Transverse Momentum Distributions of Heavy Hadrons and Polarized Heavy Quarks

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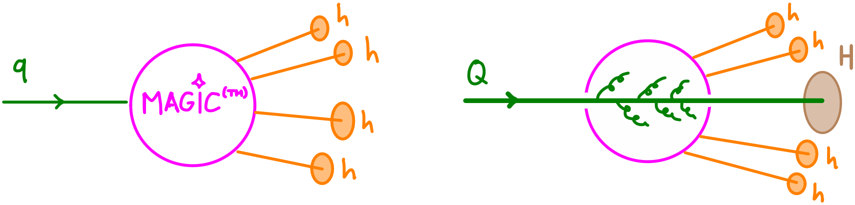
[2305.15461]

- Charm and bottom quarks with $m \equiv m_c, m_b \gg \Lambda_{\mathrm{QCD}}$ are special
- Decay & mixing of heavy hadrons: Most precisely measured [Belle, BaBar, LHCb, ...] and understood strongly coupled system in QCD $\Rightarrow \Lambda_{\text{new physics}}^{\text{flavor}} \gtrsim 100\,\mathrm{TeV}$

This talk: $\Lambda_{\mathrm{QCD}} \sim 100\,\mathrm{MeV}$

Heavy quark TMD fragmentation functions as a powerful probe of hadronization.

- Probe hadronization by "sticking in" a static color source!



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Youtube: @electricandmagneticfields2314

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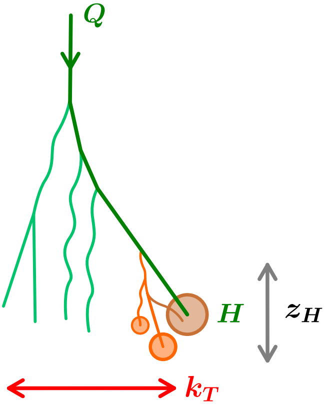
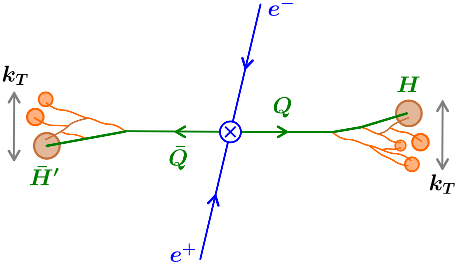


- Longitudinal z_H distribution (collinear heavy-quark FF) is well understood

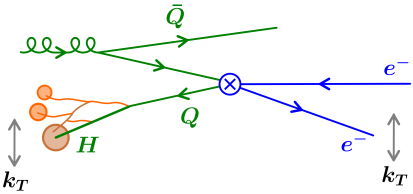
  [B. Mele, P. Nason, Nucl. Phys. B 361 (1991) 626]
  [R. L. Jaffe, L. Randall, Nucl. Phys. B 412 (1994) 79]
  [A. F. Falk, M. E. Peskin, Phys. Rev. D 49 (1994) 3320]
  [M. Neubert, 0706.2136]
  [M. Fickinger, S. Fleming, C. Kim and E. Mereghetti, JHEP 11 (2016) 095]

- Heavy-quark TMD FFs $\Rightarrow$ new!

$e^+e^- \to H\bar{H}X$



$eN \to HX$



$$d\sigma_{\text{unpol}} \propto H\, D_{1\,H/Q} \otimes D_{1\,\bar{H}/\bar{Q}}$$

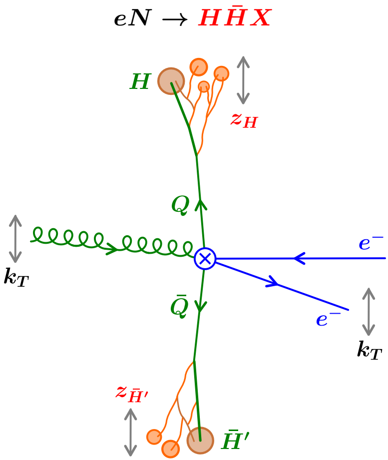
$$R_{\cos(2\phi_0)} \propto H\, H_{1\,H/Q}^{\perp} \otimes D_{1\,\bar{H}/\bar{Q}}^{\perp}$$

$$d\sigma_{\text{unpol}} \propto H\, f_{1\,Q/N} \otimes D_{1\,H/Q}$$

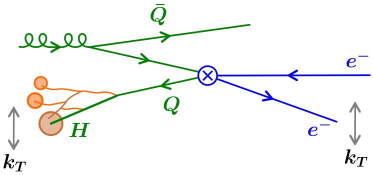
$$A_{\sin(2\phi_H)} \propto H\, h_{1L\,Q/N}^{\perp} \otimes D_{1\,H/Q}^{\perp}$$

- Form of TMD factorization is *unchanged* as long as $m, k_T \ll Q$
  - Work out how mass modifies TMD FFs/PDFs depending on hierarchy of m and k_T

... great playground for effective field theory!



- Hard heavy *dihadron/dijet* production central to EIC heavy-flavor program
- Will be used to access *gluon* TMD PDF
  [Boer et al. ’10, Zhu, Sun, Yuan ’13, Zhang ’17, Castillo et al. ’20-21, Kang et al. ’20]
- On FF side: Only sensitive to z_H!
  [Castillo, Echevarria, Makris, Scimemi ’20]
- ▶ Distinct process from $eN \to HX$
  (but similar detector requirements!)



What’s known about heavy-quark effects in TMDs

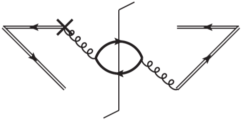
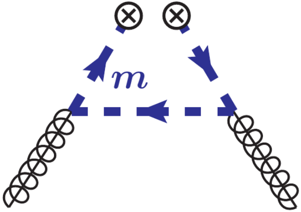
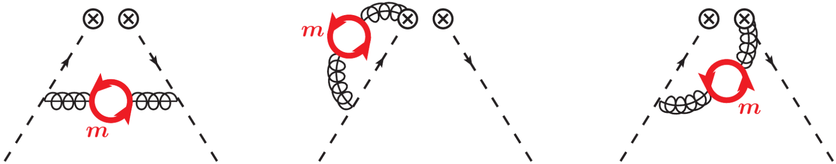


FIG. 5. Typical two-loop graph with a heavy quark loop that contributes to $\tilde{K}$. The notation is as in Ref. [9]: The cross denotes a derivative of a space-like Wilson line with respect to its rapidity.

[Pointed out in Collins, Rogers, 1412.3820]
[Calculated in Pietrulewicz et al. ’17]



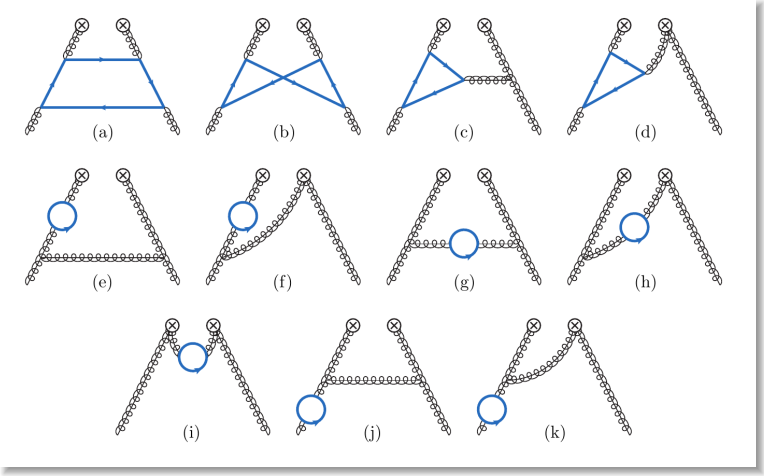
[Nadolsky, Kidonakis, Olness, Yuan, hep-ph/0210082]
[Pietrulewicz, Samitz, Spiering, Tackmann, 1703.09702]



[Pietrulewicz, Samitz, Spiering, Tackmann, 1703.09702]

- Important to get right in TMD/CS kernel extractions, but perturbative/boring …

What’s known about heavy-quark effects in TMDs

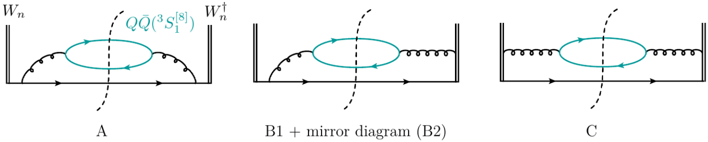


[Pietrulewicz, Stahlhofen, 2302.06623]

- Important to get Higgs p_T spectrum right at the LHC, but still perturbative …

What about TMD *fragmentation* involving heavy quarks?

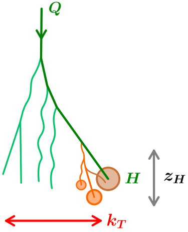
- TMD FFs for light quarks fragmenting to quarkonia (using NRQCD):



[Echevarria, Makris, Scimemi, 2007.05547]

- Light TMDs in hard quarkonium production/decay
  [Echevarria, 1907.06494; Fleming, Makris, Mehen, 1910.03586]

[crickets]



What about *a single boosted heavy quark* fragmenting to *a single heavy hadron*?

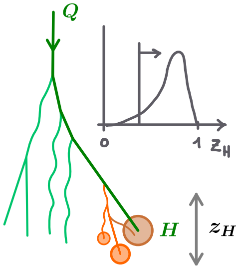
Integrate out QCD modes far off quark mass shell $\Rightarrow$ heavy-quark effective theory:

$$\mathcal{L}_{\mathrm{HQET}} = \bar{h}_v \, v \cdot D \, h_v + \mathcal{L}_{\mathrm{light}} + \mathcal{O}\Big(\frac{1}{m}\Big) \,, \quad v^\mu = P_H^\mu / M_H \,, \quad v^2 = 1$$

Spectroscopy:

- Flavor symmetry, $\mathcal{L}_{\mathrm{HQET}}(\cancel{m}) \;\Rightarrow\; m_D - m_c = m_B - m_b + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2/m)$
- Spin symmetry, $\big[\mathcal{L}_{\mathrm{HQET}}, \vec{S}_Q\big] = 0 \;\Rightarrow\; m_{D^*} = m_D + \mathcal{O}(\Lambda_{\mathrm{QCD}}^2/m)$

$$s_\ell = \tfrac{1}{2} \; : \quad D = \tfrac{1}{\sqrt{2}}\big(|{\uparrow\downarrow}\rangle - |{\downarrow\uparrow}\rangle\big) \qquad D^* = |{\uparrow\uparrow}\rangle, \tfrac{1}{\sqrt{2}}\big(|{\uparrow\downarrow}\rangle + |{\downarrow\uparrow}\rangle\big), |{\downarrow\downarrow}\rangle$$



Collinear FFs:

- $1 - z_H \gg \Lambda_{\mathrm{QCD}}/m \;:\; D_{H/Q}(z_H) = d_{Q/Q}(z_H)\, \chi_H$

  [Mele, Nason '91; general shape function case: Fickinger et al. '16]
- Flavor symmetry: $\chi_D = \chi_B$
- Spin symmetry + parity: $P\big(h_\ell = +\tfrac{1}{2}\big) = P\big(h_\ell = -\tfrac{1}{2}\big)$

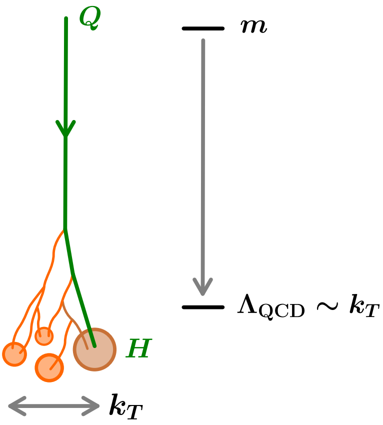
  $\Rightarrow \; \chi_{D^*} = 3\chi_D$, and similarly $\chi_{\Sigma_c^*} = 2\chi_{\Sigma_c}, \ldots$
- No interference between light helicities $\Rightarrow$ need TMDs!

Outline

1. Introduction & Review ✓
2. Heavy-quark TMD FFs
3. Polarized Heavy-quark TMD PDFs
4. Towards Phenomenology

Outline

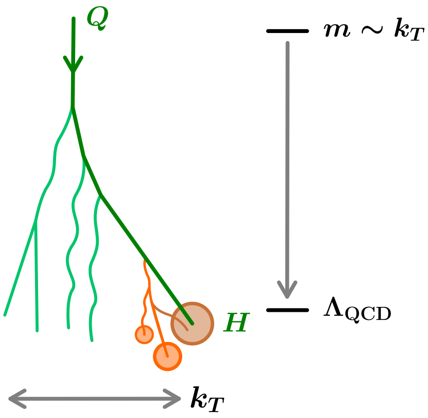
Regime 1



$\Lambda_{\mathrm{QCD}} \sim k_T \ll m$

TMD FFs *not* suppressed, unlike PDFs.

Regime 2



$\Lambda_{\mathrm{QCD}} \ll k_T \sim m$

N.B.: Continue to count $1 - z_H \sim 1$, i.e., $1 - z_H \gg \Lambda_{\mathrm{QCD}}/m$.

Regime 1 $\Lambda_{\mathrm{QCD}} \sim k_T \ll m$: Setup

Strategy: Match TMD FF correlator onto (boosted) HQET to integrate out $\mu \sim m$.

$$\Delta^{\beta\beta'}_{H/Q}(z_H, b_\perp)$$

$$= \frac{1}{2 z_H N_c} \int \frac{\mathrm{d}b^+}{4\pi}\, e^{\mathrm{i}b^+ (P_H^- / z_H)/2}\, \mathrm{Tr} \sum_X \langle 0 | [W^\dagger \psi_Q^\beta](b) | H X \rangle \langle H X | [\bar{\psi}_Q^{\beta'} W](0) | 0 \rangle$$

$$\xrightarrow{\text{HQET}} \frac{\delta(1 - z_H)}{\bar{n} \cdot v}\, C_m(m) \boxed{\frac{1}{2N_c} \mathrm{Tr} \sum_X \langle 0 | [W^\dagger h_v^\beta](b_\perp) | H_v X \rangle \langle H_v X | [\bar{h}_v^{\beta'} W](0) | 0 \rangle}$$

$$\equiv F_H^{\beta\beta'}(b_\perp)$$

Project out unpolarized and Collins TMD FF:

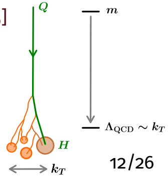
$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) = \delta(1 - z_H)\, C_m\Big(m, \mu, \frac{\zeta}{m^2}\Big)\, \chi_{1,H}(b_T, \mu, \zeta) + \mathcal{O}\Big(\frac{1}{m}\Big)$$

$$b_T M_H\, H^{\perp(1)}_{1\,H/Q}(z_H, b_T, \mu, \zeta) = \delta(1 - z_H)\, C_m\Big(m, \mu, \frac{\zeta}{m^2}\Big)\, \chi^\perp_{1,H}(b_T, \mu, \zeta) + \mathcal{O}\Big(\frac{1}{m}\Big)$$

[Matching coefficient C_m at NNLO: Hoang, Pathak, Pietrulewicz, Stewart '15]

New scalar bHQET TMD "fragmentation factors":

$$\chi_{1,H}(b_T) = \frac{1}{2} \mathrm{tr}\, F_H(b_\perp)\,, \qquad \chi^\perp_{1,H}(b_T) = \frac{1}{2} \mathrm{tr}\Big[\frac{\not{b}_\perp}{b_T} \not{z}\, F_H(b_\perp)\Big]$$



Regime 1 $\Lambda_{\rm QCD} \sim k_T \ll m$: Decoupling the heavy quarks

Field redefinition to decouple spin & color degrees of freedom of heavy quark:

$$h_v(x) = Y_v(x)\, h_v^{(0)}(x) \qquad Y_v(x) = P\Big[\exp\Big(ig\int_0^\infty \mathrm{d}s\, v\cdot A(x+vs)\Big)\Big]$$

$$\mathcal{L}_{\rm HQET} = \bar{h}_v^{(0)}(iv\cdot\partial)h_v^{(0)} + \mathcal{L}_{\rm light}$$

[Korchemsky, Radyushkin ’92]

$$h_v(x)\,|s_Q, h_Q; s_\ell, h_\ell, f_\ell; X\rangle = u(v,h_Q)\, Y_v(x)\, |s_\ell, h_\ell, f_\ell; X\rangle$$

$$\sum_{h_H} |H_v, h_H; X\rangle\langle H_v, h_H; X| = \sum_{h_H}\Big(\sum_{h_Q}\sum_{h_\ell} |h_Q; s_\ell, h_\ell, f_\ell; X\rangle\langle s_Q, h_Q; s_\ell, h_\ell|s_H, h_H\rangle\Big)$$
$$\times\Big(\sum_{h'_Q}\sum_{h'_\ell}\langle s_H, h_H|s_Q, h'_Q; s_\ell, h'_\ell\rangle\langle h'_Q; s_\ell, h'_\ell, f_\ell; X|\Big)$$

$$\Rightarrow F_H(b_\perp) = \frac{1}{2}\sum_{h_H}\sum_{h_Q,h'_Q}\sum_{h_\ell,h'_\ell} u(v,h_Q)\,\bar{u}(v,h'_Q)\,\langle\ldots|\ldots\rangle\langle\ldots|\ldots\rangle\,\rho_{\ell,h_\ell h'_\ell}(b_\perp)$$

$$\rho_{\ell,h_\ell h'_\ell}(b_\perp) \equiv \frac{1}{N_c}\,\mathrm{Tr}\sum\!\!\!\!\!\!\int_X \langle 0|[W^\dagger Y_v](b_\perp)|s_\ell, h_\ell, f_\ell; X\rangle\langle s_\ell, h'_\ell, f_\ell; X|[Y_v^\dagger W](0)|0\rangle$$

$\Rightarrow$ Parent object encoding *all* nonperturbative physics within hadron multiplet.

Taking the trace $\mathrm{tr}\big[F_H(b_\perp)\big] \propto \mathrm{tr}\big[u(v,h_Q)\bar{u}(v,h_Q')\big]$ sets $h_Q = h_Q'$:

$$D_{1\,H/Q}(z_H, b_T) \propto \chi_{1,H}(b_T) = \frac{1}{2}\sum_{h_H}\sum_{h_Q}\sum_{h_\ell} \big|\big\langle s_Q, h_Q; s_\ell, h_\ell \big| s_H, h_H \big\rangle\big|^2 \, \rho_{\ell,h_\ell h_\ell}(b_\perp)$$

$$\Rightarrow \boxed{\chi_{1,\ell}(b_T) \equiv \sum_{H\in M_\ell} \chi_{1,H}(b_T) = \sum_{h_\ell} \rho_{\ell,h_\ell h_\ell}(b_\perp)} \qquad (1)$$

M_ℓ : spin symmetry multiplet (same light spin & flavor)

e.g.: $s_\ell = \frac{1}{2}$: $\quad \chi_{1,D}(b_T,\mu,\zeta) = \frac{1}{4}\chi_{1,\ell}(b_T,\mu,\zeta)\,, \quad \chi_{1,D^*}(b_T,\mu,\zeta) = \frac{3}{4}\chi_{1,\ell}(b_T,\mu,\zeta)$

$s_\ell = 1$: $\quad \chi_{1,\Sigma_c}(b_T,\mu,\zeta) = \frac{1}{3}\chi_{1,\ell}(b_T,\mu,\zeta)\,, \quad \chi_{1,\Sigma_c^*}(b_T,\mu,\zeta) = \frac{2}{3}\chi_{1,\ell}(b_T,\mu,\zeta)$

$$\Rightarrow \boxed{\chi_1(b_T) \equiv \sum_H \chi_{1,H}(b_T) = \frac{1}{N_c}\,\mathrm{Tr}\,\big\langle 0\big|\big[W^\dagger Y_v\big](b_\perp)\,\big[Y_v^\dagger W\big](0)\big|0\big\rangle} \qquad (2)$$

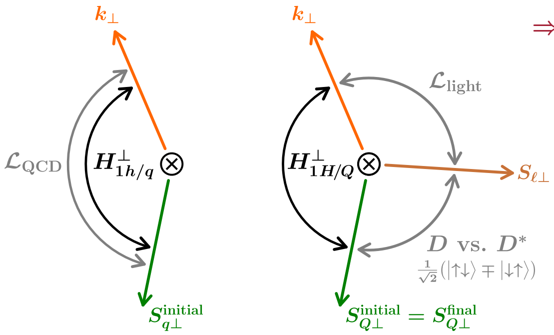
- Square & combine with soft factor $\Rightarrow$ total $e^+e^- \to H\bar{H}X$ TMD cross section
- ▶ Theoretically simplest real-life TMD observable – 100% Wilson loops!

  [Cf. Ji, Liu, Liu, 1910.11415 for extracting the TMD soft function from a similar *lattice* observable …]

$$H^{\perp(1)}_{1\,H/Q}(z_H, b_T) \propto \chi^{\perp}_{1,H}(b_T) = \frac{1}{2}\,\mathrm{tr}\Big[\frac{\not{b}_\perp}{b_T}\not{z}\,F_H(b_\perp)\Big]$$

$\Rightarrow$ Collins function given by off-diagonal entries (transverse polarization) of $\rho_{\ell,h_\ell h'_\ell}$.

e.g.: $s_\ell = 1/2\,,\ s_H = 0\ :\quad \chi^{\perp}_{1,D}(b_T) = \frac{1}{4}\big[\rho_{\ell,-+}(b_\perp) - \rho_{\ell,+-}(b_\perp)\big]$



$\Rightarrow$ $$\sum_{H\in M_\ell} \chi^{\perp}_{1,H}(b_T,\mu,\zeta) = 0$$

M_ℓ : spin symmetry multiplet (same light spin & flavor)

$$H^{\perp}_{1\,\Lambda_c/c} = 0$$
$$H^{\perp}_{1\,D/c} = -H^{\perp}_{1\,D^*/c}$$
$$H^{\perp}_{1\,\Sigma_c/c} = -H^{\perp}_{1\,\Sigma_c^*/c}$$
$$\vdots$$

- Heavy-quark limit lets us prove a much stronger result than Schäfer-Teryaev.

  (only subset of hadrons in sum, pointwise in k_T, holds at renormalized level)

Strategy: Match onto bHQET to integrate out $\mu \sim k_T \sim m$.

$$D_{1\,H/Q}(z_H, b_T, \mu, \zeta) = d_{1\,Q/Q}(z_H, b_T, \mu, \zeta)\,\chi_H + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{m}\Big) + \mathcal{O}(\Lambda_{\rm QCD} b_T)$$

- New perturbative matching coefficient: partonic heavy-quark TMD FF

$$d_{1\,Q/Q}(z_H, b_T) = \mathrm{tr}\Big[\frac{\bar{n}\!\!\!/}{2}\,\Delta_{Q/Q}(z_H, b_\perp)\Big] = \delta(1 - z_H) + \mathcal{O}(\alpha_s)$$

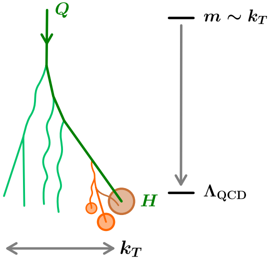
Consistency relations for $m \ll k_T$ and $m \gg k_T$:

$$d_{1\,Q/Q}(z_H, b_T, \mu, \zeta) = \frac{1}{z_H^2}\sum_i \int \frac{\mathrm{d}z}{z}\, d_{Q/i}\Big(\frac{z_H}{z}, \mu\Big)\,\mathcal{J}_{i/Q}(z, b_T, \mu, \zeta) + \mathcal{O}(m^2 b_T^2)$$

$$d_{1\,Q/Q}(z_H, b_T, \mu, \zeta) = \delta(1 - z_H)\, C_m\Big(m, \mu, \frac{\zeta}{m^2}\Big) \times C_1(b_T, \mu, \zeta) + \mathcal{O}\Big(\frac{1}{b_T m}\Big)$$

Recall: χ_H is known to satisfy spin-symmetry relations.

$\Rightarrow$ E.g. $D_{1\,\Sigma_c^*/c} = 2D_{1\,\Sigma_c/c} + \mathcal{O}(\Lambda_{\rm QCD}/m_c)$ for all k_T & to all orders in α_s.



Strategy: Two-step matching onto bHQET to integrate out $\mu \sim k_T, m$.

1. Use known twist-3 matching for light quarks to integrate out k_T:

$$b_T M_H\, H_{1\,H/Q}^{\perp(1)}(z_H, b_T) = b_T \hat{H}_{H/Q}(z_h) + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\rm QCD}^2 b_T^2)$$

$$\hat{H}_{H/Q}(z_h) \equiv \frac{z_H^2}{2N_c} \int \frac{dx^+}{4\pi}\, e^{ix^+(P_H^-/z_H)/2}\, \mathrm{Tr}\,\mathrm{tr} \sum_X \Big\{ \langle 0| W^\dagger(x)$$

$$\times\, \sigma_{\alpha-} \big[iD_\perp^\alpha(x) + g\mathcal{G}_\perp^\alpha(x)\big] \psi_Q(x) |HX\rangle\langle HX| \bar{\psi}_Q(0) W(0)|0\rangle + \text{h.c.} \Big\}$$

[Mulders, Tangerman '95; Boer, Mulders '97; for NLO at $k_T > 0$, see Yuan, Zhou '09]

2. Match twist-3 matrix element onto bHQET to integrate out m:

$$\hat{H}_{H/Q}(z_h) \to \delta(1 - z_H)\, \chi_{H,G}$$

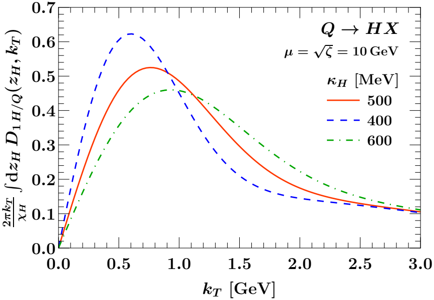
$$\chi_{H,G} \equiv \frac{1}{2N_c} \mathrm{Tr}\,\mathrm{tr} \sum_X \Big\{ \langle 0| W^\dagger \sigma_{\beta\alpha} z^\beta \big[iD_\perp^\alpha + g\mathcal{G}_\perp^\alpha\big] h_v |H_v X\rangle\langle H_v X| \bar{h}_v W|0\rangle + \text{h.c.} \Big\}$$

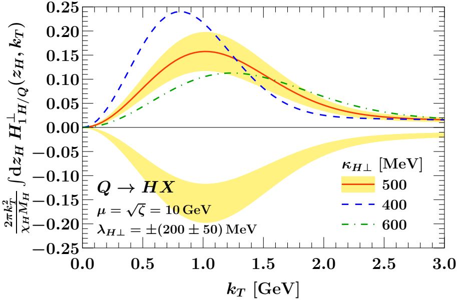
$$\Rightarrow \boxed{\begin{aligned} &b_T M_H\, H_{1\,H/Q}^{\perp(1)}(z_H, b_T) \\ &= \delta(1 - z_H)\, b_T\, \chi_{H,G} \\ &\quad + \mathcal{O}(\alpha_s) + \mathcal{O}(\Lambda_{\rm QCD}^2 b_T^2) \end{aligned}}$$

- Easy to show: $\sum_{H \in M_\ell} \chi_{H,G} = 0$

▶ New sum rule continues to hold!

Models and numerical results for TMD FFs





- At leading log, TMD FFs completely specified by $\chi_{1,H}$, $\chi_{1,H}^{\perp}$ and TMD evolution
  - Results at this order universal for $Q = c, b$

    (NLL: α_s, γ_ζ decoupling; NNLL: finite radiative terms with explicit mass dependence)

- For illustration, consider simple (weighted) Gaussian models:

$$\chi_{1,H}(b_T, \mu_0, \zeta_0) = \chi_H \exp\Big(-\kappa_H^2 b_T^2\Big)$$

$$\chi_{1,H}^{\perp}(b_T, \mu_0, \zeta_0) = \chi_H\, \lambda_{H\perp} b_T\, \exp\Big(-\kappa_{H\perp}^2 b_T^2\Big) \qquad \lambda_{H\perp} = \chi_{H,G}/\chi_H$$

Outline

Polarized heavy-quark TMD PDFs

- Heavy quarks have to be perturbatively produced from light partons w/i nucleon.

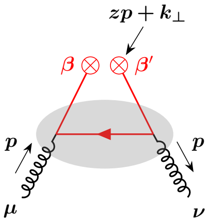
  …ignoring intrinsic charm – *TMD* intrinsic charm?

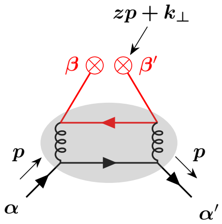
- Encoded in matching relations onto twist-2 collinear PDFs. All-order structure:

$$f_{1\,Q/N}(x, b_T) = \sum_j \int \frac{\mathrm{d}z}{z}\, C_{Q/j}(z, b_T, m)\, f_{j/N}\left(\frac{x}{z}\right),$$

$$g_{1L\,Q/N}(x, b_T) = \sum_j \int \frac{\mathrm{d}z}{z}\, C_{Q_\parallel/j_\parallel}(z, b_T, m)\, g_{j/N}\left(\frac{x}{z}\right),$$

$$b_T M_N\, h^{\perp}_{1L\,Q/N}(x, b_T) = \sum_j \int \frac{\mathrm{d}z}{z}\, C_{Q_\perp/j_\parallel}(z, b_T, m)\, g_{j/N}\left(\frac{x}{z}\right)$$

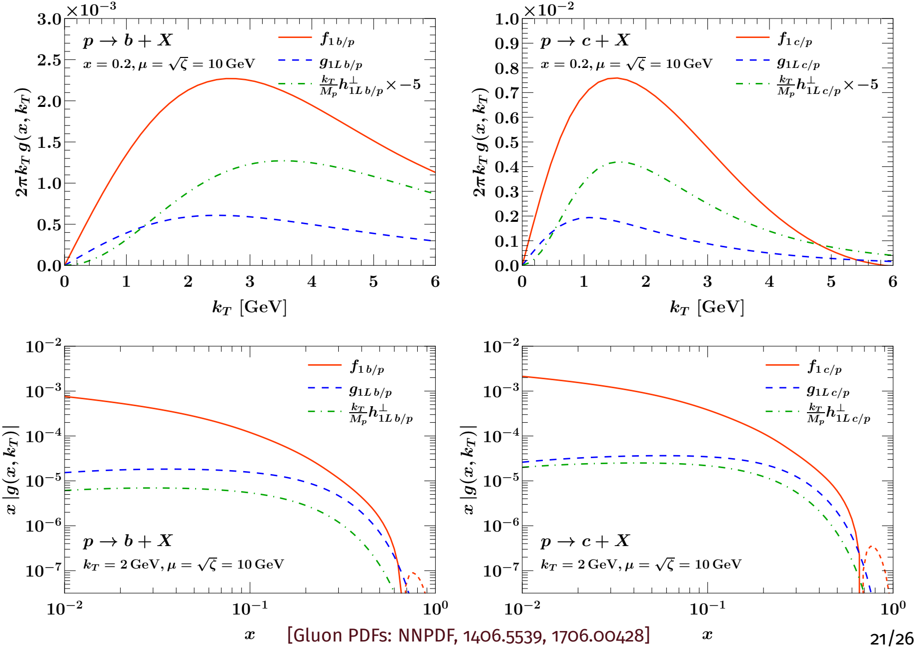




- ~~Transversity~~ by chirality & flavor conservation for light quarks.
- $C_{Q_\perp/j_\parallel} \sim m b_T$ is allowed because heavy quark mass breaks chirality.
- Calculated all matching coefficients onto gluons at leading order $\sim \alpha_s$.

  [Unpol. agrees with Nadolsky et al. '02]

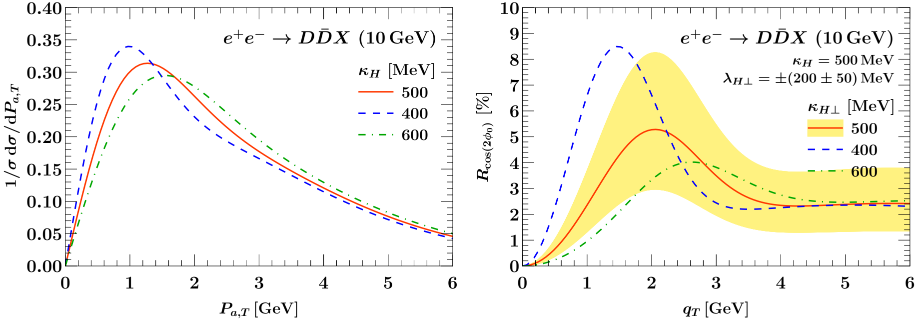
Polarized heavy-quark TMD PDFs: Numerical results



[Gluon PDFs: NNPDF, 1406.5539, 1706.00428]

Outline

Towards phenomenology: $e^+e^- \to H\bar{H}X$



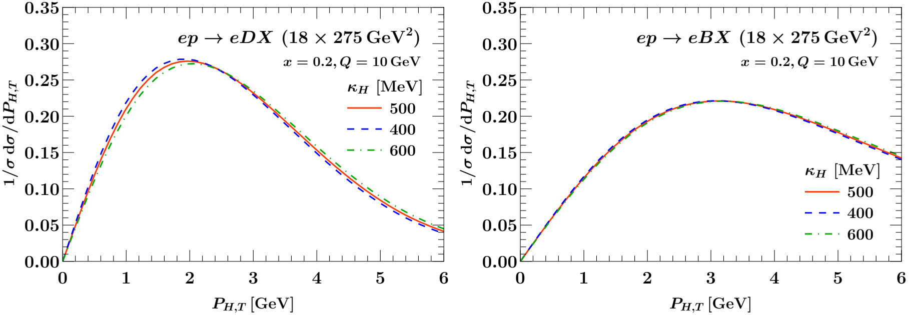
- Consider continuum charm production at $Q = 10\,\mathrm{GeV}$. say, at Belle II 😉
- Absolute sign of Collins function is lost in $R_{\cos(2\phi_0)} = H_1^\perp \otimes H_1^\perp / (D_1 \otimes D_1)$, but can access relative signs predicted by heavy-quark symmetry, e.g.

$$R_{\cos(2\phi_0)}^{\Sigma_c\bar{\Sigma}_c} = -\frac{1}{2} R_{\cos(2\phi_0)}^{\Sigma_c^*\bar{\Sigma}_c} = -\frac{1}{2} R_{\cos(2\phi_0)}^{\Sigma_c\bar{\Sigma}_c^*} = +\frac{1}{4} R_{\cos(2\phi_0)}^{\Sigma_c^*\bar{\Sigma}_c^*}$$

- For generic $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ parameters, expect Collins effect of several percent.

  N.B.: Collins effect not mass suppressed, in contrast to a claim by BaBar in 1309.5278.

Towards phenomenology: $eN \to eHX$ at the EIC

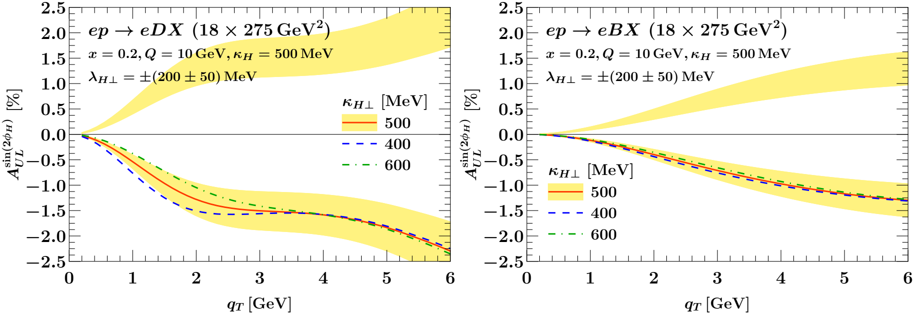


- Different size of phase space makes $P_{H,T}$ peak wider for b, reduces rate.
- Expected TMD-sensitive samples (for early $10\,\mathrm{fb}^{-1}$) are sizable in either case.

$\sigma(eN \to eHX)$ [pb]	c, $x > 0.01$	c, $x > 0.1$	b, $x > 0.01$	b, $x > 0.1$
$q_T < 2\,\mathrm{GeV}, Q > 4\,\mathrm{GeV}$	84	3.47	18	0.65
$q_T < 4\,\mathrm{GeV}, Q > 10\,\mathrm{GeV}$	16	1.45	4.9	0.42

$$0.01 < y < 0.95\,, \quad W^2 > 100\,\mathrm{GeV}^2$$

Towards phenomenology: $eN \to eHX$ at the EIC

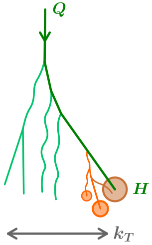


$$A_{UL^{\sin(2\phi_H)}} \propto \frac{h_{1L}^{\perp} \otimes H_1^{\perp}}{f_1 \otimes D_1}$$

- $h_{1L}^{\perp}$ is perturbatively predicted in terms of g_g.
  - Clean probe of sign of heavy-quark Collins function!

Transverse Momentum Distributions of Heavy Hadrons and Polarized Heavy Quarks:

- Initiated the study of heavy-quark TMD FFs in bHQET.
  - Goal: Understand hadronization around a static color source.
  - Demonstrated how heavy-quark TMD FFs (by spin symmetry) probe final-state spin density matrix of light degrees of freedom.
  - Proved new sum rule $\sum_{H \in M_\ell} H_{1\,H/Q}^{\perp} = \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{m_Q}\Big)$ for Collins FF.



- Extended results for unpolarized heavy-quark TMD PDFs to the polarized case.
  - Transversely polarized heavy quarks are produced from linearly polarized gluons at an appreciable & perturbatively controlled rate.
- Rich heavy-quark TMD FF phenomenology may be within reach at B factories, and is an exciting complement to the already planned EIC heavy-flavor program.

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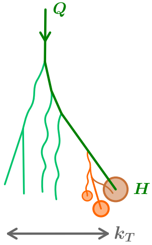

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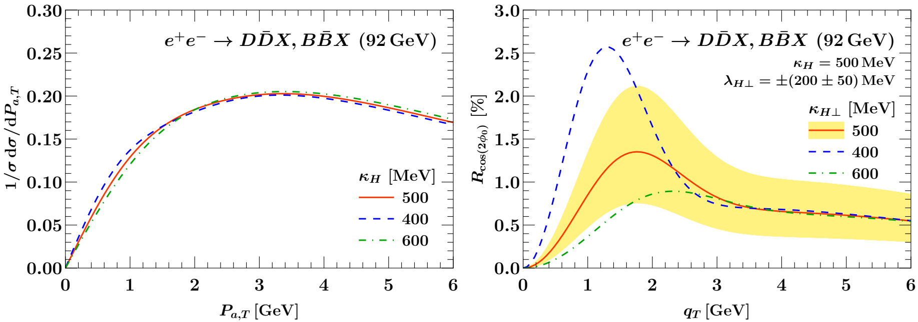
Thank you for your attention!

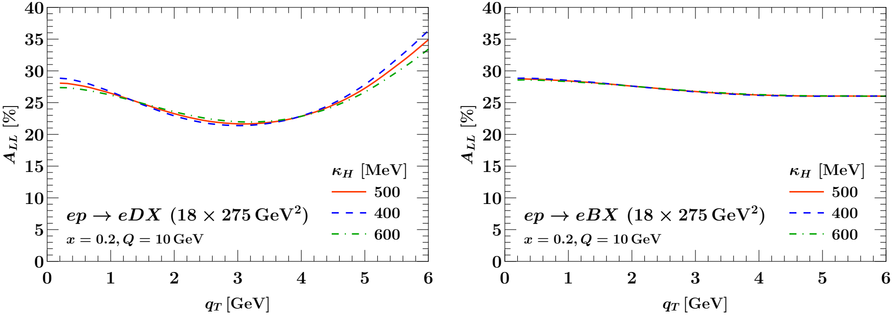
Backup

$e^+e^- \to H\bar{H}X$ at the Z pole



- $Q = m_Z$ sufficient to produce boosted bottom quarks
- Far away from either production threshold, have full universality between c and b at small $k_T \ll m_c, m_b$ (and at higher k_T at LL)

Longitudinal spin asymmetries for heavy-quarks at the EIC



$$A_{LL} \propto \frac{g_{1L} \otimes D_1}{f_1 \otimes D_1}$$

- Nonperturbative physics in $\chi_{1,H}(b_T)$ cancels out almost exactly
- Clean probe of gluon helicity PDF