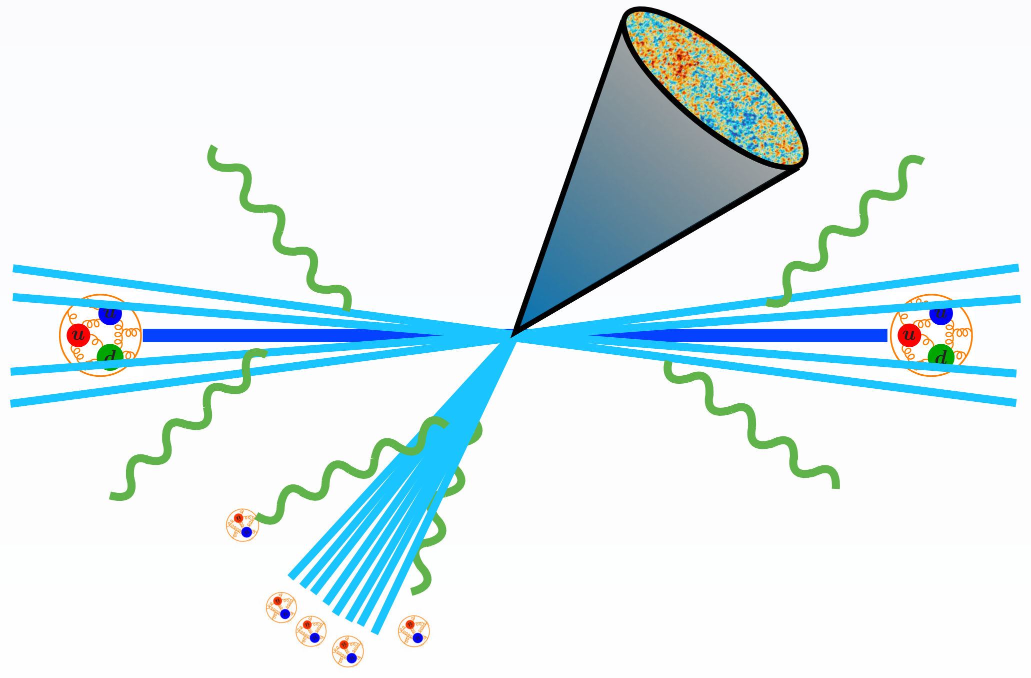
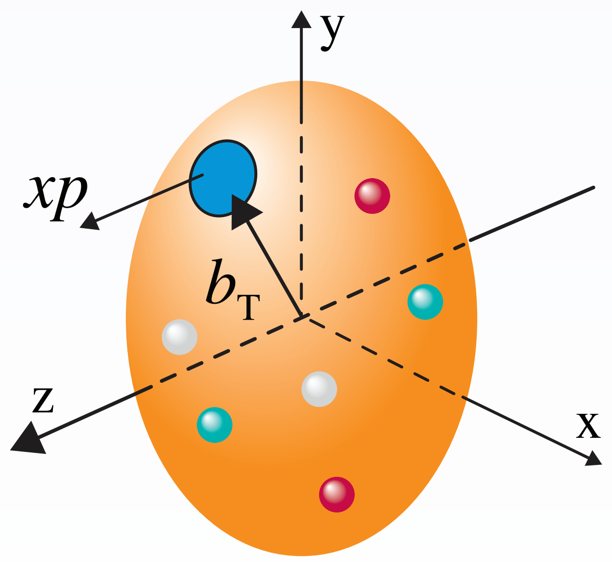
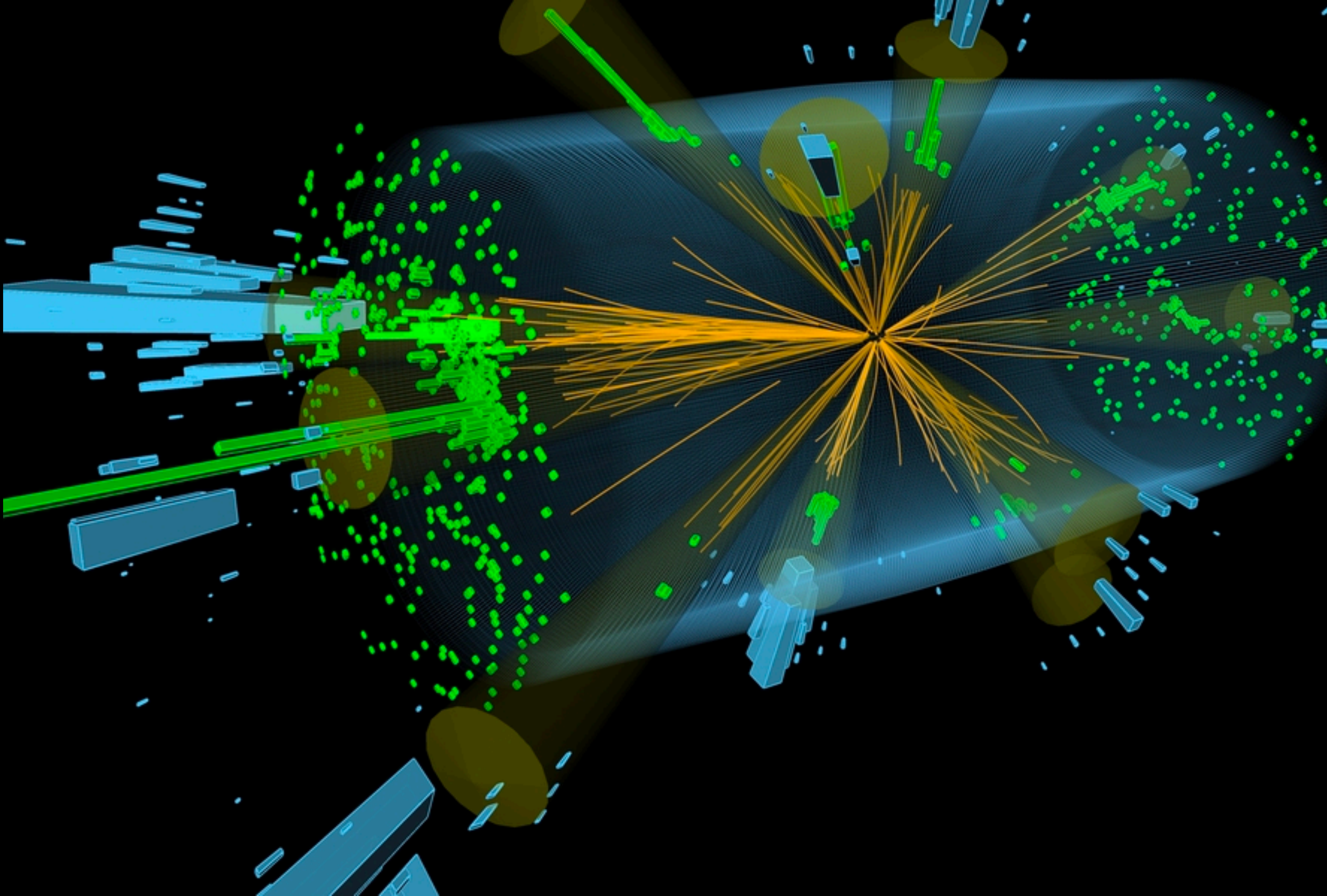
Understanding TMD structure using *jets*

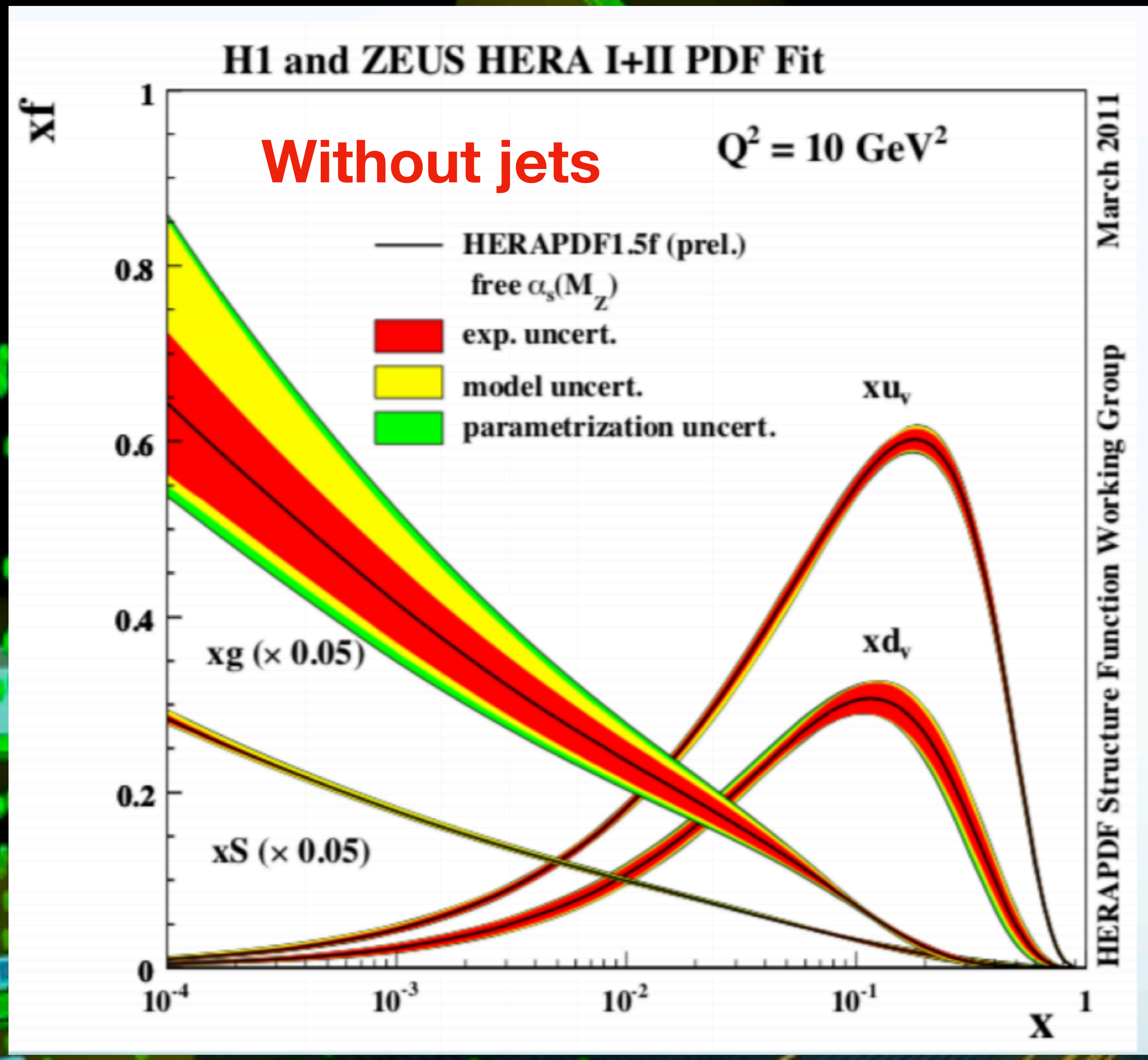
Kyle Lee
CTP, MIT

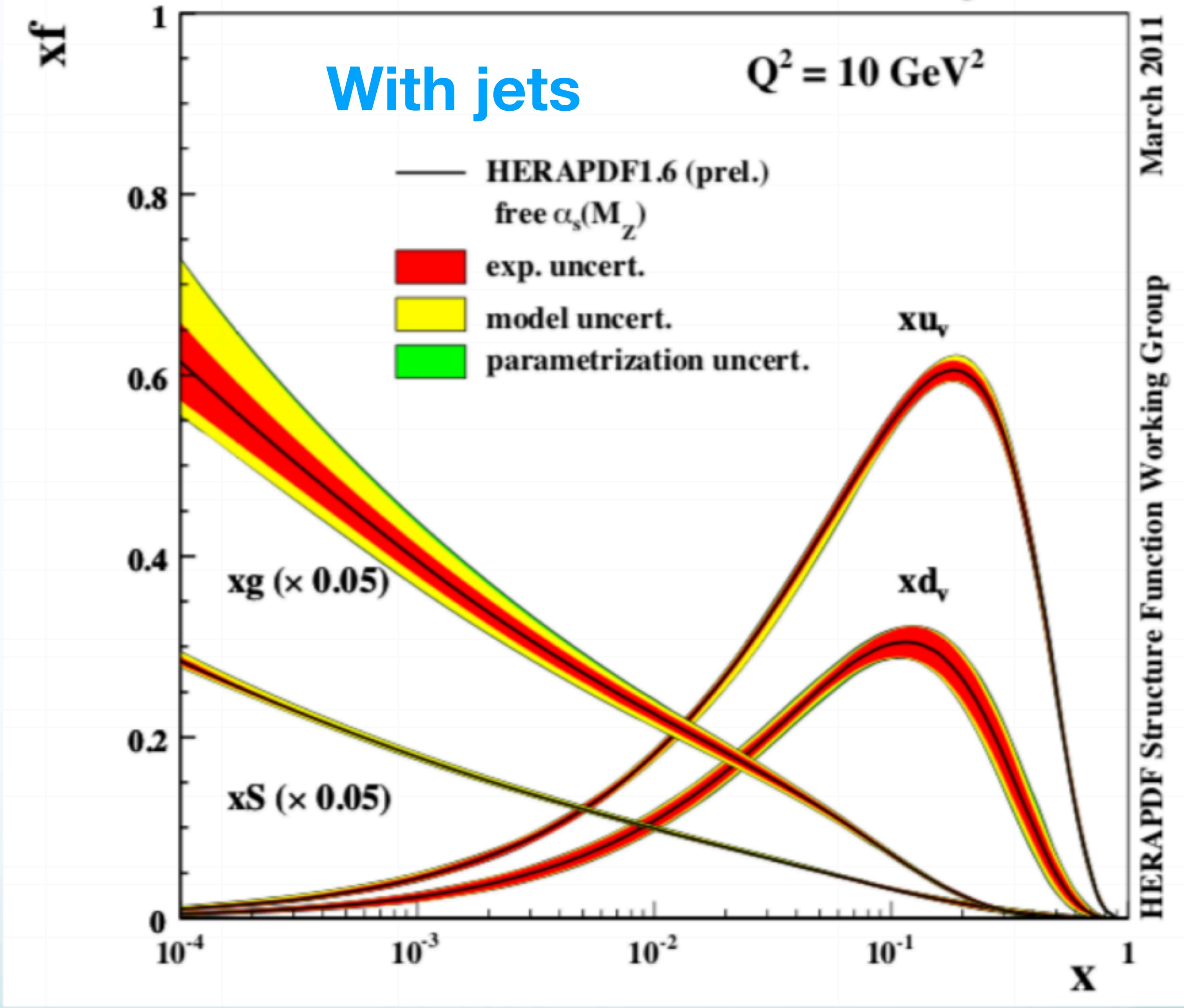




CFNS TMD workshop
June 21st - 23rd, 2023




H1 and ZEUS HERA I+II PDF Fit
Without jets
Q² = 10 GeV²
HERAPDF1.5f (prel.)
free αs(MZ)
exp. uncert.
model uncert.
parametrization uncert.
xf
xuv
xdv
xg (× 0.05)
xS (× 0.05)
March 2011
HERAPDF Structure Function Working Group
1
0.8
0.6
0.4
0.2
0
10⁻⁴
10⁻³
10⁻²
10⁻¹
1
x


H1 and ZEUS HERA I+II PDF Fit with Jets
With jets
Q² = 10 GeV²
HERAPDF1.6 (prel.)
free αs(MZ)
exp. uncert.
model uncert.
parametrization uncert.
xf
xuv
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xg (× 0.05)
xS (× 0.05)
March 2011
HERAPDF Structure Function Working Group
1
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0.6
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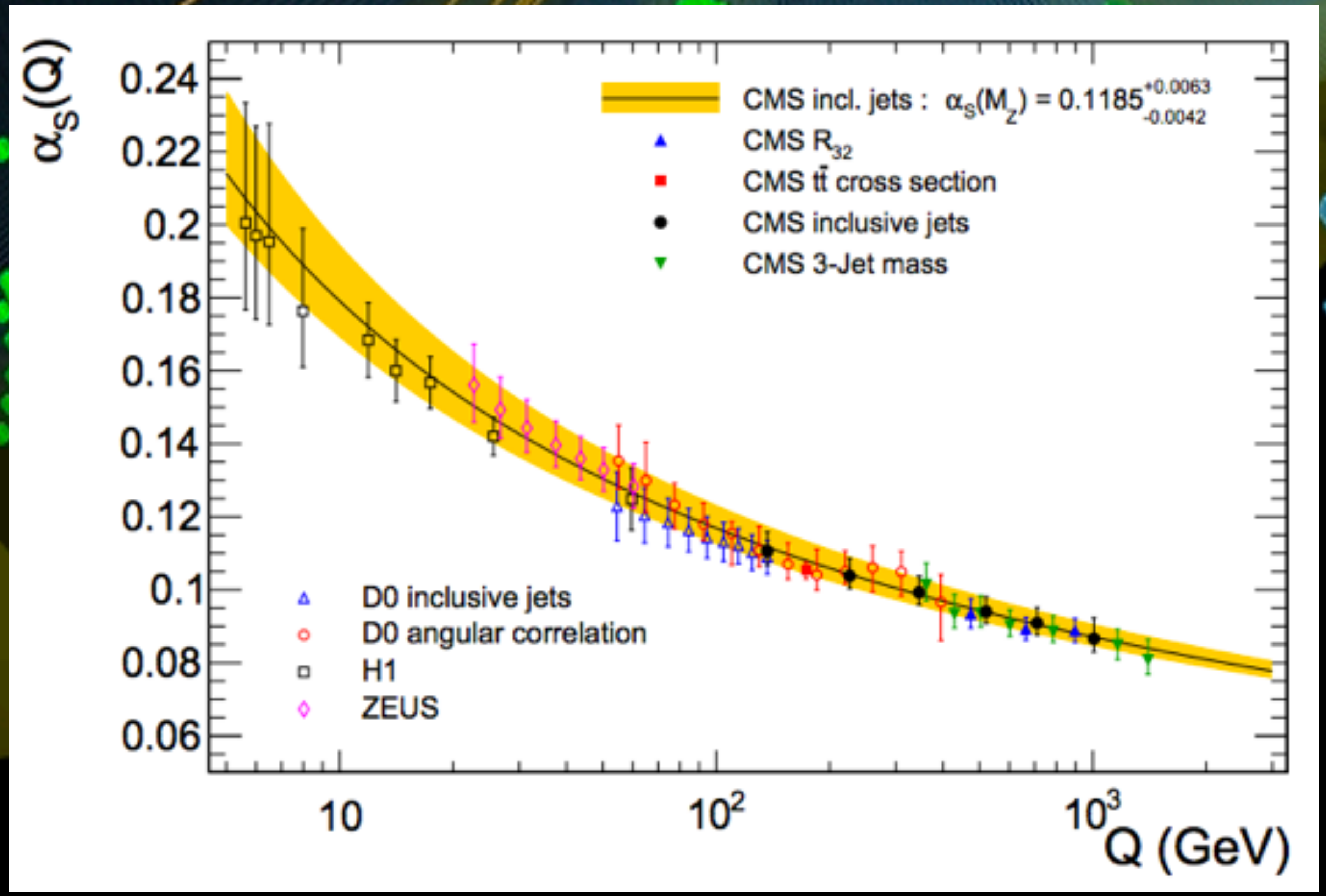
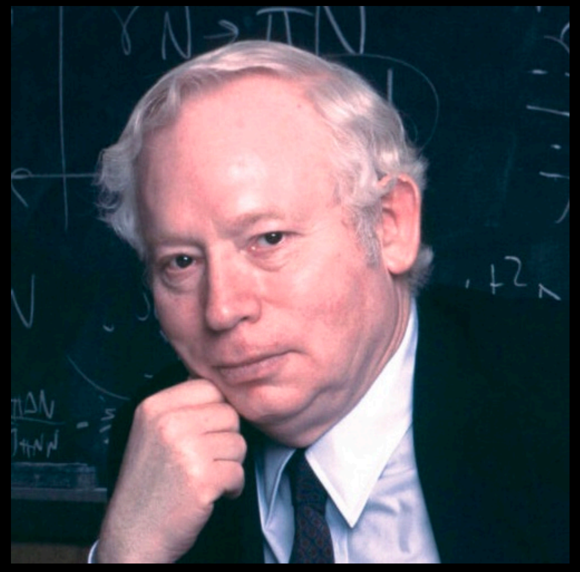
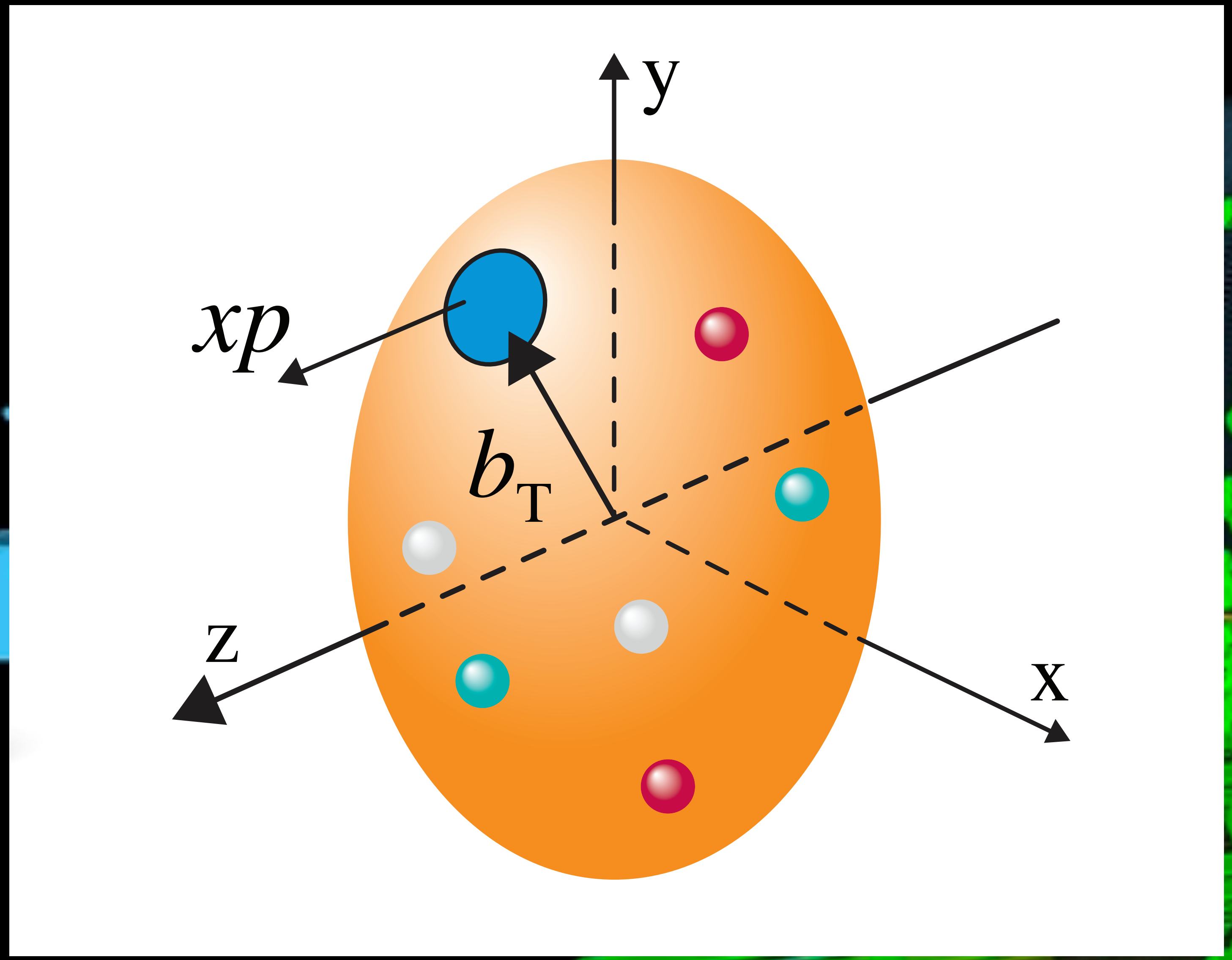
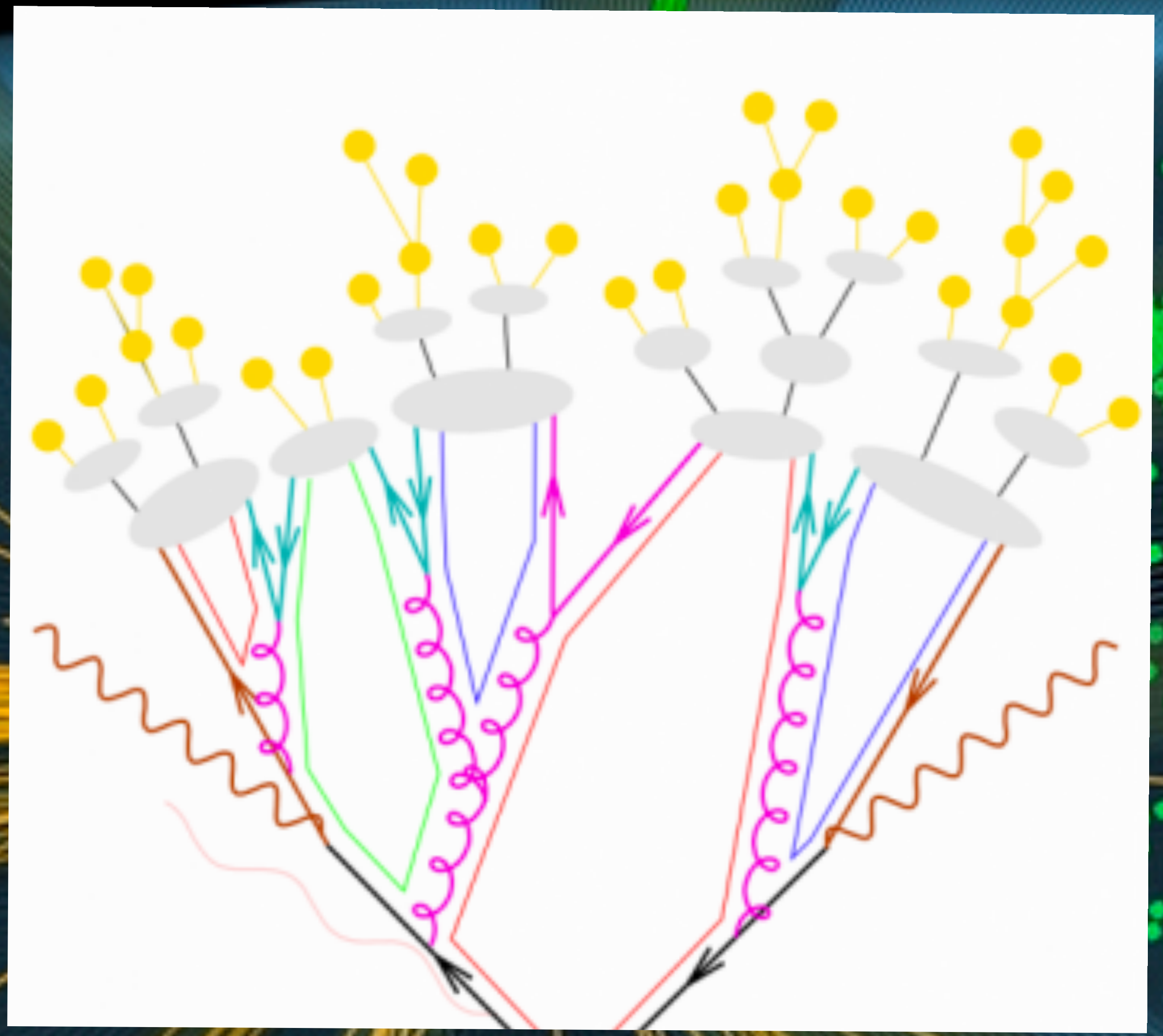
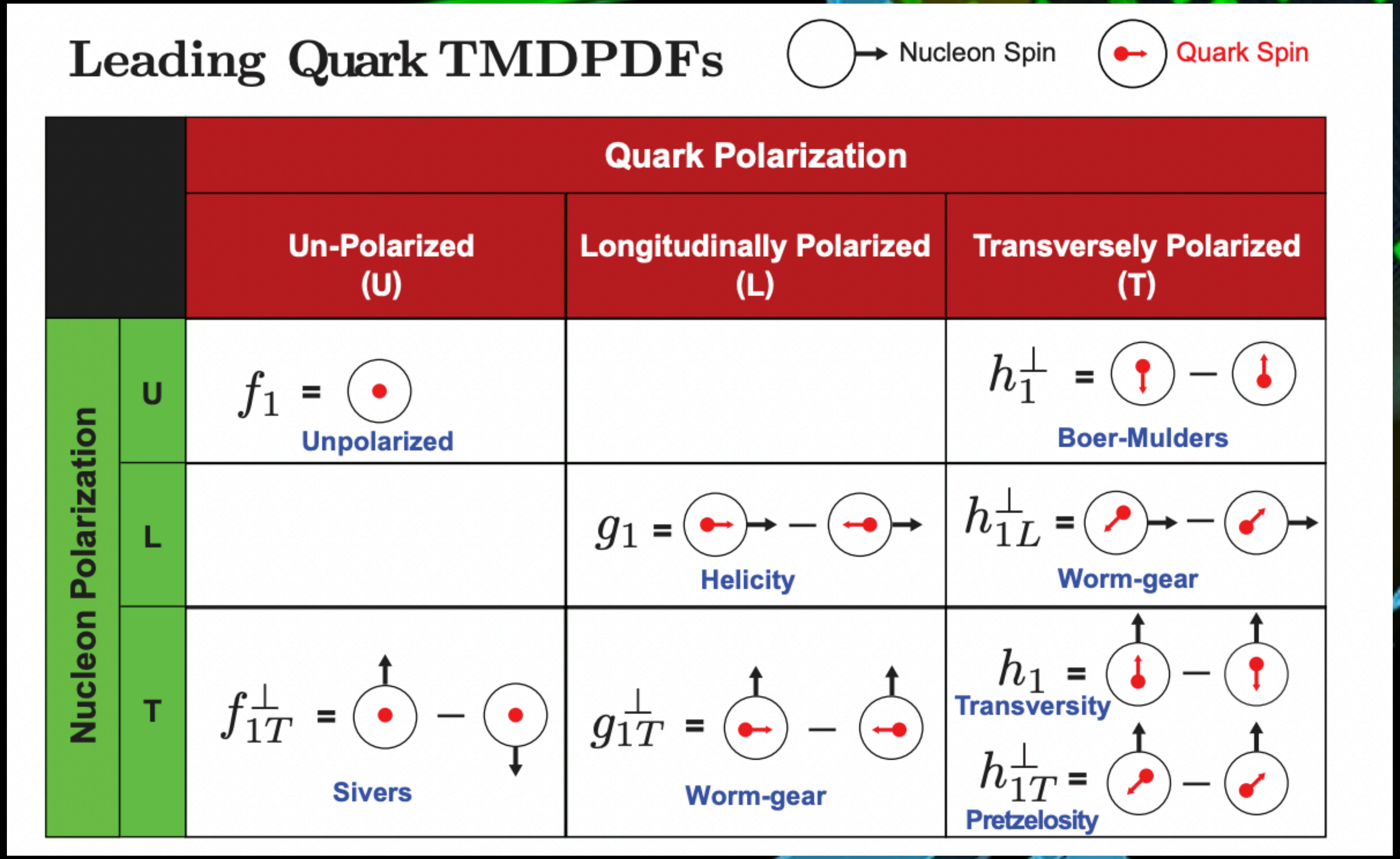

αS(Q)
CMS incl. jets : αS(MZ) = 0.1185 +0.0063 -0.0042
CMS R32
CMS tt̄ cross section
CMS inclusive jets
CMS 3-Jet mass
D0 inclusive jets
D0 angular correlation
H1
ZEUS
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
10
10²
10³
Q (GeV)





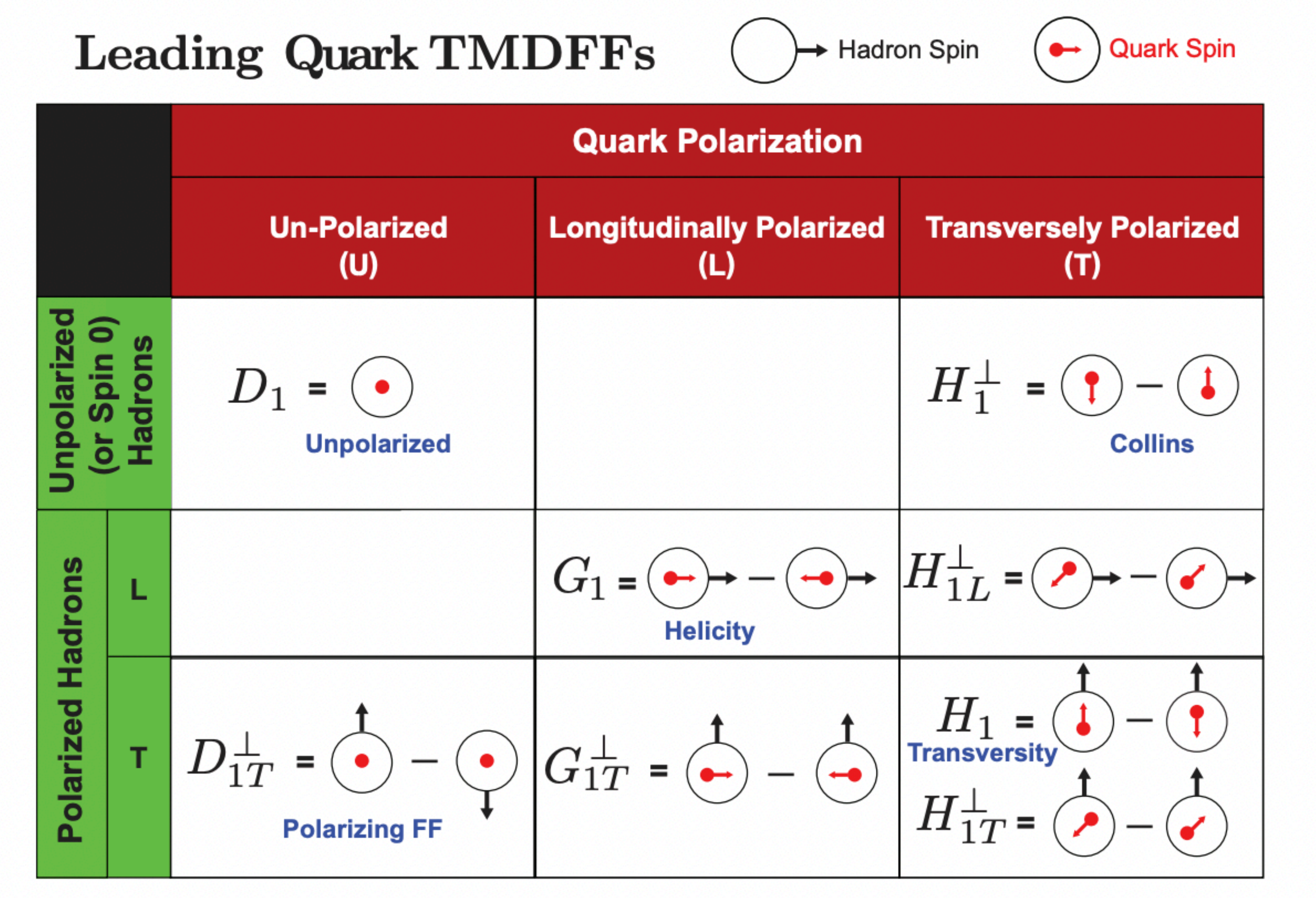




Leading Quark TMDPDFs

○→ Nucleon Spin ⊙ Quark Spin

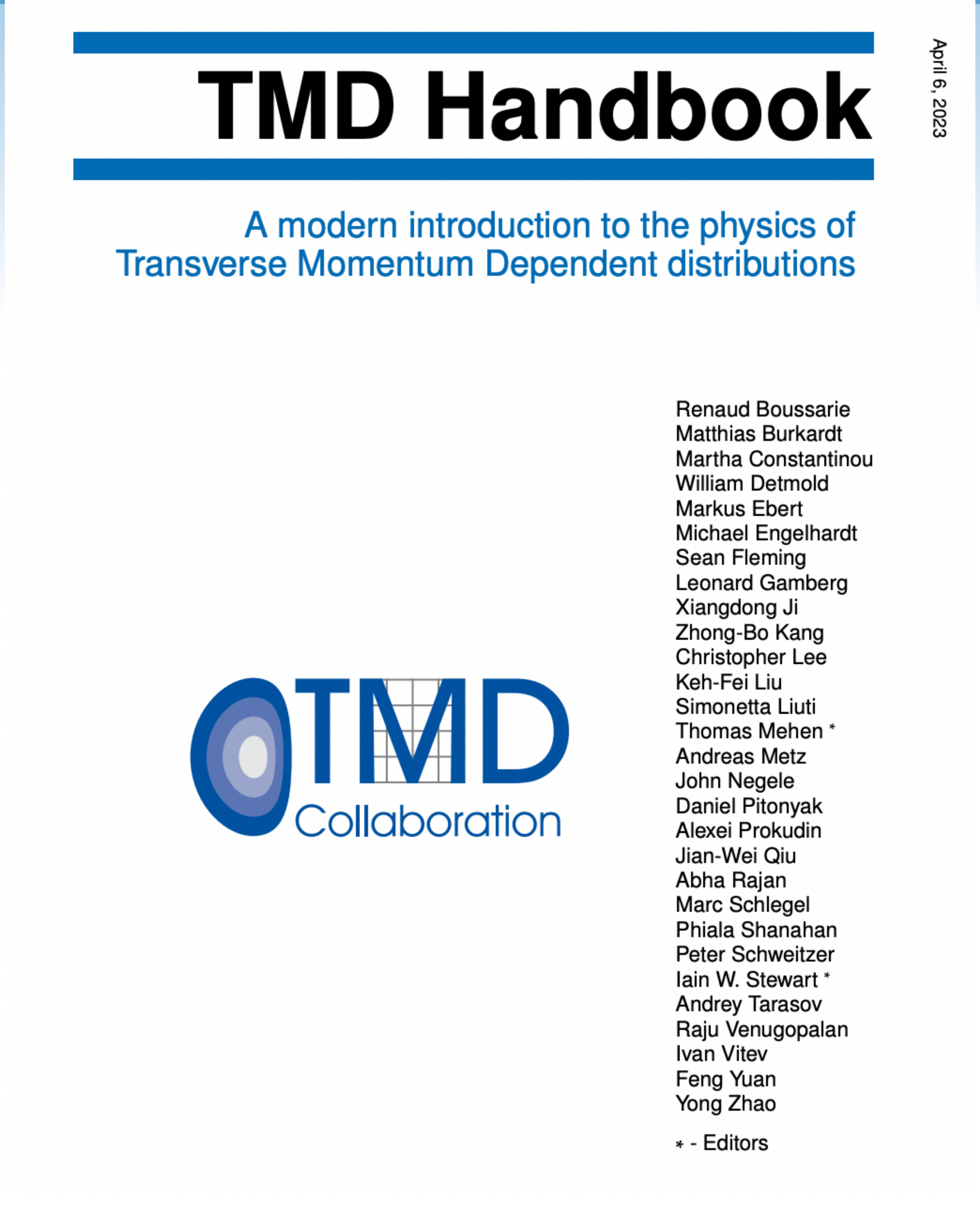
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = ⊙ Unpolarized		$h_1^\perp$ = ⊙ − ⊙ Boer-Mulders
	L		g_1 = ⊙ − ⊙ Helicity	$h_{1L}^\perp$ = ⊙ − ⊙ Worm-gear
	T	$f_{1T}^\perp$ = ⊙ − ⊙ Sivers	$g_{1T}^\perp$ = ⊙ − ⊙ Worm-gear	h_1 = ⊙ − ⊙ Transversity $h_{1T}^\perp$ = ⊙ − ⊙ Pretzelosity



Leading Quark TMDFFs

○→ Hadron Spin ⊙ Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = ⊙ Unpolarized		$H_1^\perp$ = ⊙ − ⊙ Collins
Polarized Hadrons	L		G_1 = ⊙ − ⊙ Helicity	$H_{1L}^\perp$ = ⊙ − ⊙
	T	$D_{1T}^\perp$ = ⊙ − ⊙ Polarizing FF	$G_{1T}^\perp$ = ⊙ − ⊙	H_1 = ⊙ − ⊙ Transversity $H_{1T}^\perp$ = ⊙ − ⊙



TMD Handbook

April 6, 2023

A modern introduction to the physics of
Transverse Momentum Dependent distributions

TMD Collaboration

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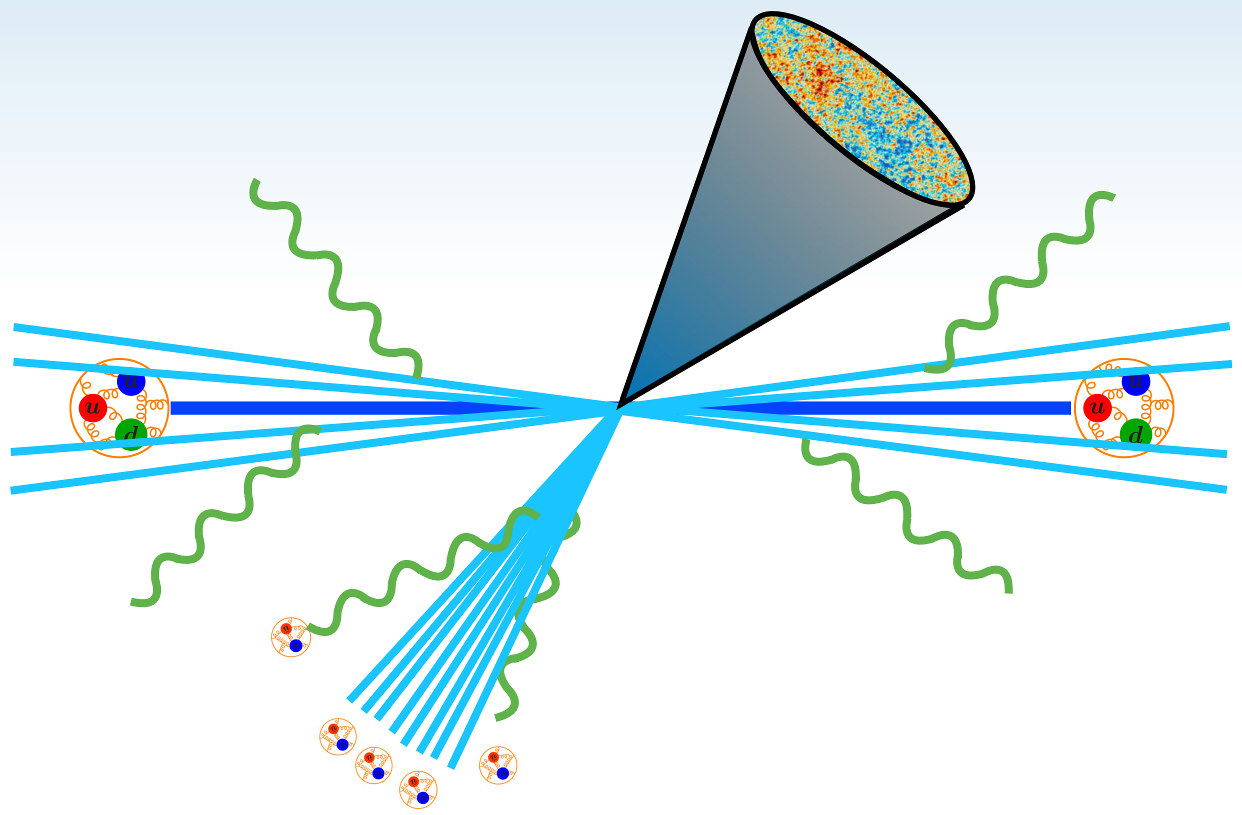
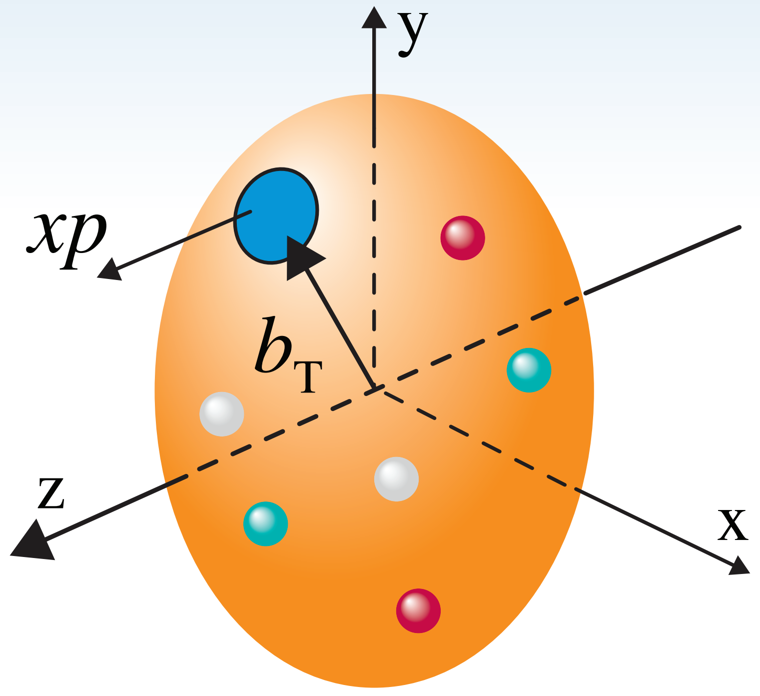
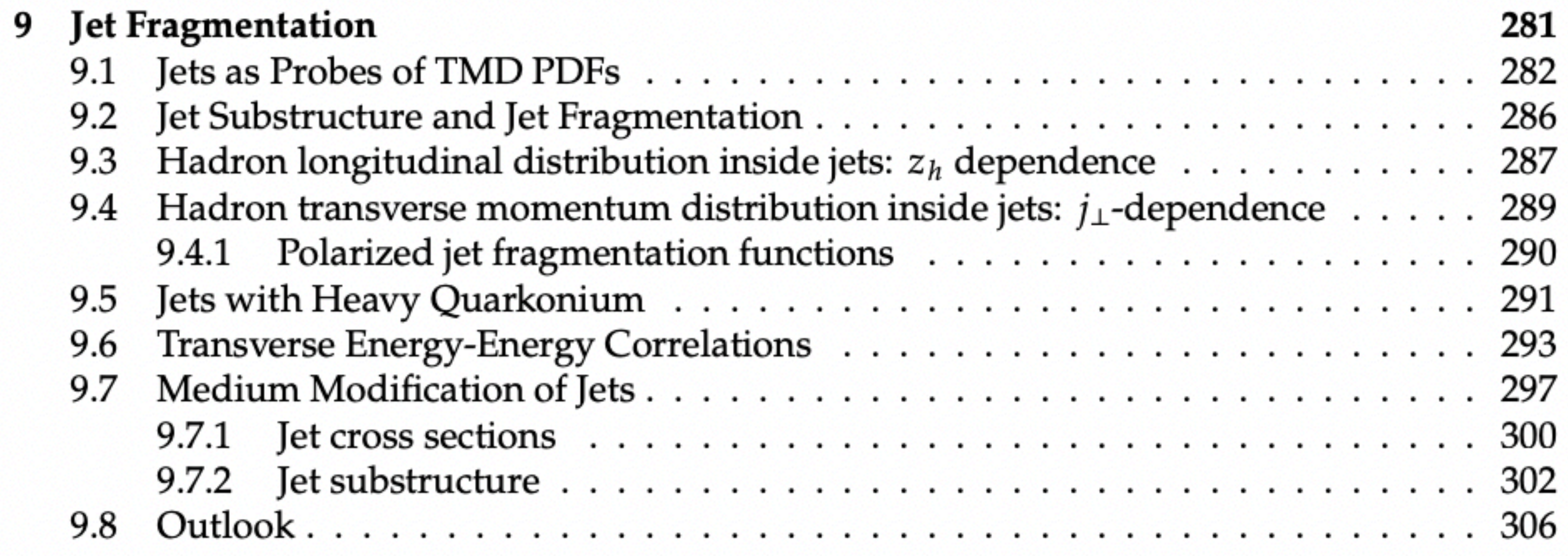




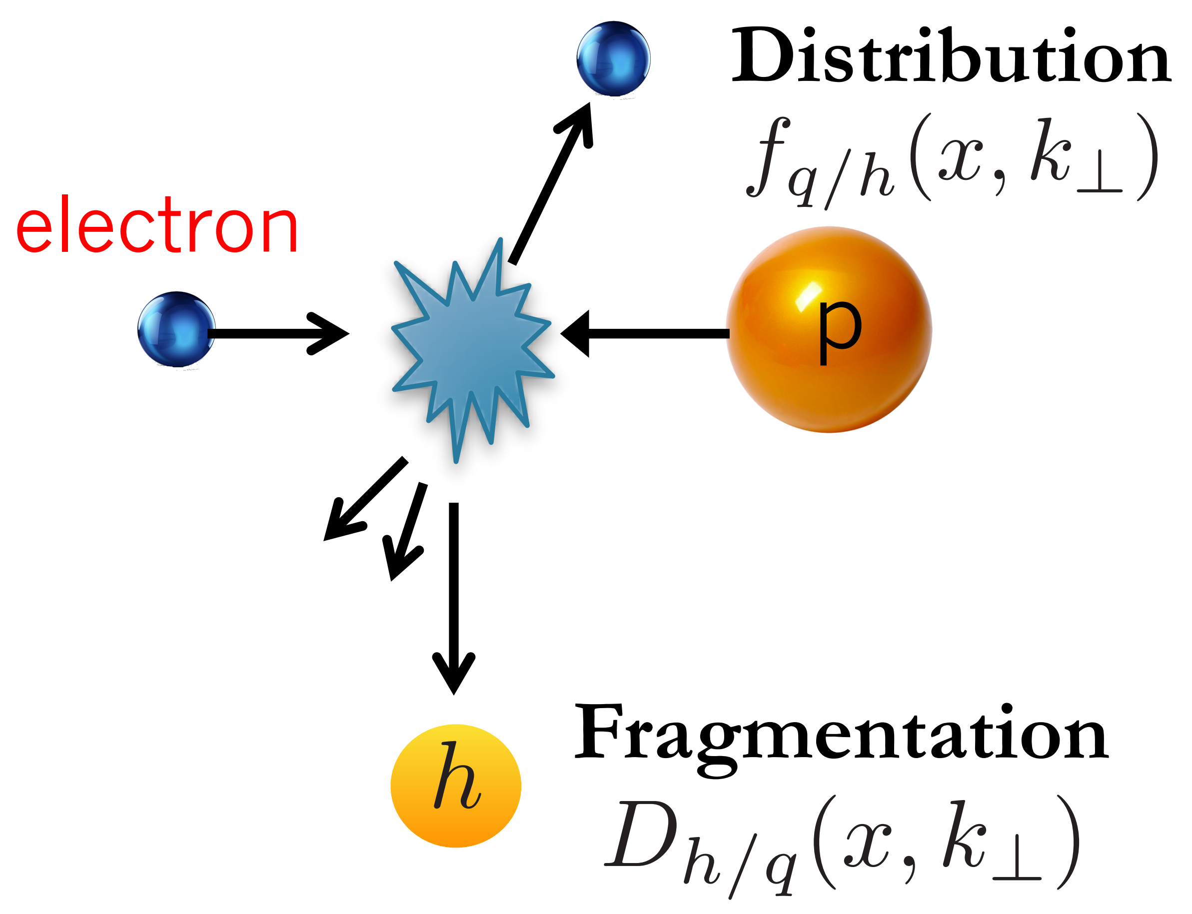


9 Jet Fragmentation **281**

Standard processes to study TMDs

Semi-Inclusive DIS (SIDIS)

$$\sigma \sim f_{q/P}(x,k_\perp)D_{h/q}(x,k_\perp)$$

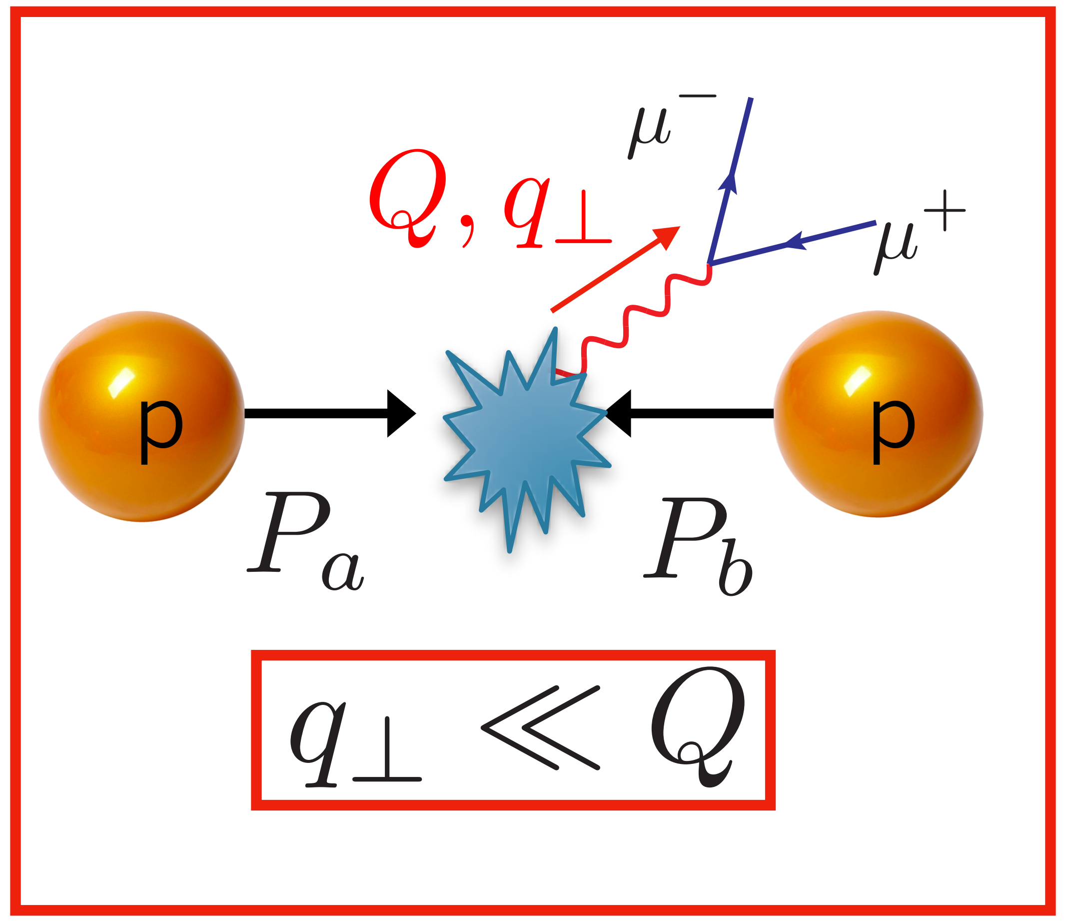


Distribution
$f_{q/h}(x,k_\perp)$

Fragmentation
$D_{h/q}(x,k_\perp)$

Drell-Yan

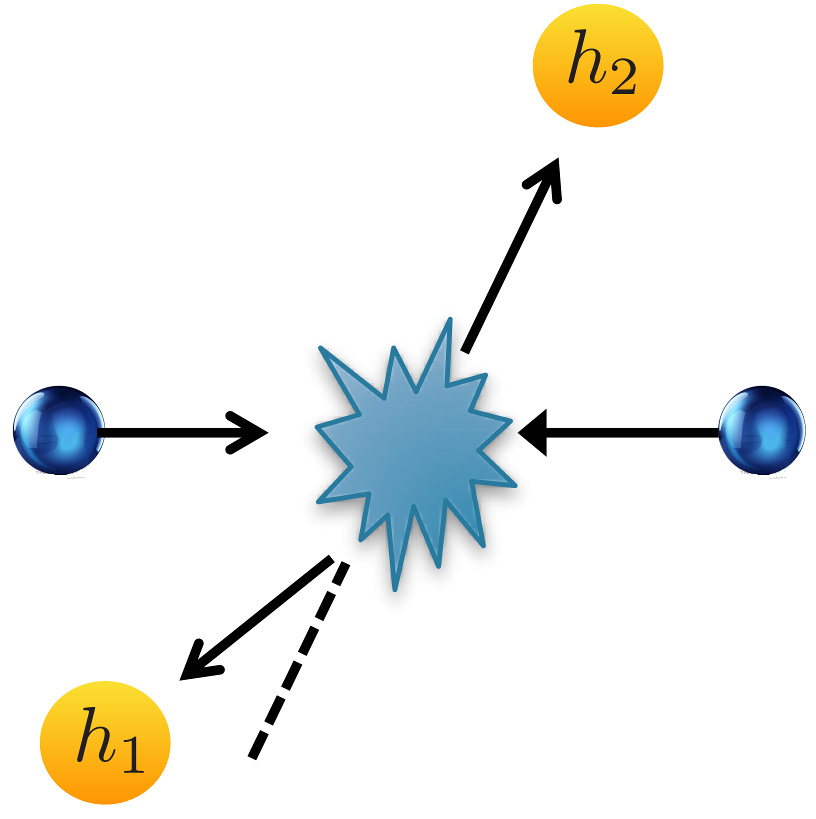
$$\sigma \sim f_{q/P}(x,k_\perp)f_{\bar{q}/P}(x,k_\perp)$$



$$\boxed{q_\perp \ll Q}$$

Dihadrons in e^+e^-

$$\sigma \sim D_{h_1/q}(x,k_\perp)D_{h_2/q}(x,k_\perp)$$



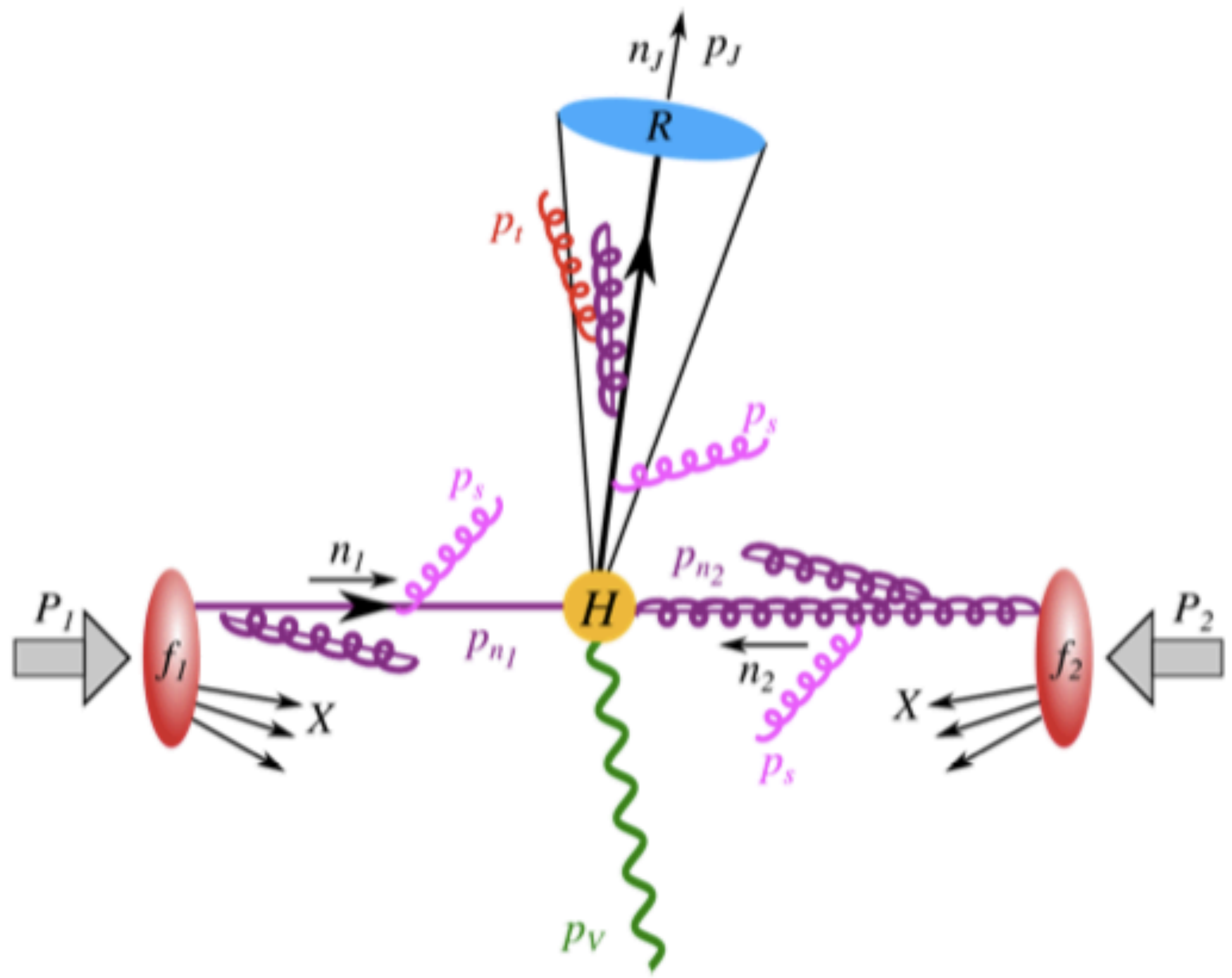
- Small transverse momentum measured with respect to an axis gives sensitivity to the intrinsic transverse momentum

- They have a well-established factorization formalism

(CSS) Collin, Soper, Sterman `81-`85
Ji, Ma, Yuan ` 04
Becher, Neubert, Wilhelm`11-`13
Echevarria, Idilbi, Scimemi `11-`14
…

Beyond the standard processes

- Many other imaginable processes involving ***jets*** with sensitivity to the TMD structure



$$PP \to J_1 + J_2 + X,$$
$$PP \to J + V + X,$$
$$PP \to J(h) + X, \ldots$$

LHC / RHIC

$$e\,P \to e + J + X$$
$$e\,P \to Q + \bar{Q} + X,$$
$$e\,P \to J(h) + X, \ldots$$

EIC

- Many experiments sensitive to such processes
- Standard processes have low sensitivity to gluon TMDs.
- Standard processes sensitive to *two* TMDs simultaneously; many involving jets will only be sensitive to a *single* TMD.

1) Lepton + jet imbalance

TMDPDFs $e\,P \to e + J + X$

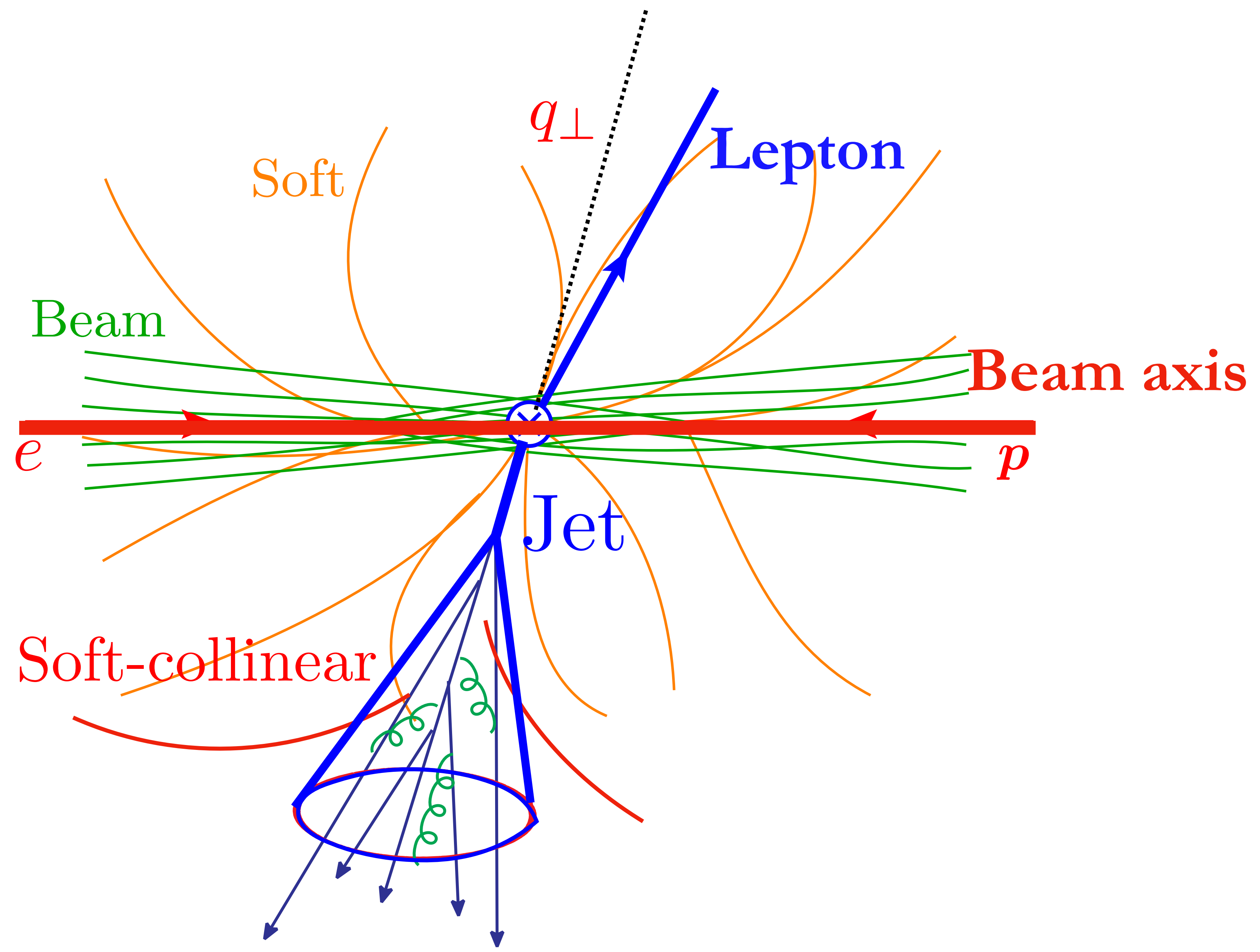
2) Fragmenting Jet Functions

TMDFFs $PP\,/\,e\,P \to J(h) + X$

3) Lepton + jet imbalance with hadron in jet

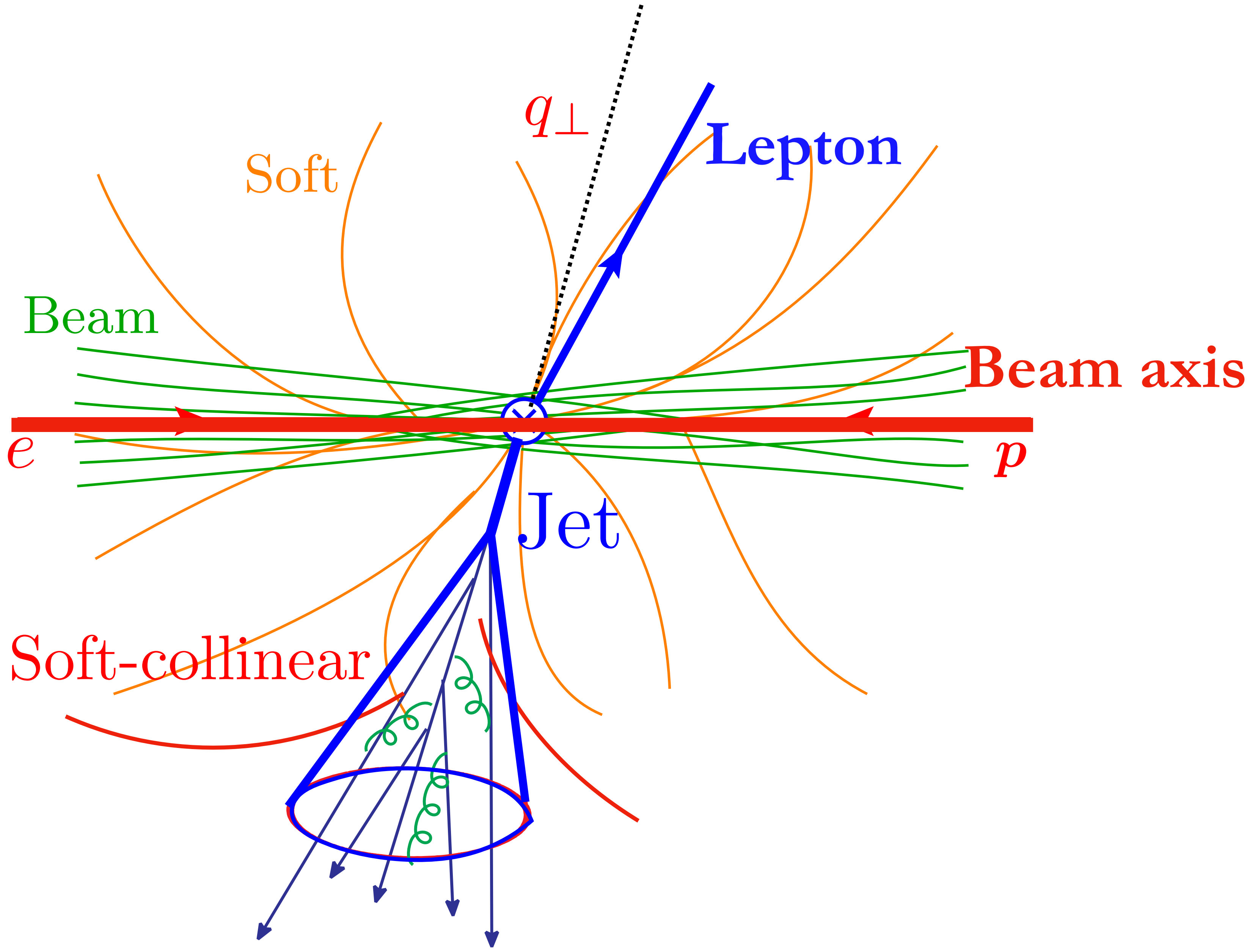
TMDFFs / TMDPDFs $e\,P \to e + J(h) + X$

Lepton + Jet imbalance



Lepton + Jet imbalance

- One of the simplest process $e\,P \to e + J + X$



$$q_\perp \equiv |\vec{p}_{e\perp} + \vec{p}_{J\perp}|, \qquad p_\perp \equiv |\vec{p}_{e\perp} - \vec{p}_{J\perp}|/2$$

$q_\perp \ll p_\perp,$ sensitive to the large logs of $\ln(q_\perp/p_\perp)$ and TMD structures of the hadrons.

$$q_\perp = p_{X,\perp} = |\vec{k}_{c,\perp} + \vec{k}_{gs,\perp} + \vec{k}_{sc,\perp}|$$

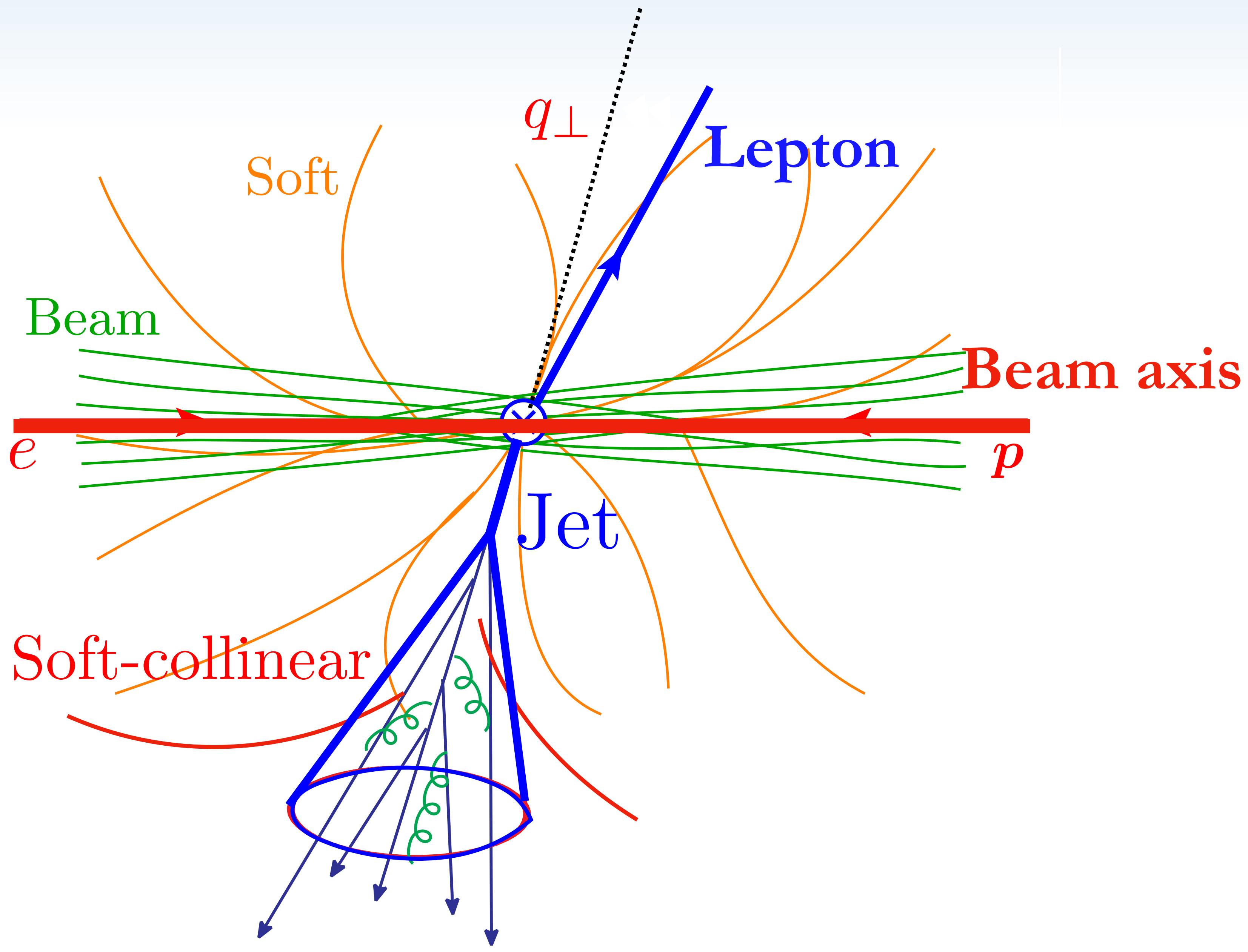
Giving relevant modes : $(+,-,\perp)$ $\lambda = q_\perp/p_\perp$

n-collinear	$k_n \sim p_\perp(\lambda^2, 1, \lambda)_{n\bar{n}}$
global soft	$k_{gs} \sim p_\perp(\lambda, \lambda, \lambda)$
soft-collinear	$k_{sc} \sim p_\perp R(\lambda R, \lambda/R, \lambda)_{n_J,\bar{n}_J}$
n_J-collinear	$k_J \sim p_\perp(R^2, 1, R)_{n_J,\bar{n}_J}$

1) Lepton + jet imbalance
TMDPDFs

Liu, Ringer, Vogelsang, Yuan `18, `20
Arratia, Kang, Prokudin, Ringer `20

Lepton + Jet imbalance

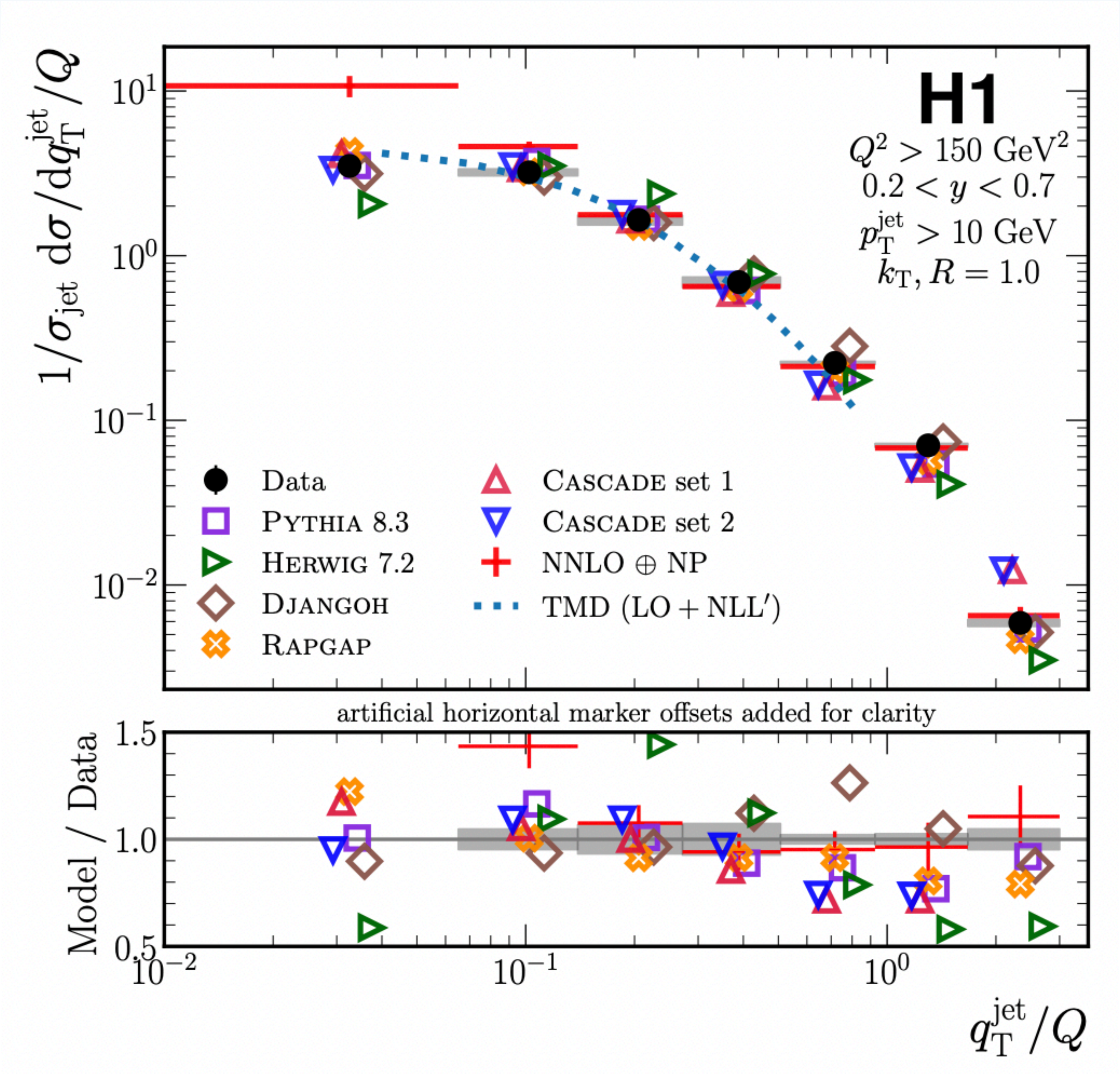


$$\frac{d\sigma_{eP\to e+\text{jet}}}{dp_\perp dq_\perp} = \int \prod_i^4 d^2k_{i\perp} H(Q)\delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{k}_{3\perp} - q_\perp)$$
$$\times f_a(x, \vec{k}_{1\perp}) S^{\text{global}}(\vec{k}_{2\perp}) S_{J_c}(\vec{k}_{3\perp}) J_c(p_\perp R)$$

n-collinear	$k_n \sim p_\perp(\lambda^2, 1, \lambda)_{n\bar{n}}$	**TMDPDFs**
global soft	$k_{gs} \sim p_\perp(\lambda, \lambda, \lambda)$	**Soft functions**
soft-collinear	$k_{sc} \sim p_\perp R(\lambda R, \lambda/R, \lambda)_{n_J, \bar{n}_J}$	
n_J-collinear	$k_J \sim p_\perp(R^2, 1, R)_{n_J, \bar{n}_J}$	**Jet functions**

Liu, Ringer, Vogelsang, Yuan `18, `20
Arratia, Kang, Prokudin, Ringer `20
Also, Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi `19, `20

Lepton + Jet imbalance



$$\frac{d\sigma_{eP\to e+\text{jet}}}{dp_\perp dq_\perp} = \int \prod_i^4 d^2k_{i\perp} H(Q)\,\delta^{(2)}(\vec{k}_{1\perp} + \vec{k}_{2\perp} + \vec{k}_{3\perp} - q_\perp)$$
$$\times f_a(x, \vec{k}_{1\perp}) S^{\text{global}}(\vec{k}_{2\perp}) S_{J_c}(\vec{k}_{3\perp}) J_c(p_\perp R)$$

n-collinear	$k_n \sim p_\perp(\lambda^2, 1, \lambda)_{n\bar{n}}$	**TMDPDFs**
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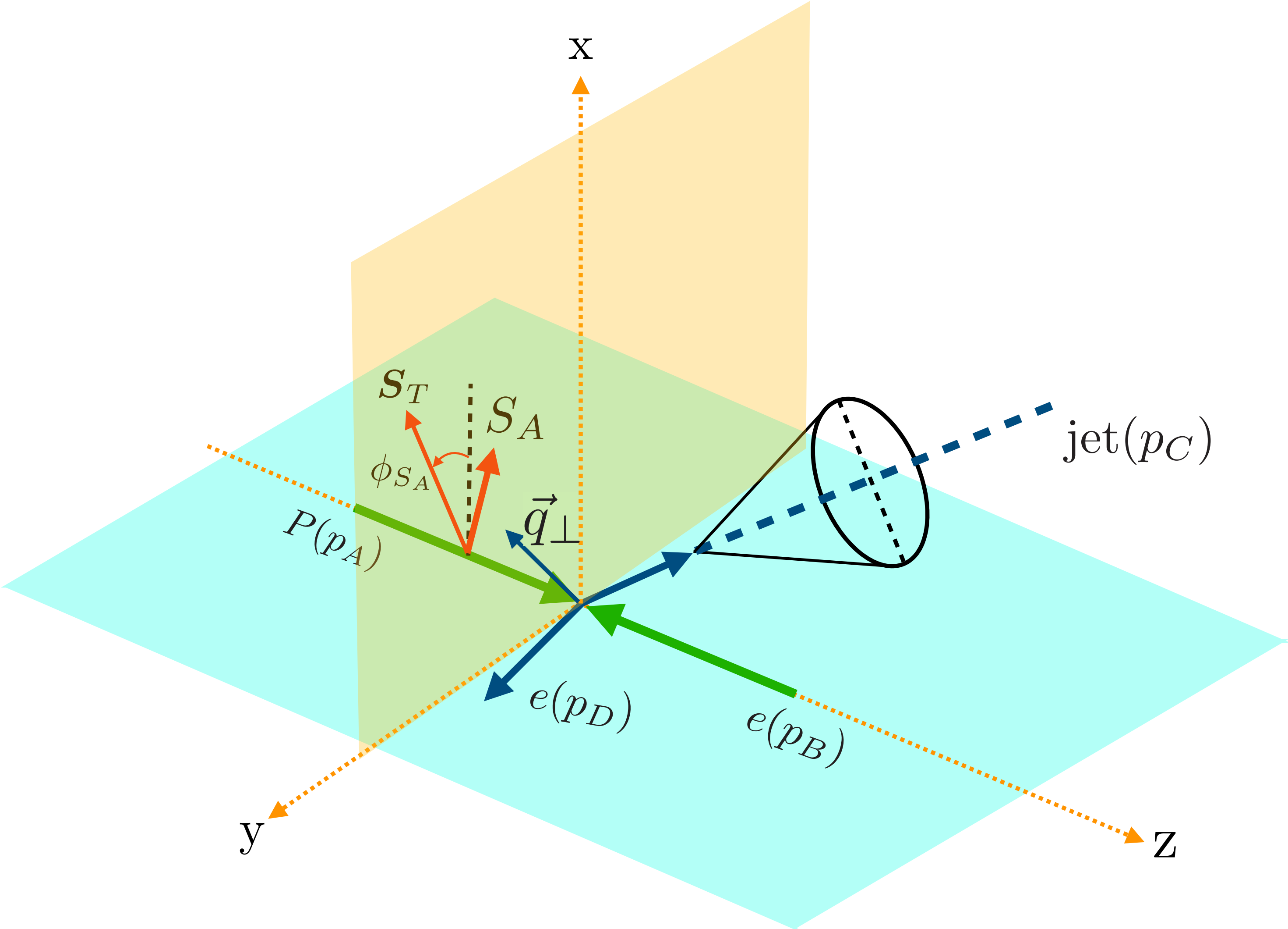
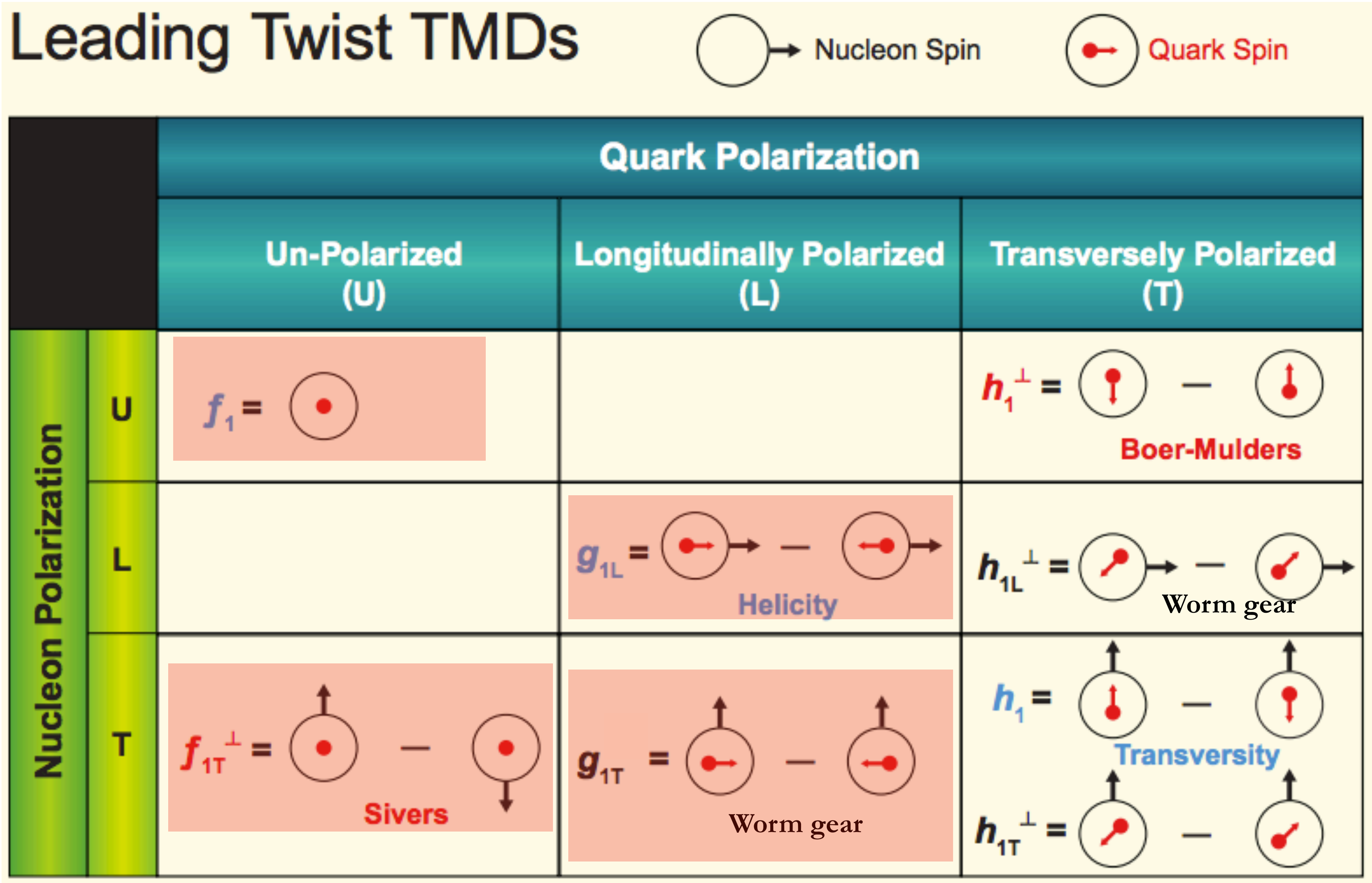
Lepton + Jet imbalance



$$\frac{d\sigma_{eP\to e+\text{jet}}}{dp_\perp dq_\perp} = \int \prod_i^3 d^2k_{i\perp} H(Q)\,\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}-q_\perp) \times f_a(x,\vec{k}_{1\perp})\,S^{\text{global}}(\vec{k}_{2\perp})\,S_{J_c}(\vec{k}_{3\perp})\,J_c(p_\perp R)$$

$$\frac{d\sigma_{e(\lambda_e)P(S)\to e+\text{jet}}}{dp_\perp dq_\perp} = \overset{\sim f_1}{F_{UU}} + \lambda_p\lambda_e \overset{\sim g_{1L}}{F_{LL}} + S_T\left\{\underset{\sim f_{1T}^\perp}{\sin(\phi_{S_A}-\phi_q)F_{TU}^{\sin(\phi_{S_A}-\phi_q)}} + \lambda_e \underset{\sim g_{1T}}{\cos(\phi_{S_A}-\phi_q)F_{TL}^{\cos(\phi_{S_A}-\phi_q)}}\right\},$$



Leading Twist TMDs Nucleon Spin Quark Spin

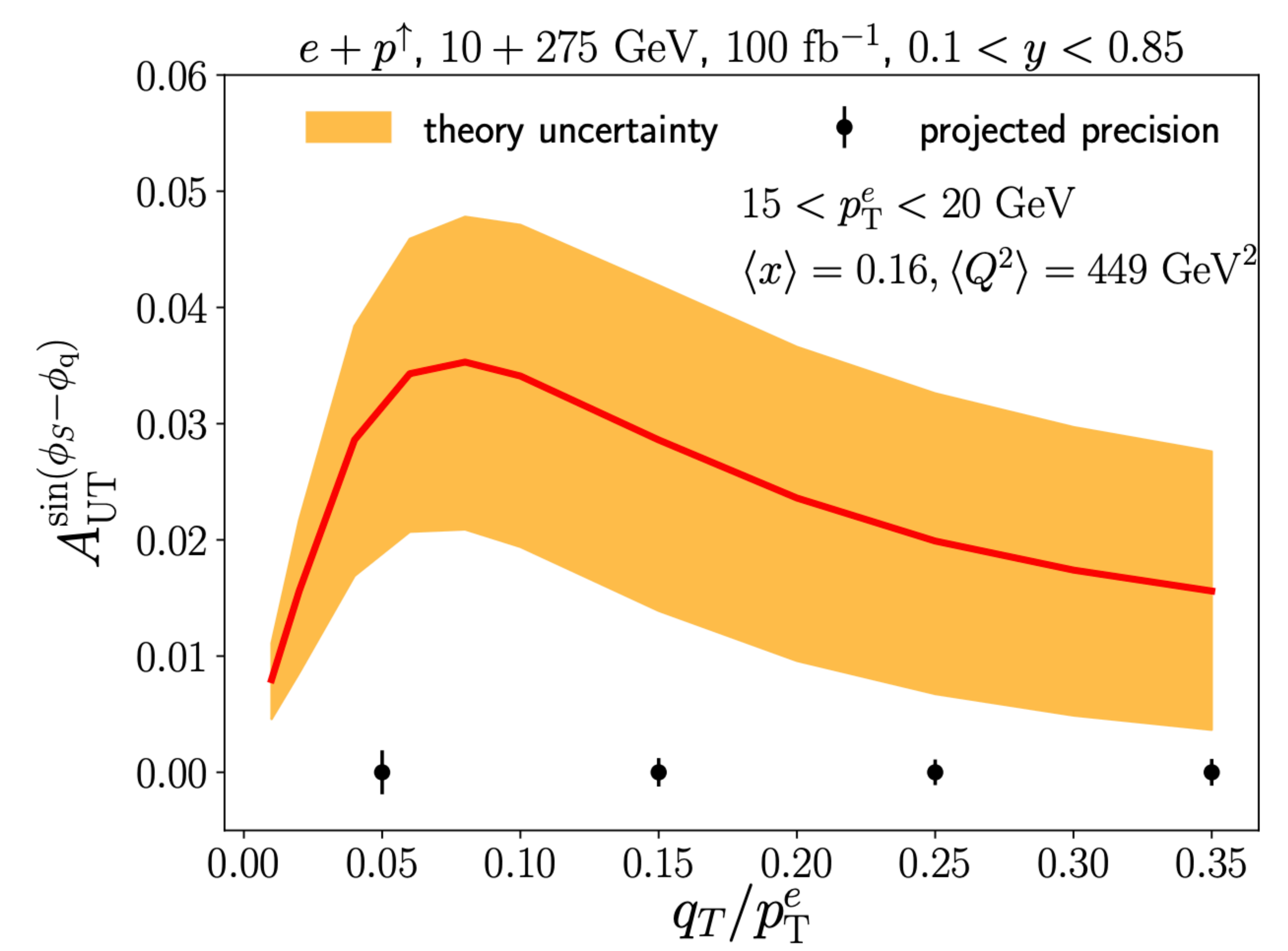
Nucleon Polarization		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
	U	f_1 =		$h_1^\perp$ = − Boer-Mulders
	L		g_{1L} = − Helicity	$h_{1L}^\perp$ = − Worm gear
	T	$f_{1T}^\perp$ = − Sivers	g_{1T} = − Worm gear	h_1 = − Transversity $h_{1T}^\perp$ = −

- With jet, only sensitive to *single* TMDs (compared to standard processes)
- Only sensitive to *chiral-even* TMDPDFs

Chiral-odd components in jets : Liu, Xing `21

Liu, Ringer, Vogelsang, Yuan `18, `20
Arratia, Kang, Prokudin, Ringer `20
Kang, KL, Shao, Zhao `21

Sivers asymmetry



$$\boxed{\frac{d\Delta\sigma_{eP\to e+\text{jet}}}{dp_\perp dq_\perp} = \frac{\epsilon_\perp^{\rho\sigma} S_{\perp\rho}}{M}\int\prod_i^3 d^2k_{i\perp} H(Q)\,\delta^{(2)}(\vec{k}_{1\perp}+\vec{k}_{2\perp}+\vec{k}_{3\perp}-q_\perp) \times k_{1\perp\sigma}\, f_{aT}^\perp(x_a,k_{1\perp})\, S^{\text{global}}(\vec{k}_{2\perp})\, S_{J_c}(\vec{k}_{3\perp})\, J_c(p_\perp R)}$$

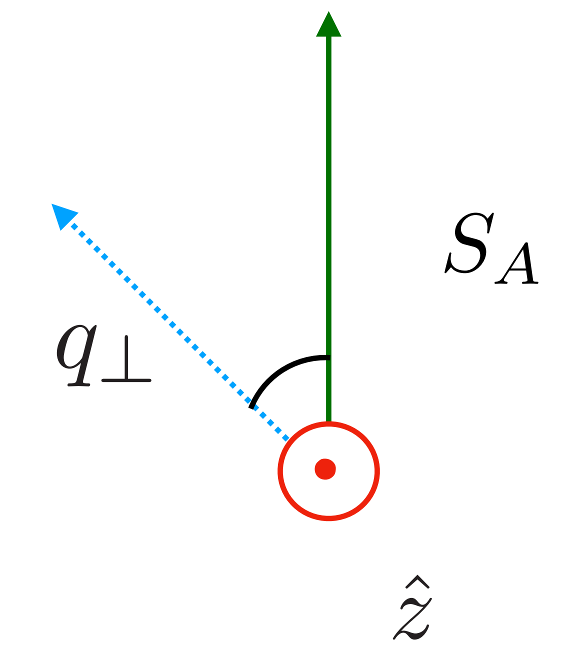
$$\frac{d\sigma_{e(\lambda_e)P(S)\to e+\text{jet}}}{dp_\perp dq_\perp} = \overset{\sim f_1}{F_{UU}} + \lambda_p\lambda_e \overset{\sim g_{1L}}{F_{LL}} + S_T\left\{\sin(\phi_{S_A}-\phi_q)\underset{\sim f_{1T}^\perp}{F_{TU}^{\sin(\phi_{S_A}-\phi_q)}} + \lambda_e\cos(\phi_{S_A}-\phi_q)\underset{\sim g_{1T}}{F_{TL}^{\cos(\phi_{S_A}-\phi_q)}}\right\}$$

Sivers from SIDIS extraction

Echevarria, Idilbi, Kang, Vitev, `14

Arratia, Kang, Prokudin, Ringer `20
Kang, KL, Shao, Zhao `21

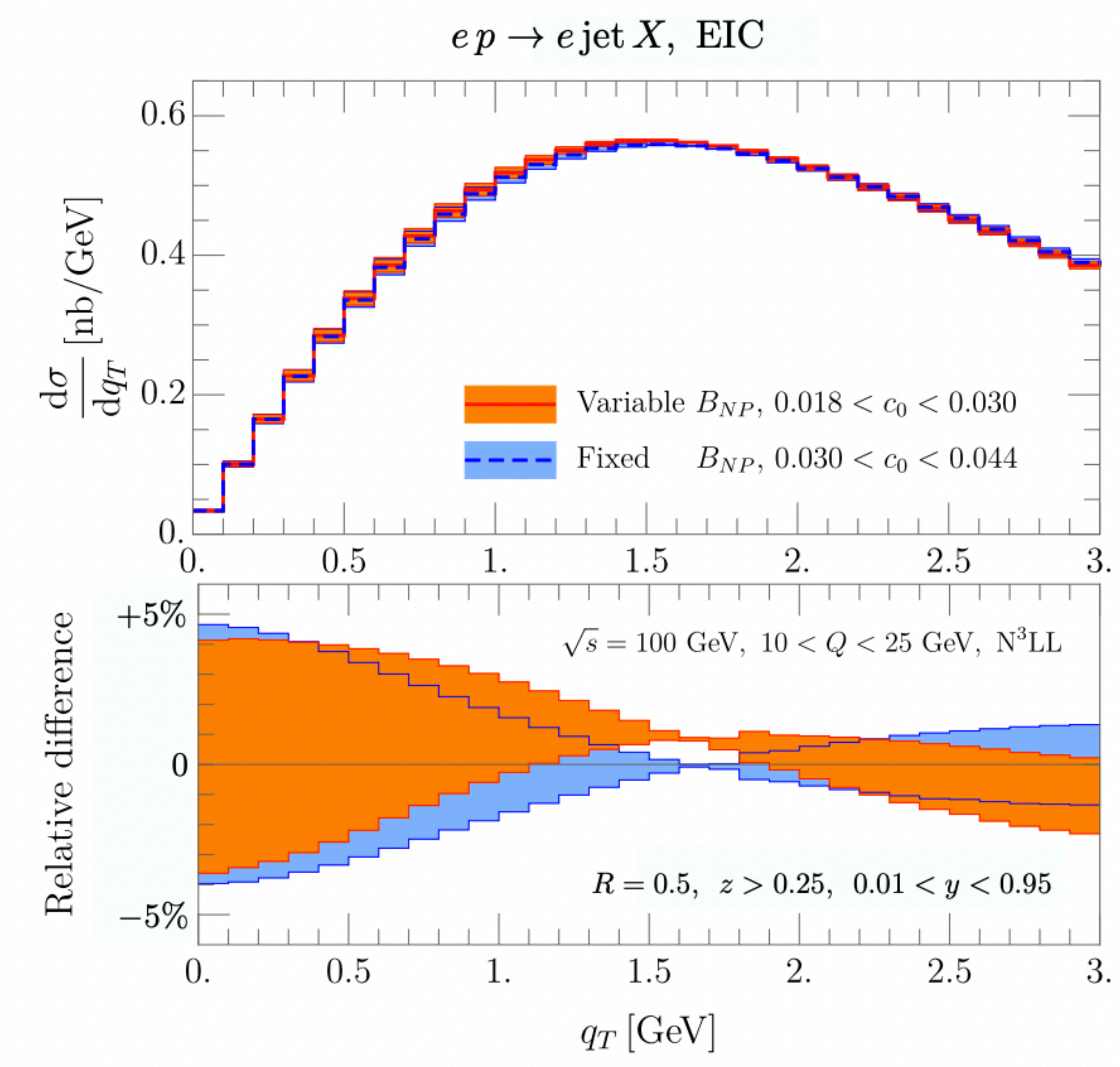
Measurement of $A_{UT}^{\sin(\phi_{S_A}-\phi_q)} = \dfrac{F_{UT}^{\sin(\phi_{S_A}-\phi_q)}}{F_{UU}}$



$\propto \sin\boxed{(\phi_{S_A}-\phi_q)}$

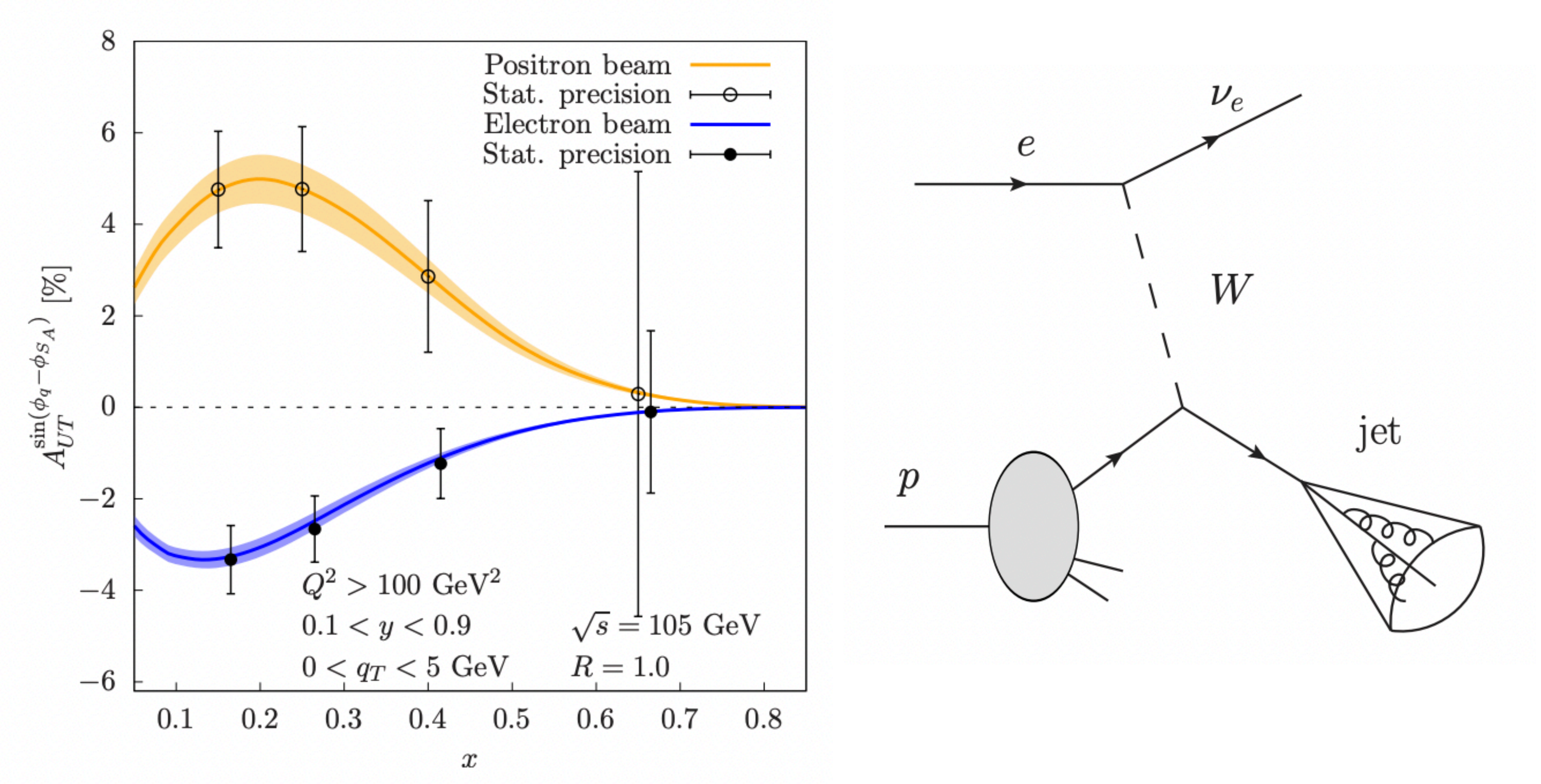
Many more…

Standard SIDIS-like process



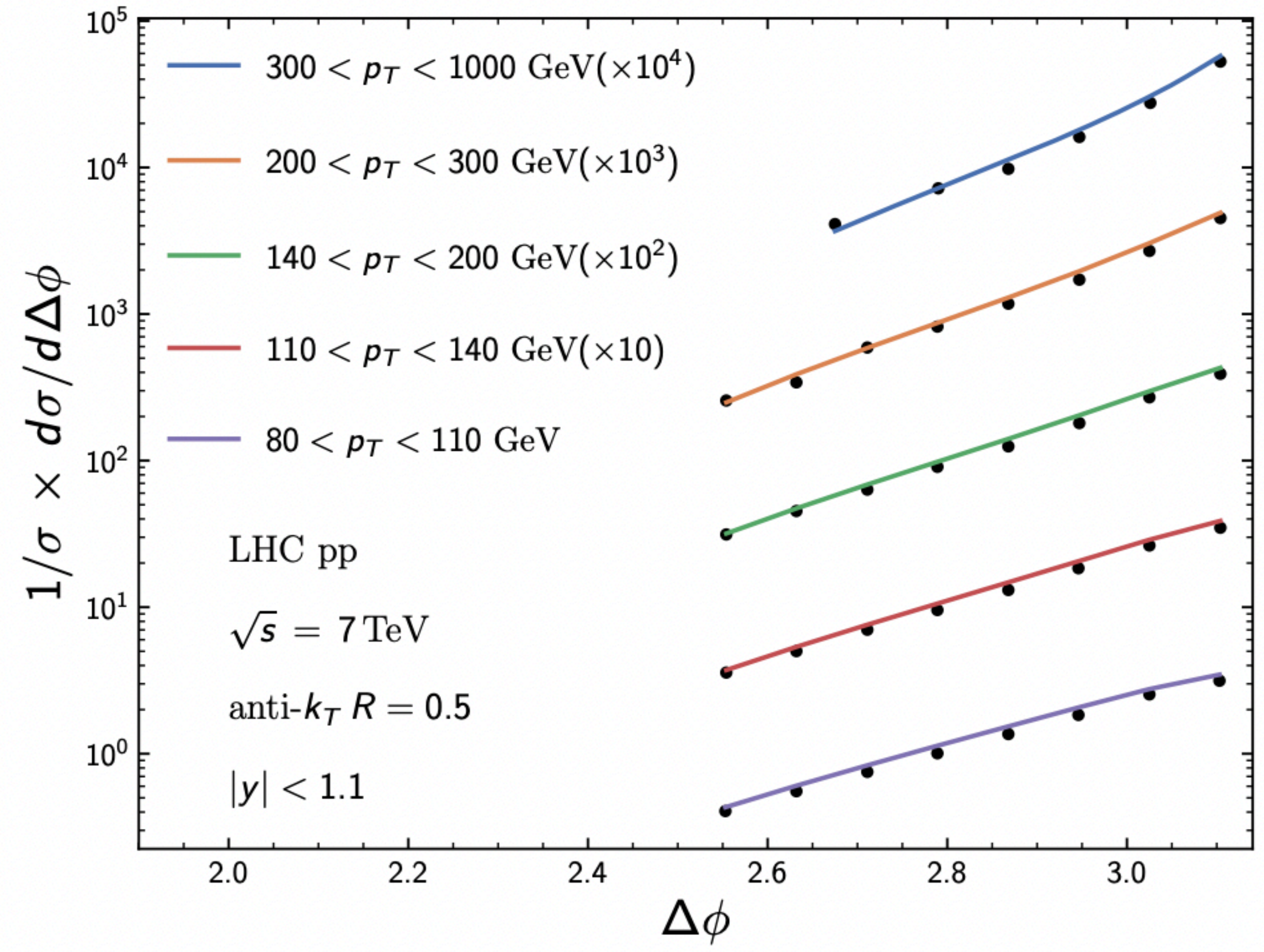
Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi `19, `20

Charged-current DIS process



Arratia, Kang, Paul, Prokudin, Ringer, Zhao `22

Hadron colliders

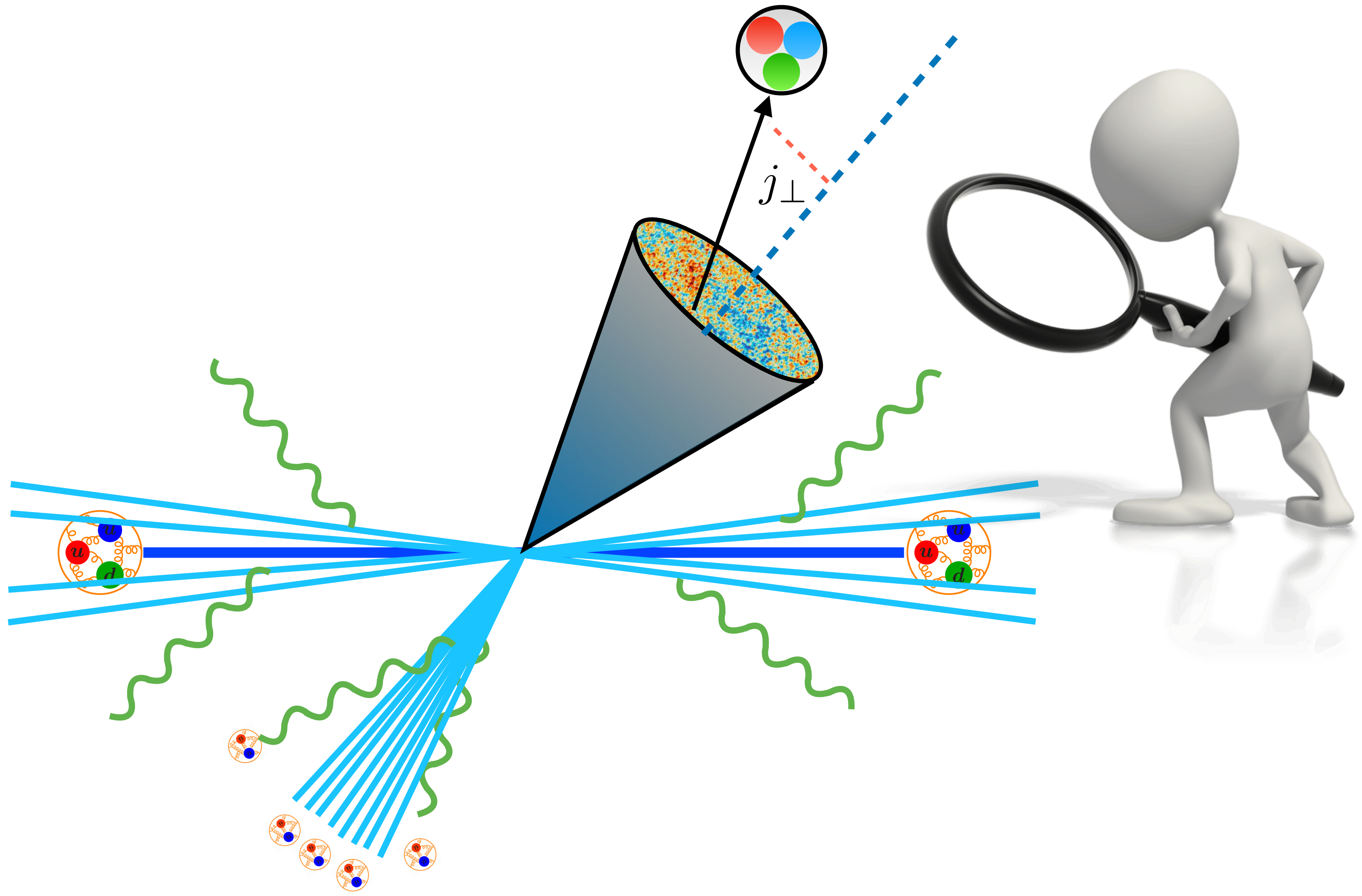


Gao, Kang, Shao, Terry, Zhang `23
Buffing, Kang, Lee, Liu `18
Kang, Lee, Shao, Terry `20
Liu, Ringer, Vogelsang, Yuan `20

Dijet DIS correlations, ***Recoil-free jets,*** ***groomed jets,*** ***etc…***

Hatta, Xiao, Yuan, Zhou `22* *Chien, Rudi Rahn, van Velzen, Shao, Waalewijn, Wu `20, 22 *Makris, Neill, Vaidya `17*

Fragmenting Jet Functions



Fragmenting Jet Functions

Unpolarized case:

$$\frac{d\sigma^{pp\to\mathrm{jet}(h)X}}{dp_T\,d\eta\,dz_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H^c_{ab} \otimes \mathcal{G}^h_c(z_h)$$

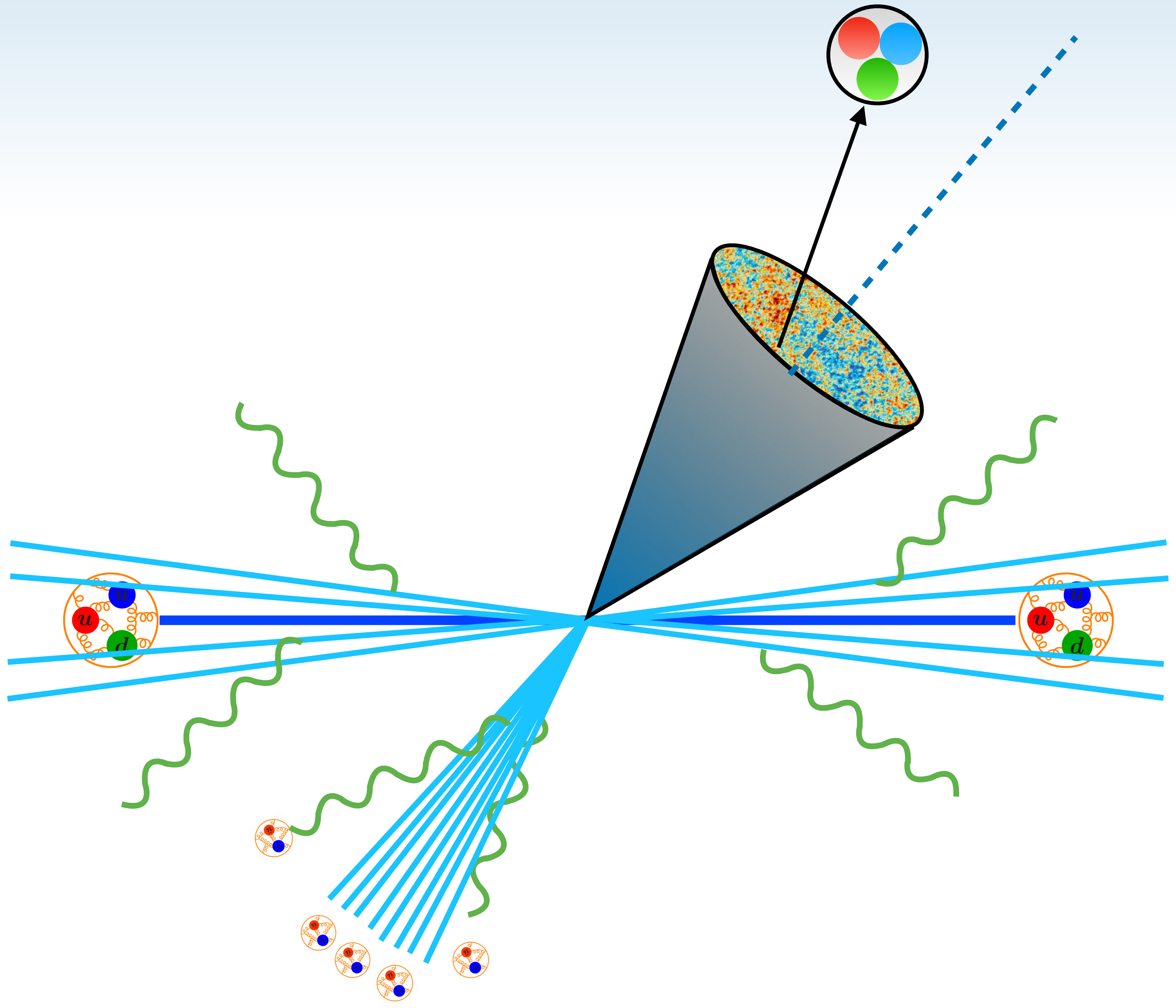
$$\Lambda_{\mathrm{QCD}} \qquad p_T \qquad p_T R \quad \Lambda_{\mathrm{QCD}}$$

where

$$z = p_T^J / p_T^c$$

$$z_h = p_T^h / p_T^J$$

$$\boxed{\frac{d\sigma^{pp\to\mathrm{jet}X}}{dp_T\,d\eta} = \sum_{a,b,c} \underset{\Lambda_{\mathrm{QCD}}}{f_{a/A} \otimes f_{b/B}} \otimes \underset{p_T}{H^c_{ab}} \otimes \underset{p_T R}{J_c}}$$



$$PP\,/\,e\,P \to J(h) + X$$

2) Fragmenting Jet Functions
TMDFFs

Procura, Stewart `10
Arleo, Fontannaz, Guillet, Nguyen `14
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16

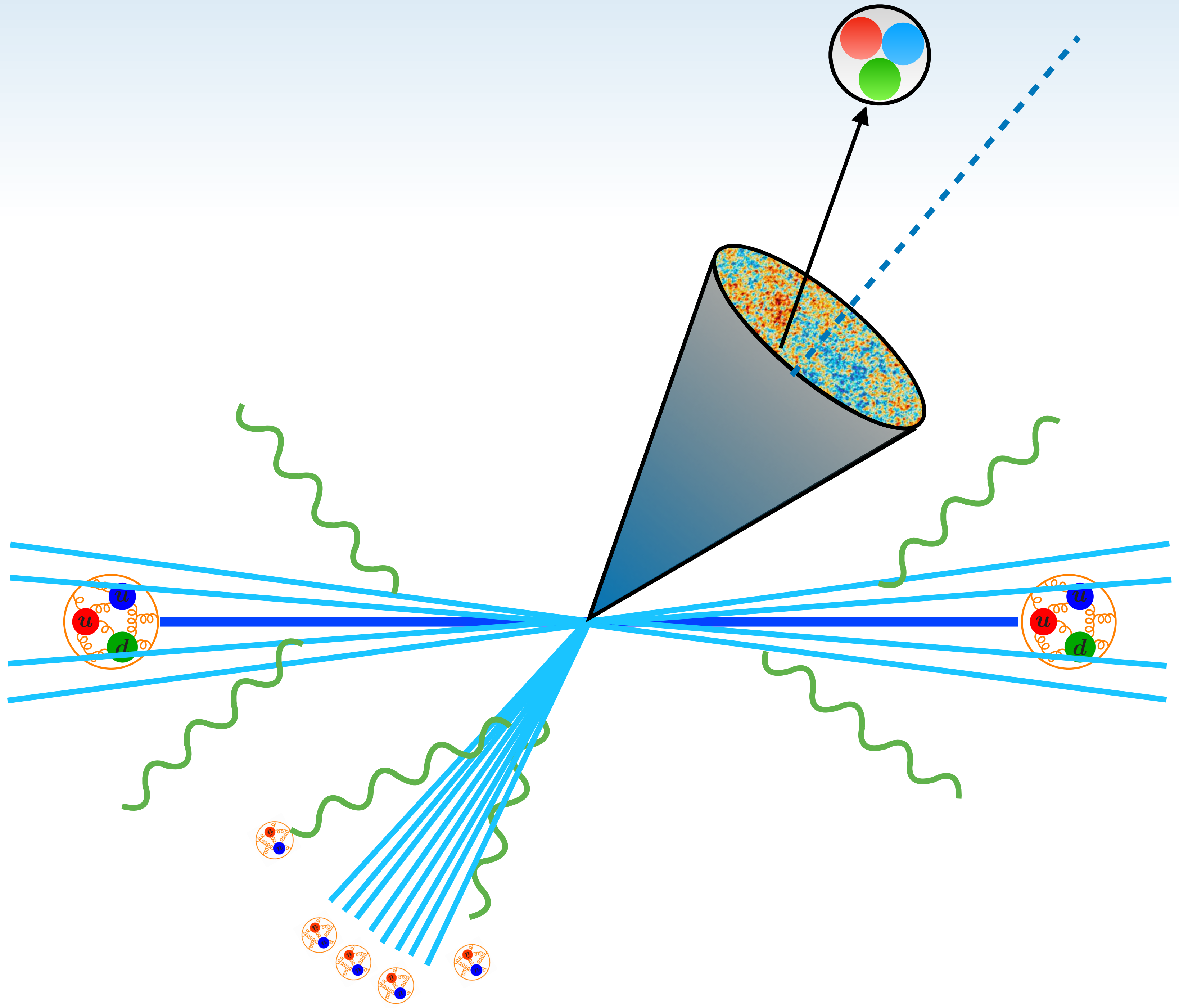
Fragmenting Jet Functions

Unpolarized case:

$$\frac{d\sigma^{pp\to\text{jet}(h)X}}{dp_T d\eta dz_h} = \sum_{a,b,c} f_{a/A} \otimes f_{b/B} \otimes H^c_{ab} \otimes \mathcal{G}^h_c(z_h)$$

Λ_{QCD} $\quad p_T \quad$ $p_T R$

Λ_{QCD}



$$PP\,/\,e\,P \to J(h) + X$$

where $z = p_T^J/p_T^c$

$z_h = p_T^h/p_T^J$

$$\mathcal{G}^h_c(z, z_h, p_T R, \mu) = \sum_j \mathcal{J}_{ij}(z, z_h, p_T R, \mu) \otimes D^h_j(z_h, \mu)$$

$p_T R$ $\quad \Lambda_{\text{QCD}}$

matching coefficients

collinear FFs

Procura, Stewart `10
Arleo, Fontannaz, Guillet, Nguyen `14
Kaufmann, Mukherjee, Vogelsang `15
Kang, Ringer, Vitev `16
Dai, Kim, Leibovich `16

Fragmenting Jet Functions

- Light charged hadrons

  Arleo, Fontannaz, Guillet, Nguyen `14
  Kaufmann, Mukherjee, Vogelsang `15
  Kang, Ringer, Vitev `16
  Neill, Scimemi, Waalewijn `16

- Photons

  Kaufmann, Mukherjee, Vogelsang `16

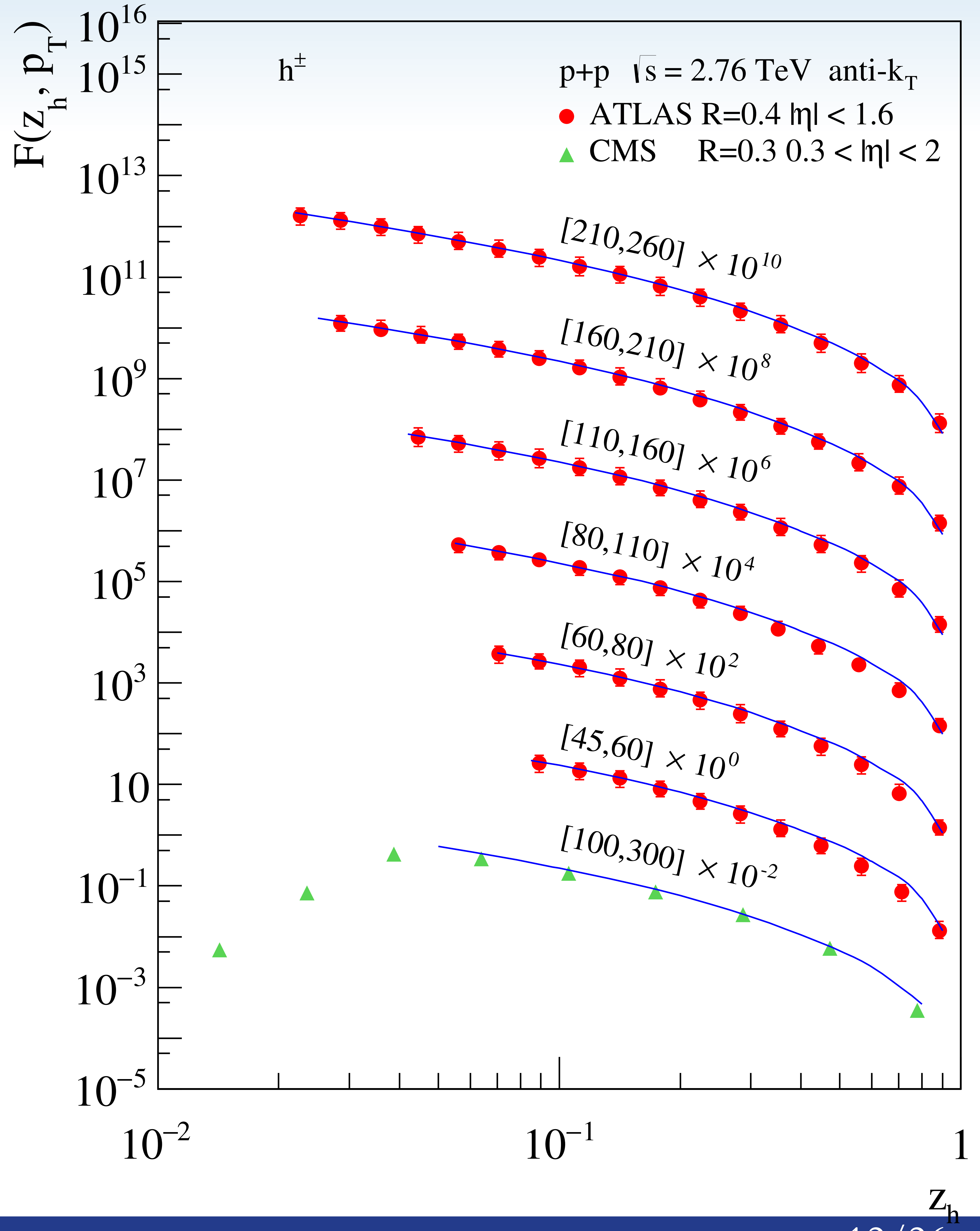
- Heavy flavor mesons

  Chien, Kang, Ringer, Vitev, Xing `15
  Bain, Dai, Hornig, Leibovich, Makris, Mehen `16
  Anderle, Kaufmann, Stratmann, Ringer, Vitev `17

- Quarkonia

  Baumgart, Leibovich, Mehen, Rothstein `14
  Bain, Dai, Hornig, Leibovich, Makris, Mehen `16
  Kang, Qiu, Ringer, Xing, Zhang `17
  Bain, Dai, Leibovich, Makris, Mehen `17

$$F(z_h, p_T) = \frac{d\sigma^{pp\to(\text{jet}h)X}}{dp_T d\eta dz_h} \Bigg/ \frac{d\sigma^{pp\to\text{jet}X}}{dp_T d\eta}$$



TMD Fragmenting Jet Functions

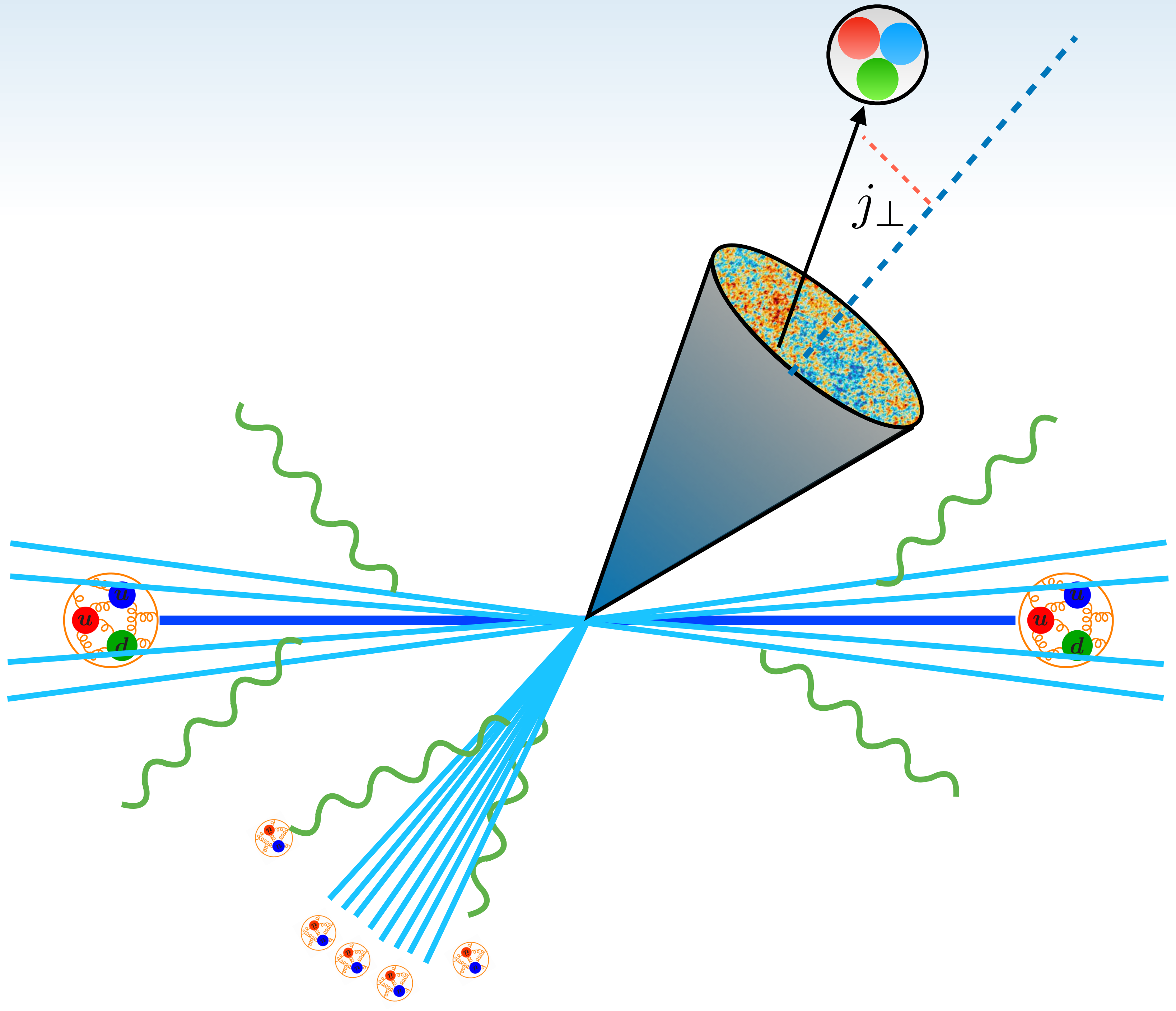
Unpolarized case:

$$\frac{d\sigma^{pp\to\text{jet}(h)X}}{dp_T\,d\eta\,dz_h\,d^2\boldsymbol{j}_\perp} = \sum_{a,b,c} f_{a/A}\otimes f_{b/B}\otimes H^c_{ab}\otimes \mathcal{G}^h_c(z_h,\boldsymbol{j}_\perp)$$

Λ_{QCD} $\quad p_T \quad p_T R$

Λ_{QCD}

$j_\perp$



$$PP\,/\,e\,P \to J(h) + X$$

$$\Lambda_{\text{QCD}} \lesssim j_\perp \ll p_T R,$$

TMD Fragmenting Jet functions can be related to the usual TMD FFs

(up to NP soft dynamics)

$$\mathcal{G}^h_c\left(z_h, j_\perp^2, \mu, \zeta_J\right) = \int_{\boldsymbol{k}_\perp,\boldsymbol{\lambda}_\perp} D_1^{h/c,\text{ unsub}}\left(z_h, k_\perp^2, \mu, \zeta'/\nu^2\right) S_c\left(\lambda_\perp^2, \mu, \nu\mathcal{R}\right) = D_1^{h/c}\left(z_h, j_\perp^2, \mu, \zeta_J\right)$$

pert

where $\zeta_J = p_T^2 R^2$

Kang, Liu, Ringer, Xing `17
Kang, Lee, Zhao `20
Kang, Lee, Shao, Zhao `21

Polarized TMD Fragmenting Jet Functions

TMD Fragmentation Functions (TMDFFs)

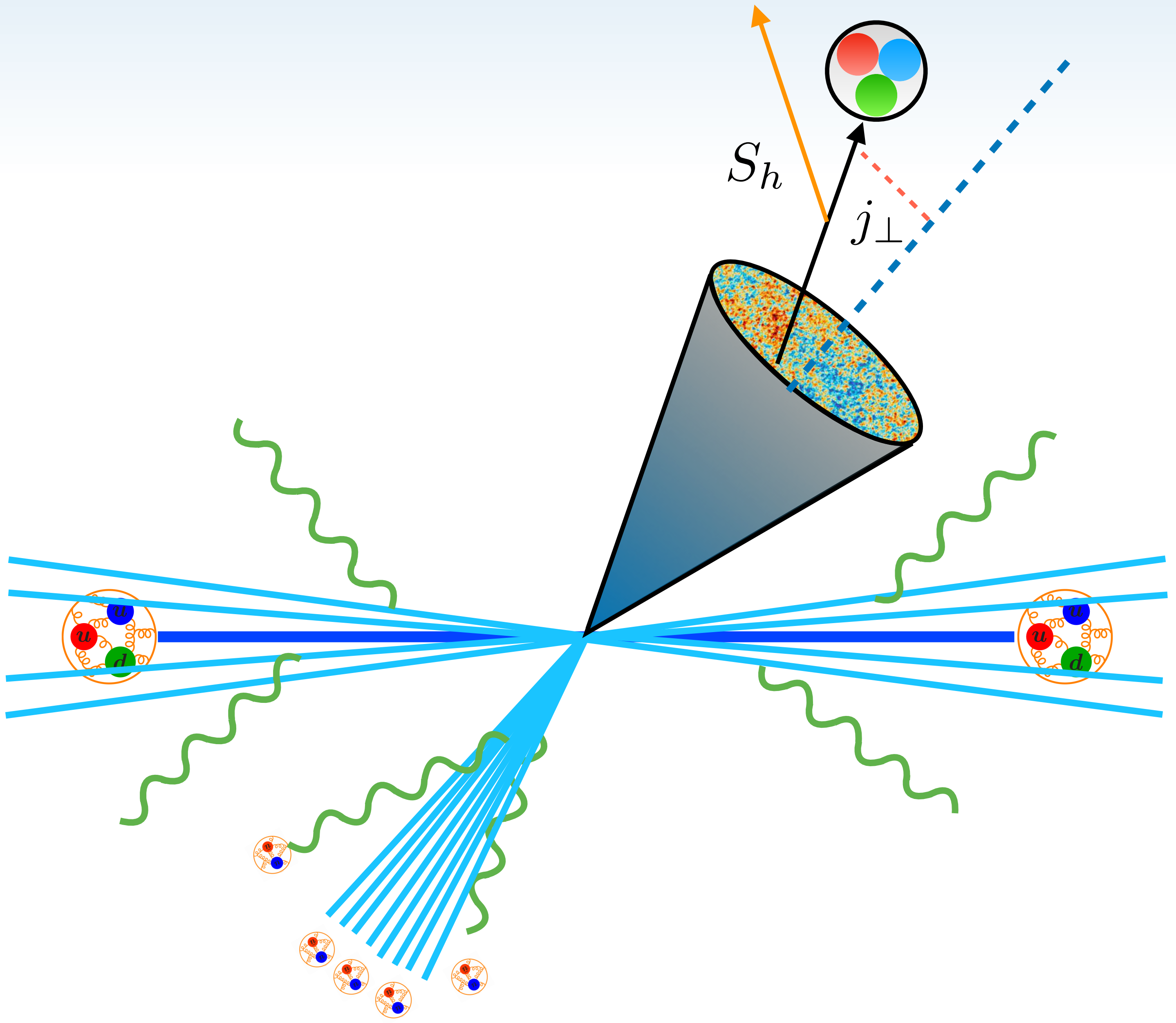
Quark polarization

Hadron polarization	U	L	T
U	$D^{h/q}$		$H^{\perp h/q}$
L		$G^{h/q}$	$H_L^{\perp h/q}$
T	$D_T^{\perp h/q}$	$G_T^{h/q}$	$H^{h/q}$ $H_T^{\perp h/q}$

TMD Fragmenting Jet Functions (TMDFJFs)

Quark polarization

Hadron polarization	U	L	T
U	$\mathcal{D}^{h/q}$		$\mathcal{H}^{\perp h/q}$
L		$\mathcal{G}^{h/q}$	$\mathcal{H}_L^{\perp h/q}$
T	$\mathcal{D}_T^{\perp h/q}$	$\mathcal{G}_T^{h/q}$	$\mathcal{H}^{h/q}$ $\mathcal{H}_T^{\perp h/q}$

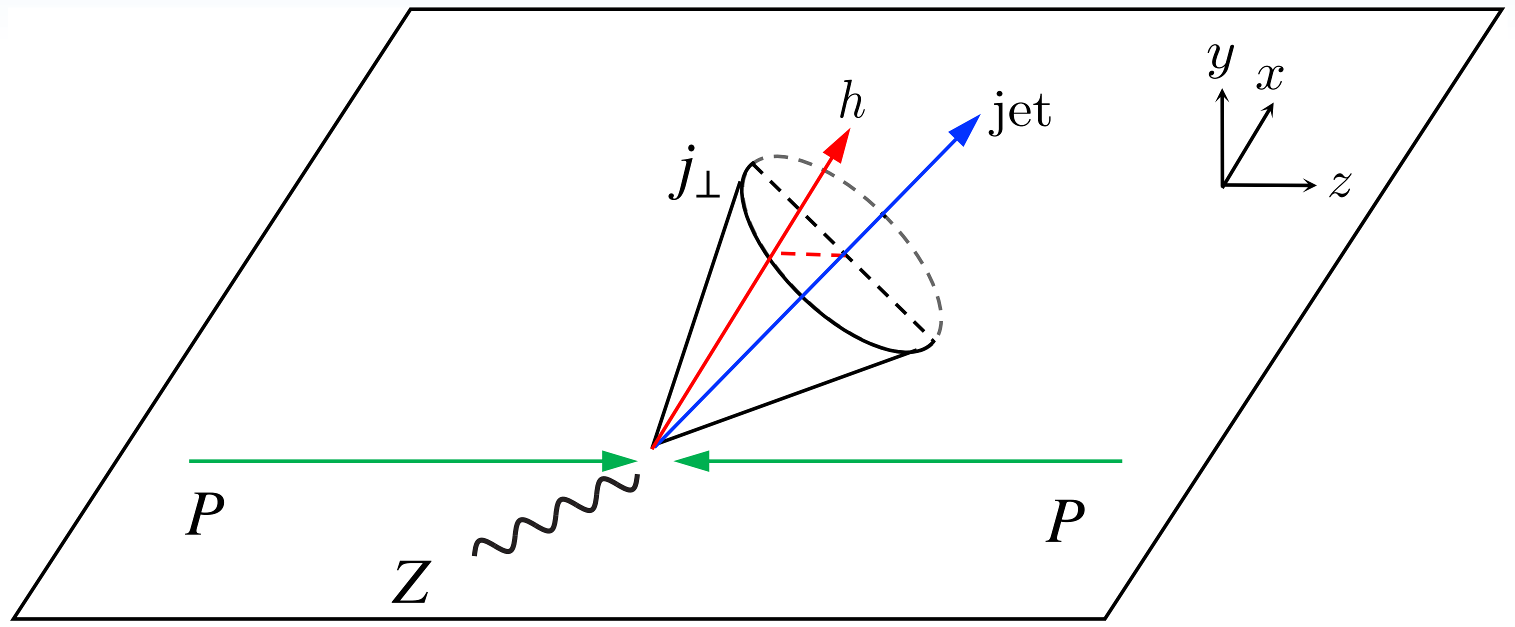


$$PP\,/\,e\,P \to J(h) + X$$

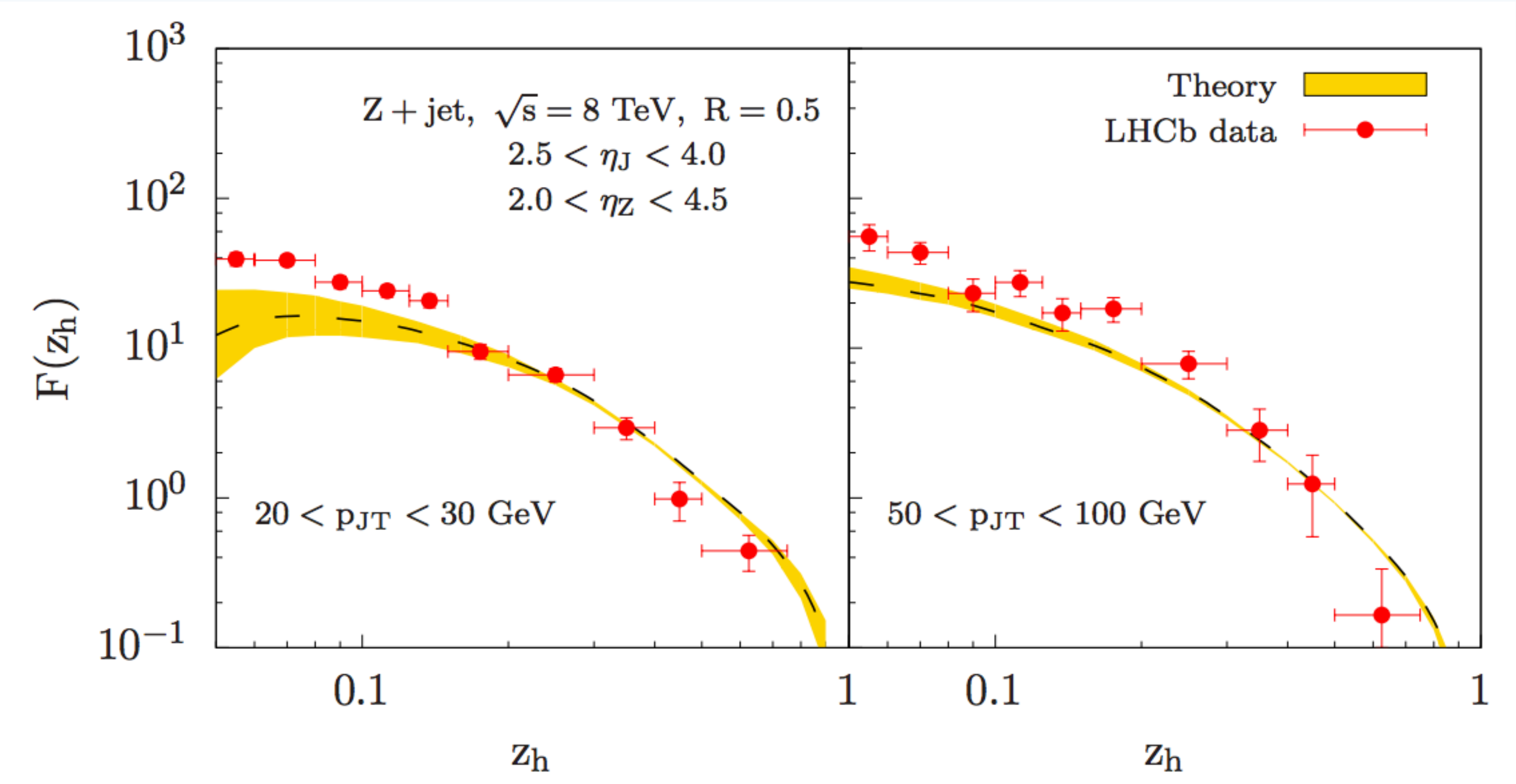
- There are analogous spin-dependent TMDFJFs relative to TMDFFs
- Red ones vanish after integration over $j_\perp$

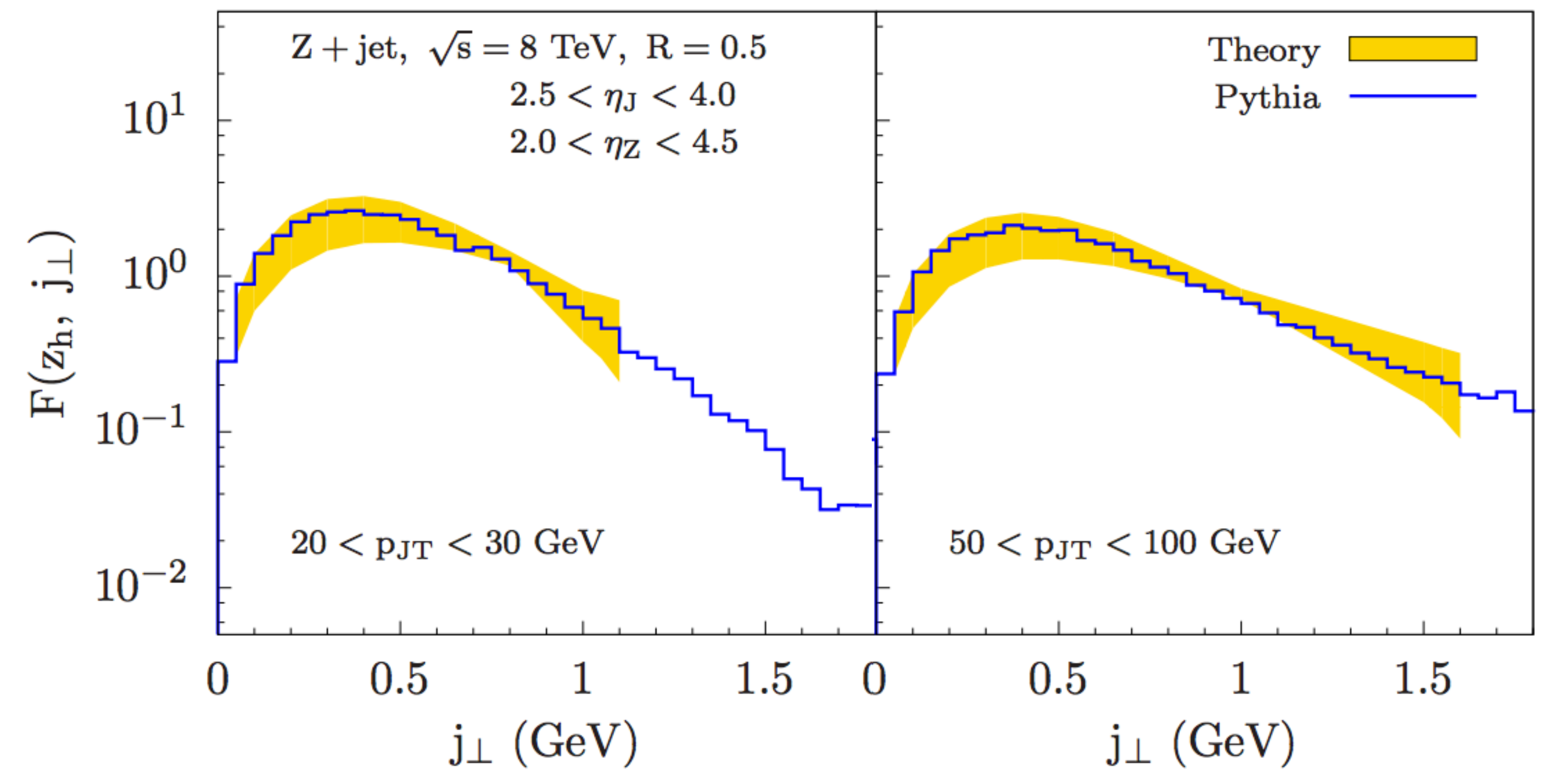
Kang, Lee, Zhao `20

Unpolarized hadron in jet



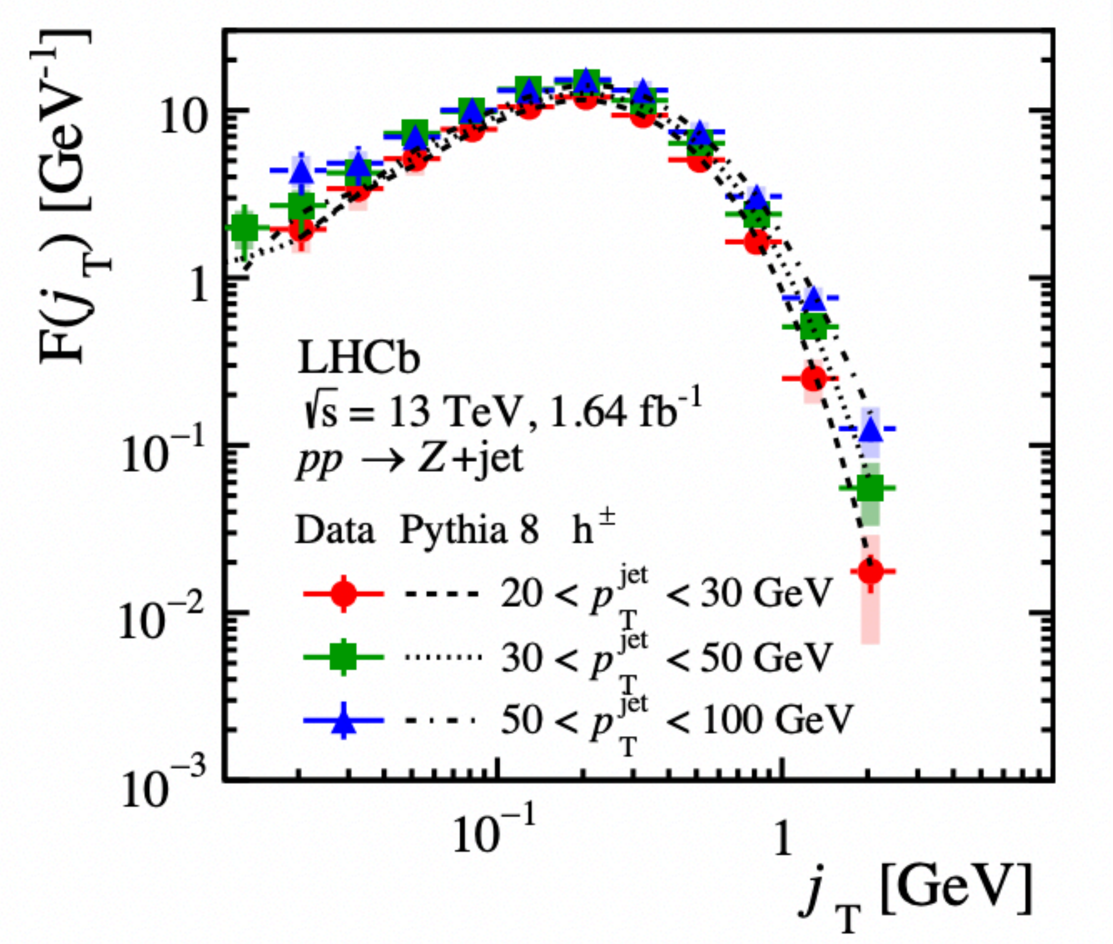
Z-tagged jets at LHCb

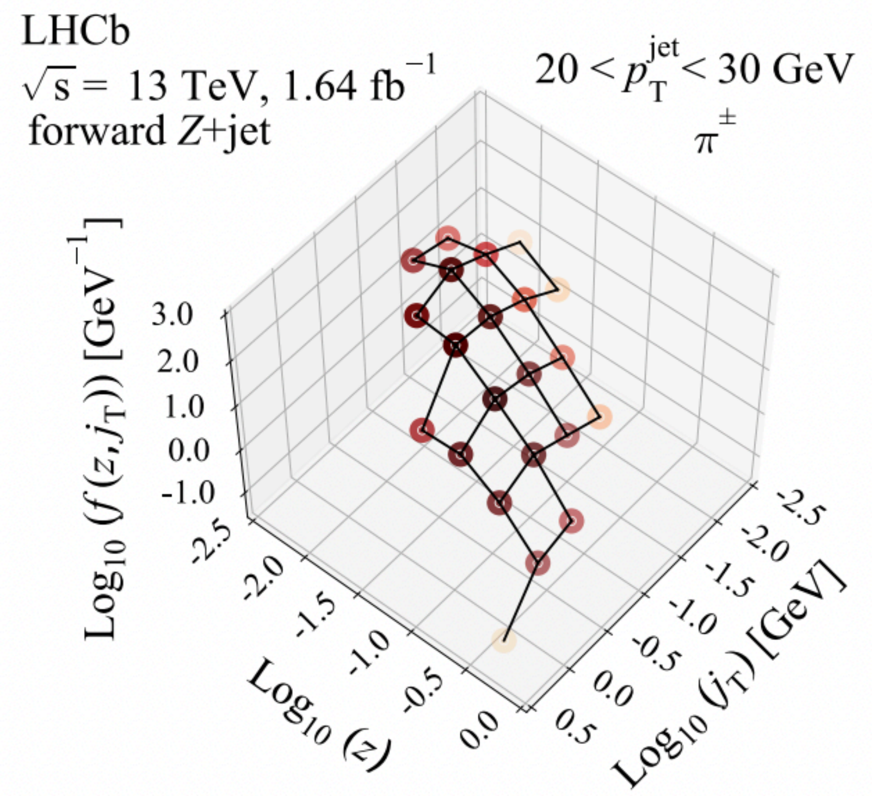




Kang, Lee, Terry, Xing `19

Find a good agreement within the intermediate z_h region.

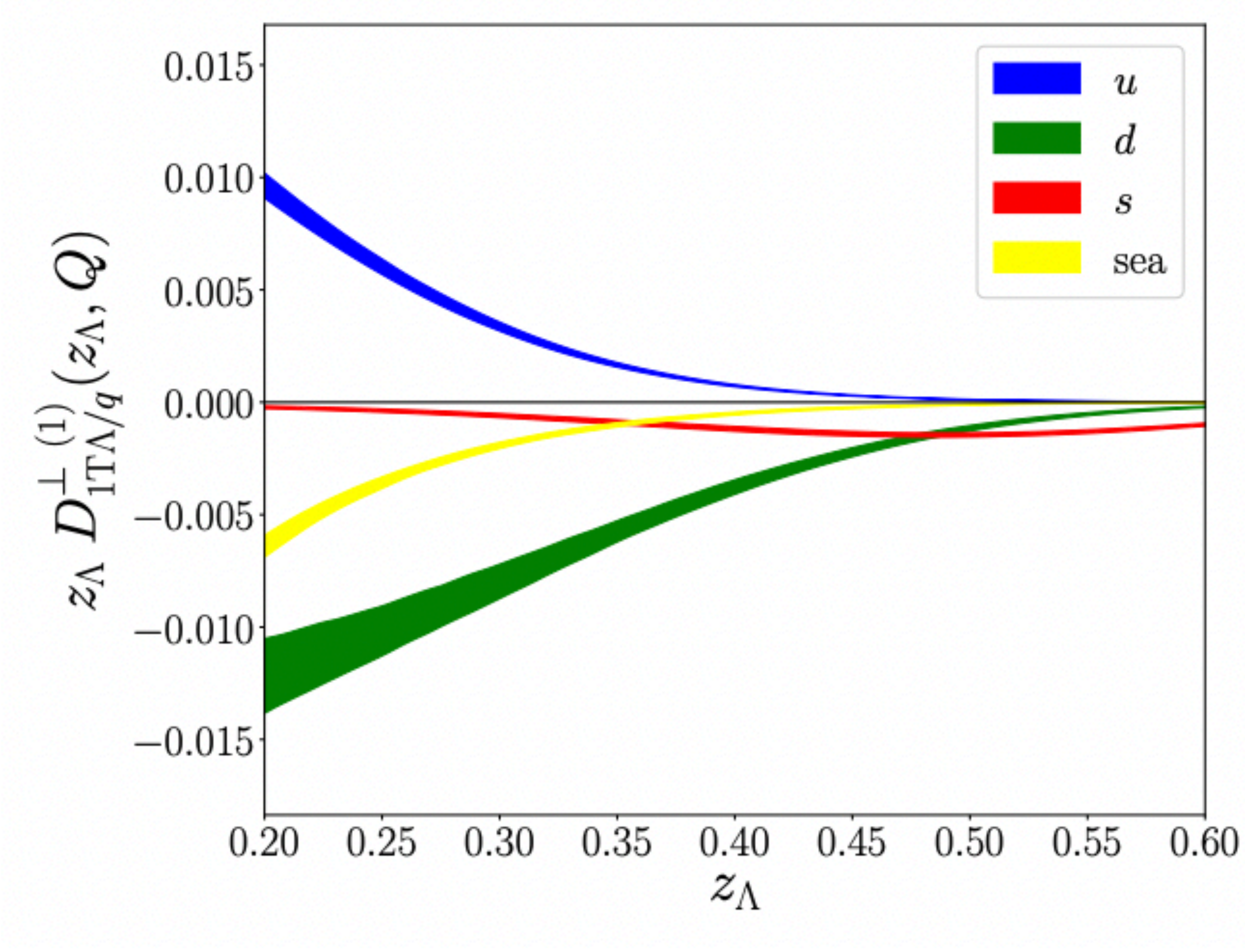




LHCb Collaboration `22

Polarizing FF / FJF

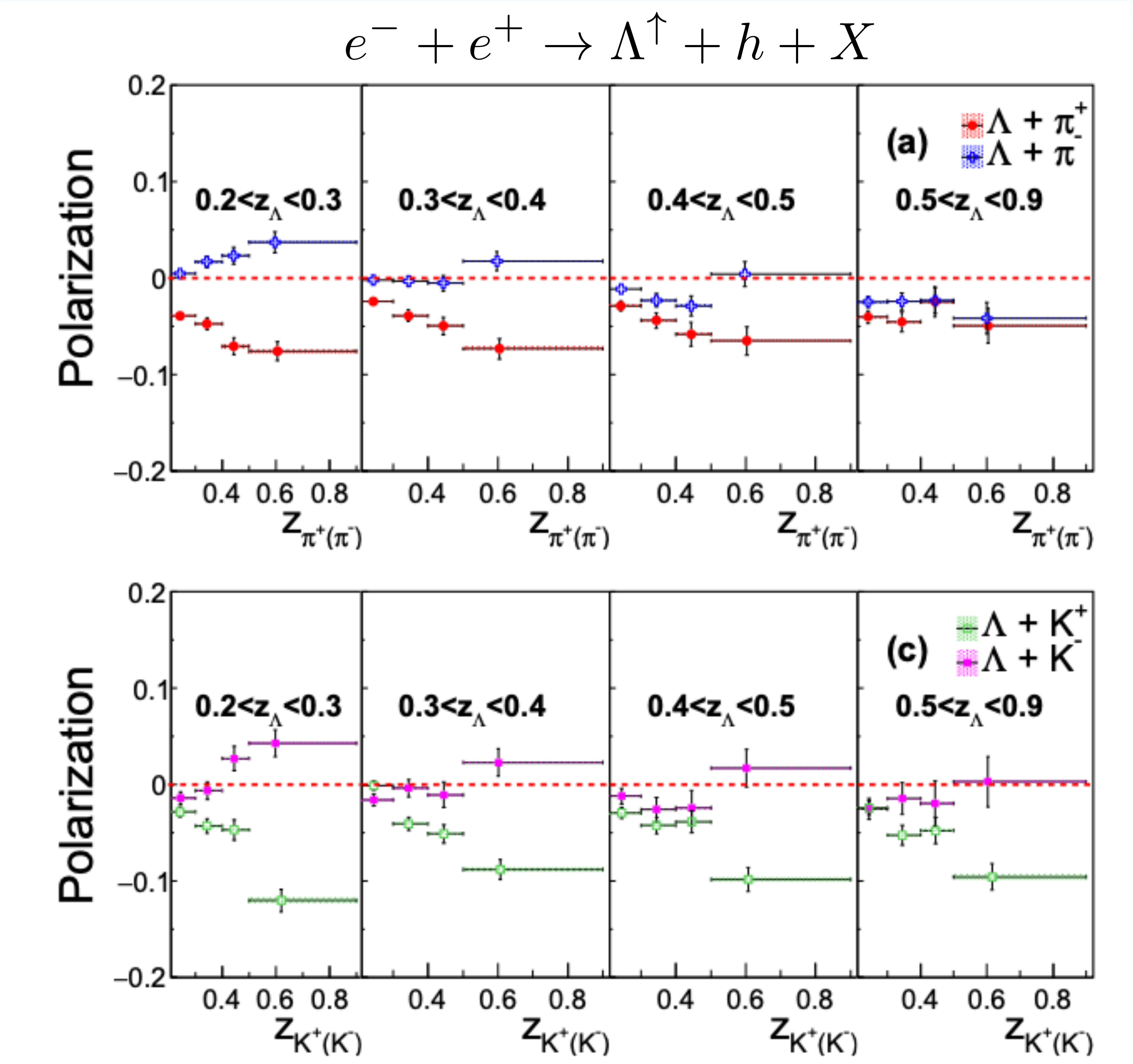
Unpolarized parton to transversely polarized hadron $D_T^{\perp h/q}, \mathcal{D}_T^{\perp h/q}$



fit from the Belle Collaboration data

Callos, Kang, Terry, `20

$e^- + e^+ \to \Lambda^\uparrow + h + X$



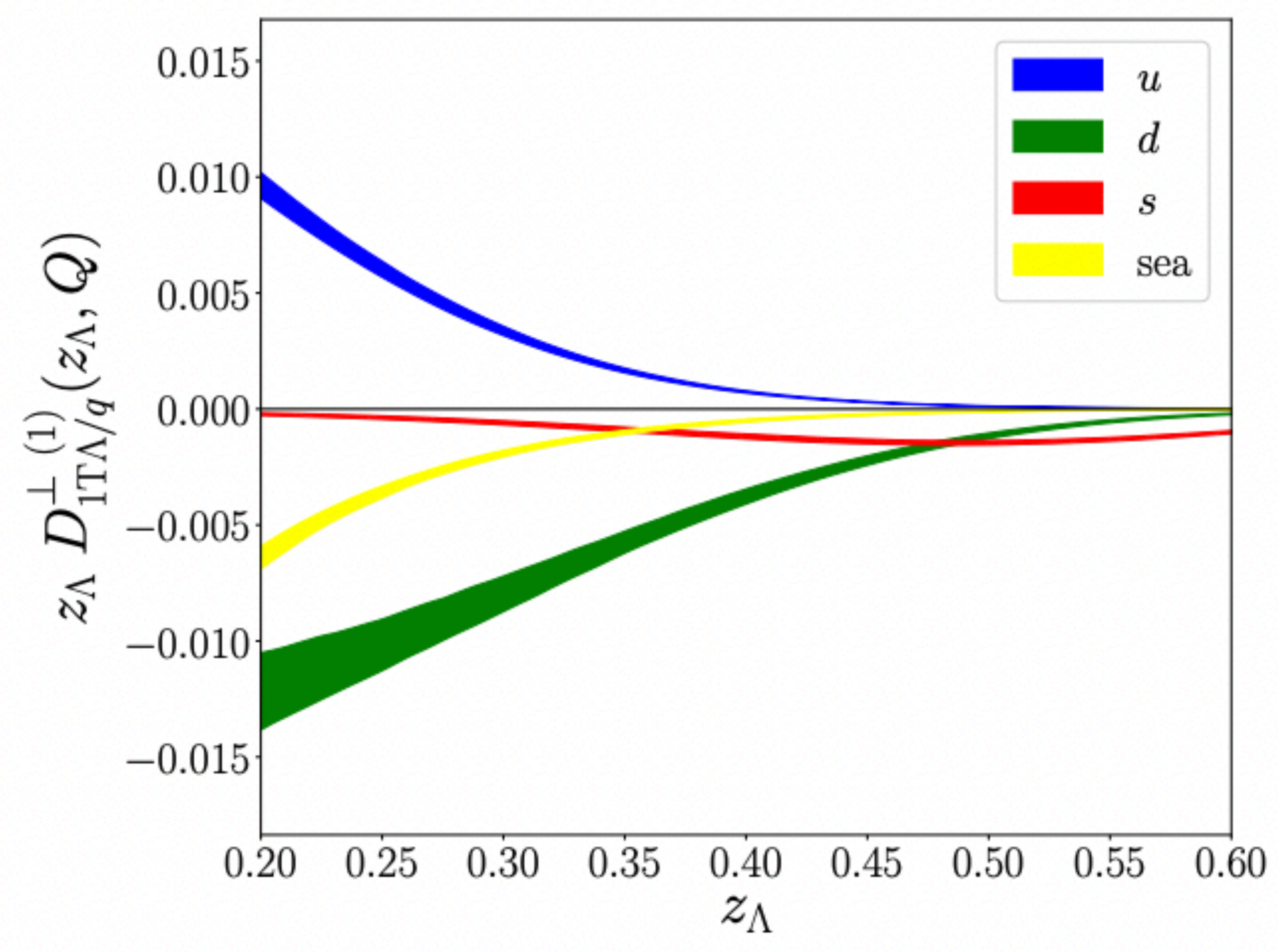
Belle Collaboration `19

$e^- + e^+ \to \Lambda^\uparrow + h + X$

Convolution between polarizing FF and unpolarized FF

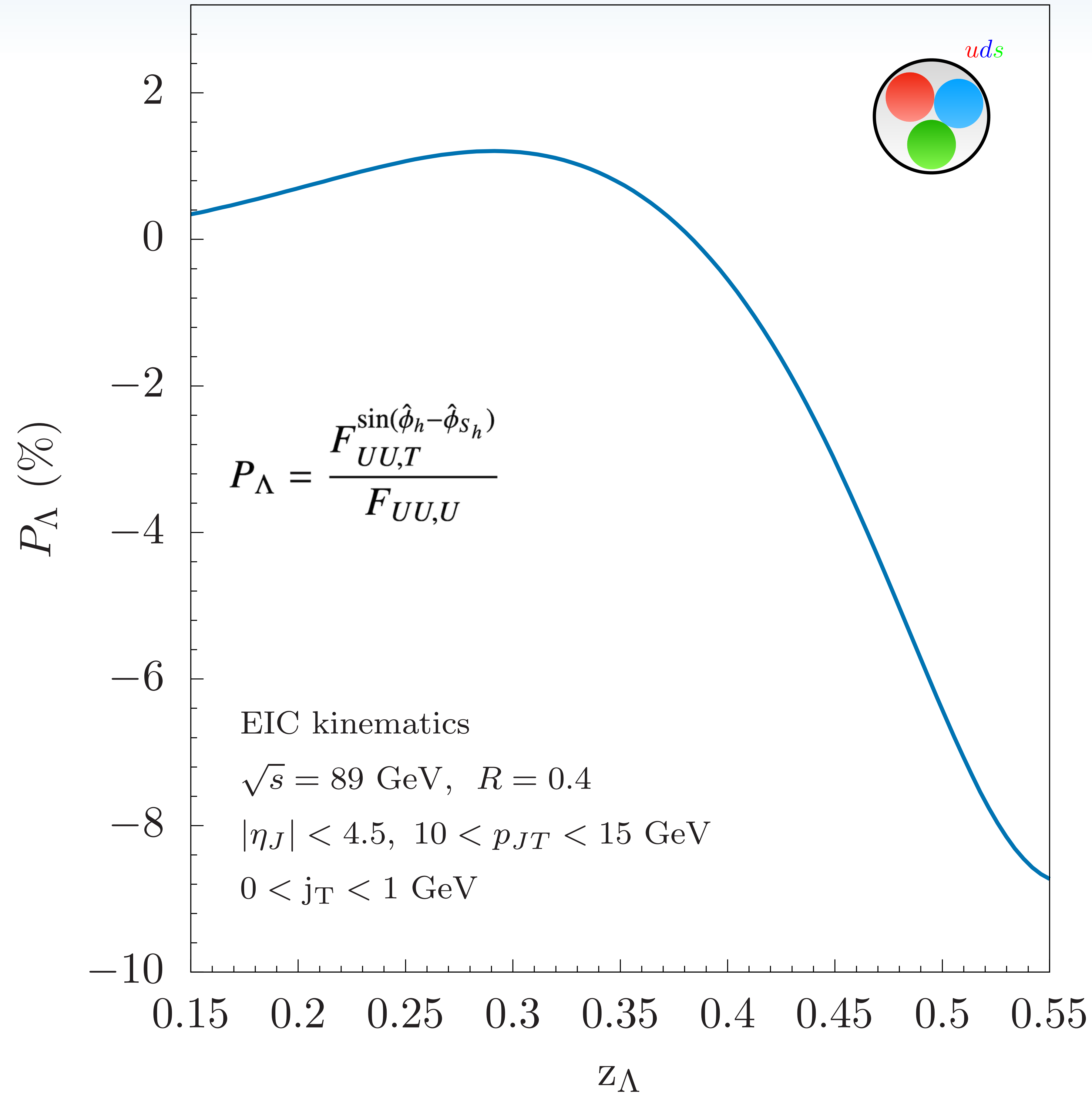
Polarizing FF / FJF

Unpolarized parton to transversely polarized hadron $D_T^{\perp h/q}, \mathcal{D}_T^{\perp h/q}$



fit from the Belle Collaboration data

Callos, Kang, Terry, `20



At the observable level, one can see where u and d polarizing FF dominates

Kang, KL, Zhao, `20

Azimuthal angular dependence

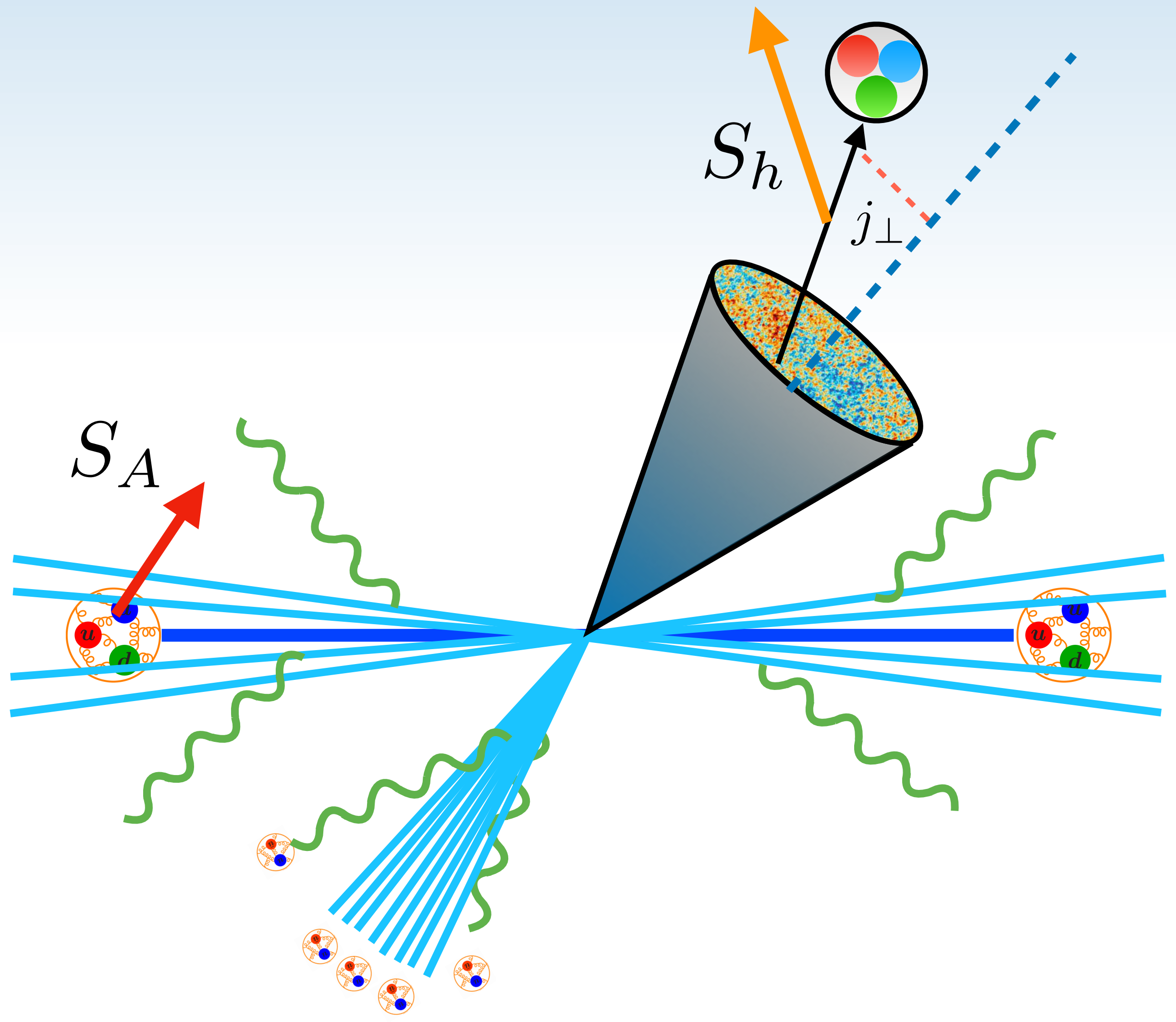
$$\frac{d\sigma^{p(S_A)+p/e\to(\text{jet}\,h(S_h))X}}{dp_{JT}d\eta_J dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + |\boldsymbol{S}_T|\sin(\phi_{S_A}-\hat{\phi}_h)F_{TU,U}^{\sin(\phi_{S_A}-\hat{\phi}_h)}$$

$$+\Lambda_h\left[\lambda F_{LU,L} + |\boldsymbol{S}_T|\cos(\phi_{S_A}-\hat{\phi}_h)F_{TU,L}^{\cos(\phi_{S_A}-\hat{\phi}_h)}\right]$$

$$+|\boldsymbol{S}_{h\perp}|\Big\{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \lambda\cos(\hat{\phi}_h-\hat{\phi}_{S_h})F_{LU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})}$$

$$+|\boldsymbol{S}_T|\Big(\cos(\phi_{S_A}-\hat{\phi}_{S_h})F_{TU,T}^{\cos(\phi_{S_A}-\hat{\phi}_{S_h})}$$

$$+\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})}\Big)\Big\},$$

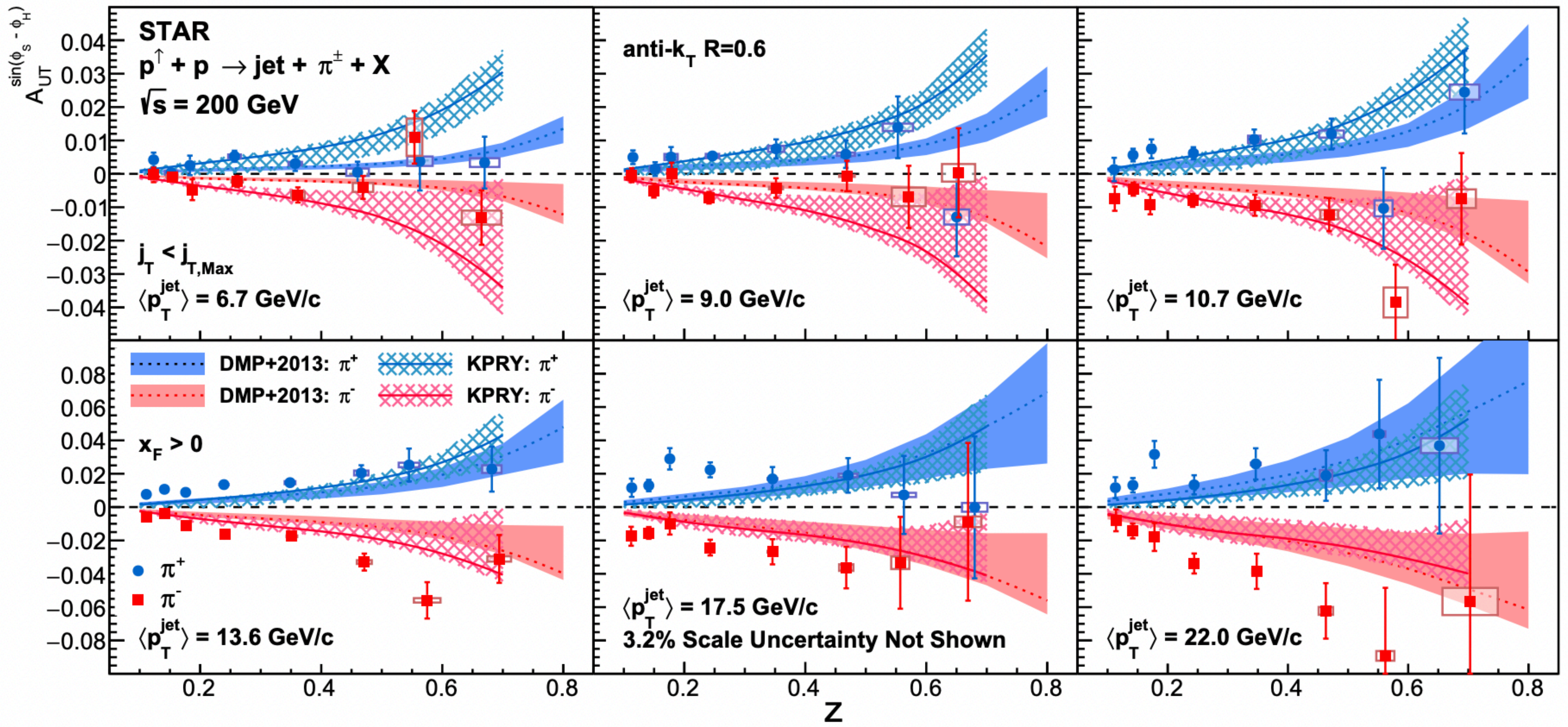


$PP\,/\,e\,P \to J(h) + X$

$$F_{S_A S_B, S_h}$$
$$\uparrow$$

Polarization of A, B, h

Collins TMD FJF

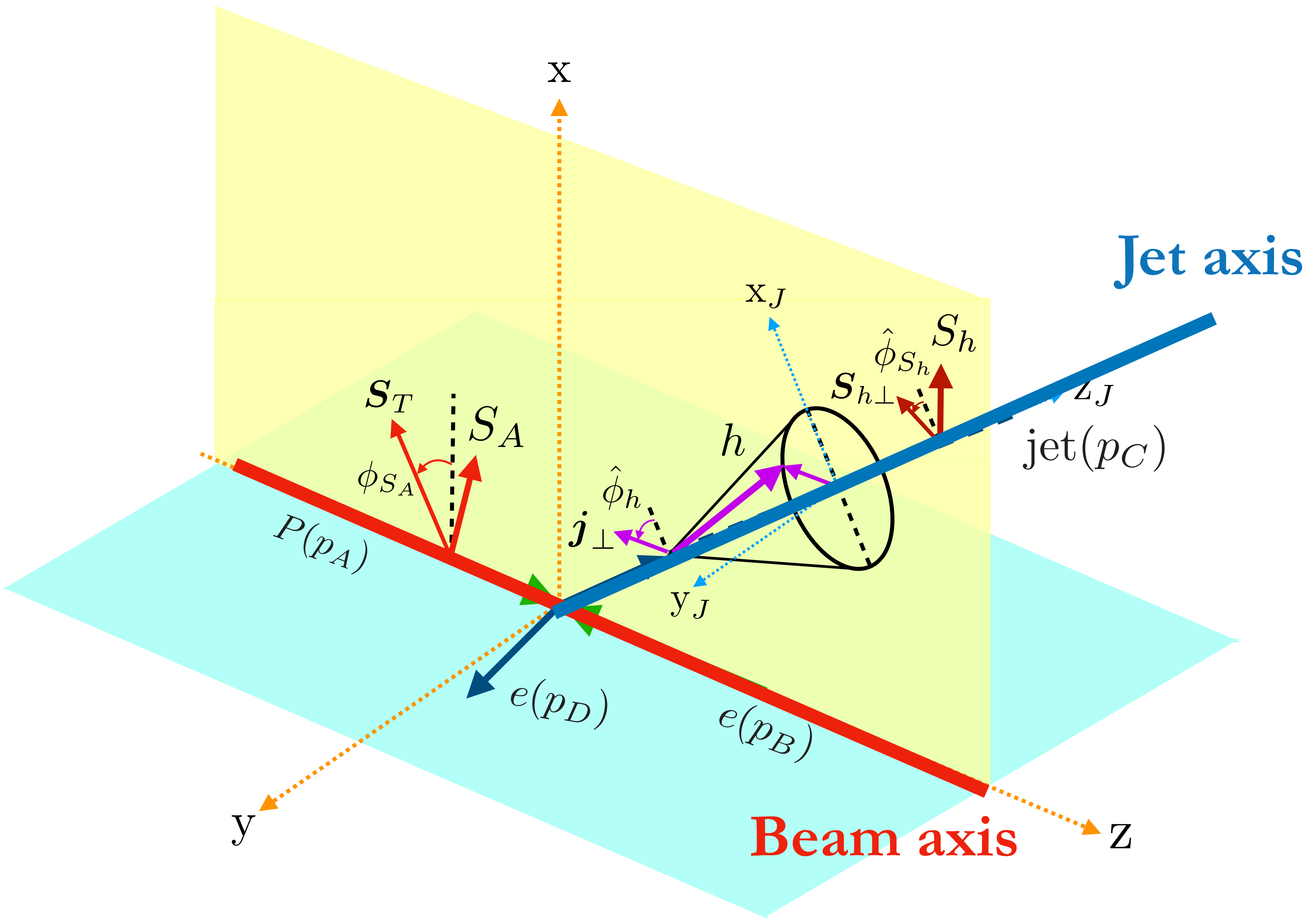


STAR Collaboration `22

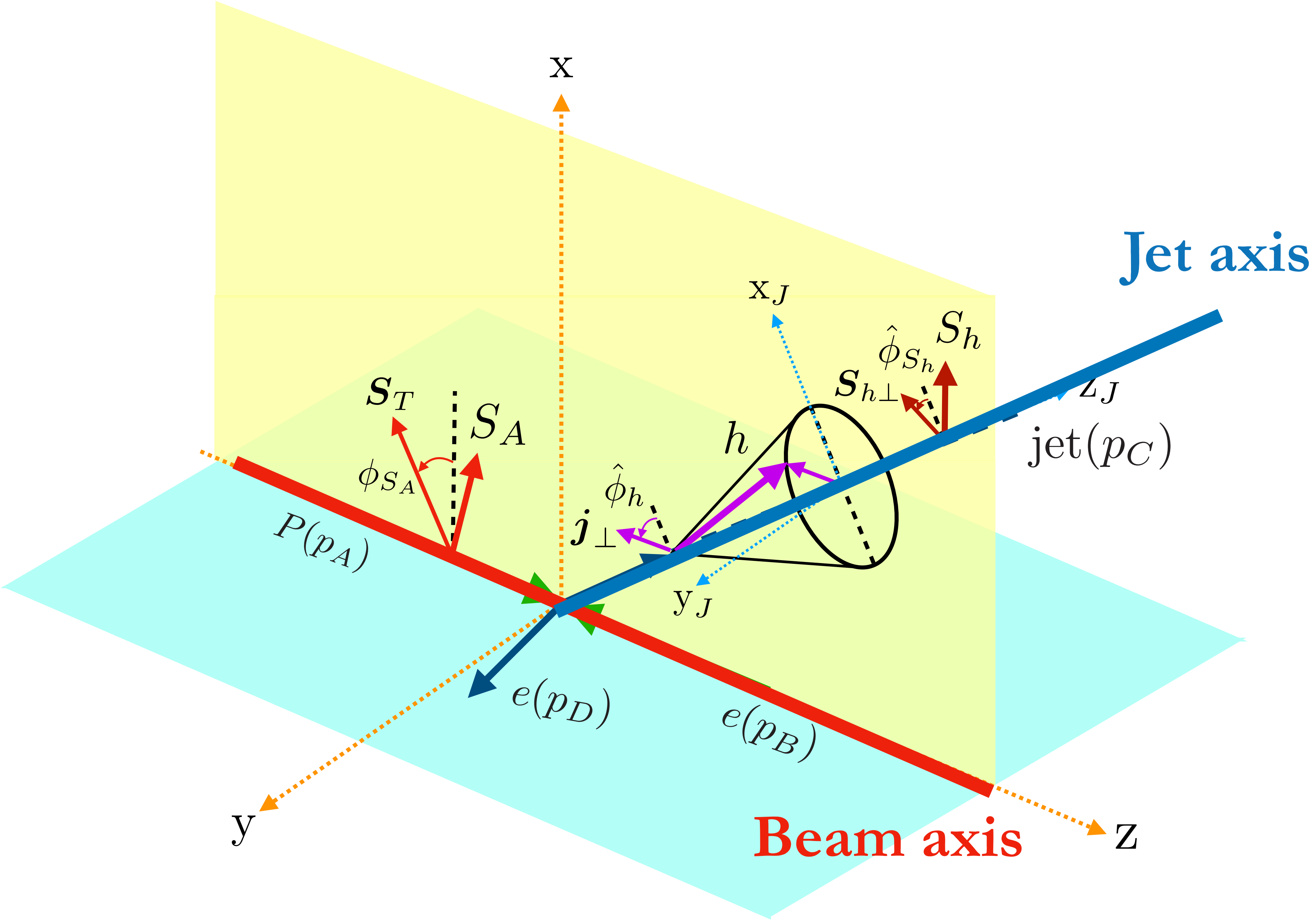
- Sensitivity to different TMD FFs with characteristic angular asymmetries

Kang, KL, Zhao, `20

Fragmenting Jet Functions + lepton-jet imbalance

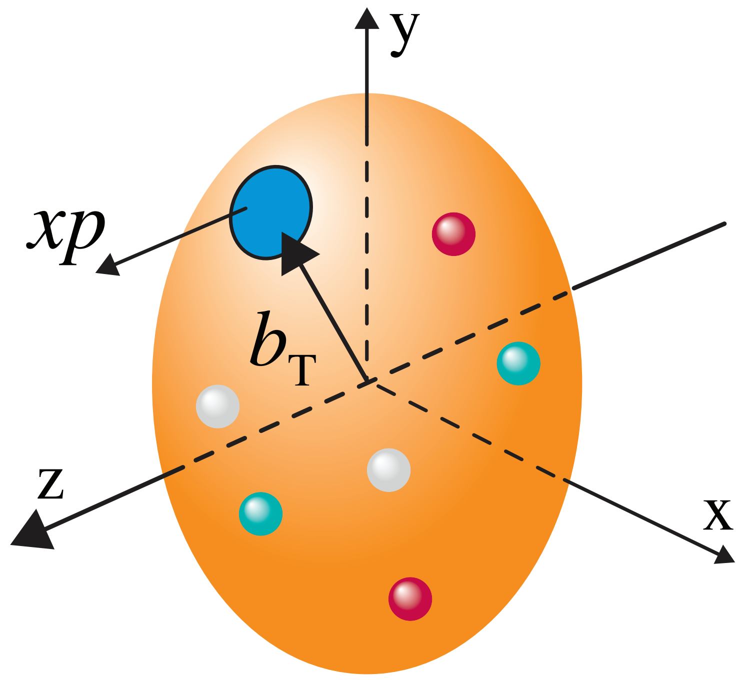


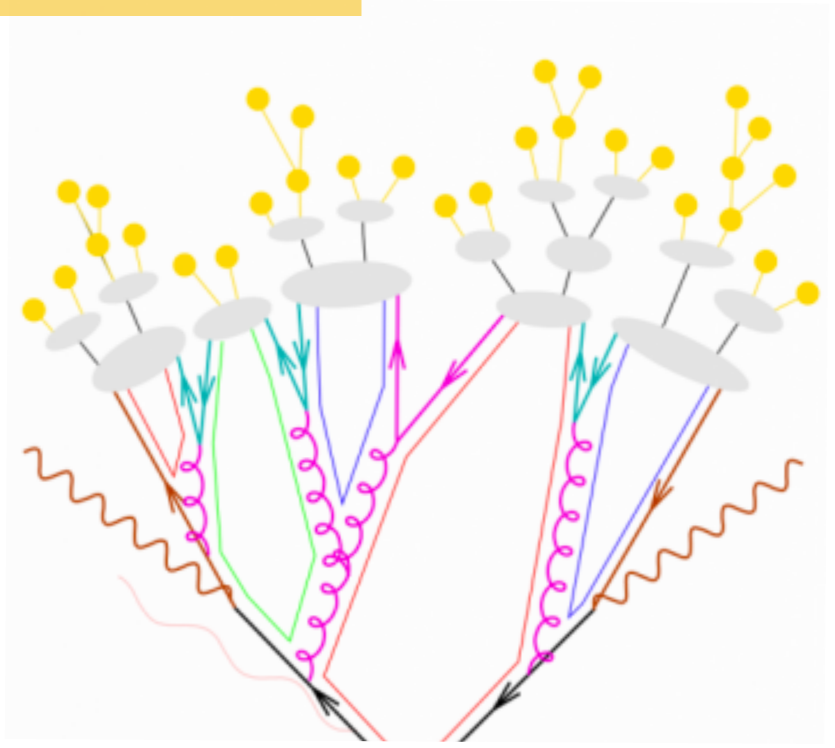
Polarized Jet Fragmentation Functions and lepton + jet imbalance



3) Lepton + jet imbalance with hadron in jet

TMDFFs / TMDPDFs





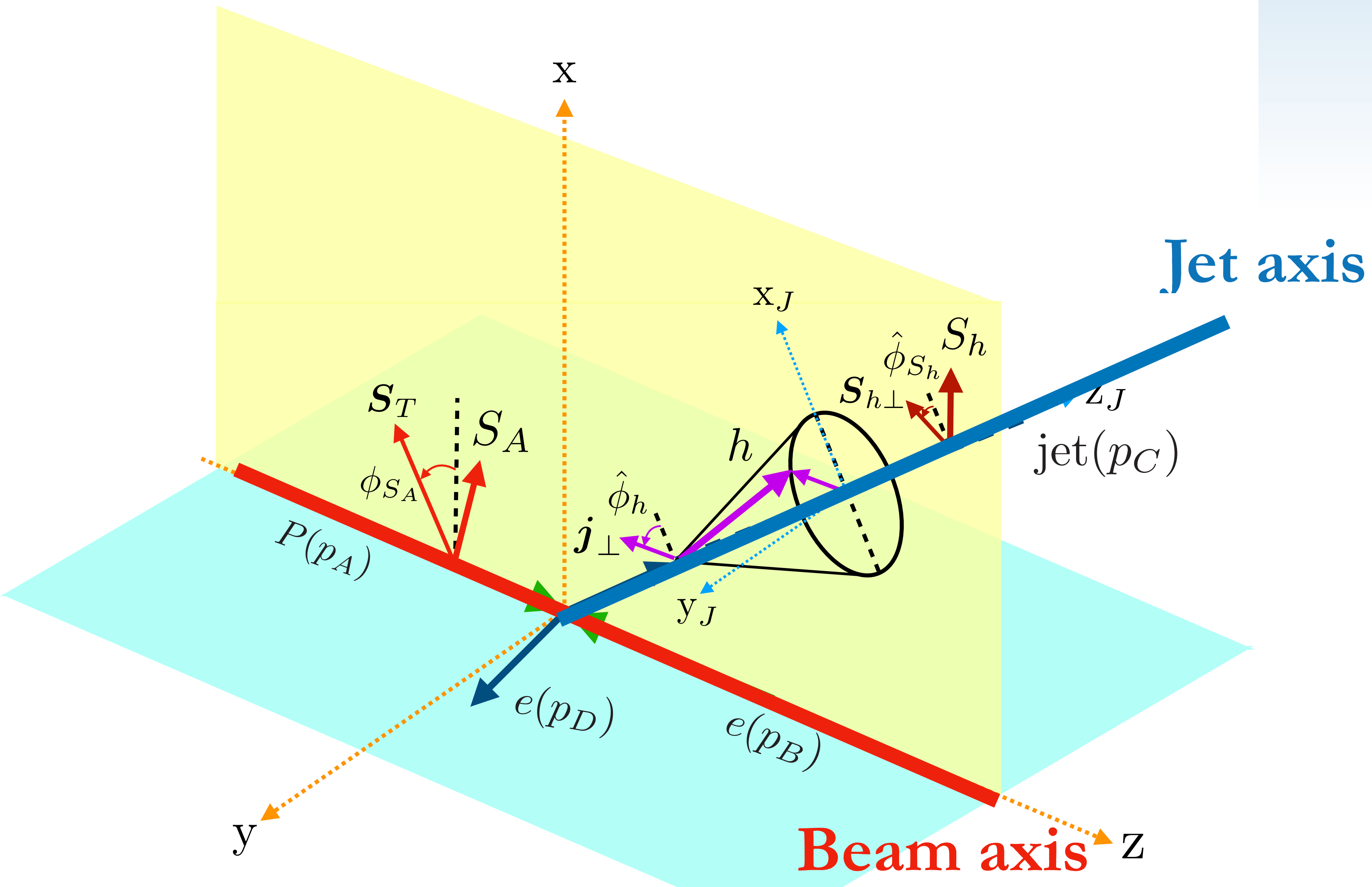
- Observation of polarized hadron inside jet gives sensitivity to all TMDPDFs and TMDFFs.
- Sensitivity to two TMDs, but sensitive to $\vec{q}_\perp$ and $\vec{j}_\perp$ separately (advantage of having two axes)

Polarized Jet Fragmentation Functions and lepton + jet imbalance

Many characteristic correlations

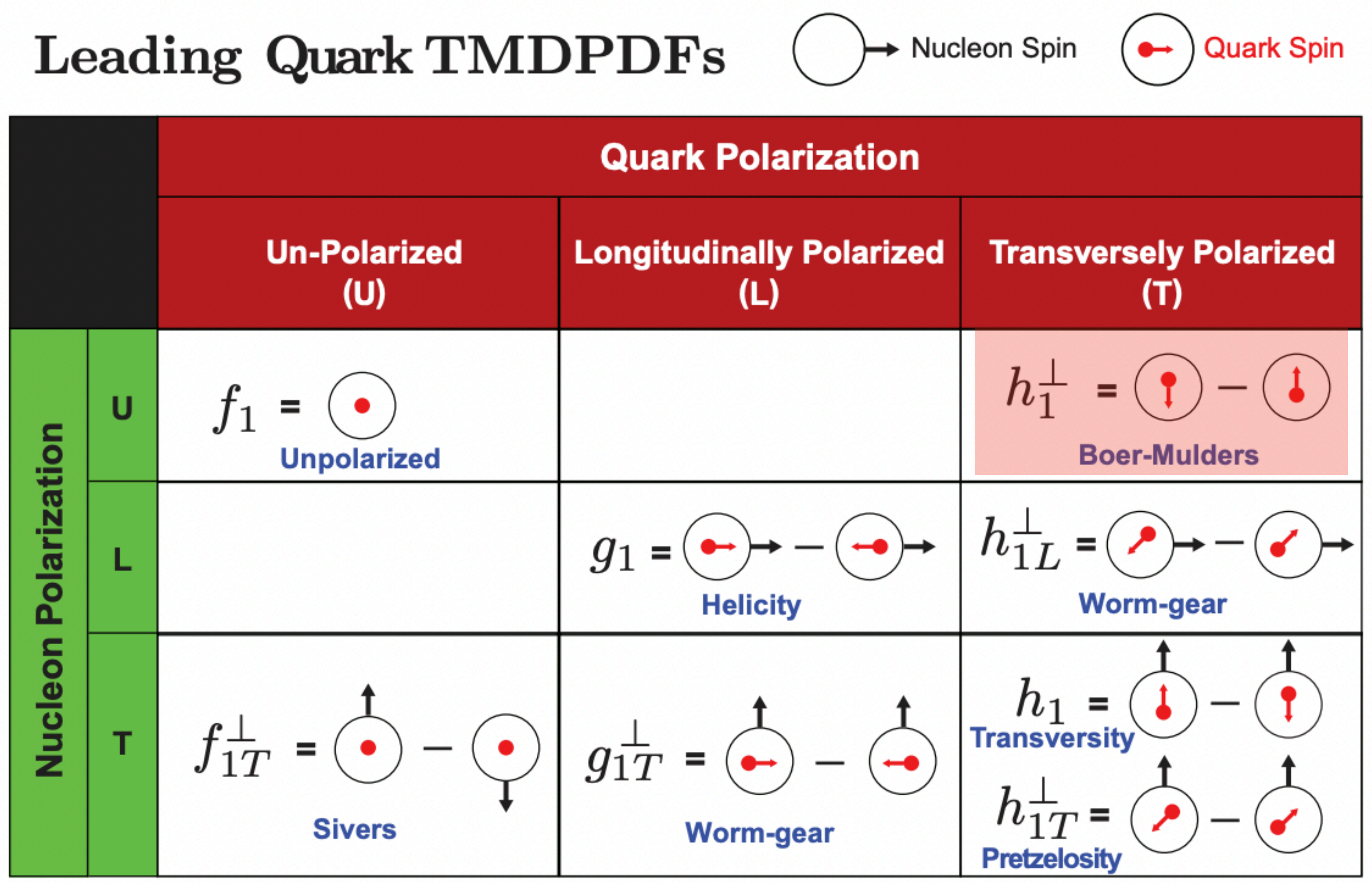
$$
\begin{aligned}
&\frac{d\sigma^{p(S_A)+e(\lambda_e)\to e+(\text{jet}\,h(S_h))+X}}{dp_T^2 dy_J d^2\boldsymbol{q}_T dz_h d^2\boldsymbol{j}_\perp} = F_{UU,U} + \cos(\phi_q - \hat{\phi}_h) F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)} \\
&+ \lambda_p \bigg\{ \lambda_e F_{LL,U} + \sin(\phi_q - \hat{\phi}_h) F_{LU,U}^{\sin(\phi_q-\hat{\phi}_h)} \bigg\} \\
&+ S_T \bigg\{ \sin(\phi_q - \phi_{S_A}) F_{TU,U}^{\sin(\phi_q-\phi_{S_A})} + \lambda_e \cos(\phi_q - \phi_{S_A}) F_{TL,U}^{\cos(\phi_q-\phi_{S_A})} \\
&\qquad + \sin(\phi_{S_A} - \hat{\phi}_h) F_{TU,U}^{\sin(\phi_{S_A}-\hat{\phi}_h)} + \sin(2\phi_q - \hat{\phi}_h - \phi_{S_A}) F_{TU,U}^{\sin(2\phi_q-\hat{\phi}_h-\phi_{S_A})} \bigg\} \\
&+ \lambda_h \bigg\{ \lambda_e F_{UL,L} + \sin(\hat{\phi}_h - \phi_q) F_{UU,L}^{\sin(\hat{\phi}_h-\phi_q)} + \lambda_p \bigg[F_{LU,L} + \cos(\hat{\phi}_h - \phi_q) F_{LU,L}^{\cos(\hat{\phi}_h-\phi_q)} \bigg] \\
&\qquad + S_T \bigg[\cos(\phi_q - \phi_{S_A}) F_{TU,L}^{\cos(\phi_q-\phi_{S_A})} + \lambda_e \sin(\phi_q - \phi_{S_A}) F_{TL,L}^{\sin(\phi_q-\phi_{S_A})} \\
&\qquad\qquad + \cos(\phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(\phi_{S_A}-\hat{\phi}_h)} + \cos(2\phi_q - \phi_{S_A} - \hat{\phi}_h) F_{TU,L}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_h)} \bigg] \\
&+ S_{h\perp} \bigg\{ \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} + \lambda_e \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{UL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} \\
&+ \sin(\hat{\phi}_{S_h} - \phi_q) F_{UU,T}^{\sin(\hat{\phi}_{S_h}-\phi_q)} + \sin(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q) F_{UU,T}^{\sin(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)} \\
&+ \lambda_p \bigg[\cos(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h})} + \cos(\phi_q - \hat{\phi}_{S_h}) F_{LU,T}^{\cos(\phi_q-\hat{\phi}_{S_h})} \\
&\qquad + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_q) F_{LU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_q)} + \lambda_e \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) F_{LL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h})} \bigg] \\
&+ S_T \bigg[\cos(\phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(\phi_{S_A}-\hat{\phi}_{S_h})} + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} - \phi_{S_A}) F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}-\phi_{S_A})} \\
&\qquad + \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) \sin(\phi_q - \phi_{S_A}) F_{TU,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h}) \sin(\phi_q-\phi_{S_A})} \\
&\qquad + \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) \cos(\phi_q - \phi_{S_A})) F_{TU,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h}) \cos(\phi_q-\phi_{S_A})} \\
&\qquad + \cos(2\phi_q - \phi_{S_A} - \hat{\phi}_{S_h}) F_{TU,T}^{\cos(2\phi_q-\phi_{S_A}-\hat{\phi}_{S_h})} \\
&\qquad + \cos(2\hat{\phi}_h - \hat{\phi}_{S_h} + 2\phi_q - \phi_{S_A}) F_{TU,T}^{\cos(2\hat{\phi}_h-\hat{\phi}_{S_h}+2\phi_q-\phi_{S_A})} \\
&\qquad + \lambda_e \cos(\hat{\phi}_h - \hat{\phi}_{S_h}) \sin(\phi_{S_A} - \phi_q) F_{TL,T}^{\cos(\hat{\phi}_h-\hat{\phi}_{S_h}) \sin(\phi_{S_A}-\phi_q)} \\
&\qquad + \lambda_e \sin(\hat{\phi}_h - \hat{\phi}_{S_h}) \cos(\phi_{S_A} - \phi_q)) F_{TL,T}^{\sin(\hat{\phi}_h-\hat{\phi}_{S_h}) \cos(\phi_{S_A}-\phi_q)} \bigg] \bigg\} ,
\end{aligned}
$$

Phenomenology : $A_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)}$



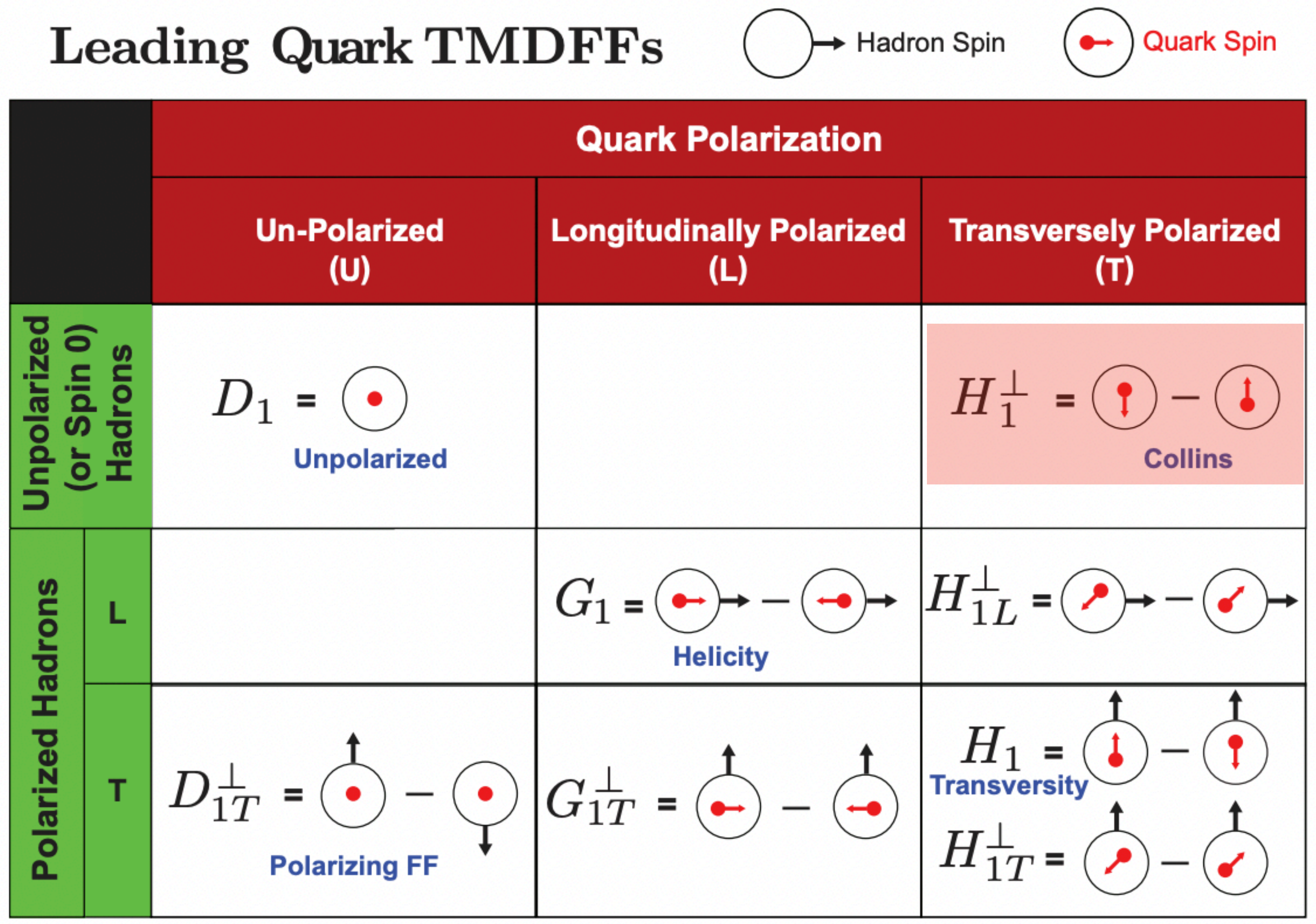
$$A_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)} \equiv \frac{F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)}(q_\perp, j_\perp)}{F_{UU,U}(q_\perp, j_\perp)} \sim \frac{h_1^\perp(q_\perp)H_1^\perp(j_\perp)}{f_1(q_\perp)D_1(j_\perp)}$$

- Boer-Mulders and Collins functions sensitive to transverse momentum measured with respect to different axes.
- "Separation" of the incoming and outgoing dynamics.



Leading Quark TMDPDFs ○→ Nucleon Spin ⊙ Quark Spin

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = ⊙ Unpolarized		$h_1^\perp$ = ⊙ − ⊙ Boer-Mulders
L		g_1 = ⊙→ − ⊙→ Helicity	$h_{1L}^\perp$ = ⊙→ − ⊙→ Worm-gear
T	$f_{1T}^\perp$ = ⊙ − ⊙ Sivers	$g_{1T}^\perp$ = ⊙ − ⊙ Worm-gear	h_1 = ⊙ − ⊙ Transversity $h_{1T}^\perp$ = ⊙ − ⊙ Pretzelosity



Leading Quark TMDFFs ○→ Hadron Spin ⊙ Quark Spin

		Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = ⊙ Unpolarized		$H_1^\perp$ = ⊙ − ⊙ Collins
Polarized Hadrons	L		G_1 = ⊙→ − ⊙→ Helicity	$H_{1L}^\perp$ = ⊙→ − ⊙→
	T	$D_{1T}^\perp$ = ⊙ − ⊙ Polarizing FF	$G_{1T}^\perp$ = ⊙ − ⊙	H_1 = ⊙ − ⊙ Transversity $H_{1T}^\perp$ = ⊙ − ⊙

Kang, KL, Shao, Zhao `21

Unpolarized π in jet (Boer-Mulders, Collins)

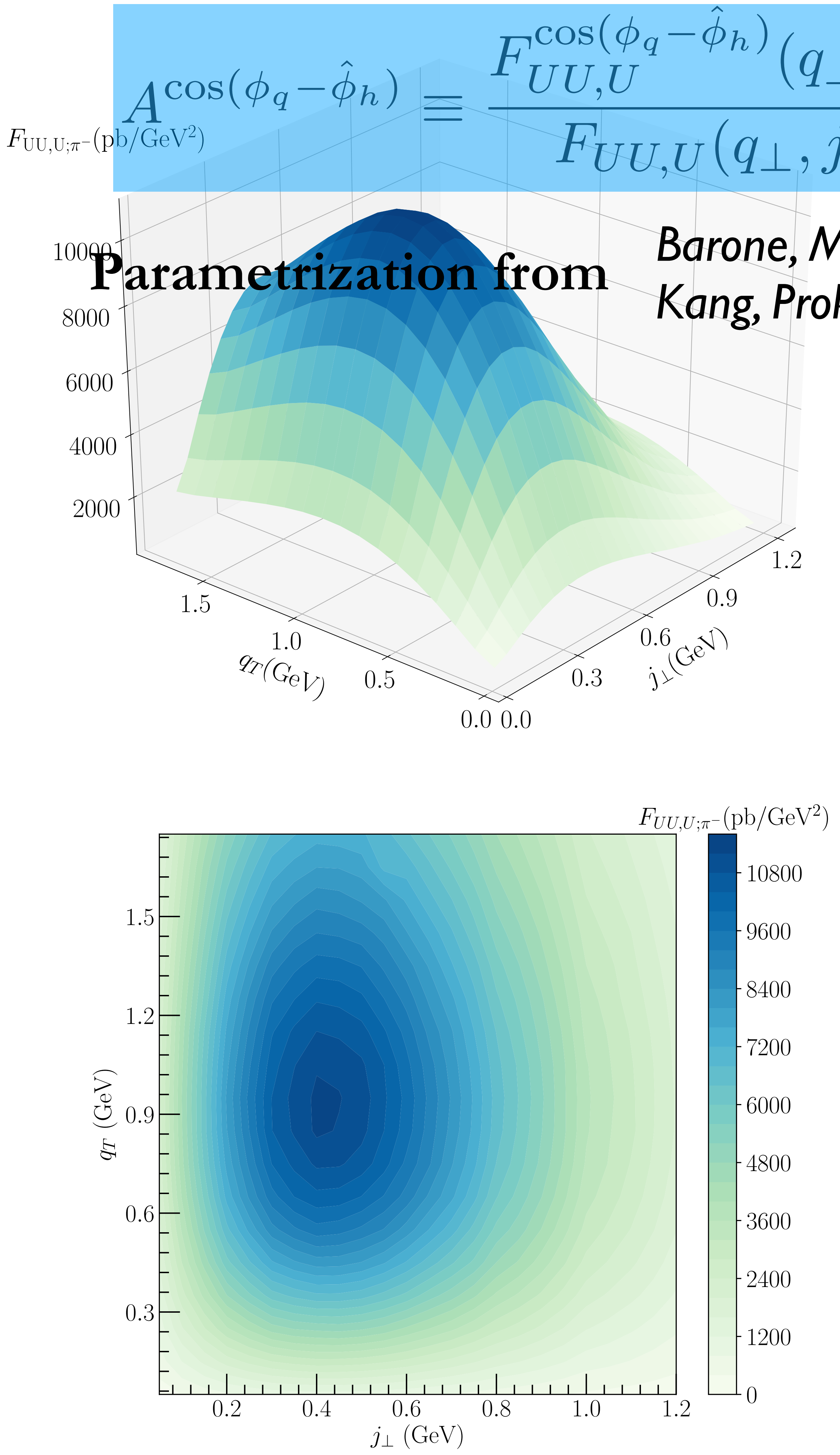
π⁻ q_T [0,**1.8**], j_T [0,1.2]

$$A^{\cos(\phi_q-\hat{\phi}_h)} \equiv \frac{F_{UU,U}^{\cos(\phi_q-\hat{\phi}_h)}(q_\perp,j_\perp)}{F_{UU,U}(q_\perp,j_\perp)} \sim \frac{h_1^\perp(q_\perp)H_1^\perp(j_\perp)}{f_1(q_\perp)D_1(j_\perp)}$$

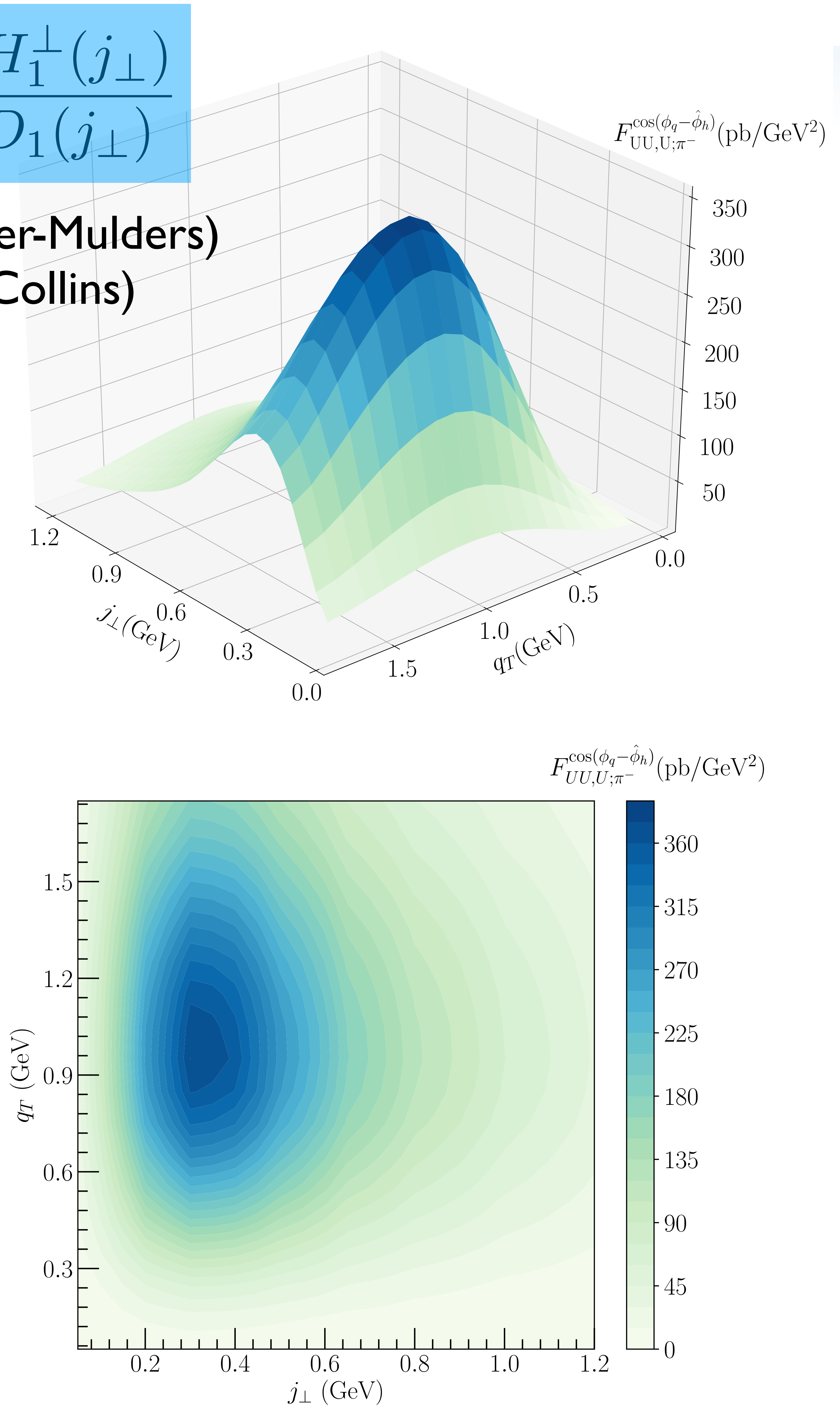
Parametrization from

Barone, Melis, Prokudin `10 (Boer-Mulders)
Kang, Prokudin, Sun, Yuan `15 (Collins)

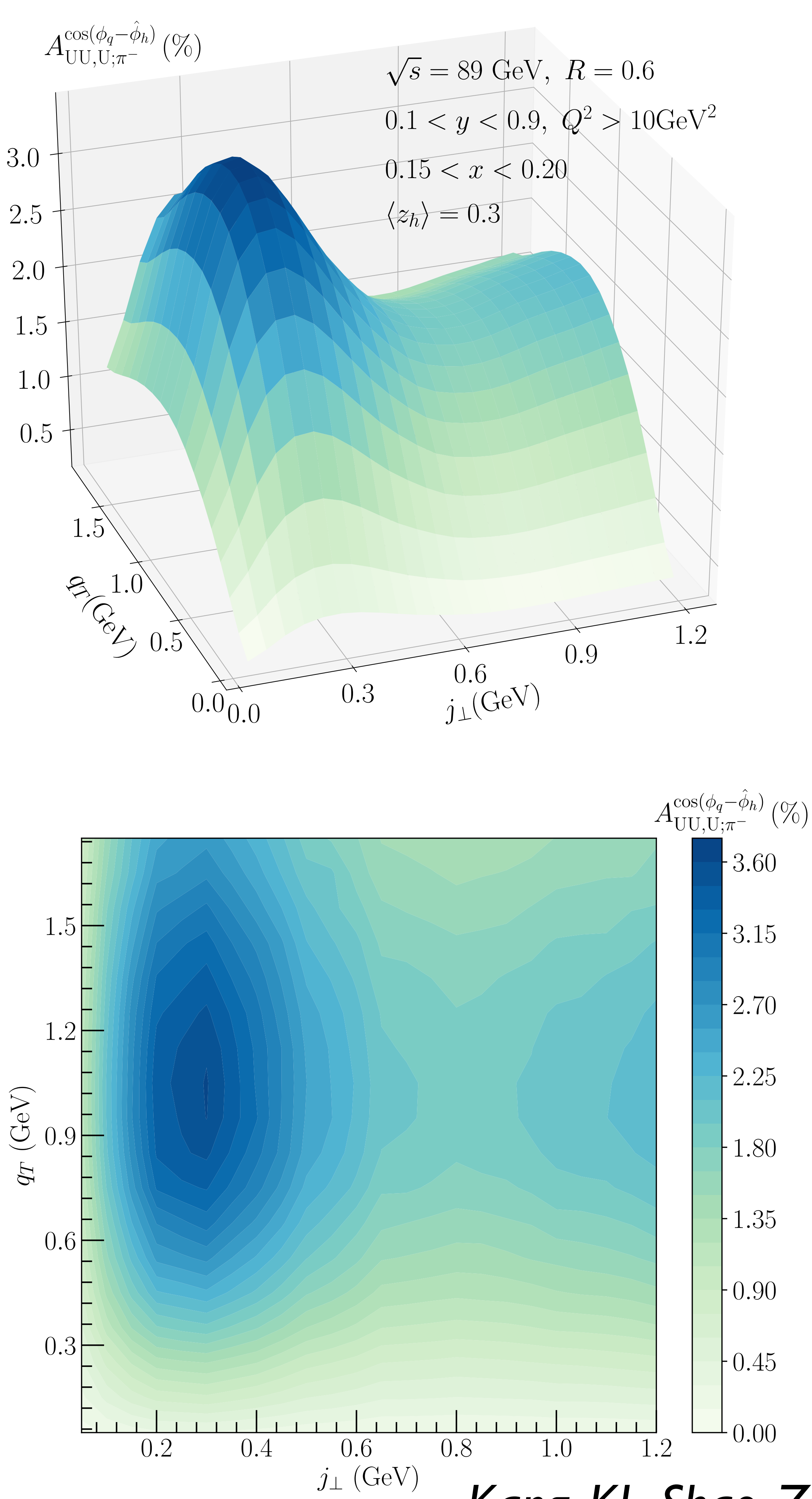
Denominator



Numerator

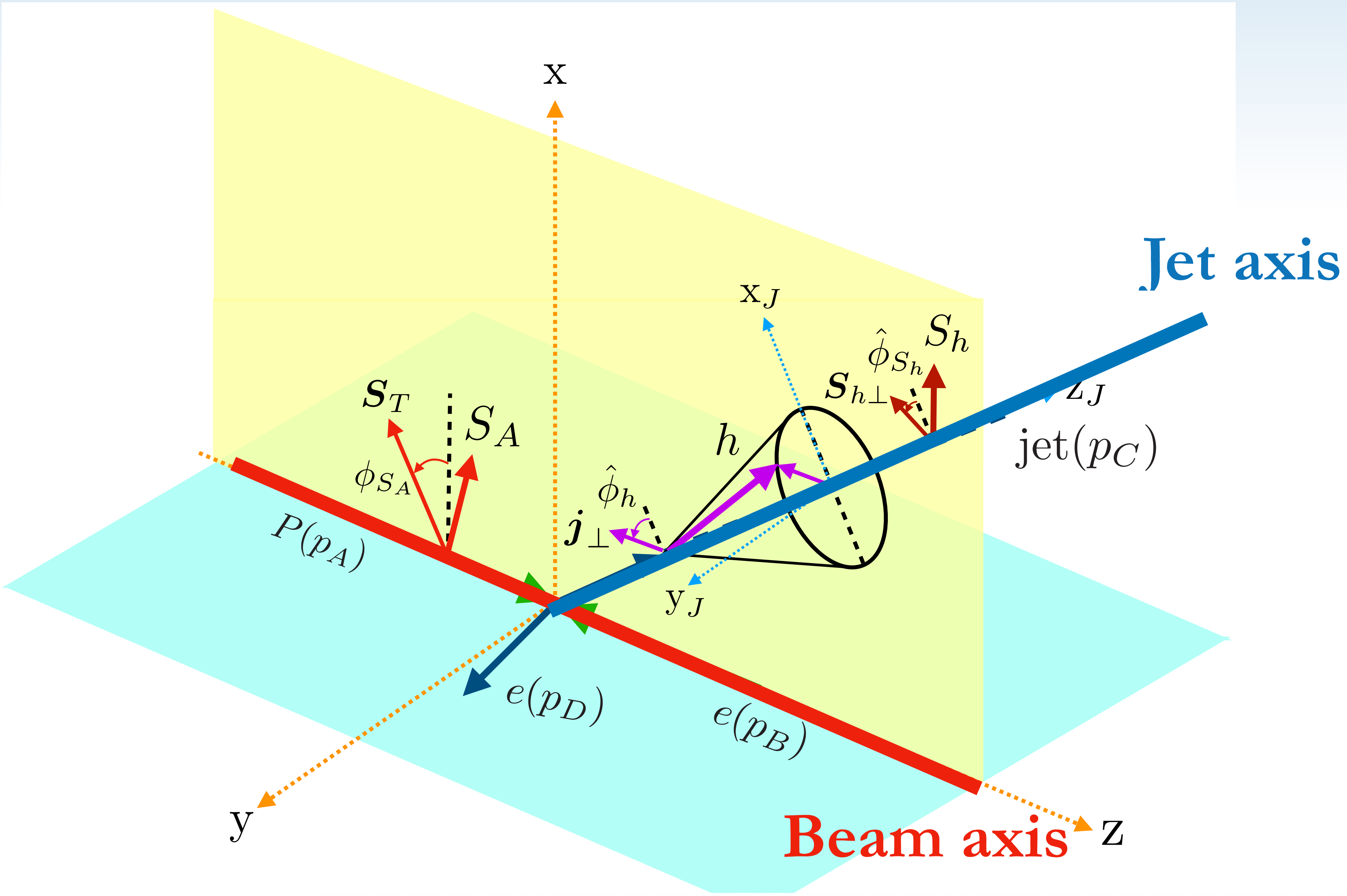


Ratio



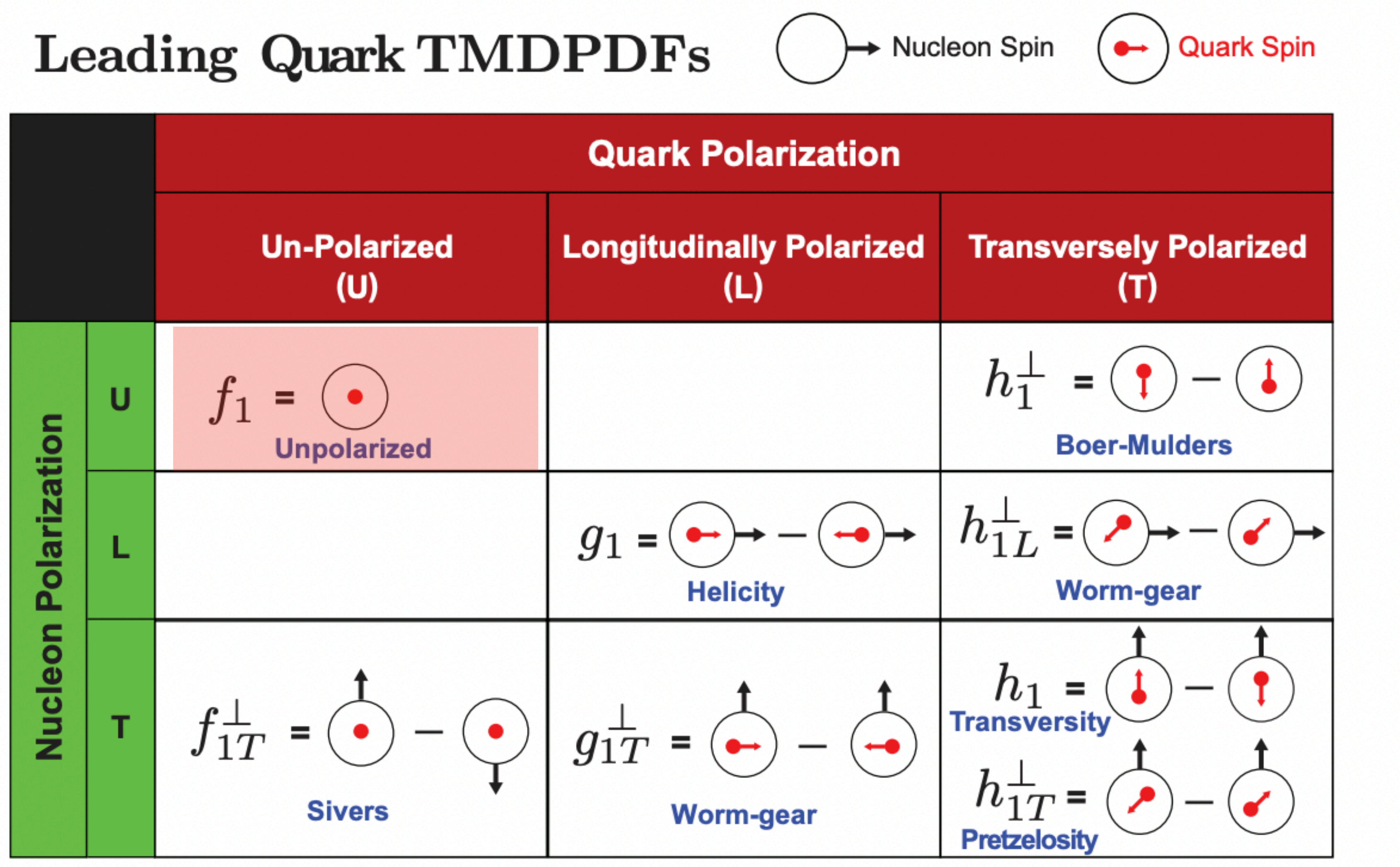
Kang, KL, Shao, Zhao `21

Phenomenology : $A_{UU,T}^{\sin(\hat{\phi}_\Lambda - \hat{\phi}_{S_\Lambda})}$



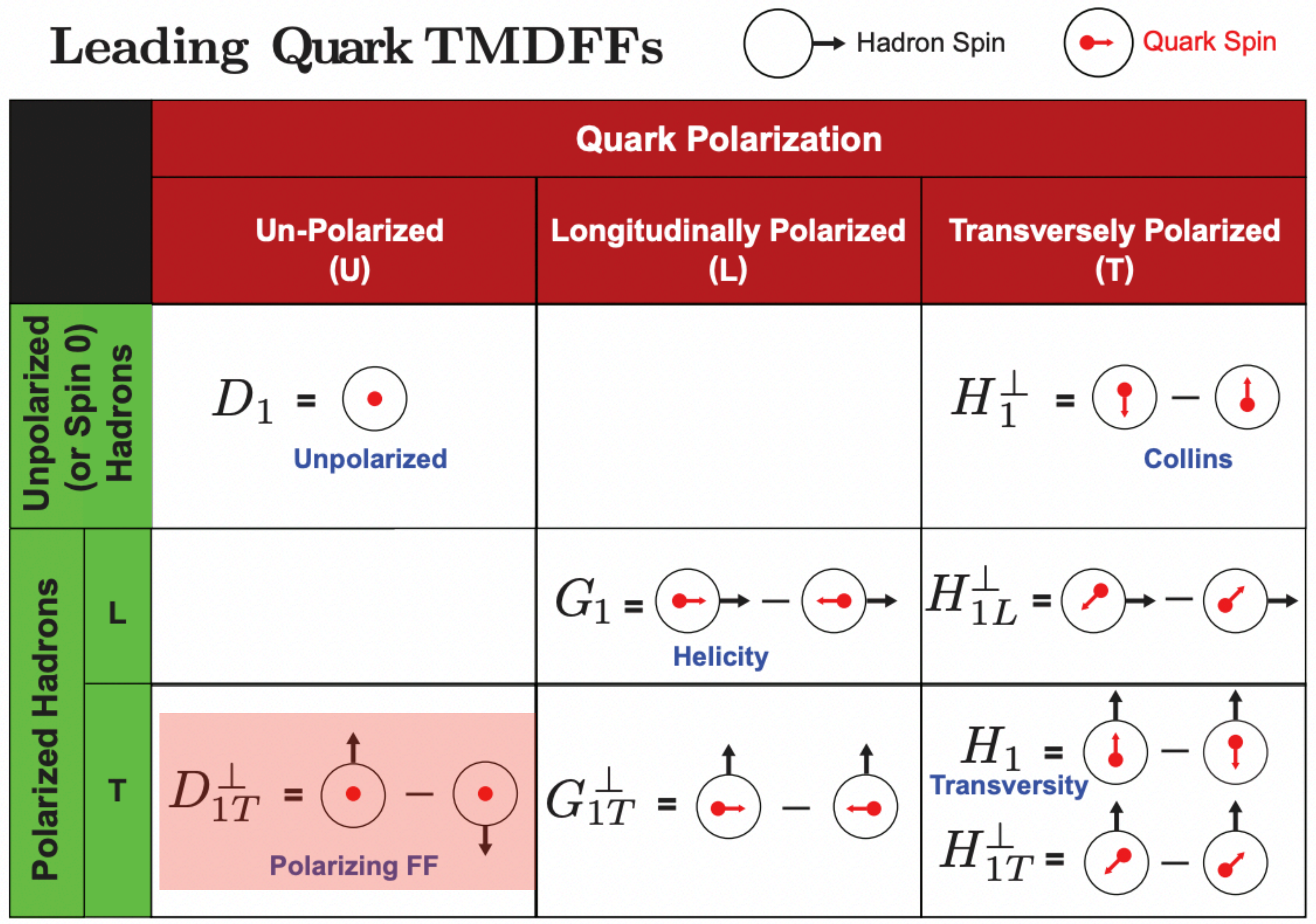
$$A_{UU,T}^{\sin(\hat{\phi}_\Lambda - \hat{\phi}_{S_\Lambda})} = \frac{F_{UU,T}^{\sin(\hat{\phi}_\Lambda - \hat{\phi}_{S_\Lambda})}}{F_{UU,U}} \sim \frac{f_1(q_\perp) D_{1T}^\perp(j_\perp)}{f_1(q_\perp) D_1(j_\perp)}$$

- "Separation" of the incoming and outgoing dynamics cancel the q_T dependence for this case.



Leading Quark TMDPDFs ○→ Nucleon Spin ⊙ Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	f_1 = ⊙ Unpolarized		$h_1^\perp$ = ⊙ − ⊙ Boer-Mulders
	L		g_1 = ⊙ − ⊙ Helicity	$h_{1L}^\perp$ = ⊙ − ⊙ Worm-gear
	T	$f_{1T}^\perp$ = ⊙ − ⊙ Sivers	$g_{1T}^\perp$ = ⊙ − ⊙ Worm-gear	h_1 = ⊙ − ⊙ Transversity $h_{1T}^\perp$ = ⊙ − ⊙ Pretzelosity



Leading Quark TMDFFs ○→ Hadron Spin ⊙ Quark Spin

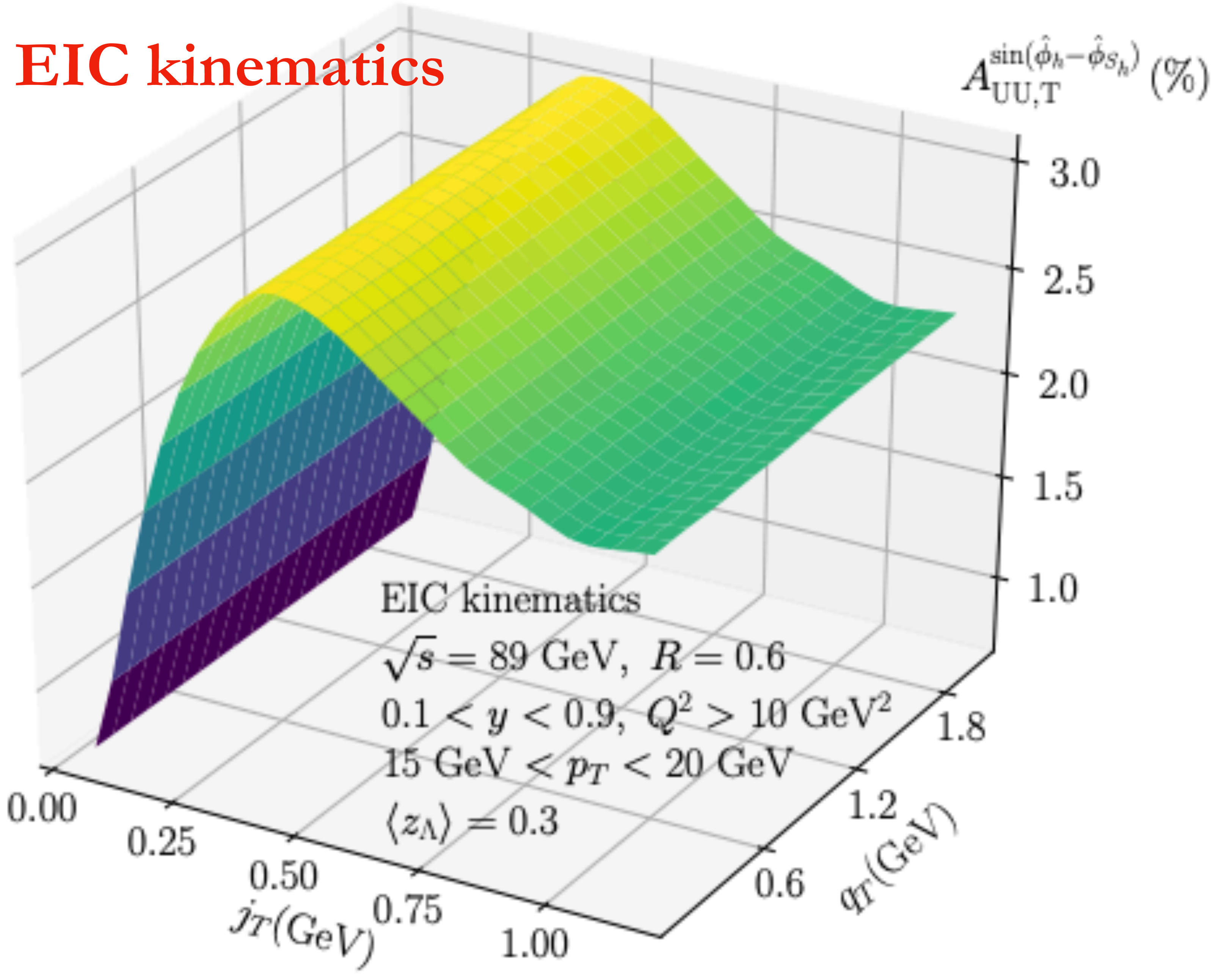
		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Unpolarized (or Spin 0) Hadrons		D_1 = ⊙ Unpolarized		$H_1^\perp$ = ⊙ − ⊙ Collins
Polarized Hadrons	L		G_1 = ⊙ − ⊙ Helicity	$H_{1L}^\perp$ = ⊙ − ⊙
	T	$D_{1T}^\perp$ = ⊙ − ⊙ Polarizing FF	$G_{1T}^\perp$ = ⊙ − ⊙	H_1 = ⊙ − ⊙ Transversity $H_{1T}^\perp$ = ⊙ − ⊙

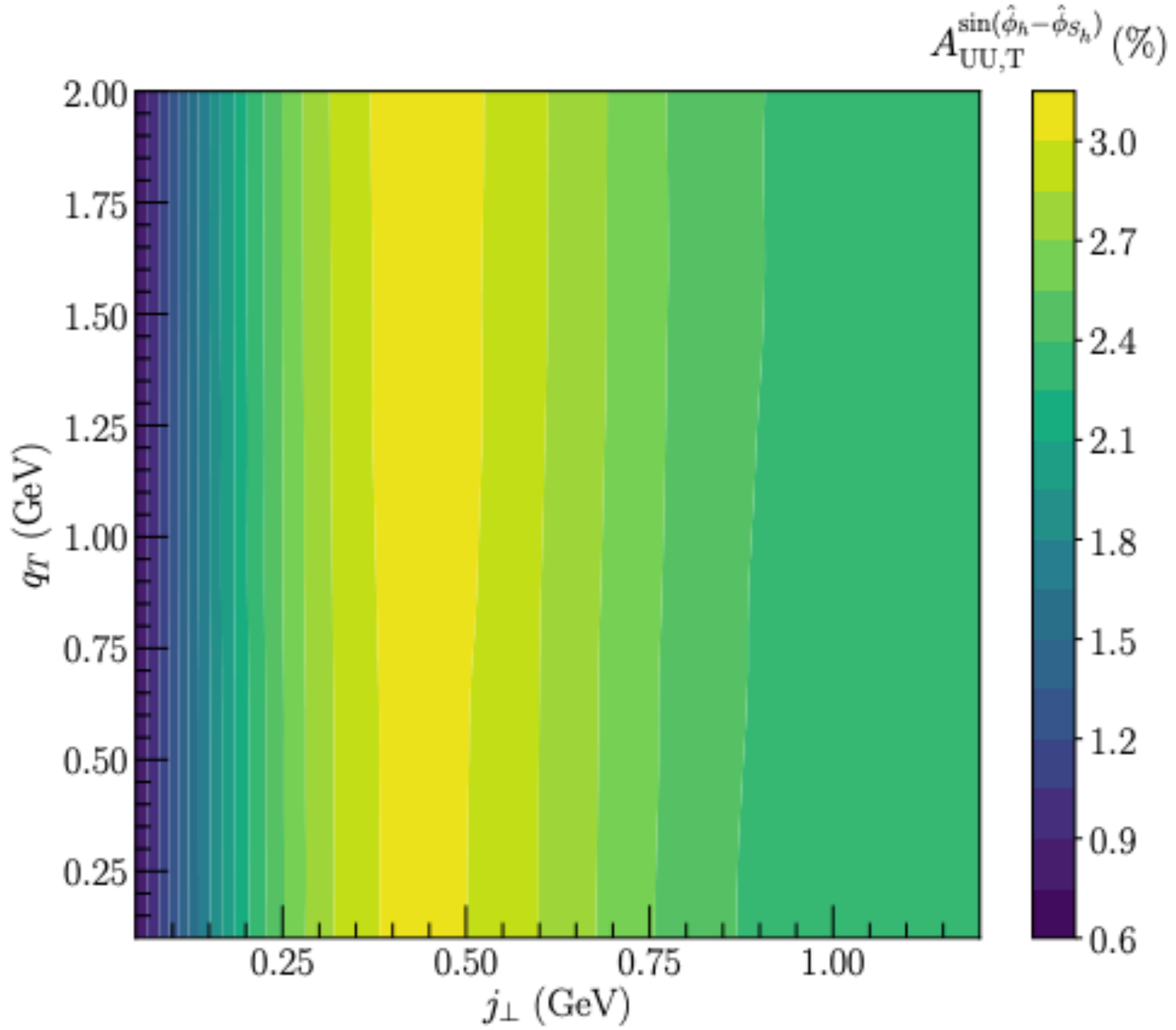
Polarizing FF

Kang, KL, Shao, Zhao `21

Phenomenology : $A_{UU,T}^{\sin(\hat{\phi}_\Lambda - \hat{\phi}_{S_\Lambda})}$

$$A_{UU,T}^{\sin(\hat{\phi}_\Lambda - \hat{\phi}_{S_\Lambda})} = \frac{F_{UU,T}^{\sin(\hat{\phi}_\Lambda - \hat{\phi}_{S_\Lambda})}}{F_{UU,U}} \sim \frac{f_1(q_\perp) D_{1T}^\perp(j_\perp)}{f_1(q_\perp) D_1(j_\perp)}$$





- q_T dependence indeed cancels and is only sensitive to TMDFFs.

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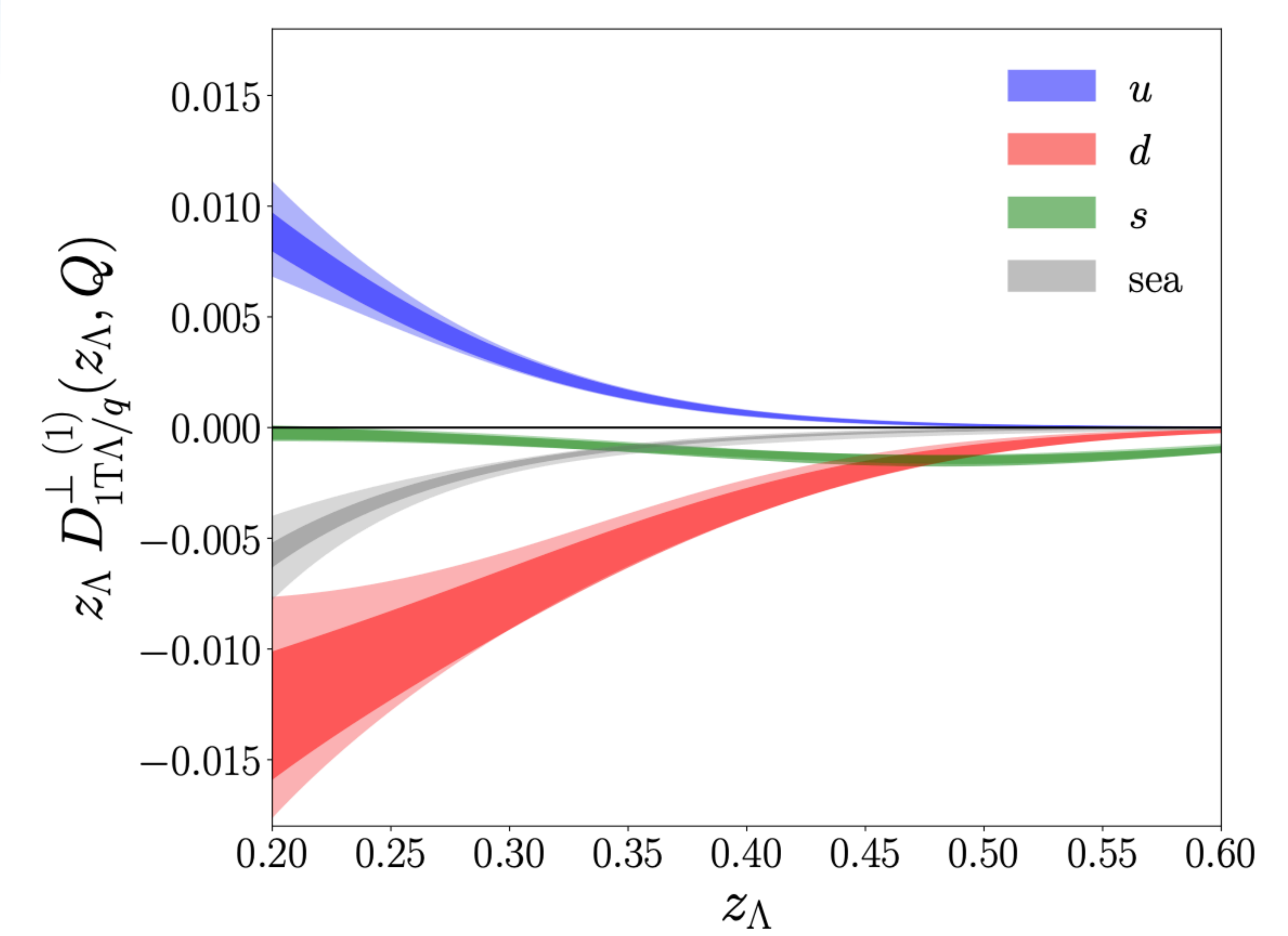
Phenomenology : $A_{UU,T}^{\sin(\hat{\phi}_\Lambda - \hat{\phi}_{S_\Lambda})}$



- EIC pseudo-data significantly decreases the uncertainties in the determination of TMDPFFs.

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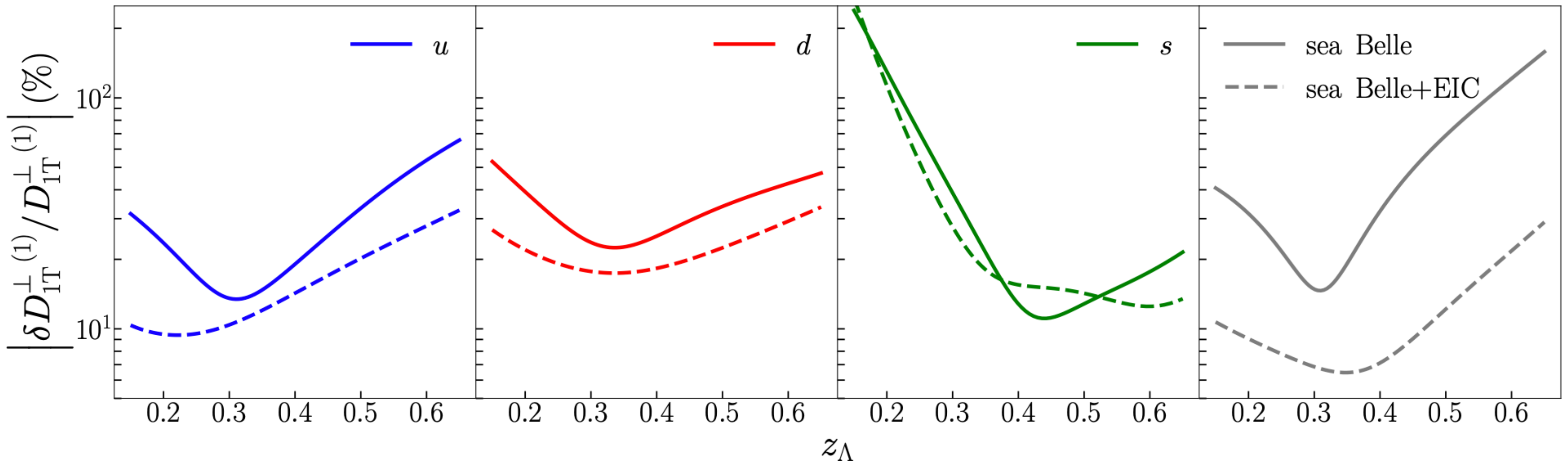

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Opening new door of opportunities