g_1(x) = \frac{1}{2P^+}\int \frac{d\lambda}{2\pi} e^{iz^- P^+ x} \langle PS | \bar{q}(0)\gamma_+\gamma_5 q(z^-) | PS \rangle$$

$$h_1(x) = \frac{1}{2P^+}\int \frac{d\lambda}{2\pi} e^{iz^- P^+ x} \langle PS | \bar{q}(0)\sigma_{+\perp} i\gamma_5 q(z^-) | PS \rangle$$



Same computational cost!

Parton distribution functions

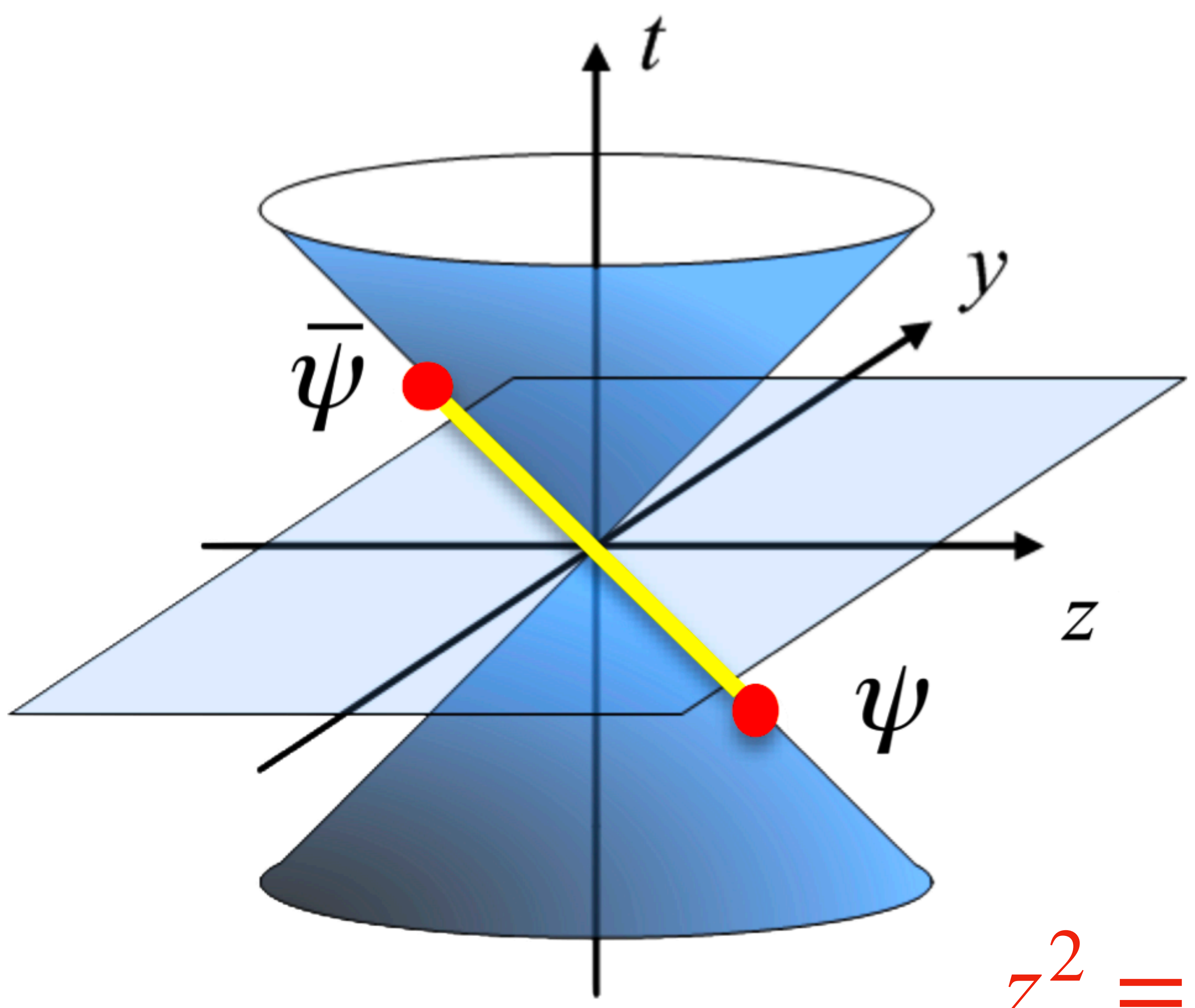
The collinear distributions are related to the target matrix elements of bilinear quark operators,

$$z + ct = 0, \quad z - ct \neq 0$$

$$f_1(x) = \frac{1}{2P^+}\int \frac{d\lambda}{2\pi} e^{iz^-P^+x}\langle PS|\bar{q}(0)\gamma_+ q(z^-)|PS\rangle$$

$$g_1(x) = \frac{1}{2P^+}\int \frac{d\lambda}{2\pi} e^{iz^-P^+x}\langle PS|\bar{q}(0)\gamma_+\gamma_5 q(z^-)|PS\rangle$$

$$h_1(x) = \frac{1}{2P^+}\int \frac{d\lambda}{2\pi} e^{iz^-P^+x}\langle PS|\bar{q}(0)\sigma_{+\perp}i\gamma_5 q(z^-)|PS\rangle$$



$$z^2 = 0$$

Same computational cost!

Light-cone correlation: Cannot be calculated on the lattice

Parton distribution functions

- Moments from leading-twist **local operators**.

$$\bar{q}\gamma^{\sigma}\overleftrightarrow{D}^{\alpha_1}\ldots\overleftrightarrow{D}^{\alpha_n}q$$

High dimensional

- **Large-momentum effective theory**: x-space matching of **quasi-PDF**.

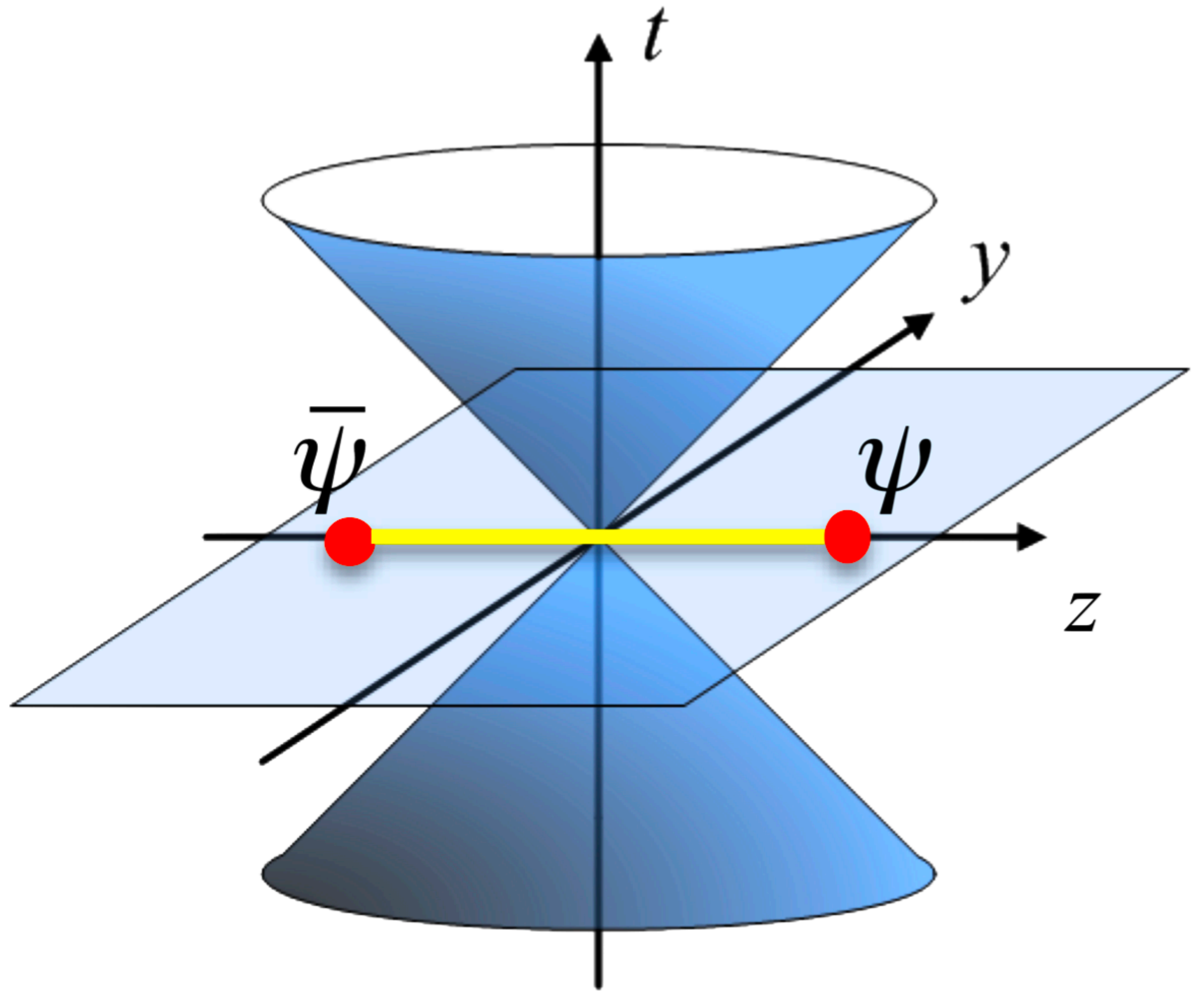
X. Ji, PRL 2013
X. Ji, et al, RevModPhys 2021

- **Short distance factorization** of the quasi-PDF matrix elements in position space or the **pseudo-PDF** approach.

A. Radyushkin, PRD 100 (2019)
A. Radyushkin, Int.J.Mod.Phys.A 2020

- …

$$t = 0, \quad z \neq 0$$



$$\langle PS|\bar{q}(-\tfrac{z}{2})\Gamma\mathscr{W}(-\tfrac{z}{2},\tfrac{z}{2})q(\tfrac{z}{2})|PS\rangle$$

$$z^2 \neq 0$$

Large momentum effective theory

Quasi-PDFs Factorization:

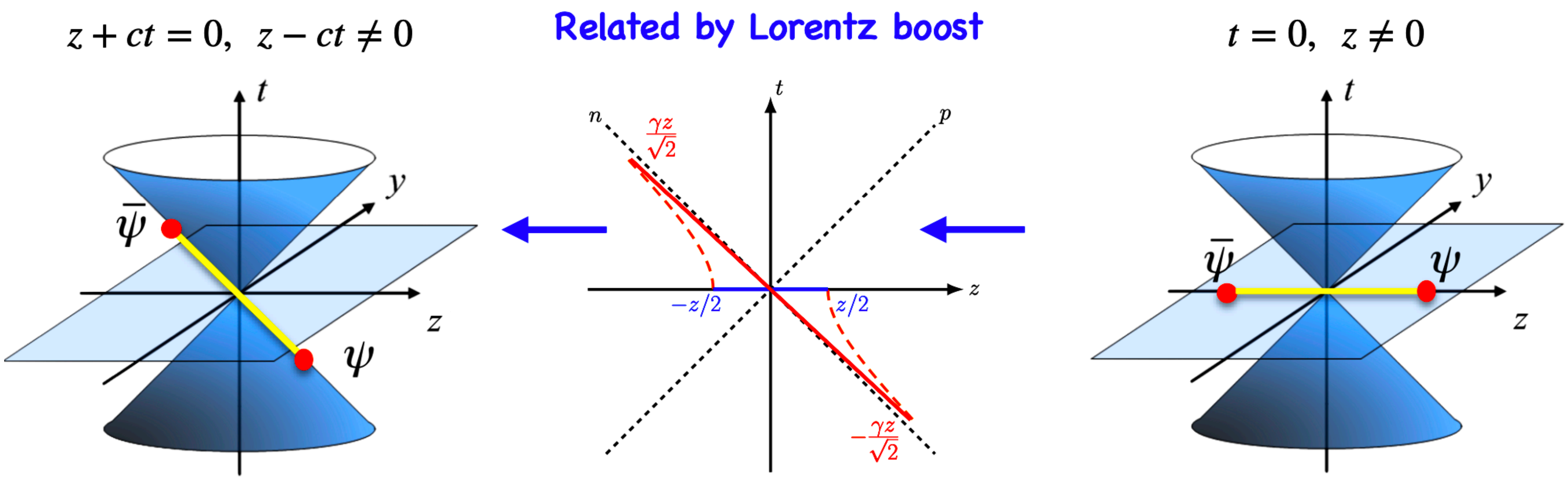
IR must cancel

$$\tilde{q}(x, P_z, \mu) = q(x, \mu) + \alpha_s(\mu)(\tilde{q}^{(1)}(x, P_z, \mu) - q^{(1)}(x, \mu))$$

$$= \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

large P_z is essential

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Xiong, X. Ji, et al, PRD 90 (2014);
- Y.-Q. Ma, et al, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, et al PRD98 (2018).
- X. Ji, Y. Zhao, et al, RMP 93 (2021).



Large momentum effective theory

Quasi-PDFs Factorization:

IR must cancel

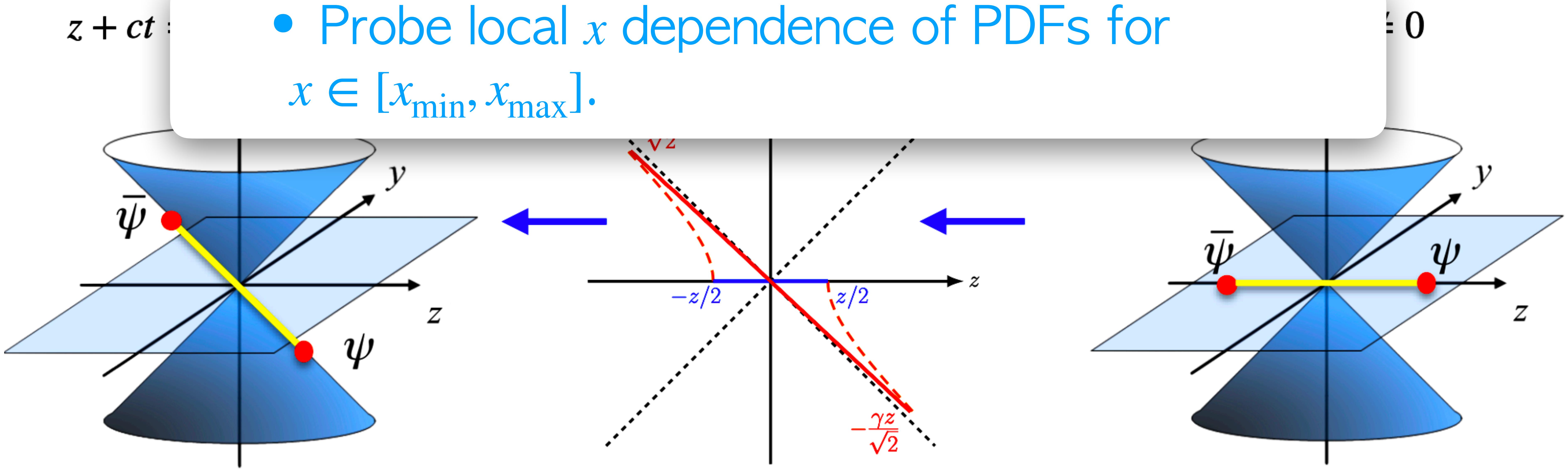
$$\tilde{q}(x, P_z, \mu) = q(x, \mu) + \alpha_s(\mu)(\tilde{q}^{(1)}(x, P_z, \mu) - q^{(1)}(x, \mu))$$

$$= \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{x^2 P_z^2}, \frac{\Lambda^2_{QCD}}{(1-x)^2 P_z^2}\right)$$

large P_z is essential

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Xiong, X. Ji, et al, PRD 90 (2014);
- Y.-Q. Ma, et al, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, et al PRD98 (2018).
- X. Ji, Y. Zhao, et al, RMP 93 (2021).

- Probe local x dependence of PDFs for $x \in [x_{\min}, x_{\max}]$.

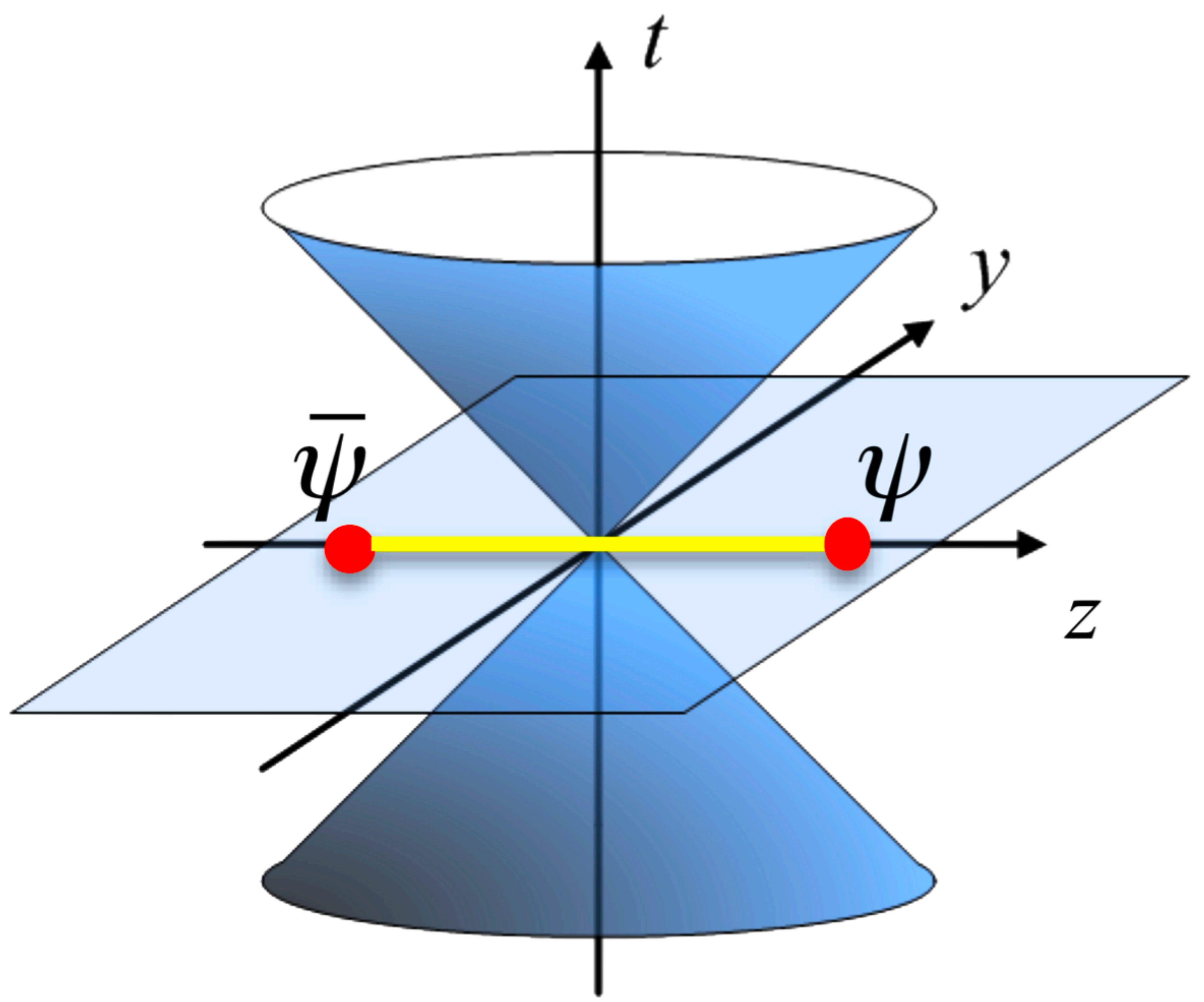


Short distance factorization

Coordinate-space factorization:

- V. Braun et al., EPJC 55 (2008)
- A. V. Radyushkin et al., PRD 96 (2017)
- Y. Ma et al., PRL 120 (2018)
- T. Izubuchi et al., PRD 98 (2018)

$$t = 0, \quad z \neq 0$$



$$h^R(z, P_z, \mu) = h^R(\lambda, z^2, \mu)$$

$$= \int_{-1}^{1} d\alpha \mathscr{C}(\alpha, \mu^2 z^2) \int_{-1}^{1} dy e^{-iy\lambda} q(y, \mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

$$= \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} C_n(z^2\mu^2) \langle x^n \rangle (\mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

Perturbative kernel

$$\langle PS | \bar{q}(-\frac{z}{2}) \Gamma \mathscr{W}(-\frac{z}{2}, \frac{z}{2}) q(\frac{z}{2}) | PS \rangle$$

- Probe global information of PDFs.

$$\langle x^n \rangle = \int_{-1}^{1} dx x^n q(x, \mu)$$

$$z^2 \neq 0$$

Short distance factorization

Coordinate-space factorization:

- V. Braun et al., EPJC 55 (2008)
- A. V. Radyushkin et al., PRD 96 (2017)
- Y. Ma et al., PRL 120 (2018)
- T. Izubuchi et al., PRD 98 (2018)

$$h^R(z, P_z, \mu) = h^R(\lambda, z^2, \mu)$$

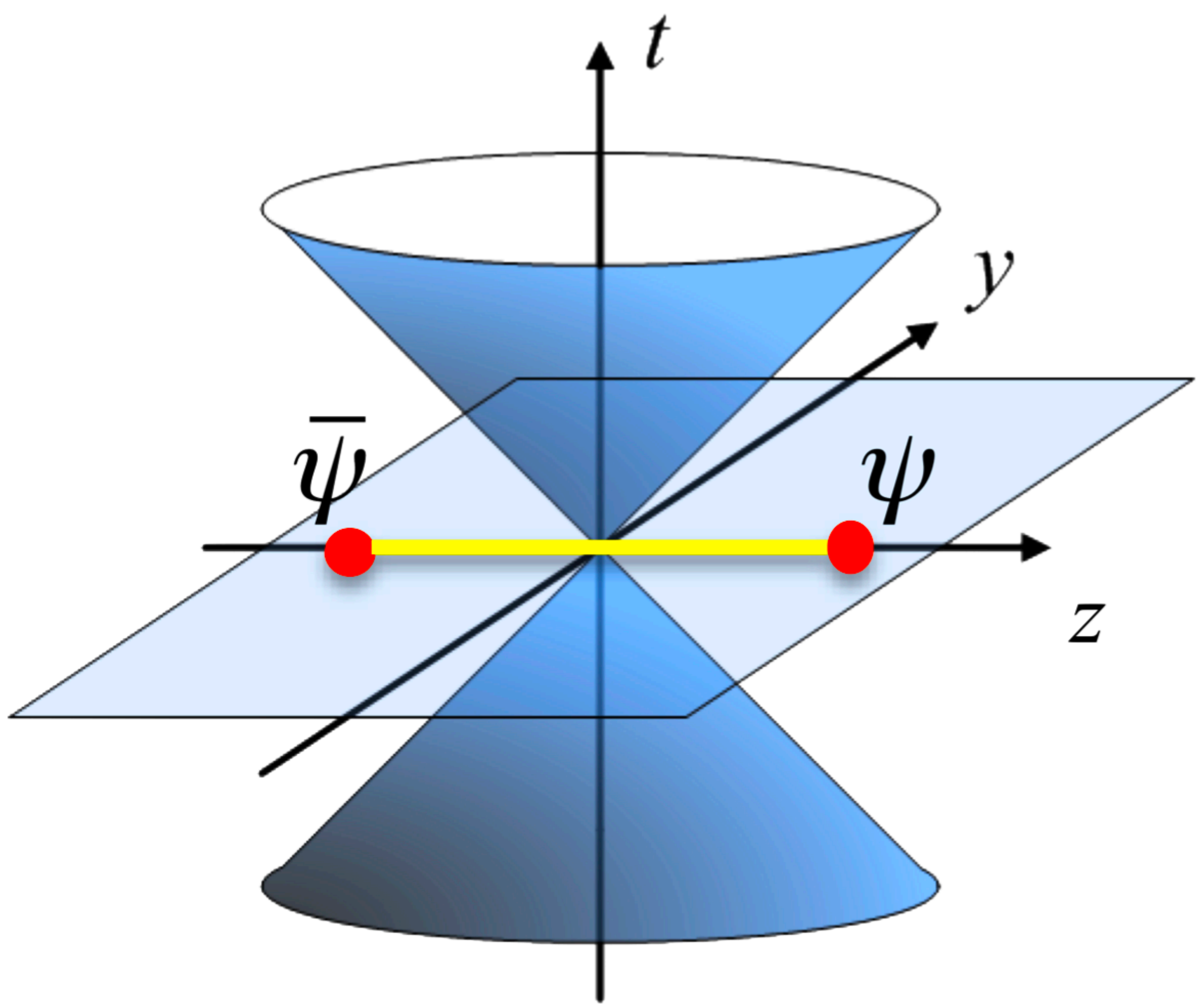
$$= \int_{-1}^{1} d\alpha \mathscr{C}(\alpha, \mu^2 z^2) \int_{-1}^{1} dy e^{-iy\lambda} q(y, \mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

$$= \sum_{n=0}^{\infty} \frac{(-izP)^n}{n!} C_n(z^2\mu^2) \langle x^n \rangle(\mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

Perturbative kernel

- The perturbative matching is valid in short range of z.
- The information is limited by the range of finite $\lambda = zP_z$.

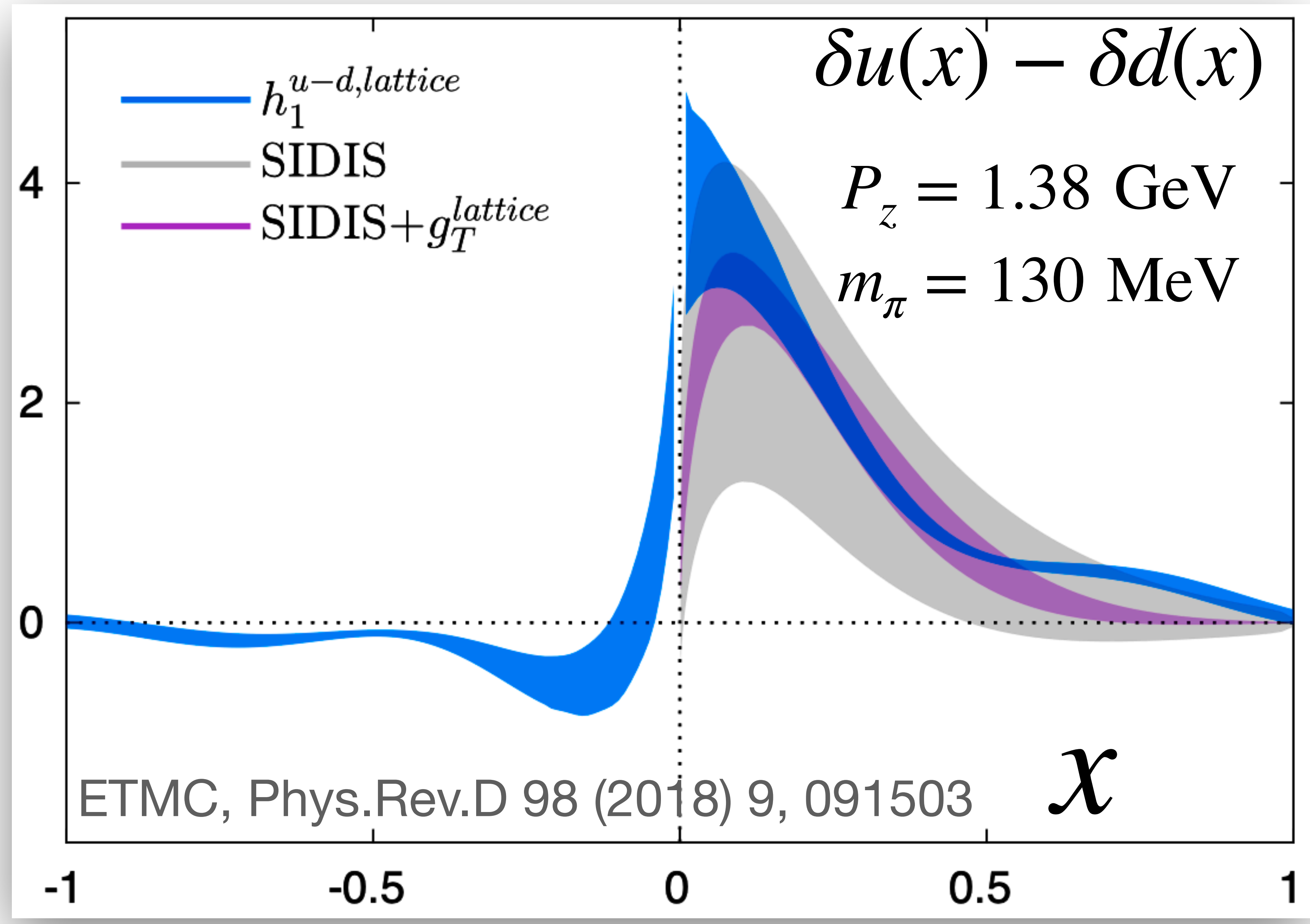
$$t = 0, \quad z \neq 0$$

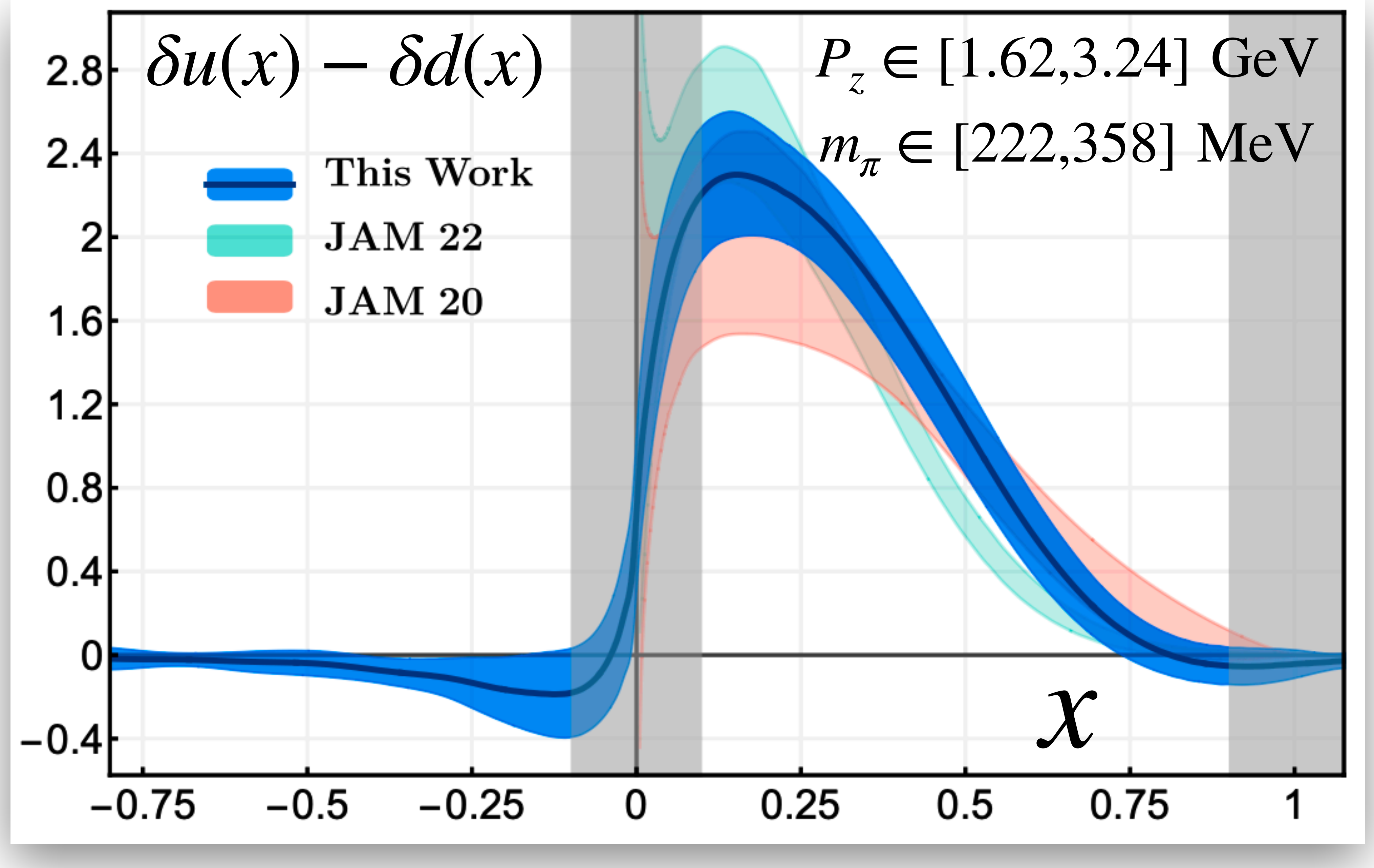


$$\langle PS | \bar{q}(-\tfrac{z}{2}) \Gamma \mathscr{W}(-\tfrac{z}{2}, \tfrac{z}{2}) q(\tfrac{z}{2}) | PS \rangle$$

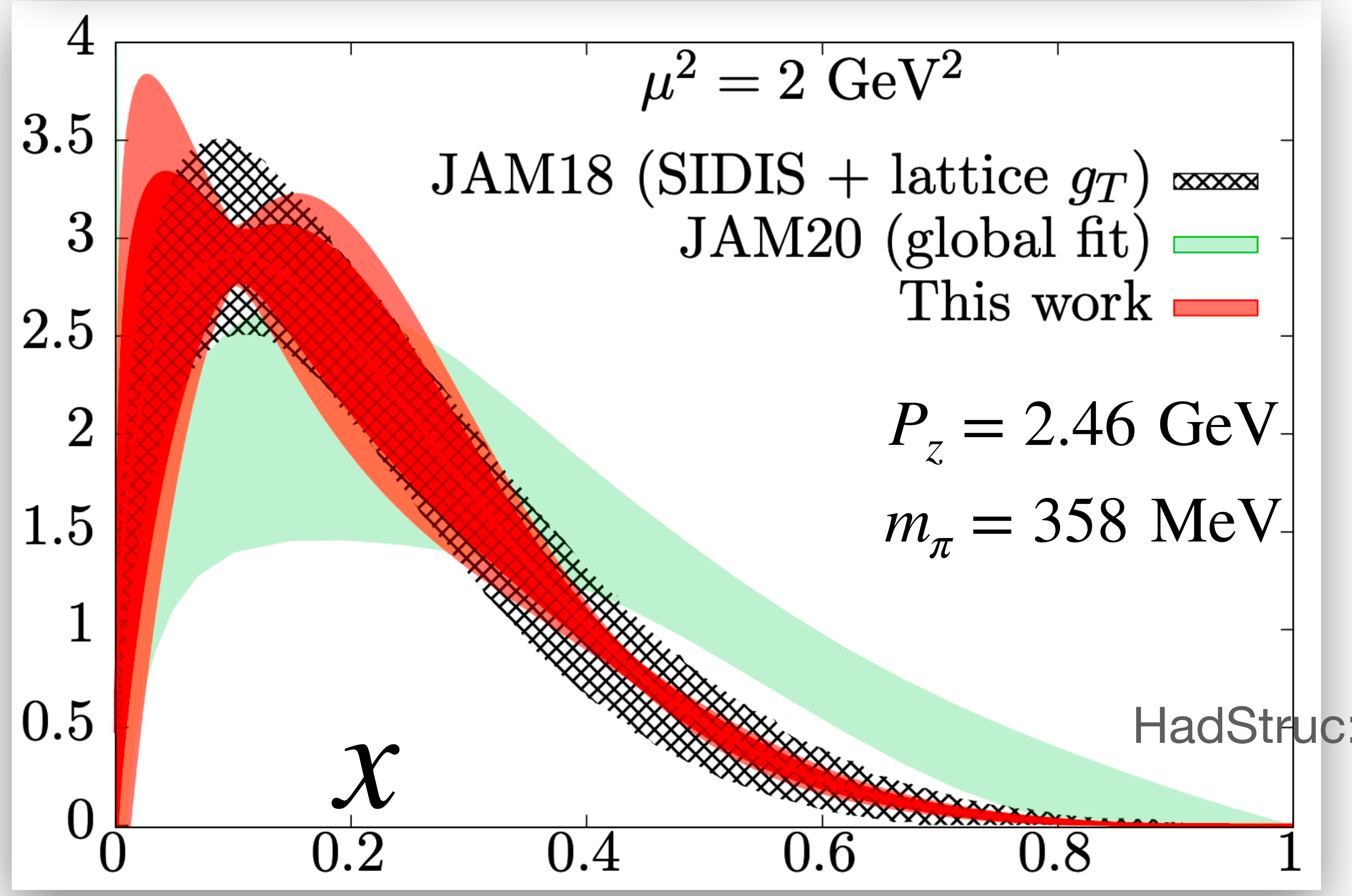
$$z^2 \neq 0$$

Transversity distribution





LPC: arXiv 2208.08008



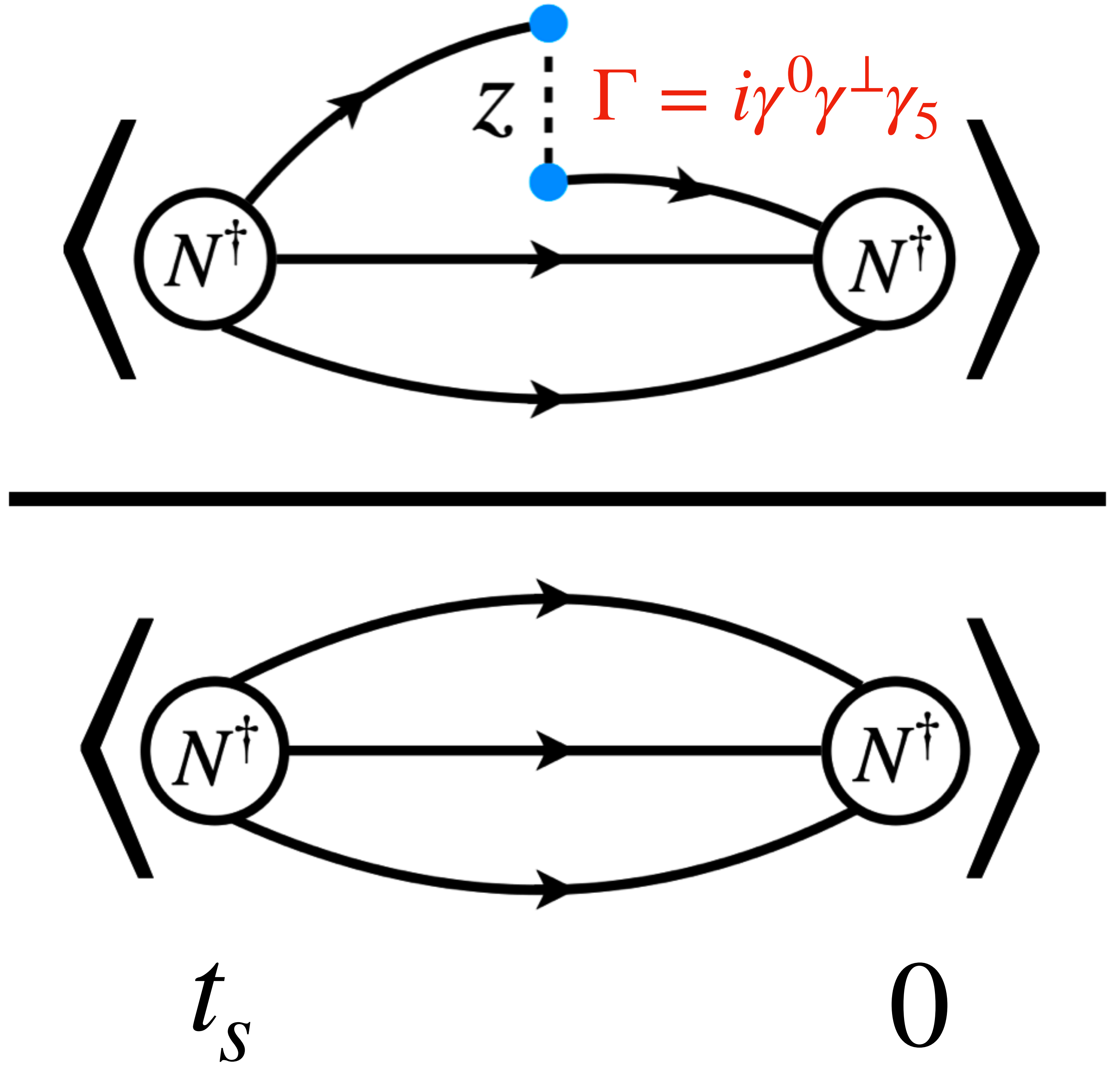
HadStruc: Phys.Rev.D 105 (2022) 3, 034507

Lattice calculations show compatible precision, which can potentially provide complementary information.

Transversity distribution

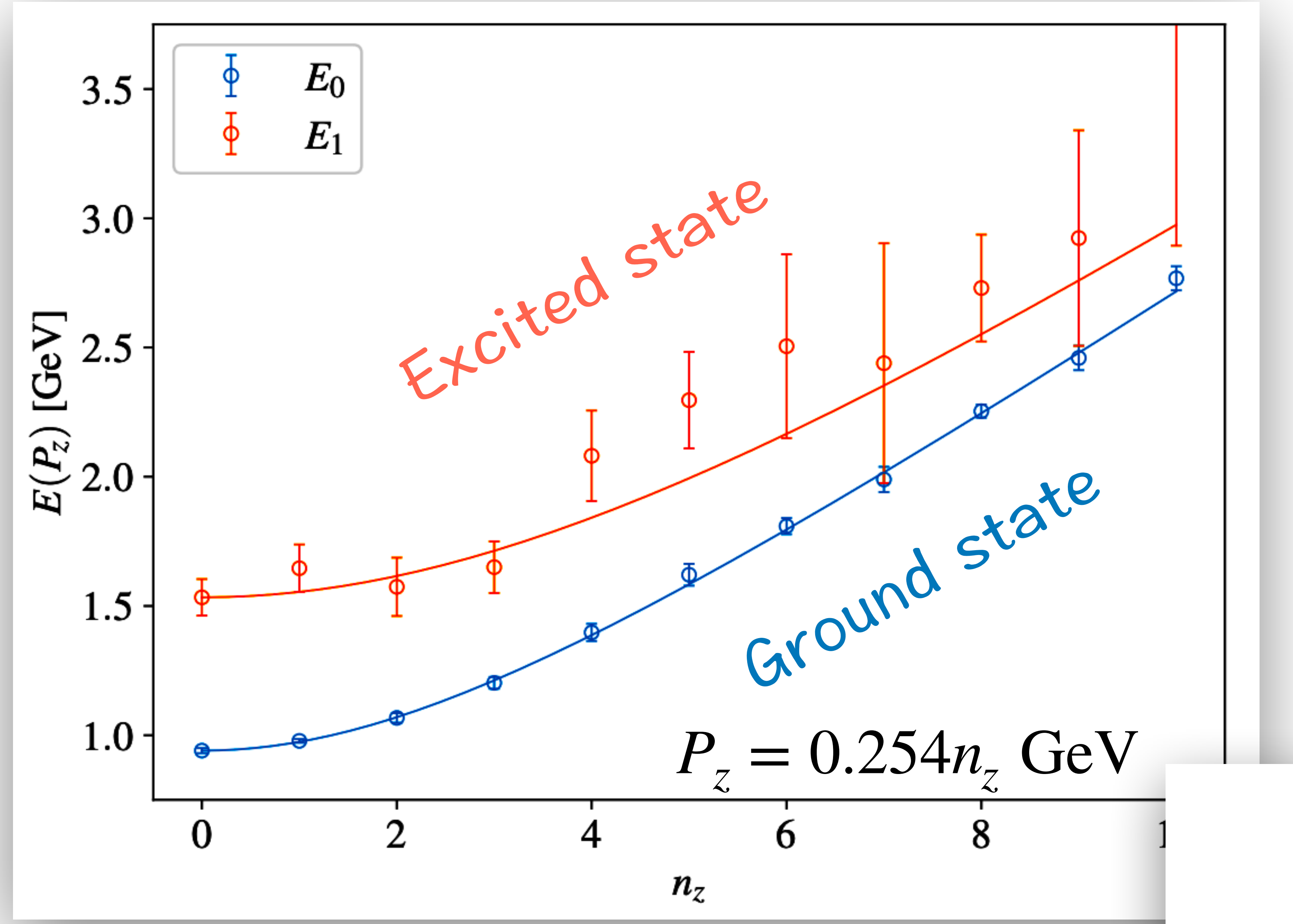
Lattice setup:

- ➡ Clover-fermion on 2+1f HISQ gauge ensembles
- ➡ $64^3 \times 64$, a = 0.076 fm, $m_\pi = 140$ MeV
- ➡ 4 momentum from 0 to 1.52 GeV using boosted smearing $P_3 = \frac{2\pi}{L_s a} n_3 \approx 0.254 \times n_3\ \mathrm{GeV}$
- ➡ 1-HYP smearing for Wilson line
- ➡ 350 configurations & AMA



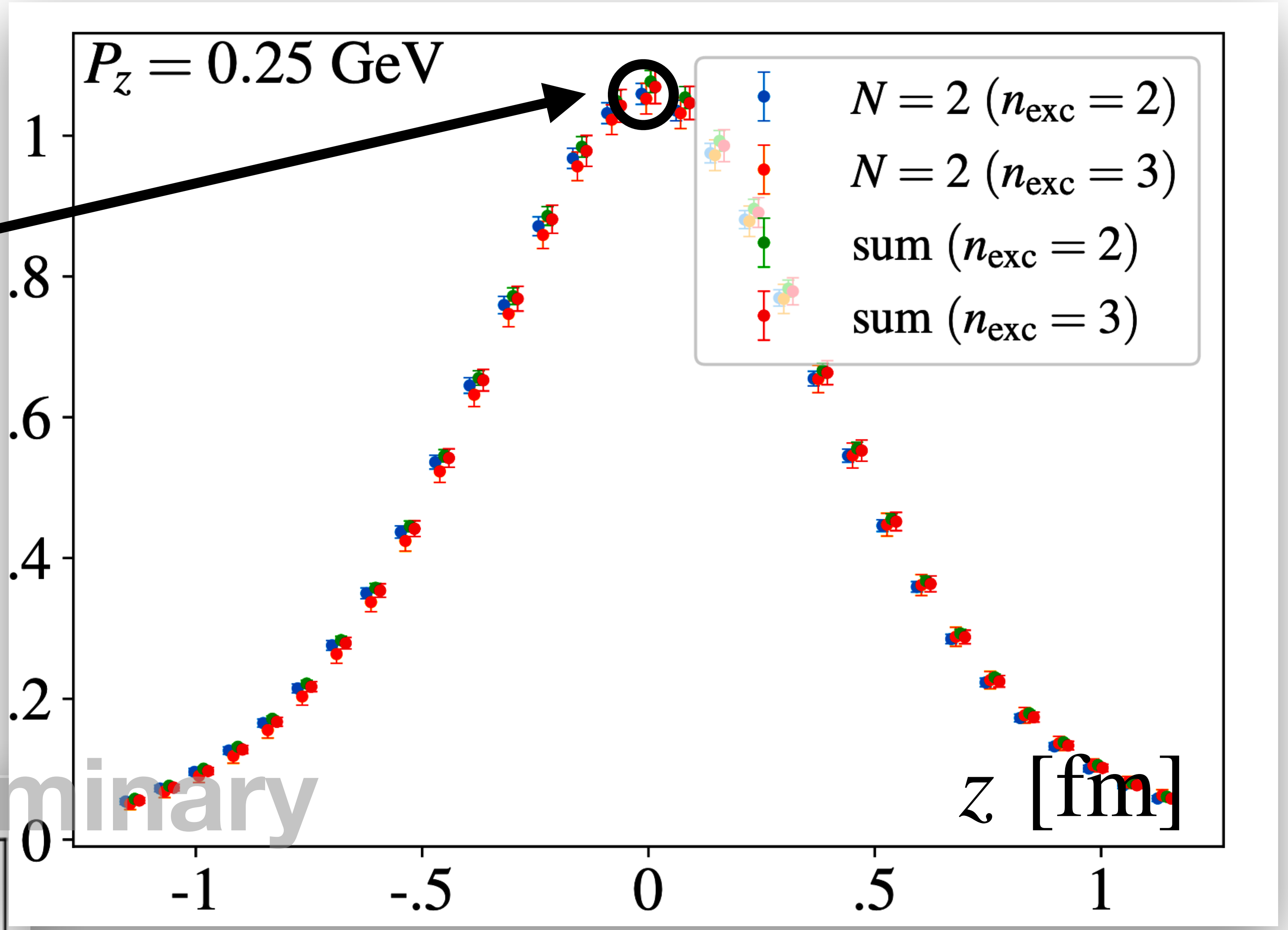
$$R(t_s, \tau) = \frac{\langle N(t_s)\mathcal{O}^f_\Gamma(\vec{z}, \tau)\overline{N}(0)\rangle}{\langle N(t_s)\overline{N}(0)\rangle} \xrightarrow{t_s \to \infty} h^B(z, P_z)$$

Bare quasi-PDF matrix elements

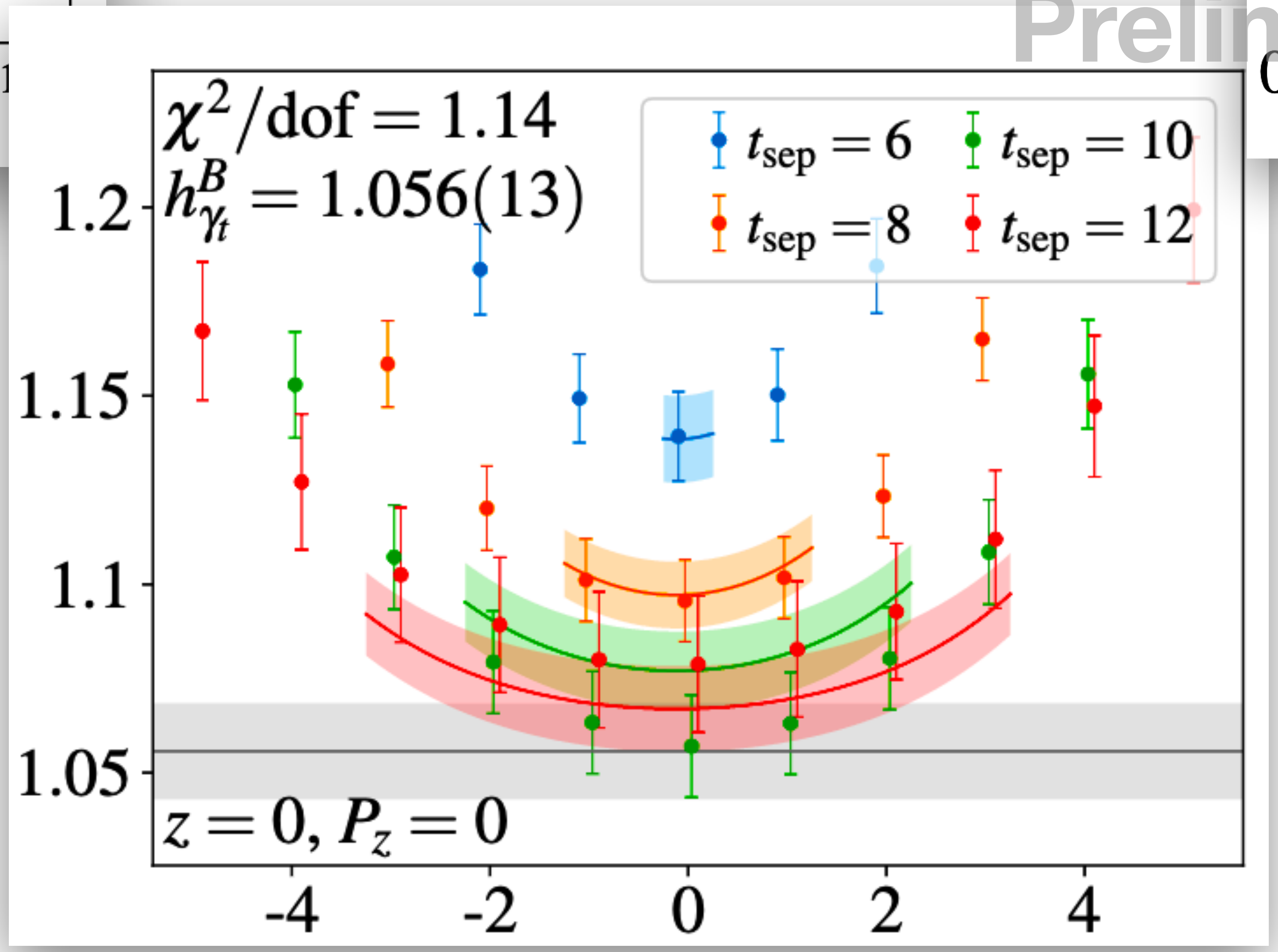


$$h^B(z=0,\,P_z) = g_T/Z_T$$

$$g_T^{u-d} = 1.046(20)_{\mathrm{stat}}$$



$h^B(z,\,P_z)$

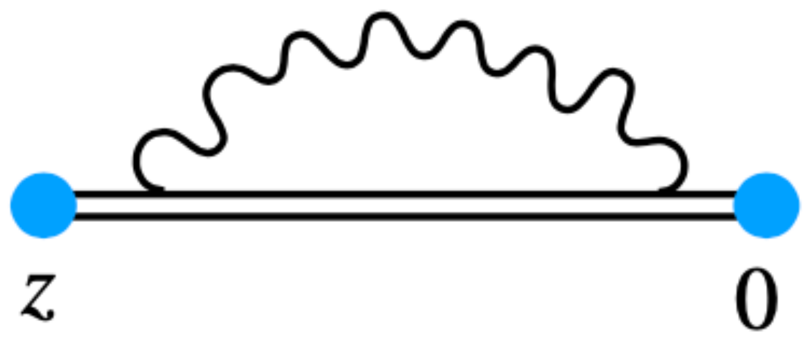


$$R(t_s,\tau) \xrightarrow{t_s\to\infty} h^B(z,\,P_z)$$

Bare matrix elements and renormalization

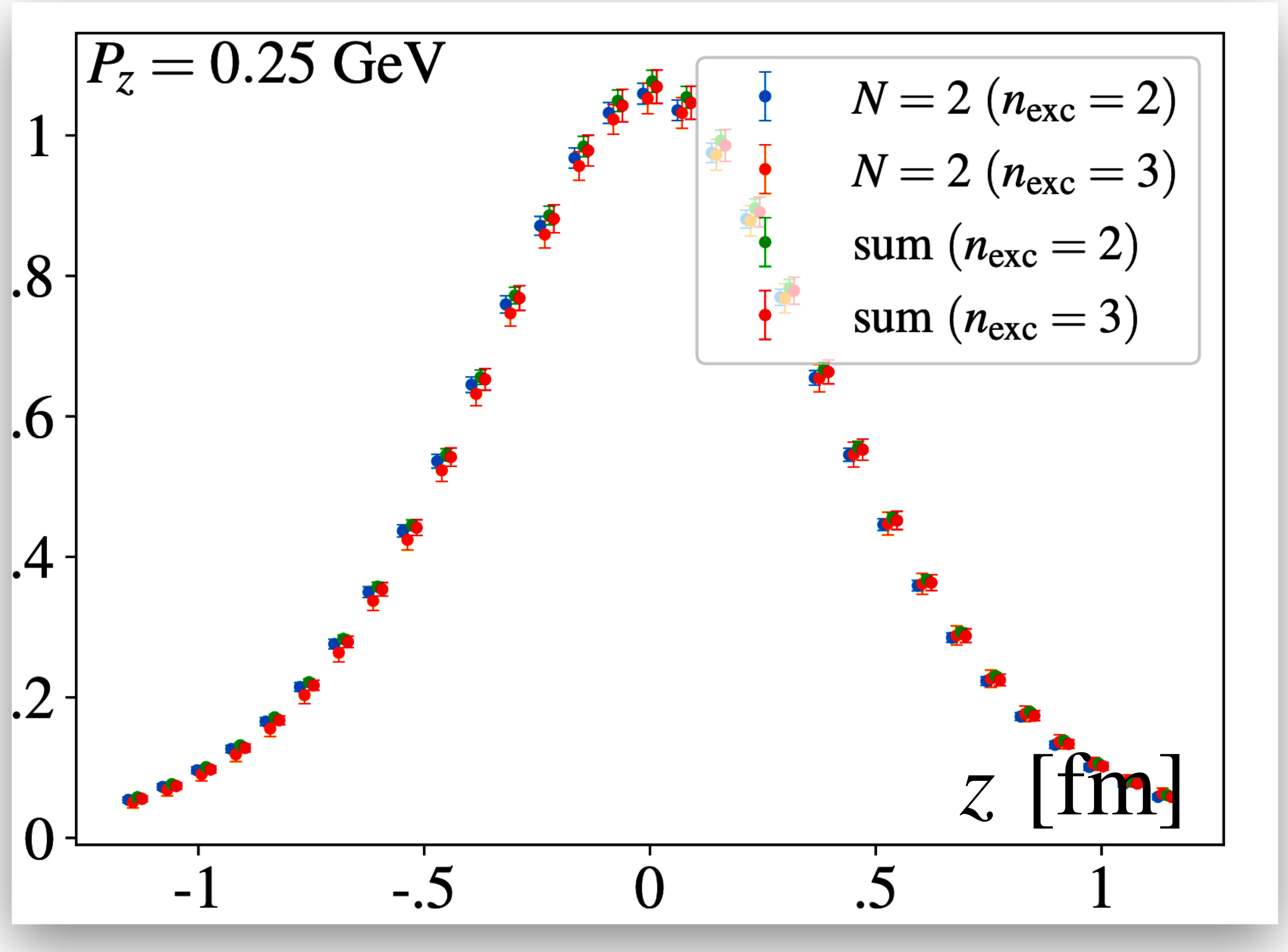
The operator can be multiplicatively renormalized:

- X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
- T. Ishikawa, et al, PRD 96 (2017)



$$= \delta m(a)|z| \propto \frac{|z|}{a}$$

Wilson-line self energy



$h^B(z, P_z)$

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m(a)|z|} Z(a) [\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

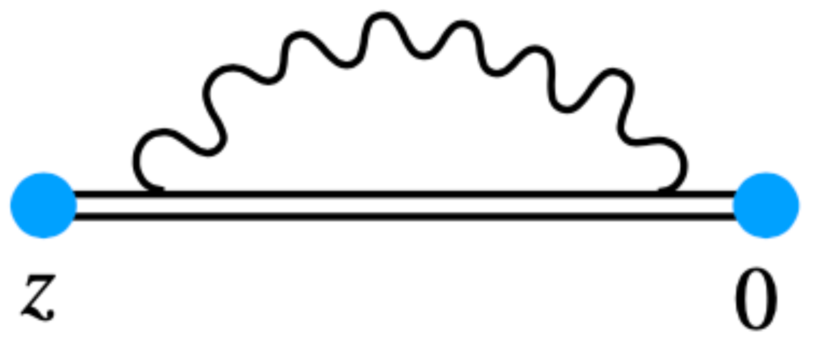
$$\delta m = m_{-1}/a + m_0$$

Renormalon ambiguity

Bare matrix elements and renormalization

The operator can be multiplicatively renormalized:

- X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
- T. Ishikawa, et al, PRD 96 (2017)



$$= \delta m(a)|z| \propto \frac{|z|}{a}$$

Wilson-line self energy

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m(a)|z|} Z(a)[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

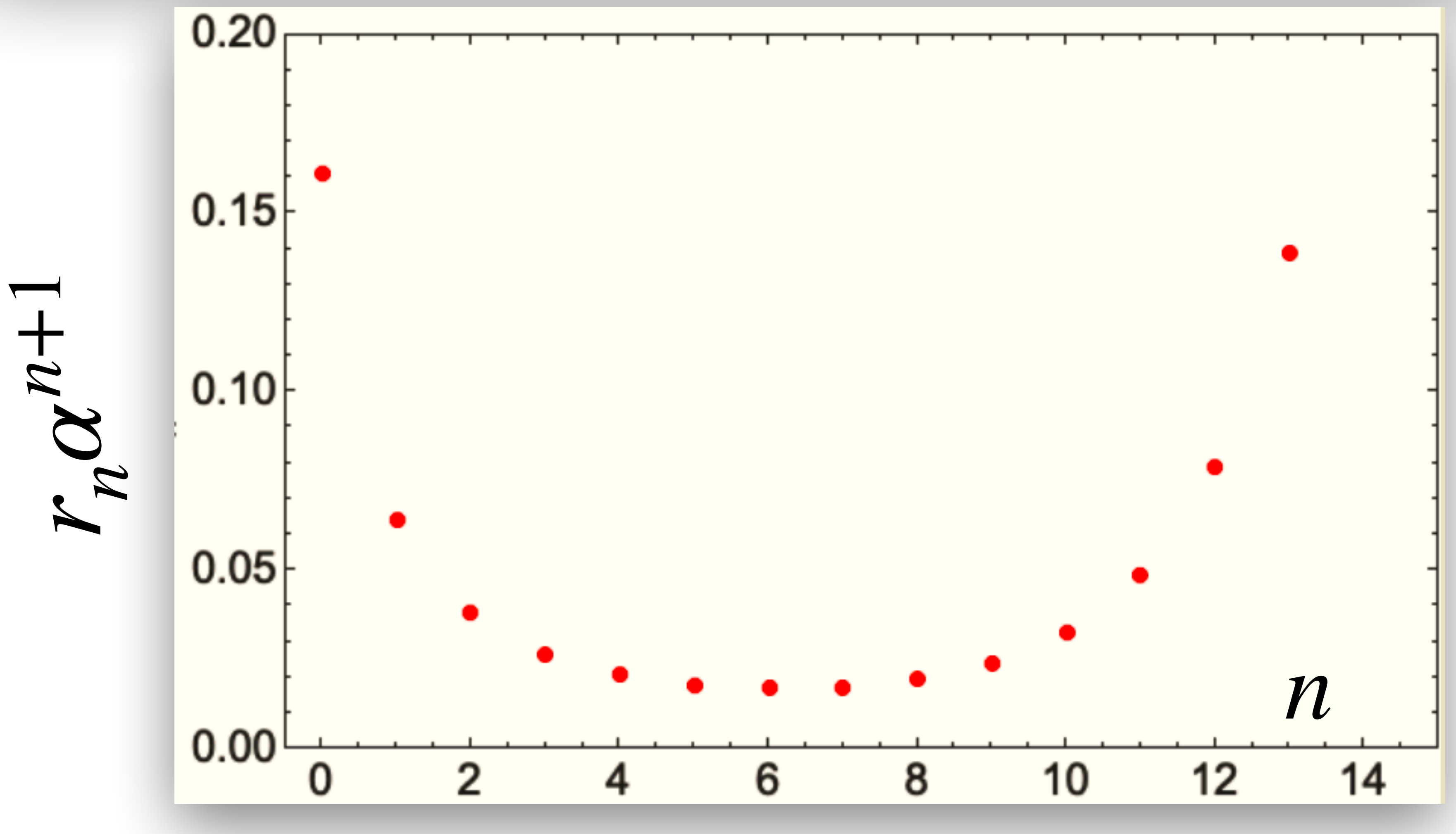
$$\delta m = m_{-1}/a + m_0$$

Renormalon ambiguity

$$\delta m = \frac{1}{a}\sum \alpha_s^{n+1}(a) r_n$$

When n is large,

- $r_n \sim n!$
- Divergent for any α_s
- No well-defined sum

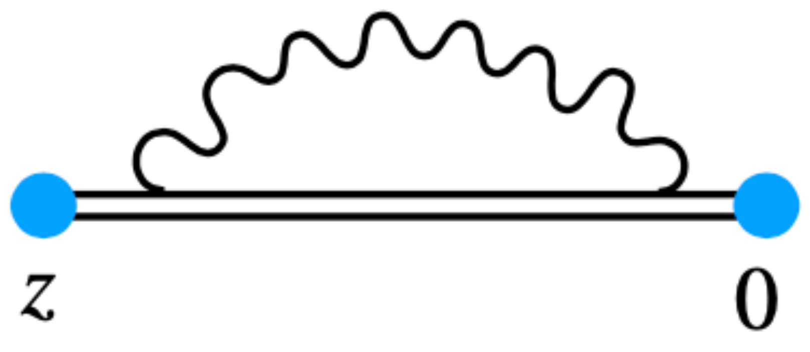


G. Bali, et al., Phys.Rev.D 87 (2013) 094517
R. Zhang, et al., arXiv: 2305.05212

Ratio-scheme renormalization and SDF

The operator can be multiplicatively renormalized:

- X. Ji, J. H. Zhang and Y. Zhao, PRL120 (2018)
- J. Green, K. Jansen and F. Steffens, PRL121 (2018)
- T. Ishikawa, et al, PRD 96 (2017)



$$= \delta m(a)|z| \propto \frac{|z|}{a}$$

Wilson-line self energy

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m(a)|z|}Z(a)[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

$$\delta m = m_{-1}/a + m_0$$

Renormalon ambiguity

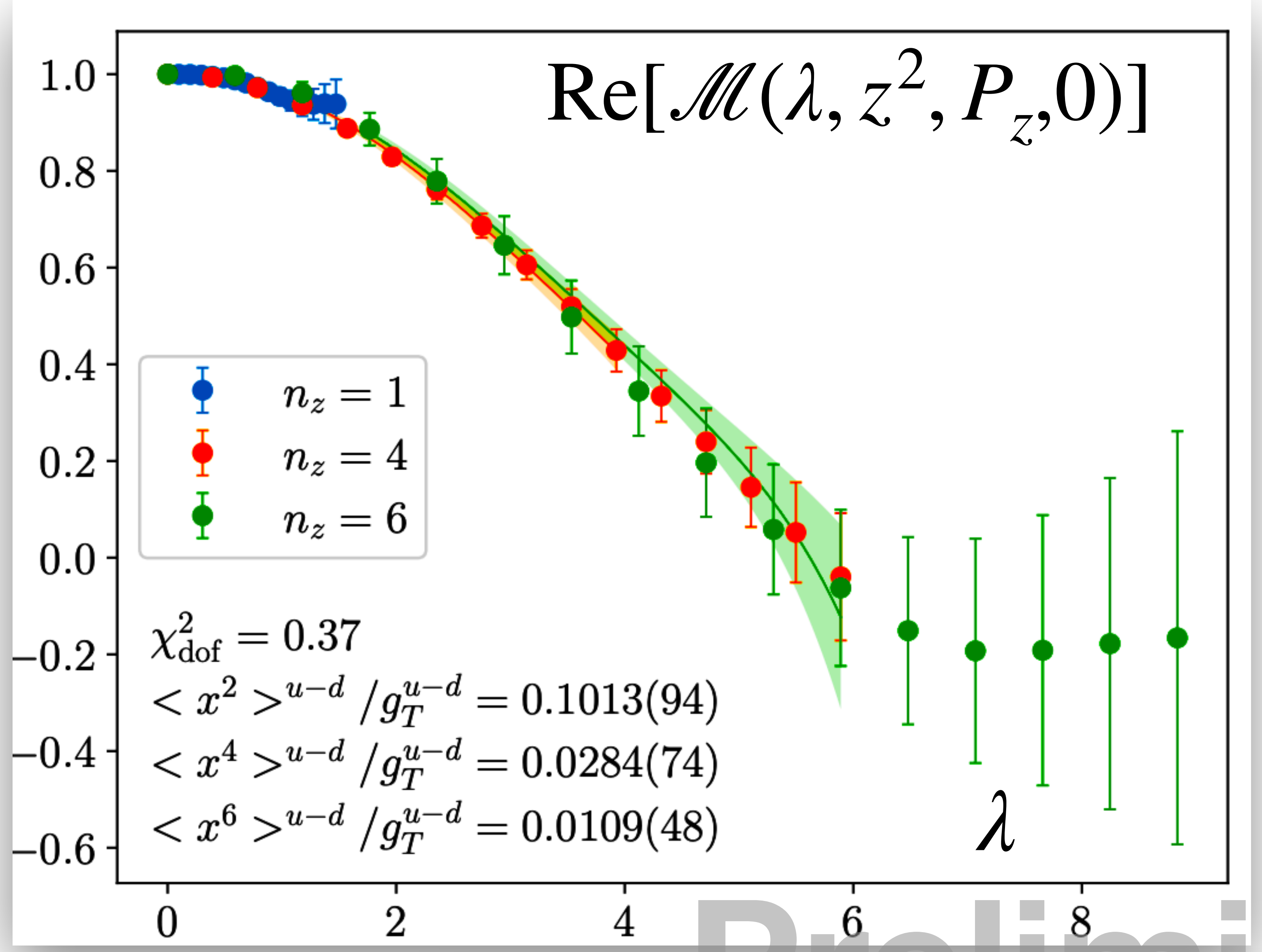
Ratio scheme renormalization

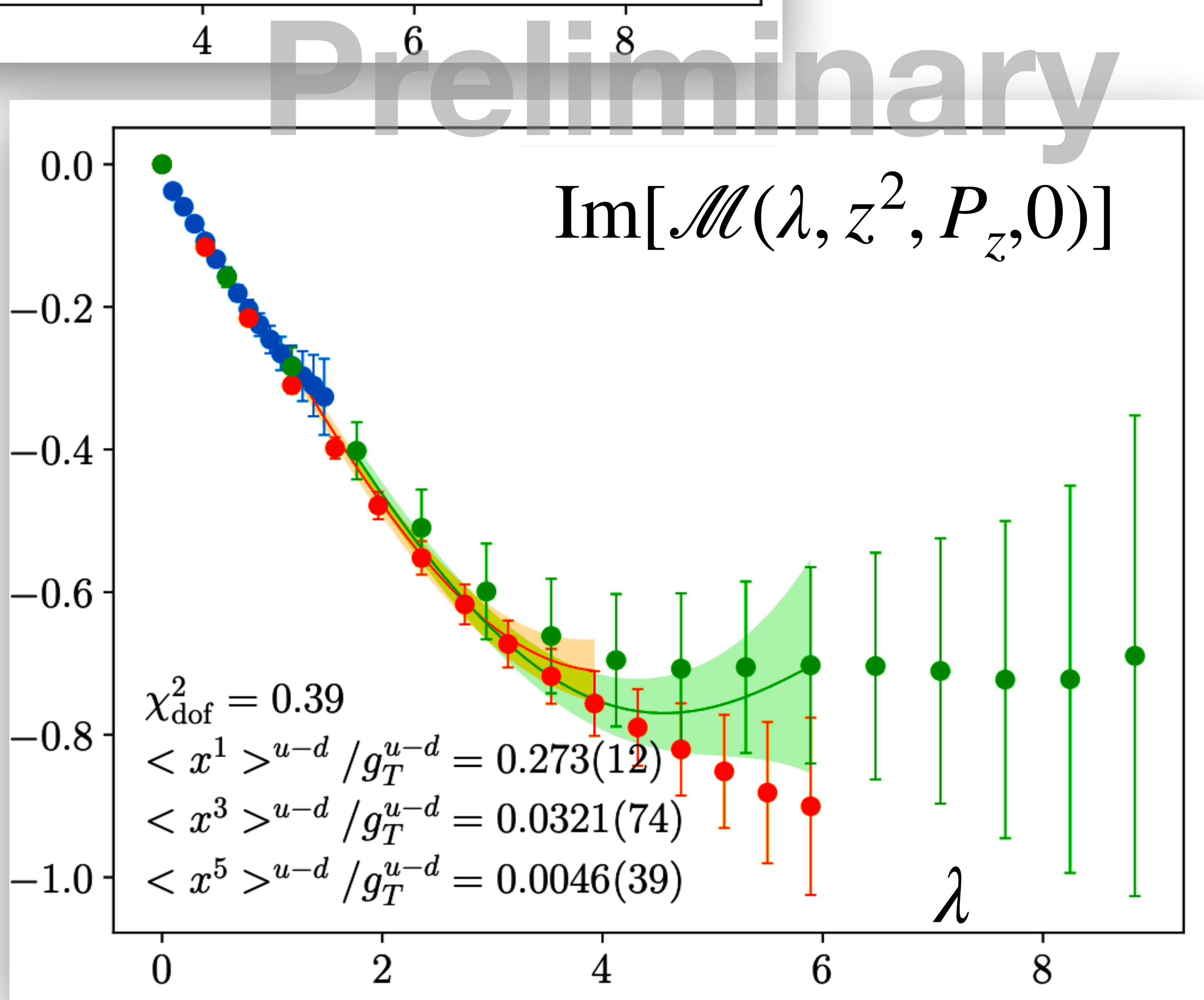
A. V. Radyushkin, PRD 2017
K. Orginos, et al, PRD 96, 2017
Bálint Joó, et al, PRL125, 2020
X. Gao, et al, PRD 102, 2020
Z. Fan, et al, PRD 102, 2020

$$M(z, P_z, P_z^0) = \frac{h^B(z, P_z, a)}{h^B(z, P_z^0, a)} = \frac{h^R(z, P_z, \mu)}{h^R(z, P_z^0, \mu)}$$

- Hadron state independent.
- Construct the RG-invariant ratio.

Ratio-scheme renormalization and SDF





- RG invariant double ratio.

$$\mathcal{M}(\lambda, z^2, P_z, P_z^0) \equiv \left(\frac{h^B(z, P_z)}{h^B(z, P_z^0)}\right)\left(\frac{h^B(0, P_z^0)}{h^B(0, P_z)}\right) = \left(\frac{h^R(z, P_z)}{h^R(z, P_z^0)}\right)\left(\frac{h^R(0, P_z^0)}{h^R(0, P_z)}\right)$$

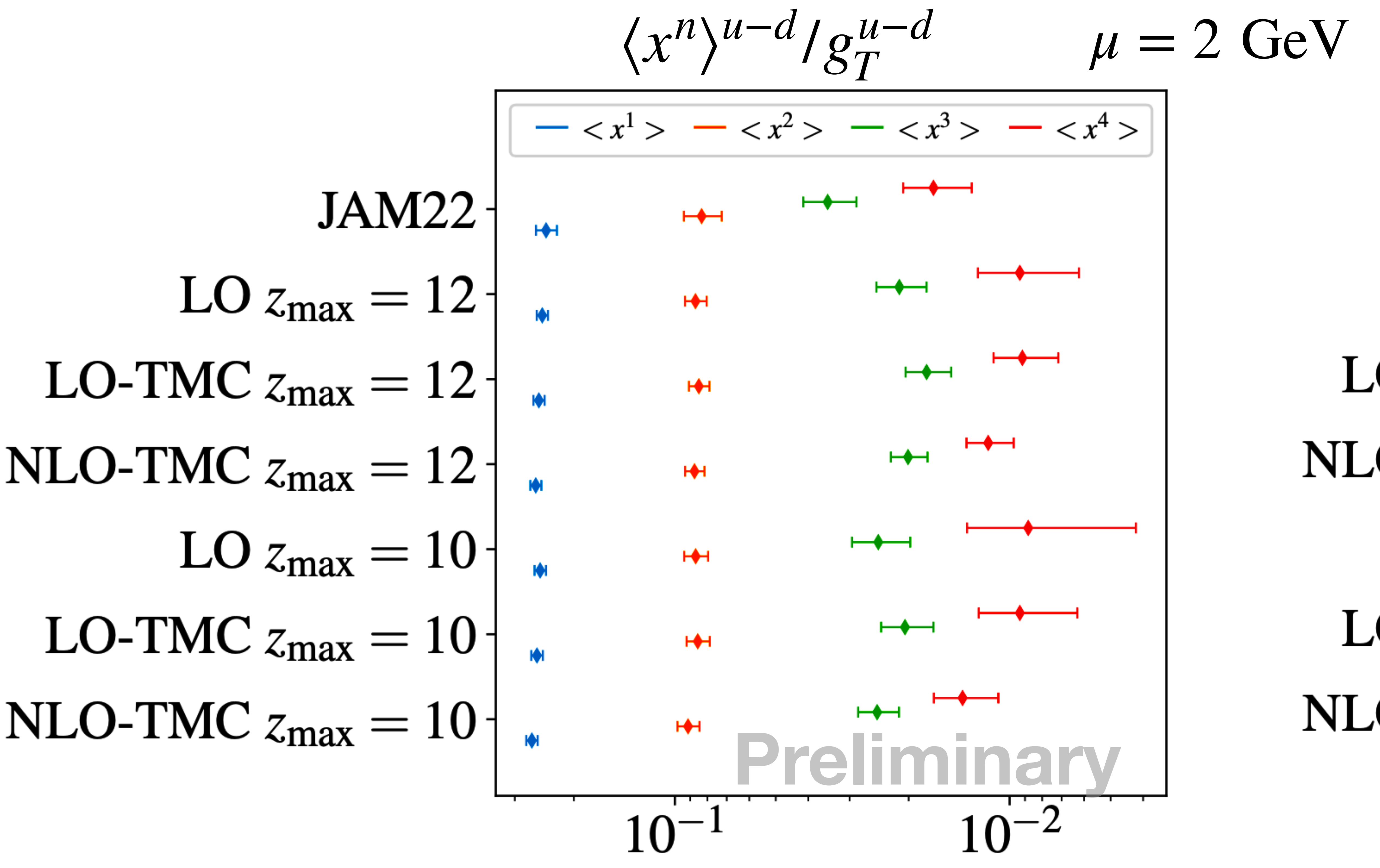
$$\lambda = zP_z$$

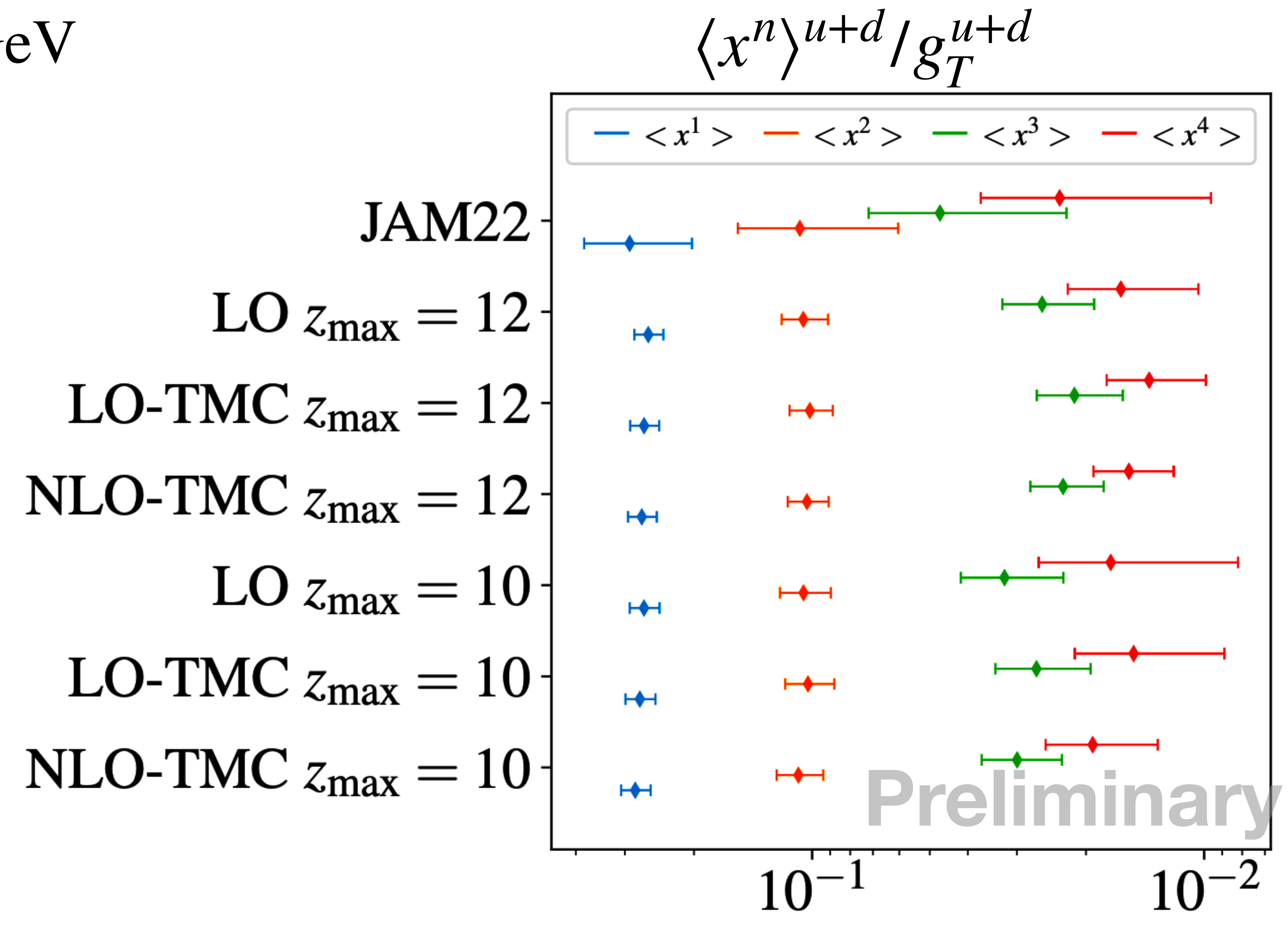
- Insert the twist-2 OPE formula.

$$\mathcal{M}(\lambda, z^2, P_z, 0) = \sum_{n=0}^{\infty} \frac{(-izP_z)^n}{n!} \frac{C_n(z^2\mu^2)}{C_0(z^2\mu^2)} \frac{\langle x^n \rangle}{g_T}$$

  - Real part: even moments.
  - Imaginary part: odd moments.

Mellin moments of transversity PDF





- Disconnected diagram and mixing with gluon neglected.

- The higher moments are factorially suppressed, but can be systematically constrained with better data quality and higher momentum P_z.

Transversity PDF from model reconstruction

Short distance factorization:

$$h^R(z, P_z, \mu) = \int_{-1}^{1} d\alpha \mathscr{C}(\alpha, \mu^2 z^2) \int_{-1}^{1} dy e^{-iy\lambda} q(y, \mu) + \mathcal{O}(z^2 \Lambda_{QCD}^2)$$

To reconstruct the x dependence of PDFs, one need to introduce additional prior knowledges or reasonable choice of models, e.g.

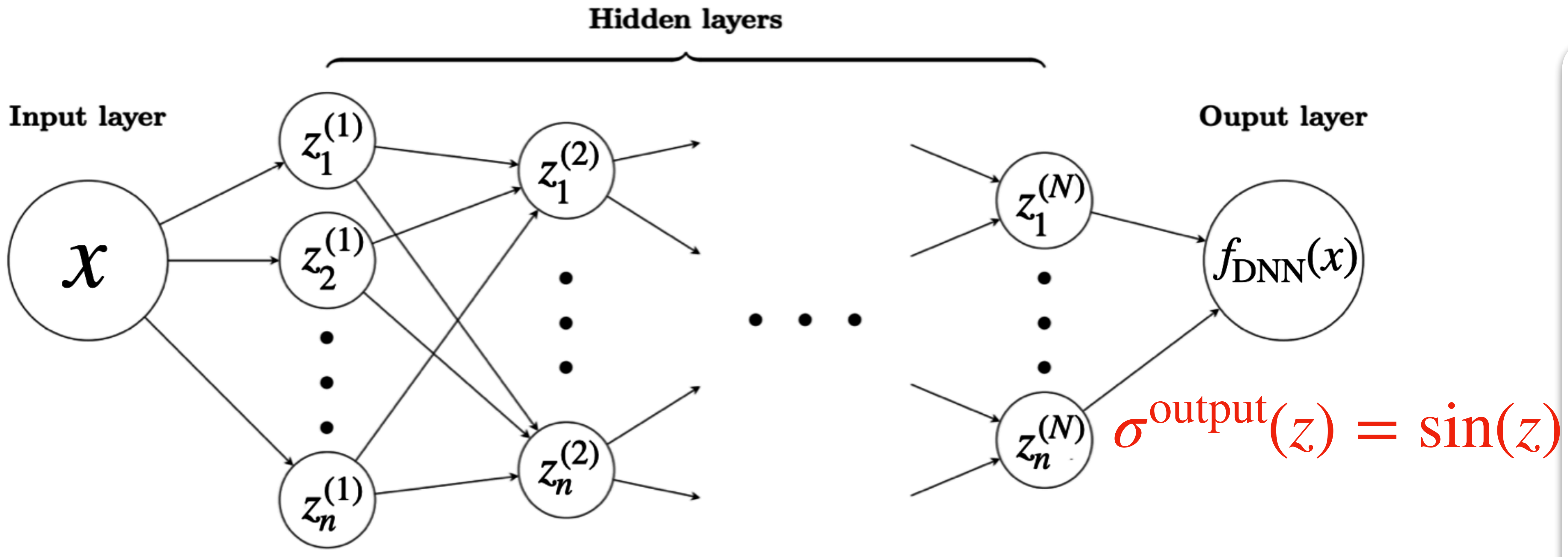
$$q(x) = Ax^{\alpha}(1-x)^{\beta}(1 + \text{subleading terms})$$

Transversity PDF from model reconstruction

To balance the model constrain and bias, the most flexible way could be the deep neural network (DNN)

$$q(x;\alpha,\beta,\boldsymbol{\theta}) \equiv Ax^{\alpha}(1-x)^{\beta}[1+\delta(x)\cdot f_{\rm DNN}(x;\boldsymbol{\theta})]$$

BNL-ANL, Phys.Rev.D 107 (2023) 7, 074509



- The contribution of DNN is limited by $|\delta(x)\cdot f_{\rm DNN}(x)| \lesssim \delta(x)$.
- By choosing a proper $\delta(x)$, one can control the size of DNN parametrized sub-leading contribution at each specific x.

Hidden layer:

Linear transformation: $z_i^{(l)} = b_i^{(l)} + \sum_j W_{ij}^{(l)} a_j^{(l-1)}$

Activation: $a_i^{(l)} = \sigma_{\rm elu}^{(l)}(z_i^{(l)}) = \theta(-z)(e^z-1) + \theta(z)z$

Transversity PDF from model reconstruction

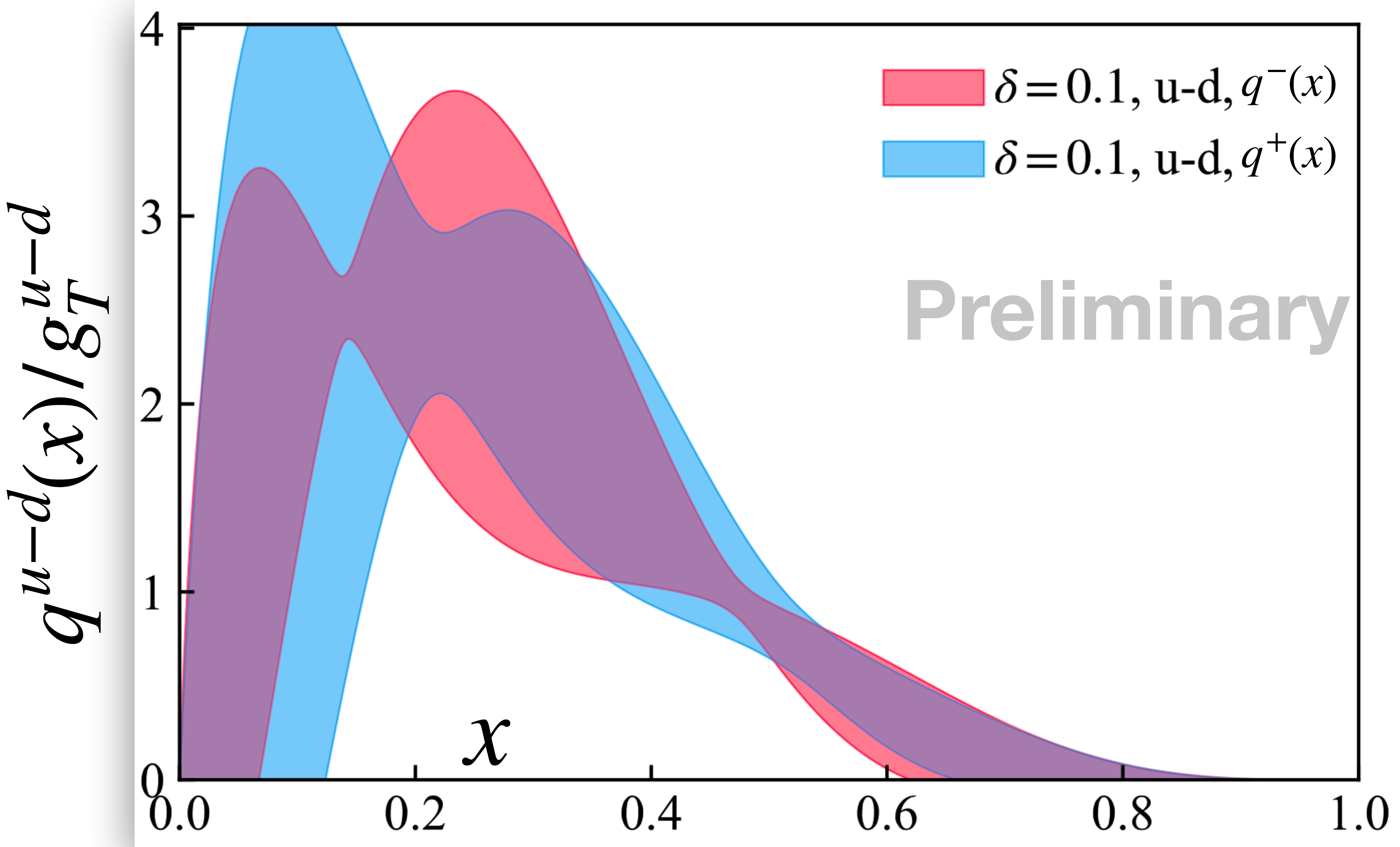
$$\mathcal{M}(\lambda, z^2, P_z, 0) = \sum_{n=0}^{\infty} \frac{(-izP_z)^n}{n!} \frac{C_n(z^2\mu^2)}{C_0(z^2\mu^2)} \frac{\langle x^n \rangle}{g_T}$$

Real part:

$$q^-(x) \equiv q^q(x) - q^{\bar{q}}(x)$$

Imaginary part:

$$q^+(x) \equiv q^q(x) + q^{\bar{q}}(x)$$

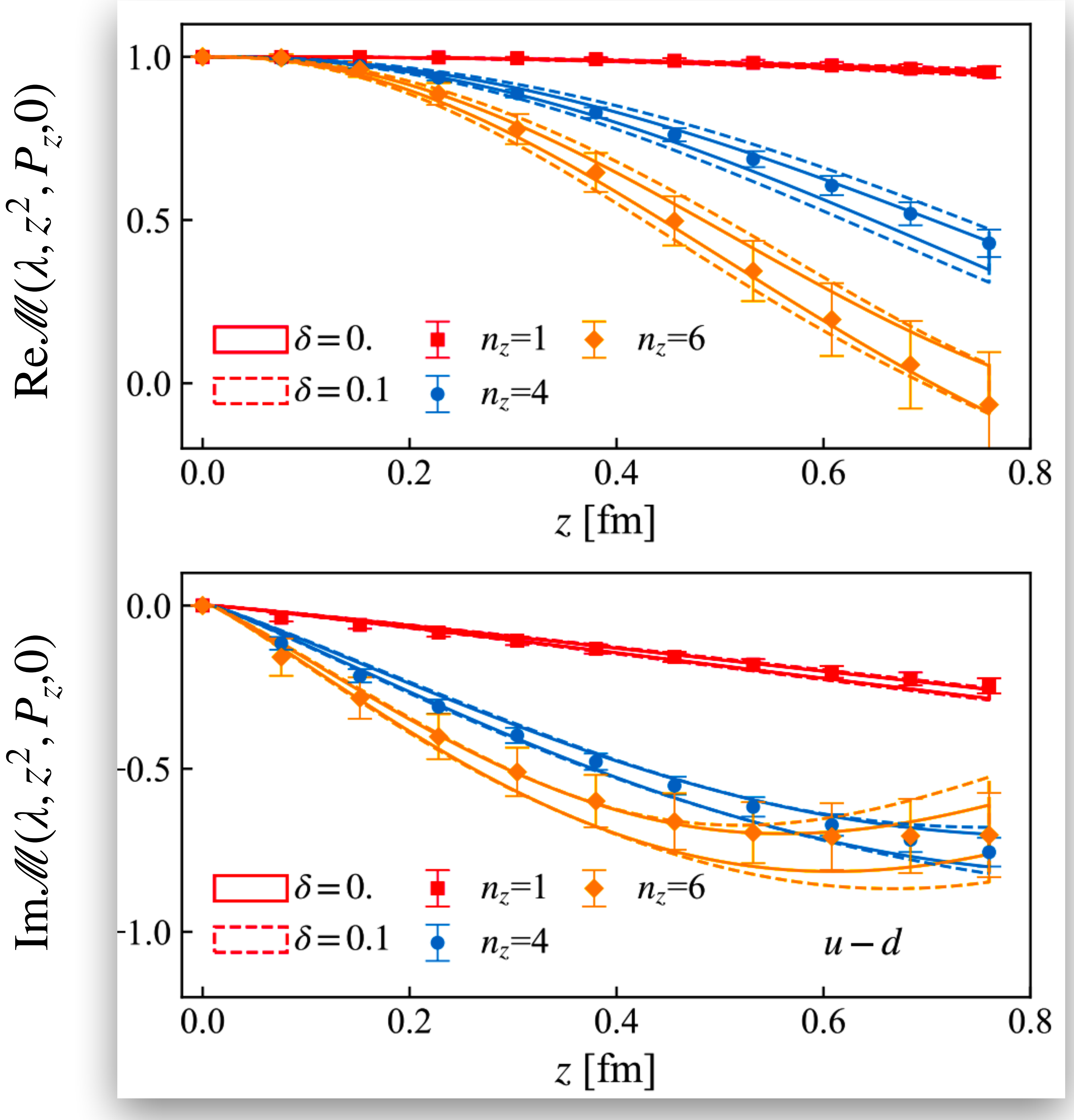


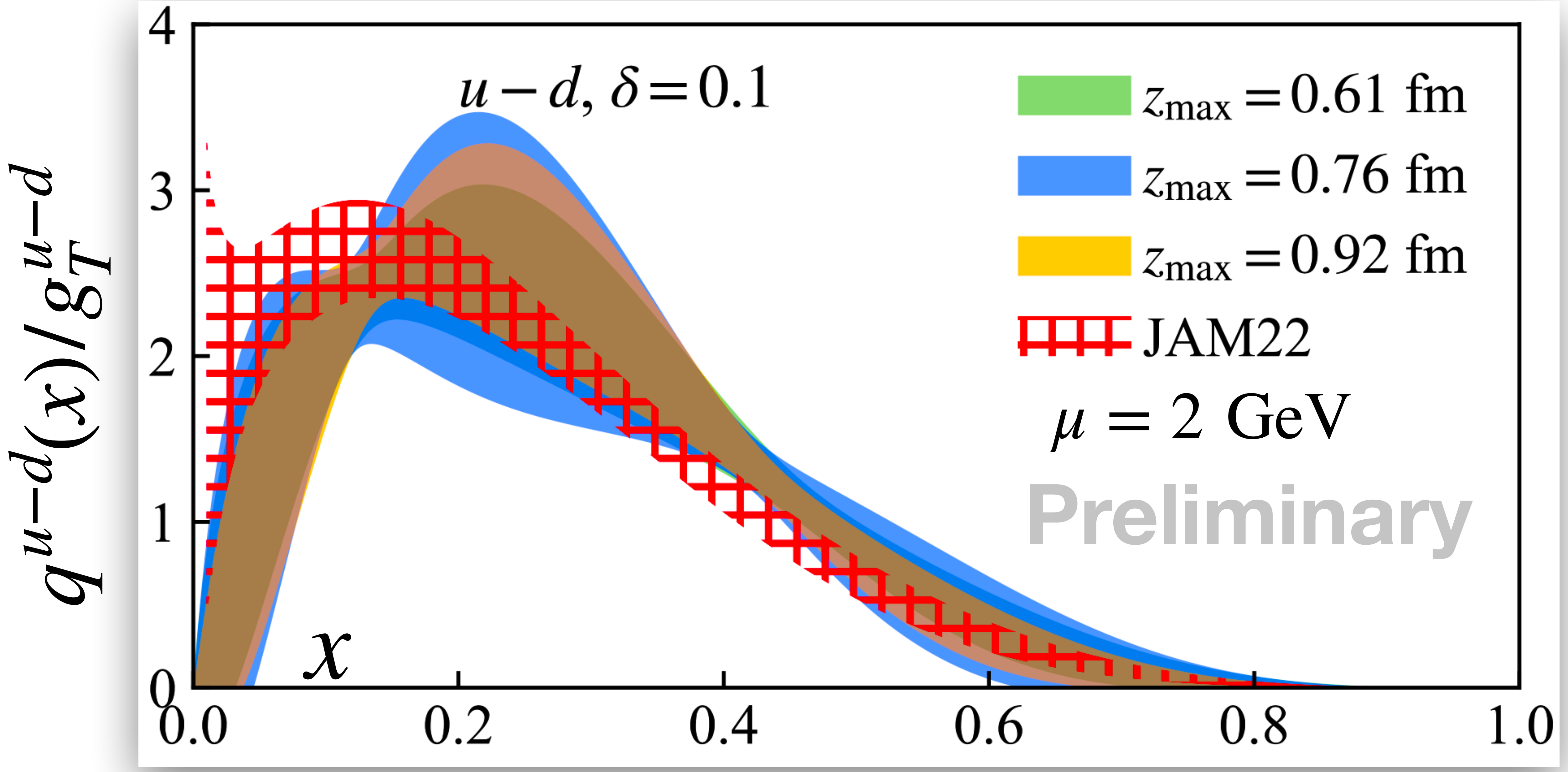
The difference:

$$q^+(x) - q^-(x) \equiv 2q^{\bar{q}}(x)$$

- Anti-quark distribution negligible.

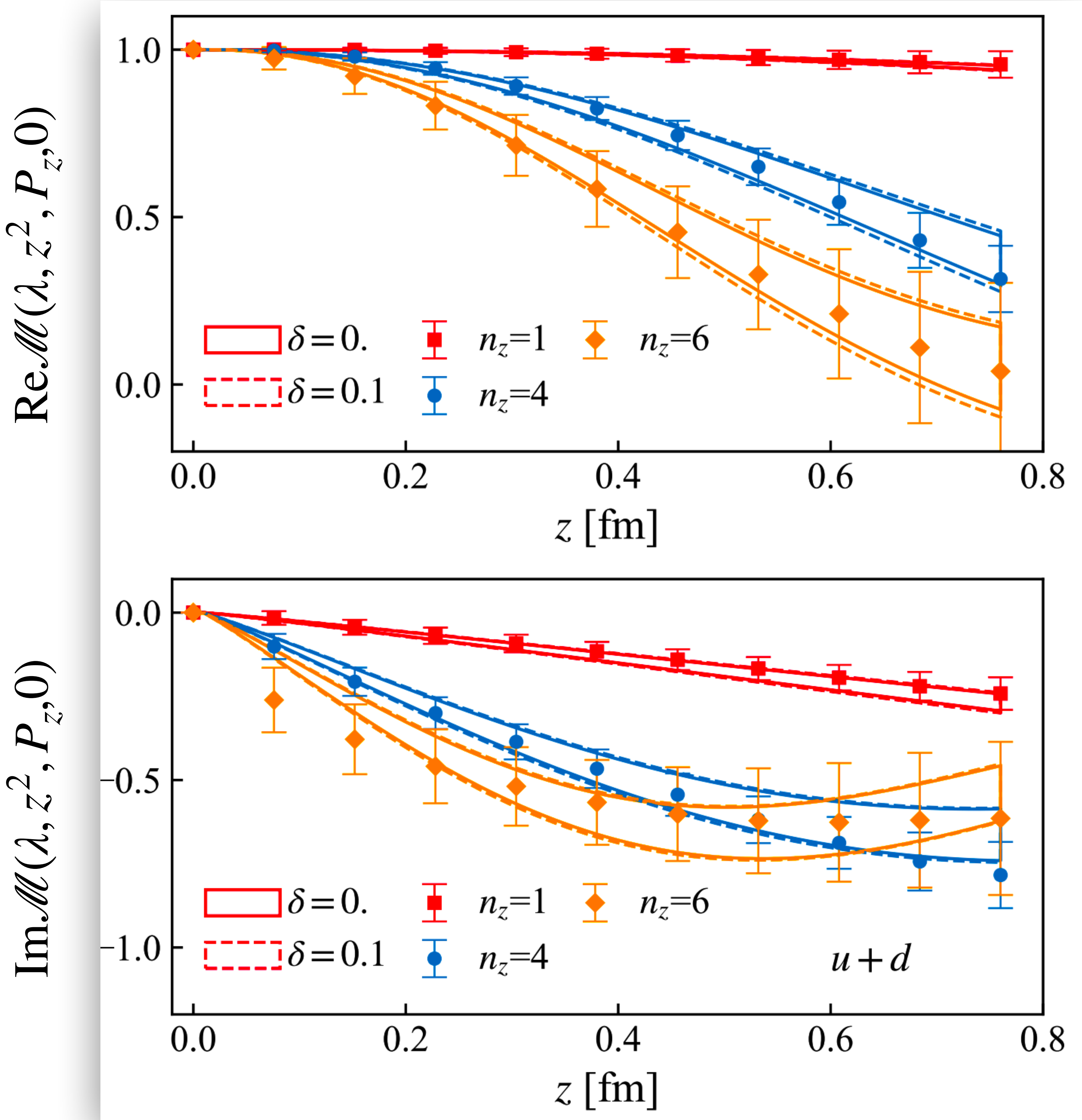
Transversity PDF from model reconstruction

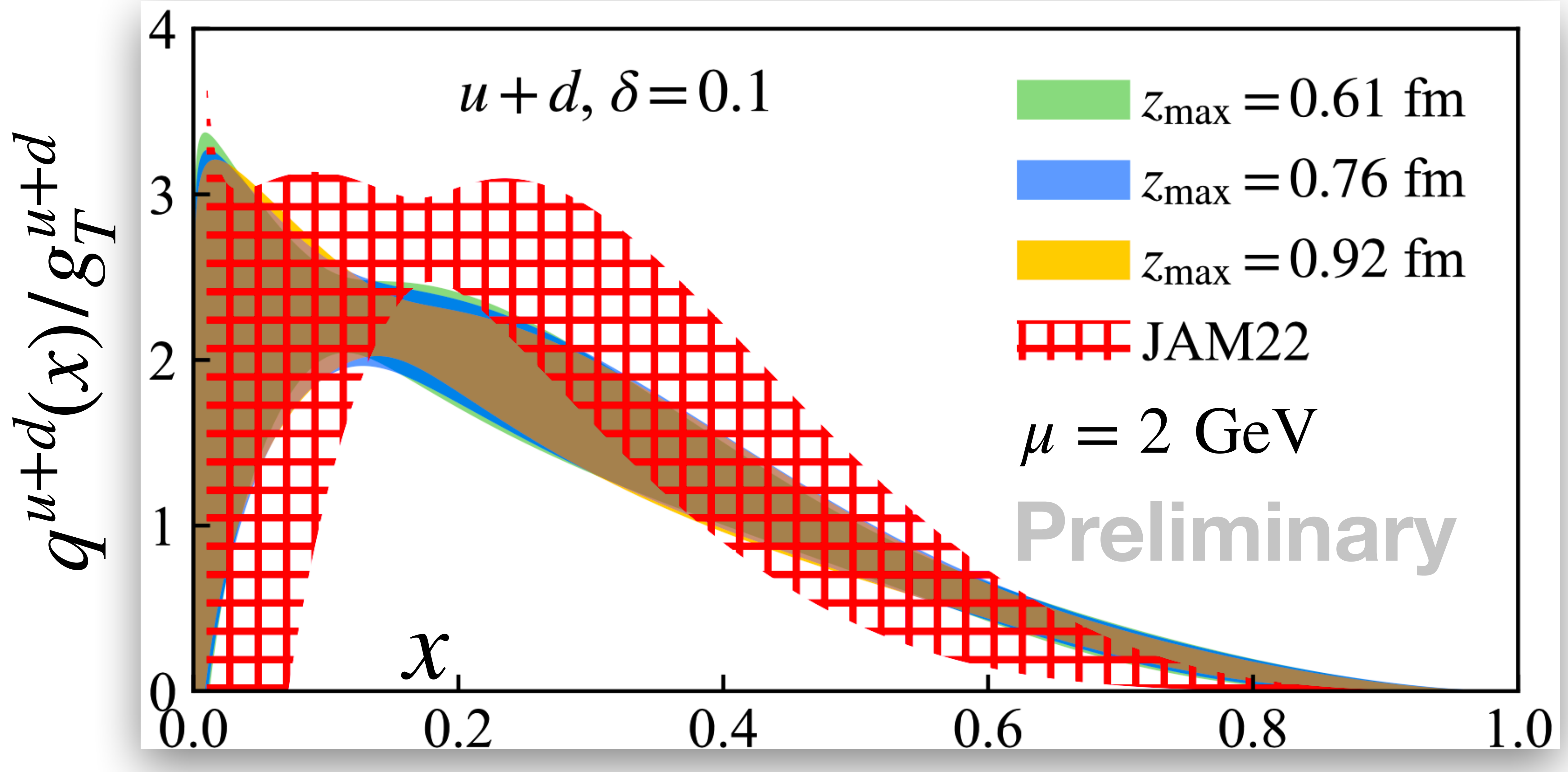




- Neglect the anti-quark contribution: $q^+(x) = q^-(x)$
- Consistent with the global analysis from JAM22.

Transversity PDF from model reconstruction





- Neglect the disconnected diagram and mixing with gluon.
- Consistent with the global analysis from JAM22.

LaMET factorization and hybrid renormalization

Direct sensitivity to local x dependence of PDFs for $x \in [x_{\min}, x_{\max}]$.

$$\tilde{q}(x, P_z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m|z|} Z(a) [\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

$$\delta m = m_{-1}/a + m_0$$

Ratio scheme renormalization

$$h^R = \frac{h^B(z, P_z, a)}{h^B(z,0,a)}$$

At large z become non-perturbative, the F.T. is not ill-posed.

LaMET factorization and hybrid renormalization

Direct sensitivity to local x dependence of PDFs for $x \in [x_{\min}, x_{\max}]$.

$$\tilde{q}(x, P_z, \mu) = \int \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{yP_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{(1-x)^2 P_z^2}\right)$$

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$

$$= e^{-\delta m|z|} Z(a) [\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

$$\delta m = m_{-1}/a + m_0$$

Hybrid scheme renormalization

- Short distance $z \in [0, z_s]$, $z_s \ll \Lambda_{\text{QCD}}$:

$$h^R = \frac{h^B(z, P_z, a)}{h^B(z, 0, a)}$$

- Long distance $z \in [z_s, +\infty]$:

$$h^R = e^{\delta m|z - z_s|} \frac{h^B(z, P_z, a)}{h^B(z_s, 0, a)}$$

LaMET factorization and hybrid renormalization

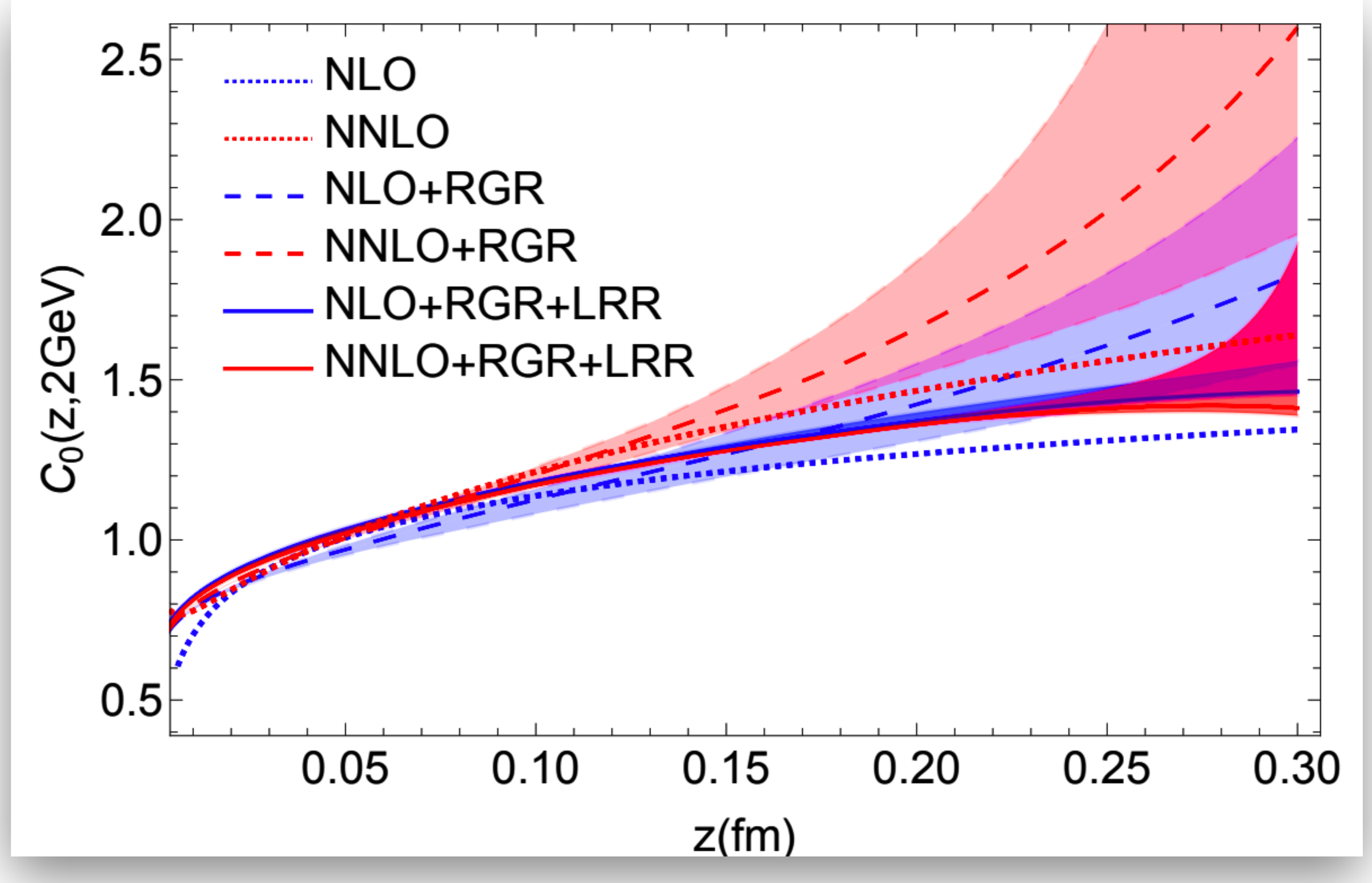
Extract δm using $P_z = 0$ matrix elements

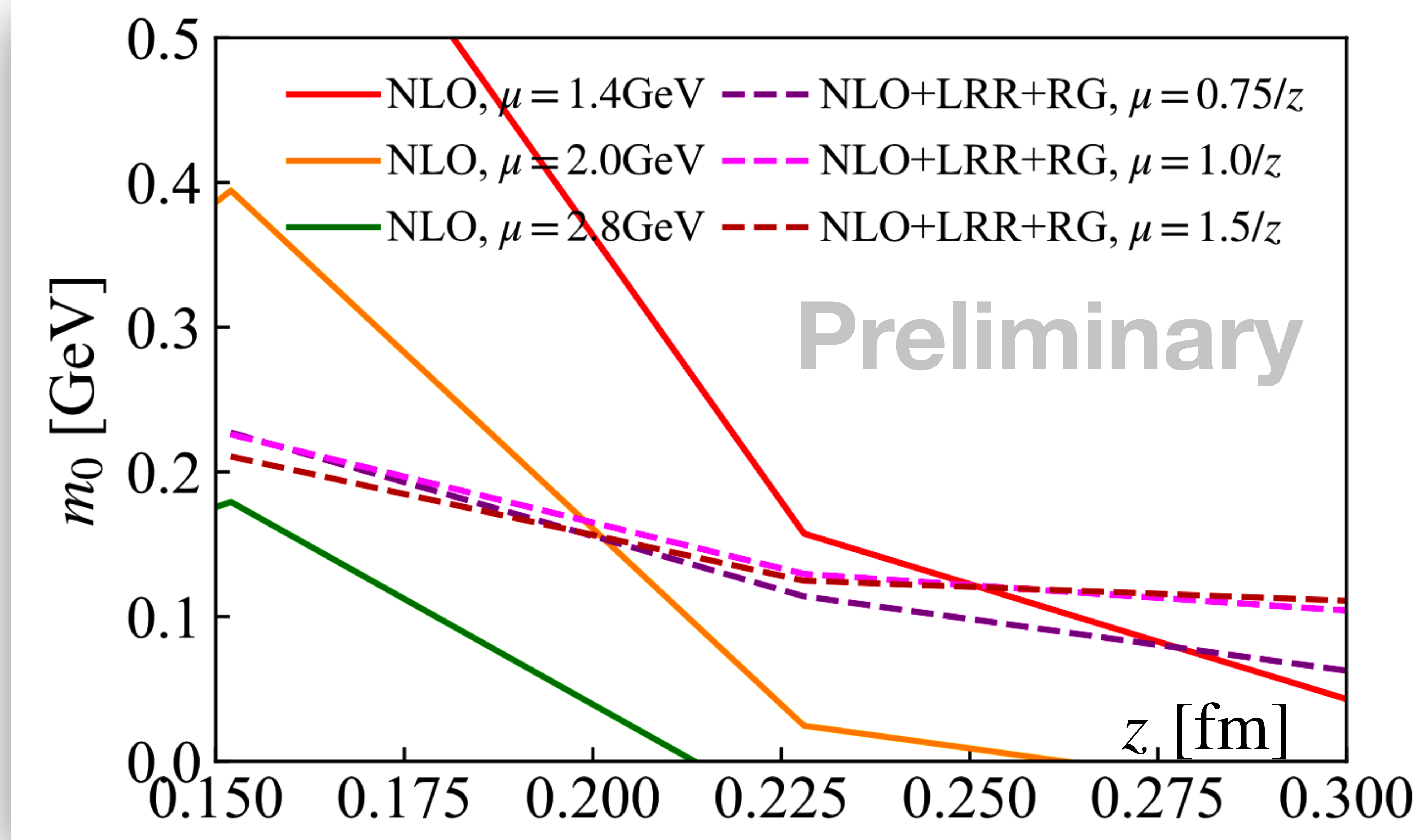
$$e^{\delta m|z|}\langle P_z = 0 | [\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B | P_z = 0\rangle$$
$$= Z(a) C_0^{\overline{\mathrm{MS}}}(z^2\mu^2)$$

$$[\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_B$$
$$= e^{-\delta m|z|} Z(a) [\overline{\psi}(0)\Gamma W_{\hat{z}}(0,z)\psi(z)]_R$$

$$\delta m = m_{-1}/a + m_0$$

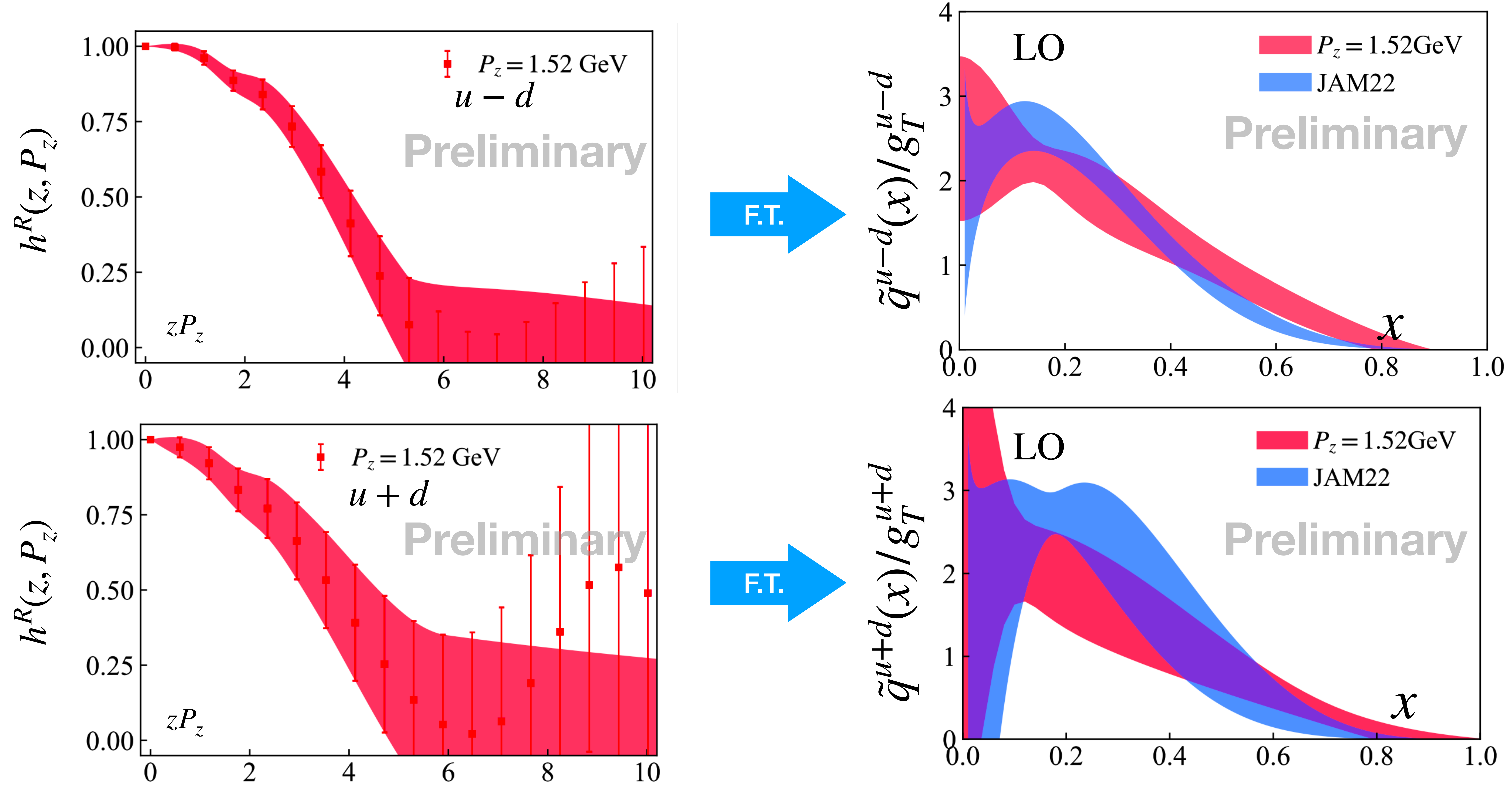
LRR: leading renormalon resummation





R. Zhang, et al., arXiv: 2305.05212

Transversity quasi-PDF



Summary

- We carried out lattice calculation of the transversity quasi-PDF matrix elements of proton.
- The first few Mellin moments were extracted using the leading-twist SDF from ratio-scheme renormalized matrix elements.
- We reconstruct the x dependence using a deep neural network which show consistent results with global analysis JAM22.
- The matrix elements are renormalized in hybrid scheme with LRR improved coefficients and we derive the x dependent quasi-PDF. The NLO+LRR+RG matching in on going.

Thanks for your attention!